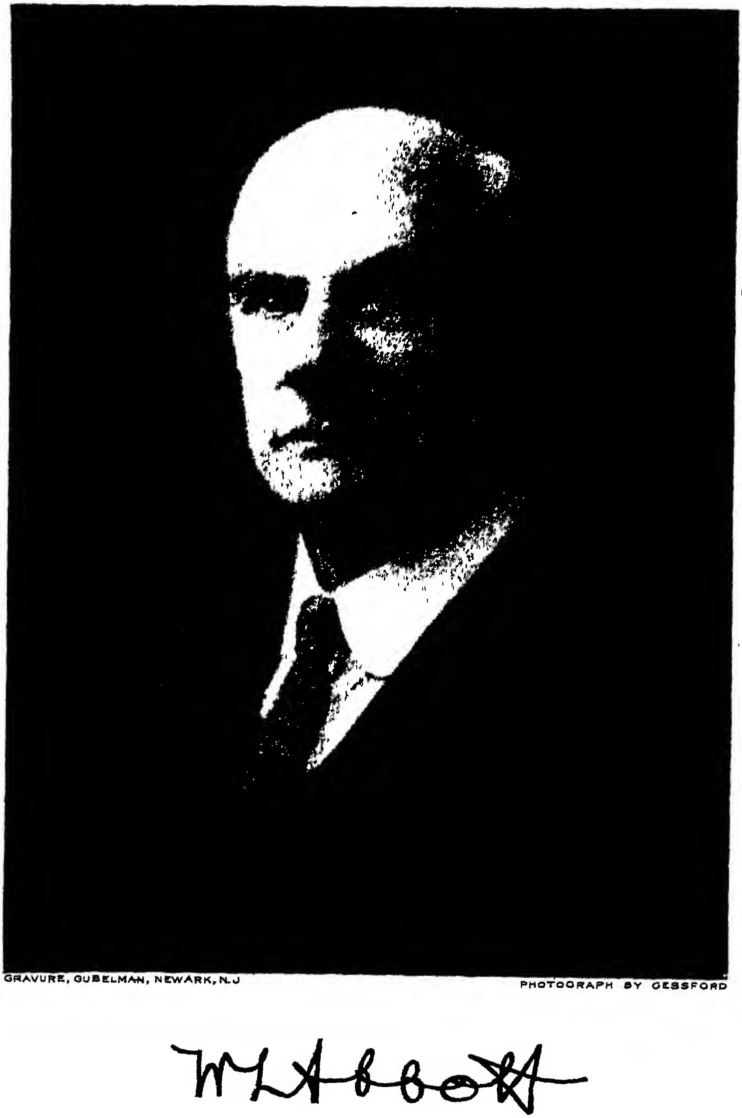


GRAVURE, GUBELMAN, NEWARK, N.J.

PHOTOGRAPH BY GESSFORD

W L Abbott

PRESIDENT 1926

OF

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

Printed in the United States of America by
The Lord Baltimore Press, Baltimore, Md.

PUBLICATIONS COMMITTEE'S FOREWORD

DURING the past few years the technical work of the Society has developed at a gratifying rate through the increased activities of its professional divisions and its technical committees. This growth has been reflected in the large volume of good material presented in papers at meetings and made available for publication. The responsibility of the Publications Committee in selecting material for the TRANSACTIONS has been correspondingly increased. Although only material which the Committee believes to be of permanent reference value has been included, and both papers and discussions have been subject to thorough editing and revision since their presentation, this volume amounts to about 1500 pages, the largest published since that for 1915.

This volume contains papers selected from the programs of the Regional Meeting in Providence, the Spring Meeting in San Francisco, and the Annual Meeting in New York, with a few others presented at local gatherings. Two-thirds of the material, however, was taken from the Annual Meeting program, which made an early publication date a difficult attainment. In spite of this fact, previous publication records have been bettered by several weeks.

Regional, Spring, and Annual Meeting papers printed in *Mechanical Engineering* but not in this volume, technical reports, and other important contributions to the monthly magazine during 1926, are listed on pages immediately following the main index to this volume.

R. E. FLANDERS, *Chairman*
O. G DALE, *Vice-Chairman*
K. H. CONDIT
E. D. DREYFUS
W. A SHOUDY.

CONTENTS OF VOLUME 48[1]

PROVIDENCE, SAN FRANCISCO, AND NEW YORK MEETINGS

[1] The Society shall not be responsible for statements or opinions advanced in papers or in discussion at meetings of the Society or of its Divisions or Sections, or printed in its publications.

WILLIAM L. ABBOTT

PRESIDENT OF THE SOCIETY FOR 1925-1926

WILLIAM L. ABBOTT, President of the Society for the year 1925-1926, was born at Union Grove, Whiteside County, Ill., February 14, 1861. He was graduated from the University of Illinois as a mechanical engineer in 1884 and became one of the pioneers in the field of electric lighting in Chicago, working at first as machinist and as draftsman with various electrical concerns. In 1887 he helped to organize the National Electric Construction Company and became its president. In 1894 the National Electric Construction Company was absorbed by the Chicago Edison Company (now the Commonwealth Edison Company). Mr. Abbott entered the employ of the Chicago Edison Company as chief engineer of its largest power house, and in 1899 was promoted to his present position of chief operating engineer, thus having a continuous record of over forty years in the electric lighting business.

Mr. Abbott served as a member of the board of trustees of the University of Illinois from 1905 to 1923, and was president of the board from 1907 to 1919 and also from 1921 to 1923. He became a member of The American Society of Mechanical Engineers in 1891, and served as a manager of the Society from 1907 to 1910. He is a past-president of the Western Society of Engineers, and a fellow of the American Institute of Electrical Engineers. He is also a member of a number of other technical and social organizations.

Mr. Abbott has made numerous speeches and addresses, many relating to economics in generating-station operation, and is considered a leading authority on the efficient and economical operation of large steam-electric generating stations and the consequent conservation of coal. He believes that present coal-mining and coal-handling methods result in much waste, and that all concerned must act in concert to stop this waste. He was chairman of the committee of the Federated American Engineering Societies, which in coöperation with the United States Government made a national study of coal storage during recent years.

No. 2000

SOCIETY AFFAIRS

ORGANIZATION AND MEMBERSHIP

ON THE following pages are given the names of those who made up the executive and administrative personnel of the Society, its representatives on joint activities, and a summary of its membership for the year 1926. The personnel of professional committees and divisions, Local Sections officers, and detailed information concerning the organization of the Society was printed in the Year Book for 1926.

OFFICERS AND COUNCIL

PRESIDENT

W. L. Abbott.......................................Chicago, Ill.

VICE-PRESIDENTS

Terms Expire December, 1926

Robert W. Angus..............................Toronto, Canada
Sherwood F. Jeter..............................Hartford, Conn.
Thos. L. Wilkinson..............................Davenport, Iowa

Terms Expire December, 1927

A. G. Christie..............................Baltimore, Md.
Wm. T. Magruder..............................Columbus, Ohio
Roy V. Wright..............................New York, N. Y.

MANAGERS

Terms Expire December, 1926

E. O. Eastwood..............................Seattle, Wash.
E. R. Fish..............................St. Louis, Mo.
Frank A. Scott..............................Cleveland, Ohio

Terms Expire December, 1927

John H. Lawrence..............................New York, N. Y.
Edward A. Muller..............................Cincinnati, Ohio
Paul Wright..............................Birmingham, Ala.

Terms Expire December, 1928

Robt. L. Daugherty..............................Pasadena, Cal.
Wm. Elmer..............................Philadelphia, Pa.
Chas. E. Gorton..............................New York, N. Y.

PAST-PRESIDENTS

E. S. Carman (1926) .... Cleveland, Ohio
Dexter S. Kimball (1927) .... Ithaca, N. Y.
John Lyle Harrington (1928) .... Kansas City, Mo.
Fred R. Low (1929) .... New York, N. Y.
W. F. Durand (1930) .... New York, N. Y.

TREASURER

Erik Oberg .... New York, N. Y.

SECRETARY

Calvin W. Rice .... New York, N. Y.

STANDING COMMITTEES

FINANCE

H. V. Coes, *Chairman and Representative on Council* (1929)
E. H. West, *Vice-Chairman* (1928) Hugo Diemer (1927)
Arthur L. Rice (1926) F. A. Schaff (1930)

Council Representatives: Roy V. Wright (1926)
John H. Lawrence (1927)

MEETINGS AND PROGRAM

C. N. Lauer, *Chairman and Representative on Council* (1926)
E. Howard Reed (1927) S. W. Dudley (1929)
R. M. Gates (1928) W. L. Batt (1930)

PUBLICATIONS

R. E. Flanders, *Chairman and Representative on Council* (1927)
O. G. Dale, *Vice-Chairman* (1926) E. D. Dreyfus (1929)
K. H. Condit (1928) W. A. Shoudy (1930)

MEMBERSHIP

C. R. Place, *Chairman and Representative on Council* (1926)
C. E. Gorton (1927) S. D. Collett (1929)
Hosea Webster (1928) L. K. Comstock (1930)

PROFESSIONAL DIVISIONS

Robt. T. Kent, *Chairman and Representative on Council* (1928)
L. B. McMillan (1926) Archibald Black (1929)
L. P. Alford (1927) J. W. Roe (1930)
H. W. Brooks, *Secretary*

Note: Dates in parentheses denote expiration of term.

LOCAL SECTIONS

JAMES A. HALL, *Chairman and Representative on Council* (1926)
WILLIAM A. HANLEY (1927) PAUL DOTY (1929)
JAMES D. CUNNINGHAM (1928) H. BIRCHARD TAYLOR (1930)

CONSTITUTION AND BY-LAWS

CHARLES H. BROMLEY, *Chairman and Representative on Council* (1927)
CLIFFORD H. MATSON (1926) W. D. ENNIS (1929)
E. E. HOWARD (1928) GEO. E. PFISTERER (1930)

RELATIONS WITH COLLEGES

L. C. MARBURG, *Chairman and Representative on Council* (1926)
W. H. KAVANAUGH (1927) A. A POTTER (1929)
H. G. TYLER (1928) S. H. LIBBY (1930)

EDUCATION AND TRAINING FOR THE INDUSTRIES

JOHN T. FAIG, *Chairman and Representative on Council* (1927)
D. C. JACKSON (1926) S. S. EDMANDS (1929)
R. L. SACKETT (1928) W. S. CONANT (1930)

AWARDS

IRA N. HOLLIS, *Chairman and Representative on Council* (1930)
R. S. RILEY (1926) L. P. ALFORD (1928)
R. H. FERNALD (1927) A. M. GREENE, JR. (1929)

LIBRARY

H. A. LARDNER, *Chairman and Representative on Council* (1927)
W. C. WETHERILL (1926) O. E. HOVEY (1929)
PERCY H. THOMAS (1928) THE SECRETARY

STANDARDIZATION

C. M. CHAPMAN, *Chairman and Representative on Council* (1926)
C. F. HIRSHFELD (1927) C. P. BLISS (1929)
A. M. HOUSER (1928) F. J. KEARNEY (1930)

RESEARCH

R. J. S. PIGOTT, *Chairman and Representative on Council* (1928)
ALBERT KINGSBURY (1926) A. E. WHITE (1929)
D. R. YARNALL (1927) ROBT. L. STREETER (1930)

NOTE: Dates in parentheses denote expiration of term.

JOINT ACTIVITIES

IN WHICH THE SOCIETY FORMS A CORPORATE PART

The President and Secretary represent the Society on the Founder Societies' Joint Conference Committee, which directs all matters of joint interest into the proper channels.

AMERICAN ENGINEERING COUNCIL:

Comprised of 28 member organizations to represent the engineers of America in matters of public welfare to the engineering and allied technical professions.

Terms Expire January, 1927

W. F. DURAND, Stanford Univ., Cal.
IRA DYE, Seattle, Wash.
W. S. FINLAY, JR., New York
DEAN E. FOSTER, Tulsa, Okla.
WILSON P. HUNT, Moline, Ill.
I. E. MOULTROP, Boston
E. N. TRUMP, Syracuse, N. Y.
W. W. VARNEY, Baltimore, Md.

Terms Expire January, 1928

W. L. ABBOTT, *Chairman*, Chicago, Ill.
JOHN T. FAIG, Cincinnati, Ohio
A. M. GREENE, JR., Princeton, N. J.
JOHN LYLE HARRINGTON, Kansas City, Mo.
IRA N. HOLLIS, Worcester, Mass.
S. H. LIBBY, Bloomfield, N. J.
D. S. KIMBALL, Ithaca, N. Y.
WILLIAM B. POWELL, Buffalo, N. Y.
H. L. THOMPSON, Waterbury, Conn.

AMERICAN ENGINEERING STANDARDS COMMITTEE:

S. G. FLAGG, JR. (1926)
C. M. CHAPMAN (1927)
C. P. BLISS (1928)
K. H. CONDIT (Alternate)
C. B. LE PAGE (Alternate)

ENGINEERING SOCIETIES' EMPLOYMENT SERVICE:

CALVIN W. RICE, Secretary A.S.M.E., *Chairman*

ENGINEERING FOUNDATION BOARD:

GEO. A. ORROK (1927)
A. M. GREENE, JR. (1928)

A third appointee is nominated from the representatives of the Society on the Board of Trustees of the United Engineering Society.

JOHN FRITZ MEDAL BOARD OF AWARD:

FRED J. MILLER (February 1927)
HENRY B. SARGENT (February, 1928)
FRED R. LOW (February, 1929)
W. F. DURAND (1930)

NOTE: Dates in parentheses denote expiration of term.

UNITED ENGINEERING SOCIETY:

Roy V. Wright (1927)
W. L. Saunders (1928)
W. S. Finlay, Jr. (1929)

SOCIETY REPRESENTATION

OTHER ORGANIZATIONS IN WHICH THE SOCIETY IS REPRESENTED BY COURTESY

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Section M, Engineering:

Alex C. Humphreys
Ira N. Hollis

JOSEPH A. HOLMES MEMORIAL BOARD:

Brigadier-General Wm. A. Bixby

NATIONAL RESEARCH COUNCIL, Division of Engineering:

R. J. S. Pigott (June, 1926)
Geo. A. Orrok (June, 1927)
Earle Buckingham (June, 1928)

SOCIETY FOR THE PROMOTION OF ENGINEERING EDUCATION, Board of Coordination and Investigation

John Lyle Harrington
Frank A. Scott
W. L. Durand (Alternate)

WESTERN SOCIETY OF ENGINEERS, Washington Award:

Charles Russ Richards (June, 1926)
James Lyman (June, 1927)

Note: Dates in parentheses denote expiration of term.

SUMMARY OF MEMBERSHIP BY RESIDENCE

United States and Possessions

Alabama	108	Nebraska	28
Alaska	1	Nevada	2
Arizona	17	New Hampshire	40
Arkansas	15	New Jersey	1200
California	871	New Mexico	2
Canal Zone	5	New York	4468
Colorado	77	North Carolina	90
Connecticut	595	North Dakota	4
Delaware	72	Ohio	1029
District of Columbia	180	Oklahoma	86
Florida	97	Oregon	69
Georgia	135	Pennsylvania	1935
Hawaiian Islands	19	Philippine Islands	24
Idaho	7	Porto Rico	25
Illinois	1060	Rhode Island	154
Indiana	260	South Carolina	28
Iowa	46	South Dakota	7
Kansas	55	Tennessee	91
Kentucky	57	Texas	156
Louisiana	107	Utah	32
Maine	36	Vermont	34
Maryland	182	Virginia	115
Massachusetts	1134	Washington	118
Michigan	531	West Virginia	47
Minnesota	103	Wisconsin	355
Mississippi	12	Wyoming	9
Missouri	313		
Montana	17	Total	16260

Other Countries

North America		
Canada	234	
Newfoundland	1	
Mexico	35	
		270
Central America		
Costa Rica	4	
Guatemala	1	
Honduras	1	
Salvador	1	
		7

West Indies		
Cuba	55	
Dominican Republic	6	
Jamaica	3	
		64
South America		
Argentina	20	
Brazil	16	
Chile	22	

South America (*continued*)		
Colombia	3	
Ecuador	1	
Peru	1	
Uruguay	1	
Venezuela	5	
		69
Africa		
Canary Islands	1	
Egypt	1	
Union of S. Africa	13	
		15
Asia		
China	23	
Dutch East Indies	1	
India	35	
Japan	24	
Manchuria	1	
Persia	1	
Siam	1	
Straits Settlements	2	
		88
Australasia		
Australia	20	
New Zealand	2	
Tasmania	1	
		23

Europe		
Austria	1	
Belgium	3	
Czechoslovakia	7	
Denmark	8	
Finland	3	
France	34	
Germany	25	
Great Britain	100	
Greece	2	
Holland	3	
Italy	6	
Lithuania	1	
Norway	2	
Poland	5	
Roumania	1	
Russia	2	
Spain	8	
Sweden	11	
Switzerland	9	
Turkey	2	
		233
Total		769

Membership in United States	16260
Membership in Other Countries	769
Present Address Unknown	7
Total Membership	17036

Summary of Membership by Grades

Honorary Members	21
Members	8136
Associates	641
Associate-Members	3978
Juniors	4260
Total	17036

REPORTS OF GENERAL MEETINGS

ONLY the Spring and Annual Meetings of the Society are reported in TRANSACTIONS, and these not in detail. More complete accounts of the general meetings, and reports of the regional and sectional meetings were published during the year in *Mechanical Engineering* and the *A.S.M.E. News*.

THE SPRING MEETING

San Francisco, Cal., June 28 through July 1, 1926

To the A.S.M.E. Transcontinental Tour, of course, considerable credit should be given for the especial success of the Spring Meeting for 1926. It carried one hundred and sixty-three members and guests from east of the Rockies to the first meeting held on the Pacific Coast in connection with a transcontinental tour since 1892 The registration for the meeting was 547, exceeding all Pacific Coast records.

The A S.M.E. Special left New York at 1: 30 p m. on Monday, June 14. On Wednesday morning the party was welcomed at Kansas City. A fifty-mile automobile tour through the boulevards, parks, and residential sections of Kansas City was followed by luncheon at the Athletic Club. Thursday was spent at Denver The program included an auto ride into the mountains, luncheon at the Elks Club, and a dinner at the Union Station dining room. The next stop was at Colorado Springs, where two delightful days were spent in sightseeing and entertainment. At Santa Fé and Albuquerque the Chambers of Commerce provided automobiles for tours of the cities. A day was spent at Grand Canyon, then the Tour proceeded through the desert to San Bernardino where waiting cars carried the party through the groves of fruits to the Mission Inn at Riverside. At Los Angeles the Local Section had prepared a comprehensive program of automobile tours during which the features of the city and the interesting surrounding country were seen. After two days the Tour proceeded to the Yosemite Valley where the party revelled in the beauties of the varied types of scenery and viewed the big trees.

The party arrived in San Francisco Sunday morning, June 27. After the Meeting the party left on the evening of July 1 for the North, travelling the entire first day in sight of Mt. Shasta, and arriving in Portland on the morning of July 3. Here the Local Section had provided automobiles for a ride down the Columbia River Highway. Independence Day was spent on Mt. Rainier, where the party engaged in a snowball fight and made a trip to the Glacier. The next morning, arriving at Seattle very early, they were met by buses provided by the A.S.M.E. Local Section and were taken on a two-hour drive about the city. They then boarded

the Canadian Pacific Steamship for the trip to Victoria and Vancouver.

An hour's ride through Victoria and an hour's ride around Vancouver gave a splendid view of the interesting estates in these two Canadian cities. In Vancouver the Tour finally turned eastward, making its first stop at Revelstoke. Here was met the first serious delay. A landslide a few miles east of Revelstoke stopped the passage of the train but gave the members of the party an opportunity to enjoy Canada's hospitality. The Canadian Pacific Railroad officials and the citizens of Revelstoke took the opportunity to provide an excellent day's entertainment which made the stop one of the most interesting of the trip. When the line was cleared the party proceeded, stopping at Lake Louise, Banff, St. Paul, and Niagara Falls on the way to New York, which was reached on Tuesday, July 13. The time spent in the Canadian Rockies and St. Paul was reduced somewhat by the delay at Revelstoke, and the members were regretful that they were not able to enjoy the beauties of these places as fully as they had hoped.

The August 7 issue of the *A.S M.E. News* was devoted to a more complete account of the Tour and contained pictures and other matters of interest contributed by members of the Tour party.

The Executive Committee for the San Francisco meeting was composed of Warren H. McBryde, *Chairman,* Frederick Birdsall, *Secretary,* C. M. Gunn, *Treasurer* and Chairman of Finance Committee, E. M. Breed, Chairman of Committee on Printing and Signs, E. B. Bumsted, Chairman of Hotel Committee, A. J. Dickie, Chairman of Publicity Committee, Ely C. Hutchinson, Chairman of Committee on Technical Events, J. A. Kinkead, Chairman of Committee on Information and Registration, Bruce Lloyd, Chairman of Excursions Committee, Elgin Stoddard, Chairman of Reception Committee, and H. L. Terwilliger, Chairman of Ladies' Entertainment Committee. Mr. McBryde also served as chairman of the Entertainment Committee, and there was a special ladies' reception committee.

There were two groups of simultaneous technical sessions during the meeting, on Tuesday and Wednesday mornings, respectively. The programs for these sessions follow:

Tuesday, June 29

PETROLEUM SESSION

(Under auspices of Petroleum Division)

Fluid Flow in Pipes of Annular Cross-Section, D. H. ATHERTON.

Mechanical Engineering in Cracking, Heating, and Cooling of Oil, B. N. BROIDO.

The Termination of Charcoal Tests for Gasoline, F. L. KALLAM.

INDUSTRIAL TRAINING AND EDUCATION SESSION

(Under auspices of Committee on Education and Training for the Industries)

The Growth of University Extension Training of the Non-College Type for the Industries of the West, JOHN L. KERCHEN.

Education and Training of Apprentices on the Pacific Coast, PAUL ELIEL.

Wednesday, June 30

FUEL AND RAILROAD SESSION

(Under auspices of Fuels and Railroad Divisions)

Combined Oil-and-Gas-Burning Furnaces for Power-Plant Use, J. GRADY ROLLOW.

Fuel Oil for Railways, J. C. MARTIN, JR.

The Development of the "Caterpillar" Tractor and Its Application to Industry, PLINY E. HOLT.

HYDRAULIC SESSION

Aspects of Steam Power in Relation to a Hydro Supply, A. H. MARKWART.

Water Power and Steam Power in California Utilities, H. A. BARRE.

Speed Changes of Hydraulic Turbines for Sudden Changes of Load, EARL B. STROWGER and S. LOGAN KERR.

OIL AND GAS POWER SESSION

(Under auspices of Oil and Gas Power Division)

Transmission of Power on Oil-Engine Locomotives, A. I. LIPETZ.

Oil Engines as a Drive for Pipe-Line Pumps, F. THILENIUS.

A Short Method of Calculating the Periodic Displacement and Oscillations in Synchronous Machines, C. W. CUTLER.

On Thursday morning there was a conference of the Boiler Code Committee with members of the California Industrial Accident Commission's Advisory Committee on Revision of Air Pressure Tank Safety Orders. Other important conferences and committee meetings were held by the Council, Local Sections delegates, and Nominating Committee. At the Council Meeting, in addition to routine business, announcement was made of a gift to the Society by John R. Freeman of twenty-five thousand dollars, to be used as determined by a committee appointed for that purpose. The Local Section delegates met in conference Monday morning, and also had a breakfast gathering with the Council on Tuesday. The Nominating Committee met Monday, Tuesday, and Wednesday and selected the nominees for officers of the Society for 1926-1927.

At the Business Meeting held Monday the two amendments to the Constitution, dealing with the number of directors and officers of the Society and junior membership age limitations, were ordered submitted to the members by letter ballot; announcement was made of the selection of White Sulphur Springs, W. Va., for the 1927 Spring Meeting; and the following standards were read by

title: Small Rivets; Tinners', Coopers' and Belt Rivets; Wrench-Head Bolts and Nuts; Track Bolts; Round Unslotted Head Bolts; Plow Bolts; T-Slots and Parts; Gib-Head Taper Keys; Plain Taper Keys; Spur-Gear Tooth Form; Steel Pipe Flanges and Flanged Fittings; Methods of Gaging and Specifications for Plain Limit Gages.

San Francisco and its surroundings offers such splendid opportunities for excursions that the Excursions Committee was able to provide an unusually enjoyable and interesting program. On Monday afternoon it offered a trip to Mt. Tamalpais and the Muir Woods, with alternate visits to the new Pacific Telephone and Telegraph building and the Hills Brothers coffee plant. The trip by special steamer around San Francisco Bay on Tuesday afternoon gave an excellent opportunity to view the waterfront and the Golden Gate. A stop was made to inspect the refinery of the California & Hawaiian Sugar Refining Corporation at Crockett, and the party were the guests of the corporation for dinner, the return trip being made by moonlight. On Wednesday afternoon there was an extensive automobile tour around San Francisco, or visits to various industrial plants, as desired. The program for Thursday offered a locomotive exhibit at the Southern Pacific Station; an all-day excursion to the East Bay district, with a choice of trips to Oakland or to the University of California; and an inspection trip to the plant of the Caterpillar Tractor Company at San Leandro. A trip to Leland Stanford, Jr., University was also arranged for Thursday afternoon. Many outstanding industrial plants in San Francisco and vicinity were open for inspection throughout the meeting.

One of the most important events of the meeting was the banquet on Wednesday evening, at which the A.S.M.E. Medal was presented to Dr. Robert Andrews Millikan. President Abbott called the meeting to order and introduced Warren H. McBryde, Chairman of the San Francisco Section, who acted as master of ceremonies. Past-President Ira N. Hollis, Chairman of the Committee on Awards, and Past-President William F. Durand presented Dr. Millikan to the President.

Dr. Durand made the address of presentation, in which he outlined the achievements of the medallist as a scientist and educator. President Abbott bestowed the medal and Dr. Millikan responded in a few appreciative remarks. Dr. Millikan then gave the address of the evening on Atomic Mechanics. Following Dr. Millikan, William Sproule, President of the Southern Pacific Company, made an interesting address in which he spoke of the responsibility of the mechanical engineer in advancing civilization.

Other social events included an informal reception and dance Monday evening, a bridge for the ladies Tuesday afternoon, and a tea Wednesday afternoon.

The Annual Meeting

New York, N. Y., December 6 through 10, 1926

The high degree of success which marked the Spring Meeting and other meetings during 1926 was fully equalled by the Annual Meeting. Compared with other Annual Meetings it surpassed all previous records with a registration of 2213, and was notable as well for the size and variety of its program.

Presidents' Night and the Annual Dinner were both occasions made memorable by the presentation of medals to distinguished engineers. On Tuesday evening, following the presidential address by William L. Abbott, the John Fritz Medal was presented to Dr. Elmer A. Sperry by the John Fritz Medal Board of Award in recognition of his development of the gyro compass and application of the gyroscope to the stabilization of ships and airplanes. In his speech of presentation, William L. Saunders, Past-President of the American Institute of Mining and Metallurgical Engineers and Chairman of the Naval Consulting Board of the United States, reviewed the thirty years of Dr. Sperry's persistent efforts to harness the gyroscope for the benefit of mankind. Incidentally he revealed something of the aerial torpedo which Dr. Sperry had perfected just at the time the World War came to an end. The complete addresses appeared in the February, 1927, issue of *Mechanical Engineering*.

On Wednesday evening, at the Annual Dinner, Capt. Ralph Earle, President of Worcester Polytechnic Institute, was presented the John Scott Medal for his invention of railway naval gun mounts and the mine barrage, which played an important part in the World War. The award was made by Dr. Louis Heiland on behalf of the Board of Directors of City Trusts of Philadelphia.

A.S.M.E. awards were presented at the Business Meeting on Wednesday afternoon. These were the Charles T. Main award to W. C. Saylor, of Johns Hopkins University, and Student Awards to R. E. Peterson, University of Illinois, and C. G. Heard, University of Toronto. Other matters on the program of the Business Meeting included the report by the tellers that the amendments to the Constitution concerning junior membership age limitations and the number of directors and officers of the Society had been adopted by letter ballot of the members of the Society; and the reading by title of the following standards: Cast Iron Pipe Flanges and Flanged Fittings for 125 lb. per sq. in.; Cast Iron Pipe Flanges and Flanged Fittings for 250 lb. per sq. in.; Malleable Cast Iron Screwed Fittings for 150 lb. per sq. in.; Cast Iron Screwed Fittings for 125 and 250 lb. per sq. in.

Council Meetings

Two meetings of Council were held during the Annual Meeting. On Monday, December 6, with President Abbott in the chair, a

large amount of routine business was dispatched, including the approval of the petition for the establishment of a Local Section at Dayton, Ohio.

On Friday, December 10, the new President, Charles M. Schwab, received his gavel and the newly elected members of the Council were introduced. Calvin W. Rice was re-elected Secretary. The following Executive Committee was chosen: President, Charles M. Schwab; Past-President, Fred R. Low; Past-President, William L. Abbott; Past-President, Dexter S. Kimball; Vice-President, Roy V. Wright; Manager, Conrad N. Lauer. The Council accepted the custody of three awards totalling $17,500 made by J. F. Lincoln for the best papers on arc welding. Following the action of the Business Session ratifying new amendments to the Constitution providing for the election of the Treasurer by Council and the addition of a Vice-President, Erik Oberg, of New York, was chosen Treasurer, and Harold V. Coes, of Chicago, Chairman of the Finance Committee, was chosen to fill the newly created office of Vice-President for the remainder of the Society year. Seattle was selected as a meeting place for the Society during the late summer of 1927.

Local Sections' Delegates Conference

The functioning of the important Local Sections' Delegates Conference was particularly smooth this year, being marked by a much better arrangement of the delegates for the transaction of business and by a keener appreciation on the part of both delegates and committee of the problems under discussion. Representatives of nearly all of the sixty-eight Sections which make up the Society were present, and they met at 9:30 Monday morning for a conference which lasted throughout the day.

Prof. James A. Hall, Chairman of the Committee on Local Sections, presided, and with him sat Col. Paul Doty, William A. Hanley, James D. Cunningham, Harry R. Westcott, and Ernest Hartford. The delegates were seated at double rows of tables surrounding the speakers' table on three sides in a sort of U-shaped formation.

The roll call was marked by brief reports of activities by the delegates. This was followed by short and inspirational addresses by men prominent in the national affairs of the Society. Among other things, an exposition was made of the work of the Professional Divisions and their important relation to the Local Sections. There was also some discussion of the Society's publications.

There was a recess at noon, at which time the delegates gathered at the Fraternity Club for luncheon with the Council of the A.S.M.E. President Abbott spoke at this luncheon. One of the lighter features was the presentation to Warren H. McBryde, delegate from San Francisco, of a huge medal. This presentation

was made by Dr. Ira N. Hollis in commemoration of Mr. McBryde's accumulation of the greatest mileage in getting to the Annual Meeting, a record which he achieved by coming by way of the Panama Canal.

On Tuesday the work of the Local Sections Delegates continued, there being an independent conference of the representatives of the various Section Groups. The work concluded with a general business meeting.

The following recommendation of the Conference of Local Sections Delegates for the personnel of the Nominating Committee for 1927 was approved by vote at the Business Meeting:

Group I — F. M. Gunby, Boston, Mass., *Chairman.* S. D. Fitzsimmons, Providence, R. I., *Alternate.*

Group II — Jas. Partington, New York, N. Y. H. H. Barnes, Jr., New York, N. Y., *Alternate.*

Group III — V. L. Sanderson, Philadelphia, Pa. Charles Schenck, Bethlehem, Pa., *Alternate.*

Group IV — E. J. Fermier, Houston, Tex. C. E. Ferris, Knoxville, Tenn., *Alternate.*

Group V — M. B. Smith, Detroit, Mich., *Secretary.* J. T. Faig, Cincinnati, O., *Alternate.*

Group VI — W. C. Lindemann, Milwaukee, Wis. W. P. Hunt, Moline, Ill., *Alternate.*

Group VII — William Lester, Denver, Col. A. L. Taylor, Salt Lake City, Utah, *Alternate.*

The group number refers to the group of local sections which selected individual members of the Committee. At a subsequent meeting of the Committee, Frank M. Gunby was chosen Chairman and Morgan B. Smith, Secretary.

The results of the letter ballot upon the officers and delegates to American Engineering Council for the next year were announced during the Presidents' Night program by Claude Hartford, on behalf of the Board of Tellers of Election. The following were declared elected:

President: Charles M. Schwab

Vice-Presidents: Charles L. Newcomb, Everett O. Eastwood, Edwards R. Fish

Managers: Paul Doty, Ralph E. Flanders, Conrad N. Lauer

Delegates to the American Engineering Council: Charles M. Schwab, New York; O. P. Hood, Washington, D. C.; Dean E. Foster, Tulsa, Okla.; W. P. Hunt, Moline, Ill.; Charles Penrose, Philadelphia, Pa.; E. N. Trump, Syracuse, N. Y.; Thomas L. Wilkinson, Davenport, Ia.; D. Robert Yarnall, Philadelphia, Pa.; Walter S. Finlay, Jr., New York, N. Y.; and Ira W. Dye, Seattle, Wash.

Professional Divisions

The work of the fourteen Professional Divisions received attention on several occasions during the meeting. One of these occasions was a dinner at the Fraternity Club of the Council and the Chairmen of the Professional Divisions. At this time the year's activities of the Divisions were presented in a graphical form which made clear comparisons possible and which brought out the points of weakness as well as the points of strength. President Abbott spoke upon the possibilities open to the Divisions. Robert T. Kent, Chairman of the Standing Committee on Professional Divisions, made use of this opportunity to praise some Divisions for worthy accomplishments and to spur others into more vigorous action. Each of the Division representatives present was called upon to give a brief report on the plans that had been laid out for the coming year.

The progress reports of the Professional Divisions were this year individually presented at technical sessions representative of each Division's work instead of being presented *en masse* at the Business Meeting, as was the case last year. They were read by the Division chairmen and were of real interest and value. They showed that they were based upon a year of close observation and careful thought on the part of those who prepared them, and not only told of marked advances in mechanical engineering but also indicated that in many cases the Divisions of the A.S.M.E. have encouraged and participated in this progress. Progress reports were presented by the professional divisions on wood industries, textiles, railroads, fuels, materials handling, steam power, management, aeronautics, oil and gas power, machine-shop practice, and petroleum.

Student Branch Conference

The work of cultivating in the Student Branches the realization that they represent the finest sort of material for the membership of the Society in the future, was continued during the meeting. A conference of delegates representing 35 of the 92 Student Branches was held Wednesday afternoon, December 8, with profitable results both for the delegates themselves and for the Committee on Relations with Colleges and the honorary chairmen, who are working in the interests of the prospective A.S.M.E. members.

On the same day the delegates of the Student Branches had lunch with President Abbott, President-Elect Schwab, and members of Council. They were privileged to hear at that time several highly inspirational talks, including one by Mr. Abbott and one by Mr. Schwab.

Many other important conferences and committee meetings were held during the five days of the meeting, covering all phases of

the Society's work — in many cases work which is not sensational as viewed by the public, but which eventually will have a profound effect upon engineering progress, upon industry, and therefore upon the public. Of such a nature is the work on Standardization.

Lectures

The presentation of the second Robert Henry Thurston and Henry Robinson Towne Lectures brought before the Society two leaders in the respective fields of science and economics. The Robert Henry Thurston Lecturer for 1926 was Dr. Cecil Howard Lander, of London.

Dr. Lander, who is widely known as a civil and mechanical engineer in England, is Director of Fuels Research, Department of Scientific and Industrial Research, in London. He spoke in a well-filled auditorium on Tuesday afternoon, December 7, upon Recent Discoveries in the Science of Coal Utilization, and described recent experiments and discoveries destined to be of great practical value to engineers.

The Henry Robinson Towne Lecture was presented to an equally large and appreciative audience in the Auditorium on Thursday afternoon, December 9. The lecturer was Dr. Davis R. Dewey, Professor of Economics and Statistics at the Massachusetts Institute of Technology, who for many years has been recognized as an authority upon these subjects. His paper was entitled The Credit Factor in the Structure of Industry.

Both papers were published in *Mechanical Engineering* for January, 1927.

Joint Sessions

Several joint sessions were held during the meeting. The program for the joint session with the American Society of Refrigerating Engineers was arranged by the Committee on Heat Transmission of the National Research Council and will be found in the following summary of the technical program. This summary also includes the program of a session on Management held jointly with the Taylor Society on Thursday afternoon. On Thursday evening a session on International Coöperation in the Development of a Science of Management was held under the auspices of the Taylor Society, A.S.M.E. Management Division, American Management Association, Society of Industrial Engineers, and the Committee on American Participation in International Management Congresses. The chief speakers were Harold B. Butler, Deputy Director of the International Labor Office, Geneva, and Stanislav Spacek, Secretary of the International Committee of Scientific Management, Prague.

Summary of Technical Program

Tuesday Morning, December 7

SESSION ON INDUSTRIAL POWER

(Under auspices of Power Division)

Properties of Boiler Tubing at Elevated Temperatures Determined by Expansion Tests, A. E. White and C. L Clark.

Relation of Stokers to Boilers, W. A Shoudy.

Power-Station Accounting for Industrial Plants, W. R. Herod.

SESSION ON WOOD INDUSTRIES

(Under auspices of Wood Industries Division)

A Study of Varnish and Lacquer Finishes Exposed to Accelerated Breakdown Tests, Paul S. Kennedy.

The Use of Wood Lacquer Finishes, Walter S. Edgar.

The Technology of Wood Stains and Fillers for Use with Lacquer, S. M. Silverstein.

Wood Finishing—A Glance Ahead, F. L. Browne.

Safety Code for Woodworking Plants, E. Ross Farra.

SESSION ON TEXTILES

(Under auspices of Textile Division)

The Cotton Textile Industry, Charles T. Main and Frank M Gunby.

Tensile Testing of Textiles, W. F. Edwards.

Tuesday Afternoon, December 7

SESSION ON SMOKE ABATEMENT

(Under auspices of Fuels Division)

Smoke Abatement, Its Effect and Its Limitations, H. B. Meller.

What is Known about the Effect of Smoke on Health, W. C. White

Present Status of the Smoke Problem, Osborn Monnett.

RAILROAD SESSION

(Under auspices of Railroad Division)

The Use of High Steam Pressure in Locomotives, E. C. Schmidt and J. M. Snodgrass.

Report of Committee on Professional Service, Presented by title by M. B. Richardson, *Chairman.*

Balancing Factors in the Use and Obligations Covering Ownership of Freight-Train Cars, L. K. Sillcox.

SESSION ON HEAT TRANSMISSION

(Joint Session with American Society of Refrigerating Engineers. Program arranged by Committee on Heat Transmission of the National Research Council)

Scope and Program of Committee on Heat Transmission, National Research Council, Comfort A. Adams.

Status of Heat-Transmission Data and Knowledge in the Refractory Field, P. Nicholls.

Determination of the Thermal Conductivities of Insulation for Temperatures up to 1000 Deg. Fahr. on Other than Flat Surfaces, R. H. Heilman.

Heat Transmission from Condensing Steam to Water in Surface Condensers and Feedwater Heaters, W. H. McAdams, T. K. Sherwood, and R. L. Turner.

Heat Transfer for Forced Flow of Air at Right Angles to Cylinders, E. L. CHAPPELL and W. H. MCADAMS.

Methods That Have Been and Are Being Used for Measuring Heat Transmission, F. G. HECHLER.

The Guarded-Plate-Heater Method of Testing Low-Temperature Insulators Compared with Several Box Methods, E. F. GRUNDHOFER.

Method of Determining the Total Transmission of Building and Insulating Materials for Built-Up Walls, F. G. HECHLER.

Heat Transfer through Insulation in the Moderate- and High-Temperature Fields: A Statement of Existing Data, L. B. MCMILLAN.

Heat Transfer for Flow of Gases inside Pipes and in Annular Spaces, W. T. DIXON and W. H. MCADAMS.

Heat Transfer in Closed Air Spaces and from Surfaces in Free Air, M. S. VAN DUSEN.

Standard Method and Apparatus for the Measurement of Thermal Conductivities of Building and Insulation Materials, M. S. VAN DUSEN.

Heat Transmission Data on Building and Insulating Materials as Employed in Dwelling Houses, Factories and Office Buildings, F. P. CARTWRIGHT.

Status of Heat Transmission Data on Building and Insulating Materials Employed under Low Temperatures, P. NICHOLLS.

GENERAL SESSION (I)

Measurement of Static Pressure, CARL J. FECHHEIMER.

The Emergency Stops of the Gearless Traction Elevator at the Terminal Landings, F. HYMANS.

A Mercury Compressor Evolved from the Archimedes Screw Pump, J. G. DEREMER.

The Lubrication of Waste-Packed Bearings, G. B. KARELITZ.

Wednesday Morning, December 8

SESSION ON BY-PRODUCT PROCESSING OF COAL

(Under auspices of Fuels Division)

The Low-Temperature Carbonization of Coal, A. C. FIELDNER.

The Distillation of Coal, WILLIAM H. BLAUVELT.

Complete Gasification of Bituminous Coal, R. S. MCBRIDE.

MACHINE SHOP PRACTICE (SESSION I)

(Under auspices of Machine-Shop Practice Division and Research Sub-Committee on Cutting and Forming of Metals)

A Research in the Elements of Metal Cutting, O. W. BOSTON.

Rough Turning with Particular Reference to the Steel Cut, H. J. FRENCH and T. G. DIGGES.

Work-Hardening Properties of Metals, EDWARD G. HERBERT.

SESSION ON MATERIALS HANDLING

(Under auspices of Materials Handling Division)

Industry's Annual Tax for Materials Handling and Suggestions for Its Elimination, HAROLD V. COES.

The Industrial Application of Conveyor Systems, C. A. BURTON.

GENERAL SESSION (II)

Tests and Theory of Curved Beams, A. M. WINSLOW and R. H. G. EDMONDS.

Internal Friction in Solids, A. L. KIMBALL and D. E. LOVELL.

Stresses Occurring in the Walls of an Elliptical Tank Subjected to Low Internal Pressures, WILLIAM M. FRAME.

Stresses and Deflections in Large Dynamo Frames Due to Dead Load, M. STONE.

Wednesday Afternoon, December 8

SESSION ON EDUCATION AND TRAINING FOR THE INDUSTRIES OF NON-COLLEGE TYPE

(Under auspices of Committee on Education and Training for the Industries)

Educational Training for Industry, MATTHEW WOLL.
Trades Training, CARL S. COLER.

STEAM TABLES RESEARCH

Progress Reports on the Work of the Steam Table Fund, L. B. SMITH and F. G. KEYES, Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology; E. S. MUELLER, U. S. Bureau of Standards, Washington, D. C.; and HARVEY N. DAVIS, Harvard University.

Thursday Morning, December 9

SESSION ON CENTRAL STATION POWER

(Under auspices of Power Division)

Operating Performance of Some Modern Surface Condensers, PAUL BANCEL.
Some Results of Condenser Operation, E. B. RICKETTS.
Steam-Condenser Practice and Performance, F. J. CHATEL.
The Influence of Radiation in Coal-Fired Furnaces on Boiler-Surface Requirements, and a Simplified Method for Its Calculation, W. J. WOHLENBERG and E. L. LINDSETH.
Accuracy of the V-Notch Weir Method of Measurement, D. ROBERT YARNALL.

MANAGEMENT (SESSION I)

(Under auspices of Management Division)

Laws of Manufacturing Management, L. P. ALFORD.
Production Control, CLARENCE G. STOLL.

MACHINE-SHOP PRACTICE (SESSION II)

(Under auspices of Machine-Shop Practice Division)

Chromium Plating, WILLIAM BLUM.
The Change of Viewpoint of the Machine Shop, A. L. DELEEUW.
Theory of Milling Cutters, N. N. SAWIN.

SESSION ON AERONAUTICS

(Under auspices of Aeronautics Division)

The Fusion-Joining of Metallic Materials in Aircraft Construction, SAMUEL DANIELS.
Development and Construction of the Standard Army Parachute, JOHN BONFORTE.
Industrial Applications of the Flettner Rotor, F. O. WILLHOFFT.

Thursday Afternoon, December 9

MANAGEMENT (SESSION II)

(Under auspices of Management Division and Taylor Society)

Railroad Organization, J. C. CLARK.
Vitalizing vs. Centralizing Organizations, ROBERT E. NEWCOMB.
An Experiment in Scientific Management in the Coal-Mine Industry, JEROME C. WHITE.
Problems of Bank Organization, H. A. HOPF.

SESSION ON OIL AND GAS POWER

(Under auspices of Oil and Gas Power Division)

Kinematics of Cams, Calculated by Graphical Study, H. SCHRECK.

The Modern Oil Engine, E. C. MAGDEBURGER.

Presentation of Award for Best Paper Delivered during Oil and Gas Power Week, 1926, to FRED THILENIUS, Mem. A.S.M.E., Assistant Master Mechanic, Prairie Pipe Line Company, Tulsa, Okla.

Ideal Gas-Engine Cycles, R. C. H. HECK.

A Temperature-Entropy Diagram for Air and the Diatomic Gases O_2, N_2, and CO, H. A. EVERETT.

The Tangent Method of Analysis for Indicator Cards of Internal-Combustion Engines, P. H. SCHWEITZER.

MACHINE-SHOP PRACTICE (SESSION III)

(Under auspices of Machine-Shop Practice Division)

Worm-Wheel Contact, EARLE BUCKINGHAM.

The Plastic Behavior of Metal in Drawing, C. L. EKSERGIAN

The Distribution of Belt Creep and Slip, R. F. JONES.

PETROLEUM SESSION

(Under auspices of Petroleum Division)

New Methods of Lubricating Steel-Mill Machinery, C. H. BROMLEY.

The Boiler House in Oil Refineries, H. A. ROSS.

Modern Fire Fighting in Oil Refineries, FRANK A. EPPS.

BOILER FEEDWATER SESSION

(Under auspices of Power Division and Joint Research Committee on Boiler-Feedwater Studies)

PROGRESS REPORTS OF NINE SUB-COMMITTEES OF THE JOINT RESEARCH COMMITTEE ON BOILER-FEEDWATER STUDIES:

Pretreatment of Boiler Feedwater, Sub-Committee No. 2 on Water Softening by Chemicals (External Treatment), C. R. KNOWLES, *Chairman.*

Present Knowledge of Foaming and Priming of Boiler Water, with Suggestions for Research, Sub-Committee No. 3 on Zeolite Softeners, Internal Treatment, Priming and Foaming, and Electrolytic Scale Prevention, C. W. FOULK, *Chairman.*

Embrittlement of Steel, Sub-Committee No. 6 on Embrittlement of Metals, A. G. CHRISTIE, *Chairman.*

Municipal Water Supplies and the Effect of Trade Wastes in Relation to the Use of Water in Power-Plant Practice, Sub-Committee No. 7 on Municipal Water Supply in Relation to Boiler Use, V. B. SIEMS, *Chairman.*

Progress Report of Sub-Committee No. 5 on Corrosion of Boilers and Effect of Treated Water in Accelerating or Relieving These Troubles, F. N. SPELLER, *Chairman.*

Progress Report of Sub-Committee No. 9 on Bibliography, G. A. STETSON, *Chairman.*

Progress Report of Sub-Committee No. 1 on Sedimentation with and without Chemicals, Pressure and Gravity Filters and Deconcentrators, Continuous Blow-Down Apparatus, R. C. BARDWELL, *Chairman.*

Progress Report of Sub-Committee No. 4 on Surface Condensers, Evaporators, and Deaerators, A. E. WHITE, *Chairman.*

Progress Report of Sub-Committee No. 8 on Standardization of Water Analysis, H. FARMER, *Chairman.*

Five-minute talks on safety were given at a number of the technical sessions by members and representatives of the National Safety Council.

The Fifth National Exposition of Power and Mechanical Engineering paralleled the Annual Meeting. It was held, as usual, at Grand Central Palace, and exhibited not only the latest developments in power-plant equipment, but showed also many examples of modern machine tools, standards, and other things related to the manufacture of power-plant equipment. It was well patronized by members of the A.S.M.E. attending the Annual Meeting.

Excursions

Excursions were made to the Kips Bay Station of the N. Y. Steam Co., and New East River Station of the N. Y. Edison Co.; the Holland vehicular tunnel, and the Harrison Gas Plant of the Public Service Company of New Jersey; the plant of the American Machine and Foundry Co., So. Brooklyn; the Hudson Ave. Station of the Brooklyn Edison Co.; the Hell Gate Station of the United Light & Power Co. and the plant of the De La Vergne Machine Co., New York; the new diesel-electric ferry boat of the Erie Railroad; and the *New York Evening Post* plant. They were well patronized and in many cases served not only to illustrate but also to strengthen facts which were brought out at the technical sessions.

Social Events

The social events of the meeting were very successful. The Open House on Monday evening, December 6, introduced a spirit of friendliness at the very outset of the meeting which affected very favorably both the ladies and the gentlemen. The ladies, under the auspices of the Ladies' Auxiliary, which did splendid work throughout the meeting, gathered on the eleventh floor of the Engineering Societies Building. Pleasant entertainment was provided, followed by refreshments. In the meantime the men gathered on the fifth floor where they listened to a burlesque investigation of a peculiar boiler explosion. Substantial refreshments followed.

Following the presidential address and presentation of the John Fritz Medal Tuesday evening, President-Elect Charles M. Schwab was presented, and the annual reception and dance was held. New members were formally introduced at the annual dinner, at which Roy V. Wright was toastmaster and President-Elect Schwab, President Abbott, and Past-President Durand were speakers. Over 800 attended this dinner, which was followed by dancing.

Other social affairs incuded the ladies' tea at the Hotel Astor Wednesday afternoon, and the college reunions on Thursday evening.

The program for the ladies, in addition to the social events already mentioned, included the annual luncheon and business meeting on Tuesday, a tour through lower New York Thursday morning, and a theater party for a performance of Don Juan, with vitaphone, Thursday afternoon.

The Annual Meeting Committees were under the general chairmanship of Emmett B. Carter, assisted by F. M. Van Deventer, Vice-Chairman. The chairmen of the sub-committees were as follows: J. W. Cox, Jr., *Reception;* A. W. Lenderoth, *Courtesy;* A. A. Adler, *Open House;* W. M. Keenan, *Catering;* John Price Jackson, *Presidents' Night;* Ernest Bramble, *Information;* R. A. Wright, *Excursions;* and Jos. W. Roe, *Dinner.* A large committee of ladies was under the chairmanship of Mrs. Roy V. Wright.

No. 2001

A MACHINE FOR TESTING STEAM-TURBINE NOZZLES BY THE REACTION METHOD

By G. B. Warren,[1] Schenectady, N. Y.
Junior Member of the Society
and
J. H. Keenan,[1] Schenectady, N. Y.
Non-Member

This paper explains in detail the apparatus used in reaction-nozzle tests, the important difficulties encountered, the means of eliminating them, and the methods of obtaining and developing test data. It discusses the underlying principles and the type of results they would indicate, and shows by sample test curves the close agreement between the expected and the actual data.

SOME years ago the General Electric Company undertook the development of apparatus for finding the relative efficiencies of various types of nozzles and the absolute values of the losses in those nozzles. Investigations using this equipment have been made continuously since that time. They have supplied a great deal of information which has been utilized wherever possible in the design of the nozzles used in steam turbines built by the General Electric Company, and have provided data which have been valuable in the analysis of results obtained from tests of the turbine as a whole.

2 A part of this development followed the type of investigation outlined by H. Loring Wirt,[2] in which nozzles were tested with air, and another part took the form of the development and operation of a reaction-nozzle testing machine. These two investigations were carried on together; the reaction-nozzle tests giving the absolute values of the nozzle efficiency throughout a large range of pressures, velocities, superheats, and nozzle forms, while the air

[1] General Electric Co.
[2] An Experimental Investigation of Nozzle Efficiency. Trans., A.S.M.E., vol. 46 (1924), p. 981.

Presented at the Midwest Power Conference, Chicago, Ill., January 26 to 29, 1926, of The American Society of Mechanical Engineers.

test permitted, as described in Mr. Wirt's paper, an analysis of the component losses and the extension of the investigation to the latter stage nozzles which were too large for testing in the reaction machine.

3 An extensive series of tests has also been made, and is still in progress, in which nozzles of various designs have been tested with bucket wheels in a steam turbine under actual operating conditions. In some cases direct comparative turbine tests have

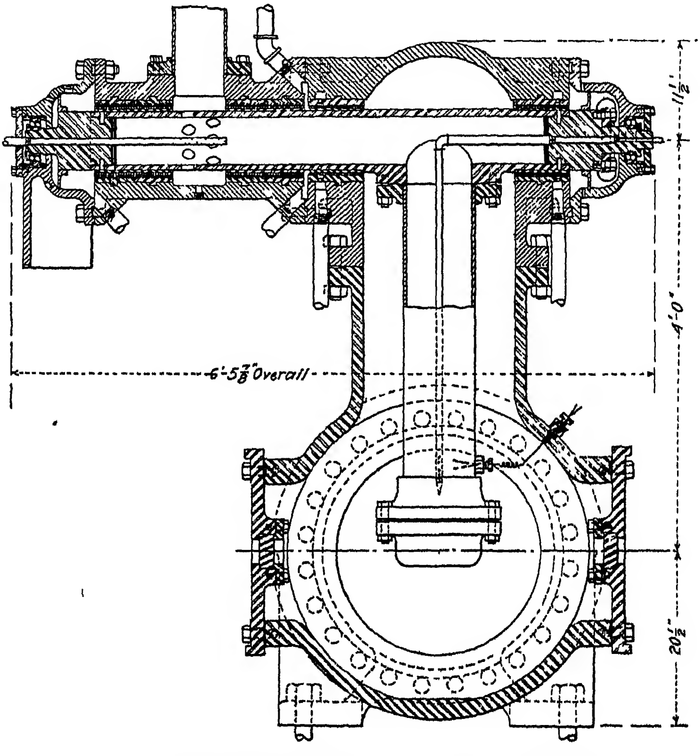


FIG. 1 NOZZLE-TESTING MACHINE—END ELEVATION

been made on nozzles tested by the air and reaction methods. In general these tests have borne out the conclusions obtained from the nozzle tests, but since so many other variables are necessarily involved when moving buckets are used, definite comparisons are somewhat complicated, and a discussion of these tests need not be given at this time.

4 This paper explains in detail the apparatus used in the reaction-nozzle tests, the important difficulties encountered, the means of eliminating them, and the methods of obtaining and developing test data. It discusses the underlying principles and

the type of results they would indicate, and shows by sample test curves the close agreement between the expected and the actual data.

The Testing Apparatus and Its Operation

5 Briefly, it may be said that the average velocity of a jet leaving a nozzle is equal to the ratio of the reaction of the jet to the mass flow per unit time. The average velocity when compared with the corresponding velocity which would be obtained from

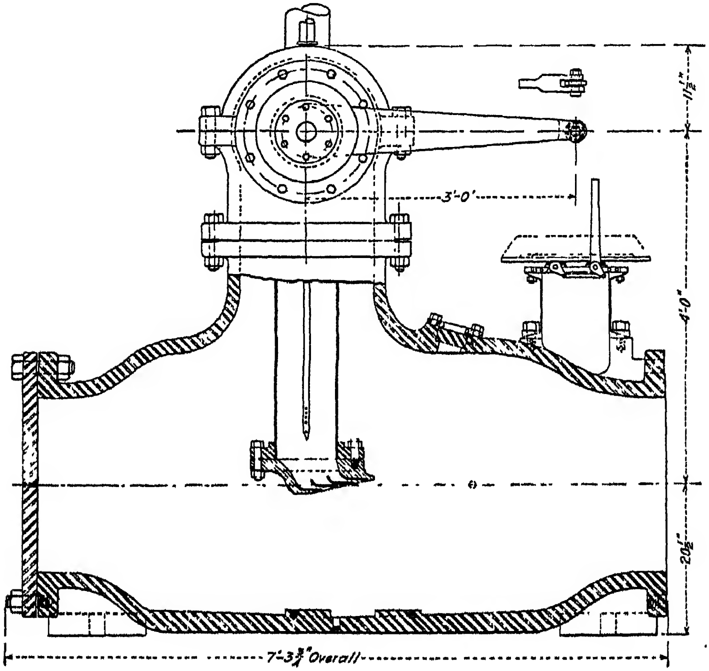


Fig. 1 Nozzle-Testing Machine—Side Elevation

frictionless adiabatic expansion from the conditions ahead of the nozzle to the pressure after it, gives a measure of the nozzle performance.

6 The nozzle-testing apparatus here described is simply a device for measuring the flow through a sample nozzle, the steam conditions before and after it, and the reaction of the issuing jet.

7 *General Description.* The nozzle-testing machine and its auxiliary apparatus are shown in Figs. 1 to 4. From these it will be seen that the nozzle group to be tested is bolted to the bottom flange of a vertical pipe in such a way as to cause the nozzle to discharge horizontally. This vertical pipe is hung from a hollow

horizontal shaft, with which it forms a vertical T. The shaft is supported at each end on ball bearings. The moment about the bearing axis of the reaction force at the nozzle is transmitted through a horizontal arm on the shaft to sensitive platform scales.[1]

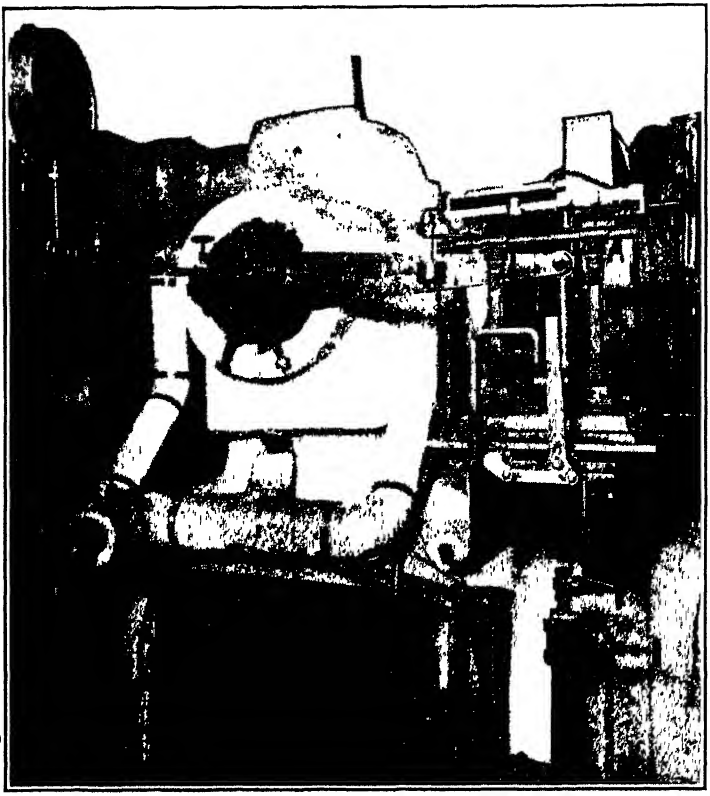


Fig. 2 Front View of Nozzle-Testing Machine

Shows the reaction-measuring scales, the arm attached to the moving element, and the piping to the low-pressure packing.

8 The exhaust chamber shell extends up to and envelopes the shaft. Steam is admitted at one end of the enveloping cylinder and it immediately enters the shaft through radial holes. It flows

[1] The general design of this machine follows to a large extent that used by Dr. Paul Christlein for similar tests made at the Technical High School in Charlottenburg, during the years 1909 to 1911. [This work was reported in the *Zeitschrift für das Gesamte Turbinenwesen*, vol. 9 (1912).] However, the machine here described is considerably larger than Dr. Christlein's and embodies many refinements and modifications which should make for increased accuracy and ease of manipulation.

down through the vertical pipe, through the nozzle, into the exhaust chamber, thence to a surface condenser.

9 From the hotwell of the condenser the condensate line leads through a pump to the weighing tanks where a quick-closing valve

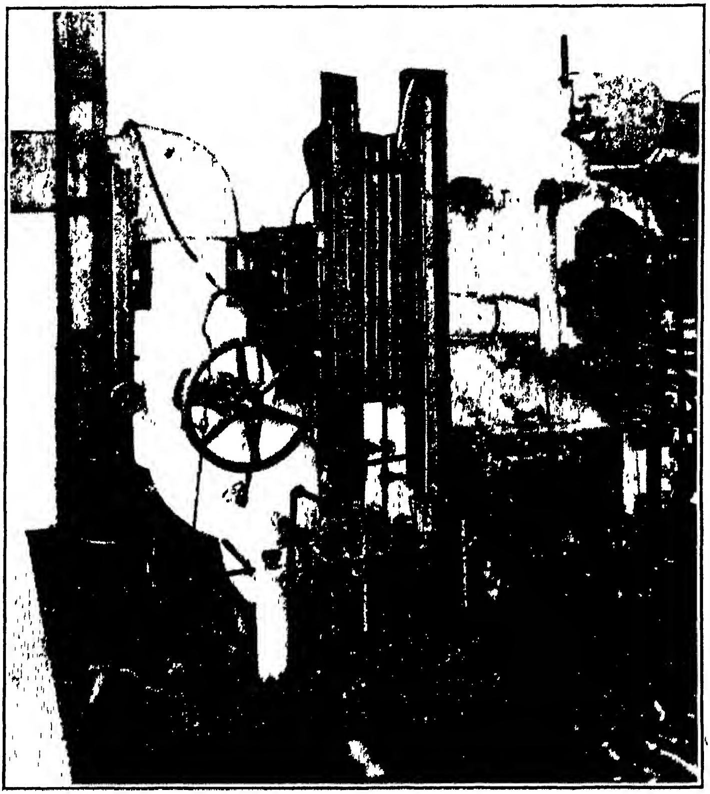


Fig. 2 Back View of Nozzle-Testing Machine

Shows the steam inlet and outlet, the piping to the high-pressure packing, and several mercury manometers for measuring the final pressure and the pressure drop across the nozzle.

operated by a precision clock automatically times the weighing operation.

Details of Machine and Its Operation

10. *Measurement of Reaction.* In order that the friction in the reaction element will not introduce appreciable systematic errors, the reaction scales are mounted on cams which allow moving the scales up and down which in turn move the element back and forth through one-half a degree. When moved in one direction the friction acts *with* the reaction, when moved in the opposite

direction it acts *against* the reaction. This method is used to obtain tare as well as gross reaction values and the difference between the means of the two sets is practically free from friction effects.

11 *Packings.* It is, of course, extremely important to insure that all of the steam which passes through the nozzle reaches the condenser and that no additional steam or water enters there without its quantity being known. The prevention of leakage into or out of the system is comparatively simple everywhere except in the condenser, as mentioned above, and at the shaft of the swinging element.

12 Along the length of the shaft at the inlet end of the machine steam must be prevented from leaking into the exhaust chamber. (See Figs. 1 to 4.) Two lengths of labyrinth packing (0.010 to

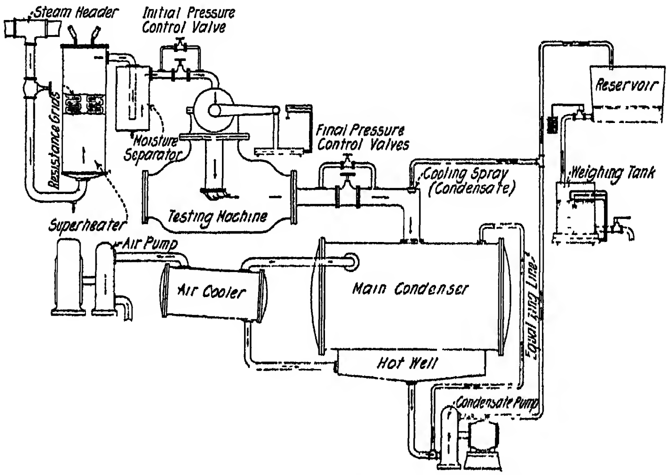


FIG. 3 DIAGRAM OF STEAM AND CONDENSATE SYSTEM OF NOZZLE-TESTING MACHINE

0.015 in. radial clearance) are provided on this section of the shaft. From the space between them sufficient steam is drawn off to an auxiliary condenser to maintain the static pressure in that space equal to the pressure in the exhaust chamber within a quarter of an inch of water. Special tests have shown that with the length of packing and the close clearances used the device is so effective that a pressure unbalance of several inches of water could exist without introducing appreciable error into the flow measurements.

13 The packing along the other end of the shaft is balanced in much the same way except that here the pressure in the packing is kept slightly below atmospheric pressure so as to have a gentle flow of air inward. Since the steam drawn from the packing in this case has come from the exhaust chamber it is necessary to lead it to the main condenser where it is mixed with the rest of the nozzle flow.

14 When the exhaust-chamber pressure is below atmospheric pressure it is necessary to bleed a small quantity of steam into this packing in order to maintain the condenser vacuum. The steam so introduced is first measured by means of a calibrated orifice and the weighed flow is corrected accordingly.

15 *The Main Condenser.* Because of the uncertainty of condenser leakage corrections main condenser leakage is never tolerated. The condenser tubes are carefully packed and a test is made twice each day for leakage. This test consists of weighing the water pumped from the condenser under full vacuum when all steam inlets have been shut off. All flow must cease within 45 minutes or the condenser is opened and inspected.

16 The air removed from the condenser while the machine is in operation is cooled below room temperature and the condensed vapor is drained to the hotwell where it is mixed with the main condensate. (See Fig. 3.)

17 *Pressure Measurements.* Pressure measurements are made in all cases with mercury columns. The ordinary U-tube is used for pressures less than about 6 in. mercury gage; above that the pressure is applied to a mercury surface in a reservoir whence the mercury is forced up into a long tube. Only a small correction for the change of level in the reservoir need be applied to the scale reading at the top of the column to obtain the column length. The pressure lines leading to the reservoir are filled with water to prevent condensation in them. Corrections must consequently be made for water columns as well as for the differential expansion between the mercury column and the scale with change in temperature.

18 The initial pressure is obtained through sampling tubes whose walls are parallel to the stream flow; the final pressure through flush holes in the exhaust-chamber wall. Great pains were taken to insure quiet conditions in the exhaust chamber. (See Par. 52.)

19 Two entirely separate measurements are made of both the initial and final pressures and a direct measurement of the pressure differential across the nozzle is made whenever it is less than 60 in. mercury.

20 *Temperature Measurements.* The initial temperature is obtained by means of carefully calibrated base-metal thermocouples which project about two inches into the supply pipe immediately before the nozzle casting. The thermocouple cold ends are maintained at the freezing point of water and the electromotive force generated is measured by a potentiometer.

21 *Steam Supply.* Steam is supplied to the machine from the power-plant steam header. On leaving the header it passes a group of moisture-injection nozzles, which may be used to lower its temperature, and then enters an upward-flow electric superheater which provides very sensitive control of the steam temperature,

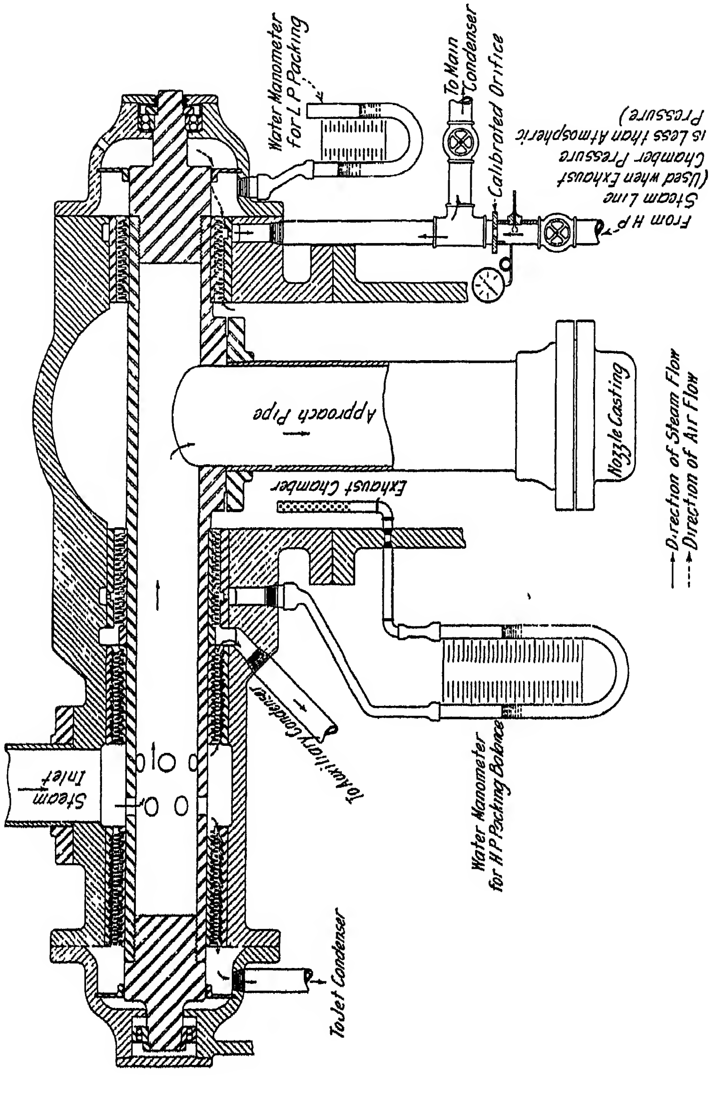


FIG. 4 PACKING DIAGRAM OF NOZZLE-TESTING MACHINE

the heating grids being especially effective in breaking up and evaporating any drops of water. In addition, between the superheater and the testing machine the steam passes through a moisture separator from the drains of which steam is always allowed to blow, thus doubly insuring dryness at the machine. Investigations from many points of view have given confidence that these precautions are effective.

The Position of the Nozzle During Test

22 The nozzle casting is so made that when it is bolted to the flange on the approach pipe the plane of the tail portion of each nozzle partition is normal to the axis of the approach pipe. This means that when the nozzles are discharging parallel to those planes the moment of the reaction about the bearing axis is the product of the jet reaction and the distance from that axis to the middle of the central nozzle passage.

23 This, of course, is no longer true once the jet deflects out of the plane of the partition tail. Above the sound velocity a downward deflection of the jet from non-diverged nozzles does occur and its effect on the reaction measurement becomes of increasing importance as the velocity increases. However, the sound velocity must be exceeded by a considerable amount before the effect of jet deflection becomes appreciable, and in this region of high velocities the tests on non-diverged nozzles are of little practical significance.

24 Below the sound velocity there is a slight upward deflection of the jet which is caused by the filling of the space immediately following the exit edge. This deflection has been carefully investigated by means of air tests which have invariably indicated that for the thin-edged nozzles used in most of these tests the effect of the deflection alone on the measured reaction would be well within 0.3 per cent and is probably nearer 0.1 per cent. This error is, however, practically counterbalanced as explained in Par. 43, by the moment about the bearing axis of the downward reaction component which slightly increases the measured reaction. The elimination of the small error involved would hardly justify complicating the test procedure by adjusting the nozzle position for jet angle.

Method of Testing

25 A single point on Figs. 15 or 16 represents a test of from one-half to one hour duration. The test is usually preceded by a period of 15 to 30 minutes' operation in order to bring the flow to an equilibrium value. During this entire time the initial pressure and temperature and the final pressure are held at the desired values. The reaction, the pressures and the initial temperature are read at three-minute intervals; the flow is measured at six-

TABLE 1 THEORETICAL VELOCITY AND FLOW VALUES FOR ADIABATIC EXPANSION FROM INITIAL CONDITIONS STATED AT TOP OF COLUMN. CALCULATED FOR REACTION-NOZZLE TEST

	31.88 lb. per sq. in. abs.				75.0 lb. per sq. in. abs.			
	100 deg. fahr. superheat		250 deg. fahr. superheat		50 deg. fahr. superheat		200 deg. fahr. superheat	
Press. ratio	Theoretical velocity, ft. per sec.	Theoretical flow, lb. per sq. in. per hr.	Theoretical velocity, ft. per sec.	Theoretical flow, lb. per sq. in. per hr.	Theoretical velocity, ft. per sec.	Theoretical flow, lb. per sq. in. per hr.	Theoretical velocity, ft. per sec.	Theoretical flow, lb. per sq. in. per hr.
0.999	66.45	110.65	72.52	101.15	65.8	261.4	72.24	237.3
0.99	210.8	348.8	230.3	318.8	208.8	823.6	229.2	747.9
0.98	299.6	491.7	327.4	449.5	296.8	1161.7	325.8	1054.9
0.97	366.5	597.2	400.6	545.8	363.1	1410.2	398.5	1280.5
0.96	424.3	685.5	463.6	626.7	420.1	1619.0	461.2	1470.7
0.95	475.3	762.8	519.9	697.3	470.8	1800.2	517.3	1636.8
0.94	521.8	830.4	570.3	758.5	516.3	1959	567.4	1781.1
0.93	565.0	891.5	617.3	814.6	558.5	2105	614.1	1912.5
0.92	605.2	947.2	661.2	865.3	599.1	2236	657.8	2032
0.91	643.4	998.0	702.6	912.0	636.8	2358	699.3	2143
0.90	679.3	1045.2	742.3	955.4	672.5	2470	738.5	2245
0.88	748.5	1130.8	816.6	1033.4	740.1	2673	812.4	2428
0.86	810.8	1205.3	885.1	1101.4	803.1	2851	881.2	2589
0.84	871.0	1271.3	951.1	1162.0	862.0	3007	946.1	2781
0.82	927.8	1328.5	1012.4	1215.5	918.5	3146	1008.0	2855
0.80	983.0	1383.5	1072.4	1263.9	972.6	3272	1067.3	2968
0.78	1036	1431.5	1130.5	1306.1	1024.5	3384	1125.3	3070
0.76	1088	1474.5	1186.5	1343.8	1075.3	3483	1181.8	3162
0.74	1139	1512	1240.5	1376.8	1125	3571	1236.2	3243
0.72	1188	1545	1293.5	1406	1172.8	3649	1289.8	3314
0.70	1235	1573	1346	1431.5	1219.8	3715	1341.7	3374
0.68	1282	1597	1396	1453.0	1266	3772	1392	3425
0.66	1327	1616.5	1447	1472.0	1311	3816	1442	3469
0.64	1372	1632.8	1496	1488.0	1356	3847	1492	3505
0.62	1417	1645.5	1544	1500.5	1401	3865	1541	3534
0.60	1461	1655.6	1592	1509.5	1446.0	3873	1590	3557
0.58	1505	1662.7	1641	1515.5	1491	3875	1638	3573
0.56	1549	1667	1689.5	1518.5	1535	3876	1687	3581
0.54	1593	1669	1738.5	1518.8	1580	3877	1735	3586

	31.88 lb. per sq. in. abs.		75 0 lb. per sq. in. abs.	
	100 deg. fahr. superheat	250 deg. fahr. superheat	50 deg. fahr. superheat	200 deg. fahr. superheat
Press. ratio	Theoretical velocity, ft. per sec	Theoretical velocity, ft. per sec.	Theoretical velocity, ft. per sec.	Theoretical velocity, ft. per sec
0 52	1636	1787 5	1624 5	1783
0 50	1681	1835 5	1669	1831
0.48	1725	1885 5	1716	1879
0.46	1770	1934 5	1761	1927
0.44	1816	1984.5	1808	1976
0.42	1862	2034 5	1854	2026
0.40	1909	2086	1902	2076
0 38	1956	2137	1952	2127
0.36	2005	2188	2002	2179
0.34	2055	2241	2054	2231
0.32	2106	2294	2107	2285
0.30	2159	2349	2161	2341
0.28	2213	2406	2218	2398
0.26	2270	2464	2276	2457
0.24	2329	2523	2336	2517
0.22	2391	2586	2400	2580
0.20	2457	2651	2466	2645
0.18	2526	2720	2537	2716
0.16	2602	2795 5	2613	2791
0.14	2683	2875.5	2697	2873
0.12	2772	2962	2790	2965
0.10	2869	3061	2892	3066
0.08	....	...	3009	3184
0.06	...	...	3148	3325
0.04	....	...	3334	3508
0.02	....	....	3601	3784
0.00	....	..	....	.
....	....	....	...	..
....	....	...	.	.

minute intervals. The deviations from each other of the five or more flow readings that are taken during a test must not exceed one per cent and are usually considerably less than one per cent. The averages of these readings taken during the test are then converted into the final form.

Calculations

26 The final form of the data obtained from the machine is the curve sheet of velocity and flow coefficients. (See Figs. 15 and 16.) The velocity coefficient may be defined as

$$C_v = V/V_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [1]$$

where

V = actual velocity
V_0 = theoretical velocity.

The actual velocity is found from

$$V = Rg/Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

where

R = reaction force at the nozzle, lb.
Q = flow in unit time, lb. per sec.
g = acceleration of gravity, 32.17 ft. per sec. per sec.

The flow coefficient

$$c_q = Q/Q_0$$

where

Q_0 = theoretical flow in unit time, lb. per sec.

27 The theoretical values (see Table 1) are calculated from the Marks and Davis steam tables when the ratio of the final to the initial pressure is less than 0.85. When the pressure ratio exceeds 0.85 the energy drop becomes so small that the deviations in the values obtained from the steam table exceed the probable error of the test. It is then assumed that the adiabatic expansion is of the form

$$p(v-b)^k = \text{constant}$$

where

p = pressure, abs.
v = specific volume
k = 1.3 for superheated steam
b = coaggregation volume, 0.016 cu. ft. per lb.

28 From this the theoretical velocity may be deduced

$$V_0 = \sqrt{2gp_1(v_1-b)\frac{k}{k-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]}$$

where

p_1 = initial pressure, abs.
p_2 = final pressure, abs.
v_1 = initial specific volume and is found from the Marks and Davis steam tables.

The Reaction Principle

29 The following discussion[1] sets forth some of the fundamental physical and mathematical bases of the determination and interpretation of reaction-nozzle coefficient curves. It will be useful in an examination of the test results.

The Normal-Outlet Nozzle of the Converging Type

30 Consider a simple converging orifice having no friction losses. If steam is expanded through it from a constant initial pressure P_1 to any final pressure P_2, a certain total reaction force R will be exerted on the chamber within which the initial pressure

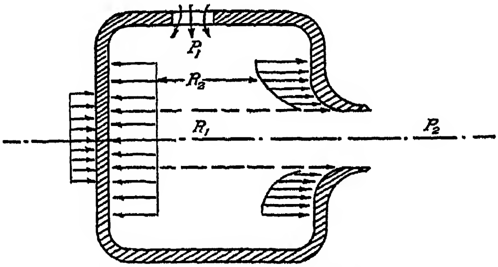


Fig. 5 Reaction Components for a Normal-Outlet Non-Diverged Nozzle

exists. (See Fig. 5.) This force may be divided into two components so that

$$R = R_1 + R_2$$

where R_1 is caused by the unbalanced pressure $(P_1 - P_2)$ on the wall of the chamber over an area projected from the orifice throat parallel to the axis of the jet. R_2, then, is caused by the unbalanced pressure on the inside walls of the chamber where the pressure is falling as the fluid is accelerated upon approaching the orifice. It is obvious that

$$R_1 = a(P_1 - P_2)$$

where a = throat area.

31 R_2 is more difficult to calculate below the sound velocity, but since for any pressure ratio and initial temperature in this region the flow and velocity for a frictionless orifice are known, it can be shown that R may be calculated from the equation

$$R = \frac{M}{t} \times V$$

[1] The basis of this discussion may be found in an article entitled Theory of Steam Turbine Nozzles, by Aug. Wewerka, *Zeitschrift für das Gesamte Turbinenwesen*, vol. 17, nos. 23, 24, and 25.

where

$$M = \text{mass flow in time } t$$
$$V = \text{jet velocity}$$

and R_2 found by subtracting R_1. Thus

$$R_2 = \frac{M}{t} \times V - a(P_1 - P_2)$$

32 For all velocities greater than the sound velocity the flow through the nozzle is constant (assuming a constant initial pressure); the entrance conditions must then become invariable and

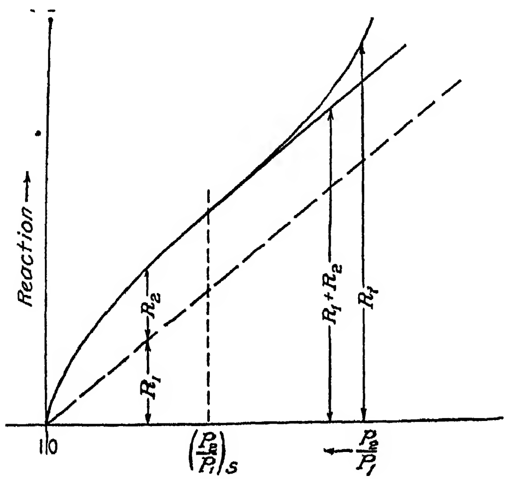


FIG. 6 COMPARISON OF THE THEORETICAL REACTION CURVE WITH THE ACTUAL REACTION CURVE FOR A FRICTIONLESS NORMAL-OUTLET, NON-DIVERGED NOZZLE

$\left(\frac{P_2}{P_1}\right)_s$ is the pressure ratio corresponding to the sound velocity. R_t is the theoretical reaction calculated from the steam tables.

R_2 becomes constant. The expression for the reaction above the velocity of sound is then

$$R = \text{constant} + a(P_1 - P_2) = \text{constant} - aP_1\left(\frac{P_2}{P_1}\right)$$

which plotted against pressure ratio $\frac{P_2}{P_1}$ is a straight line with slope $-aP_1$. However, the theoretical reaction curve calculated from the formula

$$R_t = \frac{M}{t} \times V_t$$

(where V_t is the theoretical velocity due to adiabatic expansion) begins to lift above this straight line as soon as the sound velocity is exceeded (see Fig. 6).

33 It should be noted that the slope of the reaction curve and hence of the velocity curve above the velocity of sound is independent of the amount of friction in the nozzle. (Since the flow is constant above the velocity of sound, the velocity, $V = R/Q$, varies directly as the reaction.) Hence, the velocity curve for any non-diverged normal-outlet nozzle, regardless of its efficiency, will be parallel to the theoretical velocity curve at the sound velocity. The curve of theoretical velocity against pressure ratio is very nearly a straight line for some distance beyond the sound velocity. Consequently the ratio between the actual velocity and the theoretical velocity, that is, the velocity coefficient, increases until the theoretical curve appreciably changes its slope. As the velocity is further increased the velocity coefficient decreases at a gradually increasing rate. The calculated curves and the test curves

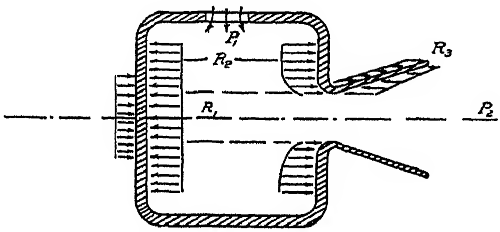


FIG. 7 REACTION COMPONENTS FOR A NORMAL-OUTLET, DIVERGED NOZZLE

of velocity coefficient given on Figs. 15 and 16 show this slight increase in the coefficient just beyond the sound velocity.

34 An explanation of these phenomena from the point of view of kinetics may be given as follows:

35 For velocities less than the sound velocity the fluid has been guided into the orifice and accelerated in the direction of its axis (normal to the plane of the opening). Above the sound velocity the pressure in the orifice throat becomes constant and all expansion below that pressure must occur, unguided, outside the orifice. The components of the acceleration normal to the orifice axis which have been produced by this extra expansion are equal in every direction and have a zero resultant; hence, the resultant acceleration and the resultant reaction are parallel to the axis. The difference between the curves of the resultant kinetic energy of the jet and the available energy from adiabatic expansion represents the energy of expansion normal to the axis. The velocity coefficient would then fall off in accordance with the ratio between the actual and the theoretical reaction.

36 Fig. 15 shows a velocity-coefficient curve obtained from tests on a simple, non-diverged nozzle. Above the velocity of sound the velocity coefficient as calculated from the straight-

line reaction — pressure-ratio curve is plotted to show the remarkably close agreement with test results.

The Normal-Outlet Nozzle of the Diverging Type

37 Consider a similar orifice with a diverging portion added (Fig. 7), the so-called expanding nozzle, and assume it frictionless. The action of such a nozzle below the velocity of sound is likely to be rather complex and not pertinent to this particular discussion. Hence, we shall ignore it for the present and consider its action only for velocities greater than the sound velocity. Here the

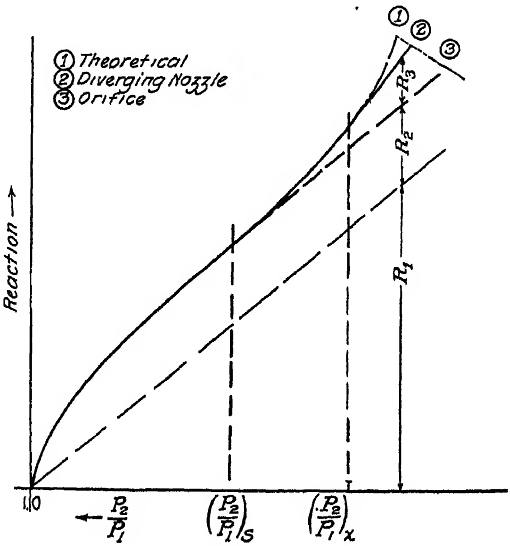


Fig. 8 Comparison of the Theoretical Reaction Curve with the Actual Reaction Curves for Frictionless, Normal-Outlet, Diverged and Non-Diverged Nozzles

$\left(\frac{P_2}{P_1}\right)_s$ is the pressure ratio corresponding to the sound velocity. $\left(\frac{P_2}{P_1}\right)_x$ is the "Characteristic" pressure ratio, the pressure ratio at which for frictionless expansion the jet fills the nozzle exit at the exhaust-chamber pressure.

pressure must drop in the diverging portion from the throat pressure to the final pressure, causing a varying pressure along the inside of the diverging walls. On the outside these walls are loaded uniformly by the pressure P_2. This pressure difference gives rise to a third component of the reaction R_3. Then for a diverging nozzle

$$R = R_1 + R_2 + R_3$$

As P_2 is decreased the value of R_3 will increase until such a pressure ratio, the "characteristic point" for the nozzle, is reached that the final velocity, final specific volume, and the maximum

flow for the nozzle require an area equal to the mouth area. If the pressure ratio is made less than this value the pressure in the nozzle mouth becomes constant and the pressure distribution within the diverging portion does not change. Under these conditions it can be seen that

$$R_3 = \text{constant} - (a' - a)P_2$$

where a' is the nozzle mouth area. This may be expressed as

$$R_3 = \text{constant} - (a' - a)P_1 \frac{P_2}{P_1}$$

from which it is evident that R_3 varies lineally with the pressure ratio, and since, from previous paragraphs, R_2 is constant and R_1 varies lineally with the pressure ratio, the summation R must bear a straight-line relationship to the pressure ratio.

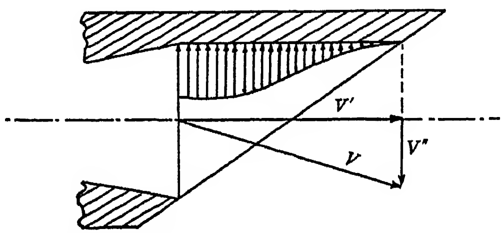


FIG. 9 UNBALANCED PRESSURES IN THE OBLIQUE PORTION OF A TURBINE NOZZLE

V is the mean velocity of the jet; V' and V'' are its components.

38 The difference between the reaction curves of the diverged and non-diverged nozzles above the velocity of sound (see Fig. 8) may be accounted for kinetically in that the diverged nozzle so guides the expansion beyond the nozzle throat that most of it occurs in the direction of the nozzle axis. The dispersion that occurs outside the non-diverged nozzle is here greatly reduced.

THE OBLIQUE-OUTLET NOZZLE

39 Consider the general case of an oblique-outlet nozzle, diverging or non-diverging, such as is used in turbine practice. For this purpose a non-diverging nozzle may be considered as a diverging nozzle for which the characteristic point is the critical pressure ratio, that corresponding to the sound velocity.

40 For all pressure ratios less than the characteristic value some expansion occurs in the oblique portion. There is then a varying pressure along the tail piece which results in a reaction component normal to the axis of the jet. The magnitude and direction of this component is determined by the pressure distribution along the tail piece. The reaction component parallel

to the nozzle axis should be the same as the reaction of a normal-outlet nozzle with equivalent divergence. Hence, the reaction of the jet from the oblique-outlet nozzle is in this case greater than that from the normal-outlet nozzle in that it is the resultant of a force equal to the latter and another force normal to it.

41 Again, the kinetic explanation is that for velocities above the characteristic velocity acceleration normal to the jet is prevented in the direction of the tail piece. Since the excess pressure along the tail piece cannot be dissipated by expansion away from the nozzle center line on this one side it accelerates the fluid away from the tail piece causing a deflection of the jet. The velocity acquired then consists of an axial component, which is equal to

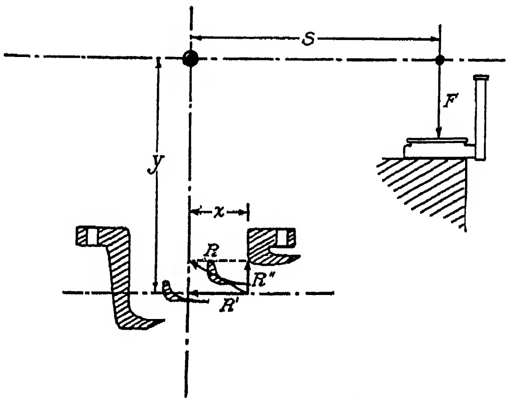


FIG. 10 DIAGRAM OF REACTION COMPONENTS AND THEIR MOMENT ARMS IN THE NOZZLE-TESTING MACHINE IN CASE OF A DOWNWARD DEFLECTION OF THE JET

the resultant velocity from the corresponding normal-outlet nozzle under the same conditions, and whatever component normal to the axis that is provided by the expansion away from the tail piece. The velocity V (Fig. 9) is the resultant of V' and V'' and hence is greater than the resultant velocity of a corresponding normal-outlet nozzle, V'.

THE EFFECT OF JET DEFLECTION ON TEST RESULTS

42 In this investigation the actual velocity is calculated from the formula

$$V_a = \frac{F}{Q} \times \frac{\text{(scale arm)}}{\text{(nozzle arm)}} \times \text{constant}$$

where F is the reaction indicated by the platform scale, "scale arm" is the distance from the axis of the bearings of the machine to the point where the force is applied to the scale, shown as

s in Fig. 10, "nozzle arm" is the distance from the bearing axis to the horizontal center line of the nozzle passages, shown as y, Q is the flow in lb. per hour. This formula gives the true value of V_a when $F \times \dfrac{\text{scale arm}}{\text{nozzle arm}}$ is equal to the resultant reaction of the jet R, which is the case only when the resultant reaction is perpendicular to the axis of the nozzle arm. In general, however, this is not the case for velocities greater than those corresponding to the characteristic pressure ratio of the nozzle, and it is not always true for velocities less than that value (for example, where deflection toward the tail piece occurs). When any deflection from the direction perpendicular to the nozzle arm occurs a reaction component R'' normal to that axis is set up, the resultant of which is applied at some distant x from the bearing axis. (Fig. 10.) The moment of this component, $R''x$, is then added algebraically to the moment of the axial component $R'y$ to give the moment of the force applied to the scale, Fs.

43 When the deflection is downward the moment of the consequent normal component of the reaction would in the case of all the nozzles tested be opposite in sign to the moment of the axial component of the reaction. Hence in such a case

$$Fs < R'y$$

and

$$F\left(\frac{s}{y}\right) < R'$$

and the V_a calculated is less than the axial component of the true V_a. The measured velocity coefficient of an oblique-outlet nozzle above the characteristic velocity is consequently less than that of the corresponding normal-outlet nozzle, though as explained in Par. 40 the true velocity coefficient of the oblique-outlet nozzle is the greater.

44 In the case of upward deflection of the jet the measured reaction would be slightly increased by the moment of the downward reaction component. But the horizontal reaction component decreases with the cosine of the angle of deflection, as that angle increases, and consequently the moment $R'y$ decreases. There are two opposing effects here which are very small and of the same order of magnitude for the thin-edged nozzles that have been tested. For the nozzle shown in Fig. 16 these errors are balanced for an upward deflection of about 3 deg. The probable value of the maximum upward deflection is about 1 deg., for which both these effects are negligible.

45 The effect of the upward component of the reaction on the measured reaction is very small for some distance above the sound velocity. It becomes of increasing importance, however, as the velocity increases.

46 In Fig. 16 the test curve is compared with the curve calculated from the linear relationship of the pressure ratio to the reaction of the normal-outlet nozzle. The difference between the two curves is the effect of the upward-reaction component.

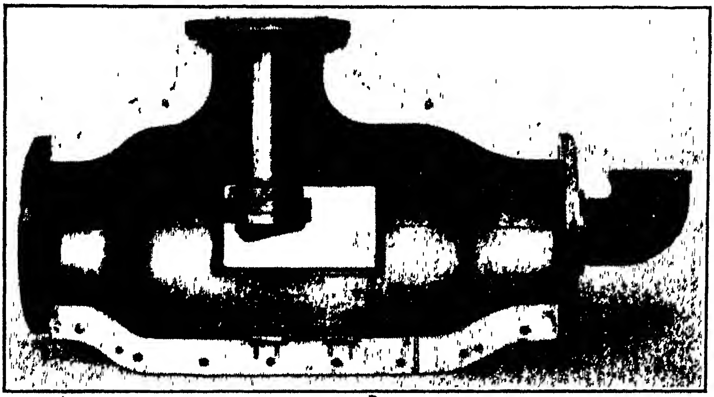


Fig. 11 Quarter-Size Model of Exhaust Chamber, Approach Pipe, and Nozzle

Mean Velocities

47 The mean velocity calculated from the reaction and flow measured by the testing machine is not the mean velocity over the cross-section of the jet. It is the velocity which that uniform jet

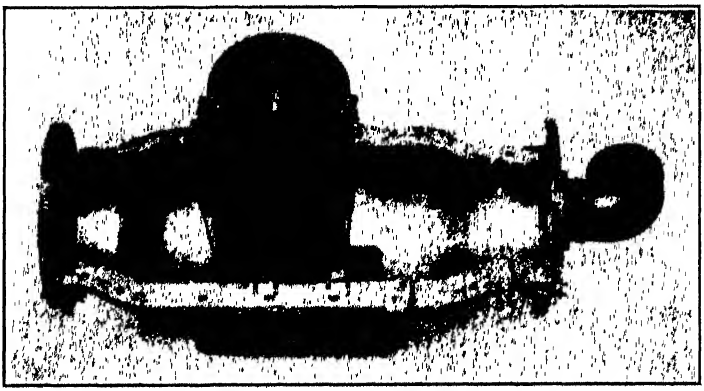


Fig. 12 Model of Exhaust Chamber with Shield in Place

would have whose mass flow and momentum in unit time were equal respectively to the mass flow and momentum in unit time of the actual jet. The square of the velocity coefficient would be the efficiency of this hypothetical jet, $\frac{1}{2}MV^2 \div \frac{1}{2}MV_0^2$.

48 Since a change from a non-uniform to a uniform velocity throughout a jet is an irreversible action, it follows from the second law of thermodynamics that such change is accompanied by a loss in kinetic energy. The momentum remains constant during such an equalization of the velocity, and so for jets of equal momentum, the one having the greatest variation of velocity across it would have the greatest kinetic energy. Since a nozzle jet is never uniform it follows that the efficiency will always be somewhat higher than the square of the reaction velocity coefficient. The difference, with the amount of non-uniformity which exists in the case of the flow of superheated steam through a

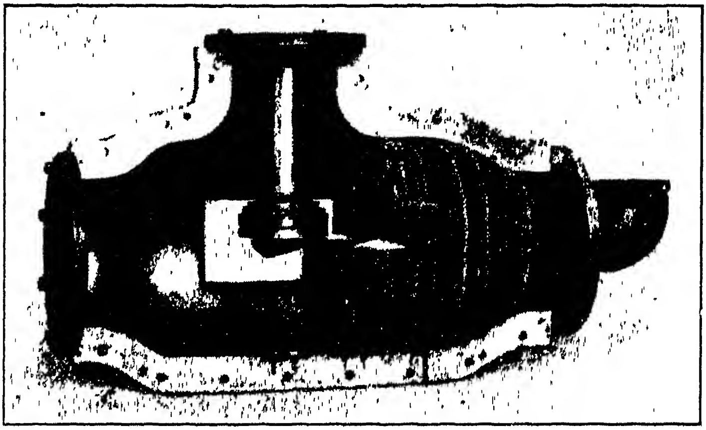


Fig. 13 Model of Exhaust Chamber with Screens Installed

Large-scale reproduction of the screens shown was used to eliminate circulation effects in the testing machine.

turbine nozzle, is, however, only a fraction of one per cent. If no moisture is present in the jet, it is probably well within 0.1 per cent for the nozzle shown in Fig. 16.

DISCUSSION OF THE EARLY RESULTS OBTAINED AND THE INVESTIGATION OF EXHAUST-CHAMBER CIRCULATION

49 The first consistent data obtained from this machine indicated a velocity-coefficient curve which had an upward trend with increasing velocity and exhibited a peak at a velocity 50 per cent in excess of the sound velocity. (See Fig. 14.) All the known factors served to indicate that the curve for a non-expanding nozzle should reach a maximum shortly after passing the sound velocity, and that when a velocity of 50 per cent in excess of the sound velocity is reached, it should be experiencing a rather rapid decline (see Pars. 29 to 41).

50 Explanations of the observed data were diligently but vainly sought until it was recognized that there was a possibility of error in the reaction measurements resulting from circulation of the steam in the exhaust chamber. The impact of a circulating stream on one side of the nozzle approach pipe accompanied by an evacuation of the space on its opposite side would add a positive or negative increment to the jet reaction. Consequently a shield was built about the reaction element as shown in Fig. 12.

51 The first test made on a non-diverged oblique nozzle subsequent to the installation of the shield showed a curve which had no high velocity peak and which behaved much as expected above the velocity of sound. Below the velocity of sound it had lost most of its slope downward with decreasing velocity, but it had acquired a curl upward which began at a little less than one-half the sound velocity and was still in progress at the lowest velocity investigated.

52 There were many reasons for entertaining suspicions concerning the validity of this low velocity rise in coefficient. A few of them are enumerated.

1 All investigations of friction losses of the nature of those found in turbine nozzles have indicated an increase in the ratio of the energy loss to the kinetic energy supplied or else a constant value of that ratio as the velocity decreased. Among them may be mentioned the National Physical Laboratory pipe friction experiments,[1] the investigations of the resistance offered to flow of air past spheres,[2] cylinders,[3] and airplane struts,[4] and the various investigations of the friction of fluids on flat plates.[5] The National Physical Laboratory experiments did show a sharp drop in the loss ratio at the change from turbulent to stream-line flow, but a very smooth and undisturbed flow was necessary to obtain this discontinuity — a condition which could hardly exist in the sharp turn at relatively high velocities in a turbine nozzle.

2 Tests of various kinds have been made on exit-edge losses and vena-contracta losses, and though they are not all in good agreement none have indicated that these losses varied in a manner which would explain a velocity-coefficient curve of the type that was being obtained.

3 Occasionally when sufficiently low velocities were reached the velocity coefficients attained the alarming magnitudes of 101 and 102 per cent.

[1] Phil. Trans., Roy. Soc. of London, Series A, vol. 214 (1914), p. 199.
[2] C. Wieselsberger, *Physikalische Zeitschrift*, vol. 22 (1921), p. 321.
[3] C. Wieselsberger, *Physikalische Zeitschrift*, vol. 23 (1922), p. 219.
[4] Applied Aerodynamics, Bairstow, Longmans Green & Co., 1920, p. 392.
[5] Froude, Report of Brit. Assoc. Adv. of Sci., 1872, p. 118, 1874, p. 249.
Zahm, *Phil. Mag.*, vol. 8 (1904), p. 58.
Thurston, *Engineering*, vol. 95, Jan. 24, 1913, p. 107.

Tests on Model of Exhaust Chamber

53 In order thoroughly to investigate the circulations in the exhaust chamber, a model of it was built (see Fig. 11) and tested with air. The tests made on the model proved to be extremely valuable. The information they yielded concerning the apparatus as used before the model tests may be briefly summarized as follows:

1 High-velocity circulations, reaching 400 ft. per sec. at times, existed in the exhaust chamber

2 The circulation was caused by splashing of the stream on the end flange of the exhaust chamber and flow backward out of the incompletely filled exhaust pipe

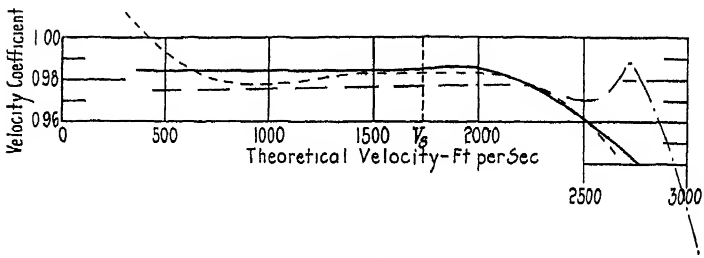


Fig. 14 Test Results Obtained from Turbine Nozzle Shown in Fig. 16

The dot and dash curve was obtained before provision was made to eliminate the effect of circulation in the exhaust chamber. The dash curve was obtained with a shield installed (as in Fig. 12). The solid curve is from the final test made after the exhaust-chamber circulation had been eliminated. The velocity of sound is indicated as V_s.

3 A part of the nozzle stream was striking inside the front face of the shield and probably was being reflected back against the nozzle pipe. This was, doubtlessly, the explanation of the low velocity rise in velocity coefficient, since all jet-angle investigations have indicated a decrease in jet angle with a decrease in velocity.

54 Various attempts were made in the model machine to eliminate the circulation or its effect. The method finally adopted for use in the testing machine was that shown in Fig. 13. It is simply a device which allows most of the jet easily to reach the exhaust pipe while it destroys the kinetic energy of any parts of the jet that might be deflected toward the nozzle. It consists of a series of coarse screen disks, with small central holes, set parallel to each other in that part of the chamber between the nozzle and the exhaust pipe. An annulus of fine screen is secured to each disk, concentric with it, and fitted closely to the chamber wall. A few half-disks of the same design were placed on the nozzle side of the series at the bottom of the chamber to prevent reflection of the deflected jet at high velocities.

55 This device was so effective that when installed a match flame would live where velocities of 300 ft. per sec. had existed in the unscreened chamber. A water manometer would indicate no differences in static pressure throughout that part of the chamber not occupied by the screens.

56 The "screen pad" appeared to be the solution of the problem. It was manufactured to the larger scale and installed in the testing machine. In order to determine whether the large-scale reproduction of the screens was as effective as the screens in the model, two types of investigations were made.

57 The first was a series of tests on a nozzle with the screen pad in place, followed by similar tests with a close-fitting shield about the nozzle pipe which would as nearly as possible shelter it from all circulation streams. Regardless of whether or not the shield was in itself a perfectly efficient device, any circulation effect in the absence of the shield would be evidenced by a change in the reaction when the shield was installed. A comparison of the reactions as determined by the two series of tests follows:

MEASURED REACTIONS WITH SCREEN PAD

Pressure ratio	Without shield	With shield
0.3225	85.45	85.43
0.3208	85.57	85.69
0.6435	53.13	53.18
0.6430	53 17	53.20
0.8612	23.34	23.43
0.8606	23.40	23.26
0.9138	14.83	14.88
0.9136	14.87	14.81

58 A second type of investigation consisted of determining the reaction for a given nozzle with atmospheric back pressure, then removing the back plate of the exhaust chamber and once more reading the reaction as well as noting carefully the lack of uniformity or the disturbances in the outward drift of the steam. The reaction comparisons for two different nozzles follow:

	Test No. 1	Test No. 2
Theoretical jet velocity, ft. per sec.	1770	2480
Flow through nozzle, lb. per hr.	6400	3500
Measured reaction with closed chamber, lb.	177.73	141.57
Measured reaction with open chamber, lb.	177.95	141.66

59 The slight rise in the reaction when the plate is removed may be accounted for by the outward drift of the steam impinging on the nozzle pipe. This outward drift was gentle and uniform at any elevation. It decreased toward the bottom where it became an inward drift of air — the normal convection type of circulation. The inward drift destroying the condenser vacuum forced a large amount of the steam out through the open end of the chamber, thus increasing the outward drift. An impact tube was used to measure the drift velocity, the exploration being made into the

chamber as far as the nozzle, and nowhere did it indicate an impact pressure greater than 0.02 in. of water though the steam issued from the nozzle at 1770 ft. per sec. in one case and 2480 ft. per sec. in the other.

The Conditions of Approach

60 The conditions in the approach pipe have not been a source of trouble, as were the conditions in the exhaust chamber. However, they have been subject to careful tests to determine whether

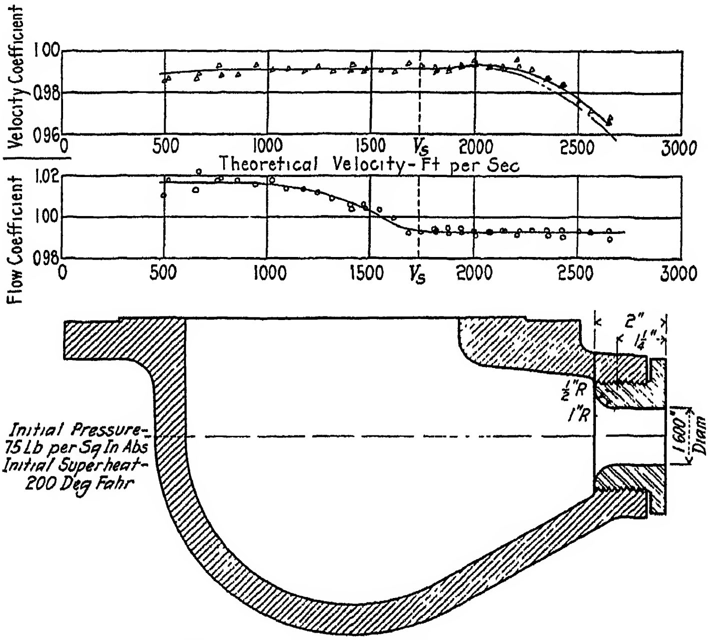


Fig. 15 Test Results on a Normal-Outlet Converging Nozzle

The sound velocity is indicated as V_s. The velocity-coefficient curve calculated from the straight-line pressure-ratio-reaction relationship is shown as a dot and dash line.

the kinetic energy of the steam before entering the nozzle was small enough to be negligible. This would be true in the case of practically all the tests made provided there was a uniform velocity distribution in the approach pipe. If, however, a high downward velocity existed in the center of the pipe with low downward or upward velocities at the sides, then the kinetic energy of approach would be an unknown, though appreciable, increment on the available energy.

61 A test was made on a high-flow nozzle with the ordinary approach conditions. Following this a test was made on the same nozzle with the approach conditions so adjusted as to give uniform distribution. The uniform distribution was obtained by inserting

in the approach pipe several groups of orifices in series, followed by baffles and screens which redistributed the flow over the entire pipe cross-section. This device was thoroughly tested with air before being installed.

62 If non-uniform distribution had introduced error in the tests without baffles, it would be such as to raise the available energy and hence the observed velocity coefficient above the true value. However, these tests made without the baffles gave an average velocity coefficient slightly lower, less than 0.25 per cent,

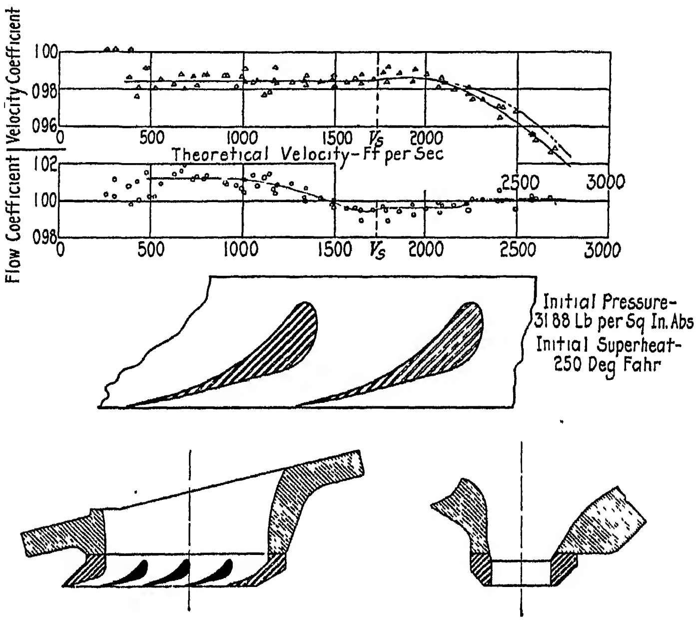


Fig. 16 Test Results on a Turbine Nozzle

The sound velocity is indicated as V_s. The velocity-coefficient curve calculated from the straight-line reaction-pressure-ratio relationship is shown as a dot and dash line. Because of the very small reaction at 300 ft. per sec. an error of 1.5 per cent could readily occur in that measurement. This probably accounts for the high velocity coefficient points at that velocity.

than the tests made with uniform distribution, a difference within the range of testing error.

63 These results were obtained with a volume flow ahead of the nozzle which has seldom been exceeded. In other words they offer especial proof of the uniformity of flow to the nozzle in all cases.

The Final Results

64 The apparatus, embodying all the improvements and devices developed to eliminate the difficulties just discussed as well as others much more numerous, though of lesser importance, is in

constant use for the analysis of nozzles used in turbine manufacture. A large variety of types has been tested. The coefficients obtained from a test of an elementary nozzle are shown in Fig. 15, and those obtained from a test of a sample turbine nozzle in Fig. 16.

65 In these results one of the outstanding features is the high velocity coefficients shown by both the elementary nozzle and the turbine nozzle. The painstaking investigation and study involved in the development of this machine to its present state would justify confidence in the absolute results it indicates. However, a further check was available in the impact-tube air test described in Mr. Wirt's paper.[1] The turbine nozzle shown in Fig. 16 was carefully tested by the impact tube method and the values obtained are compared below with the corresponding values obtained from the steam reaction test.

	Efficiency C_v^2, per cent	
Ratio of theoretical velocity to sound velocity	Reaction test steam	Impact test air
0 86	96.8	97.1
0.67	96.8	96.3
0 50	97.4	97 5

The differences between the two sets of data are well within the probable error of the impact tube method.

66 Another feature of interest is the constant-velocity coefficient shown by both the elementary nozzle and the turbine nozzle for all velocities below the velocity of sound. This characteristic is checked satisfactorily by the air-test data given above and has been further verified by a series of impact traverses taken at different velocities along the center line of similar nozzle groups.

67 A constant velocity coefficient means constant efficiency which in turn means that the friction forces involved are proportional to the square of the velocity. This is in agreement with nearly all the existing higher velocity tests on the resistance to flow of a fluid past flat plates, spheres, cylinders, airplane struts, etc.

68 Above the velocity of sound the test curves shown are compared with the values calculated from the purely theoretical considerations given in Pars. 29 to 41. For the elementary nozzle the test curve and the calculated curve almost superimpose, while for the turbine nozzle the slight disagreement between the two is a further justification of the results. It is shown in Par. 43 that the upward-reaction component that accompanies jet deflection in an oblique-exit nozzle serves to lower the actual test curve. The fact that this effect was not considered in the calculation accounts for the difference between the test curve and that calculated.

[1] Trans., A.S.M.E., vol. 46 (1924), p. 981.

69 The experience acquired in five years of almost continuous operation of this machine impresses one with the difficulties involved in the development of an apparatus of its kind which will give not only the relative efficiencies of several nozzles, but dependable absolute efficiencies as well. It also impresses one with the unwisdom of making a statement to the effect that systematic errors no longer lurk within the apparatus. But a belief that any such errors that do still exist are of minor importance is justified by the experience that has been acquired, the many difficulties overcome, and the agreement of the results throughout with logical considerations.

DISCUSSION

Gerald Stoney [1] and Telford Petrie.[2] The comparison of the results obtained in this paper with those obtained by the Steam Nozzles Research Committee of the Institution of Mechanical Engineers (England) is most interesting. This committee has now issued four reports.[3] The impulse method was used by the committee instead of the reaction method employed by the authors. It is much to be regretted that the authors give only the results obtained for two nozzles, a turbine nozzle and a straight elementary type of nozzle shown in Figs. 15 and 16. It is to be hoped that these are comparable with Figs. 17 and 18 (Figs. 8 and 12 of the fourth report by the committee). These two nozzles gave velocity coefficients in the impulse tester which are some 2 per cent below the curves given in the present paper, as can be seen from Figs. 19 and 20 (Figs. 10 and 11 of the fourth report by the committee). The band method of presenting these results represents the degree of accuracy which the committee feel has been attained at any particular velocity. Every method of testing, however, has systematic errors and there is some reason to believe that the impulse method has a tendency to give low results, while the reaction method, on the other hand, may be on the high side. A point to be noted in this paper is the very high value of the discharge coefficients between, say, 500 and 1000 ft. per sec. In the simple nozzle these appear to be about 101.5 per cent, and in the curved nozzle 101 per cent, and it is difficult to understand how this can be. The straight nozzles used by the committee averaged about 97 per cent, with a margin of ± 1 per cent at these velocities, and the curved nozzle averaged between 96 and 97

[1] Professor of Mechanical Engineering, College of Technology, University of Manchester, Manchester, England.

[2] Lecturer in Mechanical Engineering, College of Technology, University of Manchester, Manchester, England.

[3] Proc. Inst. Mech. Engrs., Jan. 1923, March 1923, May 1924, and May 1925.

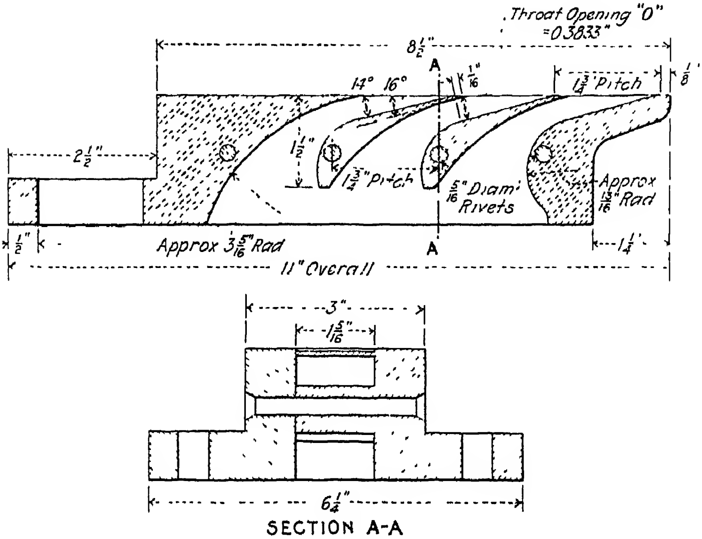


Fig. 17 Detail of Built-Up Type of Impulse Nozzles

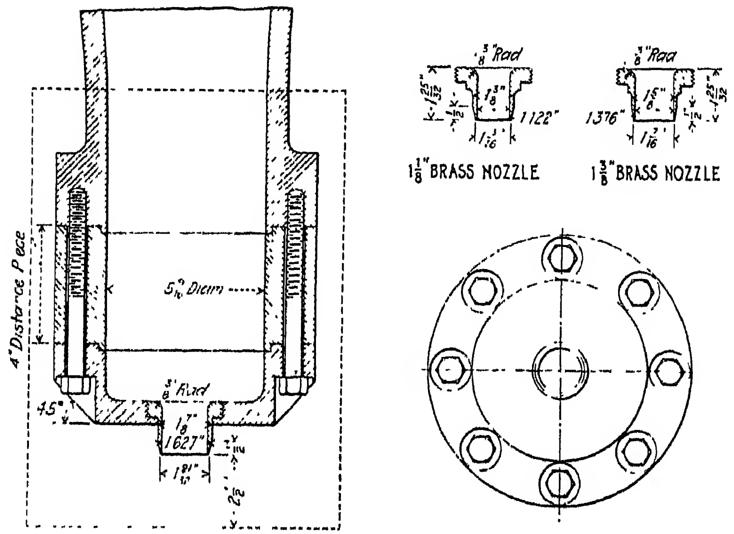
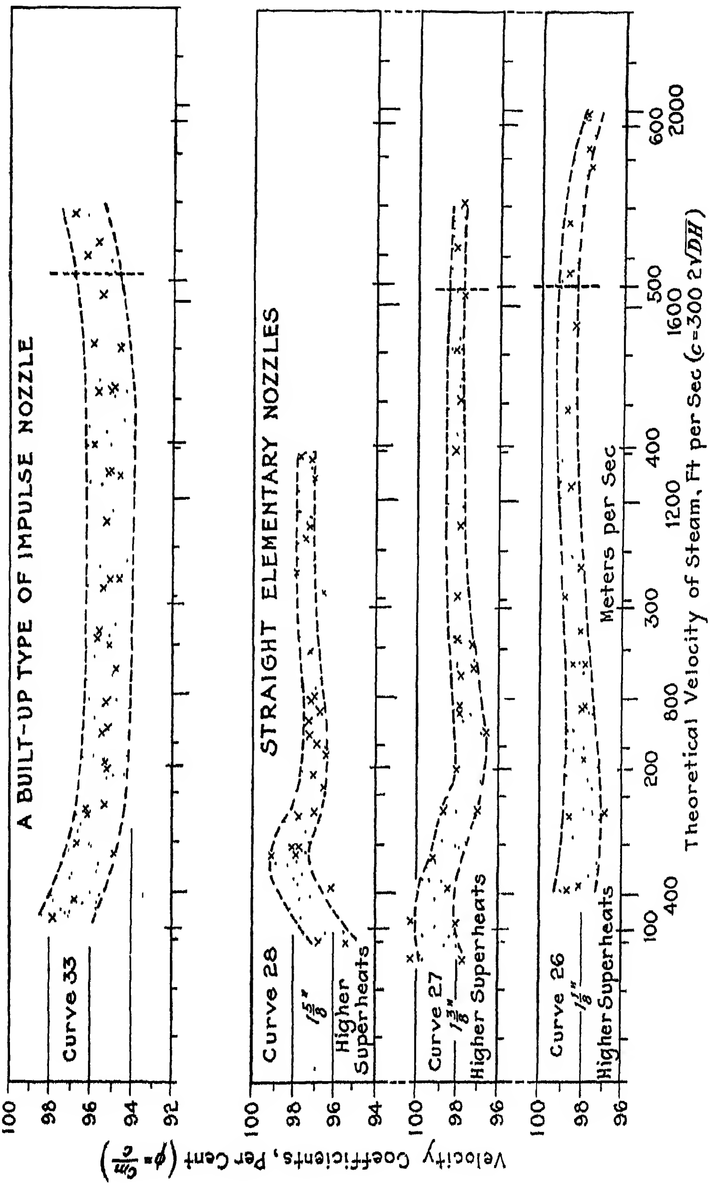


Fig. 18 Details of Three Interchangeable Straight Elementary Nozzles

(Dotted lines indicate internal dimensions of cage from top of plate.)



Figs 19 and 20

per cent.[1] If expansion due to temperature was allowed for, these results should be reduced by approximately 0.4 per cent. It is difficult to see how such a figure at 101.5 per cent can be correct, even if expansion of the nozzle was not allowed for, and the possibility of leakage is suggested. In both methods of test the discharge coefficient should be the same. The discharge coefficients beyond the critical are also higher than those calculated from the committee's tests; in fact, we are beginning to think that all nozzles of these types, when passing superheated steam, have a discharge coefficient of 98 ± 0.75 per cent at and beyond the sound velocity.

Of course, the committee used Callendar's steam tables and English values for gravity, while the authors have used the Marks and Davis tables. Where these tables do not quite agree the results would differ, as the figures obtained depend on the steam tables used.

There is little doubt that the rise in velocity coefficient at low velocities is correct, although the authors, like Stodola (see his Dampfturbinen, 5th or 6th Edition, page 125), seem to doubt their own tests. This rise has been found in nearly all cases by the Committee of the Institution of Mechanical Engineers and is confirmed by tests on Parsons turbines. In Parsons blading, as in all nozzles, at high velocity ratios of about 0.9, the carry over is practically complete, and the efficiency of the turbine is that of the nozzles when blade leakage is allowed for. The curve found by the committee for Parsons blading is given on page 327 of its second report, March 1923, and at steam velocities of from 150 to 500 ft. per sec., which are the velocities in Parsons turbines, agrees (within the limits of observation) with results obtained in such turbines.

The Authors. The reaction method of testing nozzles measures the momentum imparted to the fluid jet between its entrance to the apparatus and its leaving the nozzle. The impulse method measures the momentum of the jet after it has left the nozzle. There is no reason why these two methods should not be in perfect agreement if properly used. They demand, in common, extreme care in the disposition of the exhaust fluid in order to prevent errors in the impulse or reaction measurement. In addition, the freely swinging approach pipe necessary to the reaction method presents the problem of preventing errors in the flow measurement resulting from leakage around that pipe, while the impulse method must insure that the entire jet leaves the impulse apparatus at 90 deg. to the angle of incidence without change in its static pressure.

If there is disagreement between the results of the impulse tests of the Institution of Mechanical Engineers and these reaction tests

[1] See Mr. Ashworth's contribution to the discussion on the fourth report, page 827.

the conclusion cannot be avoided that in one or both of these tests sufficient precaution against error has not been taken.

Messrs. Stoney and Petrie suggest leakage as a probable explanation of the magnitude and characteristics of the flow coefficients shown in Figs. 15 and 16. If present, it must occur through the high-pressure packing, through the pipe walls, or through the flange packings. The high-pressure packing balance was tested at frequent intervals and, except in the early stages of the development of the apparatus, has always been found to be perfectly satisfactory. Leakage through the walls of the hollow forged shaft or the heavy vertical pipe is extremely unlikely but it has been tested for by blanking off the end of the vertical pipe and applying pressure. None was ever found.

Leakage through thermocouple glands or flange packings is the most likely of all possibilities and was experienced in a few cases. Such leakage was always readily detected owing to the throttling action in the leak which prevented the flow becoming constant with reduced back pressure until well below the critical point. There is no evidence of any such action in Figs. 15 and 16. Further, there is no reason to believe that the leakage would be a greater percentage of the total flow between velocities of 500 and 1000 ft. per sec. than at velocities in excess of the sound velocity. On the other hand, because of the throttling that would occur in practically all leaks through flange packings or thermocouple glands, there is reason to believe that the leakage would be a larger percentage of the flow at velocities higher than the sound velocity. Consequently the high flow coefficients below the sound velocity cannot be explained by leakage.

A reason for these high flow coefficients in the case of the turbine nozzle may be found in the probability of diffusion of the steam jet into the space beyond the partition edge after leaving the nozzle throat. In other words, the space just beyond the partition edge acts like the diverging part of a diverged nozzle into which the stream diffuses to the back pressure leaving a pressure in the nozzle throat lower than the back pressure. It has long been known that diverged nozzles give a flow in excess of the theoretical flow at velocities less than the sound velocity.

The high flow coefficients of the simple converging nozzle at 500 to 1000 ft. per sec. cannot be so readily explained. However, two things are of interest in this connection: First, using the Marks and Davis steam tables, the difference between the flow coefficients at 750 ft. per sec. and 1800 ft. per sec. is 2.4 per cent. If the Callendar tables are used this difference is reduced to 1.5 per cent, the coefficient being raised about 0.4 per cent above the sound velocity and lowered about 0.5 per cent below the sound velocity.

Second, the test represented by Fig. 15 was the first test made after installing screens in the exhaust chamber of the machine. A similar test had been made on the same nozzle just previous to this

with the baffle in place instead of the screens (Fig. 12). This had given the same velocity coefficient curve as the later test (the baffle apparently shielding this type of nozzle sufficiently), but the flow coefficients at 750 ft. per sec. and 1800 ft. per sec. showed a difference of only 1 per cent according to the Marks and Davis tables (or 0.2 per cent according to the Callendar tables). No reason has been found for such a radical change in the measured flow coefficient as occurred between these two tests that does not predicate a change in velocity coefficient which did not occur.

It is probable that the small ratio of the radius of the wall contour to the diameter of the $1\frac{5}{8}$-in. nozzle shown in Fig. 18 as contrasted with the large value of that ratio in the 1.6-in. nozzle shown in Fig. 15 [1] is a reason for some of the discrepancy between the results obtained by the committee and those reported here. It is likely that a vena contracta exists in the $1\frac{5}{8}$-in. nozzle shown in Fig. 18 that is absent or less pronounced in the 1.6-in. nozzle, which would make the $1\frac{5}{8}$-in. nozzle the less efficient. This would also explain the increase in efficiency with decreased diameter and constant radius of wall contour indicated by the committee's tests (Fig. 20).

The test curve shown by Stodola and referred to by Messrs. Stoney and Petrie in their discussion has a rise in velocity coefficient with decreased velocity beginning immediately below the sound velocity instead of at approximately one-third the sound velocity as in the case of the tests by the Institution of Mechanical Engineers. A point of minimum efficiency a little below the sound velocity is characteristic of nozzles with a small divergence beyond the throat or with a vena contracta at the throat. Either might readily be present in the nozzle reported by Stodola.

It is a mistake to place too much faith in conclusions drawn concerning nozzle efficiencies from tests of groups of turbine stages. There are many complex phenomena occurring in a turbine that might readily be misinterpreted. If a rise in turbine efficiency is indicated with lower jet velocities it is most likely that the explanation will be found on further search to be in something other than nozzle efficiencies.

Many reasons for being reluctant to believe in an increase in nozzle efficiency with decreased velocity are given on pages 53 and 58. The three high-velocity coefficient points in the neighborhood of 300 ft. per sec. obtained in the test of the turbine nozzle (Fig. 16) would hardly justify bending the mean curve upward when the deviation of these points from the mean curve drawn is within the probable error of the test. Like practically all the other tests that have been run on turbine nozzles of various types, the test on the simple converging nozzle (Fig. 15) gives no hint of a rising velocity coefficient with decreased velocity.

[1] The section of the nozzle given in Fig. 15 has been corrected since the publication of this paper in preprint form for presentation at the Midwest Power Conference.

No. 2002

BASIS FOR DETERMINING THE PROPORTIONS OF STANDARD T-SLOTS AND BOLTS

By Luther D. Burlingame,[1] Providence, R. I.
Member of the Society

For many years forward-looking engineers have urged the standardization of the holding elements of machine tools, such as T-bolts and slots. The importance of such standardization has been generally recognized, not only by individual manufacturers but by the National Machine Tool Builders' Association, the technical societies, and the American Engineering Standards Committee. This has resulted in the appointment of a sub-committee which for several years has been at work on the problem of providing an acceptable standard for T-slots such as are used in the tables of machine tools. A standard has now been formulated by the committee and is the subject matter of a report presented at this meeting.

A great amount of material has been collected showing past and existing practice. Tests have been conducted to ascertain the comparative strength of bolts and slots, in order that the proportions submitted might be suited to practical needs. That the report of the committee may not be loaded with the records of this investigation but instead may be concentrated on the actual proportions to be adopted that report is supplemented by this record of the underlying investigations on which it is based.

Factors Determining the Strength of T-Slots

THE elements of possible weakness which might result in failure of T-bolts, slots, and nuts and which were investigated are as follows:

A T-slot
- *a* Denting or compressing metal under heads
- *b* Springing or breaking lips

B T-bolt
- *a* Deformation or breakage of lips of head
- *b* Pulling off of head

[1] Industrial Superintendent and Patent Expert, Brown & Sharpe Mfg. Co.

Contributed by the Machine Shop Practice Division and presented at the Providence, R. I., Meeting, May 3 to 6, 1926, of The American Society of Mechanical Engineers.

c Breaking bolt through body
d Breaking bolt at roots of thread
e Stripping threads of bolt or nut

C T-nut
a Stripping threads
b Deformation or breakage of lips of nut
c Buckling by bending at tapped hole.

Other Factors to be Considered

2 Working clearances should be such as to give ample space (1) for oil and chips between the head of the bolt or the T-nut and the T-slot; (2) for clearance for the body of the bolt to slide

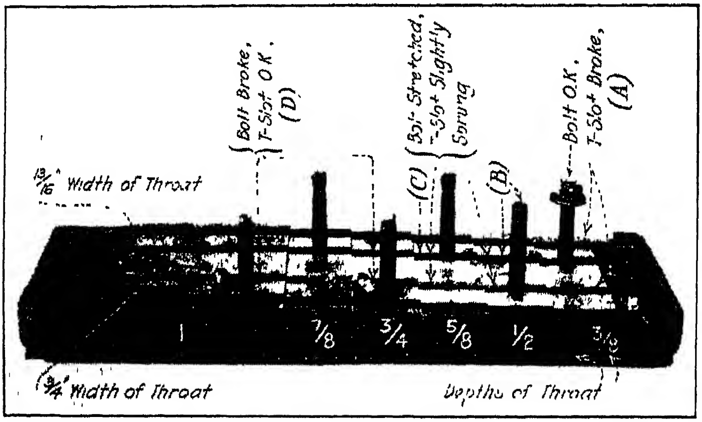


Fig. 1 Tests to Ascertain Comparative Strength of ¾-In. T-Slots and Bolts
(Table of hard cast iron)

freely in the throat of the slot even if the bolt is of maximum size or the slot is bruised; (3) for allowance for re-grinding cutters used in machining T-slots; and (4) for allowance for machining worn or bruised surface of table.

Methods of Investigation

3 A study was made of these varying conditions, and correspondence was carried on with manufacturers of machines and users of equipment requiring T-slots. This included a study of standards in use in foreign countries. Also, a series of tests was made, using milling-machine tables made of cast iron of varying degrees of hardness, bolts of varying tensile strength, and T-nuts with tapped holes for varying sizes of stud.

4 Milling-machine tables made of "hard" cast iron, such as are regularly made by the Brown & Sharpe Mfg. Co. and other milling-machine manufacturers for this purpose, were used for one series of tests. An analysis of the composition of these tables follows:

Silicon	1.25-1.50
Sulphur	0.11-0.14
Manganese	0.55-0.70
Phosphorus	0.35-0.45
Total carbon	3.20-3.60
Combined carbon	0.75-0.85

5 In the other series of tests the tables were made of gray iron, often called "soft" cast iron, with an analysis as follows:

Silicon	1.90
Sulphur	0.09
Manganese	0.60
Phosphorus	0.50
Total carbon	3.45
Combined carbon	0.60

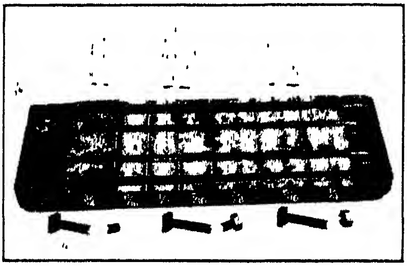


FIG. 2 TESTS TO ASCERTAIN COMPARATIVE STRENGTH OF ⅝-IN. T-SLOTS AND BOLTS
(Table of hard cast iron)

6 The terms hard and soft cast iron used in this paper will be understood to be in correspondence with these analyses. These tests were made at the Brown & Sharpe Works, and at the Engineering Laboratory at Brown University.

7 Each of these tables had two T-slots machined in it. In the tables for ¾-in. bolts, the throat of one slot was ¾ in. wide and that of the other was 13/16 in. wide. In the tables for ⅝-in. bolts the throat of one slot was ⅝ in. wide and of the other, 11/16 in. The surface of each of these tables was machined in steps so as to provide different depths of T-slot for the purpose of making tests to show the comparative strength of the bolt and the lip of the T-slot. Fig. 1 shows a table made of hard iron for ¾-in. T-bolts having a depth of throat varying from ⅜ in. to one inch, by eighths,

and illustrates the condition of bolt and slot after the test was carried to the point of destruction.

8 Fig. 2 shows a similar table of hard iron with slots for $\frac{5}{8}$-in. bolts, the throats being $\frac{5}{8}$ and $\frac{11}{16}$ in. wide respectively, and the depth of slot varying from $\frac{1}{4}$ to $\frac{3}{4}$ in., the variation being by $\frac{1}{16}$ of an inch up to and including $\frac{9}{16}$ in.

Tests of T-Bolts in Tables of Hard Iron

9 Tests were made on these tables by using case-hardened bolts

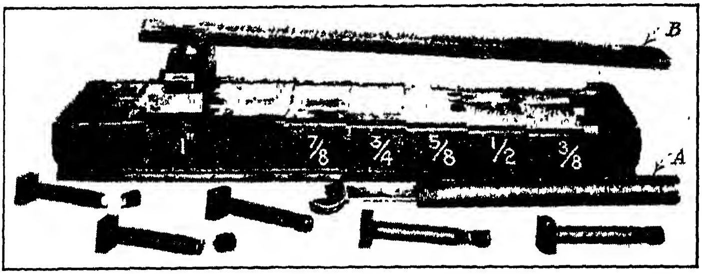


Fig. 3 Method of Making Tests

and nuts of the B. & S. standard,[1] the T-bolts being finished from forgings made of No. 2 bolt steel having an analysis as follows:

Carbon	0.10-0.15
Manganese	0.25-0.35
Phosphorus	0.04
Sulphur	0.06
Silicon	0.15

These tests were made with the surface above the T-slot where the strain was exerted unsupported, that is, there was no metal-to-metal clamping. The T-slot was therefore left in this respect in the weakest condition in which it might be used.

10 In order to secure sufficient leverage to carry out these tests an extension pipe was put on the handle of the standard wrench as shown at *A* in Fig. 3. This made it possible for two men to exert destructive force but the wrench itself would not stand this abuse and broke in service. A bar of steel (*B*), having hexagonal openings to fit the nuts, was then substituted. With this latter combination each test could be carried to the point of breakage either of the lip of the T-slot or of the bolt. In the case of the $\frac{3}{4}$-in. bolt with the throat $\frac{3}{8}$ in. deep, the metal of the lip of the T-slot was pulled out, the bolt remaining undamaged. (Shown at *A*, Fig. 1.) When the thickness of metal in the table

[1] Heads of $\frac{3}{4}$-in. bolts $1\frac{1}{4}$ in. across flats, $\frac{1}{2}$ in. thick; $\frac{5}{8}$-in. bolts $1\frac{1}{8}$-in. across flats, $\frac{3}{8}$ in. thick; diameter of bolts 0.024 in. less than nominal diameter for both sizes.

was increased to $\frac{1}{2}$ in., the lips were sprung before the bolt gave out but held sufficiently so that the bolt was stretched and would have broken before actually pulling out the metal of the table (*B*).

11 This, however, was on the border line, and represented approximately equal strength of bolt and T-slot. With the metal of $\frac{5}{8}$-in. thickness the bolt was noticeably the weaker element and stretched as shown at *C* while only slightly springing the metal of the T-slot, this latter returning to normal condition as soon as the strain was relieved. With a thickness greater than $\frac{5}{8}$ in. the bolt broke without damaging or even noticeably springing the T-slot. (See *D*.)

12 An effort was made to ascertain whether there was any difference between the strength of a slot $\frac{13}{16}$ in. and one $\frac{3}{4}$ in. wide. No noticeable difference could be observed although in theory the $\frac{3}{4}$-in. slot should have been slightly stronger. Neither did the fact that with a width of throat $\frac{13}{16}$ in. there would be less area of contact between the head and the under side of the lip of the slot develop any indication of denting or crushing the metal.

13 Similar tests were made with a $\frac{5}{8}$-in. bolt in slots having throats $\frac{5}{8}$ and $\frac{11}{16}$ in. wide. The results shown in Fig. 2 indicate that the T-slot was weaker than the bolt up to and including throats $\frac{3}{8}$ in. deep, but that with throats $\frac{7}{16}$ or $\frac{1}{2}$ in. deep, conditions were fairly balanced although favoring the strength of the T-slot in both cases, because the bolt broke before the table was sprung to a degree beyond which it would not recover. With the lips thicker than $\frac{1}{2}$ in. the bolt broke without affecting the T-slot.

14 Tests were made with studs threaded at both ends for the purpose of learning whether the stud would always break at the same end, either at the upper or lower thread, and it was found that breakage would be sometimes at one end and sometimes at the other, but always through the threaded portion.

15 From these facts it will be seen that in hard iron the thickness of lip of the $\frac{3}{4}$-in. T-slot can be a minimum of $\frac{9}{16}$ in., which is practically equal in strength to that of the bolt or stud, while for the $\frac{5}{8}$-in. slot the thickness of $\frac{7}{16}$ in. can be established as minimum.

16 A study was also made of the strength of the head of the bolt. In no case did the head spring or show weakness in other ways. The nuts used with these bolts were of cold-rolled steel, 0.10 carbon, $\frac{11}{16}$ and $\frac{9}{16}$ in. thick respectively. They showed no sign of damage or hard usage due to the tests, neither did their threads nor the threads of the bolt on the portion engaging the nuts show deformation.

17 The conclusion to be drawn is that there is sufficient width of shoulder and thickness to the heads of the bolts to distribute the pressure on the under side of the lip of the T-slot so as to avoid denting or otherwise damaging it up to the point of rupture

of the bolt; later tests show this to be so even when the bolt was made of steel of unusually high tensile strength. The further conclusion is drawn that, with the throat of the slot $\frac{1}{16}$ in. wider than the diameter of bolt, the surface of contact is not reduced to a point where it is objectionable; also that the heads are sufficiently thick and strong to withstand the greatest strain which can be put upon them before the bolt breaks through the threads.

Comparative Tests Using Hard and Soft Cast Iron for Tables

18 The tests made to ascertain the comparative strength of T-slots milled in soft and hard iron were made under the direction

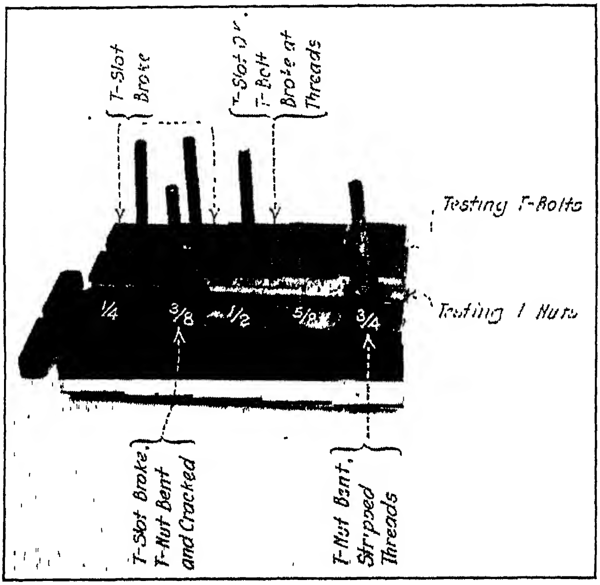


Fig. 4 Tests to Ascertain Strength of T-Slots in Soft Cast Iron and between T-Bolts and Studs with T-Nuts

of Prof. James A. Hall of Brown University, and were directed both to the question of the comparative strengths of soft and hard cast iron and to the comparative strengths of T-bolts and T-nuts with their coöperating studs. (See Figs. 4 and 5, Tables 1 and 2.)

19 The tests in hard iron showed practically the same results as for those already described. Thus for the $\frac{5}{8}$-in. T-slot, Table 2 shows that a throat $\frac{7}{16}$ in. deep provides lips stronger than the T-bolt which broke under a load of 16,600 lb. A noticeable difference between the slots in hard iron and soft iron was that the soft iron not only began to spring, but showed a permanent set after the removal of the strain, under a lighter load than in the case of the hard iron.

T-Nuts

20 When the ends of the T-slots are obstructed, or when it is desirable to insert additional holding devices after the work or attachment has been located, the heads of T-bolts are sometimes "slabbed off" so as to allow the bolt to be entered through the throat of the slot, Fig. 6. While this is a convenient method, it is not to be recommended, as it reduces the area of contact of the head with the lip of the slot to such an extent that it soon results in "chewing up" the under side of the lip.

21 For this purpose the use of T-nuts is preferable although here also there is a drawback in that the tapped hole through the T-nut weakens it. If the tapped hole and engaging stud are of the same size as the T-bolt for the corresponding size of slot, the nut is weakened to such an extent that "buckling" may result and the strain may be brought on a limited area of the lip of the slot. The

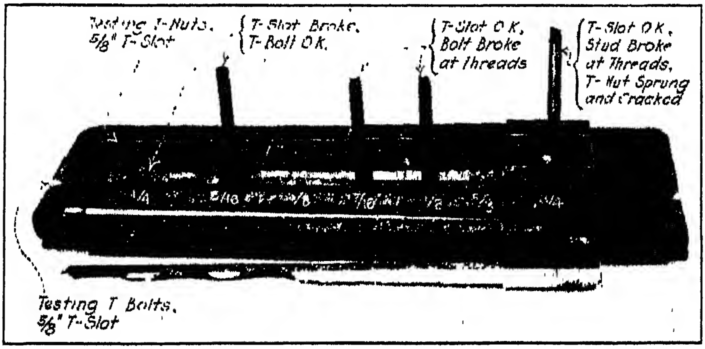


Fig. 5 Tests to Ascertain Strength of T-Slots in Hard Cast Iron and between T-Bolts and Studs with T-Nuts

T-nut may buckle to such an extent that it will fail by stripping the threads of either the nut, the stud, or both, long before the limit of strength of the stud is reached. Although, for the sake of uniformity, the T-nuts are listed as being tapped for studs of the same diameter as the corresponding bolt, it can be recommended that, where maximum strength is required, the T-nuts be tapped for smaller studs, thus giving ample strength to the nut but reducing the force which can be applied in clamping by an amount equal to the difference in strength between the two sizes of stud. This use of smaller studs can in a measure be compensated for by using studs having a higher tensile strength. It will be seen by referring to Table 2 that the $\frac{5}{8}$-in. stud in No. 8 withstood a greater load than the $\frac{3}{4}$-in. stud in No. 10 because of its greater tensile strength.

22 In most of the tests for T-nuts (Figs. 4 and 5, and Table 3), the stud, and consequently the tapped hole in the nut, were of the

TABLE 1 COMPARISON OF HARD AND SOFT CAST IRON WITH VARYING DEPTHS OF THROAT, USING T-BOLTS

No.	Type	Kind of cast iron	Bolt size, in.	Depth of throat, in.	Breaking load, lb.	Remarks
1	T-bolt	Soft	$\frac{5}{8}$	$\frac{1}{4}$	10,100	Lips of slot broke
2	T-bolt	Soft	$\frac{5}{8}$	$\frac{3}{8}$	14,300	Lips of slot broke
3	T-bolt	Soft	$\frac{5}{8}$	$\frac{1}{2}$	17,250	Bolt broke in threads
4	T-bolt	Hard	$\frac{5}{8}$	$\frac{5}{16}$	14,600	Lips of slot broke
5	T-bolt	Hard	$\frac{5}{8}$	$\frac{7}{16}$	16,600	Bolt broke in threads
6	T-bolt	Hard	$\frac{5}{8}$	$\frac{1}{2}$	17,300	Bolt broke in threads

TABLE 2 COMPARISON OF HARD AND SOFT CAST IRON WITH VARYING DEPTHS OF THROAT, USING T-NUTS AND STUDS

No.	Type	Kind of cast iron	Stud size, in.	Depth of throat, in.	Length of nut, in.	Breaking load, lb.	Remarks
7	T-nut and stud	Soft	$\frac{5}{8}$	$\frac{3}{8}$	$1\frac{11}{16}$	14,500	Both slot and nut failed
8	T-nut and stud	Soft	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{11}{16}$	30,250	Stripped threads of both stud and nut after "buckling" nut
9	T-nut and stud	Hard	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{11}{16}$	19,400	Bolt broke in threads
10	T-nut and stud	Hard	$\frac{3}{4}$	$\frac{1}{2}$	2	28,400	Bolt broke in threads

same nominal size as the bolt for the corresponding slot. They showed that the $\frac{5}{8}$-in. nuts failed by buckling through the weakened portion tapped for the stud and that this resulted in stripping the threads before breaking the stud. The condition of the nuts indicated that there was little if any added strength obtained from the upwardly projecting portion of the nut because this portion fractured as soon as a slight bending strain came upon it. Neither did the greater length of the nut, $1\frac{11}{16}$ in., provide a correspondingly increased length of bearing on the under lip of the T-slot, for, when a strain was applied and before it had reached a point approaching the strength of the stud, the buckling of the nut brought most of the pressure at the center, thus tending to spring the lip of the T-slot under a much less strain than if the load were distributed over the full length of the nut, or as compared with the use of a T-head bolt. This weakness was not so much in evidence for the $\frac{3}{4}$-in. nut in which the stud broke before noticeably bending or otherwise damaging the nut, although, if the tensile strength of the $\frac{3}{4}$-in. stud had been

in proportion to that of the $\frac{5}{8}$-in. size, it is probable that the nut would have given out first.[1]

23 Comparing the strength of the T-head with that of the T-nut, Professor Hall's tests Nos. 2 and 7 show that even though the nut is thicker and longer than the bolt head it is not stronger, if as strong, the limit of the strength of the T-head not having been reached in any of the tests because of the prior failure of the bolt, while the T-nut failed by stripping the thread after buckling the nut; and even in cases where the stud broke first the T-nut was deformed and its threads damaged before the stud broke.

24 The T-nuts and studs were provided for these tests by another manufacturer, and an analysis of their composition is not at hand; but in the case of one of the $\frac{5}{8}$-in. studs its high tensile

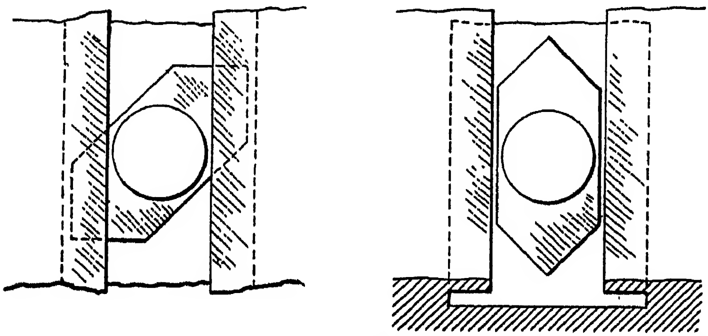


Fig. 6 Bolt Head "Slabbed Off" So that It Can be Withdrawn through Throat

strength gave an opportunity to make a severe test on the lips of the $\frac{5}{8}$-in. T-slots with throats $\frac{3}{4}$ in. deep and showed that, even with a 30,250-lb. strain concentrated in the line of the stud center, the slot was not damaged because of the buckling of the nut.

25 In order to examine the condition of the slots after the tests had been made, sections of the tables were removed as shown in Figs. 5 and 6 so that the permanent springing of the metal could be measured and a microscopic examination of the surfaces which had been under pressure could be made. Professor Hall comments on these tests, made under his direction, as follows:

The area of bolt head taking the thrust is about 0.48 sq. in. in the case of the T-bolt $\frac{5}{8}$-in. size and 0.78 sq. in. for the T-nut. At first thought it might seem that the latter would give a higher breaking load for a given size of slot, but this is not appreciably so as shown by a comparison of tests No. 2 and No. 7.

An analysis of the distribution of the forces on the T-nut shows that the bending moment at the center must be large if any considerable pressure is to be carried by the outer ends of the nut. As the

[1] The thickness of the tongue portion of the $\frac{5}{8}$-in. nut was $\frac{13}{32}$ in. and of the $\frac{3}{4}$-in. nut, $\frac{9}{16}$ in.

TABLE 3 SHOWING APPROXIMATION OF SIZES TO GEOMETRICAL PROGRESSION AND MAXIMUM AND MINIMUM CLEARANCES FOR HEAD

STANDARD T-BOLTS AND SLOTS

Bolt or Stud						Bolt Head						T-Slot																	
Diameter		Area		Threads per inch		Across flats		Area of contact		Thickness		Width of throat		Depth of throat				Width of head space						Depth of head space					
																			Clearance			Clearance			Clearance			Clearance	
Fraction	Geometrical progression	Sq. inch.	Geometrical progression	Number	Geometrical progression	Basic	Geometrical progression	Sq. inch.	Geometrical progression	Basic	Geometrical progression	Basic	Geometrical progression	Maximum	Geometrical progression	Minimum	Geometrical progression	Maximum	Max.	Min.	Minimum	Max.	Min.	Maximum	Max.	Min.	Minimum	Max.	Min.
1/4	0.25	0.049	0.049	20	20	15/32	0.47	0.088	0.09	5/32	0.16	9/32	0.28	3/8	0.38	1/8	0.13	9/16	1/8	3/32	1/2	1/16	1/32	15/64	1/32	5/64	13/64	1/16	3/64
5/16	0.32	0.077	0.077	18	17.3	9/16	0.58	0.122	0.13	3/16	0.20	11/32	0.35	7/16	0.46	5/32	0.17	21/32	1/8	3/32	19/32	1/16	1/32	17/64	3/32	5/64	15/64	1/16	3/64
3/8	0.39	0.110	0.120	16	14.9	11/16	0.72	0.172	0.20	1/4	0.26	7/16	0.43	9/16	0.56	7/32	0.23	25/32	1/8	3/32	23/32	1/16	1/32	21/64	3/32	5/64	19/64	1/16	3/64
1/2	0.49	0.196	0.190	13	12.7	7/8	0.88	0.350	0.30	5/16	0.34	9/16	0.54	11/16	0.69	5/16	0.30	31/32	1/8	3/32	29/32	1/16	1/32	25/64	3/32	5/64	23/64	1/16	3/64
5/8	0.62	0.307	0.295	11	10.9	1 1/8	1.08	0.493	0.45	13/32	0.43	11/16	0.66	7/8	0.85	7/16	0.40	1 1/4	5/32	1/8	1 3/16	3/32	1/16	31/64	3/32	5/64	29/64	1/16	3/64
3/4	0.77	0.442	0.460	10	9.4	1 5/16	1.33	0.660	0.64	17/32	0.56	13/16	0.82	1 1/16	1.04	9/16	0.52	1 15/32	3/16	5/32	1 3/8	3/32	1/16	5/8	1/8	3/32	19/32	3/32	1/16
1	0.96	0.785	0.730	8	8.1	1 11/16	1.65	1.06	1.01	11/16	0.72	1 1/16	1.04	1 1/4	1.28	3/4	0.70	1 27/32	3/16	5/32	1 3/4	3/32	1/16	53/64	11/64	9/64	23/32	1/8	1/32
1 1/4	1.20	1.14	1.55	7	7	2 1/16	2.05	1.55	1.54	15/16	0.92	1 5/16	1.28	1 9/16	1.59	1	0.94	2 7/32	3/16	5/32	2 1/8	3/32	1/16	1 9/32	3/16	9/64	1 1/32	1/8	3/32
1 1/2	1.50	2.33	2.33	6	6	2 1/2	2.50	2.33	2.33	1 3/16	1.19	1 9/16	1.56	1 15/16	1.93	1 1/4	1.25	2 21/32	3/16	5/32	2 9/16	3/32	1/16	1 11/32	3/16	9/64	1 3/32	1/4	3/32

cross-section of the T-nut at the center is greatly reduced due to the hole for the stud, the stresses due to bending at this point are apt to be very large. This was calculated, the pressure being assumed to vary uniformly from zero at the ends of the nut to a maximum at the center, and the result showed that the elastic limit of the steel of the nut would be reached long before a normal pressure can be carried by the outer ends of the nut in practice, and that practically all of the thrust is taken at the center. It is only after the slot has begun to break that the ends are under any considerable pressure. For this reason it is the opinion of the writer that these T-nuts would be stronger and more satisfactory if they were shorter.

In all cases where the pressure was not sufficient to crack or break the slot and where the bolts, therefore, broke at the root of the threads, there was no appreciable crushing of the cast iron under the bolt heads, indicating that the crushing strength of the cast iron was not reached.

Progress Report and Questionnaire

26 A progress report and questionnaire prepared under direction of the committee were sent out to a selected list of manufacturers of machinery using T-slots and to users of such machinery. This list was greatly extended through the coöperation of the National Machine Tool Builders' Association. Forty-five replies were received. The questions were:

1 As to the sizes of T-bolts and slots to be made standard

2 and 3 As to the size of head of T-bolt and the clearance to be allowed

4 and 5 As to whether the width of throat should be (*a*) equal to the nominal diameter of bolt, (*b*) wider than the nominal diameter of bolt, and if so how much, and (*c*) whether optional, either (*a*) or (*b*), so as to provide for extensive present practice, at least during a transition period.

27 In the answers some gave their present practice and others the practice which they recommended even though it might not accord with what they were using. The replies can be classified under the following heads:

1 Recommended sizes to be standardized
2 Proportions of head and head space
3 Width of throat
4 Depth of throat.

Recommended Sizes to be Standardized

28 While there was a general acceptance of the sizes proposed by the committee, a number of those replying believed that some of these sizes could be eliminated by leaving out the smaller sizes and beginning the series with $\frac{3}{8}$ to $\frac{1}{2}$ in. Others suggested leaving out some of the intermediate sizes, such as $\frac{5}{16}$, $\frac{7}{16}$, and $\frac{7}{8}$ in. The geometrical sizes to which the proposed standard conforms

accords closely with some of these suggestions. The sale of T-slot cutters has indicated that there is a wide-spread demand for the smaller sizes so they have been included.

Proportions of Head and Head Space

29 While the majority of replies showed that the proportions based on the use of commercial T-slot cutters were generally acceptable, those who did take exceptions asked for greater thickness of head and for more clearance space, especially at the bottom of the slot, so that the bolt head would slide freely even when the slot was obstructed by oil and chips, and so as to provide better for the use of T-nuts. An additional reason given was that the size of slot should be made to receive the heads of commercial bolts such as can be obtained in the market.

30 Conceding that more clearance is desirable, one of two methods for securing this seemed possible of adoption, either to make the heads smaller, continuing to use the commercial T-slot cutters, or to increase the size of slot, thus requiring new cutters. The latter plan was adopted and the size of slot has been increased so as to allow for somewhat thicker heads, especially for the smaller sizes of bolts. This has the objection that some heads of the new proportions cannot be used in present T-slots. Many manufacturers may for this reason choose to continue to make the heads of their T-bolts thinner than standard using material of sufficiently high tensile strength to insure the desired strength, so that the bolt heads can be used in either the shallower or deeper head space.

Width of Throat

31 It is generally conceded that this is the most important dimension affected by standardization and also the one which it is most difficult to standardize because of the importance of interchangeability with past product. An analysis of the replies to the questionnaire shows the following:

13 for making the throat width the same as the nominal diameter of bolts

32 using and presumably recommending a wider throat

These latter, to which may be added 4 who express willingness to change, can be grouped thus:

a Those favoring having the throat $\frac{1}{16}$ in. wider than the diameter of bolt, of whom there were 23

b Those favoring having the small sizes (usually $\frac{1}{4}$-in. and $\frac{5}{16}$-in.) $\frac{1}{32}$ in. wider and all other sizes $\frac{1}{16}$ in. wider, of whom there were 9

c Those favoring having the larger sizes $\frac{1}{8}$ in. or more wider, of whom there were 4.

32 The total number of replies came from 34 machine-tool builders, 4 machine users, 3 technical societies, and 4 professors and mechanical engineers. The thirteen who were milling-machine manufacturers divide into 7 using throat widths equal to the nominal diameter of bolt, and 6 using wider throat widths, or, when including 2 of the first group who expressed themselves as willing to change, 5 for a narrower and 8 for a wider throat.

Depth of Throat

33 The depth of throat is the least important dimension to standardize. Where the use is for strapping down work or any other purpose where the strain comes on the lips of the slot unsupported, a greater depth is required than in such cases as for clamping down a vise or for clamping "dog-bolts," where the parts are clamped together, metal to metal. In cases where T-cutters are used it is desirable to specify a maximum depth as the length of the neck of the cutter determines the depth which can be milled. Specifying a minimum depth provides a safe-guard against venturing on "too thin ice."

Comparison with Foreign Standards

34 In comparing the proposed American standard with T-slot standardization abroad, the practice in Great Britain and Germany can be taken as typical.

British Practice

35 In Great Britain, the standard for T-slot cutters published by the British Engineering Standards Association, July 1920, lists sizes from $\frac{1}{4}$ to $1\frac{1}{2}$ in. and provides for two widths of throat, these being in accord with the widths proposed for the American standard when including the supplementary width recommended on account of present extensive use and having the throat width of the same nominal width as the diameter of bolt or stud. In this respect, therefore, the product of the two countries would be interchangeable for each width, and, with provision for reversible tongues, fully interchangeable. The British standard provides for slightly deeper head space and greater maximum depth of throat than the American standard.

36 Relative to the use in Great Britain of a width of throat greater than the diameter of bolt, information was received from the British Engineering Standards Association under date of July 15, 1924, to the effect that in British practice the slots are usually made of the greater width, and the further statement is made that the T-slots were standardized on the assumption that T-headed bolts would be used. This appears to be the general

practice in that country, although it is stated that there is nothing to prevent T-nuts being used if desired.

37 Broadly speaking, the fixtures and tools adapted to be used on machines of British make and fitted to British standard T-slots can be used on American machines made to the proposed American standard, without change. This includes the use of holding bolts or holding studs and T-nuts. The reverse of this is also true, that fixtures and tools adapted to be used on machines of American make and having standard T-slots can be used without change on machines of British make

38 In some cases in either instance reversible tongues might

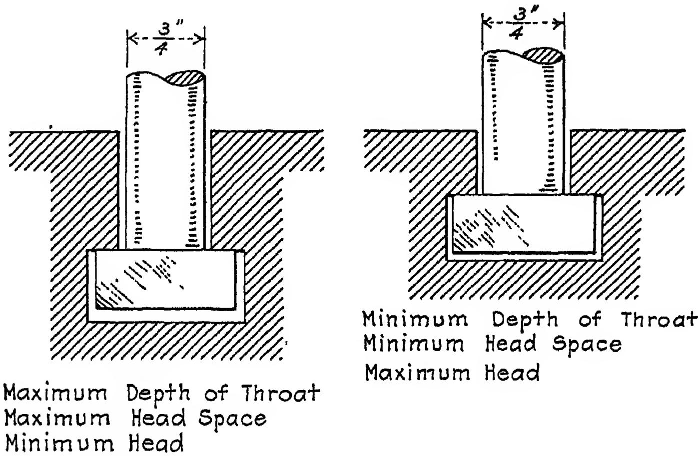


Fig. 7 Maximum and Minimum Depth of Throat and Clearances, ¾-In. T-Bolt

be required and perhaps spacing washers to provide for slight variations in the length of bolts.

German Practice

39 German standardization, as shown in $\frac{\text{DIN}}{650}$ based on the metric system, provides for a lesser depth of head space than the American standard, except for the smallest sizes, thus bringing the American standard between the German and British for the sizes most generally used.

40 In adapting to machines having metric T-slots as in the German practice, reversible tongues would be required. In the case of the bolts or studs, however, as in German practice bolts made to the inch system, Whitworth standard are commonly used. Such bolts or studs could be used interchangeably with both British and American standard T-slots.

Determining a Standard

41 Having ascertained that proportions such as indicated by the tests here described were in accord with average American practice and after comparison with the established practices of other countries, it was decided by the committee to limit the sizes standardized to those listed in Table 3 which provides a series approaching very closely to a geometrical progression of sizes. After establishing the various dimensions for the smallest and largest sizes on the basis of the above findings, the intermediate sizes were graded in approximate geometrical progression, as shown in Table 3.

42 The general demand for a greater amount of clearance space, for more room for chips and oil, and for sufficiently large slots to use commercial bolts and have their heads enter the slots, constituted a sufficient reason in the minds of the committee for departing from the established standard of T-slot cutters. This was done with more willingness because the simplified schedule of stock cutters, adopted in coöperation with the Department of Commerce, no longer lists these cutters as standard. It is believed that the proposed sizes should be obtained as readily, and at the same prices, as those formerly listed.

43 Fig. 7 illustrates the relation of the $\frac{3}{4}$-in. bolt to its T-slot by showing both the maximum and minimum clearances, also the extremes in depth of throat, and can be taken as typical of the entire series of sizes.

44 As this paper is intended to supplement the report of the T-slot committee reference is therefore made to that report for such matters as are not here fully discussed.

DISCUSSION

B. P. Graves.[1] The writer would ask Mr. Burlingame if it has been definitely decided that at such times as we can change our standards, we should make the width of the slot wider than the bolt, or should we continue with our present standard, which makes the slot nominally the same size as the bolt?

The Author. The committee has suggested that as a standard the wider throat be adopted, although during a transition stage it is recommended that the narrower throat be recognized as a supplementary standard; in other words, anyone can use the narrower throat and still be standard; but with the hope that eventually there will be uniformity in the use of the wider throat.

[1] Milling Machine Engineer, Designer, Brown & Sharpe Mfg. Co., Providence, R. I. Mem. A.S.M.E.

No. 2003

THE DEVELOPMENT OF TAP-DRILL SIZES

By A. C. Danekind,[1] Schenectady, N. Y.
Associate Member of the Society

Charts for tap-drill sizes as a rule are made up from the opinions of operators, without basic knowledge of the factors governing the selection of the proper tap drill. Different groups of operators in the same plant will develop different charts for the same set of conditions. The author has studied various existing charts and has devised one that takes into consideration the character of material being tapped, the bearing of thread desired, the thickness of the tapped section, and other essential factors.

THE proper sizes for tap drills have always been the subject of much discussion. Charts can be obtained from practically any manufacturer of taps or drills which are usually satisfactory in the majority of sizes. However, a careful study of available tap-drill charts revealed the fact that there was no suitable chart for all commonly used materials.

2 A correct tap-drill chart is very essential to good manufacturing. Too often the choice is left to machine operators who have memorized through continual use sizes satisfactory to them. However, if a canvass were made of the recommendations of different operators, much difference of opinion would be revealed. This fact has been proved in the Schenectady works of the General Electric Company, where the practices of several groups were found to be widely different.

3 In the General Electric Company's plant conditions were found exactly as they are in the general run of manufacturing concerns. Tap-drill charts were conspicuously placed about most of the tool rooms as guides for the men in selecting the proper tap drill.

4 Inquiries were made of the various plants pertaining to the use of tap-drill charts. Each plant reported the use of a chart of its own creation. Many times no two charts would agree on the

[1] General Electric Company.

Contributed by the Machine-Shop Practice Division and presented at the Providence Meeting of The American Society of Mechanical Engineers, Providence, R. I., May 3 to 6, 1926.

proper drill for a particular tap. In some cases the sizes listed represented over 100 per cent of thread, while in other cases the fullness was nearer 50 per cent. One chart called for different-sized drills for right-hand and left-hand taps of the same diameter and pitch. These charts were undoubtedly developed from infor-

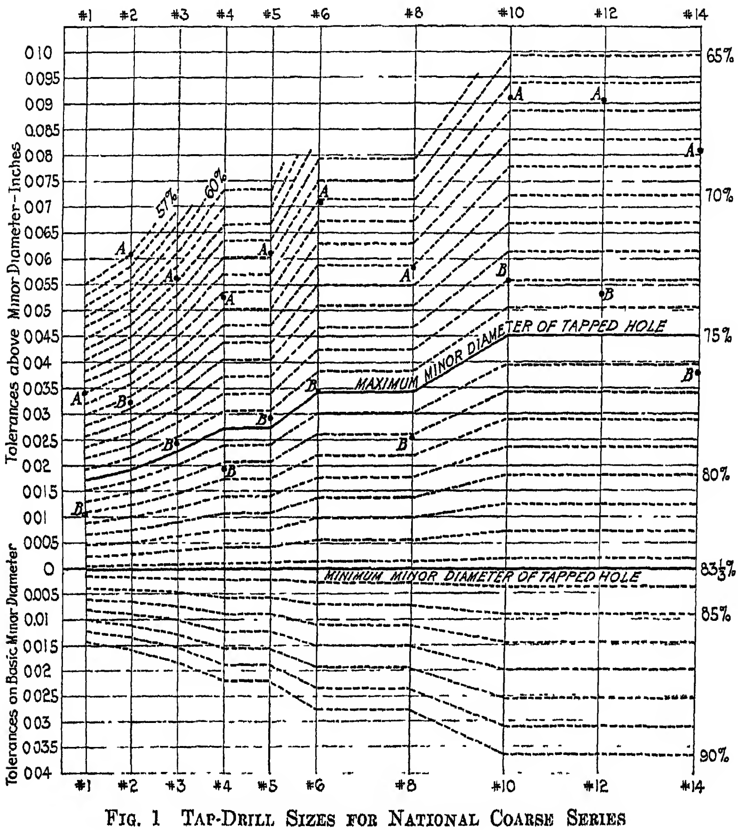


FIG. 1 TAP-DRILL SIZES FOR NATIONAL COARSE SERIES FOR MACHINE SCREWS

(Copper-Aluminum-Steel)

Size	1-64	2-56	3-48	4-40	5-40	6-32	8-32	10-24	12 24	14-24
Drill	53	49	45	43	37	35	29	23	15	5

This is a comparative graph showing relation between theoretical tap-drill sizes and actual results obtained due to "spinning up" effect on drawn sheet copper, aluminum, and steel.

"A" shows theoretical location of various tap-drill sizes. "B" shows actual location due to spinning-up effect.

Percentages refer to the percentage of full depth of thread in tapped hole.

The "minimum minor diameter of tapped hole" is represented by the line 0 or 83⅓ per cent of full depth of thread.

The "maximum minor diameter of tapped hole" is represented by line showing 75 per cent of full depth of thread, and is obtained by adding the tolerance recommended by the American Screw Thread Commission to the minimum minor diameter.

The tolerance below the minimum minor diameter shows the difference between the minimum minor and basic minor diameters of tapped holes. Basic minor diameter would indicate 100 per cent of full thread.

mation collected from handbooks or other sources, and based on mathematical calculations that did not take into consideration the spinning-up effect. The author believes it safe to assume that many manufacturing plants today are in much the same condition.

5 Taps were frequently broken off in work in process, which necessitated much extra labor and inconvenience, and investigation showed that the lack of a more nearly correct tap-drill chart

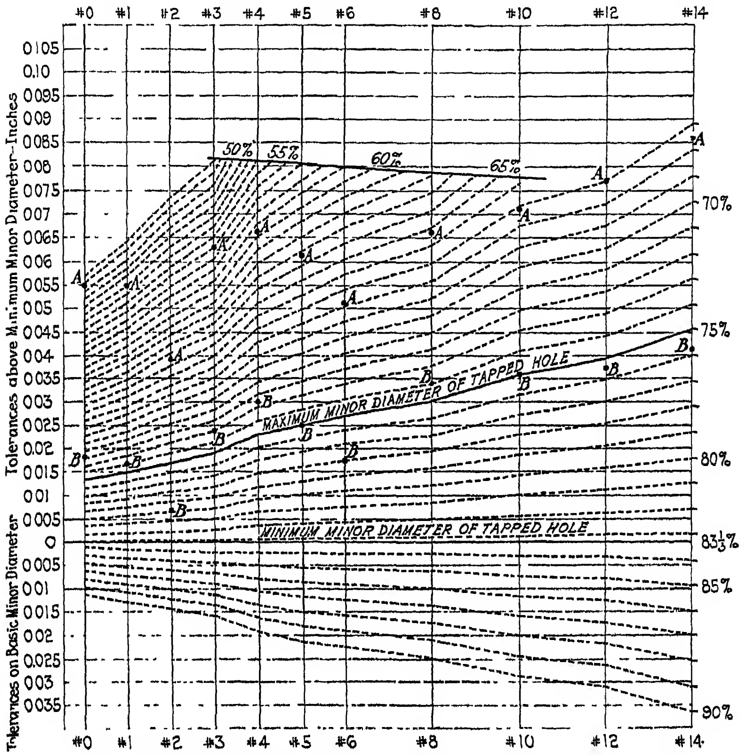


Fig. 2 Tap-Drill Sizes for G. E. Standard Machine Screws

(Copper-Aluminum-Steel)

Size	0-80	1-72	2-64	3-56	4-48	5-44	6-40	8-36	10-30	12-28	14-24
Drill	55	52	49	44	41	36	32	28	20	13	5

caused a great number of breakages through the specification of undersized tap drills. This was especially true of machine-screw sizes.

6 A study of the various charts now in existence bears out the fact that the sizes listed are generally too small for production purposes, especially those up to ⅜ in. This, the author assumes, is due to the fact that the sizes listed are calculated in most cases to produce a thread of approximately 75 per cent, but without consideration of the widely different and peculiar physical char-

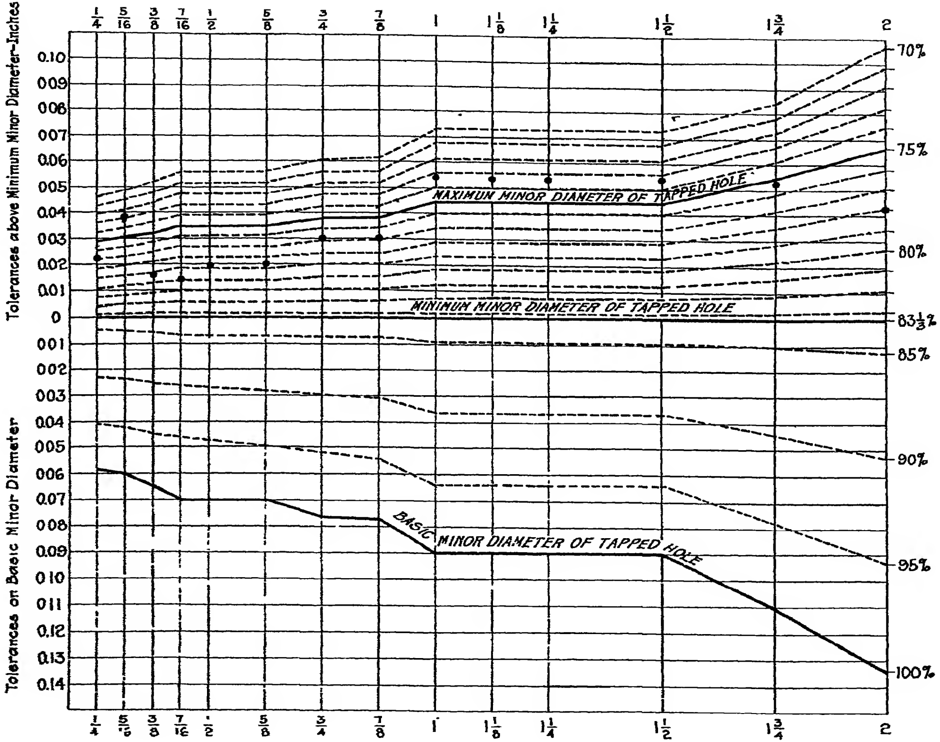


FIG. 3 TAP-DRILL SIZES FOR VERY FINE SERIES

(Copper-Aluminum-Steel)

Size	Drill
$1/4 \times 42$	No. 1 (0.228)
$5/16 \times 40$	L (0.290)
$3/8 \times 36$	S (0.348)
$7/16 \times 32$	$13/32$
$1/2 \times 32$	$15/32$
$9/16 \times 32$	$17/32$
$5/8 \times 32$	$19/32$
$3/4 \times 30$	$23/32$
$7/8 \times 30$	$27/32$
1×24	$31/32$
$1\,1/4 \times 24$	$1\,7/32$
$1\,1/2 \times 24$	$1\,15/32$
$1\,3/4 \times 20$	$1\,45/64$
2×16	$1\,15/16$

This is a graph showing actual locations from tap-drill sizes after the spinning-up effect has been taken into consideration; for drawn or rolled copper, steel, and aluminum

Percentages refer to the percentage of full depth of thread in tapped hole.

"The minimum minor diameter of tapped hole" is represented by line 0 or 83⅓ per cent of full depth of thread.

The "maximum diameter of tapped hole" is represented by line showing 75 per cent of full depth of thread, and is obtained by adding the tolerance recommended by the American Screw Thread Commission to the minimum minor diameter.

The tolerance below minimum minor diameter shows the difference between the minimum minor and basic minor diameters of tapped hole. Basic minor diameter would indicate 100 per cent of full thread.

acteristics of the materials to be tapped. In some cases cast iron is classed with steel, wrought iron, and copper, and while it is an accepted fact that the latter three materials have a tendency to spin up when tapped, such is not the case when a sand-cast material is considered.

7 An analysis of the recommendations of the American Screw Thread Commission reveals the fact that a tap drill falling anywhere between the minimum minor and maximum minor nut diameters would probably be satisfactory; in other words, between 75 per cent and 83⅓ per cent, with a mean size of about 79 per cent. This in spite of the fact that most concerns today favor the use of 75 per cent. The author disagrees with this recommendation of the Commission. On all sizes larger than ⅜ in., 75 per cent of thread is ample for all general purposes, provided of course that a good flank bearing is obtained between mating parts. This means threading tools free from excessive lead errors. Practically no additional strength is obtained from an increase of several percentage points over 75 per cent, and lead errors are much more troublesome to deal with due to this fullness of thread.

8 To be assured of uniformity in tapped holes, it is of course necessary to have uniform threading tools. With the use of these tools, however, there must exist a specific guide for tap-drill sizes. Any tap-drill chart can be offered only as a guide. This is due to the fact that tap-drill sizes must vary with the thickness of materials to be tapped.

9 The only practical way of determining the proper sizes for tap drills for any commonly used material is by actual physical research. Here particular attention can be devoted to the elimination of any inaccuracies which may appear in production tapping. Very carefully tested taps can be used, and drilled holes carefully checked before being tapped. In this way results can be obtained which can be used as a specific guide in the development of a practical tap-drill chart.

10 Wherever metal thinner than three-fourths of the tap diameter or thicker than 1½ diameters is used, a deviation must be made from the recommended chart. As an example, in thin hexagon nuts practically a full thread is necessary for a fit because of the short length of engagement, while for work of more than 1½ diameters, less than 75 per cent of thread very often is ample, due to the longer length of engagement. When commercial unground taps are used, lead errors are usually present, and a percentage of thread above 75 per cent is extremely hard to obtain and still maintain a flank bearing.

11 Considering the depth of the hole to be tapped as falling within the general classifications of from ¾ to 1½ diameters, the chart offered is correct where approximately 75 per cent of thread is desired.

MACHINE SCREW A.S.M.E. STANDARD															
		SHEET ~ ROD				CASTINGS							INSULATING	
SIZE	THDS PER IN	COPPER	ALUMINUM	STEEL	BRASS	COPPER	ALUMINUM	STEEL	BRASS	CAST IRON	MAL IRON	DIE CAST	FIBER	HARD RUBBER
0	80	55	55	55	55	55	55	55	55	55	55	55	55	55
1	72	52	52	52	53	52	52	52	53	53	53	52	53	53
2	64	49	49	49	50	49	49	49	49	49	49	49	50	50
3	56	44	44	44	45	44	44	44	44	45	45	44	45	45
4	48	41	41	41	42	41	41	41	42	42	42	41	42	42
5	44	36	36	36	37	36	36	36	36	37	37	36	37	37
6	40	32	32	31	32	32	32	32	32	32	32	32	33	33
8	36	27	27	27	28	28	28	28	28	28	28	28	29	29
10	30	20	20	20	21	20	20	20	21	21	21	20	21	21
12	28	13	13	13	14	13	13	13	13	14	14	13	14	14
14	24	5	5	5	6	5	5	5	6	6	6	5	6	6

U.S. STANDARD			
SIZE	THDS PER IN	COPPER STEEL	BRASS CAST IRON FIBER
5/16	18	17/64	F
3/8	16	O	O
7/16	14	U	U
1/2	13	27/64	27/64
9/16	12	31/64	31/64
5/8	11	17/32	17/32
3/4	10	21/32	21/32
7/8	9	49/64	49/64
1	8	7/8	7/8
1 1/8	7	63/64	63/64
1 1/4	7	1 7/64	1 7/64
1 3/8	6	1 7/32	1 7/32
1 1/2	6	1 11/32	1 11/32
1 5/8	5 1/2	1 29/64	1 29/64
1 3/4	5	1 9/16	1 9/16
1 7/8	5	1 11/16	1 11/16
2	4 1/2	1 25/32	1 25/32
2 1/4	4 1/2	2 1/32	2 1/32
2 1/2	4	2 17/64	2 1/4
2 3/4	4	2 33/64	2 1/2
3	3 1/2	2 23/32	2 23/32
3 1/4	3 1/2	2 31/32	2 31/32
3 1/2	3 1/4	3 13/64	3 13/64
3 3/4	3	3 27/64	3 27/64
4	3	3 43/64	3 43/64
4 1/4	2 7/8	3 29/32	3 29/32
4 1/2	2 3/4	4 9/64	4 9/64
4 3/4	2 5/8	4 3/8	4 3/8
5	2 1/2	4 39/64	4 39/64

NUMBER DRILLS					
1	228	29	.136	57	.043
2	.221	30	1285	58	.042
3	213	31	.120	59	.041
4	209	32	116	60	.040
5	2055	33	113	61	039
6	204	34	111	62	.038
7	201	35	110	63	.037
8	199	36	.1065	64	.036
9	.196	37	.104	65	.035
10	.1935	38	1015	66	.033
11	191	39	0995	67	.032
12	189	40	098	68	.031
13	185	41	.096	69	.02925
14	182	42	.0935	70	.028
15	180	43	.089	71	.026
16	177	44	.086	72	.025
17	.173	45	.082	73	.024
18	1695	46	.081	74	0225
19	.166	47	.0785	75	021
20	.161	48	.076	76	020
21	159	49	.073	77	.018
22	.157	50	.070	78	.016
23	.154	51	.067	79	0145
24	.152	52	.0635	80	.0135
25	1495	53	.0595		
26	.147	54	.055		
27	144	55	052		
28	1405	56	.0465		

LETTER DRILLS	
A	0 234
B	0.238
C	0.242
D	0.246
E	0.250
F	0.257
G	0.261
H	0.266
I	0.272
J	0 277
K	0.281
L	0 280
M	0.295
N	0.302
O	0.316
P	0 323
Q	0 332
R	0 339
S	0.348
T	0.358
U	0.368
V	0.377
W	0 386
X	0.397
Y	0.404
Z	0.413

FIG. 4 TAP DRILLS AS ESTABLISHED BY PHYSICAL TESTS — SAMPLE CHART (SHOWN GRAPHICALLY IN FIGS. 2 AND 3)

(Continued on following page)

G. E. FINE SERIES			
SIZE	THDS PER IN	COPPER STEEL	BRASS CAST IRON FIBER
1/4	28	#2	#3
5/16	24	I	I
3/8	24	R	Q
7/16	20	25/64	25/64
1/2	20	29/64	29/64
9/16	18	33/64	1/2
5/8	18	37/64	9/16
3/4	16	11/16	11/16
7/8	14	13/16	51/64
1	14	15/16	59/64
1 1/8	12	1 3/64	1 3/64
1 1/4	12	1 11/64	1 11/64
1 3/8	12	1 19/64	1 19/64
1 1/2	12	1 27/64	1 27/64
1 5/8	10	1 17/32	1 17/32
1 3/4	8	1 5/8	1 5/8
1 7/8	8	1 3/4	1 3/4
2	8	1 7/8	1 7/8
2 1/4	8	2 1/8	2 1/8
2 1/2	8	2 3/8	2 3/8
2 3/4	8	2 5/8	2 5/8
3	8	2 7/8	2 7/8

G.E. VERY FINE SERIES			
SIZE	THDS PER IN	COPPER STEEL	BRASS CAST IRON FIBER
1/4	42	#1	#1
5/16	40	L	L
3/8	36	S	S
7/16	32	13/32	13/32
1/2	32	15/32	15/32
9/16	32	17/32	17/32
5/8	32	19/32	19/32
3/4	30	23/32	23/32
7/8	30	27/32	27/32
1	24	31/32	61/64
1 1/8	24	1 3/32	1 5/64
1 1/4	24	1 7/32	1 13/64
1 3/8	24	1 11/32	1 21/64
1 1/2	24	1 15/32	1 29/64
1 5/8	20	1 37/64	1 37/64
1 3/4	20	1 45/64	1 45/64
1 7/8	18	1 53/64	1 53/64
2	16	1 15/16	1 15/16
2 1/4	16	2 3/16	2 3/16
2 1/2	16	2 7/16	2 7/16
2 3/4	16	2 11/16	2 11/16
3	16	2 15/16	2 15/16

RAILWAY SERIES			
SIZE	THDS PER IN	COPPER STEEL	BRASS CAST IRON FIBER
1 1/8	8	1	1
1 1/4	8	1 1/8	1 1/8
1 3/8	8	1 1/4	1 1/4
1 1/2	8	1 3/8	1 3/8
1 5/8	8	1 1/2	1 1/2
1 3/4	8	1 5/8	1 5/8
1 7/8	8	1 3/4	1 3/4
2	8	1 7/8	1 7/8
2 1/8	8	2	2
2 1/4	8	2 1/8	2 1/8
2 3/8	8	2 1/4	2 1/4
2 1/2	8	2 3/8	2 3/8
2 5/8	8	2 1/2	2 1/2
2 3/4	8	2 5/8	2 5/8
2 7/8	8	2 3/4	2 3/4
3	8	2 7/8	2 7/8

FIG. 4 (Continued)

12 Drills over $\frac{1}{2}$ in. in diameter are standardized into sizes which increase in steps of $\frac{1}{64}$ in. In cases where two drill sizes fall equidistant from 75 per cent, the size of the tap must be considered before a selection can be made. If the size in question is under 1 in. in diameter, it seems logical to recommend the use of the fuller thread, but where 70 to 72 per cent of thread is used on 1-in. or larger sizes, ample thread is obtained for most general-production work.

13 When dealing with machine-screw sizes, a closer choice of drill can be taken advantage of. Through the use of number-size drills a selection can be made which will produce practically any desired fullness of thread. However, unless the "spinning-up"

		Sheets & Rods				Castings						Insulation	
Size	Basic O.D.	Copper	Aluminum	Steel	Brass	Copper	Aluminum	Brass	Cast Iron	Mal Iron	Die Casting	Fibre	Hard Rubber
#1-64	.073	53	53	53	53	53	53	53	53	53	53	53	53
2-56	.086	49	49	49	50	49	49	50	50	50	49	50	50
3-48	.099	45	45	45	46	45	45	46	46	46	45	46	46
4-40	.112	43	43	43	43	43	43	43	43	43	43	43	43
5-40	.125	37	37	37	38	37	37	37	38	38	37	38	38
6-32	.138	35	35	35	35	35	35	35	35	35	35	35	35
8-32	.164	29	29	29	29	29	29	29	29	29	29	29	29
10-24	.190	23	24	23	24	24	24	24	25	25	23	25	25
12-24	.216	15	15	15	16	15	15	16	16	16	15	16	16
14-24	.242	5	5	5	6	5	5	6	6	6	5	6	6

FIG. 5 TAP-DRILL SIZES FOR NATIONAL COARSE SERIES FOR MACHINE SCREWS AS ESTABLISHED BY PHYSICAL TESTS—SAMPLE CHART (SHOWN GRAPHICALLY IN FIG. 1)

effect is taken into consideration, a difference of but one drill size will affect the subsequent fullness of thread by several percentage points. This, of course, is due to the shallow depth of thread of machine screws. As an illustration, a 40-pitch screw has a double depth of thread of 0.0324 in. It can readily be seen that a difference of one drill size—about 0.003 in.—will make a difference of approximately 10 per cent in the fullness of the thread. Adding to this the fact that most materials have a tendency to spin up when tapped, it is very necessary that all influencing factors be known before a tap drill is selected. Figs. 1 and 2 illustrate this point.

14 An effort was made to determine the exact amount of spin-up with an idea of devising a formula that could be used for any size. This effort was not entirely successful, due probably to the difference in the nature of the materials tapped. However, many

data were obtained, so that it was possible to determine with sufficient accuracy the effect of the spin-up of commonly used metals on a thread of any percentage of fullness.

15 The tapping was accomplished on an upright drill. A floating tap holder was used in all tests, and the taps were carefully checked for accuracy before they were used. All drilled holes were checked for size before being tapped. The materials selected were representative of the particular varieties in use in manufacturing.

16 An effort has been made to produce a practical chart for production purposes for commonly used materials. Proper drill sizes to produce approximately 75 per cent of thread have been determined, and the results are indicated by Figs. 3, 4, and 5.

DISCUSSION

Ralph E. Flanders.[1] A direct investigation of the size of the hole in tapping is exceedingly valuable. The American National Standard gives the dimensions of the finished hole, but the dimensions of the tap drills with which these holes should be drilled are not the same as those of the finished product after tapping, which the standard specifies, so that the author in his investigation has gone one step further toward the attainment of the actual results aimed at by the standard. The actual measuring of the spinning-up must be taken into account, and while the author modestly says this was not scientifically done, it nevertheless was practically done.

One interesting thing that we notice in this set-up of tap-drill sizes is that the General Electric Company has evidently decided that 75 per cent engagement is about right. The various tap-drill sizes, as a result, fall as close to that line as possible. For six or eight years the general tendency has been, of course, toward a smaller percentage of theoretical depth of engagement. The writer believes that there is no great loss of strength with the smaller percentage of engagement, so far as failure by shear is concerned. The area to be sheared remains the same within reasonable limits. Even if the engagement is 50 or 60 per cent, the area to be sheared is the same. Furthermore, the failure is almost never by shear, because the tensile strength of the bolt is a deciding element in any ordinary length of engagement, so that if it fails it is because the bolt fails, not because the thread fails in shear.

J. Howard Ayer.[2] The writer wishes to endorse the author's good work for the establishment of standards for tap-drill sizes. What we want, of course, is a thoroughly good tap hole, but of a practical size, which will reduce to a minimum the breakage of taps and speed up production. This can be obtained by the use of the author's charts.

[1] Manager, Jones & Lamson Machine Co., Springfield, Vt. Mem. A.S.M.E.

[2] Pittsfield, Mass.

No. 2004

THE SPECIFICATION AND CONTROL OF MECHANICAL SPRINGS

By Joseph Kaye Wood,[1] New York, N. Y.
Associate-Member of the Society

This paper is another of a series on the general subject of springs which the author has presented recently. In the present paper the subject of specifications for the manufacture of springs and for spring materials is discussed. A list of the available specifications for heavy springs is given and the features of these specifications are tabulated. In the absence of specifications for small springs the author discusses the outline for such specifications and illustrates his method by a hypothetical case. He introduces the idea of a "load-deflection sector" to be used in place of a load-deflection curve and employs the idea in framing his sample specification.

SPACE and weight limitations, finer control of operation, and higher speeds in modern machine design have given rise to the necessity of greater adequacy in specifications with little or no increase in commercial restrictions. In the case of mechanical springs there are a few specifications of long standing but these are confined principally to the larger types of springs used in the automotive and railway industries. Considering the great amount of spring breakages in these two fields, we can say that although the present specifications are commercial and are being met in practice, they are not adequate or suitable for the intended service. However, such specifications are written as well as the present state of the spring art will permit. The arrangement of the component parts is the result of many years of practical experience, and for this reason it would be wise to refer to them when making up specifications for medium and small-size springs. Very little publicity has been given to what few specifications there are in existence covering the latter type of springs. The usual manner of specifying such springs is by showing the essential dimensions and designation of material on the detail blueprint. It is the purpose of this paper to analyze and determine the reason for this

[1] Consulting Engineer.

Contributed by the Machine Shop Practice Division and presented at the Providence, R. I., Meeting, May 3 to 6, 1926, of The American Society of Mechanical Engineers.

condition, and to suggest a form of specification for constructive criticism.

2 Makers of springs may be divided into two general classes; first, those who manufacture springs for use in their own product, and second, those who manufacture springs for direct sale to another concern requiring the springs for use in its product. For convenience we shall hereafter refer to the makers of springs in the first class as "indirect-sale" manufacturers and to those in the second class as "direct-sale" manufacturers.

3 Direct-sale manufacturers may be classified further as those who supply their own raw material; for example, the Wickwire Spencer Steel Co., American Steel and Wire Co., Railway Steel Spring Co., etc., and on the other hand those that obtain their raw material from outside sources. The overall classification is as follows:

- Makers of mechanical springs
  - Indirect-sale
    - Automotive
    - Typewriter
    - Telephone
    - Adding machine
    - Tool machinery
    - Railway, etc.
  - Direct-sale (Spring manufacturers)
    - Supply own raw material
    - Purchase raw material

4 The specifications in all cases should be basically similar, the only difference being that in the indirect-sale type no legal proviso regarding inspection and rejection would be necessary, and in the direct-sale type of specification the manufacturing and commercial conditions of the average spring manufacturer would have to be considered, bearing in mind the classification given previously. Another condition that should be taken into consideration is the relative knowledge of the art of spring design of the direct-sale manufacturer and his customer. In this connection, however, it is to be hoped that this condition will be only a temporary one until such time as the art of spring design has been greatly clarified and more widely disseminated. In practice this problem is solved by the assumption of full responsibility by either party or by an agreed division of the responsibility.

5 Where springs are bought from a spring manufacturer, a sample mechanism may be supplied with the blueprint, in which case the manufacturer, in his capacity as a spring specialist, will either approve the design or suggest certain changes, thereby accepting full responsibility for the performance of the delivered springs. In the case of the indirect-sale manufacturer under the present state of affairs, an extreme amount of time and money is

being wasted. There is a very decided routine in this waste process, which paradoxically seems to be done in " open obscurity " because of the apparently chronic nature of the habit. The procedure is somewhat as follows: a designer may simply put the matter of spring type and size up to the draftsman, who if inexperienced, consults a handbook and uses the first formula or table on springs that confronts his eye. Then again it may happen that either the designer or draftsman will design the spring as carefully as he knows how, putting the calculated dimensions, with or without arbitrary tolerances, on the blueprint. Usually no effort is made to determine what the commercial variations due to spring and material manufacturing methods are, nor — and this is equally important — to determine what effect these variations are likely to have on the actual operation of the spring. As a result of this indifference, if the first few samples of the product manufactured are found to be satisfactory, mass production is launched. The product then goes into service by the thousands, but before a year has passed complaints begin to pour in from all sides regarding failure or faulty operation. An expensive engineering investigation is authorized and not infrequently it is found that the springs are the cause of the trouble. Careful analysis may show that the particular springs which failed are those in which manufacturing and material variations were accumulative. It might be surprising to some to know that the maximum possible range in operation, due to accumulative variations, is sometimes equal to 30 or 40 per cent. Only by providing more adequate specifications can the possibility of this high percentage be eliminated and costly troubles thus avoided, although in making up these specifications a thorough knowledge of manufacturing conditions is necessary in order not to introduce any unwise commercial restriction.

6 It sometimes happens that the engineer involved in the above procedure attempts to absolve himself by placing on the manufacturing department all blame for not producing every mechanism in exact accordance with the first sample approved by him. This is improper and unfair, because, if the engineer were as fully acquainted with manufacturing methods as he should be, he would not limit his approval to the first sample and drawings, but would satisfy himself that the product should function satisfactorily under the worst possible case of accumulative variations. He should assure himself of this by issuing an adequate specification, thus preventing abnormal manufacturing variations.

7 It is apparent, therefore, that for the proper control of spring action in machine design, adequate specifications must be provided. Before going into the construction of one of these specifications a general discussion on the fundamental meaning of a specification will be attempted.

Specifications in General

8 Standard specifications for the great variety of materials and equipment used in all fields of industrial endeavor may be divided into two general classes, namely, "manufacturing" and "purchasing."

9 Manufacturing specifications are for the purpose of assuring the production of a standard article fulfilling certain requirements of operation and cost. They involve only the principals within an individual concern or organization.

10 Purchase specifications are for the purpose of assuring the delivery of a certain quantity of a standard article fulfilling certain requirements as to operation and quality. These specifications involve two different concerns or organizations.

11 The former type of specification merely assures that the latter type will be met, and for this reason the two should harmonize for a given product.

12 The fundamental reason for the development of specifications is the same as that underlying the standard material of barter, that is, gold currency. Ethically this standard was instituted for the purpose of assuring accuracy and equality of justice in the process of evaluation. In a world containing only a single person no evaluation of material would be necessary, but with the actual plurality of persons coöperating efficiently for the purpose of bartering, the factor of service is introduced which gives value to the freely obtained material. Thus the process of evaluation became necessary and the ethics of justice required that the evaluation be accurately consistent at all times.

13 In primitive days man instinctively specified the material required for his existence and comfort. The kind of material desired was determined only in an approximate way by the various senses, the identifying characteristics being impressed in his memory, which corresponded to the present process of writing up a specification. Strange as it may sound, it is probable that even tests for hardness, breakage, and flexibility were employed instinctively in a very crude manner to aid in the recognition of the various materials specified by the memory. Quantity of material was roughly set at an armful or handful, while price was not necessary since material was free to take.

14 Just so long as the act of specifying was confined to individuals, no enforcement provision (inspection and rejection) was necessary. It soon became evident however that since more comfort and protection could be had by efficient coöperation, men would have to obtain materials for each other, and this procedure introduced the factors of service and ethics of exchange. Briefly stated, the materials, which originally are free, are given added value when service is performed upon them or in connection with them. The ethics of exchange functions in the evaluation of this

service-laden material. These two important factors form the basic groundwork of our industrial and commercial order of today. They involve every phase of industrial activity, such as the gathering, preparation, fabrication, and transportation of materials, and the application of scientific, engineering, merchandising, financial, and legal knowledge. They have given rise to the standard material of exchange, gold currency, to established standard weights and measures, and to any existing standard specifications, covering both raw materials and equipment. Greater industrial progress is made with adequate specifications; but the rate of increase in new knowledge tends to work against the standardizaton of specifications. Standardization under proper control produces greater economy, but care should be exercised so that it does not prevent the unfolding of new knowledge.

15 From the foregoing discussion it is obvious that the following procedure should be the rule in the shaping of specifications:

(1) To know what is wanted in the way of operation and service, particularly the limitations as determined by commercial considerations

(2) To determine the best possible way of supplying this need at the least cost

(3) To devise means by which the selected method may be recognized accurately

(4) To provide legal protection whereby the deliverance of only the properly recognized material or equipment is assured.

16 In the case of item (1) the words "to know" imply the "service of knowledge" which must be taken into account in the evaluation of the thing being specified. In setting the limitations for commercial reasons, all factors which add to the ultimate cost must be known, such as "rarity of component materials," and service done upon them from the source of gathering to the ultimate fabrication into a machine or other type of equipment. This principle of engineering gives the average young engineer considerable trouble because he fails to see why the ideal solution should not be accepted, due to the fact that in his training the actual commercial condition did not exist. He soon finds out, however, that every line he adds to his drawing means that a certain amount of valuable service must be devoted to the product, which more or less means increasing its ultimate cost. In other words, the function of the engineer is not merely to put his technical ideas into blueprint, but to manipulate properly the many reins he holds in his hands to direct service. In this way he can control the cost of a product and thus keep within prescribed commercial limits. For example, specifications covering steel to be used at elevated temperatures of 800 or 900 deg. fahr. require that it have a certain tensile strength at room temperature, although this figure does not in any way indicate what the tensile

strength at 800 or 900 deg. fahr. will be. The reason for this procedure is that only very few people are in a position to make elevated-temperature tests on steel, and the inclusion of such a requirement in a specification at this time would add a very expensive service to the product, thus putting it out of bounds commercially.

17 Referring to item (3), materials are of course recognized by chemical analysis and physical tests, while fabricated materials are inspected and tested by both engineers and inspectors. Material and equipment which do not meet these prescribed tests are rejected, protection in the case of purchase specification being assured by the insertion of proper legal clauses.

Specifications Covering Mechanical Springs

18 To specify a mechanical spring one must know, first, what a mechanical spring is, and second why a mechanical spring is needed. To define a mechanical spring requires thoughtful consideration of its characteristics and scope of usefulness. The following definition is given in the author's paper A Code of Design for Mechanical Springs:[1] "A mechanical spring is an elastic body whose load-deflection rate and maximum safe deflection are of values suitable for mechanical use."

19 Many varieties of materials are elastic, but most of them are not suitable for mechanical springs. It becomes necessary therefore to define a mechanical-spring material as follows: "A mechanical-spring material is an elastic material of the kind which, when made into bodies of a shape and size suitable for use in mechanical design, will function repeatedly and permanently as a mechanical spring."

20 From these definitions it follows that a mechanical spring would be needed where a difference of pull or pressure at the extremities of a given movement would be required. Since the retroactive force of springs is elastic, the pulls or pressures are usually proportional to the amount of travel, provided the proportional limit of the material is not exceeded; but this is not always the case. An exception is shown in Fig. 1 where the effective lengths of the spring change automatically.

21 Elastic hysteresis will cause a slight loop in the load-deflection curve but this is so slight that for most practical purposes it may be ignored.

22 The straight-line load-deflection characteristic is by far the common rule and, unless otherwise stated, this assumption will stand, provided of course the proportional limit of the material is not exceeded.

23 The manner in which spring action must be applied and the size and shape of space available determine the type or shape of spring needed. Thus if rotary motion in a round or square

[1] Trans. A.S.M.E., vol. 47 (1925), p. 33.

space is desired, a spiral spring would be used. If the translatory movements are to be small in comparison with a long narrow space then a flat cantilever spring would be decided upon. Helical springs would be used in a long cylindrical or prismoidal space, stressed in torsion if the motion has to be translatory and stressed in flexure if the motion has to be rotary. In both these cases, the spring axis would be coincident with the geometrical axis of the available space.

24 It would be well at this point to stress the importance of the space assigned for spring action. More often the space instead

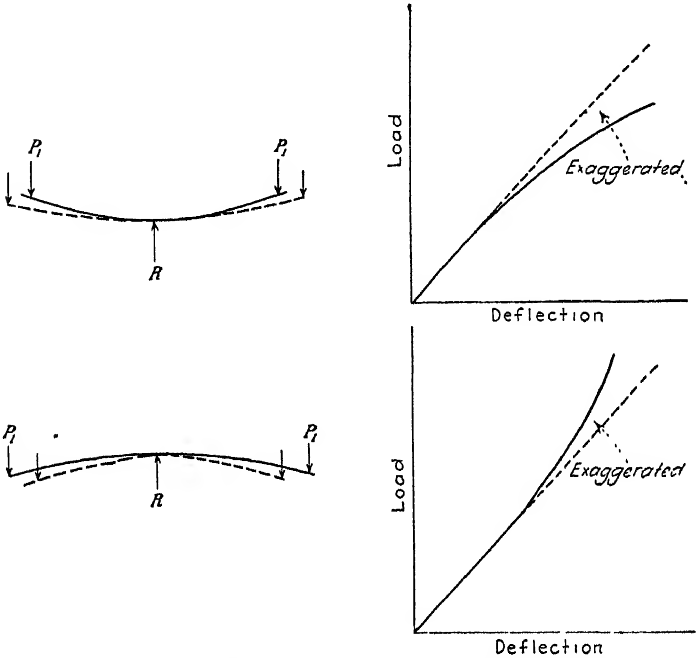


Fig. 1 Effect on Load-Deflection Curve of Change of Spring Length

of being assigned for this purpose during the design stage of a mechanism is obtained by the process of elimination. Thus space available for the installation of a proper spring often is inadequate, with the result that the product goes into service with a poorly designed spring. The usual complaint and engineering investigation follow, but due to the fact that mass production has advanced too far or standardization has been effected, the space for the spring cannot be changed. Consequently spring failures may be unavoidable for some time under these conditions. Two outstanding examples of this condition are, the motor spring of a telephone dial and the helical springs on railway cars. The spring index in the latter case ranges from 3 to 5 due to space limitations.

25 The service requirements and the proper type of spring having been determined, the specification may now be written, bearing in mind that the evaluation of the product is being built up and established.

26 Several specifications of the purchasing type will be analyzed briefly. Some of these specifications cover both the spring and the material, while others cover only the material. Spring purchasers in general require the former type, while indirect-sale manufacturers, and also direct-sale manufacturers who must purchase their raw material, require the latter type of specification. For convenience, these two types will be referred to as "spring" specification and "spring-material" specification. In some spring specifications the control of the material is accomplished by referring to a spring-material specification.

27 In general the component parts of a spring specification are as follows:

1 Title
2 Scope
3 Material
   - Reference to a spring-material specification
   - or
   - Includes the control of material
     - Process of manufacture
     - Chemical analysis
     - Method of chemical testing
     - Control of principal working heights
4 Physical properties and tests
   - Permanent-set test
   - Retest of permanent set
5 Permissible variations
6 Workmanship
7 Shipment and marking
8 Inspection
9 Rejection
10 Rehearing

28 The general construction of a spring-material specification usually is in accordance with the following outline:

1 Title
2 Scope
3 Process of manufacture
4 Chemical
   - Composition
   - Ladle analysis
   - Check analysis
5 Permissible variations in gage
   - Thickness or diameter
   - Width
   - Length
6 Finish
7 Shipment and marking
8 Inspection
9 Rejection
10 Rehearing

29 It is sometimes the practice where either the chemical or physical methods of testing are common to those included in several other types of specifications, to adopt a standard specification covering just that part and to make reference to it in either the spring or spring-material specification. Another practice is to supplement a spring specification by an appendix giving a description of a standard formula to be used in calculating the fiber stress.

30 Following is a list of some of the most important existing specifications covering both springs and spring materials.

Spring Specifications

Designation for this Paper	Title	Issued by	Designation
S-1	Specification for Helical and Elliptical Springs	Penn. R. R. System..	No. 12-H
S-2	Standard Specification for Helical Springs for Railways	American Society for Testing Materials	A61-16
S-3	Standard Specification for Elliptical Springs for Railways	American Society for Testing Materials	A62-16
S-4	Standard Specification for Elliptical Springs for Automobiles	American Society for Testing Materials	A69-18
S-5	Standard Specification for Helical Springs	American Railway Association	....
S-6	Standard Specification for Helical Springs of Chrome-Molybdenum Steel	American Railway Association	....
S-7	Elliptical Springs	American Railway Association	....

Spring Material Specifications

Designation for this Paper	Title	Issued by	Designation
SM-1	See S-1 for Carbon-steel	Penn. R. R. System..	No. 12-H
SM-2	See S-1 for Chrome-vanadium	Penn. R. R. System..	No. 12-H
SM-3	See S-1 for Silico-manganese	Penn. R. R. System..	No. 12-H
SM-4	Standard Specification for Carbon-Steel Bars for Railway Springs (with S-2 or S-3)	American Society for Testing Materials	A14-16
SM-5	Standard Specification for Carbon-Steel Bars for Railway Springs with Special Silicon (with S-2 or S-3)	American Society for Testing Materials	A68-18
SM-6	Standard Specification for Carbon-Steel Bars for Vehicle and Automobile Springs (with S-4)	American Society for Testing Materials	A58-16
SM-7	Standard Specification for Silico-Manganese Steel Bars for Automobile and Railway Springs (with S-2, S-3 or S-4)	American Society for Testing Materials	A59-16
SM-8	Standard Specification for Chrome-Vanadium-Steel Bars for Automobile and Railway Springs (with S-2, S-3 or S-4)	American Society for Testing Materials	A60-16
SM-9	Standard Specification for Spring Carbon Steel of Heavy Rolled Section	Society of Automotive Engineers	S.A.E. 1095
SM-10	Standard Specification for Spring Carbon-Steel of Light Rolled Section	Society of Automotive Engineers	S.A.E. 1080
SM-11	Standard Specification for Spring Silico-Manganese Steel of Heavy Rolled Section	Society of Automotive Engineers	S.A.E. 9260
SM-12	Standard Specification for Spring Silico-Manganese Steel of Light Rolled Section	Society of Automotive Engineers	S.A.E. 9250
SM-13	Standard Specification for Spring Chrome-Vanadium Steel	Society of Automotive Engineers	S.A.E. 6115
SM-14	Standard Specification for Spring Chrome-Vanadium Steel	Society of Automotive Engineers	S.A.E. 6150
SM-15	A. R. A. Specification for Carbon Steel Bars for Railway Springs (with S-5 and S-7)	American Railway Association	....
SM-16	See S-6 for Chrome-Molybdenum Steel..	American Railway Association	....
SM-17	Specification for Rolled or Drawn Spring Steel	U. S. Navy Dept.......	47S4d

TABLE 1 CHEMICAL COMPOSITION OF SPRING-STEEL MATERIAL

Specification (Designation for this paper)	Name of steel	Carbon Min.	Carbon Max.	Manganese Min.	Manganese Max.	S Max.	Phosphorus Max. Basic	Phosphorus Max. Acid	Silicon Min.	Silicon Max.	Chromium Min.	Chromium Max.	Vanadium Min.	Vanadium Max.	Molybdenum Min.	Molybdenum Max.
		Chemical composition, per cent														
SM-1	Carbon	0.90	1.10	....	0.50	0.05	0.05	0.05	0.25	0.50	....	....	....	....	....	....
SM-2	Chrome-vanadium	0.55	0.65	0.60	0.90	0.05	0.05	0.05	....	....	0 80	1.10	0.15	....	....	....
SM-3	Silico-manganese	0.45	0.55	0.60	0.80	0.05	0.05	0.05	1.80	2.10	....	....	....	....	....	....
SM-4 Grade A	Carbon	0.90	1.10	....	0.50	0.05	0.05	0.05	....	....	....	....	....	....	....	....
Grade B	Carbon	0.95	1.15	....	0.50	0.05	0.05	0.05	....	....	....	....	....	....	....	....
SM-5 Grade A	Carbon with special Silicon	0.90	1.10	....	0.50	0.05	0.05	0.05	0.25	0.50	....	....	....	....	....	....
Grade B		0.95	1.15	....	0.50	0.05	0.05	0.05	0.25	0.50	....	....	....	....	....	....
SM-6 Grade A	Carbon	0.85	1.05	0.25	0.50	0.05	0.05	0.05	....	....	....	....	....	....	....	....
Grade B	Carbon	0.90	1.05	0.25	0.50	0.05	0.04	0.05	....	....	....	....	....	....	....	....
SM-7 Grade A	Silico-manganese	0.45	0.55	0.60	0.80	0.045	0.045	0.05	1.80	2.10	....	....	....	....	...	....
Grade B	Silico-manganese	0.55	0.65	0.50	0.70	0.045	0.045	0.05	1.50	1.80	....	....	....	....	....	....
SM-8 Grade A	Chrome-vanadium	0.45	0.55	0.50	0.80	0.05	0.04	0.05	....	....	0.80	1.10	0.15	....	....	....
Grade B	Chrome-vanadium	0.55	0.65	0.60	0.90	0.05	0.04	0.05	....	....	0.80	1.10	0.15	....	....	....
SM-9	Carbon	0.90	1.05	0.25	0.50	0.05	0.04	0.04	....	....	....	....	....	....	....	....
SM-10	Carbon	0.70	0.90	0.25	0.50	0.04	0.04	0.04	....	....	....	....	....	...	. ..	. . .
SM-11	Silico-manganese	0.55	0.65	0.50	0.70	0.045	0.045	0.045	1.50	1.80	....	....	....	....	....	....
SM-12	Silico-manganese	0.45	0.55	0.60	0.80	0.045	0.045	0.045	1.80	2.10	....	....	....	....	....	....
SM-13	Chrome-vanadium	0.10	0.20	0.50	0.80	0.04	0.04	0.04	....	....	0.80	1 10	0.18	....	....	....
SM-14	Chrome-vanadium	0.45	0.55	0.50	0.80	0.04	0.04	0.04	....	....	0.80	1.10	0.18	..	....	....
SM-15 Class A	Carbon	0.90	1.10	....	0.50	0.05	0.05	0.05	....	....	....	....	....	....	..	....
Class B	Carbon	0.95	1.15	....	0.50	0.05	0.05	0.05	....	....	....	....	....	....	..	....
SM-16	Chrome-molybdenum	0.40	0.50	0.40	0.60	0.045	0.04	0.04	....	0.25	0.80	1.10	....	....	0 30	0 50
SM-17 Class A	Carbon	0.70	0.90	0.25	0.50	0.05	0.04	0.04	....	0.25	May be included according to supplier's judgment in which case only phosphorus and sulphur requirements need be met.					
Class B	Carbon	0.90	1.10	0.25	0.50	0.05	0.04	0.04	....	0.25						
Class C	Carbon	0.55	0.65	1.00	1.25	0.05	0.04	0.04	0.12	0.20						
Class D	Carbon	0.78	0.85	0.40	0.55	0.05	0.04	0.04	0.12	0.20						

NOTE: The materials specified by SM-1, -2, -3, -5A, -7A, -7B, -8A, -8B, and -15A may be either rolled or drawn for use in either elliptic or helical springs. Materials specified by SM-4B, -5B, -15B, -17B, -17C, and -17D may be drawn only for use in helical springs. Materials specified by SM-6A, -6B, -9, -10, -11, -12, -13, -14, -16, -17A and -17B may be rolled only for use in elliptic springs. The "A" and "B" grades or classes in all except SM-17 denote, in addition, the relative severity of stress or service to which the material in question should be subjected.

Process of Manufacture

31 All of the specifications listed above require the use of spring steel. Steel is by far the best type of material for mechanical springs and is in fact most commonly used. Spring steels are manufactured largely by the open-hearth process, although a considerable quantity is produced by the electric furnace, and a small amount is produced by the crucible process. The above specifications permit the use of any of these three processes.

Chemical Composition of Material

32 The chemical composition of the steels specified in the above specifications are given in Table 1. In connection with these steels it should be remembered that they are used entirely in large-size helical and elliptical springs.

Permissible Variations in Gage Dimensions

33 The Pennsylvania Railroad specification (S-1 in Table 4) makes no particular effort to control the dimensions of unfabricated rolled or drawn stock, except to state that "the inspector shall check the dimension of not less than 10 per cent of helical springs and not less than 25 per cent of elliptical springs." It further states that "springs shall conform to standard drawings . . . showing . . . dimensions."

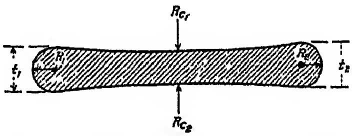


Fig. 2 Cross-Section of Double-Concave Flat Bar Stock

34 The American Society for Testing Materials make the following statement in regard to permissible variations in dimensions in all of their specifications. "The permissible variations in the width and thickness of the bar shall be agreed upon by the manufacturer and the purchaser."

35 On the other hand the Society of Automotive Engineers, the U. S. Navy, and the American Railway Association each state to a more or less degree the permissible variations in dimensions.

36 In the automotive industry flat bar stock is double concave as shown in Fig. 2. The Society of Automotive Engineers recommends that the radius of the mill roll which makes the round edges be equal to $\frac{2}{3}$ of the plate thickness. The difference in edge thickness, i.e., $\pm t_1 \mp t_2$ is specified as follows:

Width, in. Over	To, inclusive	Difference, in.
0	2	0.002
2	3	0.003
3	5	0.004

TABLE 2 PERMISSIBLE VARIATIONS IN GAGE DIMENSIONS

FLAT ROLLED STOCK

Specification	Width, in. Over	Width, in. Up to and including	Tolerances in width, in. Plus	Tolerances in width, in. Minus	Tolerances in thickness, in.: Up to 3/8 in. inclusive Plus	Up to 3/8 in. inclusive Minus	Over 3/8 including 5/8 in. Plus	Over 3/8 including 5/8 in. Minus	Over 5/8 including 1 in. Plus	Over 5/8 including 1 in. Minus	Over 1 including 2 in. Plus	Over 1 including 2 in. Minus	Over 2 in. Plus	Over 2 in. Minus
SM-17 U. S. Navy	..	1	$\frac{1}{64}$	$\frac{1}{64}$	0.007	0.007	0.010	0.010	0.012	0.012	....	....	....	....
	1	2	$\frac{3}{128}$	$\frac{3}{128}$	0.010	0.010	0.012	0.012	0 015	0.015	0.020	0.020	....	....
	2	4	$\frac{1}{32}$	$\frac{1}{32}$	0.010	0.010	0.012	0.012	0.015	0.015	0.020	0.020	$\frac{1}{32}$	$\frac{1}{32}$
	4	6	$\frac{1}{16}$	$\frac{1}{16}$	0.010	0.010	0.012	0.012	0.015	0.015	0.025	0.025	$\frac{1}{32}$	$\frac{1}{32}$
	6	8	$\frac{5}{64}$	$\frac{5}{64}$	0.012	0.012	0.015	0.015	0.020	0.020	0.025	0.025	$\frac{1}{32}$	$\frac{1}{32}$

Specification	From (Inclusive)	To (Inclusive)	Plus	Minus	Up to 3/8 in. inclusive Plus	Up to 3/8 in. inclusive Minus	Over 3/8 to 3/4 in. inclusive Plus	Over 3/8 to 3/4 in. inclusive Minus
SM-15 American Railway Association	1	2	0.015	0.015	0.015	0.005	0.015	0.010
	2	4	0.047	0.015	0.015	0.005	0.015	0.010
	4	5	0 047	0.032	0.015	0.005	0.015	0.010
	5	6	0 062	0.032	0.015	0.005	0 015	0.010
Society of Automotive Engineers	0	2¼	$\frac{1}{32}$	0	All thicknesses plus or minus 0.005 in.			
	2¼	3	$\frac{3}{64}$	0	All thicknesses plus or minus 0 006 in.			
	3	5	$\frac{1}{16}$	0	All thicknesses plus or minus 0.007 in.			

37 Leaf spring-steel bars shall not have more than 1 inch curvature in 20 ft. or 1¼ inches in 25 ft. or 1½ inches in 30 ft. The concavity, or difference between the thicknesses at the edges and at the center of the bar shall be specified as follows:

Width, in.	Normal concavity, in.	Max. concavity, in.	Min. concavity, in.
1½	0.007	0.009	0.004
1¾	0.008	0.010	0.005
2	0.010	0.012	0.006
2¼	0.011	0.013	0.007
2½	0.013	0.015	0.009
3	0.016	0.018	0.012
3½	0 018	0.020	0.013
4	0.021	0.023	0.016
5	0.029	0.031	0.023

Round or Square Drawn Stock
(American Railway Assn.)

Size	Diameter or thickness, in. Over	Under
Up to 5/16 in., inclusive	0 005	0.005
Over 5/16 to 7/16 in., inclusive	0.007	0.005
Over 7/16 to ⅝ in., inclusive	0.009	0.005
Over ⅝ to ⅞ in., inclusive	0.010	0 006
Over ⅞ to 1 in., inclusive	0.011	0.007
Over 1 to 1⅛ in., inclusive	0.012	0.008
Over 1⅛ to 1¼ in., inclusive	0.013	0.009
Over 1¼ to 1⅜ in., inclusive	0.014	0.010
Over 1⅜ to 1½ in., inclusive	0 016	0.011
Over 1½ to 2 in., inclusive	0.019	0.012

38 The U. S. Navy in the case of round bars allows a variation of 0.02 in., plus or minus, in diameter.

Physical Tests of Springs

39 In nearly all cases the physical properties or necessary heat treatments to obtain these properties for the spring materials in question, are not specified, but instead, certain physical tests on the finished springs are required. These tests aim to assure that the physical properties of the material and the design of the spring are adequate. From the practical standpoint, this is the best method, but it is not an exact scientific method, since such tests do not give any indication as to the probable life of the spring, and, in the case of less than 100 per cent inspection, no control is had over springs with maximum accumulative variations.

40 For helical springs the following physical tests are made (see Fig. 3):

1 Solid height
2 Free height
3 Loaded height
4 Permanent set

41 The solid height is the perpendicular distance between the plates of the testing machine when the spring is compressed solid with a test load of at least one and one-quarter times that necessary to bring all coils in contact.

42 The free height is the height of the spring when the test load specified for determining the solid height has been released,

and is determined by placing a straight-edge across the top of the spring and measuring the perpendicular distance from the plate on which the spring stands to the straight-edge at the approximate center of the spring.

43 The loaded height is the difference between the plates of the testing machine when the specified working load is applied.

44 The permanent set is the difference, if any, between the free height and the height after the spring has been compressed solid three times in rapid succession with the test load specified for determining the solid height, measured at the same point and in the same manner.

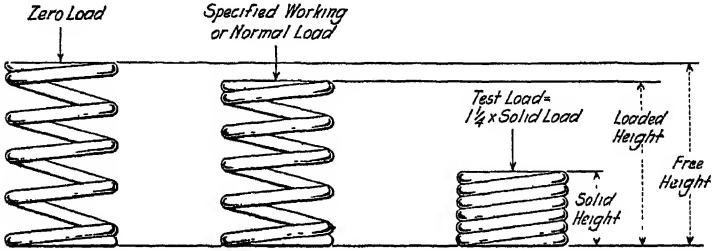


FIG. 3 ILLUSTRATION OF MEANING OF SPRING-HEIGHT TERMS, HELICAL SPRING

45 The limits allowed for these physical tests by the various specifications listed are given in Table 3.

TABLE 3 ALLOWABLE LIMITS FOR PHYSICAL TESTS, HELICAL SPRINGS

	Following heights shall not vary from those specified						Permanent set shall not exceed
	Solid height		Free height		Loaded height		
Specification	Plus	Minus	Plus	Minus	Plus	Minus	
S-1 (Penn. R. R.).	$\frac{1}{16}$ in.	....	$\frac{1}{16}$ in.	....	....	0	$\frac{1}{32}$ in.
S-2 (A.S.T.M.).....	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{16}$ in.	$\frac{1}{32}$ in.
S-5 (A.R.A.).......	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	$\frac{1}{16}$ in.	$\frac{1}{32}$ in.

46 For elliptical springs the following tests and permissible results are specified. (See Fig. 4.)

Railway.

a The free height is the height from the center line of the spring-eyes to the face of the leaf at the center of the spring seat after a test load of $1\frac{1}{2}$ times the specified normal load (working load) has been applied and fully released.

b The loaded height and loaded length are respectively the height and length when the test load in *a* has been applied and is slowly released to the specified normal load. The loaded height shall not be less, but may be $\frac{3}{8}$ in. more, than that specified. The loaded length shall not vary more than $\frac{1}{4}$ in. from that specified.

c The permanent set is the difference, if any, between the free height and the height after the test load has been applied again

and fully released. If the permanent set is less than $\frac{1}{32}$ in., a second test for permanent set after two additional applications of the test load shall not show any further permanent set.

Automotive.

a The maximum test load shall be twice the specified normal load (working load) provided the corresponding deflection is possible from the standpoint of interference under the car. Otherwise the load corresponding to the maximum possible deflection shall be the maximum test load.

b The loaded height is the height when the specified normal load is applied after the maximum test load of *a* has been applied and fully released three times. The loaded height in the case

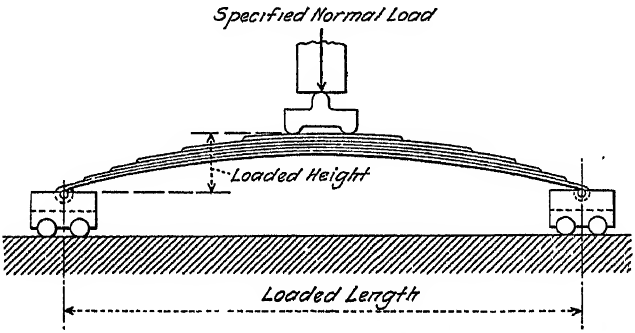


FIG. 4 ILLUSTRATION OF MEANING OF FLAT SPRING TERMS

of pleasure cars, unless otherwise specified, shall not be less, but may be $\frac{1}{4}$ in. and $\frac{3}{8}$ in. more than that specified for front and rear springs respectively.

c The permanent set is the difference, if any, between two successive measurements of the height under a load equal to 75 per cent of the specified normal load. This load shall be applied for the first time after the maximum-load test has been applied and fully released three times, and for the second time after the maximum test load has been applied again and fully released. The permanent set shall not exceed $\frac{1}{16}$ in., but if to the contrary, the maximum test load shall be applied again and fully released two additional times and a load equal to 75 per cent of the specified normal load applied a third time. No additional permanent set shall be allowed after this second test.

d The flexibility, in pounds per inch of deflection, shall be determined by measuring the height under 75 per cent of the specified normal load and the height under 125 per cent of that load and dividing the difference in loads by the difference in heights. The flexibility shall not vary from that specified more than 5 per cent.

47 The above physical tests are effective only to the extent that they assure the attainment of certain operative requirements of load and corresponding deflection within reasonable limits. In the case of certain specifications they further assure that the springs will withstand a maximum static load corresponding to a fiber stress of 90,000 lb. per sq. in. for helical springs, and 127,500 lb. per sq. in. for elliptical springs, irrespective of the type of steel (unless otherwise agreed upon). The formulas recommended by the A.S.T.M. for calculating test loads based upon these stress figures are as follows:

Helical springs

$$P = \frac{3.1416Sd^3}{8D_m}$$

Elliptical springs

$$P = \frac{2Snbh^2}{3L}$$

where P = load, S = fiber stress, d = diameter of material, h = thickness of material, b = width of material, D_m = mean diameter of helical spring, L = length of elliptic spring and n = number of leaves in semi-elliptic or half the number of leaves in a full-elliptic spring. Units are pounds and inches.

48 The physical tests in question do not take into account the probable life of the spring or, in the case of less than 100 per cent inspection, the effect of accumulative variations on both the operation and life of the spring. More adequate control of design and data on fatigue of metals will enable the proper consideration of these factors. The former may be accomplished by the specification of heat treatment and expansion of knowledge on the design of springs in general. The Society of Automotive Engineers and the American Society for Steel Treating furnish heat-treatment specifications for spring steels similar to some of those spring steels already considered. The A.S.S.T. steels and their heat treatments are given in Tables 4 and 5.

Workmanship

49 The various specifications considered cover workmanship by a statement ranging from a simple reference to "injurious defects" and "finish in a workmanlike manner" to a more lengthy paragraph on inspection of general dimensions and workmanship. Some of the points covered are as follows:

Helical Springs. The springs shall be of uniform pitch with ends tapered to give a reasonably square, firm bearing. The points of bars shall not protrude beyond the outside diameter of the springs.

The outside dimensions of the springs, excepting the height, shall not vary more than $\frac{1}{16}$ in. from those specified. In the A. R. A. and Penn. R. R. specifications this requirement is covered under "permissible variations." In the latter section of the A. R. A. specification it also states that when

TABLE 4 COMPOSITION OF A.S.S.T. SPRING STEELS

Designation for this paper	Steel	Carbon	Manganese	Phosphorus	Sulphur	Silicon	Chromium	Vanadium	Molybdenum
SM-18	Carbon	0 90	0 30	0 040	0.050	0.10	....	....	....
		1.05	0.50	....	....	....	....	....	....
SM-19	Chrome	0.45	0.80	0.040	0.050	0.10	1.00	....	....
		0 55	1.00	....	....	0.20	1.20	....	....
SM-20	Chrome-	0.45	0.70	0.040	0.050	0.10	1.00	0.15	....
	Vanadium....	0.55	0 90	....	....	0 20	1.20	0.25	....
SM-21	Chrome-	0.45	0.70	0.040	0.050	0 10	0.70	....	0.20
	Molybdenum	0 55	0.90	....	....	0 20	0.90	.. .	0.30
SM-22	Chrome-Silico-	0.45	0.70	0.040	0.050	0.40	0.70	....	....
	Manganese...	0.55	0.90	....	....	0.60	0.90	....	....
SM-23	Silico-	0.55	0.70	0.045	0.050	1.80	....	....	....
	Manganese...	0.65	0.90	....	....	2.10	....	....	....

TABLE 5 HEAT TREATMENTS FOR A.S.S.T. SPRING STEELS

Designation for this paper	Steel	Normalizing temperature, deg. fahr.	Hardening temperature in deg. fahr. Quench in oil. Time of heating = diameter × 60	Approx. tempering temperature, deg. fahr., to give Brinell in adjacent column	Brinell hardness number
SM-18	Carbon	1575–1625	1525	600–950	352–415
SM-19	Chrome	1600–1650	1525	650–950	363–444
SM-20	Chrome-Vanadium	1600–1675	1550–1600	675–975	363–444
SM-21	Chrome-Molybdenum	1600–1675	1550–1600	675–975	363–444
SM-22	Chrome-Silico-Manganese...	1600–1650	1600–1650	675–975	363–444
SM-23	Silico-Manganese	1600–1650	1600–1650	650–950	363–444

under the specified normal load, at the loaded height, the spring shall be of uniform pitch within 15 per cent of the bar diameter throughout its length, excluding the tapered section.

Elliptical Springs. Dimensions which affect contour only and do not affect the interchange or service of the springs need only be approximated.

The springs shall have the leaves properly graduated in length, properly bent, and fitted to reasonably true circular arcs.

The bands of the springs shall not vary from the specified dimensions more than $\frac{1}{16}$ in. in width and $\frac{1}{32}$ in. in thickness of straps, nor more than $\frac{1}{8}$ in. in width across the spring.

Markings

50 In most cases it is specified that the markings of manufacturer or inspector shall be so placed as not to be detrimental to the life or service of the spring.

Specifications Covering Smaller Size Springs

51 The foregoing discussion was in connection with the specifications covering large helical springs of the compression type

and elliptical springs used in the U. S. Navy and the railway and automotive industries. In the case of smaller mechanical springs no general or standard form of specification exists. To develop such a specification, the more generalized features of the large spring specifications may be followed, although certain important deviations must be made in order to satisfy a vastly different set of conditions. Different production problems are met with, since small springs are usually made in larger quantities at one time and the unit price is also very much less.

52 The layout of a specification for small springs would, in a general way, follow the topical outline given in Par. 27. It should be brief, however, with items concerning only the production department isolated from those concerning only the sales or purchasing departments as the case may be. Thus the first part alone would serve as a manufacturing specification for use by both direct-sale and indirect-sale manufacturers, while the two parts together would serve as a purchasing specification to be submitted only to direct-sale manufacturers. Spring-material specifications of the purchasing type would also have to be developed for use by indirect-sale manufacturers and direct-sale manufacturers who do not produce their own raw materials.

53 The title of a specification exclusively for manufacturing would be very brief and would have a "piece-part" code number as for example,

Helical Spring (Extension) P-145012

54 The scope or use to which the spring is to be submitted would consist of a brief reference to the assembly or partial-assembly code number of the mechanism in which the spring is to operate, as for example,

MT-1 Typewriter A-4567.

55 In all other types of specifications the title would not include any code numbers, but would give some indication of the general use to which the spring is to be put, as for example, Standard Specification for Helical Springs for Precision Instruments.

56 The scope should be somewhat more explanatory than in the case of exclusive manufacturing specifications or in the case of the standard specifications for large springs. The approximate limits of the proposed service should be stated briefly, such as the probable number of operations and accidental overloads, the relative certainty of these limits, severity of corrosion, elevated temperatures, and the relative importance of the spring to the operation of the mechanism in which it is assembled. Reference to factors which are definitely controlled by the body of the specification should not be included. The number of operations bears a very important relation to the fatigue breakdown of the

spring, although the exact relation is not known sufficiently at the present time to put it into practice. Enough fatigue data are at hand, however, to show that a spring which is to deflect one million times should be designed differently from one which is to deflect 12 million times or 50 thousand times. The following examples will illustrate the constructive features of different types of scopes.

1 This specification covers helical springs to be used for keeping the filaments of high-powered vacuum tubes taut at a temperature of approximately 500 deg. cent.
2 This specification covers helical springs to be used in safety pop valves. (This brief statement implies that the spring material must be non-corrosive, that the spring is constantly under load, and that the number of operations are very few, because the service conditions in this case are fairly common knowledge.)
3 This specification covers small, flat, cantilever springs for use in mechanisms operating at high speeds in which the required number of spring deflections (completely reversed) ranges between 5 and 50 million.

Spring Materials for Small Springs

57 Aside from the method of spring manufacture, the factor having the greatest influence in shaping the special features of a specification for small springs is the question of materials.

58 In general the raw material is referred to as "wire" which replaces the term "bar" used in large-spring practice. The cross-section, which may be round or rectangular (flat, but occasionally square), is obtained by the repeated drawing of a bar through a round die. Rolling of a round wire then produces a flat wire. In certain cases flat springs are punched out of a rolled sheet, but this is not considered good practice unless the width contour is not straight with parallel edges. Drawing the wire cold, annealing within the necessary intermediate draws, and leaving the last draw unannealed introduces strain-hardening or cold-work into the material, thereby giving it the desired spring temper. Increasing the carbon content in steel up to 0.85 per cent, the tin content in phosphor bronze up to 10 per cent, or similar manipulation of the chemical analysis of other spring materials further increases the spring temper thus obtained. Strain-hardening of this kind must not be carried too far, however, or the material will be damaged internally, which fact makes the cold-drawing of high-carbon steel (music wire) very difficult, since the number of intermediate annealings are greater than is required for lower-carbon steel. Aside from damaging the material in this manner, the introduction of considerable strain, while increasing the proportional limit, decreases the ductility to an unsafe amount. Thus so-called soft springs (lower spring temper or proportional

limit) are best for severe vibration service, particularly when the fiber stress is high. Steel wire made in the above manner and having the following typical analysis

Carbon	0.85 per cent
Manganese	0.45 to 0.50 per cent
Sulphur	0.02 per cent
Phosphorus	0.025 per cent
Silicon	0.15 per cent

is known as music wire. Such material in 0.024-in. round wire will give tensile strengths as high as 400,000 lb. per sq. in. and a tensile proportional limit of about 200,000 lb. per sq. in. For calculation of helical springs it is the custom to use a torsional fiber stress of 100,000 lb. per sq. in. for ordinary service and as low as 60,000 lb. per sq. in. for very severe service. It is possible, however, by the manufacturing process of "surging" a helical spring of small spring index, to develop a torsional fiber stress as high as 200,000 lb. per sq. in.

59 Fortunately the heat-treating properties of steel allow of better methods whereby spring temper may be imparted to this metal. One method is to hot-draw a 0.65 per cent carbon steel, harden in oil, and temper to give the desired proportional limit. Although this wire, which commercially is known as oil-tempered wire, has a much lower tensile strength than music wire, being in the order of 200,000 lb. per sq. in., its tensile proportional limit is relatively higher, being about 75 per cent of the tensile strength. Oil-tempered wire is cheaper than music wire and is used in greater quantities for springs than is any other type of wire.

60 The other method of imparting spring temper to wire is to wind or make the spring first from the wire in the annealed state and then to harden and temper the formed spring. Alloy-steel wire springs are made in this manner.

61 A cheaper grade of steel wire, known as Premier wire, is cold-drawn from a 0.60 per cent carbon steel enriched with manganese.

62 A summary of the common materials used in the manufacture of small mechanical springs is as follows:

1 Music wire (steel), cold-drawn to give desired temper, average per cent of carbon = 0.85
2 Premier wire, cold-drawn, 0.60 per cent carbon steel
3 Oil-tempered wire, hot-drawn, 0.65 per cent carbon steel. By continuous process wire is hardened and tempered
4 Annealed steel wire, heat treated after forming of spring
5 Alloy steels, hot- or cold-drawn, annealed and then formed into springs, which are then hardened and tempered. Same analysis as for large springs
   Chrome-vanadium steel
   Silico-manganese steel
   Chrome-molybdenum steel

6 Phosphor bronze, rolled or drawn to temper; 4 to 8 per cent tin
7 Nickel silver, rolled or drawn to temper; 55 per cent copper, 18 per cent nickel, 26 per cent zinc
8 Monel metal, rolled or drawn to temper.

63 It is obvious that the physical properties of the above materials are likely to vary widely with the different manufacturers. It should be possible, however, to secure limiting values in the important properties of the more or less standard materials from each manufacturer. These figures should not be included in the specification unless they can be easily checked by some simple commercial test, such as swedging, twisting, bending, or hardness tests. For calculating purposes the commercial limits in the proportional limit, modulus of elasticity, ultimate strength, and, if possible, the endurance limit should be known. The object is not so much to hold the raw-material manufacturer to a given set of physical property values or to close limits of these values as it is to know what the commercial limits are, in order to know the limitations in design and for making up adequate but commercial specifications.

64 Direct-sale manufacturers of springs who furnish their own raw material have a decided advantage in this respect for two reasons, first, they know their own material very well, and second, they may manipulate their wire-making processes within reasonable limits to produce properties most suited to a given type of spring.

Gage and Gage Variations in Spring Materials

65 There are many systems of wire gages in existence but for ordinary steel wire used in the manufacture of springs, the Washburn and Moen system is used most commonly. Steel music wire has a special gage of its own, while the Birmingham wire gage is used for specifying the thickness of flat steel wire. Phosphor bronze and nickel silver, in both sheet and wire form, are specified by the Brown & Sharpe system. Considerable confusion is caused by these many systems, so that it is considered essential always to state the decimal equivalent of the gage along with the gage number. This matter is now before the A.E.S.C. for clarification as to which is the most appropriate standard to adopt for general use. To the author's mind, if the use of several wire standards is allowed to continue, it would be more convenient and less confusing to state wire sizes in decimals of an inch only. For example, it is less work to write "wire diameter 0.0720 in." than to write "No. 15 W. and M. wire (0.0720 in. diameter)," or more briefly, "0720 wire" as against "No. 15 W. and M. wire," and there is no chance for confusion.

66 The music-wire gage covers 39 sizes, ranging from 0.004 to 0.095 in. although music wire is made in larger sizes, possibly up to $\frac{3}{16}$ in. In the W. and M. gage there are 46 sizes ranging from 0.0070 to 0.4615 in. but all of these sizes are not used in spring manufacture. Springs are made from oil-tempered wire in sizes ranging from No. 21 W. and M. (0.0317 in.) to the largest size No. 000000 (0.4615 in.). Premier spring wire is made in sizes ranging from the No. 21 W. and M. to the No. 4 (0.2253 in.). The Brown & Sharpe gage also covers 46 sizes ranging from 0.0031 to 0.5800 in., all of which sizes apply to sheet manufacture.

67 The usual variations in thickness specified by any of the above gages, whether it be in round wire, flat wire, or sheet form, in commercial practice is 2 to 3 per cent. The width of flat wire may vary by a similar amount, but in the case of sheet stock it may be as follows:

Width	Variation
$\frac{3}{8}$ in. and under	0.007 in.
Over $\frac{3}{8}$ in. to $\frac{3}{4}$ in. inclusive	0.008 in.
Over $\frac{3}{4}$ to 1 in. inclusive	0.010 in.
Over 1 in.	$\frac{1}{64}$ in.

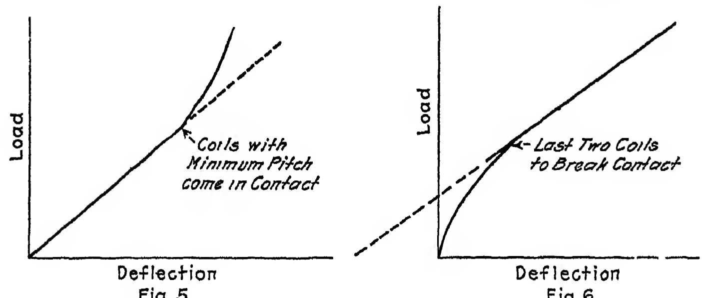


FIG. 5 COMPRESSION SPRING WITH VARIABLE PITCH
FIG. 6 EXTENSION SPRING WITH VARIABLE PITCH (INITIAL TENSION)

DIMENSIONAL VARIATIONS DUE TO MANUFACTURE

68 The method of manufacture must be considered in the shaping of a commercial specification for small mechanical springs, because the method to be commercial must unavoidably introduce certain variations in the finished springs. The number of springs and the rate of production required determine the method of manufacture. Indirect-sale manufacturers in most cases make all their springs, irrespective of quantity, by the methods which direct-sale manufacturers use only for small lots of springs, say one thousand or less. The latter, being engrossed in only one task, have developed automatic machinery and modern tools for quantity production, which, in addition to reducing manufacturing costs, has resulted in better control over variations. The direct-

sale manufacturers supplying their own raw materials have carried these developments out to a greater advantage. Only a few of the large indirect-sale manufacturers have adopted these modern methods, and it would pay many other large concerns to segregate all their spring manufacturing into an efficient spring department.

69 *Small Quantities*. Helical springs of both the extension and compression type are wound cold over an arbor set in a lathe. Since considerable tension has to be applied to the wire, stretching will reduce the wire diameter, and if the tension is insufficient the spring diameter will increase. Both of these variations tend to decrease the load-deflecton rate of the finished spring. The pitch of the spring which is controlled by the lead screw of the lathe may also vary slightly. In a compression spring this variation would cause a stiffening of the spring as it is being deflected, resulting in the type of load-deflection curve shown in Fig. 5, while in an extension spring initial tension would be introduced and the load-deflection curve shown in Fig. 6 would be obtained.

70 *Large Quantities*. In manufacturing large quantities of helical springs, rollers actuated automatically by cams upset the wire on the outside of the finished coil, constituting a method which might be called "outside winding" as compared with "inside winding" by the arbor method. The entrapped stresses in these two methods are quite different in type and magnitude, which probably accounts for the different variations in springs made by either method.

71 In either outside- or inside-wound springs, torsional overstrain is introduced by the commercial process known as surging, for the purpose of raising the torsional proportional limit. Helical springs, particularly those wound with low bending stresses (large indices), are heated under a low temperature to eliminate the dominating effect of entrapped stresses.

72 Springs wound from annealed wire and then hardened and tempered are subject to considerable variations in dimensions, when not carefully controlled.

73 From the foregoing discussion it is obvious that the physical properties of the material, whether it is steel, phosphor bronze, nickel silver, or monel metal, play a considerable part in the variations introduced during the manufacture of the spring. Each type of spring, such as the spiral, flat, helical, just covered, or the many special types, presents its own problems as regards commercial manufacturing variations. In general, the total actual range in commercial variation of the load-deflection rate is about 8 per cent, but the possible maximum range due to accumulative variations is somewhat greater. These variations will be more or less, depending upon whether indirect-sale or direct-sale manufacturers or direct-sale manufacturers who furnish their own raw material are considered.

74 To shape a specification to control the purchase of springs subject to the above maze of manufacturing variations is more difficult than in the case of large springs consisting of only two general types (helical and elliptical) and a smaller variety of materials (all steels). The measurement of the principal dimensions and permanent set of small springs is not so practicable, and in

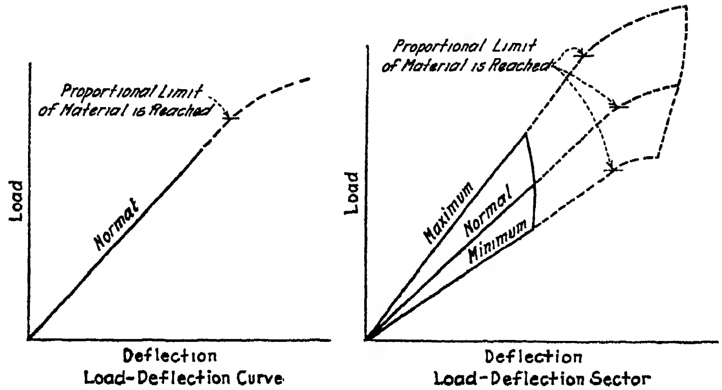


FIG. 7 LOAD-DEFLECTION CURVE AND LOAD-DEFLECTION SECTOR

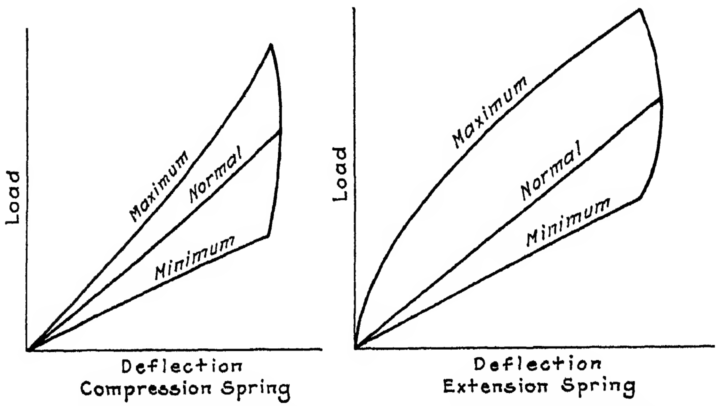


FIG. 8 LOAD-DEFLECTION SECTORS FOR CASES ILLUSTRATED IN FIGS. 5 AND 6

places where it is done on a 10 per cent basis a special laboratory with a fairly skilled personnel is provided for the purpose. The specification devised, however, must satisfy the two following essential requirements, which in a way are antagonistic to each other: (1) they must be adequate, and (2) they must be commercial.

75 Only a considerable knowledge of the art of spring design, spring materials, and the various manufacturing conditions will

enable a spring specialist to meet these two requirements. To meet requirement No. 1, the maximum possible accumulative variations, not the average maximum variations, should be definitely related to the operating characteristics of the spring. Based upon this principle the author wishes to introduce in this paper his idea of a "load-deflection sector" to be used in place of the load-deflection curve and to employ this idea in framing a sample specification. Fig. 7 illustrates the meaning of the load-deflection sector. Fig. 8 shows a sector including the type of variation shown in Figs. 5 and 6.

Load-Deflection Sectors Showing Effect of Variations Due to Non-Uniform Pitch

76 By encouraging engineers when designing springs to think in terms of load-deflection sectors instead of curves, we will be training them to control the over-all condition including manufacturing methods and commercial limitations, instead of narrowing themselves down to just the mere technical requirements. Engineers deal with actual conditions and facts, not merely with splendid ideas and ideals. With this general principle in mind, the following specification has been drawn up for a helical spring in which commercial considerations of cost required very liberal variations.

SPECIFICATION

John Doe Mfg. Company, Chicago, Ill.

Steel Helical Spring — Part 11-F-14

Scope. This specification covers a steel helical compression spring to be operated under a continual load that is to be oscillated about one million times in normal air during a period of about 5 years.

Specifications.

1 Operation:

a Load-deflection sector .... $70 \, {}^{+20}_{-15}$ lb. per in.
b Solid load .... As per test ($235 \, {}^{+90}_{-60}$ lb.)
c Test load .... 175 lb.
d Working load .... 130 lb.
e At working load, permanent set shall not exceed............... $\frac{1}{32}$ in.

2 Material:

Spring steel, oil-tempered wire, acid or basic open-hearth, carbon, 0.60 to 0.75 per cent

3 Dimensions:

a Gage .... $\frac{5}{32} \, {}^{+0.003}_{-0.002}$ in.
b Outside diameter .... $1 \pm \frac{1}{32}$ in.
c Solid height[1] .... $3\frac{7}{16} \pm \frac{1}{8}$ in.
d Loaded height[2] .... $5\frac{5}{16} + \frac{1}{8} - \frac{1}{16}$ in.
e Free height .... $6\frac{3}{4} \pm \frac{1}{8}$ in.
f Pitch .... $\frac{5}{16} \pm \frac{1}{64}$ in.

---

[1] After being compressed solid thrice with at least $1\frac{1}{4}$ times the test load (1*c*).

[2] Height corresponding to working load (1*d*).

4 Number of active turns..............$21 + 1 - 1\frac{1}{4}$
Number of inactive turns (ends).....1
5 Style of ends......................Squared only
6 Heat treatment of springs..........None
7 Finish ....Black enamel.

Inspection Tests.

8 Chemical analysis: Test samples shall be taken at random from the shipment and analyzed for carbon. The steel shall be free from slag inclusions, and otherwise of sound crystalline structure.

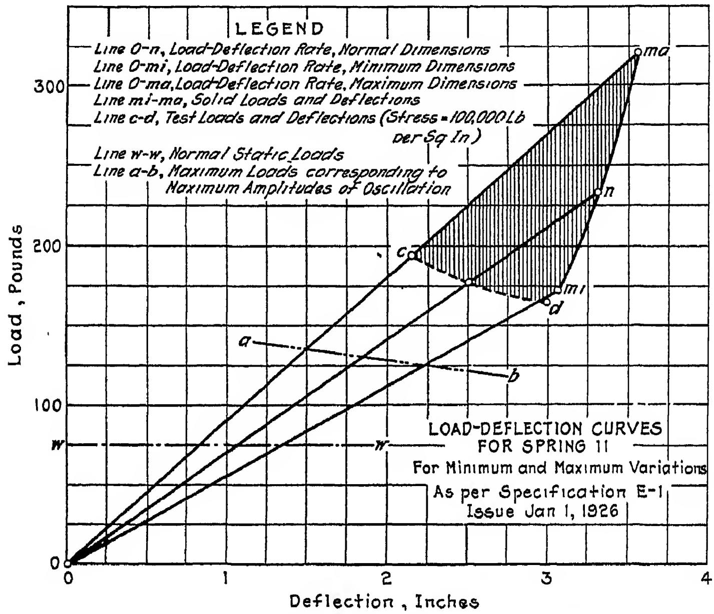


FIG. 9 LOAD-DEFLECTION CURVES TO ACCOMPANY SAMPLE SPRING SPECIFICATION

9 Physical tests: 10 per cent of the shipment chosen at random shall be submitted to the following tests:

a Springs shall be compressed solid 3 times with $1\frac{1}{4}$ times the actual test load (1*c*) before test load is applied again and released to free height

b Difference, if any, from free height (3*e*) as received from manufacturer is permanent set

c Permanent set shall not exceed the amount specified (1*e*)

d Load-deflection rate must be within specified limits (1*e*).

10 Guarantee: The manufacturer shall guarantee operating characteristics that fall within the load-deflection sector below the line *a-b*. He shall further guarantee that permanent set for loads and deflections below the line *a-b* will not be greater than $\frac{1}{32}$ of an inch and that failure by breakage will not occur within the sector below the line *c-d* for the conditions pointed out in this specification. If the manufacturer by testing a normal spring can show that the actual load-deflection

curve does not coincide with the normal line *o-n* he shall have the right to shift the entire sector until the normal line coincides with the actual test line. He shall then base his physical test requirements on the sector in revised position if agreeable to both manufacturer and purchaser (see Fig. 9).

Rejection.

11 Shipments which do not comply with the above specifications and inspection tests will not be accepted.

12 No exception to rejection will be considered after 10 days' notice by the purchaser.

13 Unless otherwise agreed upon all expenses involved in the rejection of a shipment will be charged to the manufacturer or supplier.

77 The above sample specification for the control of small mechanical springs, which is merely suggestive, would probably have to be modified or expanded to suit the particular purpose at hand. In general, however, the author feels that some step should be taken to make spring specifications more effective and adequate but with due consideration to the necessary commercial restrictions. If purchaser and supplier both have a common understanding as to what limits in operating characteristics and life of a spring are to be allowed, even though they may be very liberal, then a better evaluation of the product may be set. The author hopes that with the proper use of the load-deflection sector this end may be attained.

DISCUSSION

E. P. Graves.[1] Can the author give any information as to the advantage of making springs of formed wire in preference to round wire? When we have used a particular round-wire spring, with ample clearance, and it has failed, the manufacturer has made up a formed-wire spring as a substitute, and it has given satisfactory service.

W. T. Donkin.[2] The spring industry needs the coöperation of the spring user and manufacturer if the desired results are to be obtained. As the author points out, the spring maker must know certain facts about the use to which the spring is to be put. Too often he is given only a blueprint with no information as to the functions of the spring. The manufacturer cannot intelligently design a spring if he does not know the rate of vibration, the loads, and whether it is necessary to allow for use over a rod or in a hole. If he is given complete information he can tell the user just what spring can be made to meet the requirements.

Referring to the question of defective springs, raised by Mr. Graves, spring makers have been doing considerable research work

[1] Milling Machine Engineer, Designer, Brown & Sharpe Mfg. Co., Providence, R. I. Mem. A.S.M.E.

[2] Engineer, Cleveland Wire Spring Co., Cleveland, Ohio. Jun. A.S.M.E.

in regard to the relationship of failure and defects in steel. Naturally the first defect looked for was dirty steel. The responsibility for failure, however, could be traced only in a few instances to inclusions. By the aid of the deep-etch test, the reason for failure was traced to surface defects. Microscopic examination showed that practically every spring that broke had a visible surface defect.

Coöperation with the wire makers has given us wire that is absolutely free from surface defects. The use of this laboratory inspected wire has cut our breakage from about 90 to 10 per cent of its former value.

THE AUTHOR. In reply to a question asked by P. A. Porter,[1] the author does not believe that bending a helical spring until its ends are brought together will be a wise test to adopt. It may be called a safe test, however, because it will be effective in cases where the springs are being worked up to their maximum capacity by loads which tend to buckle the springs.

As to the advantage of making springs of specially formed wire, concerning which Mr. Graves inquired, a special wire of rectangular cross-section has been substituted for round wire in the motor spring used in telephone dials, which devices are coming into service in greater numbers every day. The designer of the dial apparently thought that the spring was an insignificant matter and left only a limited amount of space for it. The spring was then designed, tested, and improved upon in different ways to meet a very severe service. The best kind of spring material, steel music wire, was used, hence the best possible spring using round-section wire was obtained. However, a considerable number of the dial springs broke in service, which was a serious matter. In reconsidering the design it was found that a 24 per cent improvement was possible, provided a specially formed steel wire having a rectangular section and equal physical properties were used. The author went to a wire manufacturer and found that he was not anxious to supply such a wire unless a certain accomplishment could be guaranteed with it. Thus the matter resolves down to a commercial problem in which specially formed wire can be satisfactorily specified in cases where the spring is important enough to bear the expense.

In the case of round wire, practice is fairly well established. The round-wire spring can be easily wound on a mandrel, but in the case of a spring wire having a rectangular section the wire has a tendency to twist. Specifying special springs of this kind involves certain manufacturing difficulties, but such springs can be and are being made. The essential thing is that such a spring costs more.

[1] Engineering Department, Morgan Construction Co., Worcester, Mass. Mem. A.S.M.E.

No. 2005

BOILER AND STOKER PERFORMANCE AT HELL GATE POWER STATION OF THE UNITED ELECTRIC LIGHT AND POWER CO.

By H. W. Leitch,[1] New York, N. Y.
Member of the Society

The object of this paper is to show the trend of development of stoker and boiler installations in a modern power house whose extensions have been made at fairly regular intervals, and to give the effect of these yearly additions on the overall efficiency and maintenance. There is included a comparison of the essential features of the tests of the several types of installation, with a detailed test of the latest.

THE Hell Gate Station of the United Electric Light and Power Company now has an installed main-turbine capacity of 285,000 kw., 50,000 kw. of which has been installed within the last five months, and an installed boiler horsepower of 35,000, of which 3770 boiler hp. has been installed within the same length of time. It has sent out from the bus over a billion kilowatt-hours in the last twelve months. This is at the average rate over the 12 months of 3.53 kw. per installed boiler horsepower, although the load factor was only 50 per cent.

Original Boilers

2 The original installation of twelve 1890-hp. boilers was placed in service between the middle of November, 1921, and the early part of 1922. These boilers, as shown in Fig. 1, are twenty tubes high with interdeck superheaters separated from the furnace by a bank of six tubes. The tubes are 3 in. in diameter, 20 ft. long, and have a pitch of 15 deg. They are fired from each end by 14-retort stokers, 17 tuyeres long. The clinker pit and grinding rolls are located at the center of the furnace between the ash-discharge ends of the two stokers. These boilers are not provided

[1] General Superintendent of Power Plants, United Electric Light and Power Company.

Contributed by the Power Division and presented at the Providence, R. I., Meeting, May 3 to 6, 1926, of The American Society of Mechanical Engineers.

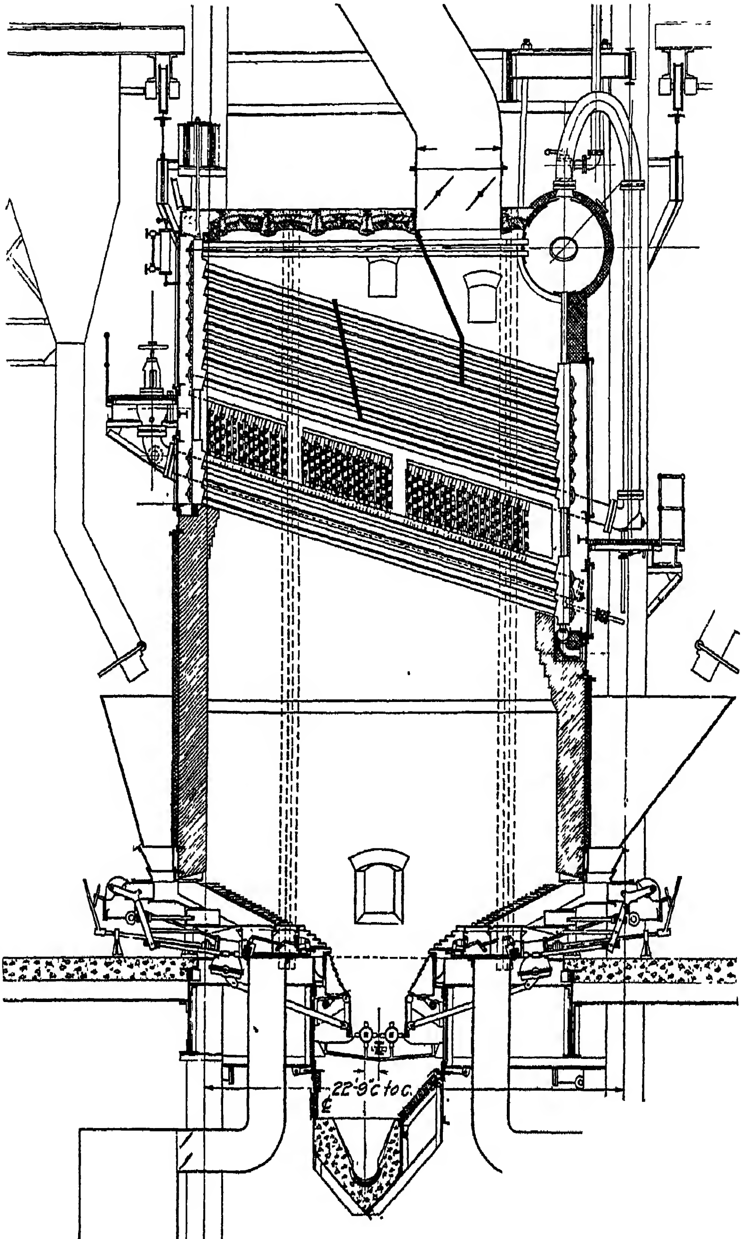


Fig. 1 Cross-Section of Original Boiler Setting, Hell Gate

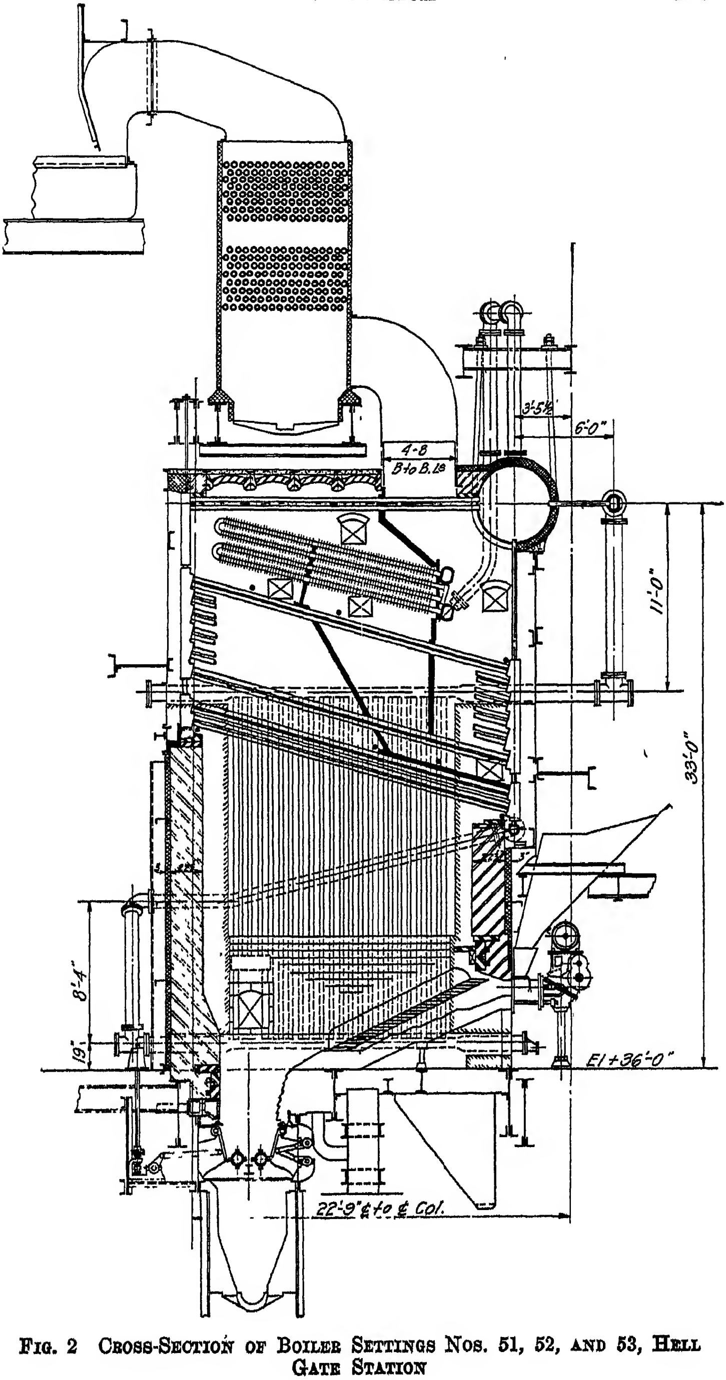


Fig. 2 Cross-Section of Boiler Settings Nos. 51, 52, and 53, Hell Gate Station

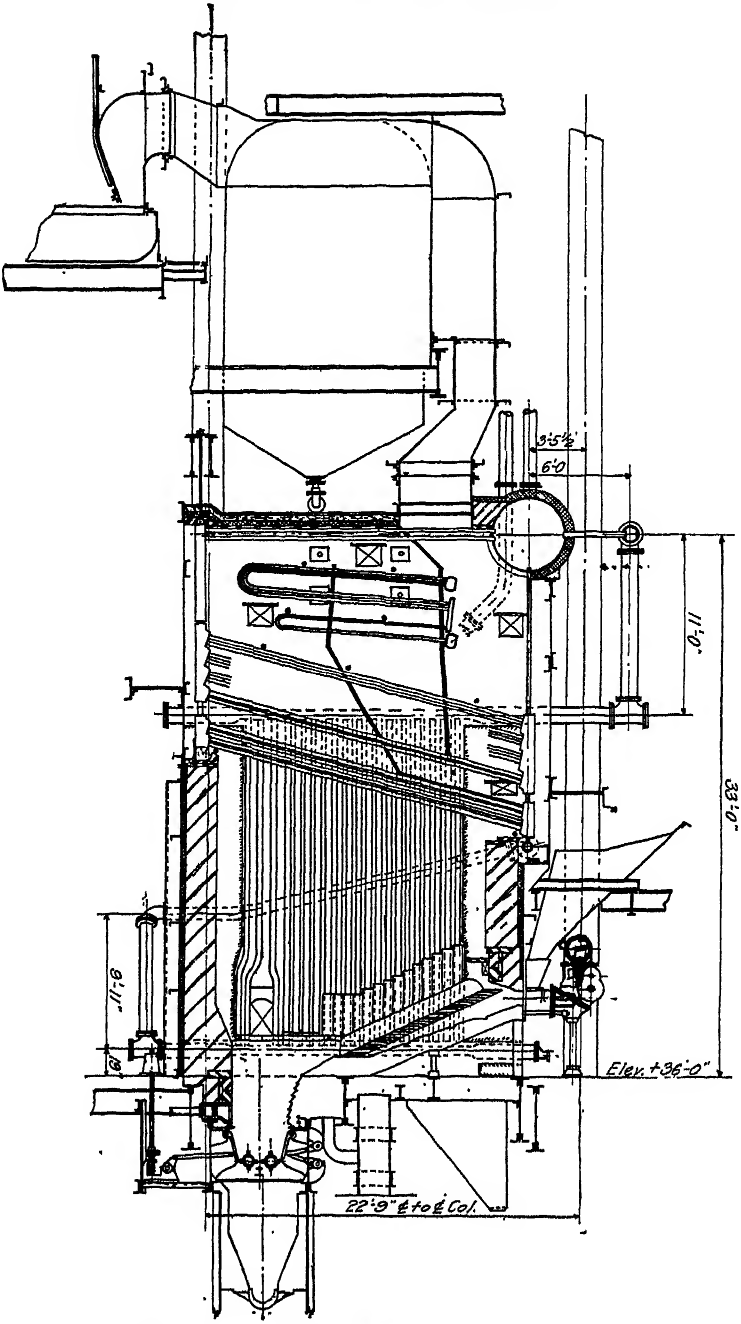


FIG. 3 CROSS-SECTION OF BOILER SETTINGS NOS. 61, 62, AND 63, HELL GATE STATION

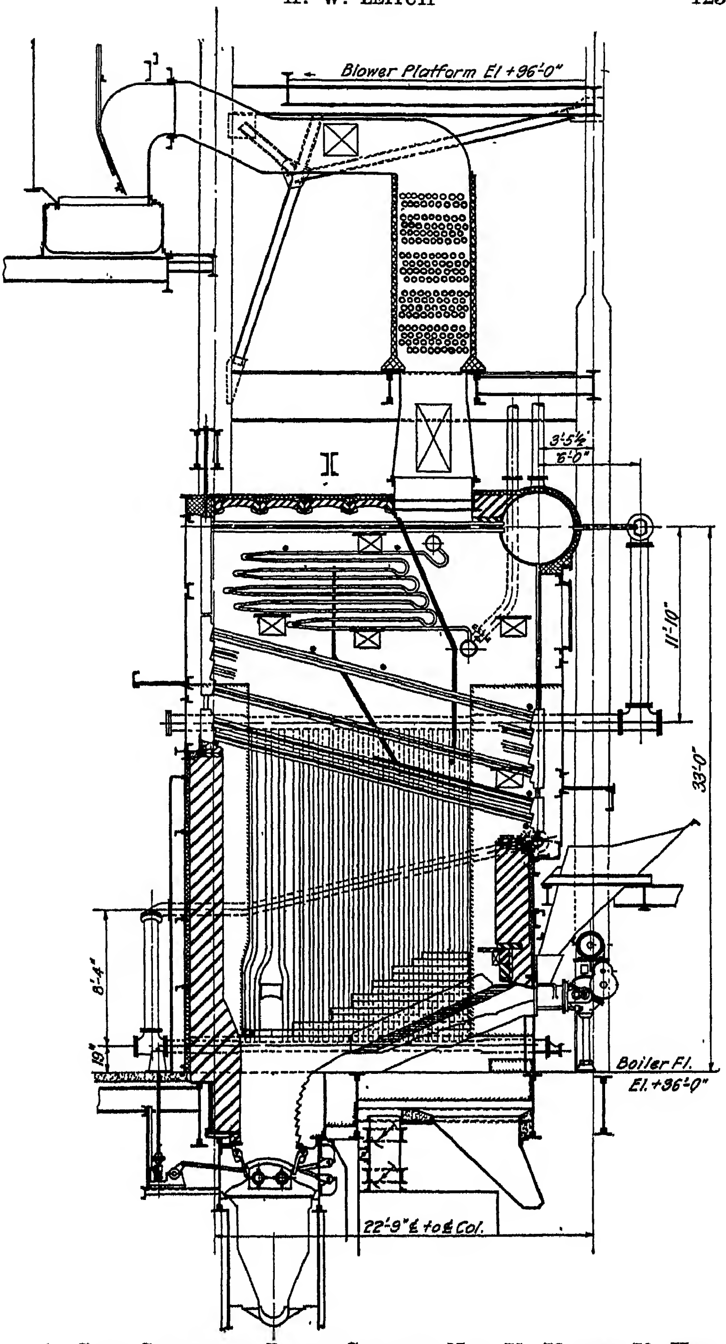


FIG. 4 CROSS-SECTION OF BOILER SETTINGS NOS. 71, 72, AND 73, HELL GATE STATION

with economizers, although they are equipped with induced-draft fans for operation at high ratings. These fans are adequate to accommodate economizers, should their installation later be indicated.

Boilers Nos. 51, 52, and 53

3 At the time of preparing the layout for the original twelve boilers the short stoker had reached a high state of development, while the long stoker was still in somewhat of an experimental stage. When, however, it became necessary to increase the boiler

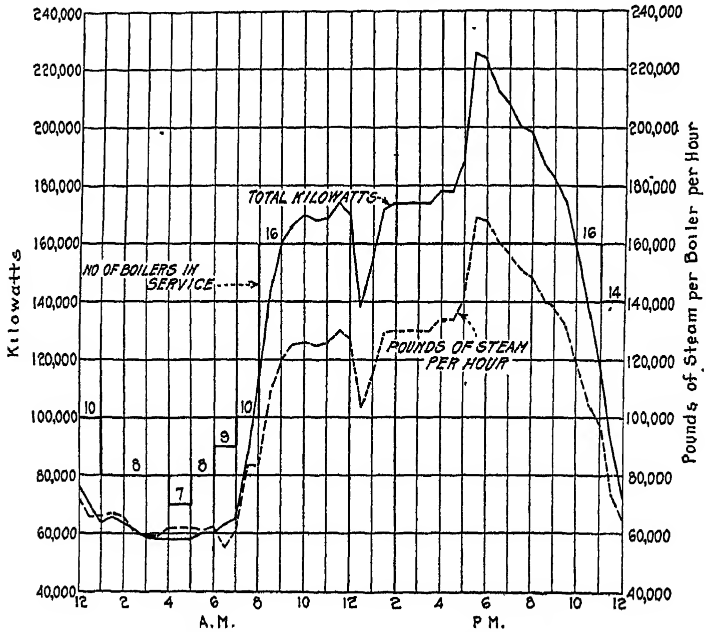


Fig. 5 Typical Daily Load Curve, Hell Gate Station — Station Output, 3,141,000 Kw-Hr.

capacity, there had been further developments in stoker design, so that three boilers were installed with economizers and fired at one end only by 14-retort stokers, 33 tuyeres in length. These boilers are known as Nos. 51, 52, and 53, and a cross-section of them is shown in Fig. 2. The clinker pit, equipped with a two-roll crusher, is located in the pocket formed at the rear wall of the boiler and the ash-discharge end of the stoker. The superheater is located on top of the first pass. The side walls of the furnace are water-cooled by means of fin-type tubes connected into the boiler circulation, an innovation at that time. The portion of these water tubes along the fire line of the stoker is covered by protective tiling.

4 When these boilers were first placed in operation some difficulty was experienced with the breakage of stoker rams and with burning of certain parts of the stoker. These difficulties, however, were soon overcome by the manufacturer, and the reliability of this type is now equal to that of the older types. These boilers are operated at high rates of driving, but the water-cooled side walls have reduced the furnace-brickwork maintenance to a minimum. The superheaters had been purchased before it was

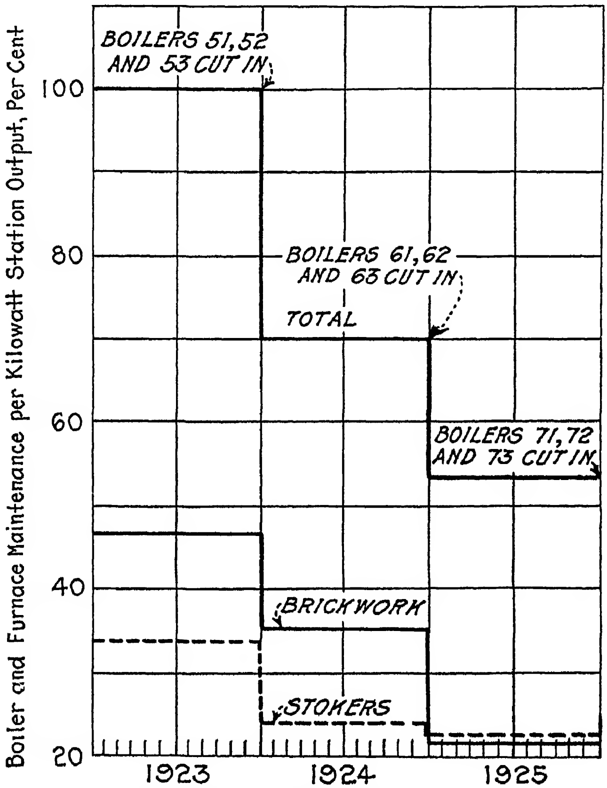


Fig. 6 Boiler-Furnace Maintenance Costs, Hell Gate Station

decided to install water walls, and owing to the fact that data were not available it was decided to install the superheaters as purchased, although it was realized that the desired temperature would not be obtained.

Boilers Nos. 61, 62, and 63

5 One year later, late in 1924, three more units were placed in operation and are known as Nos. 61, 62, and 63; see Fig. 3. These are similar in many respects to those of the second installa-

TABLE 1 CHARACTERISTICS OF HELL GATE BOILERS

Boilers				
Year installed	1921-22	1923-24	1924-25	1925-26
Number installed	12	3	3	3
Number of tubes wide	54	54	54	54
Number of tubes high	20	16	12	12
Water-heating surface, sq. ft.	18900	15500	12100	12100
Water-Cooled Side Walls				
Surface, sq. ft.	....	460	460	460
Make	....	Combustion	Combustion	Combustion
Type	....	Fin	Fin	Fin
Superheater				
Surface, sq. ft.	2135	7410	10450	4000
Make	Superheater Co.	Power Specialty	Power Specialty	Superheater Co.
Type	Elesco	Foster	Foster	Elesco
Location	Interdeck	Top first pass	Top first pass	Top first pass
Economizers				
Surface, sq. ft.	....	13824	12100	10692
Make	....	Power Specialty	B F. Sturtevant	Power Specialty
Type	....	Foster	Lead-alloy coated	Foster
Stokers				
Number per boiler	2	1	1	1
Number of retorts per stoker	14	14	14	14
Number of tuyeres	17	33	33	33
Projected grate area, sq. ft.	472	378	378	378
Make	American Engrg.	American Engrg.	American Engrg.	American Engrg.
Type	Taylor AA7	Taylor HC7	Taylor HC7	Taylor HC7
Furnace volume, cu. ft.	8000	6500	6500	6500

tion, which have proved so efficient and flexible, except that instead of being 16 tubes high these are 12 high, with the superheater, as before, located at the top of the first pass. This effectively provides the superheat and increases the overall boiler and economizer efficiency, except at very high rates of driving. The superheat is slightly higher than was expected, but at no rate of driving does it exceed temperatures considered safe. The stoker wind box is divided into four compartments so that the air pressure to any one of the four sections may be controlled independently. This feature

TABLE 2 CHARACTERISTIC RATIOS OF HELL GATE BOILERS

Year installed	1921–22	1923–24	1924–25	1925–26
Ratio of total water-heating surface to grate area	40	42.1	33.3	33 3
Ratio of furnace volume to total water-heating surface	0.423	0 41	0.517	0.517
Ratio of furnace volume to projected grate area	16.9	17.2	17.2	17 2
Ratio of superheating surface to total water-heating surface	0.113	0.466	0.332	0.317
Ratio of economizer surface to total water-heating surface	....	0.87	0.965	0.85

has been found valuable, particularly at high rates of driving. The economizers in this installation incorporate some new features in design. The tubes and cast-steel headers are coated inside and out with a lead alloy by a process very similar to tinning. The tubes are held in the headers by taper fits similar to the method employed with cast-iron economizers.

Boilers Nos. 71, 72, and 73

6 In the latter part of 1925 a further addition of three boilers was made, Nos. 71, 72, and 73 — see Fig. 4. It was realized that if the stoker could be lengthened the efficiency curve would have less drop at high rates of driving, and studies were made to incorporate a stoker 37 tuyeres long. The design, however, indicated such weaknesses due to the limitations imposed by the boiler-house structure, which was erected as a unit, that the proposition was abandoned and stokers of the same length as those previously installed — 33 tuyeres — were purchased. The stoker which was tested has certain refinements, notably the ability to adjust the travel of the lower rams. It is thought that this will be of particular value with coals of poorer grades than are now being used at the station.

7 The characteristics and ratios of these four types of boiler installation are given in Tables 1 and 2.

8 Referring to the cross-section of the most recent installation, Fig. 4, the greater simplicity of the layout is apparent. The economizers on the end boilers of the row are installed directly above the boiler uptake, eliminating the usual cinder pan and one elbow, and covering the entire width of the boilers. The tubes are 22 ft. long, an increase of 6 ft. over any previously installed in

the station, permitting a much more economical distribution of surface, with a consequent reduction in the number of headers required. The greater width as compared with the previous installations is favorable to the improvement of distribution of gas flow through the unit, and there is no tendency for the economizer to cause an uneven distribution of gas flow through the boiler. The economizer on the inside boiler of the row is offset sufficiently so that the tubes may be withdrawn from any economizer without

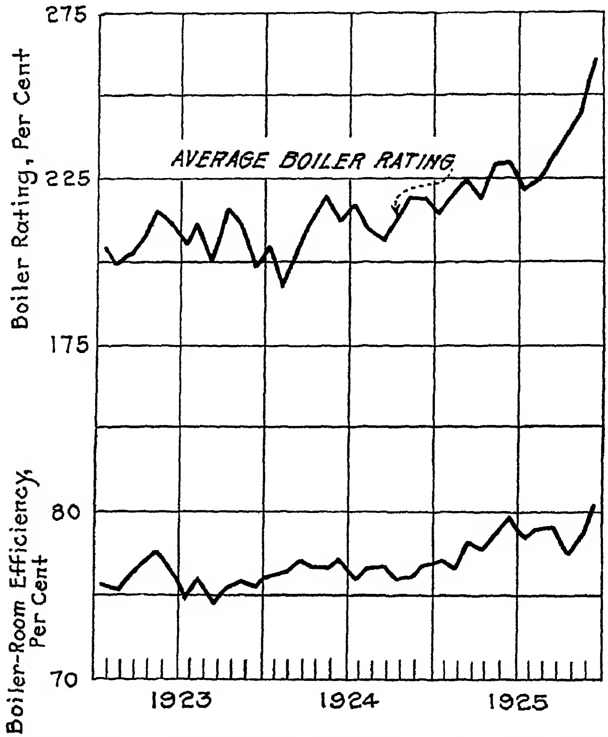


Fig. 7 Increase in Boiler-Room Efficiency with Introduction of Improved Boiler Units, Hell Gate Station

interference. Washing the economizer has been superseded by soot blowing.

Influence of Later Boilers on Maintenance and Efficiency

9 In order to show the character of the service from this station, there is shown in Fig. 5 a typical autumn daily load curve with the number of boilers steaming and the steam generated per boiler in pounds per hour.

10 It has been demonstrated that the improvement in design of each row of boilers has contributed materially to the operating

results of the station. Fig. 6 shows the decrease in boiler and furnace maintenance over the past three years with the dates of the successive additions indicated. The two major items of this maintenance — stokers and brickwork — are given in the lower curves. The marked decrease in brickwork maintenance is apparent, due entirely to the installation of the water-cooled walls on all of the newer boilers.

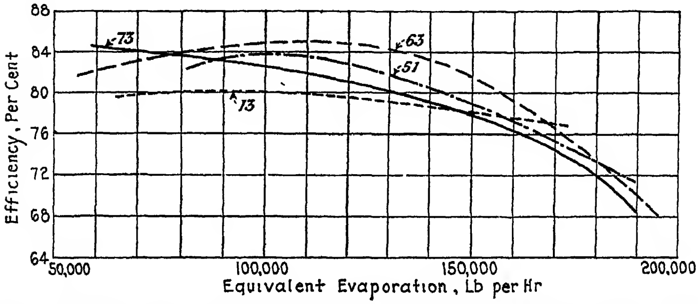


FIG. 8 EFFICIENCY CURVES, BOILERS AND SUPERHEATERS, HELL GATE STATION
(Equivalent evaporation of boiler and superheater.)

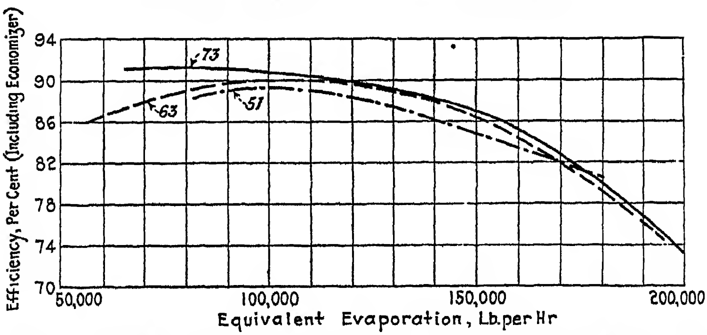


FIG. 9 OVERALL EFFICIENCY CURVES, BOILERS, SUPERHEATERS, AND ECONOMIZERS, HELL GATE STATION
(Equivalent evaporation of boiler and superheater.)

11 In the lower curve of Fig. 7 the gradual increase of boiler-room efficiency is shown. This is in spite of the increase in average rate of driving as shown in the upper curve.

12 The boiler and superheater efficiency curves of the four types of installation are given in Fig. 8. The flatness of the curve for No. 13 boiler is very pronounced, but as an operating proposition it is impossible to maintain as high rates of driving on boilers of this type for sustained periods as on the more recent type, due to clinker difficulties. Had it been possible to maintain the same grate area with the more recent additions, the performance curves would have been elevated somewhat at the higher rates. It

was necessary, however, to sacrifice some of the grate area in order to increase the capacity of the ash-disposal mechanism as a means toward greater reliability and reduced maintenance.

13 The apparent superiority of boiler No. 63 over the other boilers is probably due to certain structural differences which were incorporated in the furnace of this boiler for experimental purposes. Air-admission tuyeres were provided just above the fire in

TABLE 3 TESTS OF BOILER NO. 73

General							
1	Test No.	1	2	3	4	5	6
2	Date of test, 1926	2/18–19	2/19–20	2/23–24	3/3–4	3/9–10	3/30–31
3	Duration, hr.	24	24	25	24	22	24
4	Rate of driving, per cent of builder's rating	153	223	275	275	343	427
5	Coal fired per hr., lb.	5512	8338	10430	10354	13777	18200
6	Dry coal fired per hr., lb.	5200	7960	9913	9820	12960	17202
7	Actual water per hr., lb.	59957	85716	105940	105040	131300	161580
8	Equiv. water per hr., boiler and superheater, lb.	66500	96700	119200	118800	148900	184040
9	Equiv. water per hr., boiler, superheater and economizer	72500	105500	132800	132000	153400	205207
10	Actual water per lb. coal fired, lb.	10.87	10.28	10 15	10.15	9.54	8.88
11	Quality of steam entering superheater, per cent	99.2	98 8	98.6	99.1	98.9	98.9
Temperatures, Deg. Fahr.							
12	Boiler room	77	68	63	60	68	76
13	Air for combustion	72	63	65	65	72	72
14	Feedwater entering economizer	149	150	140	140	141	142
15	Feedwater entering boiler	245	258	264	263	274	268
16	Flue gases leaving boiler	458	523	576	612	653	659
17	Flue gases entering economizer	437	494	569	557	602	...
18	Flue gases leaving economizer	186	198	268	230	254	300
19	Superheated steam — east outlet	559	611	624	641	671	664
20	Superheated steam — west outlet	560	590	630	628	651	661
21	Average superheat	145	185	212	220	245	247
Pressure, Lb. Per Sq. In. Gage							
22	Water at economizer — inlet	342	343	346	342	342	342
23	Water at economizer — outlet	340	338	337	333	323	314
24	Steam at superheater — inlet	277	282	281	280	284	291
25	Steam at superheater — outlet	277	279	278	276	279	281
Drafts, Inches of Water							
26	Air duct	+3.0	+3.1	+3.2	+3.3	+4.5	+5.1
27	Wind box	+ .25	+1.1	+2.1	+1.9	+3.3	+3 7
28	Extension grate	+ .1	.0	+ .3	+ .4	+ .6	+ .6
29	Over fire	— .02	— .00	— .02	— .02	— .01	— .03
30	Leaving boiler	— .04	— .23	— .44	— .44	— .72	—1.17
31	Leaving economizer	— .21	— .73	—1.33	—1.22	—2.08	—2.99
32	Induced-draft-fan inlet	— .21	— .83	—1.48	—1.45	—2.28	—3.41
33	Loss through boiler	.06	.23	.42	.42	.71	1.14
34	Loss through economizer	.17	.50	.89	.78	1.36	1.82
35	Loss through boiler and economizer	.23	.73	1.31	1.20	2 07	2.96
36	Induced-draft-fan speed, r.p.m.	0	123	199	207	285	385
37	Stoker-motor speed, r.p.m.	249	457	522	588	663	955

the front wall and segregated in zones provided with manually-controlled dampers. The side-wall tuyeres were also arranged somewhat differently, but it is not believed that this had any appreciable effect. While from the standpoint of tests the efficiency appears superior, it is doubtful whether any material gains could be effected in ordinary operation, due to the human element.

14 In the curves of overall efficiency, however, in Fig. 9, the successive improvement in the last three installations is shown.

15 In Fig. 10 are shown the furnace efficiencies of the four types of installation based on saturated-steam temperature. In

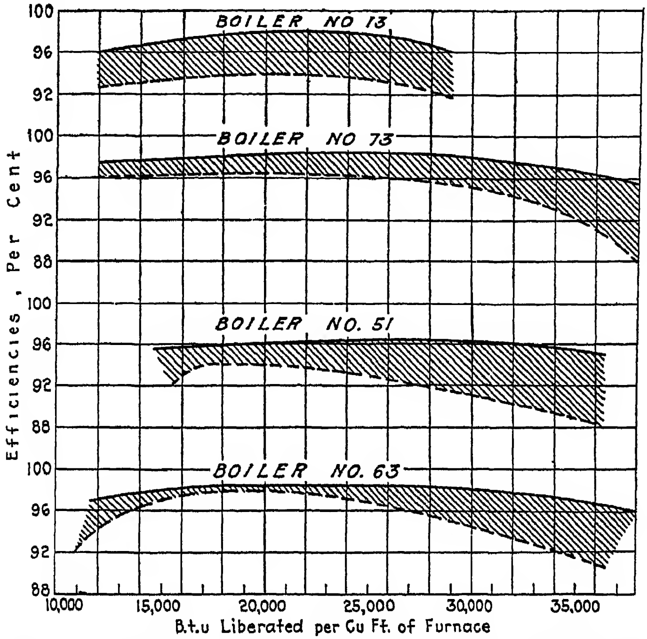


Fig. 10 Furnace Efficiencies Based on Saturated-Steam Temperatures, Hell Gate Station

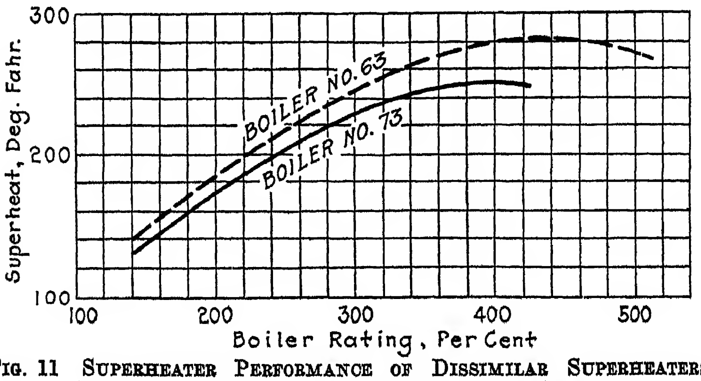


Fig. 11 Superheater Performance of Dissimilar Superheaters Installed on Similar Boilers, Hell Gate Station

each case the shaded area represents the radiation and unaccounted-for losses, so that the actual furnace efficency lies somewhere in this band and is not subject to accurate measurement.

16 Fig. 11 is self-explanatory, but in considering the curves it should be borne in mind that the superheater of boiler No. 63 was of the extended-surface type and had a rated effective surface of 10,450 sq. ft., while that of boiler No. 73 is of the plain-tube type with a surface of 4000 sq. ft. Each superheater, however, occupies substantially the same volume.

Tests of Boiler No. 73

17 The tests of boiler No. 73, reported in Figs. 12, 13, and 14, and Table 3, were conducted shortly after the boiler was placed

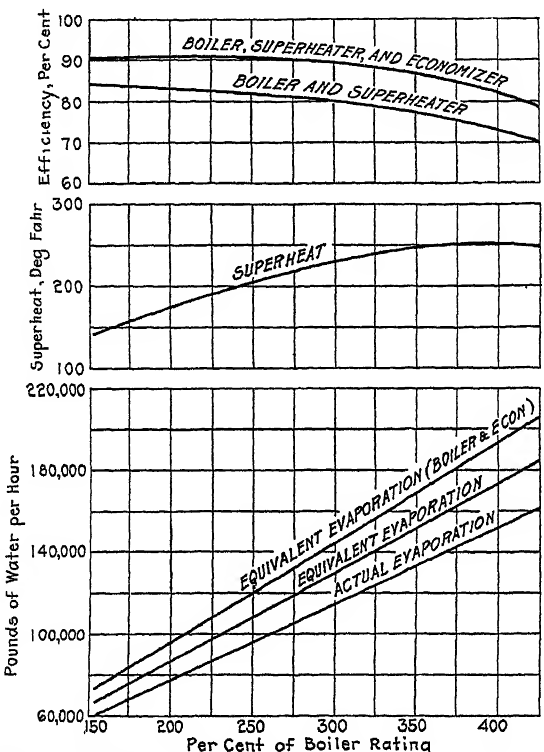


Fig. 12 Tests of Boiler No. 73, Hell Gate Station

in service and the usual care was exercised to secure high accuracy. The coal was weighed by weighing lorries, the accuracy of which had been checked by dead weights. The water evaporated was measured by means of a venturi meter and mercury manometer which had been standardized by the use of weighing tanks. The steam flow from the boiler was also measured by a Bailey meter and used as a check against the measured water.

18 In conducting the tests the feedwater flow was maintained constant by regulating the feed valve in accordance with the

indications of the manometer. The level of the water in the boiler was in turn controlled by regulating the combustion air supplied by the forced-draft fans in accordance with the height of the water in the gage glass. This was accomplished simply by maintaining the air-duct pressure slightly in excess of that required by the

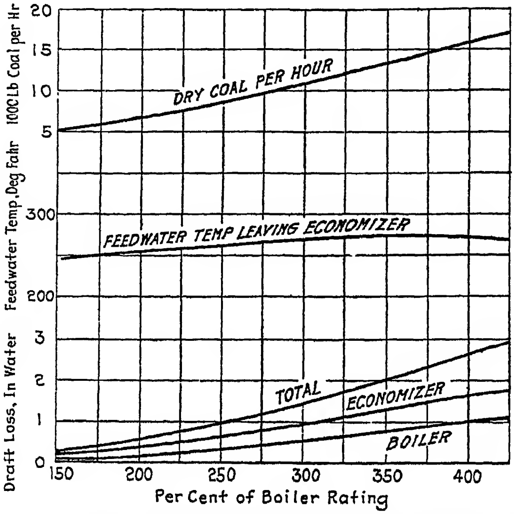


FIG 13 TESTS OF BOILER NO. 73, HELL GATE STATION

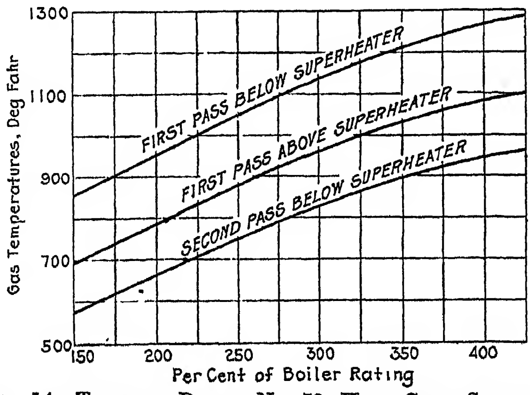


FIG. 14 TESTS OF BOILER NO. 73, HELL GATE STATION

average rate of driving, and slight fluctuations were taken care of by means of the hand dampers at the stoker. Briefly, the flow of water to the boiler was fixed, while the amount of heat imparted to the boiler was regulated to maintain an average rate of evaporation corresponding with the rate at which the feedwater was admitted. The steam pressure was controlled by means of the other boilers in the station.

19 Efforts were directed toward securing a representative ash sample from each test for the purpose of determining the quantity of combustible in the refuse. In spite of the elaborate precautions which were taken, it is felt that at least in one instance the sample was not as representative as had been hoped for. This applies particularly to test No. 6 which shows a loss due to radiation and

TABLE 4 COAL, FLUE-GAS, AND ASH ANALYSIS

Test No.	1	2	3	4	5	6
Rate of driving—Per cent of builder's rating	153	223	275	275	343	427
Proximate Analysis						
38 Moisture, per cent	5.87	4.61	5.05	5.20	5.85	5.48
39 Volatile matter, per cent	21.75	18.85	20.00	19 00	17.27	19.10
40 Fixed carbon, per cent	66 37	67.89	68.54	69.20	68.13	69.76
41 Ash, per cent	6.01	8.65	6.41	6.60	8.75	5.66
42 Fusion temperature of ash, deg. fahr.	2540	2675	2490	2780	2650	2592
B.T.U. Per Pound						
43 As fired	13840	13608	13940	13864	13397	13938
44 Dry	14704	14268	14681	14625	14223	14748
45 Combustible	15700	15690	15750	15710	15710	15690
Ultimate Analysis						
46 Sulphur, per cent	0.54	0.53	0.60	0.68	0.60	0.61
47 Hydrogen, per cent	4.53	4.42	4.48	4.45	4.32	4.51
48 Carbon, per cent	83.47	80.71	83.33	83.00	80.70	83.95
49 Nitrogen, per cent	1.21	1.22	1.23	1.19	1 25	1.19
50 Oxygen, per cent	3.87	4.04	3.61	3.79	3.83	3.78
51 Furnace refuse, per cent of coal fired (dry)	6.64	9.55	7.86	7.38	9.78	8.15
52 Combustible in furnace refuse, per cent	3 78	4 92	14.12	5.75	4 95	30.55
52a Carbon burned per lb. of coal.	0.8322	0.8024	0.8221	0.9257	0.8022	0.8133
53 Carbon dioxide, per cent, boiler outlet	13.7	14.1	14.5	14.7	14.5	15.3
54 Oxygen, per cent, boiler outlet	5 2	4.9	4.3	4.2	4.4	3.2
55 Carbon monoxide, per cent, boiler outlet	0.1	0	0.3	0	0	0.4
56 Carbon dioxide, per cent, economizer outlet	13.2	13.4	14.0	13.6	13.8	14.9
57 Oxygen, per cent, econ. outlet.	5.8	5.8	5.2	5 6	5 3	3.7
58 Carbon monoxide, per cent, economizer outlet	0.1	0	0	0	0	0.3
59 Ratio gas to coal by weight, boiler outlet	15 06	14.44	14.12	14.28	14.06	13.19
60 Ratio gas to coal by weight, economizer outlet	15.56	15.16	14.9	15.42	15.11	13.62
61 Excess air, per cent, boiler outlet	26.3	28.5	22.5	23.6	25.3	15.7
62 Excess air, per cent, economizer outlet	30.5	34.8	29.3	25.0	34.6	19.5

unaccounted-for of 8.3 per cent. It is believed that in this test the combustible in the refuse, as indicated in the table, was higher than actually found by analysis. Any one familiar with modern stokers with large-size clinker pits will readily appreciate the difficulty of securing a truly representative sample, especially when cooling water is employed on the rolls and aprons, and the ashes are sluiced by hydraulic means. The ash samples for the determination of combustible were withdrawn just below the clinker rolls, and there was little difficulty involved while the spray water to the clinker rolls was not employed. However, at high rates of driving it was necessary to employ the spray water to protect the

TABLE 5 CONDENSED SUMMARY OF RESULTS

Test number	1	2	3	4	5	6
Coefficient of heat transmission, economizer	7.53	7.45	6.03	6.44	7 82	7.53
Equivalent evaporation, boiler and superheater	66500	96700	119200	118800	148900	184040
Boiler horsepower developed, boiler and superheater	1930	2800	3460	3450	4300	5335
Per cent of rating, boiler and superheater	153	223	275	275	343	427
Equivalent evaporation, boiler, superheater and economizer	72500	105500	132800	132000	166900	205200
Boiler horsepower developed, boiler, superheater and economizer	2100	3050	3850	3830	4835	5950
B.t.u. liberated per cu. ft. of combination volume per hr	11750	17400	22150	22000	28190	37600
Efficiency of boiler and superheater, combined	84.6	82.2	79.5	80.3	78.3	70 5
Efficiency of boiler, superheater and economizer, combined	92.2	90 4	88.6	89.3	87.8	78.6

COMBINED HEAT BALANCE

	B.t.u.	%	B t.u.	%	B t.u.	%	B.t.u.	%	B.t u.	%	B t.u.	%
Heat per pound of dry coal	14704	100.0	14268	100.0	14681	100.0	14625	100.0	14223	100.0	14748	100.0
Heat absorbed by water in boiler	11330	77.1	10353	72.6	10360	70.6	10330	70.6	9650	67.8	9010	61.1
Heat absorbed by steam in superheater	1102	7.5	1375	9.6	1311	8.9	1423	9.7	1489	10.5	1381	9.4
Heat absorbed by water in economizer	1117	7.6	1164	8.2	1336	9.1	1316	9.0	1350	9.5	1194	8.1
Heat loss due to moisture in coal and combustion air	83	0.6	67	0 5	75	0.5	79	0 5	84	0.6	93	0.6
Heat loss due to water from combustion of hydrogen	450	3.1	444	3.1	465	3.2	455	3 1	443	3.1	570	3.9
Heat loss due to dry chimney gases	407	2.8	473	3.3	733	5.0	690	4.7	677	4.8	732	4.9
Heat loss due to escape of carbon monoxide	61	0.4	...	...	...	...	...	...	...	...	162	1.1
Heat loss due to combustible in furnace refuse	36	0.2	69	0.5	162	1.1	63	0.4	72	0.5	382	2.6
Heat loss due to radiation and unaccounted for	116	0.7	323	2 2	239	1.6	279	2.0	456	3.2	1223	8 3

equipment, and it is possible that some of the finer particles of coke and cinder were washed out of the sample as it was being withdrawn.

20 In referring to the test data in Table 4, indicating the composition of the flue gases leaving the boiler and leaving the economizer, it is at once obvious that the casing and communicating ducts were tight. Due to past experience with air infiltration between the boiler and economizer outlet, special precautions were taken in the design of this equipment to eliminate the waste which results from this source. In the condensed summary of results, Table 5, the coefficient of heat transmission for the economizer is indicated, but for some reason tests Nos. 3 and 4 did not show performance in line with the other tests. It is felt, however, that this may be due to difficulties in measuring the gas temperature entering the economizer indicating a higher mean temperature difference than actually existed, but in any event the performance of

the economizer from the heat-transfer standpoint appears very satisfactory.

Conclusions from Tests

21 Assuming the theoretical efficiency of a perfect boiler as 100 per cent, the efficiency ratios obtained by tests on these later Hell Gate boilers, superheaters, and furnaces run as high as 94 per cent. This is a ratio similar to the Rankine-cycle efficiency-ratio for a turbine. It is questionable, therefore, whether any considerable investment to improve this best-point efficiency is warranted. There remain, however, three important goals of endeavor. The first is to make the operating efficiency equal the test efficiency. This entails a perfect system of combustion control and a design requiring a minimum of maintenance. The second is to increase the range of high efficiency to allow of the accommodation to practical conditions of load variation and eliminations of banking losses. The third is to reduce the investment by increasing the overload capacity of the evaporating surfaces and coal-burning equipment, by reducing the furnace volume, and by more skilful design of the arrangement of apparatus.

DISCUSSION

B. N. Brodo.[1] When the first few pulverized-fuel installations were put into operation and high capacities and efficiencies were obtained, it was generally believed that this was due to the method of firing, i.e., to the use of pulverized fuel. The advocates of stokers have contended, however, that if stoker furnaces were provided with large combustion space and sufficiently large grate area, high efficiencies and capacities could also be obtained with stokers. The high efficiencies obtained with the latest boilers at Hell Gate, including superheater and economizer, of up to 92 per cent, have shown that with stokers, also, high efficiencies are possible. The only question that arises is whether or not these efficiencies can be obtained continuously in operation. We know that high efficiencies have been obtained in every-day operation with pulverized fuel.

The Hell Gate boilers, on which the tests given by the author were made, are provided with two completely water-cooled side walls, and the complete combustion and high efficiency obtained with rates of driving up to 430 per cent show that water-cooled walls do not have the bad effect on combustion that it is contended they have. To get the best results it has been stated that a temperature of at least from 2000 to 2200 deg. fahr. is necessary in a

[1] Consulting Engineer, The Superheater Company, New York, N. Y. Mem. A.S.M.E.

furnace, and that this cannot be attained if the furnace is lined with bare tubes containing water. The excellent results obtained at Hell Gate disprove this statement and should be used as an example that it is not necessary to cover water tubes located in the furnace. In the opinion of the writer, even more cooling surface could be placed in this furnace without detracting from the excellent results. This opinion is supported by the fact that high steam temperatures could be obtained with the superheater located between the first and second pass, which would indicate that the furnace temperatures were rather high and that even more cooling surface could be placed in the furnace and complete combustion still be obtained.

Only a few years ago, when the maximum rate of driving that could be obtained with a boiler was about 200 per cent of rating, engineers were wondering what could be done in order to increase the capacity of a boiler. It is well known that the first few tubes generate most of the steam, perhaps as much as 70 to 80 per cent, while the heat absorption in the last tube of the boiler is very low. The problem has been to make the last boiler tubes more effective. We can now operate boilers, even stoker-fired ones, up to 400 per cent of rating and over, and, as the author's tests show, with a boiler efficiency of more than 78 per cent, and a flue-gas temperature leaving the boiler of about 300 deg. fahr.

This high rate of driving could not be obtained if the last boiler tubes did not do more work as compared with the work done at the lower rate. It seems, therefore, that this additional work was performed only by reason of the high velocity in the last pass. While such high velocity requires additional draft, it makes the part of the boiler which usually does very little work more effective, and is an indication of how to increase capacities.

Attention is called to Fig. 11, showing superheater performances for boilers Nos. 63 and 73. Boiler No. 63 had been in operation for over a year, while No. 73 had been in operation for less than a month when the tests were made. The performances of these two superheaters can therefore hardly be compared. Experience with superheaters installed in new boilers has shown that the superheat gradually increases with the time the boiler is in operation. With a new boiler the boiler heating surface is clean inside and outside, and the temperatures are lowered considerably by the time the gases reach the superheater. After the boiler has been in operation for a considerable period, however, the heating surface becomes less effective, so that the gases reaching the superheater are usually at a higher temperature than at the beginning. This results in higher superheat. In order to compare the performances of the two superheaters, therefore, both should be in operation for approximately the same length of time.

L. J. Levit.[1] It would not be proper to credit all the improvement in the combined efficiency of boiler plant in recent years to the combustion system alone. Were it possible to determine accurately the various furnace losses for use in comparisons, several factors, which are not directly affected by the method of combustion, would be eliminated. In order to simplify the analysis of performance of the boiler, furnace, superheater, and economizer (or air preheater), the losses should be divided into three groups: necessary losses, boiler losses, and furnace and grate losses.

The first group, necessary losses, includes:

a Heat loss due to moisture in coal and vapor in the theoretical amount of air, up to the temperature of saturated steam

b Heat loss due to water formed by combustion of hydrogen, up to the temperature of saturated steam

c Heat loss due to the theoretical amount of dry gases, up to the temperature of saturated steam.

Necessary loss, as its name implies, is independent of the combustion system and is affected only by the grade of coal and the steam pressure in the boiler. It is analogous to the unavailable heat of the temperature-entropy chart. This analogy, however, is very rough. Consequently the necessary loss can be disregarded as far as the comparison of the combustion system is concerned.

The boiler losses, in a narrow sense, include:

a Heat loss due to the temperature elevation of the dry gases above the temperature of saturated steam

b Heat loss due to the temperature elevation of the water formed by the combustion of hydrogen above the temperature of saturated steam

c Heat loss due to the temperature elevation of vapor formed from the moisture in the coal, and from vapor in the combustion air, above the temperature of saturated steam.

The losses due to the moisture in coal and vapor accompanying the air for combustion are lumped together because their magnitudes are small as compared with the dry gas and hydrogen losses, even though they are treated differently in calculation.

Boiler loss is affected only in an indirect way by the furnace conditions, such as excess air and carbon monoxide. The former has a bearing on the exit-gas temperature, due to its effect on the gas velocity. Carbon monoxide will raise the flue-gas temperature if accompanied by secondary combustion.

In a perfect boiler the exit-gas temperature would be equal to that of saturated steam. Hence the loss due to the temperature

[1] Assistant Engineer, The United Elec. Light & Power Co., New York, N. Y. Jun. A.S.M.E.

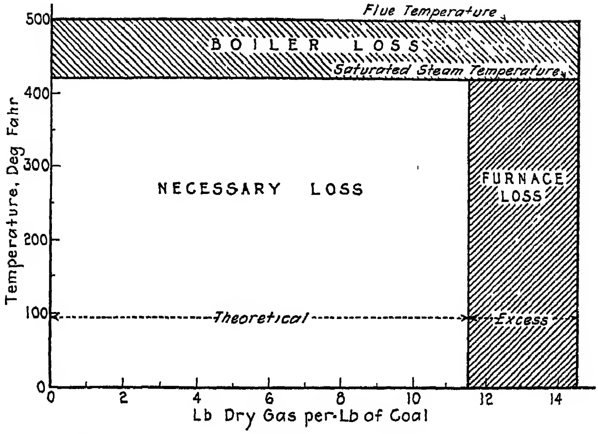


Fig. 15 Apportionment of Dry-Gas Losses

elevation of the products of combustion above the saturated-steam temperature should be charged to the boiler proper. This loss can be divided into two parts, one due to the temperature elevation of

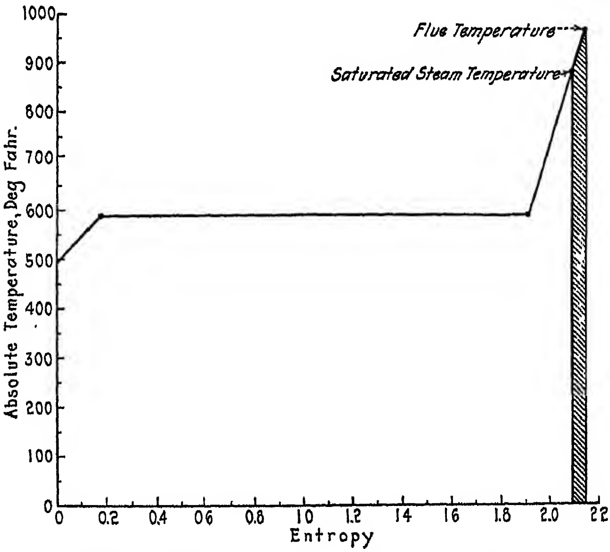


Fig. 16 Apportionment of Losses Due to the Combustion of Hydrogen

the theoretical amount of dry gas, and the other due to the excess dry gas on the same temperature-elevation basis. Some question might arise as to the status of the latter, i.e., whether it should

be charged to the boiler or to the furnace. Inasmuch as this loss would be equal to zero in a perfect boiler, it should be charged to the boiler proper. This is more or less arbitrary, however, and an argument in favor of charging this loss to the furnace has its merits.

The furnace and grate losses comprise:

a Heat loss due to excess air up to the temperature of saturated steam

b Heat loss due to combustible in the refuse

c Heat loss due to carbon monoxide

d Heat loss due to vapor in excess air for combustion

e Radiation and unaccounted for.

There is some dispute as to where the radiation and unaccounted for loss should be charged. Probably, a large portion of the loss, but not all of it, should be charged to the furnace. This loss includes combustible in refuse and combustible in cinders, as well as the loss due to hydrocarbons which frequently accompany high carbon monoxide in the flue gas.

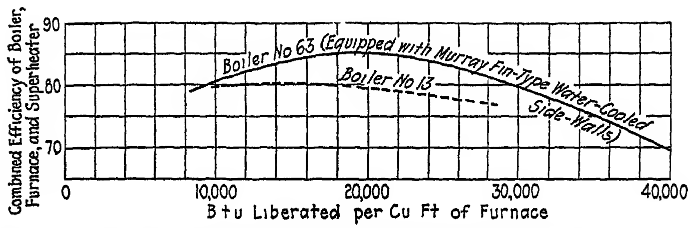


FIG. 17 RELATIVE EFFICIENCY OF BOILERS WITH REFRACTORY FURNACE AND WITH WATER-COOLED SIDE WALLS

This loss, however, also includes radiation and errors in estimating the furnace conditions at the beginning and the end of the test.

Fig. 15 shows how the dry-gas loss was apportioned as between necessary losses, boiler losses, and furnace losses. As before mentioned, there is some doubt as to the status of the small area which represents the loss due to the temperature of excess gas above the temperature of saturated steam.

If an economizer is used, the temperature of the entering feedwater can be substituted for the saturated-steam temperature. The introduction of an economizer has an effect which is roughly analogous to that of the condenser of a steam turbine.

Fig. 16 shows the apportionment of the loss due to the combustion of hydrogen between necessary losses and boiler losses. No portion of this loss is chargeable to the furnace, as the furnace conditions affect it only indirectly. An examination of the diagram shows that the boiler loss due to water formed by the combustion of hydrogen is very small indeed. Similar procedures can be resorted to in apportioning the losses due to moisture in the coal and

to the vapor in the combustion air. Inasmuch as the magnitudes of these losses are small as compared with the accuracy of other figures, it is hardly worth while to apportion them.

In order to bring out the improvement effected in boilers equipped with water-cooled side walls, the curves in Fig. 17, showing combined efficiencies of boiler, furnace, and superheater of boiler No. 13 and boiler No. 63, are given. As in the case of the curves shown by the author, which were plotted on the equivalent-evaporation basis, these curves show a remarkable improvement in the combined efficiency, as well as an extension of the economical operating range of boilers equipped with Murray fin-type side walls, as compared with the older installation.

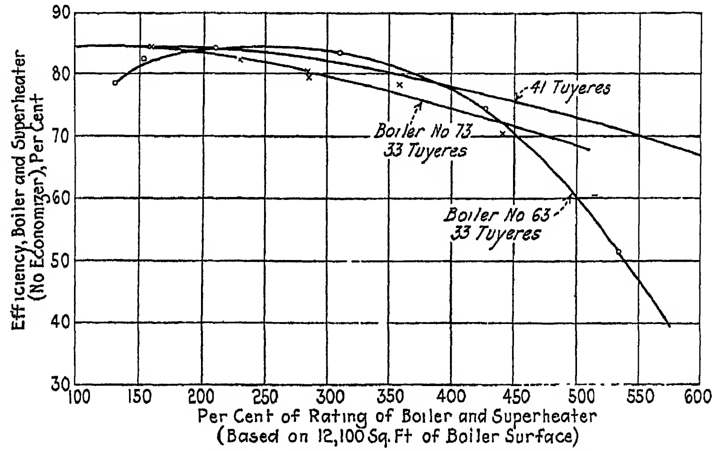


FIG. 18 EFFECT OF STOKER SELECTION ON EFFICIENCIES AT HIGH AND LOW RATES OF DRIVING

JOS. S. BENNETT, 3RD.[1] It is popularly believed that the fuel-burning equipment installed at Hell Gate represents the very latest and best in stoker practice. The author, however, points out that this is not the case, and that in adding the most recent boilers, consideration was given to installing 37-tuyere stokers instead of 33-tuyere stokers. However, on account of structural limitations, the shorter stoker was used.

Fig. 18 shows how the selection of the stoker proportions affects boiler performance. Boiler performance without an economizer is plotted for boilers Nos. 63 and 73, and a curve has been added showing the performance with a 41-tuyere stoker. It will be noted that the performance of boiler No. 63 exceeds that shown for a 41-tuyere installation up to 400 per cent of rating, while boiler No. 73

[1] Assistant Engineer, American Engineering Company, Philadelphia, Pa. Assoc-Mem. A.S.M.E.

falls somewhat below the 41-tuyere performance, but shows a curve with a flatter characteristic. These two curves are significant, showing as they do the development in improving the performance curve at higher rates of operation.

It is obvious that stoker performance does not depend on the boiler rating at all, since with a given steam output the boiler rating is determined entirely by the size of the unit installed. The author recognizes this fact in presenting his curves on the basis of hundreds of thousands of pounds of steam plotted against efficiency.

If the size of the stoker is increased, for example, to give 400 per cent of boiler rating with the same fuel-burning rate as would

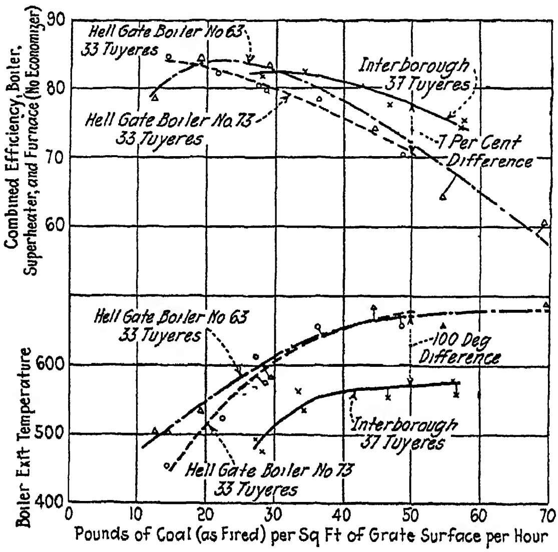


FIG. 19 COMPARISON OF EFFICIENCIES OF 33- AND 37-TUYERE STOKERS AT THE SAME FUEL-BURNING RATES

exist at 300 per cent with a smaller stoker, it is evident that the boiler efficiency should be at least as high, except for greater stack losses, when running at 400 per cent of rating.

The fact must be borne in mind that results such as are shown for the 41-tuyere stoker are not secured by the simple addition of grate area. If this were the case, the problem of the stoker designer would be very simple. With the longer stoker, the average energy-release rate per unit of area is increased. The " dead " or inactive areas are reduced in proportion to the whole area. The velocity of the coal flow is changed. Present designs offer means for reaching and burning out the combustible in the ash in hitherto dormant portions of the ashpit. The net result is the ability of the longer stoker to burn more coal and burn it more efficiently than the

shorter one. This fact has been realized by the industrial as well as the public-utility purchasers of stokers, and as a result an increasing number of industrial plants are taking advantage of the gains to be effected by the long stoker.

As a concrete example of the superiority of the long stoker, it is interesting to compare the tests at the Interborough Rapid Transit Company on a 37-tuyere stoker reported in the March, 1926, issue of *Mechanical Engineering,* page 246.[1] At a fuel-burning rate of 50 lb. per sq. ft. per hour, the efficiency on the Interborough tests was 77 per cent, and at Hell Gate 71 per cent for boiler No. 73. At that fuel-burning rate the boiler exit temperature at Hell Gate was 665 deg. fahr. and at Interborough it was 570 deg. fahr. Allowing a reduction of 2½ per cent for the difference in stack losses, this would show a net gain in efficiency for the 37-tuyere stoker over the 33-tuyere stoker of 3½ per cent.

Data for the above results were obtained by plotting boiler efficiency against pounds of coal per square foot per hour for both sets of tests. See Fig. 19.

The writer has dwelt largely on the improvement in boiler efficiency at the higher percentages of rating, but the fact must be recognized that for a considerable portion of the time, a boiler operates at a very low percentage of rating. This problem has been attacked by the stoker designer by means of air control, so that efficiencies can now be obtained at very low rates of operation that are comparable to results obtained at what was formerly regarded as the most efficient point.

The author has presented the true furnace efficiency in a very striking fashion in Fig. 10, in pointing out that the true furnace performance lies somewhere within the shaded area.

Efforts of power-plant engineers should be devoted to a study of the best means of reproducing the excellent results which have been demonstrated to be feasible. Adjustments which now depend upon the judgment of the fireman will in the future be made automatically, while the designer will arrange the furnace and select the fuel-burning equipment so that a smooth, uniform flow of the fuel will take place, making poor operating practically impossible.

The Author. Referring to Mr. Broido's contribution, it might be well to point out that the tests on boiler No. 63 were conducted after about the same period of operation as were those on boiler No. 73. Superheat readings taken about six months after the tests given in the paper, on boiler No. 73, and with similar rates of driving and conditions of flue gas, show a slightly lower, rather than a higher, value.

[1] New Boiler Equipment at I. R. T. Co.'s 59th St. Plant, by H. B. Reynolds, J. M. Taggart, and R. S. Lane.

The theoretical analysis of boiler performance as brought out by Mr. Levit contains considerable material which it would be well to bear in mind when making comparisons of furnaces and boilers of different characteristics.

The statement of Mr. Bennett that a boiler operates for a considerable portion of time at a very low percentage of rating would be more widely applicable if the performance curve of the stoker had a less drooping characteristic at the lower rates of driving. Modern methods of combustion control make it possible to operate boilers without difficulty considerably below normal rating, but as is frequently the case it is more economical to bank some of the boilers than run at the lower efficiency.

No. 2006

FLUID FLOW IN PIPES OF ANNULAR CROSS-SECTION

By D. H. Atherton,[1] Berkeley, Cal.
Associate-Member of the Society

The object of the investigations reported in this paper was to determine the actual values of the friction coefficients for the flow of air, oil, and water through pipes of annular cross-section. The apparatus and method of testing are described and the data and results of the tests are given in graphic and tabular form. It appears that the coefficient of friction corresponding to any given turbulence factor for both turbulent and viscous flow has a value for pipes of annular cross-section slightly higher than for pipes of circular cross-section when determined by the equivalent-diameter formula.

In problems of fluid flow, which involve the determination of the pressure drop or loss of head due to friction, the formula most commonly used for conditions of turbulent flow in pipes of circular section is the one variously attributed to Chézy, Fanning, or D'Arcy,[2]

$$\Delta p = \frac{4fwlV^2}{2gd} \quad \ldots\ldots\ldots\ldots \quad [1]$$

and for sections other than circular

$$\Delta p = \frac{fwlV^2}{2gm} \quad \ldots\ldots\ldots\ldots \quad [2]$$

in which

V = velocity, ft. per sec.
w = density, lb. per cu. ft.
u = viscosity, poundals-sec. per sq. ft.
l = length, ft.
d = diameter, ft., $4/d = 1/m$ for circular sections
f = coefficient of friction
m = mean hydraulic radius, ft.
Δp = drop in pressure, lb. per sq. ft.

[1] Assistant Professor of Mechanical Engineering, University of California.

[2] Equation [1] is often given by various authorities as

$$\Delta p = \frac{fwlV^2}{2gd}$$

in which the value of f is four times that given above.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

2 Values of the coefficient of friction f for flow in smooth pipes of circular section have been shown by the principle of dimensional homogeneity to be a function of the so-called turbulence factor or modulus, dwV/u. Data confirming this have been obtained by various authorities from extensive experiments covering a wide range of values for all of the variables concerned and show it to be true for all fluids whether liquid or gaseous, such as water, oil, air, steam, etc. This function is modified by the roughness factor of the pipe, but at present there is no method by which it may be evaluated. The values which have so far been established and which may be used with assurance are those for pipes which

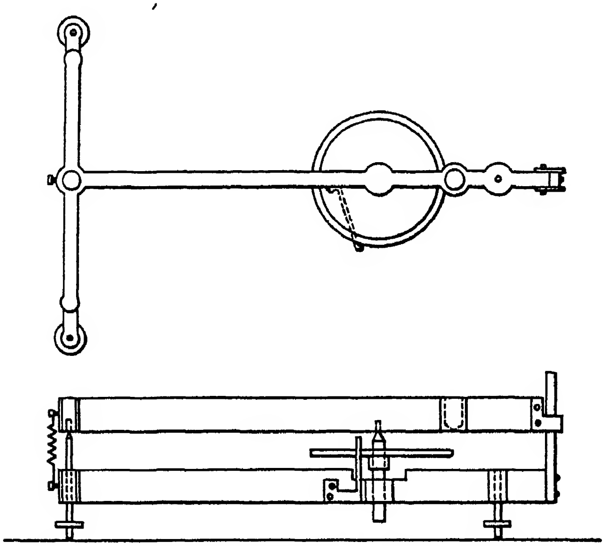


FIG. 1 CHATTOCK TILTING MANOMETER ASSEMBLY

have a smoothness or roughness factor equivalent to that of commercial brass tubing.

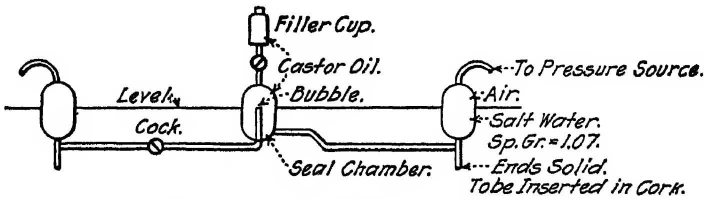
3 As far as known there are no experimental data available as to the value of the coefficient of friction f for use in the Chézy equation when it is applied to sections other than circular. It has been customary to determine from the hydraulic radius of the section in question, the diameter of a circular section having the same hydraulic radius and then substitute this equivalent diameter in dwV/u to obtain the corresponding turbulence factor. This having been obtained, the value of f for the same turbulence in circular sections is then used in the Chézy formula.

4 The object of the research reported in this paper is to determine the actual values of f for annular sections, there being need of such data in problems of fluid flow through such sections, as in heat-exchange apparatus of the double-tube type.

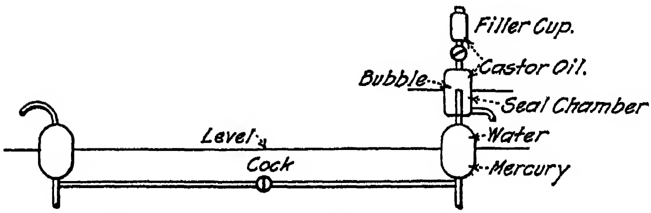
Design and Construction of Apparatus

5 In order to avoid the influence of the roughness factor, seamless brass tubes were used. These tubes were placed one inside of another thus forming an annular cross-section through which the fluid could flow. Provision was made so that there was no deflection of the inner pipe or rod which would cause a change in shape of the annular section.

6 *The Manometer.* A Chattock tilting manometer, shown in Fig. 1, was constructed following closely the description and data in London *Engineering* of Sept. 12, 1913. The glass parts, shown in Fig. 2, one for air and another for water, were constructed by an expert glass-blower. The instrument could be read, if desired, to 0.00006417 in. of water or 0.000002315 lb. per sq. in.



MANOMETER FOR DETERMINING AIR PRESSURE



MANOMETER FOR DETERMINING WATER PRESSURE

Fig. 2 Glass Work for Chattock Tilting Manometer

7 *The Test Pipe.* In order to insure a pipe of exact and uniform annular cross-section, it was necessary to devise a method by which one pipe would be supported centrally within another in such a manner that minimum disturbance of stream-line flow would result.

8 The supports could not be placed too far apart due to possible deflection of the inner pipe, and yet had to be placed as far as possible from the points at which readings were taken in order to avoid flow disturbances. No angular pipe fittings could be used near the test pipe due to the effect on stream-line flow.

9 After considering all factors, it was decided that the method given herein came nearest to meeting the requirements.

10 *Pipe Flanges and Piezometer Rings.* Small bronze female flanges, see Fig. 3, were made to fit standard $2\frac{1}{2}$-in. pipe. A flange (male both ways) was made to fit accurately between the two female flanges. This later flange was fitted with a small, cast, acorn-shaped center held in place by three cast arms. The cross-section of the arms was that of a flat ellipse, sharp at the edges. Two flanges each (1) and (2) were required for screwing on to the test pipe and connecting pipes. One flange each (3) and (4) were required for suspending the inner pipe. The cast, acorn-shaped center (4) of the flanges for supporting the inner pipe was drilled and tapped for an adjusting screw.

11 The inner pipe ends were plugged with brass and both pipe and plugs were then turned and carefully sized. The ends were

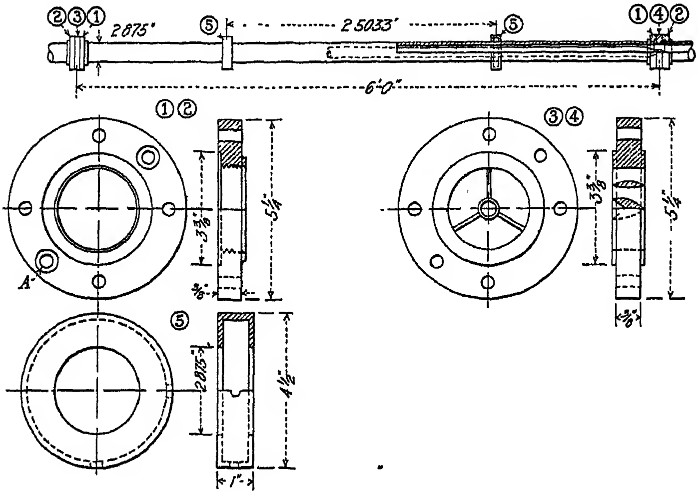


FIG. 3 FLANGES, PIEZOMETER RINGS, AND TEST-PIPE ASSEMBLY

turned to a 60-deg. acorn-shaped point in order that they might fit into the supporting flanges. This also insured perfect centering of one pipe within the other. All test pipes were of seamless, drawn, brass tubing. All flanges were carefully finished and rounded in order that stream-line flow might not be affected.

12 Cast piezometer rings (5) were soldered to the outer pipe at sufficient distance from the supporting flanges to give at least fifteen diameters distance from the very slight obstruction introduced by the supporting flanges. The supporting flange containing the adjusting screw was placed at the exit end of the test pipe.

13 In order to make sure that all the holes through the outer pipe into the piezometer ring would be in the same plane, they were drilled with the pipe in a lathe and the drill fastened to the tool rest. By means of machine screws the flanges (3) and (4) supporting the inner pipe could be independently connected to the

flanges (1) screwed on the outer pipe. The other two flanges (2) of the two sets were screwed to lengths (at least 20 ft. each way) of standard 2½-in. pipe.

14 The object of this arrangement was to facilitate the changing of the inner pipes for various test runs. Any one of the inner pipes could be inserted and adjusted to position, after which the two could be placed in the line. The bolts which passed through all three flanges of a set could then be inserted and tightened.

15 In order to facilitate tabulation of data the various pipes were designated by numbers as follows:

Pipe No. 1, the containing pipe on which were mounted the flanges and piezometer rings, inside diameter = 2.3125 in.

Pipe No. 2, the largest inside pipe, outside diameter = 1.85 in.

Pipe No. 3, the intermediate-sized inside pipe, outside diameter = 1.049 in.

Pipe No. 4, the smallest inside pipe, outside diameter = 0.840 in.

Method of Procedure

16 Tests were made with water, oil, and air flowing at various velocities through pipes forming various annular cross-sections.

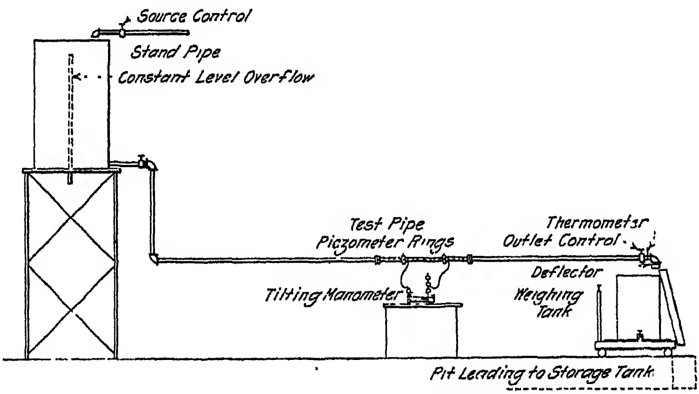


Fig. 4 Piping Diagram for the Flow-of-Water Investigation

Experiments with Flow of Water

17 In the experiments with water as the fluid, a standpipe of considerable volume was used, as shown in Fig. 4, provided with an overflow pipe and supply valve, so that a constant head could be maintained. The supply valve was controlled by one of the observers. A 3-in. pipe with control valve was led out of the bottom of the standpipe and then down to within 5 ft. of the floor. From that point a straight line of 2½-in. pipe 30 ft. in length led to the test pipe. From the test pipe 20 ft. more of 2½-in. pipe led to the outlet. At a point 18 in. from the outlet a control

valve was placed, and slightly farther toward the outlet a thermometer was inserted directly in the water. A deflector was arranged on the outlet so that the water could be directed either into the weighing tank or into the waste pit.

18 *The Water Measuring Scale.* A standard weighing tank was used consisting of an ordinary portable platform scale on which was placed a cylindrical metal tank of 20 cu. ft. capacity. The rate of flow was controlled by the valve at the outlet end of the pipe line, so that no obstruction to flow was introduced until after the fluid had passed through the test pipe. As has already been stated, an adequate length of straight pipe immediately preceded and followed the test pipe. This piping arrangement was used in all of the various tests with water, air, and oil.

19 A considerable number of test runs were made, the last being that of full flow with the control valve wide open, after which this valve was closed and static conditions were rechecked

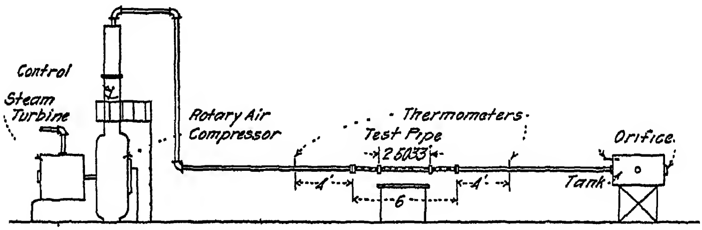


FIG. 5 PIPING DIAGRAM FOR THE FLOW-OF-AIR INVESTIGATION

against those obtaining at the beginning. All combinations of pipes together with various rates of flow were used, thereby obtaining data on various sizes of annular cross-section. As a check all tests were again run with an ordinary manometer substituted for the Chattock tilting manometer.

EXPERIMENTS USING AIR

20 In the experiments with air (see Fig. 5), the test pipe already described was connected to a turbo-compressor. Practically steady flow was secured by this means. In this case thermometer wells were introduced into the pipe line 4 ft. from the test pipe on either end. These wells formed a slight obstruction to stream-line flow but they were necessary in order that the temperature of the fluid might be obtained.

21 Varying rates of flow were secured by a damper control in the discharge line of the turbo-compressor and also by changing the orifices.

22 After considering several methods which might be employed in measuring the flow of air, it was decided that the method given

by R. J. Durley[1] exactly fitted the conditions. By this method the pounds per second discharged are

$$0.6299 \times Cd^2\sqrt{i/T}$$

in which

C = coefficient of discharge for various heads and diameters
d = diameter of orifice in inches
i = pressure drop between the air inside and outside of the tank in lb. per sq. ft.
T = absolute temperature in deg. fahr.

23 *Air Tank and Orifices.* A drum, the cross-section area of which was considerably more than twenty times the area of the largest orifice, was used. The drum was also long enough to take care of any velocity of approach resulting from the entrance of the air at the opposite end. One end was fitted with a flange so that it could be connected to the test-pipe discharge. The other

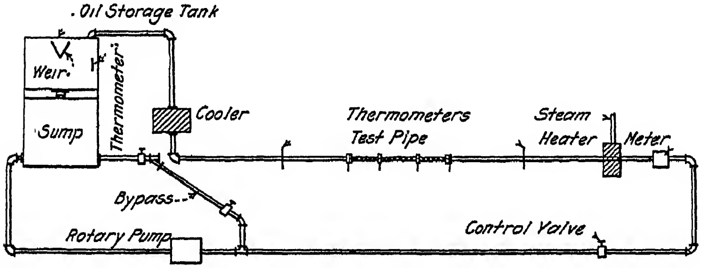


FIG. 6 PIPING DIAGRAM FOR THE FLOW-OF-OIL INVESTIGATION

end was cut out to receive the orifice plates. They were made of No. 15 B. & S. gage iron. Five orifices were used, their diameters being respectively 2.515, 2.006, 1.522, 1.002, and 0.500 in. A manometer connection was made on one side of the drum. A thermometer was inserted in a hole in the drum end.

24 In the experiments using air all possible combinations of test pipes were used.

EXPERIMENTS USING FUEL OIL

25 When using fuel oil the pipe line containing the test pipe was connected to a motor-driven rotary pump. Referring to Fig. 6 it will be seen that for regulating purposes the pump was piped to deliver oil either to the test pipe or through a bypass to the sump. Previous to reaching the test pipe, the oil passed through a heater in which its temperature could be regulated within reasonable limits, thereby controlling the viscosity.

26 After leaving the test pipe the oil passed through a cooler from which point it returned to the sump. The weir shown

[1] Trans., A.S.M.E., vol. 27 (1906), p. 193.

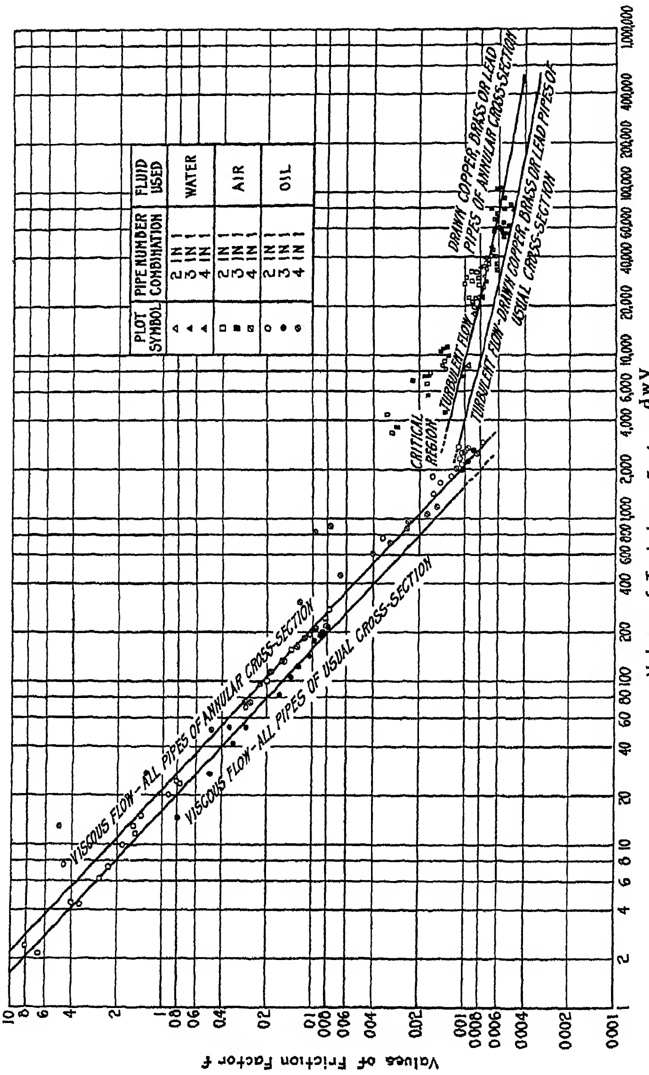


FIG. 7 RESULTS OF INVESTIGATIONS OF RELATION BETWEEN FRICTION AND TURBULENCE FACTORS OF FLUIDS FLOWING IN PIPES OF ANNULAR CROSS-SECTION AS COMPARED WITH SUCH FLOW IN PIPES OF USUAL CROSS-SECTION

(See note at bottom of p 153)

in Fig. 6 performs no function other than that of allowing the fluid to return quietly to the sump. The quantity of flow was measured by a calibrated Worthington piston-type meter.

27 The temperature of the oil entering and leaving the test pipe was measured by means of thermometers placed in the wells located 4 ft. each way from the test pipe as in the case of tests made with air. Extreme care and watchfulness were necessary in order to prevent any change of temperature during a run. The heater was equipped with a bypass which was an invaluable aid to this end.

28 Starting with the oil at room temperature, runs were made using pipe No. 2 in the containing pipe No. 1. The temperature

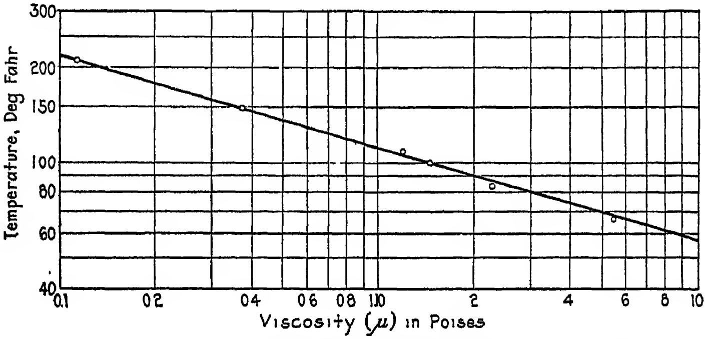


FIG. 8 VISCOSITY-TEMPERATURE CHART OF FUEL OIL, SAYBOLT VISCOSIMETER

$\mu = w\left(0.0022t - \frac{1.80}{t}\right)$
$= 1.453$ at 100 deg. fahr.
$= 0.1133$ at 210 deg. fahr.
$= 5.4718$ at 66 deg. fahr.
$(0.0672\mu = u)$

$\mu = 2.28$ at 84 deg. fahr.
$= 1.19$ at 108 deg. fahr.
$= 1.45$ at 101 deg. fahr.
$= 0.373$ at 148 deg. fahr.

was then raised 20 deg. fahr. and when constant conditions obtained another run and check run were made. This method was used throughout the desired temperature range, making use of all pipe combinations. The upper temperature limit was that at which evaporation of the oil began to be evident. The lower limit was that of room temperature.

29 A sample of fuel oil, as used in each run, was secured, and its viscosity determined by use of the Saybolt viscosimeter, while the specific gravity was determined by the Westphal balance. (See Fig. 8.)

NOTE TO FIG. 7. — The calculations necessary for solving the test results of this investigation are based on Equation [2] as developed by Chézy. This is the equation for turbulent flow inside sections other than circular. For use herewith the formula becomes, with proper substitutions,

$$f = \frac{2gm\Delta p}{wlV^2} \quad \text{and} \quad TF = \frac{4mVw}{\mu \times 0.0672}$$

In these formulas

TF = turbulence factor
u = viscosity poundals = μ in poises $\times 0.0672$

Manometers Used

30 All values of Δp in the data on water flow were obtained with the Chattock tilting manometer and were checked with an ordinary manometer. All values of Δp in the data on air and oil flow were obtained by using ordinary manometers. This was possible due to the selection of test-pipe size and the rates of flow.

Discussion of Results

31 Referring to Fig. 7, upon which have been plotted the results of the tests with air, water, and oil as shown in Tables 1 to 7, it will be seen that the points fall upon a line closely approaching that for similar conditions in circular pipes.[1] See Table 8.

32 The results obtained for the flow of air under various pressures, with corresponding changes in density, and for the flow of oil at different temperatures, with corresponding changes in viscosity, plot in consistently with the values obtained for the flow of water.

33 The curves plotted through the experimental points so obtained are throughout their length, both in stream-line and turbulent flow, parallel to and above those plotted for the values which have been established in the past for flow under similar conditions in smooth circular pipes.

Conclusion

34 In conclusion it would appear that the coefficient of friction corresponding to any given turbulence factor for both turbulent and viscous flow has a value for pipes of annular cross-section slightly higher than for pipes of circular cross-section, when determined by the equivalent-diameter formula.

35 The ratio between these two values, for any given turbulence factor, determined from the graph in Fig. 7, is

1 to 1.26 for turbulent flow

and

1 to 1.36 for viscous flow.

If then f is the friction factor of fluids flowing in pipes without inside cores and if f' and f'' are respectively the friction factors of turbulent and viscous flow in pipes containing inside cores, then

$$f' = 1.26f \text{ for turbulent flow}$$

and

$$f'' = 1.36f \text{ for viscous flow}$$

for use in formulas dealing with flow of fluids through pipes of annular cross-section.

[1] W. F. Durand, Hydraulics of Pipe Lines, p. 261; Flow of Fluids through Commercial Pipe Lines, *Journal of Industrial and Engineering Chemistry*, vol. 14 (1922), pp. 105-119, R. E. Wilson, W. H. McAdams, and M. Seltzer.

TABLE 1 DATA AND RESULTS OF WATER FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Weight of water, lb.	Cu. ft.	Time, sec.	Cu. ft. per sec.	V ft. per sec.	V^2 ft. per sec.	Δp in. of Hg	Δp lb. per sq. ft.	Temp. deg. fahr.	μ poises	w lb. per cu. ft.	f	TF
Pipe 2 in Pipe 1													
1	312.5	5	206.8	0.02416	2.31	5.34	0.2	13.05	70	0 00975	62.32	0 00952	8,520
2	312 5	5	99.0	0.0505	4.82	23.23	0.8	52.18	70	0.00975	62.32	0.00873	17,750
3	312 5	5	78.2	0.0639	6.10	37.21	1.2	78.3	70	0 00975	62.32	0.00818	22,450
4	312.5	5	65.5	0.0763	7.28	53.0	1.6	104.3	70	0.00975	62.32	0.00767	26,150
5	625.0	10	108.0	0.0926	8.83	77.97	2.3	150 0	68	0.01	62.32	0.00749	30,500
6	625.0	10	98.0	0.1020	9.74	94.87	2.7	176.1	65	0 0104	62.34	0.00723	33,650
7	625.0	10	91.4	0.1093	10.44	109.0	3 0	195 6	65	0.0104	62.34	0.00698	36,100
8	625.0	10	86.2	0.1160	11.07	122.30	3.4	221.7	65	0.0104	62.34	0.00706	38,200
9	625.0	10	81.6	0.1226	11.70	137.0	3.7	241.1	65	0 0104	62.34	0 00685	40,380
Pipe 3 in Pipe 1													
1	312.5	5	90.0	0.0556	2.40	5.76	0.064	4.18	65	0 0104	62 34	0 00787	22,550
2	312.5	5	52.0	0.0962	4.16	17.30	0.18	11.74	65	0 0104	62.34	0.00738	39,100
3	625.0	10	85.0	0.1178	5.08	25.81	0.24	15.66	62	0 0109	62.35	0.0066	45,500
4	625.0	10	65.0	0.1538	6.67	44.49	0.40	26 1	61	0 0111	62.35	0.00637	58,750
5	937.5	15	83.0	0.1806	7.82	61.15	0.540	35.23	60	0.0112	62.36	0.00625	68,300
Pipe 4 in Pipe 1													
1	312.5	5	61.0	0.082	3.24	10.50	0.1	6.52	64	0 01068	62.33	0.00787	34,600
2	312.5	5	36.5	0.137	5.42	29.38	0 2	13.05	64	0.01068	62.33	0 00565	57,800
3	625.0	10	55.0	0.182	7.20	51.84	0.3	19.56	64	0.01068	62.33	0.00480	76,700

Limit was total flow from source to standpipe.

TABLE 2 DATA AND RESULTS OF AIR FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Orifice diameter in.	w lb. air per sec.	Cu. ft. per sec.	C coef. dischg.	V ft. per sec.	V^2 ft. per sec.	Δp in. of water	Δp lb. per sq. ft.	Temp. air in tank deg. fahr.	Ave. test-pipe temp. deg. fahr.	Vol. 1 lb. air, cu. ft.	Tank manom. in.	$\mu \times 10^6$ poises	w lb. per cu. ft.	f	TF
								Pipe 2 in Pipe 1								
1	2.515	0.1127	1.6	0.5995	152.7	23400	12.9	67.0	79	103.5	14.18	1.2	192.6	0.0705	0.00994	32,200
2	2.515	0.1103	1.57	0.5995	149.7	22450	11.5	59.7	81	106.0	14.25	1.15	193.2	0.0670	0 00972	29,800
3	2.515	0.1002	1.43	0.5995	136.3	18600	9.9	51.4	82	106.0	14.25	0.95	193.2	0.0670	0.0101	27,250
4	2.515	0.000	....	0.60	....	....	8.9	46.25	81	102.0	14.15	0.0	192.2	0.0707	....	...
5	2.006	0.0484	0.202	0.60	19.27	371	0.7	3.64	68	80.0	13 6	0.05	186.6	0.0735	0.0327	4,360
6	2.006	0.0754	1.03	0.60	98.3	9650	5.2	27.0	68.5	82.0	13.65	1.8	187.0	0.0733	0.00933	22,100
7	2.006	0.0956	1.31	0.60	125.0	15625	8.1	42.5	70	84.5	13.71	2.1	187.7	0.0731	0.00903	27,900
8	2.006	0.1102	1.525	0.60	145.5	21200	10.3	53.6	71	89.0	13.82	2.8	188.9	0 0723	0.00854	32,050
9	2.006	0.1213	1.71	0 60	163.0	26569	11.5	59.8	74	98.0	14.05	3.4	191.2	0.0711	0.00776	34,800
10	1.522	0.0316	0.442	0.6005	42.2	1780	1.35	7.02	75	94.0	13.95	0.7	190.1	0.0718	0.0134	9,110
11	1.522	0.073	1.02	0.6025	97.4	2500	4.8	24.9	76	95.5	13.97	3.7	190.5	0.0714	0.00898	21,000
12	1.522	0.0932	1.31	0.6035	125.0	15625	7.1	36.9	76	97.0	14.02	6.0	190.9	0.0712	0.00812	26,850
13	1.522	0.1036	1.46	0.6046	139.0	19321	8.85	46.0	78	101.0	14.13	7.4	191.9	0.0708	0.00822	29,500
14	1.002	0.0286	0.401	0.604	38.2	1450	1.1	5 72	76	96.5	14.01	8.0	190.8	0.0713	0.01345	8,220
15	1.002	0.0678	0.95	0.622	90.6	8200	4.0	20.8	76	96.0	14.0	15.9	190.7	0.0716	0.00867	19,550
16	1.002	0.0732	1.025	0.6255	97.8	9560	4.75	24.65	76	97.0	14.02	18.36	190.9	0.0712	0.00885	21,000
17	1.002	0.0776	1.09	0.6285	104.0	10816	5.1	26.5	76	98.0	14.05	20.40	191.1	0.0711	0.00848	22,250
18	0.500	0.01153	0.1605	0.619	15.3	234	0.4	208.0	75	92.0	13.9	7.475	189.6	0.0720	0 0302	3,345
19	0.500	0.02315	0.321	0.6675	30.65	940	0.95	4.94	75	91.0	13 87	25.9	189.3	0.0721	0.01785	6,710
20	0.500	0.0267	0.372	0.6845	35.3	1247	1.2	6.24	74	90.0	13.85	32.7	189.1	0.0722	0 01696	7,740
21	0.500	0.0295	0.41	0.688	39.1	1530	1.2	6.24	73	89.0	13.82	34.6	188.8	0.0723	0.0138	8,612

$$W = 0\ 6299 C d^2 \sqrt{\frac{i}{T}}$$

Various readings per orifice were obtained by varying the air-compressor damper opening.

TABLE 3 DATA AND RESULTS OF AIR FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Orifice diameter in.	w lb. air per sec.	Cu. ft. per sec.	C coef. dischg.	V ft. per sec.	V^2 ft. per sec.	Δp in. of water	Δp lb. per sq. ft.	Temp. air in tank deg. fahr.	Ave. test-pipe temp. deg. fahr.	Vol. 1 lb. air, cu. ft.	Tank manom. in.	$\mu \times 10^6$ poises	w lb per cu. ft.	f	TF
								Pipe 3 in Pipe 1								
1	2.515	0.214	2.9	0.5982	125.2	15680	2.22	11.54	65	77.0	13.52	4.25	185.8	0.0738	0.00676	78,200
2	2.515	0.1845	2.505	0.599	108.2	11700	1.54	8.0	68	79.0	13.57	8.15	186.8	0.0736	0.00631	67,000
3	2.515	0.10	1.352	0.60	58.4	3412	0.02	0.104	68	77.5	13.53	0.0	185.9	0.0737	0.028	3,640
4	2.006	0.05	0.67	0.60	29.0	481	0.03	0.156	63	72.5	13.41	0.10	184.6	0.0747	....	18,400
5	2.006	0.155	2.085	0.60	90.1	8110	0.99	5.15	64	74.0	13.45	5.4	185.0	0.0745	0.0174	56,850
6	2.006	0.1755	2.38	0.60	103.0	10609	1.24	6.44	66	76.5	13.51	7.0	185.6	0.0738	0.00557	64,300
7	2.006	....	....	0.60	....	....	1.75	9.10	68	82.5	13.65	10.4	187.2	0.0733	..	....
8	1.522	0.1227	1.68	0.6005	72.6	5270	0.15	0.0779	67	79.0	13.57	0.3	186.8	0.0736	0.0136	4,490
9	1.522	0.1365	1.858	0.6075	80.3	6450	0.75	3.9	67	80.5	13.60	12.55	186.7	0.0735	0.00557	55,500
10	1.522	0.1525	2.086	0.6094	90.2	8140	0.90	4.75	69	83.5	13.68	15.5	187.2	0.0731	0.00532	55,850
11	1.522	0.1786	2.406	0.6102	104.0	10816	1.14	5.93	72	90.0	13.85	20.36	190.1	0.0722	0.00514	62,000
12	1.002	0.0207	0.282	0.6022	12.18	148.5	0.00	0.0	67	81.5	13.64	1.55	186.9	0.0734	...	....
13	1.002	0.0904	1.235	0.6308	53.35	2840	0.32	1.62	68	82.5	13.65	27.2	187.2	0.0733	0.0054	32,700
14	1.002	0.1005	1.38	0.6405	59.7	3565	0.39	2.03	69	84.5	13.71	32.6	187.6	0.0730	0.00733	36,500
15	0.500	0.01434	0.1945	0.6209	8.4	70.6	0.00	0.0	67	78.0	13.55	11.35	186.0	0.0737	....	5,225
16	0.500	0.0274	0.2715	0.6808	11.73	137.8	0.03	0.156	65	76.0	13.50	34.0	185.5	0.0743	0.0103	7,380
17	0.500	0.0282	0.38	0.6902	16.4	269.0	0.035	0.182	64	75.0	13.46	35.8	185.2	0.0744	0.00614	108,300
18	0.500	0.02872	0.386	0.6904	16.7	279	0.035	0.182	64	74.0	13.45	36.7	185.0	0.0745	0.00593	106,000
19	0.500	0.0289	0.389	0.6905	16.8	282	0.035	0.182	63	74.0	13.46	37.0	185.0	0.0745	0.00586	106,000

TABLE 4 DATA AND RESULTS OF AIR FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Orifice diameter in.	w lb. air per sec.	Cu. ft. per sec.	C coef. dischg.	V ft. per sec.	V^2 ft. per sec.	Δp in. of water	Δp lb per sq. ft.	Temp. air in tank deg. fahr.	Ave. test-pipe temp. deg fahr.	Vol. 1 lb air, cu. ft.	Tank manom. in.	$\mu \times 10^6$ poises	w lb. per cu. ft	f	TF
								Pipe 4 in Pipe 1								
1	2.515	0.249	3.69	0.5974	0.146	21316	13.5	10.2	72	129	14.83	5 8	199.3	0.0674	0.00561	90,280
2	2.515	0.201	2.995	0.5982	118.3	13970	9.4	7.275	102	132	14.90	4.0	200 0	0.0671	0.00613	72,600
3	2.515	0.156	2.32	0.599	91.7	8415	5.9	4.575	103	131	14.87	2.4	199 8	0.0672	0.00641	56,400
4	2.006	0.0204	0.297	0.60	11.74	138	0.25	0.2075	93	118	14.55	0 1	196 4	0 0686	0 0173	7,490
5	2.006	0.1174	1.71	0.60	67.6	4568	5.25	2.6	90	117	14.53	3.3	196.2	0.0687	0 00654	43,300
6	2.006	0.1634	2.38	0.60	94.2	8880	9.8	4.52	93	117.5	14.54	6.4	196 3	0.0687	0.00586	60,200
7	2.006	0.2126	3.12	0.60	123.2	15180	16.4	7.23	96	122:5	14.66	10.9	197.6	0.0681	0.00553	77,750
8	1.522	0.0265	0.383	0.6005	15.13	229	0.6	0.26	90	113	14.43	0.5	195.0	0.0693	0.01294	9,830
9	1.522	0.1075	1.553	0.605	61.4	3772	9.8	2.08	88	113	14.43	8.1	195.0	0.0698	0.0063	39,900
10	1.522	0.1426	2.06	0.609	81.4	6640	16 75	3 225	90	114	14.45	14.1	195.4	0.0692	0.00555	52,700
11	1.522	0.1781	2.61	0.614	103	10609	1.8	4.675	92	121	14.63	21.75	197.2	0.0683	0.00506	82,300
12	1.002	0.0195	0.286	0.6023	11.35	128.8	1.65	0.2075	88	109	14.33	1.55	194.0	0.0697	0.01825	7,470
13	1.002	0.0755	1.08	0.631	42.7	1825	1.55	1.17	86	108	14 30	20.35	193.7	0.0698	0 00726	28,100
14	1.002	0.0885	1.263	0.6172	50	2500	2.1	1.455	86	107	14.27	27.2	193.1	0.0699	0.006125	33,100
15	1.002	0.0962	1.372	0.62	54 3	2946	2.4	1 61	85	107	14.27	31.3	193.1	0.0699	0.006175	35,900
16	0.500	0.01843	0.595	0.6465	10 25	105	1.4	0 2075	78	95 5	14.08	17.7	190 5	0 0710	0 022	6,980
17	0.500	0.0277	0.387	0.692	15.3	234	2.6	0.3115	77	94	13.95	34.7	190.1	0 0718	0 01463	10,550
18	0.500	0.0283	0.395	0.6945	15 6	243.8	2.7	0.3115	76	93	13.93	36.05	189.9	0.0719	0.01405	10,800
19	0.500	0.0291	0.406	0 698	16.05	258	2.8	0.3115	76	93	13.93	37.4	189.9	0.0719	0.01325	11,100

TABLE 5 DATA AND RESULTS OF OIL FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Meter, cu. ft.	Time, sec.	Cu. ft. per sec.	Cu. ft. per sec., corrected	V ft. per sec.	V^2 ft. per sec.	Δp in. Hg	Δp lb. per sq. ft.	Ave. temp. deg. fahr.	μ poises	Sp. gr.	w lb. per cu. ft.	f	TF
							Pipe 2 in Pipe 1							
1	0.9	1800	0.0005	0.0005	....	....	....	....	....	....	....	....	....	..
2	0.7	360	0.00194	0.00225	0.2145	0.046	2.7	176.2	66	6.05	0.9405	58.6	15.9	1 197
3	0.5	240	0.0028	0.0037	0.353	0.125	3.06	199.8	68	5.5	0.9397	58.5	6.63	2.16
4	0.7	240	0.002915	0.003835	0.366	0.134	4.08	262.6	69	5.15	0.9393	58.5	8.135	2.39
5	0.6	120	0.005	0.0067	0.621	0.385	5.04	329.0	70.5	4.85	0.9388	58.4	3 55	4.32
6	0.9	180	0.005	0.0067	0.621	0.385	5.7	372.0	71	4.70	0.9387	58.4	4 015	4 44
7	1.6	240	0.00667	0.00872	0.832	0.693	6.66	435.0	72	4.50	0.9383	58.4	2.61	6.225
8	1.4	180	0.00778	0.0102	0.974	0.95	8.04	525.0	72	4.50	0.9383	58.4	2.295	7.28
9	1.3	180	0.00778	0.0102	0.974	0.95	9.24	603.0	72.5	4.35	0.9382	58.4	4.535	7.54
10	1.8	180	0.01	0.0132	1.26	1.585	10.62	693.0	73	4.25	0.938	58.4	1.815	9.95
11	2.1	180	0.0117	0.0155	1.48	2.19	12.19	795.0	73	4.25	0.938	58.4	1 506	11.7
12	2.4	180	0.0133	0.0171	1.63	2.66	15.11	986.0	73	4.25	0.938	58.4	1.54	12.9
13	2.7	180	0.015	0.0198	1.89	3.58	18.1	1180.0	73	4.25	0.938	58.4	1.37	14.94
14	2.4	120	0.020	0.0264	2.52	6.35	21.2	1384.0	73.5	4.2	0.9378	58.425	0 907	20.2
15	2.8	120	0.0233	0 0306	2.925	8.55	23.9	1560.0	73.5	4.2	0.9378	58.425	0.758	23 45
16	2.9	120	0.0242	0.0318	3.03	9.18	27.2	1775.0	73.5	4.2	0.9378	58.425	0.804	24.3
1	8.4	120	0.07	0.0934	8.92	79.6	22.224	1450	108	1.08	0.9263	57.65	0.07735	274
2	7 7	120	0.0642	0.0877	8.37	70.0	20.82	1360	107	1.13	0.9267	57.65	0.0824	242.5
3	7.2	120	0.06	0.08	7.63	58.15	19.8	1292	106 5	1.21	0 9268	57.65	0 0943	211
4	6.3	120	0.0525	0.07	6.68	44.6	16.74	1091	107	1.15	0.9267	57.65	0.104	193
5	5.1	120	0 0424	0.0565	5.39	29.05	14.40	940	107	1.15	0.9267	57.65	0.1373	156
6	4.3	120	0.03585	0.04775	4.56	20.8	11.46	748	107	1.15	0.9267	57.65	0.1525	132
7	3.3	120	0.0275	0.0367	3.5	12.25	8.7	567.5	107	1.15	0.9267	57.65	0.1965	101
8	3.4	180	0.0189	0.0252	2.406	5.79	5.76	375.3	107	1.15	0.9267	57.65	0.276	69.5
1	9.5	60	0.1583	0.211	20.14	406	15.69	1023	170	0.235	0.9054	56.4	0.01092	2785
2	8.6	60	0 1433	0.1911	18.23	332	12.24	800	170	0.235	0.9054	56.4	0 01043	2520
3	8.0	60	0.1332	0.1776	16.94	287	10.92	714	170	0.235	0.9054	56.4	0 01077	2340
4	7.4	60	0.1232	0.1643	15.7	243.2	9.00	588	170	0.235	0.9054	56.4	0.01045	2170
5	6.2	60	0.1033	0.1377	13.12	172.5	7.56	493	170	0.235	0.9054	56.4	0.01237	1815
6	5.4	60	0 090	0.120	11.45	131.2	6.60	437	170	0.235	0.9054	56.4	0 01443	1660
7	4.6	60	0 0767	0.1023	9.77	95.5	5.46	356	170	0.235	0.9054	56 4	0 01615	1415
8	5.5	120	0.0458	0.0611	5.715	32.7	4.02	262.5	168	0.245	0.906	56 4	0 0347	764

TABLE 6 DATA AND RESULTS OF OIL FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Meter, cu. ft.	Time, sec.	Cu. ft. per sec.	Cu. ft. per sec., corrected	V ft. per sec.	V^2 ft. per sec.	Δp in. Hg	Δp lb. per sq. ft.	Ave. temp. deg. fahr.	μ poises	Sp. gr.	w lb per cu. ft.	f	TF
							Pipe 3 in Pipe 1							
1	13.5	120	0.1124	0.150	6.48	41.9	4.5	293.3	83.5	2.71	0.9345		0.0814	218
2	6.7	120	0.1116	0.1488	6.43	41.6	4.32	282.0	83.5	2.71	0.9345		0.0792	216
3	6.2	60	0.1033	0.1378	5.96	35.5	4.02	262.0	83.5	2.71	0.9345		0.086	200.3
4	11.8	120	0.0983	0.1311	5.665	32.1	3.78	245 0	83.5	2 71	0.9345		0.0888	190.4
5	10.8	120	0.090	0.120	5.185	26.85	3.42	223 0	83.5	2.71	0.9345		0.0967	177.7
6	9.8	120	0.0817	0.109	4.71	22.2	3.06	199.5	84.0	2.65	0.9343		0.1046	142.6
7	8.5	120	0.0708	0.0944	4.07	16.6	2.7	176 0	84.0	2.65	0.9343	58.15	0.1235	123.4
8	7.3	120	0.0608	0.0811	3.5	12.25	2.22	144.8	84.0	2.65	0.9343		0.1376	106.0
9	5.7	120	0.0475	0.0633	2.73	7.46	1.62	105.8	84.0	2.65	0.9343		0.165	82.6
10	3.6	120	0 030	0.040	1.726	2.975	1.38	90.1	84.0	2.65	0.9343		0.353	52.8
11	3.6	120	0.030	0.040	1.726	2.975	1.08	70.5	84.0	2.65	0.9343	.	0.276	52 8
12	2.9	120	0.02415	0.032	1.381	1.91	0.84	54.8	84.0	2.65	0.9343		0.334	41.9
13	2.8	180	0.01554	0.0207	0.894	0.8	0.504	32.9	84.0	2.65	0.9343		0.479	27.1
14	1.0	120	0.00833	0.01113	0.481	0.231	0.24	15 65	84.0	2.65	0.9343		0.788	14.57
1	9.0	60	0.150	0.20	8.64	74.7	0.84	54.8	158.5	0.294	0.9092	56.6	0 00878	2610
2	8.0	60	0.1332	0.1776	7.67	58.8	0.72	47.0	157.5	0.305	0.9097	56 6	0 00958	2225
3	7.0	60	0.1167	0.1556	6 72	45.15	0.60	39.1	158.0	0.30	0.9095	56.6	0.01037	1990

Blow-out of tubes leading from test pipe to manometer at run 7 in this group made it advisable to discard remainder of data.

TABLE 7 DATA AND RESULTS OF OIL FLOW IN PIPES OF ANNULAR CROSS-SECTION

Run No.	Meter, cu. ft.	Time, sec.	Cu. ft. per sec.	Cu. ft. per sec., corrected	V ft. per sec.	V^2 ft. per sec.	Δp in. Hg	Δp lb. per sq. ft.	Ave. temp. deg. fahr.	μ poises	Sp. gr.	w lb. per cu ft.	f	TF
							Pipe 4 in Pipe 1							
1	6.0	60	0.10	0.1333	5.27	27.8	3.0	195 7	84	2 65	0 9343		0.0953	211.3
2	5.4	60	0.09	0.120	4.75	22.55	2.88	188.0	83	2.75	0.9347		0.113	184.0
3	5.0	60	0.0834	0.1113	4.41	19.41	2.76	180.0	82	2 9	0.935		0.1256	161.7
4	4.3	60	0.0717	0.0956	3.775	14.25	2.58	168.2	81	3.0	0.9354		0.160	133.8
5	3.7	60	0.0617	0.0823	3.25	10.58	2.28	148.8	81	3.0	0.9354		0.1907	115.0
6	4.8	90	0.0534	0.0712	2.815	7.93	1.98	129 2	80	3.1	0.9357	58 25	0.221	96.6
7	3.8	90	0.0422	0.0563	2.225	5.065	1.62	105.7	80	3.1	0.9357		0 283	76 25
8	4.0	90	0.0412	0.055	2.173	4.72	1.38	90.0	80	3 1	0.9357		0 2585	74 5
9	2.5	120	0.0280	0.0373	1.474	2.173	1.14	74.4	80	3.1	0.9357		0.472	50.6
10	2.7	180	0.015	0.020	0.791	0.625	0.9	58.65	80	3.1	0.9357		1 273	27.15
11	1.7	240	0.0071	0.0095	0.3752	0.141	0.78	50.9	80	3.1	0.9357		4 90	12.88
1	9.3	60	0.155	0.2067	8.17	66.8	1.14	74 4	122	0.73	0.9217		0.0153	1172
2	8.4	60	0.14	0.1867	7.38	54.5	1.08	70 5	122	0.73	0 9217		0 0178	1063
3	7.5	60	0.125	0.1667	6 585	43.4	1 14	74.4	122	0 73	0 9217		0 02355	948
4	6.8	60	0.1133	0.1511	5.98	35.75	0.96	62 64	122	0 73	0.9217	57.5	0.0241	862
5	5.6	60	0.0933	0.1244	4.92	24.25	0.84	54.8	122	0 73	0.9217		0 03105	708
6	4.8	60	0.08	0.1067	4.215	17.86	0.803	52.4	122	0.73	0.9217		0.04035	607
7	7.1	120	0.0592	0.079	3.123	9.76	0.72	47.0	122	0.73	0.9217		0.0662	450
8	3.6	90	0.040	0.0533	2.105	4.44	0.60	39.1	121	0.76	0.922		0.121	303
1	8.4	60	0.14	0.1867	7.38	54.4	0.6	31.9	158	0.300	0.9095		0 01003	2545
2	9.6	60	0.16	0.2134	8.45	71.4	0.6	31.9	158	0.300	0.9095		0.00765	2913
3	8.6	60	0.1433	0.1911	7 565	57 25	0.6	31.9	159	0.292	0.909		0 00953	2675
4	8.0	60	0.1332	0.1776	7.03	49.4	0.456	29.7	159	0.292	0.909	56.63	0.00838	2490
5	6.6	60	0.11	0.1467	5.8	33.64	0.42	27.4	159	0.292	0.909		0.01137	2050
6	5.8	60	0.0967	0.130	5.14	26.45	0.48	31 3	158.7	0.292	0.9092		0.0165	1820
7	9.0	120	0.075	0.100	2.528	6.4	0.54	35.2	159	0.292	0.909		0.0767	895
8	5.3	120	0.04415	0.0589	2.33	5.43	0.576	37.55	159	0.292	0.909		0.0966	825

TABLE 8 TURBULENT FRICTION FACTORS FOR VISCOUS AND TURBULENT FLOW

From Hydraulics of Pipe Lines, Vol. 261, by W F. Durand of Stanford University.

TF	*f*	*f*/4	*TF*	*f*	*f*/4
200	0.3200	0.0800	25000	0.0249	0.0062
400	0.1600	0.0400	30000	0.0238	0.0059
600	0.1067	0.0267	35000	0.0228	0.0057
800	0.0800	0.0200	40000	0.0219	0.0055
1000	0.0640	0.0160	45000	0.0213	0.0053
1200	0.0533	0.0133	50000	0.0208	0.0052
1400	0.0457	0.0114	60000	0.0200	0.0050
1600	0.0400	0.0100	70000	0.0195	0.0049
1800	0.0355	0.0089	80000	0.0190	0.0047
2000	0.0320	0.0080	90000	0.0185	0.0046
2500	0.0442	0.0110	100000	0.0180	0.0045
3000	0.0426	0.0106	150000	0.0168	0.0042
3500	0.0412	0.0103	200000	0.0158	0.0039
4000	0.0400	0.0100	250000	0.0150	0.0037
4500	0.0390	0.0097	300000	0.0144	0.0036
5000	0.0382	0.0095	350000	0.0140	0.0035
6000	0.0364	0.0091	400000	0.0137	0.0034
7000	0.0350	0.0088	450000	0.0134	0.0033
8000	0.0340	0.0085			
9000	0.0330	0.0082			
10000	0.0320	0.0080			
12000	0.0304	0.0076			
14000	0.0292	0.0073			
16000	0.0280	0.0070			
18000	0.0271	0.0068			
20000	0.0264	0.0061			

TF = turbulence factor
f = friction factor

Divide *f* by 4 to get *f* in $\Delta p = \frac{flV^2w}{2gm}$

DISCUSSION

Lewis F. Moody.[1] The author states that the object of the investigation was to determine the actual values of the friction coefficients for pipes of annular cross-section. If the work were limited to merely finding the friction coefficients for certain pipes of a particular material, the conclusions would be of only limited interest, as they would not throw any light on general laws of resistance as affected by the form of the cross-section.

What would be of real interest would be to determine the effect of the shape of the section on the coefficients of resistance for pipes of exactly the same material. The author attempts to reach a conclusion on this point as indicated by his statement that: "It appears that the coefficient of friction . . . has a value for pipes of annular cross-section slightly higher than for pipes of circular cross-section." The writer does not find, however, in the data presented, any basis for such a conclusion. No data are given for any experimental determination of the loss in a pipe of equivalent diameter and of plain circular cross-section with which the loss in the pipes of annular cross-section could be compared, and in fact there are in the paper no data even for the loss in the outside pipe actually used, with the central cores omitted.

[1] Consulting Engineer, I. P. Morris Dept., Wm Cramp & Sons Ship & Engine Building Co., Philadelphia, Pa. Mem A.S.M E.

Apparently the work was limited to measurements of three different annular cross-sections, and no test was made on a plain circular pipe either of the full outside diameter or of equivalent diameter. To form a proper basis for the conclusions stated, such reference tests would be necessary.

The proper basis of comparison would be furnished if, corresponding to each annular cross-section, a pipe of equivalent circular section, and having the same hydraulic radius, had been tested. The diameter of the equivalent circular pipe would then be equal in each case to the difference of the diameters of the outer and inner surfaces of the annular pipe.

It is not proper to compare the results with those calculated from previous tests of other observers under different conditions. A slight difference in the pipe material, the condition of the surface and other conditions of measurement could readily introduce wide errors, and the only proper basis for comparing the annular cross-sections with circular cross-sections would be to test both shapes of section under identical conditions and with identical material.

As far as the tests themselves would indicate, there is no consistent change in the coefficient due to different sizes of core or differences in the annular shape as distinguished from a change in the hydraulic radius. The writer has plotted on logarithmic cross-section paper the observations for the flow of water in the pipes tested as shown in Table 1. These results agree closely with the exponential form of equation, the loss of head varying directly as the velocity to the 1.8 power and inversely as the diameter to the 1.2 power, using for the diameter the equivalent diameter of a plain circular pipe of the same hydraulic radius. This conclusion is not inconsistent with the variation of frictional loss in pipes of plain circular section.

From the above considerations, the writer is not able to accept the conclusion mentioned in the paper, that " the coefficient of friction . . . has a value for pipes of annular cross-section slightly higher than for pipes of circular cross-section " because he does not find in the paper experimental data which would justify any conclusion as to this point one way or the other.

W. R. Eckart.[1] The experimental work carried out in the laboratory at Stanford University and reported in Professor Atherton's paper was performed, at the writer's suggestion, as a thesis subject. He therefore makes the following comments bearing upon Mr. Moody's discussion.

It would be desirable, of course, if possible, in making comparisons of this kind to test tubes of both cross-sections, annular and circular, under identical conditions and with identical materials,

[1] Professor of Mechanical Engineering, Stanford University, Cal. Mem. A.S.M.E.

each such annular section being compared with a circular section of the same mean hydraulic radius. But are such requirements absolutely essential?

Tubing to meet such requirements and having the exact necessary inside and outside diameters would be difficult to obtain, unless of special manufacture, and even then it would probably not be possible to secure identical conditions of surface.

To obtain identical effects in pipes or tubes of different diameters, the surface irregularities would have to be geometrically similar in all cases. That is, a certain degree of roughness in a pipe of small diameter would have a greater effect than the same degree of roughness in a pipe of larger diameter. Even in the same pipe, the degree of roughness or its influence has been found to be different with the same conditions of flow, if the direction of flow is reversed.

It is recognized that in the application of the laws of dimensional similarity to an investigation of this kind,[1, 2, 3] the coefficient of friction f is dependent upon five factors, namely, mean velocity of flow, V; density of fluid, w; viscosity of fluid, u; diameter of pipe, d; and the roughness factor. There is at present, however, no way of defining or measuring the character of the surface or the degree of roughness.

If we omit the roughness factor as indeterminate and consider only tubes of the same degree of roughness, then f is some function of dwV/u.

With the latter fact in mind, and in order to minimize the influence of roughness as much as possible, the experiments were conducted, as stated, with seamless brass tubes. A large amount of experimental data is available, showing extremely consistent results for fluid flow in commercial tubing of this material. This would not be so if the character of the surface varied materially or if the degree of roughness was great enough to have any decided influence.

With reference to this, Stanton and Pannell,[4] of the National Physical Laboratory of Great Britain, state:

> For very accurate comparison the surfaces of the tubes should have been precisely geometrically similar, as regards roughness, but as this condition could not be fulfilled, the experiments were all made on commercially smooth-drawn brass pipes. From the general agreement of the results of different pipes it does not appear that slight irregularities in this respect have a marked effect on the resistance within the range of the diameters here used.

These experimenters used four different diameters of tubing and further compared their results with those of investigations

[1] Durand, Hydraulics of Pipe Lines, p. 257.

[2] Buckingham, Trans. A.S.M.E., vol. 37, 1915, p. 266.

[3] Wilson, McAdams, and Seltzer, *Jl. Ind. & Eng. Chem.*, vol. 14, no. 2, Feb. 1922, p. 105.

[4] Phil. Trans. of Royal Society, vol. 214A (1914), p. 202.

made by Saph and Schoder[1] at Cornell University some years previously, who used tubes of seven other diameters.

In this case the tubes were undoubtedly of different manufacture, of slightly different material, slightly different surface conditions, the test conditions not exactly the same, and the observers different.

As to the concordance of their own results, Stanton and Pannell[2] state:

Throughout the whole of this range in vd/ν of from 2500 to 470,000, with the exception of a few individual determinations due possibly to errors of observation, the variation of $R/\rho v^2$ for either fluid, in any of the four pipes, from its mean value does not exceed 2.0 per cent, so that the similarity of motions over this range is fully demonstrated.

In the above, $\frac{vd}{\nu} = \frac{Vdw}{u}$

where $v = V =$ velocity of flow

$\nu =$ kinematic viscosity $= \frac{u}{w}$

and $\frac{R}{\rho v^2} = \frac{R}{wV^2} = \frac{f}{2}$

where $R =$ resistance per unit of surface

$\rho = w =$ density of fluid

$v = V =$ velocity of fluid.

The fluids were air and water.

As to the comparison of their results with those of Saph and Schoder,[3] they state that "the plottings of these, with very few exceptions, lie within the limits of the present experiments, of which they form an excellent check."

The present investigations cover the regions of stream line or viscous flow, turbulent flow, and the critical region between. In the stream line region an additional fluid, oil, was used. In the stream line region the influence of the character of the surface and material is in any case practically negligible, as the flow is purely viscous and independent of the nature of the surface.

Wilson, McAdams, and Seltzer[4] state:

There can be no real doubt as to the essential validity of Poiseuille's formula for viscous flow in pipes of commercial size and roughness. Since the same law has been checked up very closely for all types of liquids in extremely smooth glass and metal tubes, it is evident that

[1] Proc. A.S.C.E., 1903, vol. 51, p. 253.

[2] Ibid., p. 207.

[3] Some of the data from the Saph and Schoder tests were used by Buckingham, Trans. A.S.M.E., 1915, p. 266, in plotting his Fig. 1, p. 268.

[4] *Jl. Ind. & Eng. Chem.*, vol. 14, no. 2, Feb. 1922, p. 105.

the roughness of the pipe is a comparatively small factor in determining the resistance to viscous flow—as might indeed be anticipated on theoretical grounds, since there is in any case a practically stationary and fairly thick film of oil in the inside surface of the pipe.

Professor Durand[1] states that "regarding the influence of roughness on these values, it appears that for stream line flow the character of the surface of the pipe seems to have but slight influence."

The data for circular tubes of seamless-drawn brass, on which the author based his comparisons and which are published in Table 8, in which the turbulence factor TF is dwV/u and the values for f for the present paper are those in the column headed $f/4$, were taken from Professor Durand's book.[2]

These data were in turn compiled from curves plotted by the writer in 1916-1917 on a working chart of large scale, upon which all the data available at that time, relating to the flow of fluids, including air, water, and oil, in pipes of all degrees of roughness were recomputed on this basis. The present tabulation represents the mean of the results for seamless-drawn brass tubes, in which the data of Stanton and Pannell predominate. In Fig. 7 of the author's paper these curves are reproduced on logarithmic paper, being the lower of the two sets of curves. They represent the variation of the coefficient of friction f with the turbulence factor dwV/u assuming that, for practical and engineering purposes at least, we may consider that the surface characteristics of seamless-drawn brass tubing do not vary sufficiently to have a marked influence on the results.

As to the necessity for conducting tests with any particular diameters, we may take into consideration that one of the most important purposes of applying the laws of dimensional similarity to investigations of this nature is that it enables us to make allowance for the effect of all variables which enter into our problem. In this case, since the coefficient of friction f is some function of the turbulence factor of dwV/u, we have four variables to consider, each of which has equal weight. Equivalent relative changes in the variation of any of these variables, which give the same numerical value of the turbulence factor will result in the same value for the coefficient of friction, and this holds no matter what the fluid, provided that in the critical region or in turbulent flow the roughness of the surface is the same. That is, we can produce effects or conditions of turbulence, upon which the coefficient of friction depends, in a pipe of any diameter similar to those which would occur in another pipe of some particular diameter by a relative modification of the other three variables either independently or together, as we desire. We can do this

[1] Hydraulics of Pipe Lines, p. 257.
[2] Hydraulics of Pipe Lines.

by varying the velocity of flow, or changing the temperature, which will change both the density and viscosity, or we can change the fluid which will also change the order of the density and viscosity involved, and with the new fluid can further modify these by change of temperature. That is, we can produce a wide range of effects, with a single diameter of pipe which would be similar to those which would be due to relative variations in diameter.

In viscous flow it will be noted that the curve, Fig. 7, is a straight line sloping to the right at an angle of 45 deg. The slope of the line u is -1 and the intercept b on the y axis, which is constant, is 16.

$$f = b\left(\frac{dwV}{u}\right)^{-1} = 16\left(\frac{u}{dwV}\right)$$

if we substitute this value of f in the formula for pressure drop we have

$$\Delta p = \frac{4fwlV^2}{2gd} = 16\left(\frac{u}{dwV}\right)\cdot\frac{4wlV}{2gd} = 32\,\frac{ulV}{gd}$$

which gives us the equation for Poiseuille's law for viscous flow and "which may be derived[1] from the definition of absolute viscosity by the aid of calculus and which has been experimentally confirmed for diameters ranging from those of capillary tubes up to 12-in. pipe."

In the turbulent region the line is curved and the equation for f would probably have a form

$$f = c + b_1\left(\frac{dwV}{u}\right)^n$$

where n is again negative and c and b_1 are again constants.

In this connection Stanton and Pannell[2] state that "it may therefore be taken as fully demonstrated that an index law for surface friction cannot be devised which will express the facts with any accuracy, except over a comparatively small range in the value vd/ν."

Only over small ranges of value of the turbulence factor dwV/u, then, is it possible to make the assumption that within that range the curve is a straight line having a slope n (negative), so that the equation for this range may be taken as

$$f = b_2\left(\frac{dwV}{u}\right)^n$$

[1] Walker, Lewis, and McAdam, Principles of Engineering Chemistry, p. 74.

[2] Phil. Trans. of Royal Society, vol. 214A, p. 202.

The values of b_2 and n will depend upon the particular section of the curve taken, n decreasing as the turbulence increases until it finally becomes zero, when the curve becomes parallel to the x-axis $\left[\left(\frac{dwV}{u}\right)^0 = 1\right]$, and f becomes constant and independent of temperature changes and equal to b_2, the intercept on the y-axis, and

$$\Delta p = \frac{4fulV^2}{2gd} = \frac{4b_2wlV^2}{2gd}$$

In the critical region the situation is very unstable, the value of f depending upon the indeterminate variation in the conditions, such as whether the turbulence is gradually increasing or decreasing, upon the roughness factor, or any other disturbing factor such as vibrations, etc. When the flow is gradually increasing the tendency will be for the stream line condition to persist below the lower critical point, until a sudden change occurs causing the value of f to increase rapidly. The general slope of the curve in this region, when conditions permit, is approximately perpendicular to the stream line curve, so that n is positive and may be of the order of 1; then,

$$f = b_3\left(\frac{dwV}{u}\right)$$

and

$$\Delta p = \frac{4b_3}{2g}\,\frac{lw^2V^3}{u}$$

With the turbulence factor gradually decreasing, the turbulent condition may continue along even until the lower critical velocity is reached, or it may break and drop to stream line at any time between the upper and lower critical points.

Mr. Moody, plotting the values from Table 1 for water flowing in the annular section, found that these results agree closely with the exponential form of the equation, the loss of head varying directly as the velocity to the 1.8 power and inversely as the diameter to the 1.2 power, using for the diameter the equivalent diameter of a plain circular pipe of the same hydraulic radius.

This is correct, the flow of water in this case being in the region of turbulent flow (TF varying from 8520 to 76,700), and the loss of head varies as stated. Similarly for the same data, it will be found that the coefficient of friction f varies directly as the 0.2 power of the viscosity and inversely as the same power of the density, diameter, and velocity. That is, plotting the values of f as ordinates and dwV/u as abscissas on logarithmic paper, as in Fig. 7, a straight line may be drawn for the short range of dwV/u given, which may be considered as fairly representative

of the variation of f with dwV/u. The slope of this line then will be $n = -0.2$ and

$$f = b_4 \left(\frac{dwV}{u}\right)^{-0.2} = b_4 \left(\frac{u}{dwV}\right)^{0.2}.$$

Substituting in the formula for pressure drop we have

$$\Delta p = b_4 \left(\frac{u}{dwV}\right)^{0.2} \cdot \frac{4wlV^2}{2gd} = \frac{4b_4}{2g} u^{0.2} w^{0.8} l \frac{V^{1.8}}{d^{1.2}}$$

In hydraulic work, with water at ordinary temperatures, the values of the viscosity and density are generally considered sensibly constant, and are then incorporated in the general constant, so that the equation takes the form

$$\Delta p = k \frac{lV^{1.8}}{d^{1.2}}$$

Since the curves for the annular section have been drawn by the author as parallel to those for the circular sections, the exponent n would be the same in both cases for the same range of turbulence factor, but the intercepts b_4 and b_5 on the y axis would be different, having higher values in the annular sections.

In hydraulic problems, where the water temperature varies considerably, or in all problems where the physical properties of the fluid change materially with temperature, which is true in so much of the equipment of the oil industry, it is impossible to ignore the effect of changes in viscosity and density. The only rational method of attack is that under which these tests were conducted. The viscosity of water, for instance, at 130 deg. fahr. is only one-half of that at 68 deg. fahr.

One of the factors which might have affected the results, but which cannot be avoided, is that of the spider supporting the central tube. This was designed to have as small an influence as possible, and has none in the stream line region.

As to the methods and instruments used, they were such as to present no difficulties, which would make a comparison of the results with those of Stanton and Pannell inconsistent. The Chattock tilting manometers were designed similar to those used by them. As the set-ups, methods of determining volume of flow, etc., had to be different for the different fluids, and as the effect of temperature upon the density and viscosity had widely different ranges in each case, the general consistency of the results appear to speak for their accuracy within the limits of what might be anticipated. Attention might be called to the fact that while the viscosity of water and oil decrease with increase of temperature the reverse is true with air.

Another factor, which has not been mentioned, but which possibly has some bearing upon the variation in the results

between the annular and circular sections, and depends upon the shape, is the relation between the surface velocity V_s and the mean velocity V. In what has been considered, the resistance R per unit area of surface has been taken as a function of the mean velocity. But the mean velocity is different from the relative velocity of the water and the surface of the pipe, and, if possible, it would be better to express R as a function of V_s. The surface velocity, however, cannot be directly determined, as it depends upon the roughness and turbulence, etc. Therefore it is customary to express R as a function of the mean velocity, assuming that there is a more or less definite ratio between V_s and V. Whether or not the relationship between the surface and mean velocities in circular and annular sections having the same mean hydraulic radius is the same is a question.

The writer knows of no investigations showing the variation of velocity across annular sections, and similar investigations relating to circular pipes are not altogether concordant. Probably the best are those of Stanton and Pannell heretofore noted in which it is demonstrated that for pipes of equal diameters the ratio of the mean velocity to the maximum velocity is a function of $\frac{dwV_{max}}{u}$. For stream line flow $V_s/V_{max} = 0.50$, giving for V_s/V a value of 0.60, the curve being an ellipse. For turbulent flow the curve flattens out, giving higher values.

Bazin found for different values of C in the Chézy formula

$C =$	80	100	120
$V_s/V =$	0.552	0.642	0.702

Some experiments by the writer[1] upon the variation in the distribution of velocity in a jet of water issuing from a needle nozzle, and for four different nozzle openings, while not directly comparable either qualitatively or quantitatively with flow in straight sections of annular pipe, indicate that the results in the annular and circular sections would be of similar order. The jet varied in diameter from about 4⅝ in. to 6$\frac{1}{16}$ in. at full opening, and was under a head of about 808 ft. At the center of the jet, where the velocity was that of the water just after leaving the surface of the needle (which would correspond to that on the outside surface of the inner tube of an annular pipe), the ratios V_s/V were 0.739, 0.744, 0.688, and 0.932 (the last at full opening). It was not possible to get readings as close to the outer edge of the jet (corresponding to the inside surface of the outer pipe) as desirable, but for the second size jet, the ratio $V_s/V = 0.812$ was obtained, and the trend of the curve was such that it would give at the outer diameter a value about equal to that at the

[1] Proc., Inst. of M. E., Jan., 1910.

center. The distribution curve showed a slightly more rapid rise from the velocity at the outside diameter of the jet than from that at the center, but otherwise the variation was very similar to that in pipes of circular section, with high turbulence.

Had the author had more time at his disposal it would have been desirable to have conducted tests on the outside pipe without the central core, so as to tie this in with the data as given for the circular sections, and to have also carried out additional tests with a change in the diameter of the outside pipe, which he had made up.

The need for such data as are here presented is illustrated in the paper on Mechanical Engineering in Cracking, Heating, and Cooling of Oil by B. N. Broido,[1] where the pressure drop in the outside annular section of the double-tube type of heat exchanger, illustrated in Fig. 14, is to be calculated, and where the effect of the turbulence factor in such exchangers is to be considered in connection with the determination of the heat transfer rate.

SANFORD A. MOSS.[2] The results of the present experiments as plotted in Fig. 7 really agree very closely with the mean line from all previous tests which Professor Eckart has mentioned and which is the lower line in the figure. All previous curves, including the English ones, are drawn from data which have experimental errors, and the actual individual observational points differ from each other about as much as the points in the experiments described in the paper. It therefore seems that with the inevitable irregularities in both the annular and circular pipes one should not pay too much attention to any slight difference between the averages. In Fig. 7 the actual percentage of difference between the two curves is really very slight, and it seems to the writer that the conclusion can be drawn that the coefficient is practically the same for annular pipes as it is for circular pipes.

DISCUSSION IN ENGINEERING.[3] It is usual in hydraulic engineering to assume that the frictional resistance of two waterways will be similar when the hydraulic radius is the same for both. This hydraulic radius is defined as the area of a channel divided by its wetted perimeter, and is thus equal to $D/4$ in the case of a simple circular pipe, and to $(D-d)/4$ in the case of an annulus having inner and outer diameters equal to D and d respectively. The experiments showed that this rule gives too low a result, both when the flow is viscous and when it is turbulent. In turbulent flow the friction was 26 per cent higher than for a circular pipe

[1] *Mechanical Engineering*, vol. 48, no. 6, June 1926, p. 673.
[2] Engineer, Thomson Research Laboratory, General Electric Co., West Lynn, Mass. Mem. A.S.M.E.
[3] Engineering (London), vol. 122, no. 3158, July 23, 1926, p. 111.

with the same hydraulic radius, and below the critical velocity, when the flow becomes viscous, the friction was 36 per cent higher for the annulus. This latter case, it may be added, can be dealt with mathematically, but the author does not enter into this. If, however, u be the mean velocity of flow and μ the coefficient of viscosity of the fluid, then for viscous flow and a simple circular pipe the pressure gradient $\frac{dp}{dx}$ is equal to $-\frac{32\mu u}{D^2}$, while the corresponding expression for an annular pipe is

$$\frac{dp}{dx} = -\frac{32\mu u}{D^2 + d^2 - \dfrac{D^2 - d^2}{\log_e D - \log_e d}}$$

From this it follows that for viscous flow the increase of friction for the annuli experimented on should have been 49 per cent for the narrowest and 43 per cent for that formed with the smallest pipe as core. The experimental value averaged as stated 36 per cent. The difference is possibly due to slight errors in centering, since it is known that with narrow annuli and a stated difference of pressure, the flow obtained with the core touching the outer wall is two and a half times as much as with the core concentric. For very narrow annuli of this kind the assumption that the passage can be treated as equivalent to a circular pipe of the same hydraulic radius leads to enormous errors.

THE AUTHOR. Some of the experimental data for brass tubes and pipes of circular cross-section to which reference is made in the paper and upon which the table given on page 158 was based are as follows:

These data are offered to satisfy the request on the part of certain of those discussing the paper for experimental confirmation of the data for the mean values already quoted.

Water — Brass Pipe No. 16 — 1.255 cm (0.494 in.)

(Stanton and Pannell)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00299	29180	0.00337	17800
0.00288	33800	0.00348	16600
0.00278	38800	0.00356	14900
0.00280	38900	0.00334	18700
0.00278	38800	0.00339	17600
0.00298	29200	0.00348	16200
0.00255	56600	0.00358	14800
0.00262	52200	0.00380	12800
0.00268	47800	0.00374	12800
0.00271	43700	0.00356	14200
0.00288	33800	0.00355	13700
0.00263	50600	0.00371	12700
0.00255	55400	0.00381	11600
0.00247	59700	0.00397	10500
0.00250	62400	0.00388	9750
0.00247	66000	0.00404	8780
0.00245	69200	0.00413	7600
0.00242	71400	0.00444	6400
0.00238	76200	0.00456	6000
0.00238	79000	0.00461	5480
0.00234	82500	0.00482	4700
0.00235	82500	0.00481	4000
0.00230	85600	0.00526	3360
0.00231	90000	0.00449	6120
0.00252	59200	0.00437	6950
0.00301	26200	0.00430	7280
0.00300	27500	0.00433	6980
0.00320	22300	0.00514	3740
0.00342	17100	0.00488	4360
0.00323	20700	0.00530	3200
0.00308	24600	0.00529	2960
0.00314	21700	0.00521	2610
0.00318	21900	0.00472	2220
0.00323	21000	0.00235	77000
0.00327	20400	0.00232	87200
0.00332	18900	0.00227	96600
0.00223	105200	0.00197	178000
0.00219	120000	0.001845	238000
0.00220	118200	0.001813	289000
0.00486	4370	0.001816	330000
0.00536	2970	0.001748	373000
0.00526	2660	0.001795	407000
0.00429	2200	0.001950	178000
0.00530	3090	0.001788	243000
0.00536	2800	0.001790	380000
0.00351	15800	0.001750	418000
0.00331	19350	0.001738	430000

Water — Brass Pipes No. 1 — 2.855 cm. (1.124 in.)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00309	25820	0.00339	18800
0.00303	27360	0.00350	16500
0.00294	30100	0.00363	14100
0.00295	32620	0.00380	12000
0.00294	30000	0.00390	10300
0.00282	34200	0.00396	9600
0.00272	39500	0.00408	8580
0.00270	44100	0.00433	7440
0.00261	47400	0.00449	6120
0.00327	21000	0.00481	5060
0.00316	23000	0.00440	6700
0.00328	21000		

Water — Brass Pipes No. 17 — 0.7125 cm (0.2805 in)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00375	13000	0.00348	17170
0.00392	10260	0.00302	27200
0.00403	9060	0.00316	23400
0.00430	7740	0.00337	19500
0.00457	6120	0.00292	31500
0.00353	15270	0.00282	35100
0.00278	37600	0.00456	5960
0.00273	43100	0.00173	415000
0.00264	48200	0.00170	412000
0.00260	52600	0.001725	416000
0.00253	57700	0.00188	236000
0.00247	67600	0.00176	304000
0.00248	62500	0.00176	331000
0.00406	9130	0.00210	132000
0.00435	7510	0.00175	317000
0.00498	4180	0.00172	402000
0.00508	3680	0.00170	394000
0.00528	3140	0.00174	344000
0.00465	5470	0.00177	354000
0.00535	2660	0.00172	367000
0.00540	2530	0.00203	156000
0.00536	2660	0.00234	88500
0.00538	2420	0.00221	107100
0.00518	2400	0.00211	117500
0.00473	5150	0.00206	140600

Water — Brass Pipe No 18 — 0.361 cm (0.142 in)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.001975	202000	0.00208	156200
0.001990	195300	0.002120	143200
0.002045	182800	0.001972	202500
0.002040	175000	0.00346	17700
0.002110	159700	0.00325	21200
0.002100	139200	0.00309	25300
0.002180	132600	0.00302	28700
0.00222	113300	0.00290	32800
0.002235	106200	0.00283	35600
0.002090	167600	0.00323	22000
0.002050	177400	0.00341	17050
0.002070	167000	0.00339	17050

Air — Brass Pipe No 1 — 2.855 cm. (1.124 in)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00506	4480	0.00434	6810
0.00491	4870	0.00457	6000
0.00515	4070	0.00378	11700
0.00514	3780	0.00374	13000
0.00525	3590	0.00362	14560
0.00517	3280	0.00348	19000
0.00528	3440	0.00325	22000
0.00482	5370	0.00302	31700
0.00468	5820	0.00303	29200
0.00458	6320	0.00307	28100
0.00437	6670	0.00310	27200
0.00468	5290	0.00313	26000
0.00394	9210	0.00320	23800
0.00422	7780	0.00323	23000
0.00407	9410	0.00327	21200

Air — Brass Pipe No. 16 — 1.255 cm. (0.494 in.)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00476	5560	0.00493	5140
0.00534	3250	0.00519	2740
0.00553	2970	0.00509	3730
0.00532	3570	0.00459	6490
0.00509	4090	0.00434	8300
0.00452	6810	0.00411	9780
0.00440	7490	0.00397	11180
0.00509	3850	0.00381	12430
0.00490	4840	0.00423	8910
0.00458	6210	0.00428	8350
0.00491	4550	0.00382	12980
0.00531	3430		

Air — Brass Pipe No. 12A — 12.62 cm (4.968 in.)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00205	185000	0.00175	341000
0.00188	228000	0.00220	114000
0.00180	269000	0.00238	85000
0.00175	321000		

Air — Brass Pipe No. 18 — 0.361 cm. (0.142 in.)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00391	2060	0.00454	6800
0.00566	1440	0.00445	7670
0.00970	838	0.00421	8550
0.01210	667	0.00476	5010
0.01756	465	0.00512	3910
0.00326	2630	0.00480	5250
0.00344	3100	0.00527	3390
0.00376	3190	0.005	4300
0.00477	3200	0.00534	3100
0.00469	5950		

Air — Brass Pipe No. 17 — 0.7125 cm. (0.2805 in.)

$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$	$\frac{R}{wV^2} = \frac{f}{2}$	$\frac{VD}{\nu} = TF$
0.00374	2215	0.00436	1860
0.00514	1510	0.00455	1800
0.00373	2170	0.00484	1712
0.00530	2780	0.00554	1468
0.00505	3377	0.00479	1670
0.00530	3185	0.00768	1012
0.00519	2780	0.00582	1370
0.00510	2770	0.00548	1440
0.00493	2715	0.00627	1288
0.00484	2768	0.00598	1310
0.00481	2655	0.00464	1722
0.00447	2600	0.00490	1620
0.00403	2425	0.00496	1570
0.00408	2010	0.00552	1445
0.00433	2440	0.00480	5030
0.00478	2580	0.00467	5585
0.00489	2680	0.00456	5890
0.00503	2645	0.00451	5995
0.00395	2120	0.00439	6655
0.00388	2390	0.00497	4130
0.00449	2540		

No. 2007

THE TERMINATION OF CHARCOAL TESTS FOR GASOLINE

By F. L. Kallam,[1] Los Angeles, Cal.
Junior Member of the Society

One of the most important features of testing natural gas for gasoline content by means of activated charcoal is the knowledge of when to terminate the test. This paper deals with a method by which the maximum gasoline content may be determined without danger of under- or over-saturating the charcoal when even the approximate content of the gas is unknown. It points out that the correct termination point is determined from the temperature rises and falls in the charcoal due to the adsorption and displacements of the various hydrocarbons. The modifying effect of the condensing phase of the problem is also discussed and conclusions drawn.

In testing natural gas for gasoline content a metered amount of the gas is allowed to pass through a tube containing activated charcoal. The gasoline adsorbed by the charcoal is then recovered by means of distillation with glycerine and the volume of the condensate noted. With this information at hand the gallons of gasoline per thousand cubic feet of gas is readily calculable.

2 To arrive at the maximum gasoline content of any particular gas it is necessary to pass the correct volume of that gas through the charcoal. The knowledge of this volume, relative to the amount of charcoal used, or when to terminate the test, has been more or less optional with the various operators, with the result that check tests are infrequent. By standardizing upon a method for determining this maximum test point, considerable advance will have been made in determining gasoline yields from natural gas.

The Heating Phenomenon

3 When wet gas is first passed through a tube filled with activated charcoal, rapid adsorption takes place, as is evinced by the heat generated at the first point of contact. This zone of heat

[1] Superintendent, Mutual Gasoline Co.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

gradually moves along the tube, leaving cool the layer of charcoal first in contact with the gas. On very rich gas the heat generated is often so intense that it is impossible to keep one's hand on the metal tube. If the gas is allowed to flow through the tube for a long enough period, the heat zone will traverse the entire length of the tube, leaving the body of the charcoal at the original temperature. This phenomenon is familiar to all operators.

4 In view of this peculiarity, it has been assumed that this heat wave is related in some manner to the saturation of the charcoal with the constituents of the gas. However, no indication is given as to the degree of gasoline adsorbed relative to the other components of the gas. Consequently, the conditions existing in the charcoal at the point of gas exit have been investigated. As this region of the charcoal is saturated last, it is logical to presume that the best indication for stopping the test will be obtained at this point. This can be accomplished by placing a properly scaled thermometer in the open end of the tube, so that the bulb is entirely covered with the charcoal, and noting the temperatures. The packing of glass wool between the thermometer and the tube wall will prevent loss of carbon due to gas velocity.

The Temperature Rises

5 Passing gas through a tube arranged as just described shows several interesting temperature changes. Fig. 1 gives the results of temperature measurements obtained when passing various amounts of a rich gas through a tube containing 250 cc. of 8 to 14 mesh charcoal. The four characteristic temperature rises and falls, typical of all wet gases, are clearly illustrated. In some cases even a fifth rise and fall has been observed, although for such a point the rise, instead of being obtained as a distinct increase in temperature, is found as a period of constant temperature followed by a gradual decrease. In fact, the above may be said to be true for the fourth rise in some cases, especially with very dry gases. It is believed that with suitable apparatus, the fifth rise might more readily be detected, as perhaps might a sixth and even seventh point. Because of the consistency of these characteristic temperature changes with the various gases, it is assumed that the number and intensity of the rises are governed by the number and percentage of members of the paraffin hydrocarbons present in the gas.

Constant-Volume Adsorption

6 From numerous tests it has been determined that charcoal will adsorb practically a constant volume of hydrocarbons from a gas regardless of the volume passed through the carbon. This is of course assuming that a sufficient quantity of gas is first passed through the charcoal to furnish hydrocarbons in volume to fill the

surface and capillaries of the charcoal. The passing of more gas than this amount will not increase or change the volume of hydrocarbons adsorbed. With these ideas in mind it is now in order to correlate the temperature rises and the gasoline adsorbed when using activated charcoal.

The Replacement of Air and Moisture

7 Referring again to Fig. 1, it will be noted that with the initial passage of gas through the charcoal a slight drop in temperature takes place. This may be attributed to the fact that the gas is replacing air and moisture contained in the charcoal. In

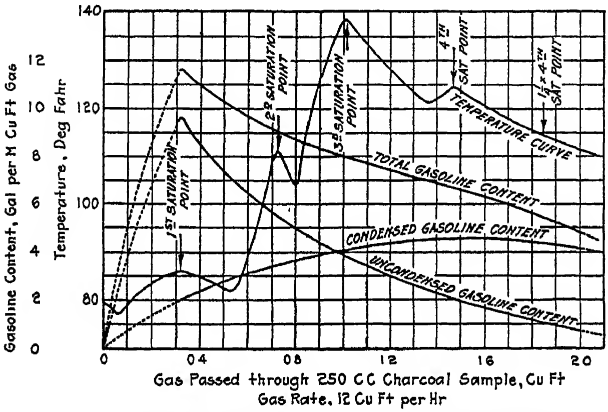


Fig. 1 Relation Between Saturation Points and Gasoline Content

no case should this initial momentary drop be mistaken for a temperature rise-and-fall point.

The First Saturation Point

8 Charcoal possesses the inherent property of adsorbing the gasoline constituents of a gas most readily, showing marked preference for heptane, hexane, and pentane in the order named. Analysis of numerous gases shows that these hydrocarbons are always present to a lesser degree than the lighter members of the paraffin series, as butane, propane, ethane, and methane. Of these latter, methane generally predominates, and for this hydrocarbon, charcoal shows the least inclination to adsorb and retain. Thus when gas first comes in contact with charcoal, there is not enough heptane, hexane, and pentane present to satisfy the capillary capacity of the charcoal, with the result that butane, propane, ethane, and methane are also adsorbed. In other words, with the first fractional volume of gas exposed to the adsorbent, all of the hydrocarbons present are adsorbed and condensed within the

pores of the charcoal. The charcoal continues to adsorb until the condensed hydrocarbons equal in volume the volume of the capillaries, and up to this point, no hydrocarbons escape adsorption. As more and more gas enters the charcoal, beyond this point, a greater amount of heavier hydrocarbons are presented for adsorption; but in order for the charcoal to hold these, it is necessary to provide the capillary volume. This it can do only by releasing some already adsorbed constituents, and these will be the ones for which it has the least affinity.

9 This is the condition existing in the charcoal at the top of the first temperature rise (Fig. 1), or first saturation point, as it will hereafter be called. The rise in temperature up to this point can thus be said to be due to the heat of adsorption of the various hydrocarbons, and in particular to methane, because of the latter's greater predominance in the gas.

10 It is at this first saturation point that the maximum gasoline content will be found, bearing in mind the relation between gasoline and gas volumes. This content we shall consider as one hundred per cent of that attainable. However, due to the composition of this gasoline as previously explained, exorbitant condenser pressures would have to be applied, accompanied by low temperatures, in order to secure complete condensation. Such gasoline, even if produced, would at present have no commercial value because of its volatility.

11 Consider the example given, Fig. 1, and assume the test terminated at the first saturation point, or, after 0.31 cu. ft. of gas had passed through the charcoal. Upon distillation of the charcoal, to recover the gasoline adsorbed from this amount of gas, and condensing the vapors given off in the usual laboratory ice-bath condenser at atmospheric pressure, the condensate would be such as to show a gasoline content for the gas tested of 2 gal. per 1000 cu. ft. This would be misleading information unless it were realized that a greater part of the vapors were being uncondensed, in this case enough to give a content of 9.6 gal. per 1000 cu. ft. Hence if all the vapors were condensed the total gasoline content would be 11.6 gal. per 1000 cu. ft. of gas. This means that at the first saturation point for the particular gas examined, and standard conditions of condensing, a content of only 17 per cent of that adsorbed can be shown.

The Second Saturation Point

12 It now remains to be seen what hydrocarbons will be released after the first point of saturation is reached in order that the charcoal may avail itself of the more desirable gasoline constituents. Obviously, this will be methane, for which the charcoal has the least attraction. This expulsion of methane is clearly shown in Fig. 1 by the fall in temperature after the first saturation

point, showing that the heat of adsorption during this period is utilized to revaporize the methane. Adsorption is all this time taking place, except that of methane which from now on passes unmolested through the charcoal. This is shown by the temperature again rising, signifying adsorption of heavier hydrocarbons. To secure the required volume, the charcoal selects ethane to a greater degree than before, where previously it adsorbed methane. The extent of these replacements will of course be controlled by the individual constituents and composition of the gas under test. Thus at the second saturation point we find the condition that for the charcoal to adsorb further, it will be necessary to replace the lightest hydrocarbon already adsorbed, ethane, in order to provide capillary volume.

13 The gasoline now available in the charcoal for extraction, by virtue of the absence of all methane, is slightly more stable. From the example it will be seen that the total maximum content is now 9 gal. per 1000 cu. ft., and that by atmospheric condensation, a content of 3.4 gal. This latter amount is 38 per cent of that obtainable.

The Third Saturation Point

14 Passing the second saturation point, we find ethane being replaced by heavier hydrocarbons, of which propane is in the greatest volume. Now, two constituents, previously adsorbed by the charcoal, methane and ethane, are allowed to pass through unadsorbed. The result is that the third saturation point designates that for further adsorption to take place, the propane will have to be replaced.

15 The gasoline adsorbed and held by the charcoal at the third saturation point is still very "wild," due largely to the amount of propane present. Continuing the example, the total gasoline content is seen to be 8 gal., while the condensed gasoline yields a content of 4 gal. per 1000 cu. ft. of gas. It can readily be seen that 50 per cent of the available gasoline content is now obtainable by atmospheric condensation.

The Fourth Saturation Point

16 As before, passing a saturation point designates that the lightest hydrocarbon held up to this point is being replaced, in this case, propane. All methane, ethane, and propane coming in with the new gas now passes untouched through the charcoal, while all the butane, pentane, hexane, etc., is adsorbed. Here again there is a deficiency in the volume of the heaviest hydrocarbons, so that a very large amount of butane is adsorbed in order to satisfy the charcoal as regards volume of hydrocarbons held. When this volume is obtained, the fourth saturation point

has been reached as shown by the fourth temperature rise in Fig. 1.

17 Sixty-seven per cent of the total gasoline content, or 4 5 gal., is the result obtained by condensation at this point in our example. This content, however, is only 37 per cent of the total content obtained at the first saturation point.

The Total-Saturation Point

18 Continuing past the fourth saturation point, the butane is replaced by the heavier hydrocarbons in the incoming gas, of which pentane is in the greater percentage. Hexane (and other heavier hydrocarbons if they are present) will be adsorbed more readily than pentane, but the latter is required to furnish the necessary volume. The fifth point of saturation will thus correspond to the point where all the pentane will have to be replaced in order for further adsorption to take place.

19 This process of replacement continues until a sufficient volume of the heaviest hydrocarbon present in the gas to fill the capillaries of the charcoal has been passed through the latter. From this point on, no hydrocarbons will be adsorbed, all passing unadsorbed through the charcoal. This condition is the total-saturation point, and the gasoline content based on this volume will perhaps be only 5 per cent of that shown at the first saturation point. But of course, practically all of the gasoline adsorbed at this point will be condensed. The stability of this product is very great, but not as great as one would expect. It seems that while charcoal replaces or selects the heavier hydrocarbons in preference to the lighter ones, some of the latter are absorbed or dissolved in the heavier members of the series, which in turn are retained by the charcoal. This secondary absorption is naturally small, but does account for some of the unexpected vapor pressures encountered at times in charcoal gasoline.

Commercial Natural Gasoline

20 From the analyses of many samples of natural gasoline, it has been found that very few, if any, hydrocarbons lighter than propane are present. This is due of course to their exceptionally low boiling points. Propane is found to the extent of 1 to 5 per cent, and while it, too, has a low boiling point, it cannot be freed from the gasoline without carrying away with it other heavier and more valuable gasoline constituents. Hence propane is a serious and objectionable constituent of gasoline, being of such nature as to be commercially detrimental. By eliminating propane, a greater amount of butane may advantageously be contained in gasoline without abnormal losses in handling.

21 At a risk of repetition it is desired to point out that at the fourth saturation point the charcoal contains butane and all the

heavier hydrocarbons present in the gas. These constitute a gasoline in which dissolved gases are minimized, with the result that losses are due mainly to the evaporation of true liquid constituents. This being a highly desirable commercial product, it follows that the fourth saturation point will always assure the adsorption of the maximum commercial natural gasoline constituents from a gas by charcoal. This then, is the proper point at which to terminate the test.

The Distillation Effect

22 However, inasmuch as a gasoline determination is composed of three distinct operations, adsorption of the gasoline (as has already been discussed) and the subsequent distilling and condensing, it is necessary to consider these latter two factors before drawing final conclusions as regards the proper termination point for the test.

23 The function of distilling the gasoline from the charcoal may be briefly discussed and dismissed. All of the gasoline may be removed from the charcoal, regardless of the point of saturation at which the adsorption test was stopped. This is easily accomplished by the application of heat and some vaporizing agent, as glycerine, mercury, oil, etc., which will replace the gasoline in the pores of the charcoal. The heat applied must be of such intensity as to vaporize and carry the gasoline into the condenser. Distillation may take place under any convenient or desired pressure.

The Condensing Effect

24 By stopping the test at the fourth saturation point we are assured of condensing, within the pores of the charcoal, a volume of desired gasoline which will give a maximum gasoline content for the gas. But in order to obtain this content, it is necessary to recover this adsorbed gasoline by distilling and condensing. As a result of countless tests, however, using 250 cc. of 8 to 14 mesh charcoal and distilling and condensing equipment having the same dimensional ratio to each other in every instance, it has been found that the maximum gasoline content is not always obtained at the fourth saturation point.

25 This is because condensing at atmospheric pressure and 32 deg. fahr. it would only be possible to obtain approximately complete condensation if the vapors handled were 100 per cent butane. Naturally, with the vapors composed of butane plus heavier hydrocarbons, together with air and other fixed or uncondensable gases, the partial pressures on each constituent in the condenser will be lowered to such an extent that condensation is not as effective as it would otherwise be. It can be seen from this that the development of a general law covering all cases of

testing is impossible because of our present lack of knowledge regarding the composition of the various individual gases.

26 One remedy for such a condition is to run the test past the fourth saturation point, or in other words, displace, at least to some extent, the butane with pentane. It has been found, however, that the maximum test always lies within a narrow limit, that is, between the fourth saturation point and a point which corresponds to 1.25 times the volume at the fourth temperature rise. This latter point may be defined as the saturation of the charcoal secured by passing 1.25 times the volume of gas required to reach the fourth saturation point through the charcoal. This rule holds for wet gases ranging from ½ gal. upward. For gases with ½ to 2½ gal. content the 1.25-point has been found very satisfactory for the maximum yield, but for the richer contents, the fourth saturation point will give slightly higher results. This is probably true because these gases contain greater percentages of pentane, hexane, etc., and hence smaller amounts of butane, so that it is unnecessary to displace any of the latter.

27 By resorting to pressure condensation, it is possible to recover 95 to 97 per cent (by weight) of the gasoline adsorbed by the charcoal at the fourth saturation point. This makes it convenient for field testing, as the adsorption can be stopped in every case at the fourth saturation point. A pressure of 45 lb. per sq. in. absolute and a temperature of 60 deg. fahr. will assure this degree of condensation in practically all cases. Though here again, dimensions of condenser and still, relative to the size of sample used and vapors handled, will have a bearing upon consistent check results. Even after weathering condensate obtained by this method to atmospheric conditions, with a loss of from 10 to 15 per cent, a higher yield will be obtained for the same gas than will be given by atmospheric condensation. This advantage is due to the vapor pressure-temperature relation gained by increasing the pressure of condensation.

Conclusions

28 The following conclusions may be drawn from what has been presented:

a It is possible to fractionate the hydrocarbon constituents of natural gas by the use of activated charcoal

b Commercial natural gasoline contained in a gas should be construed to mean gasoline containing butane and all heavier hydrocarbons present in the gas

c There is a definite relation between the volume of gas sampled and amount of charcoal used in order to obtain a maximum test of the commercial natural gasoline contained in the gas

d For atmospheric pressure and 32 deg. fahr. condensation the maximum test for wet gas will be obtained in all cases between

the fourth and 1.25 times the fourth saturation point for any particular gas and any particular amount of charcoal. This assures neither under- nor over-saturating the charcoal. For gases ranging between 0.5 and 2.5 gal. content, the test should be terminated at 1.25 times the fourth saturation point. For gases ranging from 2.5 gal. and up, the test should be terminated at the fourth saturation point.

e When condensing under 45 lb. per sq. in. absolute and 60 deg. fahr. all tests may be stopped at the fourth saturation point. This method will give higher results than will atmospheric condensation.

DISCUSSION

A. F. Semino.[1] The method of determining the gasoline content of natural gas described by the author possesses inherent possibilities. It heretofore has been the practice for different companies to make haphazard estimates of the gasoline content of natural gas, so that it has been utterly impossible to obtain check results by the various methods used.

The temperature control method of gasoline determination has been used successfully by the Shell Company of California for the past two years. Consistent check results are readily obtainable without resorting to a series of tests. The gasoline plant production can be estimated from the field tests within a satisfactory degree of accuracy.

A word of caution should be given in regard to the saturation point. While it is true that ordinarily 1.25 times the volume at the fourth saturation point will give satisfactory saturations for natural gas containing from 0.5 to 2.50 gal. of gasoline per 1000 cu. ft., it is not always the case. Occasionally a gas having a gasoline content within this range is found which will test highest at the saturation point corresponding to the fourth temperature rise. This happens when the gas is proportionally richer in the heavier hydrocarbons, since the gasoline condensed is of lower Baumé gravity than is usually obtained. Furthermore, for gases of gasoline content between 0.5 and 1.0 gal. per 1000 cu. ft., it may be necessary to saturate the charcoal to a point corresponding to from 1.3 to 1.4 times the volume at the fourth temperature rise in order to recover the maximum yield. However, when the ratio has been once determined, reliable results are obtainable even though the gasoline content varies quite appreciably between tests. Consequently, without a knowledge of the gasoline constituents contained within the gas, statements pertaining to preferable saturation points should be accepted cautiously.

Distilling and condensing under pressures above atmospheric seems to offer the most logical solution of the problem of gasoline content determination. However, the same situation as above

[1] Gas Engineer, Shell Company of California, San Francisco, Cal.

arises, in that the same pressure will not suit every case. Assuming a pressure sufficiently high, then the fourth temperature rise should give reliable results for gases usually experienced in practice. This pressure has been found to be in the neighborhood of 45 lb. per sq. in. abs.

G. V. D. Marx.[1] There may be some question about the curves for the total, the condensed, and the uncondensed gasoline content as shown in Fig. 1. The condensed gasoline content is obviously what was condensed during the distillation. The uncondensed gasoline content was obtained by putting absorbers at the end of the condensers and leading the vapors through a mineral oil vat holding about 2 in. of water pressure on the mineral oil, thus obtaining the uncondensed gasoline content, from the increase in gravity of the mineral oil, together with its increase in volume. In this manner, the total gasoline content curve was obtained; that is, the sum of the above volumes.

The Author. The ultimate and final solution of the problem of the proper termination point lies in the development of a simple and commercial method for determining quantitatively the various hydrocarbons in the gas. Until this is accomplished, the point emphasized by Mr. Semino will necessitate in some cases the making of a series of tests covering a range of terminating points, and the selection of the point which gives the maximum content. However, where time is an item, as in commercial work, the method of termination as given in the paper will be found very satisfactory even from a standpoint of accuracy and consistency.

The fact that in all cases the maximum test is not obtained at the fourth saturation point cannot be attributed to the failure of the charcoal to make the proper selection, but to the relative proportions of the various hydrocarbons adsorbed. These may be in such proportions as to enhance condensing, while again, if the relation of the lighter fractions to the heavier constituents is large, condensing will be impaired. At present, these ratios cannot be determined from the saturation points, and for this reason the fourth and 1.25 times the fourth saturation point was selected as most nearly fitting all gases containing hydrocarbons.

In a personal communication Dr. Max Latshaw[2] called attention to the omission of data concerning the test charcoal. In all cases, the tests were conducted using 50-min., 8-14 mesh, cocoanut charcoal.

[1] Engineer, Standard Oil Company of California, San Francisco, Cal. Jun. A.S.M.E.

[2] The Silica Gel Corporation, Baltimore, Md.

No. 2008

ASPECTS OF STEAM POWER IN RELATION TO A HYDRO SUPPLY

By A. H. Markwart,[1] San Francisco, Cal.
Non-Member

Steam power, in a regional power system having a hydro supply as its basis, either because it was started that way or because fuel may for a time have been costly, has greater significance than is generally conceded. Steam power becomes increasingly important as hydro sources become more remote and costly of development; as the annual load factor may tend to lower; as the public may demand higher standards of service; as those manufactures requiring greater continuity of service may grow; as hydro generating and transmission units may increase in capacity; as stream-flow plants without storage regulation become more numerous; and as cyclic changes in water run-off, becoming revealed, may indicate hydro deficiency.

Briefly, steam power is needed

1 *To effect the best system economy*
2 *For stand-by*
3 *For meeting the seasonal hydro deficiency during the short-water period on non-regulated streams*
4 *For meeting the dry-year hydro deficiency.*

The above phases, together with hydro capacity and cost of power, are broadly discussed in what follows.

System Economy

WHENEVER a system load factor is less than unity and when the capital cost of steam plants at load centers with relatively short transmission is less than that of hydro plants with long transmission, it will be economical to carry a portion of the system peak by steam plants. In general, the cost of energy from steam plants at high load factors exceeds the cost of that from hydro plants because of the fuel charges, whereas the cost of energy from hydro plants at low factors exceeds the cost of that from steam plants on account of the higher fixed charges.

[1] Vice-President in Charge of Engineering, Pacific Gas and Electric Company.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

2 Fig. 1 shows a load curve having an annual load factor of 67 per cent. It is typical of those of the Pacific Gas and Electric Company which has an annual load factor varying from 60 to 64 per cent. This curve takes the form of twelve daily load curves which are characteristic of the average day of each of the twelve months of the year. It approximately represents the load conditions which have obtained in the past and its form was determined by averaging the loads of the half-hour periods for the days including Sunday of the second week in each month of a selected year. Therefore, in effect, there would be 31 such curves for January; likewise 30 for April. From such a load curve, therefore, may be obtained by an area measurement the kilowatt-hours for an average day for any month, because the area under the monthly curve represents the kilowatt-hours required for the average day of the month in question. By multiplying such kilowatt-hours by the days of the month, the kilowatt-hour demand for the month, and finally for the year, may be obtained.

3 From this curve was determined the load factor and kilowatt-hours from base and peak for percentages of system peak from 100 per cent to 32.5 per cent. This is shown by Table 1. This curve indicates, for instance, that if 60 per cent of the annual peak from base is carried on hydro plants, 81.8 per cent of the required energy will be supplied on load factor 91.7 per cent. If the remaining 40 per cent of the peak is carried on steam plants, what is left, or 18.2 per cent of the required energy, will be supplied at a load factor of 30.6 per cent.

TABLE 1—LOAD FACTORS AND PERCENTAGES OF TOTAL OUTPUT FOR VARIOUS PERCENTAGES OF THE ANNUAL PEAK BOTH FROM BASE AND PEAK [1]

Per cent of annual peak	From base		From peak	
	Load factor	Per cent of kw-hr	Load factor	Per cent of kw-hr.
100	67.3	100.00	0.0	0.0
95	70.8	99.96	0.3	0.02
90	74.6	99.89	0.8	0.11
85	78.6	99.36	2.9	0.64
80	82.3	97.91	7.0	2.09
75	85.4	95.23	17.8	4.77
70	87.6	91.18	19.8	8.82
65	89.7	86.73	25.5	13.27
60	91.7	81.81	30.6	18.19
55	93.6	76.56	35.0	23.44
50	95.5	70.98	39 0	29.02
45	97.4	65.17	42.6	34.83
40	98.8	58.75	46.2	41.25
35	99.8	51.96	49.7	48.04
32.5	100.	47.95	51.7	52.05

[1] Computed for the load curve of Fig. 1, having an annual load factor of 67 per cent.

4 With this information the cost per kilowatt-hour from steam plants and from hydro plants may be obtained under various assumptions of oil, money, operation, maintenance, and deprecia-

tion costs, and of capital cost of steam plants and hydro plants including transmission.

5 Without attempting to develop this, it may be stated that the best theoretical economy in kilowatt-hour cost under present-day conditions with such a load curve is had by carrying approximately 80 per cent of the maximum demand from the base on hydro, and the remaining 20 per cent from the top on steam. But, as a practical matter, steam in excess of this may be carried without material increase in the cost of the mingled kilowatt-hours, as curves which can be prepared will show that there is comparatively little variation in cost over quite a range either side of the minimum point. Hydro to 75 per cent of the demand, and steam to 25 per cent, may be considered within the range of practical

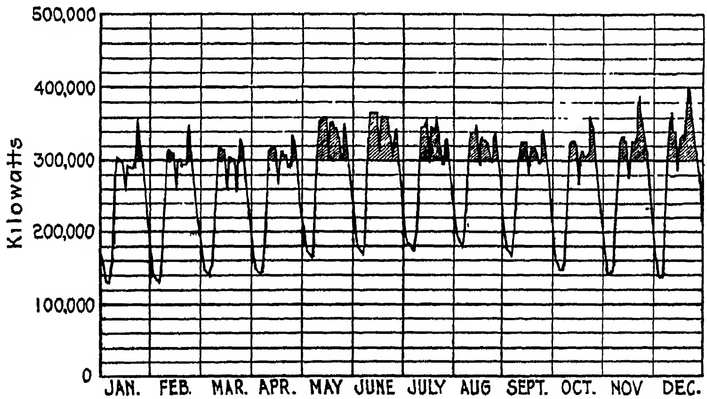


Fig. 1 Typical Daily Load Curve

economy. Under this condition the hydro will produce 95.2 per cent of the total annual energy at a load factor of 85.4 per cent, and the steam 4.8 per cent at a load factor of 17.8 per cent.

6 The foregoing remarks apply exclusively to the idea of economical production of kilowatt-hours, all other considerations being neglected. These statements have as their basis primary hydro power and neglect consideration of years of deficient water supply. They also neglect consideration of the amount of energy which might be generated while running steam plants regularly at reduced loads in order that the steam units shall be in the proper operating condition to render possible such service as stand-by implies.

7 Were it safe to assume the present-day low fuel prices for an indefinite period, steam power for the base load of a regional supply like that of Northern California could compete on very favorable terms with the cost of a combined steam and hydro supply, or a hydro supply. Broadly viewed, however, it would appear to be hazardous to assume such a favorable condition, and

unsound from a standpoint of fuel conservation, for obviously it is highly proper in a region where water power is available to use such water power as far as it will go, creating for it as favorable a position as possible by the judicious use of steam power.

Steam Stand-By

8 Various considerations help to fix the extent of stand-by for a regional hydro power supply. The proper amount of steam capacity to be installed in a regional power system for stand-by purposes should be determined by the amount of preferred service required at metropolitan centers served from such a power system, by the character of the load carried, by the kind of general service which is desired, and by other policy considerations. For a metropolitan load center the maximum amount of stand-by which might be provided would be 100 per cent of the expected metropolitan peak at all times. This would be an exceedingly conservative provision. It is probable that 75 per cent protection would be considered adequate in most instances, for as a rule there is more than one transmission line into such load centers. An installation to furnish this protection would have the capacity to reduce the total peak, permitting hydro to furnish the bulk of kilowatt-hours on the base at an improved hydro load factor. Experience in Northern California shows that a regional system equipped with steam capacity based on 75 per cent of the metropolitan peak also will be a system possessing steam reserve sufficient to supply steam kilowatt-hours on a base load in dry years when the hydro supply is deficient.

9 Energy production from stand-by plants will vary with the size of the units which are installed or which may be installed from time to time. It will also vary with the hours of operation, or the duration of the hydro system interruption, the amount of stand-by load carried, or the capacity of the hydro or transmission unit which temporarily may be out of service, and the quantity of load that the plant may be required to carry at any instant.

10 The larger the unit the greater the number of kilowatt-hours generated, even when standing by to take full or partial loads, as units of 15,000 kva. may be operated to carry a minimum load of 1000 kw. while a 35,000-kva. high-pressure unit probably cannot be operated feasibly with a reduced load of less than, say, 3000 kw.

11 As a large system may have several steam plants, the kilowatt-hours produced from such stations will vary with the number of stand-by plants and the number of units that may be operating in the several plants.

12 The experience of the Pacific Gas and Electric Company with four steam plants having a total of twelve units, none larger than 15,000 kva., indicates a kilowatt-hour output of 3 per cent

of the total system load as a minimum. This is in addition to that percentage of output which is required to effect the best hydro-steam system efficiency and does not include the steam generation necessary to make up for interruptions to the hydro service. This latter may be large or small depending upon the nature and gravity of the failure in the hydro system.

Steam for Short-Water Season

13 When the hydro supply is put on a suitable load factor by reason of combing off a portion of peak on steam, the amount of which has been determined by economic study, ideal conditions for the hydro do not even then necessarily exist. Conditions would be ideal if the stream-discharge curve were identically the shape of the hydro-load curve which remains after the steam has supplied its complement of peak and energy. For the purposes herein we may neglect the daily fluctuation of load placed on hydro, because, as a rule, this is cared for by forebay capacity. However, monthly conditions are not so easily cared for, because generally storage sites of sufficient capacity do not exist to equate the natural flows on streams where hydro plants are situated. It then becomes necessary during the short-water season, extending from three to six months, depending upon character of water shed and the year, to build up the difference between the actual and the desired monthly hydro block by pulling steam. This does not mean that steam-plant capacity would be provided in addition to that determined by other considerations, such as stand-by at load centers or steam capacity to meet years of deficient run-off.

14 It behooves the power companies to use every effort to effect all possible storage to meet the seasonal short-water condition because it means a saving of fuel which is an important matter, particularly in those years when fuel cost is high.

15 The foregoing statements relating to seasonal stream flow refer largely to primary hydro power which is available the larger portion of the time. It should be noted, however, that under some circumstances it may be economical to make use of additional steam to permit the absorption of secondary power. Under such a plan of using stream flow, steam-plant operation would be required to an even greater extent than would be required with primary hydro power during the short-water season.

16 Seasonal steam-generated energy will tend to increase because of the desirability, in the public interest, of absorbing by-product power from storage works erected primarily for irrigation. Since agriculture is fundamental, land for it should be developed to the greatest extent that can be shown economical, proper balance of crop production and demand considered. Such by-product plants must be increased in power capacity to absorb the power of the annual irrigation flow, 60 per cent of which occurs during

the three summer months. Power from such plants is largely secondary, as the flow of water is determined wholly on the basis of irrigating the maximum number of acres of land. Under such a condition the seasonal make-up from steam would be greater than that where water is held in such reservoirs to a desired level and discharged at a rate designed especially to answer the power needs rather than the irrigation demand of the maximum number of acres.

Dry-Year Steam

17 The power companies in California have been operating for about thirty years and reliable water run-off records for most of the streams are available for about twenty years of this time. It appears that the last eight or nine years which cover the period of greatest expansion in the industry have been deficient in precipitation, in fact a number of these latter years have been abnormally dry.

18 While the run-off records are not available for a long time, fairly reliable precipitation records are. Fig. 2 is a monthly and annual precipitation record for the City of Sacramento for 76 seasons, commencing with the 1849-50 season and closing with the 1924-25 season. While this chart cannot deal quantitatively with the power problem because it does not indicate run-off, it does give a qualitative picture of the situation and serves to emphasize future low water yields, as the run-off of the streams of the State of California depend exclusively upon precipitation, either in the form of water or snow, except as some of its rivers may be influenced by underground contributions outside of the state, as is probable in the case of the Pit. Even drier years may obtain because it is believed that a period of at least 100 years is necessary in order to measure the effect of all the cyclic changes which might be expected. At any rate, a study of the chart is of peculiar interest at this time. Briefly, the chart says this: The normal rainfall for the 76 seasons is 18.5 in. During the period named the minimum precipitation was 4.7 in. occurring in the season 1850-51, and the maximum was 36.4 inches occurring in the season 1852-53. There were 46 seasons out of the 76 in which the precipitation was below normal, and 30 in which the precipitation was above normal. Of the 76 seasons there were only 24, or one-third, that were substantially above normal. A further disappointing disclosure is that the normal as now determined from the available record is going down, this being particularly emphasized by reason of the abnormality of precipitation of the eight seasons preceding the 1924-25 season, two of which were less than one-half normal.

19 It must be inferred then that power would have had to be supplied to make up the hydro deficiency during those dry years when the water supply fell substantially below normal, if

the capacity of hydroelectric developments is in general based on flows of years of normal run-off, which is likely. It appears that the last eight seasons have been the longest continued dry period in the 75 on record. As a matter of fact this dry period extends back the last 10 seasons if we neglect the 1915-16 precipitation of 18.3 in. The other dry periods may be noted to be from two years to not more than seven years long. Of course it is recognized that quantity of precipitation is not the only index to the water yield of a stream For instance, the distribution of precipitation in the form of rain and snow in a season having, say, 18 in., might be such as to produce far more beneficial and sustained run-off than in one having 25 in., largely rain, with a bulk of it concentrated in any one month, and especially if such precipitation were to occur in the early fall before the cold weather

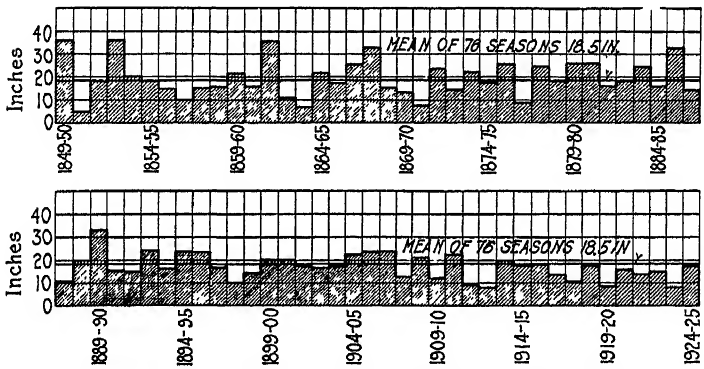


FIG. 2 ANNUAL PRECIPITATION RECORD, SACRAMENTO, CAL.

comes. Actual run-off, of course, is the real criterion of the power yield of any stream, but as stated above, the chart does give a picture which will suggest a deficiency of hydro energy during the years of deficient precipitation.

20 Roughly, a system like the Pacific Gas and Electric Company is deficient one-third in energy output on the basis of dry years of severity that have thus far obtained. Actually, the annual run-off of streams in the northern part of the state was deficient a greater amount than this, probably from 40 to 60 per cent, taking 1923-24 as the criterion. The difference in the deficiencies comes about because of the extent to which the streams have been developed. In other words, they have not been developed for the high flow which is available for only a few months of the year but rather for the flow that is reasonably sustained throughout the greater part of the year as this was known from records available when developments were to be made.

21 What actually takes place in dry years as contrasted with normal years is that the steam power takes its position on the base of the load immediately above the load that can be carried by stream-flow plants which are not subject to daily regulation. The portion of the load lying above the steam will be carried by those hydro plants which have forebay regulation, as far as they can, and the final balance by the steam plants, if the steam plants are not already carrying their capacity on the bottom. This normal- and dry-year contrast is shown by actual daily load curves of

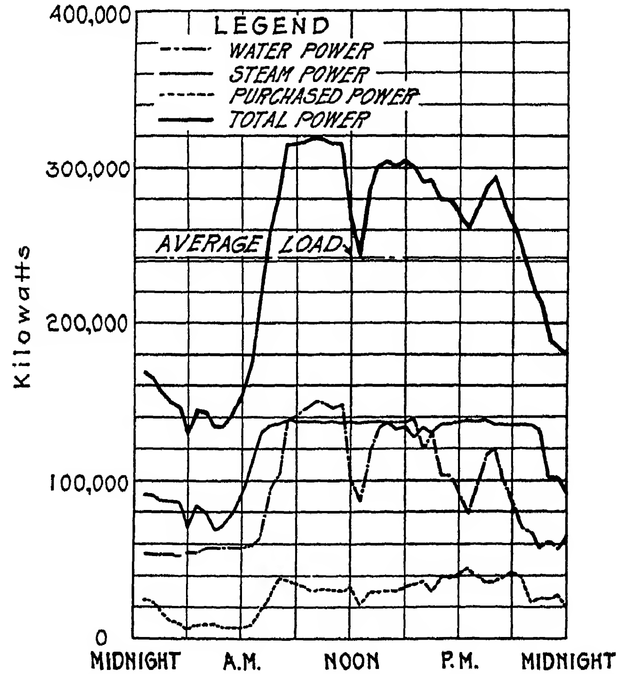


FIG. 3 DAILY LOAD CURVE, PACIFIC GAS AND ELECTRIC CO., AUG. 21, 1924

the Pacific Gas and Electric Company in Figs. 3 and 4, the former being for August, 1924, under conditions of drought and the latter being for May, 1925, when water conditions were fairly normal. In both cases, however, the peak pulled was about the same.

22 Thus, if power consumption is not to be curtailed, power companies must provide the steam-plant capacity required in the dry years, because this is a condition — not a theory. In other words, were it assumed for one reason or another that steam stand-by protection at load centers was superfluous, or combing off the peak because of good economical showing on the hydro was

unnecessary, steam would in any event be required to that extent necessary to meet hydro deficiency in these extremely dry years. This steam-plant capacity, as determined by experience, should be in this region of the order of 40 per cent of the anticipated system peak, and it may even be greater than this in other portions of the state.

23 As a coincidence resulting from the relation which exists between the metropolitan and system peaks on the system of the Pacific Gas and Electric Company, a steam-plant capacity of 40

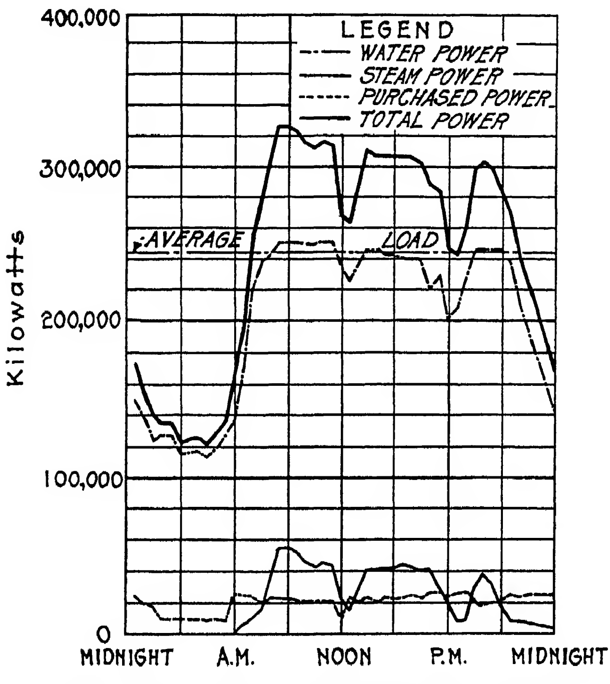


Fig. 4 Daily Load Curve, Pacific Gas and Electric Co., May 5, 1925

per cent of the system peak also will approximate 75 per cent of the metropolitan peak. This will of course change as the relation changes, and as more hydro plants are built on streams of sustained flow like the Pit River or on streams which may be regulated by storage works. As has been stated, the kilowatt-hour production of the hydro system as now developed in Northern California is deficient, during dry years, to the order of one-third. This deficiency may be made up readily from steam-generating stations having a capacity of 40 per cent of the system peak or 75 per cent of the metropolitan peak. For example, with a system

peak of, say, 400,000 kw., of which 213,000 kw. is the load of metropolitan centers, steam capacity of 160,000 kw. at load centers would be sufficiently great to furnish the deficient hydro kilowatt-hours to the general system in dry years when employed on the base, and normally furnish suitable protective capacity at metropolitan centers. Likewise, this steam capacity would permit combing off the peak under normal operation to any amount that was desired to improve the hydro load factor so as to obtain the best system economy.

24 As an operating problem in dry years, if during the precipitation period it appears that the stream yield for the coming run-off season is likely to be deficient, it becomes necessary to use good judgment in the holding of stored waters and the burning of oil during the spring months so that, in the late summer, the power available from steam plants and used on the base, together with power available from reserved waters when used in the top, will be sufficient to meet the anticipated load.

25 Under such conditions the steam plants, up to their capacity, must take from the total load, a block of a size as determined by the amount that the hydro of waters held in storage will meet. Thus during a dry year, the variations in loads are taken up by stored-water plants, whereas, in normal years, variations are taken by steam plants and stored water, except as daily variations are concerned, serving together with the stream flow to meet base-load requirements.

26 While the maximum overall economy of a regional system in normal-water years is secured at present by generating the maximum of hydro energy at a high load factor and the minimum of steam energy at a low load factor, it is desirable in the interests of economy to use stored water through hydro plants in a manner which will permit the highest possible daily load factor on steam for the obvious reason that the kilowatt-hours from a unit of fuel increase as the load factor improves. In modern steam-turbine plants, with boilers of 400 lb. pressure, it is possible to produce 400 kw-hr. per barrel of oil at load factors of 70 per cent and above, with less as the load factor lowers, down to perhaps 250 kw-hr. per barrel with a load factor as low as 10 per cent. While in normal years it may not be possible to operate for ideal fuel consumption, it is quite possible to do so in dry years. In such years, when annual costs become an important consideration the load factor on steam will be high with corresponding steam-plant efficiency.

27 Normally, steam plants must be used to supply kilowatts of peak, if this function can be imagined, instead of kilowatt-hours of energy, leaving the hydro to supply the bulk of the energy requirements. Under such condition there will be but a minimum of oil consumed, as the amount of energy required from steam when supplying the selected portion of the peak is relatively small.

28 In dry years, however, steam plants must be used to supply the kilowatt-hours of energy of the base load. At this time operating costs become a very important consideration because of the unproductive investment in hydro plants. Therefore, to improve the situation economically, the hydro plants must be operated in a manner that will produce the highest possible load factor on the steam plants, with corresponding steam-plant efficiency.

Hydro Capacity

29 The desired steam-plant capacity in a combined system will be determined by the various considerations elsewhere mentioned. Hydro capacity, on the other hand, is involved solely with the question of economy in kilowatt-hour production. With the increase in distance of transmission, larger capacities of generating and transmission units, and higher capital costs of such over steam plants, it becomes necessary to install hydro plants to operate on the highest possible load factor. This is accomplished by combing a proper amount off the peak with steam plants, leaving the base load, containing by far the greatest number of kilowatt-hours, for hydro. The hydro-plant capacity must then be that which is necessary to meet the remaining and larger proportion of the load. And streams should be developed for the flows regulated or unregulated as may be possible in normal years, with steam to make up the hydro deficiency in subnormal years.

30 To illustrate again with the example of peak load of 400,000 kw., neglecting the question of spare capacity which becomes a matter of judgment, a system with an annual load factor of from 60 to 65 per cent would have a hydro capacity of 75 per cent of the peak from the base, or about 300,000 kw. This, together with the steam-plant capacity of 160,000 kw., brings the system capacity to 460,000 kw., with kilovolt-amperes higher, as may be required by power-factor conditions.

Cost of Power

31 The cost of power from such a regional system naturally will vary from year to year. The total of fixed and other annual charges on hydro and steam-plant capacity will remain substantially the same irrespective of the output, whereas the fuel charge will vary, depending upon the hydro deficiency. In dry years a portion of the hydro-plant capacity will be non-productive, and in wet years this condition will obtain with respect to steam-plant capacity. Because of this, the cost of power is therefore not constant, even though unit fuel cost and fixed charges be stationary over a long period of time. However, the average cost of energy may be ascertained with reasonable accuracy, but only if a period of years which contains a dry cycle is considered. It is possible

to make a summation of annual costs for such a period and apply these to the total of kilowatt-hours expected from hydro and steam plants for the period, thus obtaining a period average unit cost of energy. The total cost will be made up of the fixed charges on the capital invested in hydro, including transmission, and in steam plants, plus operation, maintenance, depreciation, and similar costs on both, plus the fuel costs over the period considered.

Conclusion

32 In the final analysis the relative development by a utility of hydro and steam generating capacity is a problem of economy; and the natural resources which are available, the character of the load to be carried, the rate at which energy must be supplied or the load factor, the extent to which other utilities may depend upon it for stand-by, and other related elements furnish the criteria for its solution. This can be illustrated by reference to the measure of steam capacity prevalent with several of the utilities of this region. The steam-plant capacity of the California-Oregon Power Company, a utility with a small load, is nil, as it serves a sparsely settled territory and its hydro plants enjoy a fairly dependable water supply. The Great Western Powèr Company has a steam-plant capacity of approximately 20 per cent of its system peak, relatively small but doubtless justified because of its interconnections and extensive storage capacity which is provided for regulating the stream on which its hydro plants are situated. The Pacific Gas and Electric Company, and the Southern California Edison Company, who jointly furnish about 70 per cent of the energy for the state, each have a ratio of steam to hydro capacity which corresponds fairly closely with that outlined in the foregoing. The Los Angeles Gas and Electric Company operates steam plants exclusively, as it serves a city with a high load density, and cheap fuel in the form of natural gas and oil is readily available.

33 The general tendency seems to be for the steam-plant ratio to increase slightly. The central-station construction program for eleven western states for the three-year period 1924 to 1926 inclusive is as follows:[1]

	Hydro kva.	Steam kva.
1924	243,223	139,500
1925	314,625	87,500
1926 (projected)	194,550	145,000
Total	752,398	372,000

34 For these three years, the ratio of steam-plant construction to the total is 33 per cent. The installed generator capacity on

[1] From *Journal of Electricity,* Feb. 1, 1926.

Jan. 1, 1925, for eleven western states is given as 3,084,974 kw., of which 30 per cent or 935,783 kw. was steam.

35 The steam ratio will probably remain fairly constant, but with a tendency to increase slightly until the majority of the more favorably situated hydro projects are developed. To predict further into the future is impossible on account of the uncertainty of the cost of fuel. At the present rate of growth the hydro resources will not approach complete development for another generation. The cost of fuel oil may and probably will increase, but the supply of low-grade coal which can be substituted will keep the cost of steam within reason, thus permitting fuel to continue to be a factor in the economical development of the remaining hydro resources.

DISCUSSION

R. L. THOMAS.[1] The paper seems to confirm the following remarks made at the recent Atlantic City Convention of the National Electric Light Association in closing a summary of the year's work of the Hydraulic Power Committee of that association:

> Notwithstanding this list of physical works and achievements, I believe that the most important recent and present tendency in hydroelectric engineering is the intensive study of the economics of water-power development and more especially of combined hydro and steam generating systems. Conservation of natural resources is a much-to-be cherished ideal, but when all is said and done a hydro project must stand on its own bottom economically. The great progress which has recently been made in the art of transforming the energy stored up in fuel into electrical energy has made it necessary to study carefully the most advantageous methods of combining and coördinating water power with steam power and to utilize and evaluate the benefits that may be obtained, even in the case of run-of-river plants, from hydro power, other than the saving of a certain amount of fuel. The latter alone will as a rule no longer justify the development of hydro projects.

It seems to the writer that there are several things that hydro-electrical engineers can do, such as

(1) See that the hydro plant is given a fair chance to prove its right to a place in the sun. In the East, at any rate, in the past there has been a tendency to determine the maximum value of a hydro development or of hydro supply by subtracting a liberal profit margin from the cost of equivalent all-steam supply. Although, as already stated, a hydro project must stand on its own bottom, economically speaking, it should not be required to do more than that to justify its existence. If the costs of hydro power and steam power are approximately the same, the desirability of conserving our natural resources, which means *utilizing* our water powers, should throw the balance in favor of hydro power.

[1] Assistant to General Superintendent, Pennsylvania Water & Power Co., Baltimore, Md.

(2) Try to cut down the first cost of water-power development. This has been mounting rapidly. In particular, an effort should be made to reduce the preliminary costs of investigating, planning, and promoting. Projects are today being peddled from banker to banker and, like a boy's snowball rolled along in wet snow, are accumulating successive "layers" of preliminary costs which are prohibitive.

(3) Study and secure a recognition of the advantages which hydro supply may have in addition to the saving of a certain amount of fuel. Even at run-of-river developments on streams of extremely variable flow the available hydro energy at minimum flow may be utilized to "skim off" the peaks of the system load at a very low daily load factor, and the hydro plant may be credited with the saving of a certain amount of steam-plant capacity and investment. Among other advantages of hydro supply there might be mentioned what was called by the Conowingo engineers at recent commission hearings the "A. C. storage battery" effect of pondage and quick-starting turbines; the convenience and economy with which maintenance work may be carried out at the steam plants during high flow periods; the diversity in generating station difficulties, interruptions due to cable failures, etc., obtained by an independent source of supply; saving in distribution expense (both initial and annual) effected by bringing the hydro supply in to the city systems at strategic points; more favorable position of the electric company at times of mine or railroad strikes; the stabilization of operating costs; the mechanical reliability of hydraulic units; and the almost magical popular appeal of water power.

(4) Design and operate the plant so as to make the most of such advantages as the quick starting characteristics and mechanical ruggedness of hydraulic turbines as compared with steam turbines. In the latter connection hydro power should be given its due. At a symposium on stand-by steam generating units held by the Prime Movers Committee of the National Electric Light Association last winter, it was an interesting fact that in the many written and spoken discussions even hydroelectric engineers and operators unconsciously spoke of steam turbines acting as stand-by for *hydro power* when they meant stand-by for *transmission lines.* That is, probably in all of these cases the steam stand-by unit at the load center would have been deemed more necessary if there had been a steam plant at the other end of the transmission line. This does not refer to stand-by, or more properly supplementary, steam required on account of shortage of water.

Sanford A. Moss.[1] The use of white fuel is so much in the lime-light that it is refreshing to see that the author finds that

[1] Engineer, Thomson Research Laboratory, General Electric Co., West Lynn, Mass. Mem. A.S.M.E.

black fuel is not so black as has been painted and continues to have a very important place as a power producer. The hydro plant enthusiasts who think that fuel cost forbids steam plants are as wrong as the steam enthusiasts who see nothing but the fixed charges of interest and depreciation on hydro plant investments. A middle course is the right one, and steam and hydro plants must be tied together with a proportion depending upon the individual circumstances. The use of fuel is often deplored because

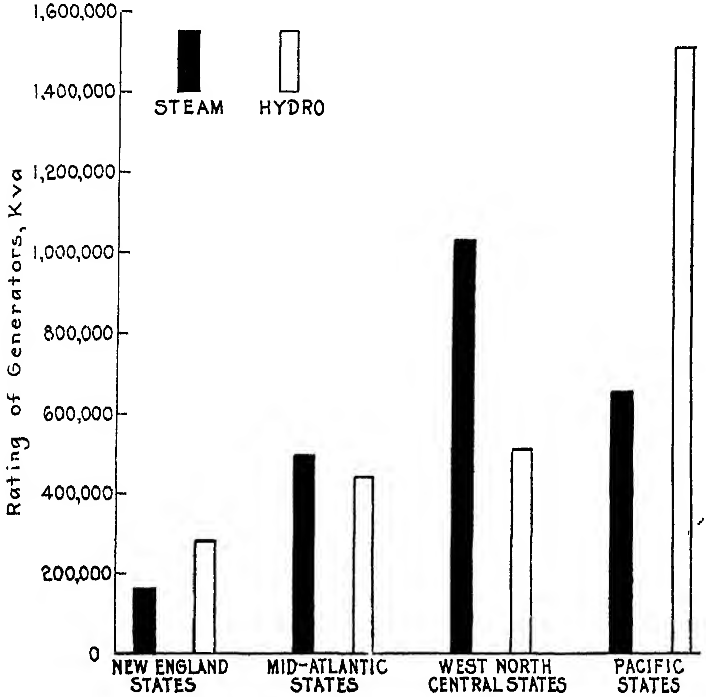


FIG. 5 RELATIVE SIZE OF HYDRO AND STEAM PLANTS OF STEAM-HYDRO COMPANIES

fuel once burned is gone forever; but it must also be borne in mind that capital once invested in a hydro plant is gone forever, and the consequent fixed charges are a continuous drain just as is fuel cost. The high efficiency of the modern steam turbine has done a great deal to decrease the fuel charge. There is also, of course, a fixed charge on capital investment for a steam plant, but recent advances in output of a given amount of steam plant apparatus, due to the use of higher pressures and temperatures, increased ratings, and increased speeds, has decreased this greatly.

In connection with the balancing of the relative amounts of steam and hydro power to be tied in together, it is interesting

to note what has occurred in some of the older communities. In some cases industries were located where water power was available, but as the years have gone on, all of the other factors mentioned by the author have been at work and the actual integration of these factors is shown by the present situation. This is exhibited

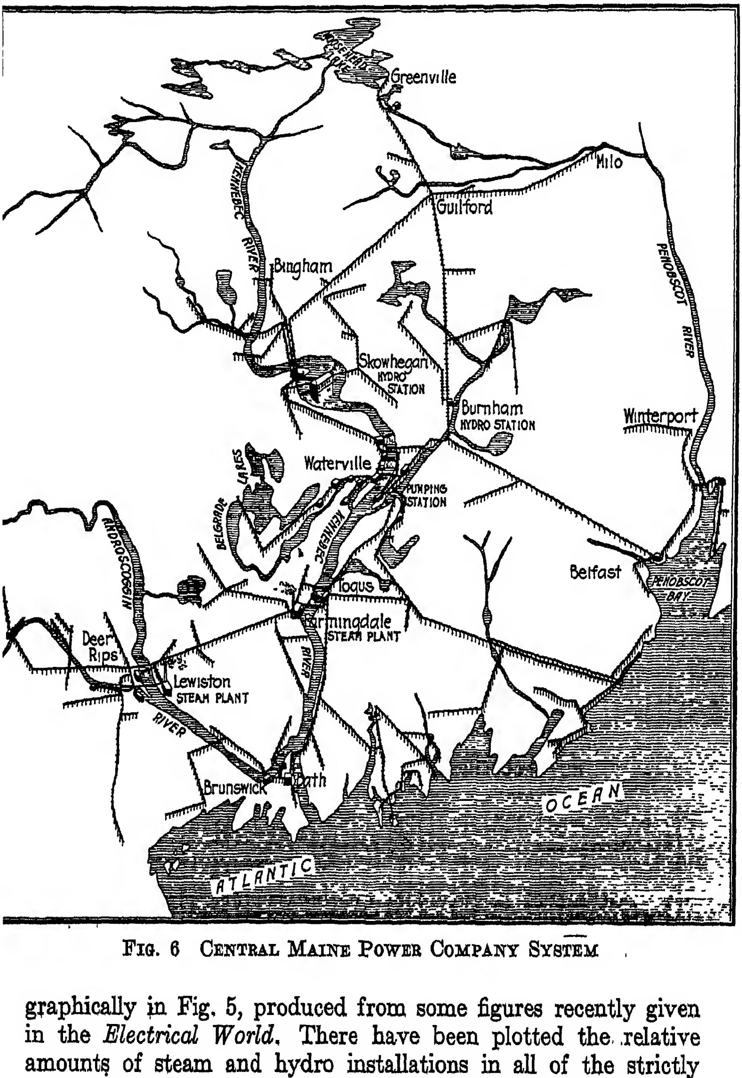


FIG. 6 CENTRAL MAINE POWER COMPANY SYSTEM

graphically in Fig. 5, produced from some figures recently given in the *Electrical World*. There have been plotted the relative amounts of steam and hydro installations in all of the strictly hydro companies. Systems where steam only is used have not been included. The black area represents stations using black fuel and the white area represents those using white fuel. As will be seen, in three of the older communities the amount of black fuel

used is nearly equal to or exceeds that of white fuel. At the present time in the Pacific States the amount of black fuel used is but half that of white fuel. Time only can tell whether the situation will ever be the same as now exists in the other parts of the country. In many cases in the East, industries using water power have grown beyond the economical extension of the water power, and steam stations have been necessarily added. Of course, in none of these cases has the available power been anything like the amount available on the Pacific Coast. However, the power obtainable from the low falls of the New England rivers was once as much ahead of the demand as seems to be the present situation on the Pacific Coast. We may therefore look forward to the time —

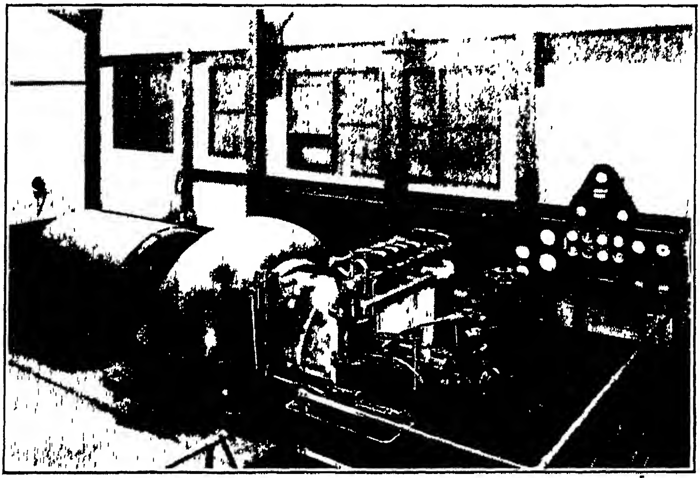


Fig. 7 5000-Kw. Steam Turbine (General Electric Company) Arranged with Governor and Stage Openings for Steam Extraction

possibly a long way in the future — when the industrial growth of the Pacific Coast will pass the available water power and require steam power in the same proportion as in the East. Another factor which has been at work in the East is the fact that in many cases the high efficiency of modern steam plants has balanced the fixed charges on hydro investments, so that the steam plants have become directly competitive.

The various circumstances surrounding Eastern hydro networks have often been such that the steam plants, instead of being large stations such as is the case so far on the Pacific Coast, have been comparatively small stations distributed about the country. Fig. 6 shows a map of the Central Maine Power Company where this has occurred. Such stations use turbines of from 2,000 to 10,000 kw., often of the type shown in Fig. 7. The advances in economy

and capacity in turbines of such sizes have been as great as the advance in large turbines. Such turbines can now be furnished with efficiencies nearly equal to those of the larger units. Time will tell as to whether or not the Pacific Coast conditions will ever economically require such comparatively small units.

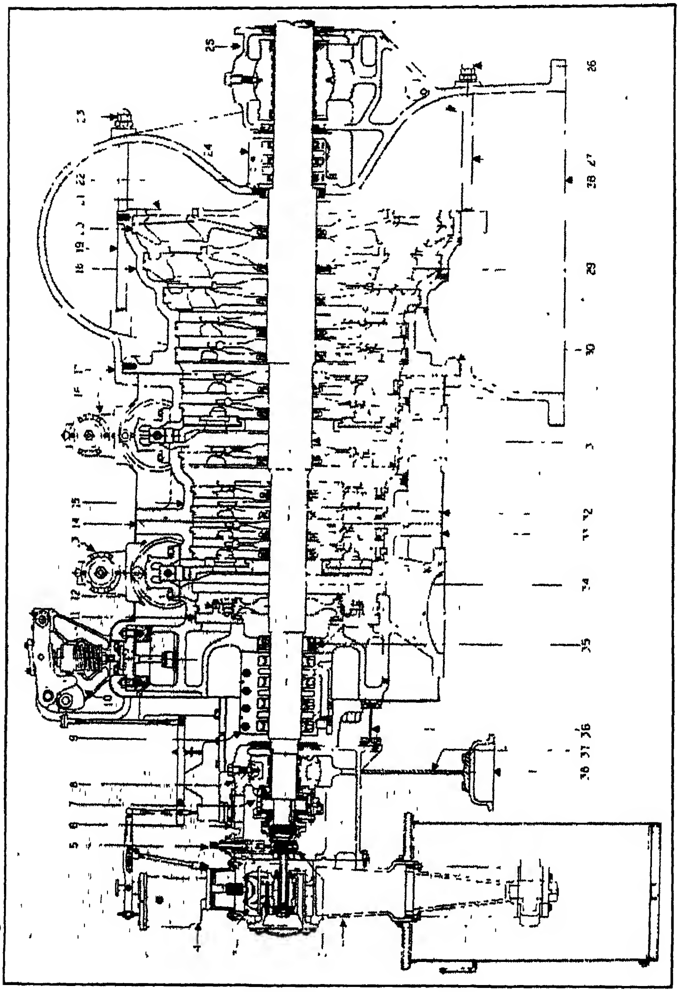


FIG. 8 3500-KW. TURBINE (GENERAL ELECTRIC COMPANY) WITH TWO OPENINGS FOR STEAM EXTRACTION

One matter which has extended the use of such small units is the tendency on the part of hydro companies to take on the business of supplying steam for industrial purposes because power can be generated as a by-product. Such installations are of two types: one, where steam is required for industrial processes, such as refineries, textile mills, and paper mills; the other, where steam

is required for heating, including central-station heating distribution. In all such cases the steam can be generated at a pressure suitable for use in a steam turbine at nearly the same cost as if the steam were used directly. Hence the power which is obtained from the steam on its way through the turbine, instead of being obtained from white fuel or black fuel, is obtained with almost no fuel at all. Such turbines are directly tied in with the hydro network. The steam enters the turbine at the usual boiler pressures and some of it is extracted in one or more places part way down the turbine where the stage pressure is that required in the industrial process. Fig. 8 shows the type of turbine used for this purpose.

In some cases the remainder of the steam passes to a condenser. In other cases non-condensing turbines are used, exhausting at such a back pressure as is needed in the industrial processes, pressures even up to 60 lb. per sq. in. being common. Governing systems have been developed so that such turbines operate in parallel with hydro stations while maintaining the desired steam conditions. They may deliver power up to the limit of the industrial steam needed. If the power demand requires it, they may deliver a less amount of power, in which case high-pressure steam is used through a reducing valve. Or, they may deliver a greater amount of power than corresponds to the industrial steam, in which case some steam passes to a condenser.

If there is ever realized the vision that the industrial growth of the Pacific Coast will require steam power as it has in the East, there will be many such turbines tied in with hydro networks.

W. L. Abbott.[1] A study of the problem of supplementing steam plants with hydro power or using hydro power instead of steam power in the Middle West brought the writer to the conclusion that if there were a water power at a distance of, say, 150 miles from a center where the power was to be used and if alongside of the water power were a coal mine, it would be cheaper to build neither the hydro plant nor the mine steam plant but to build a steam plant in the city or center where the power is to be used, shipping coal to the city plant by rail. One considerable reason for that conclusion is the great cost of a right-of-way through the thickly settled communities of the prairie states.

Even here in the home of hydro power where the cost per mile of a transmission line right-of-way is much less than in the East it is questionable if engineers and financiers have adjusted their sense of proportion to the lower fuel cost of modern steam plants, their lower overhead charges, and the psychological and practical ad-

[1] Chief Operating Engineer, Commonwealth Edison Company, Chicago, Ill. President, 1926, A.S.M.E.

vantages of having the source of power near the load to be carried.

With all factors in the problem given proper weight it would appear that in connection with water power, steam power is entitled to a zone on your load curve reaching down much nearer to the base load than it is now allowed to go.

The Author. The author believes that at equal cost between hydro and steam we must, of course, develop our hydro resources. There are a number of incidental advantages in so doing. When we develop a hydro plant we not only obtain power, but frequently water for irrigation as well. The Great Western Power Company is a case in point. A great deal of their power comes from Lake Almanor, one of the largest reservoirs in the world, which incidentally was built at a very low cost. Following the generation of power, a part of this water is re-used for irrigation. This would not have been possible with a steam development.

In this connection also there is a certain amount of enlightened selfishness on the part of the power company, in that it receives a very substantial reward from the growth of the territory through which its transmission lines pass. Thus a hydro development with its transmission network presents beneficial aspects aside from the furnishing of power and may be classed as a public improvement of the highest order, particularly in a semi-arid country. There are certain other factors also in a hydro development which are not controlled solely by economic considerations.

Such may not be said of a steam plant in the same measure, since it is constructed to meet a given situation on a purely economic basis.

Referring particularly to the discussion by Mr. Thomas, the author is in fullest accord with the first point mentioned. Even though the hydro plant merely pays for itself at the point of distribution, its installation would seem to be justified by those other advantages mentioned subsequently by Mr. Thomas.

In his third point Mr. Thomas mentions the use of hydro power from run-of-river developments, during periods of minimum flow, to "skim off" the peaks of the system load at a very low daily load factor, and thereby to save steam plant capacity. While this is exactly the reverse of conditions in the West it may be desirable as a temporary measure required by an abnormally low minimum on a very dry year when the minimum flow is too small to permit carrying any portion of the base load, however slight, on the hydro at a high load factor. Such a situation the author feels would never be economical as a basis upon which to build a hydro plant unless the hydro plant was of very low capital cost and the economy in the steam-plant operation could be improved greatly

through the higher load factor from the base which doubtless would obtain for the steam plant.

The general principles suggested in the opening paragraph of the paper, as to the suitability of steam for low-load factors and hydro for high-load factors, may perhaps be restated with more direct reference to conditions in the East discussed by Mr. Thomas.

As indicated in Pars. 2, 3, 4, and 5 of the paper, the best economy in central-station-generation costs must of necessity be determined through a consideration of the character of the system load curve, the capital cost of steam and hydro plants, and the cost of money and fuel. The author is confident that it will be found economical under practically any set-up, where hydro development is a possibility, to carry a portion of the base load on hydro. What portion of the base load of a given steam system may economically be carried on hydro is dependent upon the interrelation of the above factors. For the best economy in a steam supply, the percentage of the load which can be carried on hydro will increase as the hydro capital costs decrease, and as the price of fuel increases.

For a regional system like that of the Pacific Gas and Electric Company, oil would have to be permanently priced at about 50 cents per barrel for the company even to consider a 100 per cent steam supply. On the basis of heat equivalents this would mean coal at around $2.80 per ton.

Even were this the case, probably savings could be made on this system, were a portion of the base load still carried on hydro, because some of the hydro plants would be near some of the load, with consequent savings in transmission capital, thus giving the hydro less capital for the accumulation of fixed charges. Moreover, when steam in comparison with hydro for a regional system is under consideration, one must not forget the cost of the back transmission from the steam plants. Neither of these features would obtain, with the load entirely concentrated in metropolitan centers.

As an illuminating example, closer to Mr. Thomas' situation, let us take an exclusively metropolitan load under northern California conditions. The full cost of hydro power delivered at the plant switchboard may be taken at about $22 per kw-yr. on the basis of capital cost which is obtaining for hydro plants at present. On the other hand, the cost of steam power at its switchboard, without the fuel cost, is close to $16 per kw-yr. Both costs include appropriate fixed charges. The difference of $6 per kw-yr. is what can be paid for fuel, to effect a standoff in cost, on the basis that transmission was unnecessary. On the basis of 65 per cent load factor, or 5700 hours of use, $6 will allow for the payment of about one mill per kw-hr. for fuel. Running an economy of 400

kw-hr. per barrel, which is possible in modern plants, the fuel oil would have to be available at 40 cents per barrel to have a fuel cost of $6 per kw-yr. If transmission were necessary to the metropolitan district, it would cost about $10 more per kw-yr. on the basis of present capital costs of transmitting large blocks of power over the distance which now obtains on the average. This $10 will allow for the payment of 1.75 mills more per kw-hr., or a total of about 2.75 mills per kw-hr. for fuel. This will permit the purchase of oil at $1.10 per barrel.

Using the above figures, but assuming a 30 per cent load factor, or, say, 2600 hours of use with the same economy, the $16 ($10+$6) margin will allow a fuel cost of 6.15 mills per kw-hr., or for the payment of oil at a price as high at $2.46 per barrel. If transmission were unnecessary oil would have to sell at 92 cents per barrel.

From the foregoing it will be observed that at a fairly high load factor the margin of cost, between the cost of steam power without fuel and the total cost of delivered hydro power, which is available for the purchase of fuel, is small. This margin becomes greater as transmission is required for the hydro, and as the load factor decreases. This fact accounts for the building of many steam plants. In other words, for low percentage demand, cheap power from steam plants becomes possible with efficient machinery, even though fuel costs may tend to be high. As stated in the beginning, however, the author believes, in general, that the proper plan is to divide the load to hydro and steam so that each may do that portion of the work which it can perform cheapest.

In addition to the general advantages of a partial hydro development in a steam-power system, the author would amplify Mr. Thomas' thought that a partial hydro development tends to stabilize operating costs. Steam costs fluctuate considerably with variation in the cost of fuel. On the other hand, hydro costs are fairly fixed, the capital charges accounting for about two-thirds of the total costs, while fluctuations in the cost of labor and incidental material, which constitute the other one-third, are relatively slight.

No. 2009

SPEED CHANGES OF HYDRAULIC TURBINES FOR SUDDEN CHANGES OF LOAD

By Earl B. Strowger,[1] Niagara Falls, N. Y.
Non-Member
and
S. Logan Kerr,[2] Philadelphia, Pa.
Associate-Member of the Society

This paper shows how the variation in speed of hydraulic turbines due to sudden load changes may be determined by the application of the fundamental principles of water hammer with allowances for variation of turbine efficiency. The methods of determining governor time are noted. The relation of generator flywheel effect to speed variation is established and methods are indicated for the proper selection of these characteristics. Numerical examples are shown illustrating the speed changes for various load changes on a 70,000-hp. hydroelectric unit. The application to practical cases and the accuracy of approximate formulas are discussed.

Introduction

The ability of a hydraulic-turbine governor to accomplish its function of regulating satisfactorily the speed of the unit depends upon three fundamental considerations:

1 Mechanical correctness
2 Proper field adjustment
3 Complete coördination of the governor characteristics with the physical constants of the installation.

Of these three considerations, the first two will not be discussed in this paper, although they play a very vital part in securing good speed regulation. The third consideration is most important of all, since the selection of governor time and generator flywheel

[1] The Niagara Falls Power Company.
[2] Assistant Hydraulic Engineer, Wm. Cramp & Sons Ship & Engine Building Co.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

effect in conjunction with the hydraulic conditions will fix the amount of speed variation resulting from given load changes on the units.

2 The ability to predetermine the speed changes on a hydraulic turbine in terms of the maximum change in speed or the variation in speed with respect to time, permits the complete study of regulation problems in advance of the actual construction of a hydroelectric project. It is often desirable to know the amount of speed change occurring at various times following a load change, in order to make adjustment of the electrical protective devices, or to determine the possibility of over-voltage conditions in the stator of the generator or on transmission systems[1] due to sudden load changes.

3 This paper discusses the methods of predetermining the speed changes resulting from sudden load changes, and indicates the methods to be employed in selecting the proper generator flywheel effect and the correct time of governor movement.

4 The paper is based on the application to the problems of speed regulation of the theory of rise and fall in pressure due to velocity changes in the penstock. Complete discussions of these phenomena have already been published in the technical press.[2]

Physical Constants of Installation

5 The physical constants of the installation, comprising the head on the plant, the capacity of the units, the length of the conduit and the velocities in the conduit, are usually fixed by local considerations.

6 With these physical constants determined, the speed regulation obtained becomes a question of compromise between the time of governor action, the allowable pressure variation in the penstock, the flywheel effect of the generator and turbine, and the permissible speed variation for given load changes on the unit. Of these four functions, the governor time may be determined independently by fixing the maximum allowable rise or fall in pressure in the penstock. The two variables remaining, comprising the flywheel effect of the rotating element and the speed variations for given load changes on the unit, may then be found.

[1] See Over-voltage on Transmission Systems Due to Drop of Load, by E. J. Burnham, *Journal*, A.I.E.E., May 1925.

[2] See Pressure in Penstocks Caused by Gradual Closing of Turbine Gates, by N. R. Gibson, Transactions, A.S.C.E., vol. lxxxiii (1920), p. 707.

Fall in Pressure in Hydraulic Turbine Penstocks Due to Acceleration of Flow, by S. Logan Kerr, in Hydraulic Power Committee Report (1924), National Electric Light Association.

Determination of Governor Traversing Time

7 With the length, diameter, and profile of the penstock determined, considering, in the case of high heads and long water conduits, that a properly designed surge tank or open forebay constitutes a point of relief as defined in the study of water hammer, then the minimum time of closure of the turbine gates depends upon the maximum rise in pressure allowable for the strength of the penstock or upon the maximum fall in pressure permissible to avoid the formation of a vacuum in the penstock due to acceleration of flow. It is usual to design hydroelectric plants with the maximum rise in pressure not in excess of 50 per cent of the normal head on the turbine, but it is frequently found desirable to limit this maximum rise to 25 per cent of the normal head, or even less, in order to avoid excessive stresses in the penstock which may be repeated from time to time due to the changes in load on the unit. Recent experiments have indicated that excessive water hammer occurring at frequent intervals may cause fatigue of the materials in the penstock and increase the danger of rupture following water-hammer surges.

8 In certain installations the rise in pressure is taken care of by providing an auxiliary relief valve opened by the governor as the gates are closed, thus bypassing the water and reducing the rate of change in velocity of the water to a minimum. In calculating the speed rise for such cases the quantity of water discharged by the relief valves is deducted from the total flow in computing the rise in pressure and change in velocity.

9 In the case of oncoming loads, the profile of the penstock should be considered and the maximum fall in pressure calculated, the acceleration gradient determined, and the minimum allowable opening stroke of the governor established in this manner.

10 In determining the flow through the conduit the rated capacity of the units should be taken and the discharge corresponding to this output at an efficiency of 85 to 88 per cent calculated. Although the actual efficiency may be greater than this, there is some overload capacity allowed beyond the rating and the use of the lower efficiency will compensate for this as an added margin of safety.

11 The definition of governor traversing time is taken as the time required for the governor to move the gates from the rated-capacity position to the speed no-load position, including an allowance for the dead time of the governor. The governor dead time is the period that elapses between the rejection or application of load and the first movement of the turbine gates. This time is usually less than $\frac{1}{2}$ second for modern governors and, when considered in computations, is included as shown in Fig. 2c.

12 Fig. 4 illustrates the proportions of full gate time required for intermediate gate strokes. The rated capacity will occur at

some gate opening less than full as there is always a margin allowed in power. The amount of gate opening required for a given increment of power is much greater beyond the point of the maximum efficiency than below this value. This tendency grows more pronounced as the specific speed increases, hence the assumption regarding governor time as noted above is specified. It is usually safe to assume that rated capacity will occur between 70 and 80 per cent of full gate travel, although on some units it may be more

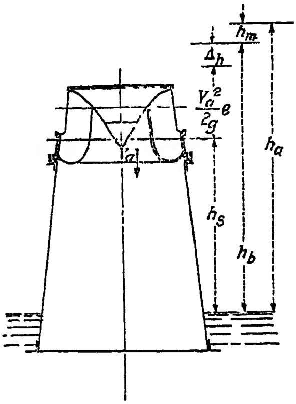


FIG. 1 CALCULATION OF DRAFT-TUBE SURGE

h_a = atmospheric pressure
h_m = vapor pressure
h_b = barometric limit water column

or less than this amount depending upon the amount the manufacturer allows as a margin over the guaranteed power output.

13 Fig. 3 illustrates, for a unit having a specific speed of 34.7 in the foot-pound system, a curve of percentage of gate opening plotted against percentage of rated capacity.

14 In the case of installations involving a length of penstock which would permit a governor time of less than two or three seconds, it is necessary to investigate very carefully the change of flow occurring in the turbine draft tube. If the governor stroke is made too rapid, the water column below the runner may break, due to excessive vacuum followed by a return surge striking the runner with considerable force, which may be dangerous in certain installations.

15 In a closed conduit it is assumed that the deceleration of flow produces the same effect whether it is accomplished by means of a valve at the lower end of the conduit causing a rise in pressure above normal at this point, or whether it is accomplished by means of a valve at the upper end of the conduit and is manifested by a fall in pressure below normal at the valve, equal in magnitude to the rise in pressure which would result in the first instance. In the

case of the draft tube, the discharge is completely sealed against admission of air, and hence the inertia of the water column is subject to rational calculation by the application of the water-hammer theory to the change of flow occurring in the draft tube.

16 It is possible to calculate the average velocity of the water passages in the draft tube and, since the length is known, the rise in pressure due to a given governor time may be calculated. This rise in pressure will be manifested as an increase in vacuum, and the sum of this increase plus the absolute velocity head regained in the top of the draft tube at the discharge from the runner and the elevation in feet of this point above tailwater should not exceed the barometric limit.

$$\Delta h = h_b - \frac{V_a^2}{2g} e - h_s$$

Δh = allowable increase in vacuum in feet of water caused by governor time T

h_b = barometric limit of water column in feet

V_a = absolute velocity of water at top of draft tube, ft. per sec.

e = efficiency of energy regain of draft tube

h_s = static elevation of top of draft tube above tailwater in feet

g = acceleration due to gravity.

Fig. 1 indicates the method of determining this feature.

17 The calculation of the permissible governor time may be based on a load change from 50 per cent to zero without breaking the draft-tube vacuum, since the condition for full load thrown off is sometimes too severe and results in too long a governor time, although in certain cases it may be justifiable to utilize the full-load condition for design.

18 In case both the above methods of computing the governor time show a permissible rate of movement of the turbine gates for full load change of less than two or three seconds, it is advisable to limit the governor motion to a slower rate of closure for purely practical reasons. With large units having a rapid governor stroke, the governor itself must be quite large and the piping leading from the governor to the operating cylinders must be made over-size to avoid high velocities of the governor fluid resulting from a full stroke in a very short time. It is rarely necessary to go below two seconds governor time in order to meet ordinary commercial regulation practice in regard to speed variation and, if a governor time of three seconds is utilized for plants having appreciable length of penstocks even under medium heads, it will be found that more satisfactory hydraulic conditions will result than from the use of a much shorter time. In high-head plants, it may be desirable to go as high as four or five seconds for a full stroke, and possibly more than this in certain cases.

Theory of Speed Changes

19 Having fixed, tentatively, the time of governor movement, it is possible to calculate the resulting variation of power input and the energy change in the rotating element; and with this established, the speed variation from normal caused by this change in energy is limited only by the flywheel effect of the rotating parts.

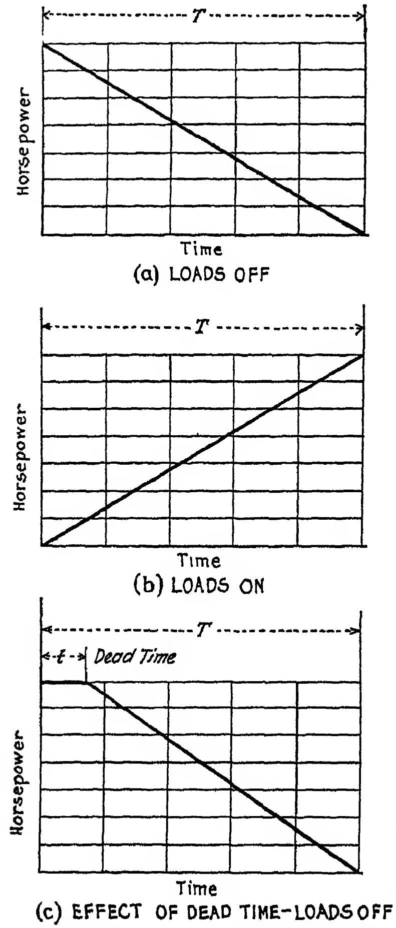


Fig. 2 Uniform Variation of Horsepower with Respect to Time

20 In all the calculations in this paper, it is assumed that the unit is isolated from the system without securing any benefit of connected flywheel effect of other generating units or of the load which it is carrying. In actual practice, for small load changes which do not change the speed sufficiently to trip the relay devices, the connected flywheel effect of the system assists materially in reducing the speed fluctuations. However, in the cases where the unit is separated from the system by the interruption of service, no assistance is possible in reducing the amount of speed variation which occurs on the particular unit. The actual amount of flywheel effect supplied by the connected load on the system is very difficult to determine, although some tests have been made recently in this regard.[1]

21 Neglecting the effect of water hammer and assuming that the power input to the rotating element decreases uniformly with respect to time as indicated in Fig. 2*a*, the excess power during the time of closure of the turbine gates will be the average between the initial excess amount and zero,

[1] Practical Aspects of System Stability, by Roy Wilkins, *Journal*, A.I.E.E., February 1926, p. 142.

or one-half of the amount of load dropped. The dead time of the governor is assumed to be zero. The kinetic energy in the rotating mass is:

$$\frac{W(2\pi nr)^2}{2g3600} = \frac{Wr^2n^2}{5870}$$

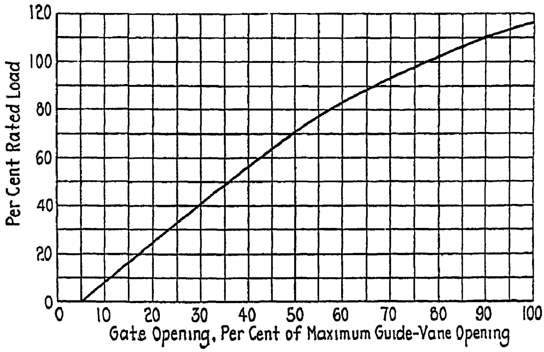


FIG. 3 RELATION BETWEEN GATE OPENING AND RATED CAPACITY, SPECIFIC SPEED 34.7

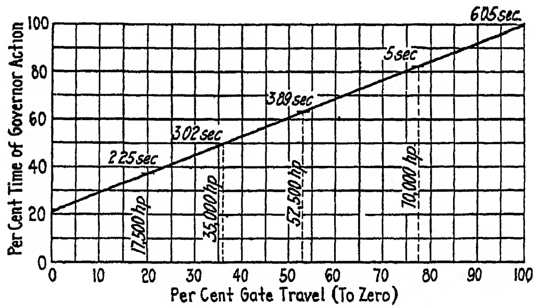


FIG. 4 PROPORTIONS OF FULL-GATE TIME FOR INTERMEDIATE GATE STROKES

where W = weight of the rotating mass in pounds
n = revolutions per minute
r = radius of gyration in feet
g = acceleration due to gravity, 32.2 ft. per sec. per sec.

If N_1 is the normal value of speed, and N_2 the value of speed at the end of the governor stroke, then the energy which has been absorbed by the rotating mass in increasing the speed of the rotating element is

$$\frac{Wr^2N_2^2}{5870} - \frac{Wr^2N_1^2}{5870} = \frac{Wr^2}{5870}(N_2^2 - N_1^2)$$

We may then equate this energy to the amount of energy given up by the water during the closure:

$$\text{av.hp.} \times T \times 550$$

where T = governor time in seconds
av.hp. = the average horsepower during closure.

Then

$$\frac{Wr^2}{5870}(N_2^2 - N_1^2) = \text{av.hp.} \times T \times 550 \quad . \quad . \quad . \quad . \quad . \quad [1]$$

Assuming the following constants:

$$Wr^2 = 70{,}030{,}000 \text{ pounds feet squared}$$
$$T = 5 \text{ sec.}$$
$$N_1 = 107 \text{ r.p.m.}$$
$$\text{av.hp.} = \frac{70{,}000}{2} = 35{,}000 \text{ hp.}$$

22 Neglecting the energy changes in the penstock due to water hammer and the variation in turbine efficiency during closure, and consequently assuming that as soon as the load is dropped from the generator the energy delivered to the water wheel from the water column decreases uniformly to zero in a certain time T, then N_2 is equal to 139.7 r.p m. and the percentage of speed rise 30.6 per cent above normal.

23 In the case of full load thrown on the unit from zero, the situation is completely reversed, that is, the demand is greater than the supply, and the energy is given up by the rotating element until the supply has increased sufficiently to equal the demand. In this case the following formula will give the N_2 for oncoming loads. With the energy which is in the penstock, due to water hammer, neglected as before, and the assumption made that the energy delivered to the water wheel from the water column increases uniformly from zero to the required amount in a time equal to T (Fig. 2*b*), then

$$\frac{Wr^2}{5870}(N_1^2 - N_2^2) = \text{av.hp.} \times T \times 550 \quad . \quad . \quad . \quad . \quad . \quad [2]$$

where av.hp., in this case, is the average deficiency in power during the opening stroke. Assuming the same constants as before we find that N_2 by this method is equal to 58.4 r.p.m., and the percentage of decrease in speed is 45.4 per cent below normal

Effect of Water Hammer

24 In considering first the case of the rise in speed following a complete loss of load, as the turbine gates close, reducing the velocity in the penstock, an increase in pressure will result, and the horsepower delivered to the generator at any time during the stroke may be obtained from the following formula:

$$\text{hp.} = VHC \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [3]$$

where V = velocity in the penstock in ft. per sec.
H = head on the turbine in ft.
C = constant equal to $0.1136AE$

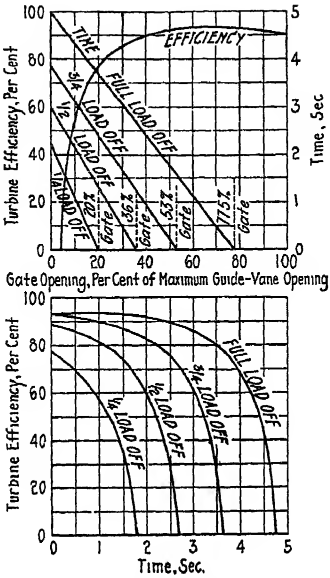


FIG. 5 VARIATION OF TURBINE EFFICIENCY WITH TIME FOR LOADS THROWN OFF TO ZERO

(Full load = 70,000 hp.)

C is derived from the relation

$$VHC = \frac{QwHE}{550}$$

$$C = \frac{62.5QE}{550V} = 0.1136\,\frac{QE}{V} = 0.1136AE$$

in which Q = discharge in cu. ft. per sec.
A = average area of the penstock in sq. ft.
E = the efficiency of the turbine at any particular gate opening
w = weight of water in lb. per cu. ft.

Since the average penstock area is known, the constant C becomes a function of the turbine efficiency and is therefore a "variable constant" to be determined at each interval of time. The curve of efficiency with respect to gate opening and the curve of gate opening with respect to time may be determined from tests of similar turbines with a reasonable amount of accuracy and hence the various values of C found in this manner. Fig. 5 gives these

relations for a unit having a specific speed of 34.7 based on the movement of the gates being uniform with respect to time.

25 The rise in pressure due to the reduction in velocity in the penstock may be computed by the method of arithmetical integration as set forth in Mr. Gibson's article mentioned previously, and it is found that the time intervals must be taken equal to $2L/a$ seconds, where L is the length of the penstock and a is the velocity of the pressure wave. For very long penstocks with correspondingly long intervals of time, it may be desirable to take fractions of intervals, although this necessitates a greater number of computations in order to secure the pressure-time curve. The variation of velocity with respect to time is also obtained simultaneously with the variation of pressure and time, and hence by the application of the constant C, the horsepower delivered to the rotating element at any time during the complete closure may be determined.

26 Equation [1], given above, may then be applied to the increments of change in speed caused by the increments in change of horsepower delivered to the rotating element, and in this case N_1 indicates the speed at the beginning of each interval of time selected and N_2 the speed at the end of that particular interval, which, for the next interval, becomes N_1 and contributes to the calculation of a new N_2 at a subsequent interval. The variation of speed with respect to time can then be plotted directly, and the speed at any time during the governor stroke determined, as well as the maximum rise in speed attained. The following numerical example for full load thrown off to zero illustrates in tabular form (Table 1) the computations necessary to secure these curves, and Fig. 6 illustrates graphically the results of these computations. The following data apply to this numerical example as well as to those covering fractional load changes and loads thrown on from zero:

Wr^2 = 70,030,000 lb. ft.^{2}
T = 5 sec.
t = dead time, equal to zero
h_0 = 213.5 ft.
Q = 3160 cu. ft. per sec. at 70,000 hp.
Load = 70,000 hp.
Penstock length = 690 ft. (including draft tube)
Penstock diameter = 21 ft.
N_1 = 107 r.p.m.
V_0 = 10.25 ft. per sec. (weighted average velocity as to length of penstock)
Average area = 308 sq. ft.

Working equation for pressure rise:

a = velocity of pressure wave = 4680 ft. per sec. (in this case)

$$\text{One interval} = \frac{2L}{a} = \frac{2 \times 690}{4680} = 0.2949 \text{ sec.}$$

$$B_0 = \frac{V_0}{\sqrt{H_0}} = \frac{10.25}{\sqrt{213.5}} = 0.7015$$

$$B = \frac{V}{\sqrt{H}}$$

$$\Delta H = \frac{a\Delta V}{g} = \frac{4680}{32.2}\Delta V = 145.25\Delta V$$

ΔH = increase in head during one interval

ΔV = destruction of velocity during same interval

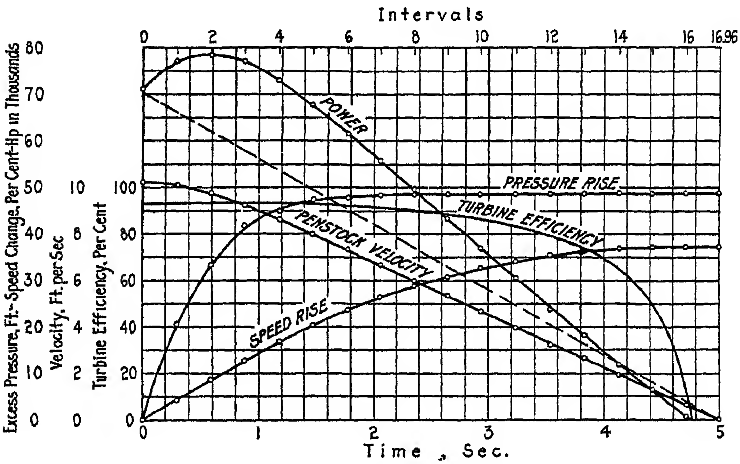


FIG. 6 SPEED RISE FOR FULL LOAD THROWN OFF TO ZERO, UNIFORM CLOSURE

Working equation for speed change:

$$C = 0.1136AE = 0.1136 \times 308 \times E = 34.99E$$

$$\text{hp.} = VHC$$

$$\frac{70{,}030{,}000}{5870}(N_2^2 - N_1^2) = \text{hp.} \times 0.2949 \times 550$$

$$N_2 = \sqrt{0.01359 \text{ hp.} + N_1^2}$$

27 The pressure rise consisting of the first seven columns of Table 1 is readily computed and the curves of pressure rise and velocity decrease are drawn as in Fig. 6. The variation of turbine efficiency with respect to time is obtained from the relation of turbine efficiency with respect to gate opening and the relation between gate opening and time as shown on Fig. 5. The curve

TABLE I

Full load thrown off (70,000 hp.)
Head, 213.5 ft.
Governor time, 5 sec.
Uniform gate closure.

Penstock length, 690 ft.
Penstock diam., 21 ft.
$N_1 = 107$ r.p.m.
$Wr^2 = 70,030,000$ lb. ft^2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
											Ave power for interval					
Interval	Time, sec.	Gate B	$H = h_0 + h$	V / ΔV	$\Delta H = 145.25\Delta V$	$\Sigma\Delta H = h$	VH	E %	$C = 34.99e$	hp. $= VHC$	Delivered by turbine	Given to rotating parts	0.01359 hp.	N_1^2	R.p.m.	Speed change, per cent
0	0.0000	0.7015	213.50	10.250	....	00.00	2188	92.8	32.48	71000	..	.	..	11449	107.0	0 00
			20.63	0.142	20.63	....	....	....	....	....	74060	73803	1004	....	....	....
1	0.2949	0.6601	234.13	10.108	....	20 63	2368	93.1	32 58	77120	..	..	.	12453	111 6	4.30
			33.27	0.371	53.90	....	....	....	....	....	77760	77503	1054	....	....	....
2	0.5897	0.6188	246.77	9.737	....	33.27	2402	93.3	32 64	78400	...	....	..	13507	116 3	8.69
			41.97	0.517	75.10	....	....	....	....	....	77700	77443	1053	....	....	....
3	0.8846	0.5774	255.47	9 220	....	41.83	2356	93.5	32.70	77000	..	....	....	14560	120.7	12.80
			44.92	0.597	86.75	....	....	....	....	....	74900	74643	1041	....	....	....
4	1.1795	0.5360	258.42	8.623	....	44.92	2229	93.4	32.68	72800	....	....	....	15601	125.0	16 82
			47.48	0.636	92.40	....	....	....	....	....	70250	69993	950	....	....	....
5	1.4744	0.4946	260.98	7.987	....	47.48	2083	93.0	32.53	67700	....	....	....	16551	128.7	20.28
			47.97	0.657	95.45	....	....	....	....	....	64665	64408	875	....	....	....
6	1.7692	0.4533	261.47	7.330	....	47.97	1917	92.0	32.18	61630	....	.	....	17426	132.5	23.82
			48 48	0.664	96.45	....	....	.	....	....	58705	58448	794	....	....	....
7	2.0641	0.4119	261.98	6.666	....	48.48	1746	91.3	31.94	55780	....	....	....	18220	135.1	26.25
			48.40	0.667	96.88	....	....	....	....	....	52690	52433	713	....	....	....
8	2.3590	0.3705	261.90	5.999	....	48.40	1570	90.3	31.60	49600	....	....	....	18933	137.7	28 69
			48.65	0.668	97.05	....	....	....	....	....	46435	46178	627	....	....	....
9	2.6538	0.3292	262 15	5.331	....	48.65	1398	88 5	30.96	43270	...	...	....	19560	139.9	30.74
			48.52	0.669	97.17	....	....	....	....	....	40080	39823	541	....	...	..
10	2.9487	0.2878	262.02	4.662	...	48.52	1221	86.4	30.22	36890	..	....	....	20101	141 9	32.60
			48 65	0.669	97.17	....	....	....	....	....	33745	33488	455	....	....	....
11	3.2436	0.2464	262.15	3 993	....	48.65	1047	83.5	29.21	30600	....	....	....	20556	143.4	34 01
			48.67	0.670	97.32	....	....	....	....	....	27090	26833	365	....	....	....
12	3.5385	0.2051	262.17	3 223	....	48.67	848	79.5	27.80	23580	...	....	....	20921	144.7	35.22
			48.65	0.670	97.32	....	....	....	....	....	20800	20543	279	....	....	....
13	3.8333	0.1637	262.15	2 653	....	48.65	696	74.0	25.89	18020	..	...	....	21200	145.6	36 07
			48.67	0.670	97.32	....	....	....	....	....	14920	14663	213	....	....	....
14	4.1282	0.1223	262.17	1.983	....	48.67	520	65.0	22.73	11820	....	...	....	21413	146.5	36.90
			48 65	0.670	97.32	....	..	....	....	....	8920	8663	118	....	....	....
15	4.4231	0.0809	262.15	1.313	....	48.65	344	50.0	17.50	6020	....	...	....	21531	146.7	37.10
			48.67	0.670	97.32	....	....	....	....	....	3365	3108	42	....	....	..
16	4.7179	0.0396	262.17	0.643	....	48.67	169	12.0	4.20	710	....	....	...	21573	146 8	37 20
			....	0.643	....	....	....	....	....	....	355	98	1	....	...	....
16.96	5.0000	0.0000	....	0.000	....	....	00	0 0	0 00	0	..	...		21574	146 8	37 20

of gate opening and time in this case is a straight line assuming uniform closing. The gate-opening values are in per cent of maximum guide-vane opening. The power quantity 71,000 hp. in column 11 is the power at the turbine shaft at the beginning of the shut-down.

28 In this example the power consumed by the unit in friction and windage has been considered and has been subtracted from the average power for each interval. This is a refinement which changes the result by only 0.2 per cent and may therefore be neglected in most cases. If friction and windage are neglected the speed change at the end of five seconds becomes 37.4 instead of 37.2 per cent.

29 This value of 37.4 per cent shows that the speed rise of 30.6 per cent, obtained above by neglecting the effect of water hammer and assuming that the curve of power with respect to time during closure is a straight line, is in error by 6.8 per cent of the normal speed or 18.3 per cent of the change in speed.

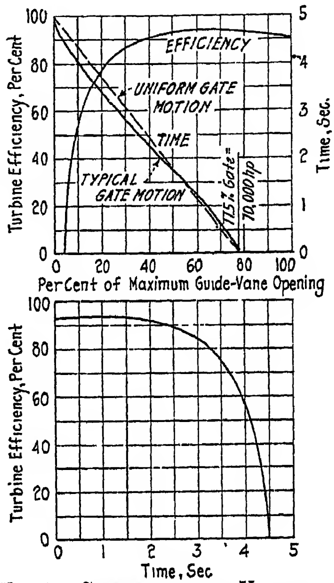


FIG 7 COMPARISON OF UNIFORM AND NON-UNIFORM GATE MOTION

30 It is also worthy of note at this time that the usual movement of the turbine gates with respect to time is not uniform as the mechanism has an inertia effect at the beginning of the stroke, and at the end of the stroke a cushioning effect is present. The curve of typical gate motion with respect to time is compared with the uniform movement in Fig. 7. This figure also gives the variation of turbine efficiency with respect to gate opening. From these two relations is obtained, as before, the variation of turbine efficiency with respect to time required in the determination of the constant C.

31 To illustrate the effect of non-uniform gate motion, calculations have been made for full load thrown off to zero, similar to that illustrated in Table 1, but using the typical gate-motion curve in Fig. 7. The results[1] are shown in Fig. 8. It will be noted that the maximum pressure rise has been increased con-

[1] Tables of calculations, similar to Table 1 and containing the data plotted in Figs. 8 to 14 inclusive, were submitted by the authors and may be consulted at the Society headquarters where they are on file.—EDITOR.

siderably, but that the final speed rise remains about the same, namely 36.2 per cent as compared with 37.2 per cent above normal with the uniform closure. In the remaining discussion, the use of uniform gate closure will be adhered to throughout, as, for practical purposes, the difference between the two computa-

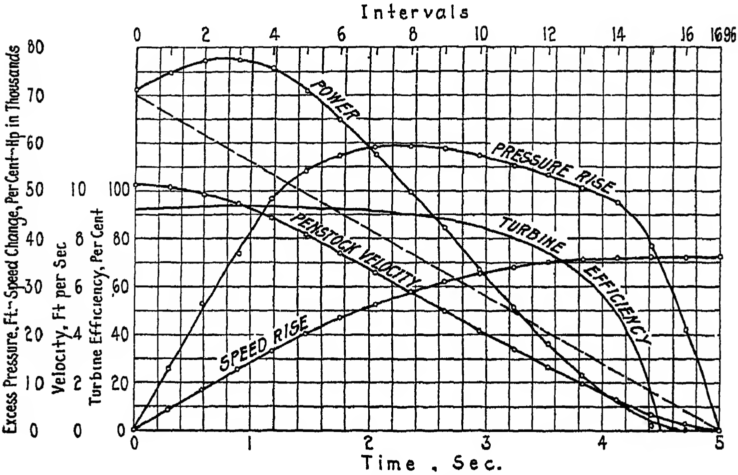


FIG 8 SPEED RISE FOR FULL LOAD THROWN OFF TO ZERO, NON-UNIFORM CLOSURE

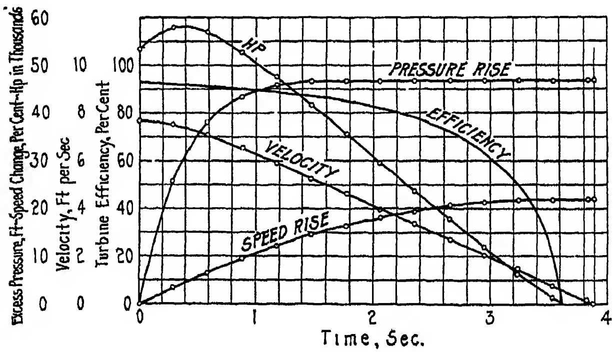


FIG. 9 SPEED RISE FOR 75 PER CENT LOAD THROWN OFF TO ZERO, UNIFORM CLOSURE

tions is so small that it need not be considered except in special investigations where such small differences are important.

COMPUTATION OF FRACTIONAL LOAD CHANGES

32 It is possible to compute readily the speed changes resulting from fractional load changes by the same method, using the power and velocities corresponding to the desired load. The time

of governor action is not necessarily proportional to the percentage of gate motion, owing to the fact that certain time elements enter into the problem caused by the physical limitations of the governor itself. A typical curve of percentage of time of full stroke, corresponding to the percentages of gate travel, is shown in Fig. 4.

33 Figs. 9, 10, and 11 illustrate the results of investigation for $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of full-load changes, based on uniform gate motion, together with allowance for friction and windage of rotating element.

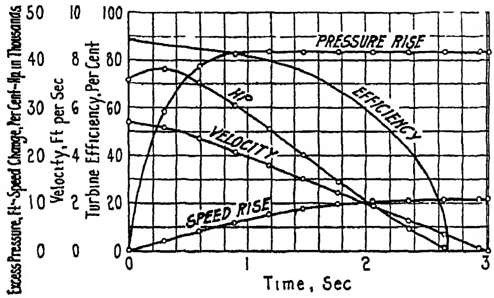


FIG 10 SPEED RISE FOR 50 PER CENT LOAD THROWN OFF TO ZERO, UNIFORM CLOSURE

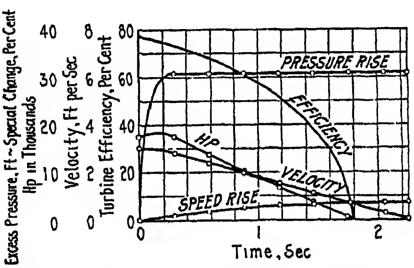


FIG. 11 SPEED RISE FOR 25 PER CENT LOAD THROWN OFF TO ZERO, UNIFORM CLOSURE

COMPUTATIONS FOR ONCOMING LOADS

34 The same general method may be applied to the increase in load on the unit from zero, but it must be remembered that the deficiency in energy input to the rotating element must be considered, rather than the excess energy as in the case of the rise in speed for rejected loads. This in turn complicates the calculation slightly, but the same fundamental theory obtains. The fall in pressure with respect to time is computed in about the same manner as the rise in pressure, although the characteristics of these two phenomena are somewhat different. The increase in velocity is then obtained and the power input with respect to time secured as the product of the velocity, the effective head,

and the constant C determined as before. This amount of power input to the rotating element is then deducted from the demand, leaving a value of horsepower deficiency which is supplied by the energy of the rotating element. This change in energy is manifested by a reduction in speed in the same manner as the excess energy causes an increase in speed in the computations for the rise in pressure.

35 Figs. 12, 13, 14, 15, and 16 show the corresponding calculations for the reduction in speed with respect to time for full, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ loads thrown on from zero.

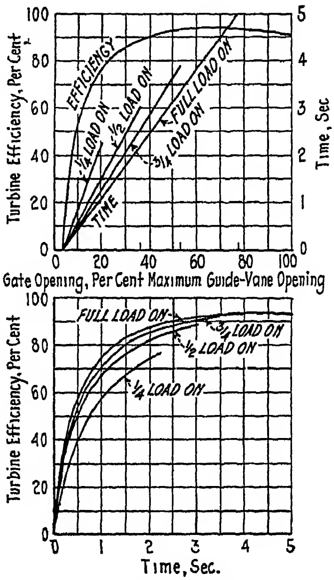


FIG. 12 VARIATION OF TURBINE EFFICIENCY WITH RESPECT TO TIME FOR LOADS THROWN ON FROM ZERO

(Full load = 70,000 hp.)

36 It will be noted in the calculations for loads thrown on that the flow has not reached its normal value by the time the gates have completed their stroke. The speed would continue to drop a small amount in addition to the maximum shown but would be compensated for by an additional opening of the gates beyond the normal position corresponding to the new demand.

37 By plotting the results obtained, as shown on Fig. 17, the maximum speed change for any load off to zero or on from zero may be read from the curve.

GENERAL EQUATIONS AND DETERMINATION OF FLYWHEEL EFFECT

38 Referring to the formula previously given illustrating the change in speed resulting from a change in power input, we find

that the expression

$$\frac{Wr^2}{5870}(N_2^2 - N_1^2)$$

represents the energy change in the rotating element, caused by

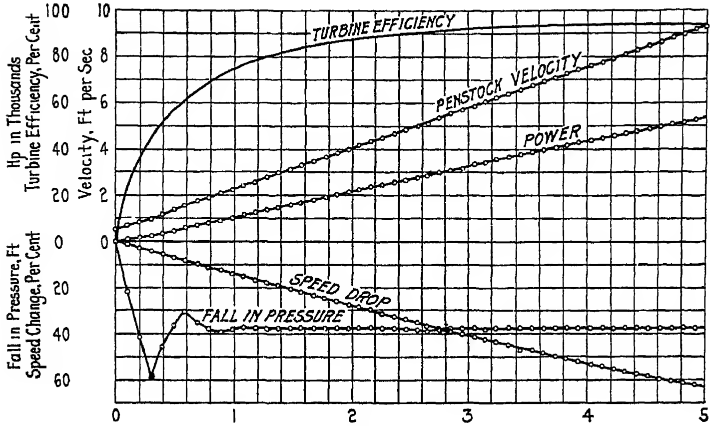


FIG 13 SPEED DROP FOR FULL LOAD THROWN ON FROM ZERO

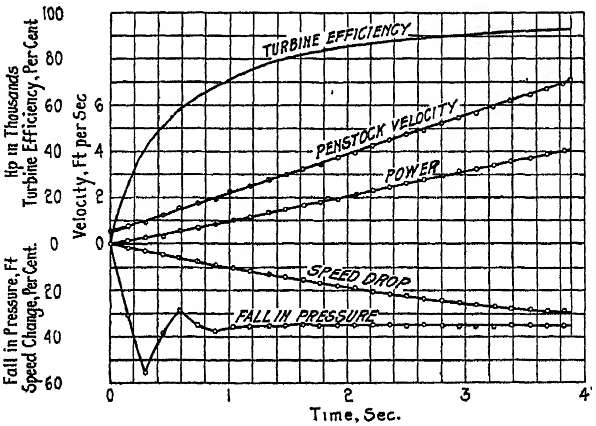


FIG. 14 SPEED DROP FOR 75 PER CENT LOAD THROWN ON FROM ZERO

a variation of horsepower with respect to time during the gate movement. As the governor time is fixed in accordance with the previous discussion, the variation of horsepower with respect to time is therefore established independently of the flywheel effect of the generator, or of the resulting speed change. We may then compute the summation of horsepower with respect to time and

equate that to the energy change as noted above. In this case N_2 becomes the maximum speed attained and N_1 the normal speed of the unit. If the permissible speed variation is known, the Wr^2 of the rotating element is automatically determined, from which should be deducted the Wr^2 of the turbine runner and shaft to secure the net Wr^2 in the generator. On the other hand, if it is desired to utilize a given Wr^2 of the generator, the maximum rise in speed is automatically fixed. For any given conditions,

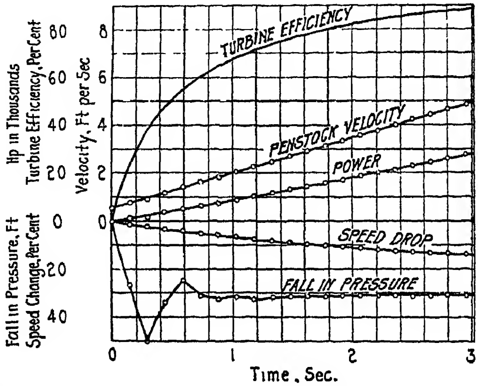


FIG. 15 SPEED DROP FOR 50 PER CENT LOAD THROWN ON FROM ZERO

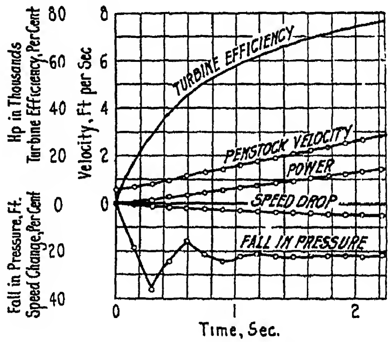


FIG. 16 SPEED DROP FOR 25 PER CENT LOAD THROWN ON FROM ZERO

it is necessary, therefore, only to compute the variation in horsepower with respect to time, take the summation of the various intervals, and substitute them in the general equation given below:

$$\frac{Wr^2}{5870}(N_2^2 - N_1^2) = \Sigma(\text{hp.} \times \Delta t) \times 550 \quad . \; . \; . \; . \quad [4]$$

The same method may be utilized in connection with the decrease in speed for oncoming loads as given below:

$$\frac{Wr^2}{5870}(N_1^2 - N_2^2) = \Sigma(\text{hp.} \times \Delta t) \times 550 \quad . \ . \ . \ . \ [5]$$

where hp. represents the horsepower deficiency at any time during the opening stroke. It is then possible, by assuming different values of Wr^2 of the rotating element, to compute the corresponding speed variations which will result with a given governor time, and hence establish a curve of Wr^2 versus speed change for any

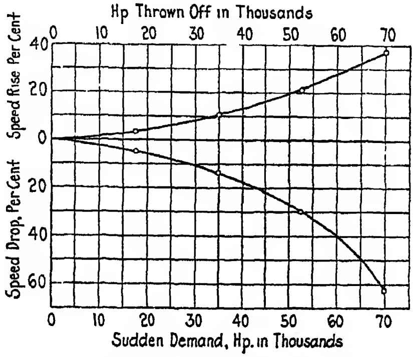


FIG. 17 MAXIMUM VALUES OF SPEED CHANGE FOR VARIOUS LOAD CHANGES ON 70,000-HP. TURBINE

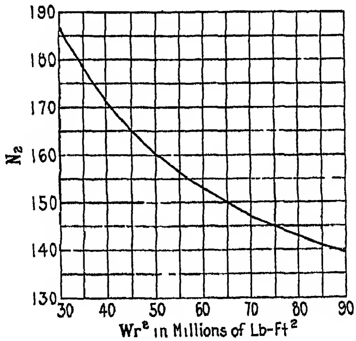


FIG. 18 RELATION BETWEEN MAXIMUM SPEED AND TOTAL Wr^2 FOR 70,000 HP. THROWN OFF TO ZERO IN 5 SECONDS

given set of conditions, and from this curve establish the Wr^2 of the generator desired. Fig. 18 illustrates such a curve for the determination of the flywheel effect based on conditions used in the example in Table 1.

39 The economic features of the problem in regard to the cost of increased flywheel effect of the generator and the evaluation in terms of money of the speed variation for given load changes determine the allowable flywheel effect to be used. In certain

cases, where a large number of plants are operating in parallel, the requirements for speed regulation are not as severe as in the case of an isolated plant operating without assistance from other stations. Certain industries require closer speed regulation than others, some examples of which are electrically driven paper machines, silk mills, other textile mills, and centrifugal ball mills for ore crushing.

40 It is usually possible to estimate from observation of similar equipment what the probable maximum and normal load changes will be on a given system, or on a given unit. Then, with this determined, the permissible speed variation should be fixed, and having these two values established, the generator flywheel effect can be determined in the manner described previously to limit the speed variations to the values desired.

41 Fig. 19 shows the typical hydraulic-turbine speed variation for loads thrown off to zero obtained by averaging a number of the regulation guarantees on units having capacities up to 10,000 hp. under heads ranging from 15 to 90 ft. Fig. 20 illustrates the same relations for loads thrown on from zero. For large-capacity units or plants involving a moderate length of penstock the general trend of speed changes will be about the same as shown in Figs. 19 and 20. For special cases involving high heads, long penstocks or special regulation requirements, the guarantees may vary considerably from the usual range.

Application to Special Cases

42 In certain problems where it is necessary in connection with electrical computations to determine the amount of voltage rise on transmission lines, the variation of speed with respect to time becomes a very important factor. The time settings on the protective relays on the unit also require a knowledge of the time in which it is necessary for them to act in order to protect the generator.

43 Another case in which a detailed study is essential is found in the design of an automatic generating station which must be placed in service in a very short time, where calculations are necessary to establish the minimum possible time for building up full load on the unit. In the following example the unit was equipped with a very long conduit leading up to a surge tank and from there the water was led through a steel penstock to the Johnson valve at the entrance to the turbine casing. The plant would be started by means of an automatic centrifugal device which would come into service in case the system frequency fell below a limiting value. The unit would be operating at synchronous speed with the Johnson valve closed, and the automatic devices would open the Johnson valve and turbine gates, throwing full load on the unit in a period of a few seconds.

44 Allowance was made for the change in level of the surge tank, the throttling effect through the partially opened Johnson valve, the fall in pressure due to acceleration of flow, and the resultant change in speed of the unit determined, considering that full load was required. Two values of Wr^2 were selected, and Fig. 21 illustrates the result of the study, indicating that it was possible to limit the drop in speed to 31.7 per cent with full load thrown

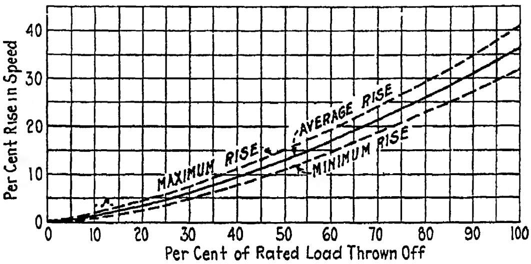


Fig. 19 Typical Speed Variation Loads Thrown Off to Zero

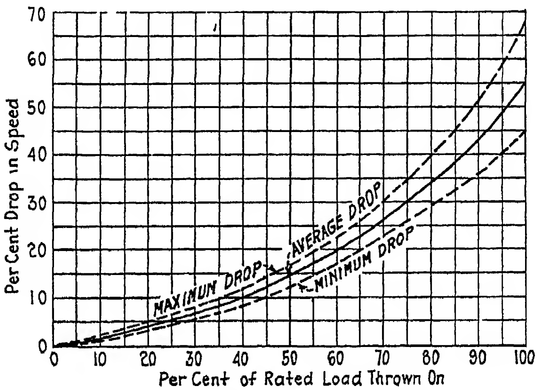


Fig. 20 Typical Speed Variation Loads Thrown On from Zero

on from zero with a Wr^2 of the generator of 5,000,000 pound-feet squared.

Excess-Power Ratio

45 Referring to the general Equations [4] and [5], the expression $\Sigma(\text{hp.} \times \Delta t)$ represents the integration of excess or deficiency in horsepower with respect to time. By substituting for this expression the product of the actual load thrown off or on, as the case may be, and the full governor stroke T, the water-hammer effect may be compensated by the addition of a suitable constant K.

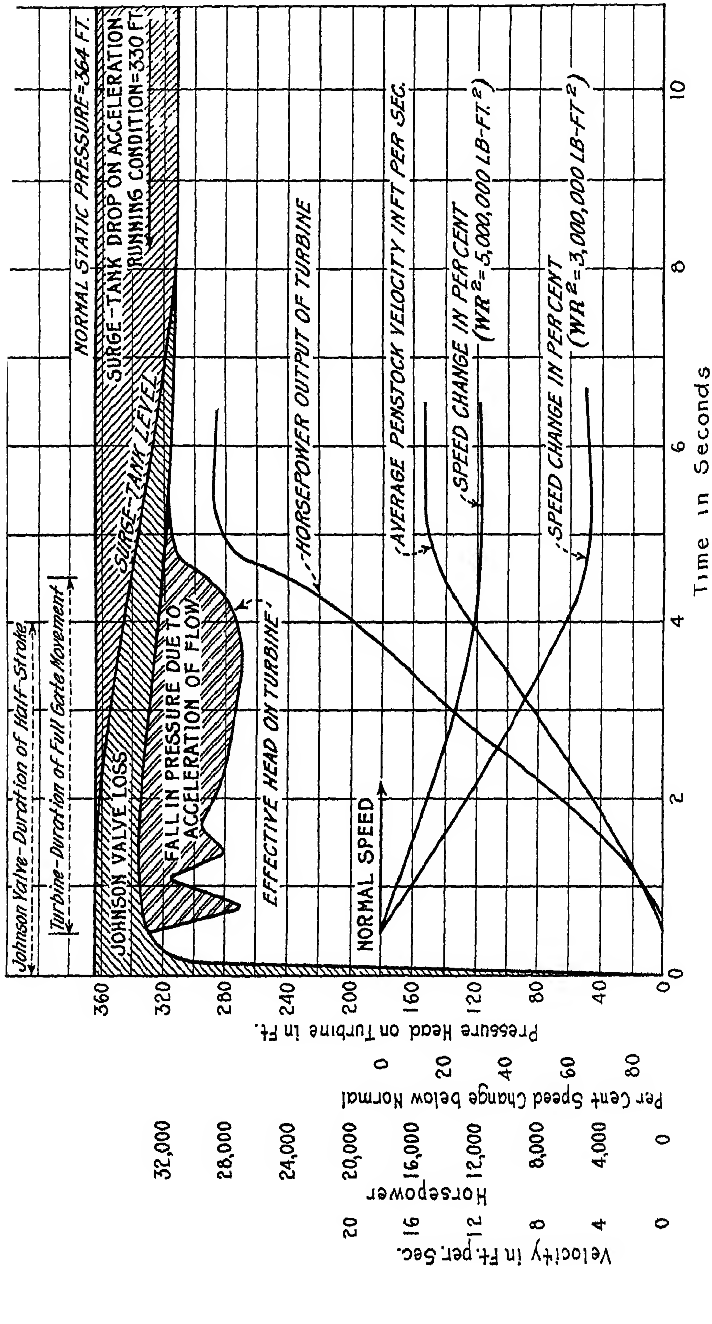


FIG. 21 EFFECT OF FULL LOAD THROWN ON AUTOMATIC GENERATING STATION, HEAD 330 FT., CAPACITY 28,500 HP

46 For loads thrown off to zero the equation will be

$$\frac{Wr^2}{5870}(N_2^2-N_1^2) = \frac{hp.}{2} \times T \times 550 \times K_1 \quad . \quad . \quad . \quad . \quad [6]$$

The constant K_1 may be called the excess-power ratio, which is in reality the ratio of the actual summation of horsepower with respect to time, considering the water-hammer effect as compared to the assumption that the power input to the rotating element varies directly with the time.

47 For loads thrown on from zero the equation will be

$$\frac{Wr^2}{5870}(N_1^2-N_2^2) = \frac{hp.}{2} \times T \times 550 \times K_2 \quad . \quad . \quad . \quad . \quad [7]$$

The constant K_2 in this case is the ratio of the summation of the deficiency in power with respect to time as compared to the uniform variation previously assumed.

48 By establishing these factors for various conditions, based on detailed calculations, it will be possible to combine them in the equation and solve for the speed variation directly.

49 It will be noted that the efficiency of the turbine also comes into the determination of the constants K_1 and K_2, and since the efficiency will vary with the load, and consequently with the gate opening, it is necessary to determine the variation in efficiency with respect to time and find the weighted mean thereof for the full stroke in order to apply it to the excess-power ratios determined without respect to efficiency. For part-load changes the efficiency of the transfer of energy differs from that of full-load changes, since the initial output is usually at a lower efficiency than at full load, and consequently the energy absorbed by the rotating element is less than it would be where the efficiency is higher, as in the case of the change from full load to zero.

50 The efficiency curve changes in shape materially with the specific speed of the turbine, and with high- and medium-head units the average efficiency throughout the range of gate movement is higher than in the case of low-head turbines with lower part-load efficiencies.

51 With these factors determined by detailed study, the authors believe it possible to arrive at an approximate formula without the necessity of performing the arithmetical integration method of calculation for the variation in horsepower with respect to time.

Conclusions

52 *a* The subject of speed variation on hydraulic turbines lends itself readily to accurate computation by the application of the fundamental principles of water hammer and by making an allowance for the efficiency of the turbine.

b For a given set of conditions with the variation in power with respect to time determined, it is possible to obtain a direct relation between generator flywheel effect and speed variation in order to facilitate the selection of the proper flywheel effect.

c The method has its application to numerous practical problems involving the speed changes on hydraulic turbines with respect to time and also for maximum changes for any particular load change which may occur.

d The accuracy of an approximate formula depends upon the refinements used to integrate the variation in horsepower with respect to time, and secure the net excess or deficiency in horsepower during the governor stroke.

DISCUSSION

P. F. Kruse.[1] Many problems in connection with hydraulic turbine installations would be very difficult, if not impossible of solution, except by arithmetical integration. Speed regulation of hydroelectric units is in this category. If the writer were to make any criticism of the paper, it would be that certain phases of the technique of solving one of these problems has not been given sufficiently in detail, for example, the manner of obtaining the physical constants, especially in the scroll case, and also in connection with the calculation of the draft tube surge to determine the maximum draft head. At what point during the gate closure does the maximum draft occur, and is the allowable increase in vacuum in feet of water, designated by Δh in the formula given in Par. 16, exceeded for any given governor time T or part thereof, thus endangering a breaking of the water column in the draft tube? Hereinafter is described the method the writer has been using to obtain these factors.

In the plant to which the authors' example pertains, the draft tube is a relatively small part of the total water conduit. In many low-head and short-penstock installations, however, the draft tube is a large part of the water conduit. The writer has recently been connected with the design of a hydroelectric station containing four 20,000-hp. units, for a head of 81.5 ft., wherein the inertia of the draft tube is a large part of the total inertia of the water column, and the question of surge in the draft tube was an important consideration in its design. He submits the following data pertaining to this installation as descriptive of the points raised above:

Fig. 22 shows a plan and section of the total water conduit, including penstock, scroll case, and draft tube. The table in the caption below this illustration gives the values of LV for each of the three parts of the waterway. The total LV of the waterway

[1] Sanderson & Porter, New York, N. Y.

is 1980, of which total amount the draft tube constitutes over 50 per cent. The design of the draft tube represents a modified design of an old-type, short-elbow tube, as shown by dotted lines in Fig. 22, which was cast into the power house foundations for each of the four units referred to above, and constructed 13 years ago. After the power-house foundation up to the wheel setting had been

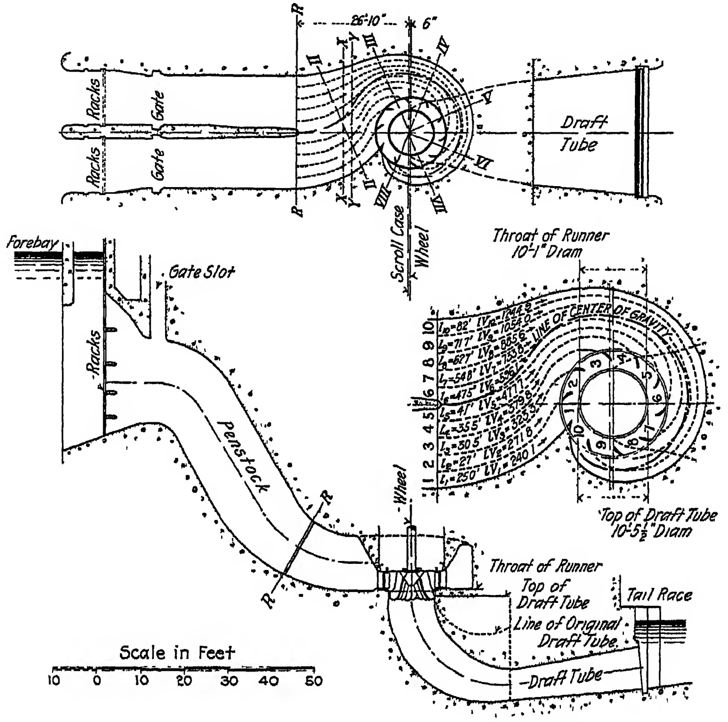


FIG. 22 PLAN AND SECTIONAL ELEVATION OF INSTALLATION WHEREIN DRAFT TUBE CONSTITUTES LARGE PORTION OF WATERWAY

Waterway	L (Feet)	Mean V (Ft. per Sec.)	LV
Penstock to Section R	63.182	5 415	342.14
Scroll Case	47.775	13.00	620.71
Draft Tube	69.000	14 73	1016.25
Total	179 96	11.00	1979 10

constructed, the project was abandoned and was not resumed until 1925. The original plans and construction contemplated four 12,000-hp. units under 81.5 ft. net head.

At the time of making new designs for the station and renewing construction work, it was desired to increase the capacity of the units to at least 18,000 hp. In view of this desired increase in size of units, the inefficient design of the existing draft tubes, and the necessity of utilizing the existing foundation concrete, an

elaborate series of laboratory tests was made, using models of runners and draft tubes. As will be noted in Fig. 22, the final design of draft tube involved a contraction of the existing tube and a long horizontal extension to obtain the desired regain of energy. The overall length of the draft tube from the throat of the

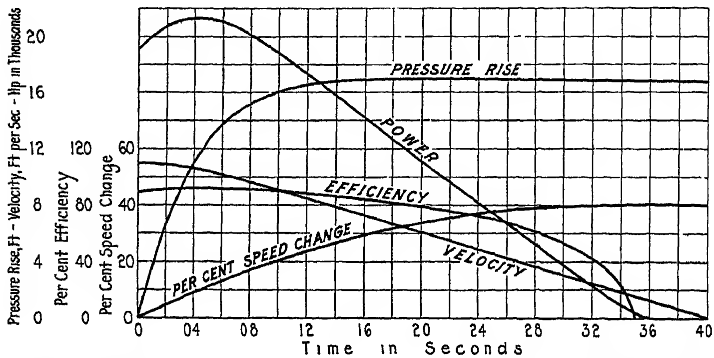


FIG. 23 PRESSURE RISE, VELOCITY, POWER, AND SPEED CHANGE RELATIONS OF UNIT SHOWN IN FIG. 22

Wr^2 = 8,250,000 lb. ft. sq.
T = 4 sec.
h_0 = 81.5 ft.
Q = 2300 cu. ft. per sec.
Load = 19,000 hp.
V_0 = 11.0 ft. per sec. (weighted average velocity as to length of total waterway)
Length of waterway = 180 ft. (including draft tube)
N_1 = 133.8 r.p.m.

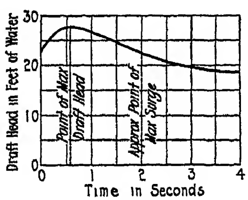


FIG. 24

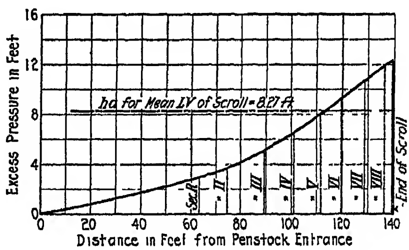


FIG. 25

FIG. 24 RELATION OF TOTAL DRAFT HEAD TO TIME
FIG. 25 PRESSURE RISE IN PENSTOCK AND SCROLL CASE

runner is 69 ft., and the mean velocity, when the unit is operating at a load of 19,000 hp., with a corresponding discharge through the wheel of 2300 cu. ft. per sec., is 14.73 ft. per sec. It will be seen, therefore, that the question of draft-tube design in this case involved not only regain of energy but also the question of regulation of the unit as affected by the inertia of the water column in the draft tube. In the example herein given, the manner in which the maximum draft head was computed is given, in addition to the other factors affecting the regulation of the unit.

Fig. 23 shows the relation of the power given up by the wheel, the excess pressure, efficiency, velocity change, and per cent rise in speed for the duration of closure. The excess pressure shown in Fig. 23 is that for the total water conduit both above and below the runner. After having obtained the pressure rise curve for the duration of closure, the surge in the draft tube and the maximum

TABLE 2 COMPUTATIONS FOR DRAFT TUBE SURGE AND MAXIMUM DRAFT HEAD

1	2	3	4	5	6	7
Time	Surge p, Ft.	Velocity of Throat, Ft. per sec.	hv_t, Ft.	$hv_t \times 0.80$, Ft.	Static Draft Head, Ft.	Total Draft Head, Ft.
0.0	0.0	32 0	15.91	12.73	10 0	22 73
0 2	3 34	31 65	15 56	12 44	10 0	25 78
0.4	5.62	30 78	14 70	11.76	10.0	27.38
0.6	7.04	29 44	13.46	10 77	10 0	27.81
0 8	7 79	27.92	12 10	9.68	10.0	27.47
1.0	8 24	26.27	10 71	8 57	10 0	26.81
1 2	8 50	24.61	9.41	7.53	10.0	26.03
1 4	8 61	22.90	8.14	6.51	10 0	25.12
1.6	8 66	21.10	6.91	5.53	10.0	24.19
1.8	8.69	19.32	5.79	4.63	10.0	23.32
2 0	8 70	17.55	4.78	3 82	10.0	22.52
3 0	8 67	8.75	1.19	0.95	10.0	19.62
4 0	8.62	0.00	0 00	....	10.0	18.62

$$\text{Ratio}\cdot \frac{\text{Draft Tube } LV}{\text{Total } LV} = \frac{1016}{1980} = 0\ 513.$$

draft head occurring at any time during the gate closure were obtained in the following manner:

The surge in the draft tube, at various times throughout the gate closure, was obtained by direct ratio of the LV of the draft tube to that of the total waterway. This ratio is $1016 \div 1980 = 0.513$.

TABLE 3 COMPUTATIONS FOR PRESSURE RISE AT VARIOUS POINTS THROUGHOUT PENSTOCK AND SCROLL CASE (SEE FIG. 22)

1	2	3	4	5	6
Section	LV	ΣLV	$r = \frac{\text{Column 3}}{1980}$	$p = r \times 17.0$	ΣL
R	...	342.14	0 173	2.94	63.182
II	81.3	423.4	0 214	3.64	74.47
III	164.7	588 1	0.297	5.05	89.47
IV	159.8	747.9	0.378	6.42	100.08
V	166.7	914 6	0.462	7.85	110.14
VI	169 0	1083.6	0.547	9.30	119.77
VII	158.2	1241 8	0.627	10.66	128.56
VIII	141.7	1383.5	0 699	11.88	136.43
End of Scroll	54.7	1438.2	0.726	12.34	139.47

Referring to Table 2, the values of surge in the draft tube in the second column were obtained by multiplying corresponding values on the pressure-time curve of Fig. 23 at various points throughout the duration of closure by the factor 0.513. Column 3, Table 2, gives the axial velocities at the various points throughout the gate travel. Column 4 gives the velocity head corresponding to the velocities given in column 3. In computing the amounts of velocity head regained as shown in column 5, an efficiency of the draft

tube of 80 per cent was used. On the basis of the axial velocities, this draft tube efficiency may be slightly high. On the other hand, no allowance was made in this example for the whirling component of the water discharging from the runner which tends to increase the actual velocities over those shown in the table. With one unit operating, the minimum tailwater level was estimated to be 10 ft. below the throat of the runner. It may be stated that this low tailwater level will occur infrequently and that under average operating conditions the static draft head will be more nearly 6 ft. The last column of the table gives the total draft head, which includes the static head, the velocity head regained, and surge at various times throughout the duration of closure.

These total draft heads were then plotted against time, as shown in Fig. 24, and the maximum value of draft head during the gate closure was obtained. As will be noted, the maximum draft head does not occur at the point of maximum surge, but very soon after the gates begin to close. The significance of this lies in the fact that the maximum draft head may occur with relatively small percentages of drop in load from full load and therefore occur more frequently than might otherwise be expected. This emphasizes the importance of limiting the maximum draft head to safe amounts to obtain good regulation, to reduce the tendency of the runners to pit, and to prevent the possibility of frequent breaking of the water column in the draft tube.

In obtaining the physical constants of the waterway, the *LV* of the scroll case was obtained in the following manner:

Referring to Fig. 22, the scroll case and lower part of the penstock was divided into 10 parts corresponding to the number of openings in the turbine guide ring. The mean *LV* of each one of these stream paths was obtained and averaged to give the mean *LV* of the scroll case. Incidental to the regulation problem, it is sometimes desired to obtain the excess pressure at various points throughout the scroll, for use in connection with the reinforcing and structural design of scroll case. This may be done by obtaining the proportional amount of pressure rise to that of the total waterway. For this purpose, in the example herein given, the *LV*, at various points throughout the scroll, was obtained on the basis of the length of waterway measured along the line of center of gravity of the scroll case. The computation for obtaining the curve of Fig. 25 giving pressure rise at various points through the penstock and scroll case is shown in Table 3.

The writer wishes to suggest another point for consideration in connection with the combining of the waterway above the turbine gates with that of the draft tube below the turbine gates. It is the writer's impression that whereas upstream of the turbine gates the pressure waves are reflected from the gates to the point of relief at the forebay and back, the pressure waves in the draft tube are reflected in the opposite direction from the turbine gates to the

tailrace. This point is believed to be of minor significance, but it has a bearing on the value of $2L/a$, the interval of time for the travel of the pressure waves from the gates to the point of relief and return.

From tentative studies the writer has made in this connection, he believes that the method of obtaining the value of $2L/a$ from the total length of waterway, as used by the authors and by himself in the example herewith, is sufficiently accurate for all practical purposes. It would seem to be a subject for an interesting laboratory experiment with models.

ROY WILKINS.[1] This discussion is confined to the Theory of Speed Changes in Pars. 19 to 23.

Total Separation: Full Governor Closing. As is stated in the paper, the speed rise on separation is dependent on Wr^2 of the unit, the load being carried, and the rate at which the driving force can be removed The Wr^2 in a hydroelectric generator is originally determined by the generator designer on the basis of speed and voltage rise. On units having direct-connected exciters it is almost wholly dependent on voltage rise, and many governor settings are determined by voltage rise, rather than by penstock pressure rise.

For present-day generators with voltage regulators and direct-connected exciters it is possible to hold the voltage rise to approximately the same percentage as the speed rise, provided the governor closing time is brought down to the order of about two seconds.

Such generators have a Wr^2 proportioned to accomplish this, and the pressure rise in the penstock and scroll case must be taken care of either in the relief valve or by increasing the allowable pressure by using a stronger penstock. The problem of the proportion of cost to be borne by the generator and the penstock is purely economic and as noted is usually determined by local conditions.

Governing: Partial Closing. Speed changes during governing are dependent principally on three things:

(1) The system. Characteristics made up of the effects of impedance between all of its rotating equipment, which limits the amount of and the rate at which corrective energy can be transferred from one point to another, and so determines the rate of speed change for a given load change.

(2) The flywheel effect of all the rotating equipment, both generating and load.

(3) The characteristic of the load with varying speed, i.e., centrifugal pumps have one type of speed load curve, fans and blowers another, and incandescent lamps still another.

[1] Assistant Engineer, Division of Hydroelectric and Transmission Engineering, Pacific Gas & Elec. Co., San Francisco, Cal.

The rate of change necessary for governing a system, therefore, depends on the system.

There is still another consideration necessary for a complete and satisfactory solution which has to do with the performance of the unit during system disturbances, and the present tendency to consolidate smaller transmission networks may and probably will cause the generator designer to demand a drop in output for a drop in speed under certain conditions, in order to make the system stable. The necessary increase in cost of penstock and scroll case due to higher pressure demands will be more than offset by the saving in lines and synchronous condensers.

The larger the system and the more diversified the load the less are the governing demands except as they affect stability and overspeed on total separation. It is quite possible to spend several times the price of a complete penstock for a plant in equipment to maintain stable operation, so that there will be more and more demanded of the hydraulic unit by the electrical end of the design on economic grounds as system networks increase and operation becomes more difficult. The hydraulic element is only one part of the generating unit, which, in itself, is only a small part of the system. The ideal to be hoped for is the maximum reliable output for a minimum total cost.

Ray S. Quick.[1] This method, for an accurate determination of the speed change where the maximum variation is moderate, may be extended to determine the time required to regain normal speed, following a sudden load change, providing the characteristics of the residual load and the governor overtravel are known as well.

Modifications in this analytical method, while considered negligible by the authors for moderate speed changes, may enter in, to an appreciable extent, under certain conditions. These modifications include consideration of:

a Variation in no-load losses with speed

b Discharge-speed characteristic with head and gate setting constant

c Efficiency-speed characteristic with head and gate setting constant.

The no-load losses of a unit will vary as some function of the speed. However, as they total but a small proportion of the energy considered, it is questionable whether this refinement is essential. It would be of interest to take it into account, however, where the speed change is considerable, or in determining the time required to regain normal speed following a sudden total loss of load.

Another modification in the study involves consideration of the variation in flow through a turbine for different speeds, the operat-

[1] Sales Engineer, Pelton Water Wheel Co., San Francisco, Cal. Assoc-Mem. A.S.M.E.

ing head and gate setting being constant. While negligible for underspeed, appreciable overspeed for a turbine such as the 70,000-hp. unit discussed tends to reduce the discharge, which in effect is the same as partial gate closure controlled by the speed rather than by the governor. As the specific speed of the turbine is increased, the overspeed discharge may not decrease but actually increase slightly, thus indicating the necessity of being familiar with the particular characteristics of the unit under consideration.

The typical efficiency-speed characteristic with operating head and gate constant, is similar in form to a parabola, the efficiency being zero when the runner is locked or at runaway, and maximum at an intermediate value which usually is normal speed. This characteristic changes slightly with the gate setting, having a tendency to shift toward reduced speeds as the gate opening is decreased.

A sudden loss of load, resulting in a rise in pressure and increase in speed, will first introduce compensating modifications, as the correct normal speed should be increased slightly to suit the momentary higher head. As the speed increases beyond this new normal value, the efficiency will be reduced until it reaches zero at runaway speed, the net tendency being to decrease the total excess energy available for acceleration. This reduces the speed change or improves the speed regulation. While this change is negligible in the average case where the maximum speed rise is moderate, such as 25 per cent, it may be the determining factor for large values of overspeed resulting from having a very slow governor stroke, or when operating the unit entirely without governor as has been adopted in a number of cases for large systems.

In the case of suddenly applied load, the initial drop in pressure reduces slightly the normal speed for initial efficiency, thus again compensating at the start for the loss which otherwise would result. As the speed is reduced appreciably beyond this point, the loss in efficiency decreases the energy delivered by the turbine, increasing the deficiency between the constant demand and power, and thus resulting in a still greater drop in speed as the stored kinetic energy of the rotating system is used up. In the extreme case, if the initial stored energy is not greater than the deficiency during gate opening, the unit will come entirely to rest. Obviously, this introduces a situation difficult to conceive in the practical case as it is evident that the load demand would have to be of a very special nature to maintain its original value during such a severe drop in both frequency and generator voltage. It is probable that the generator load would decrease with the speed, thus actually resulting in a much smaller drop in speed. If a study of this nature is to be made to permit of the logical design and adjustment of actual apparatus to meet operating conditions, would it not be well to modify our understanding of load demand so as to be more in accordance with the probable facts of the case?

Mr. Wilkins[1] has found that the load on the system of the Pacific Gas & Electric Company has various characteristics, part being independent of the frequency while part varies as the cube of the frequency. This applies to very small changes of speed and probably steady voltage but it serves to indicate a modification of our definition of demand. Further comments on this matter would be appreciated by those having to do with studies of speed regulation. If the electrical industry will establish the relation between generator output and speed, it will be a great step toward a better understanding and more logical determination of speed change to meet the practical case. At present we are limited to the condition where the generator rejects load to zero and becomes disconnected from the system, thus placing the various factors on a definite basis.

O. V. Kruse.[2] This paper is of interest chiefly because it brings to light another important application of arithmetical integration. The use of this method of analysis has been altogether too limited in the engineering profession, probably because the mathematical work has appeared somewhat tedious. Those accustomed to its use have found it extremely interesting, and tedious only in that it takes a somewhat longer period of time to obtain a definite result. In the writer's experience it has been used for various hydraulic studies, such as the discharge through a relief valve during simultaneous closure of turbine gates, studies of the action of a differential surge tank with respect to sudden demand or rejection of load, actual pressure changes in penstocks caused by the gradual closure of turbine gates or valves, and various other applications. Arithmetical integration allows all variables to be taken into account, so that results obtained in any study of this nature are unquestionably accurate.

For example, in this paper the non-uniform motion of the turbine gates, together with the variation of penstock pressure, can be accurately considered in determining the speed change on the unit. This latter factor, more commonly known as water-hammer, has ordinarily been neglected in studying speed changes, but there is no good excuse for omitting the penstock pressure change. In fact, it should not be omitted where long penstocks exist. The example cited in Par. 22 shows a speed rise of 30.6 per cent, neglecting the change in penstock pressure. This speed rise becomes 37.4 per cent when the effect of the penstock is incorporated. Where installations have still longer penstocks under comparatively low heads the "flywheel effect" of the penstock might show an important difference.

The economic value of accurate studies of speed change with respect to load applied is questionable except in isolated cases.

[1] Practical Aspects of System Stability, *Journal* A.I.E.E., Feb. 1926.
[2] Hydraulic Engineer, Larner Engineering Co., Philadelphia, Pa.

Most hydroelectric plants are now a part of a general system. A sudden load application would therefore be distributed on various units. In the case of sudden load rejection the study is more important, as the unit may be cut loose from the system and would therefore reach a speed above normal, depending primarily on the governor time and flywheel effect. Even in this case, however, the economic value of such a detailed study is seldom of importance, as most units are designed to run safely at so-called runaway speed for a short space of time. The relation of governor time to penstock design is a different problem, and the setting of the governor frequently influences the cost of the penstock.

An illustration is cited beginning at Par. 42, which is intended to emphasize the importance of these speed change studies. The writer is familiar with this particular study, and it might be of interest to mention two or three points which are not entirely clear. It is stated that the unit is operating at synchronous speed. The unit, therefore, is tied in with the general system but is delivering no power. The turbine casing under these circumstances is full of water but at a very low pressure, the water being supplied through a small by-pass around the Johnson valve. A sustained drop in speed on the system sets in motion the mechanism to open the Johnson valve, thus building up instantly full penstock pressure in the turbine casing. While the valve is opening the turbine gates are also opening, thus supplying water under pressure to the turbine runner. The curves illustrated in Fig. 21 are chiefly of interest to show the change of pressure in the penstock and the horsepower delivered to the system while the gates are being opened. It will be noted that all variables, such as the drop in level in the surge tank, inertia effect of the water column between the surge tank and unit, and temporary resistance to full load discharge through the Johnson valve while it is opening, are taken into account. None of these factors should rightfully be neglected when accurately determining the power delivered or when studying the stresses in the penstock following such a severe operating requirement. Arithmetical integration is the only accurate method which could be used to secure these results.

The changes in speed for various flywheel effects incorporated in Fig. 21 are perhaps somewhat confusing, as the unit is synchronized on a general system. Speed drops such as those noted apply only to the single unit, and, of course, such speed drops are impossible, since the unit is synchronized to begin with and the flywheel effect of the entire system must be considered. Actually, the capacity of the unit is automatically thrown onto the system to supply a shortage of power and has the effect of bringing the speed back to normal. The study merely illustrates

what drop in speed would occur should the unit be operating alone and full load be applied suddenly.

William F. Uhl.[1] The mathematical solution of definite speed regulation problems has been quite fully developed in recent papers before the engineering societies and the paper under discussion is a distinct contribution toward the simplification of the mathematics in theories involved in such solutions. As usual in practical engineering problems, the real difficulties now are not the solutions of problems in speed regulation for definite assumptions, but the determination of practical assumptions.

It is difficult to decide what speed variations are permissible under different conditions, and still more difficult to establish what the worst conditions may be under which a particular hydroelectric unit may be called upon to operate and maintain satisfactory speed regulation.

If a single unit must carry a known load by itself, the problem is comparatively simple, but as soon as it becomes a part of a comprehensive system, the effect of all the connected inertia of the interconnected units and the machinery comprising the load carried by the system becomes important, and should receive consideration in determining the dimensions of the physical constants permissible to obtain satisfactory speed regulation. In such cases the problem is one of safety rather than one of determination of satisfactory speed regulation. The speed regulation is of little importance when the total load on the unit is suddenly lost, and the safety of the various features of the hydroelectric development is then the real problem to solve. It is wasteful to design pipe lines and other conduits to obtain speed regulation for conditions that will never occur in practice, or under which, if they should occur, the speed regulation is of no importance.

E. E. Halmos.[2] The writer desires enlightenment on the meaning of the last two sentences of Par. 18, which are quoted as follows:

> It is rarely necessary to go below two seconds governor time in order to meet ordinary commercial regulation practice in regard to speed variation and, if a governor time of three seconds is utilized for plants having appreciable length of penstocks even under medium heads, it will be found that more satisfactory hydraulic conditions will result than from the use of a much shorter time. In high-head plants, it may be desirable to go as high as four or five seconds for a full stroke, and possibly more than this in certain cases.

The above statements seem to convey the impression that the authors wished to point out that the governor time must be lengthened when the head on the plant becomes larger, on account of the increasing importance of water-hammer under the increas-

[1] Hydraulic Engineer, Chas. T. Main, Boston, Mass. Mem. A.S.M.E.
[2] Parsons, Klapp, Brinckerhoff & Douglas, Engineers, New York, N. Y.

ing heads. If this interpretation is correct, the writer dissents from the conclusions reached by the authors.

The percentage amount of pressure increase or decrease, due to more or less rapid closing or opening of the turbine gates, is proportional to one-half of the square root of the ratio of the kinetic and potential energy of the water column contained in the penstock. As the penstock velocity is independent of the head, as a rule, it is evident that the water-hammer will be proportionately less in a high-head plant than in a low-head plant. The writer never has had any trouble with short governor time in high-head plants, but he has had to specify three seconds and more in low-head plants where the penstocks were more than 500 or 600 ft. long. In a recent case with a length of conduit of not more than 1000 ft., the governor time became so slow that it was imperative to introduce a large surge tank in order to bring the speed regulation within practical limits.

F. M. Wood.[1] The following discussion has for its major theme the development of a simple graphical method of determining the pressure changes at the turbine gates as they are regulated to the new load, and hence the expected speed regulation. The formulas for pressure rise and pressure drop as enunciated by N. R. Gibson, and later by S Logan Kerr, are solved by means of a chart, and in this latter are incorporated sufficient data to determine at the same time the horsepower developed throughout the period of gate movement. When the average excess or deficiency of horsepower has been so determined, it is simply a matter of substitution in the well-known formula

$$\pm (N_1^2 - N_2^2) = \frac{550 \times 5870}{\text{Total } Wr^2} \times \text{governor time} \times \text{excess or deficiency in hp.}$$

to obtain the speed change to be expected.

Let us consider in detail first the case of full load thrown off to zero. The performance curves for the unit under consideration should be plotted as in Fig. 26. As will be noted, the argument taken here is the gate opening, measured from the speed no-load position to the rated load position. The question arises as to how this opening is to be measured, whether in angular turn, in inches of opening, or in inches of stroke of the servomotor. Whichever basis is used for computation, it should be borne in mind that the relationship between governor time and gate opening is the important point, and in case it is desired to use a non-uniform gate movement curve, as shown in curve E, this latter must be based on the same unit of opening as the characteristic curves A, B and C. The writer has used his charts for solutions based on both

[1] Lecturer in Civil Engineering, McGill University, Montreal, Canada.

non-uniform and uniform gate movement and has reached the same decision as the authors, apparently, i.e, that the refinement does not compensate for the added labor involved. It will be seen later that the use of non-uniform gate movement complicates the work for cases of part loads thrown on and off. For these reasons, all studies given in the following paragraphs are based on uniform movement.

When the characteristic curves for any new runner are being plotted, the unit of gate opening should be chosen which most

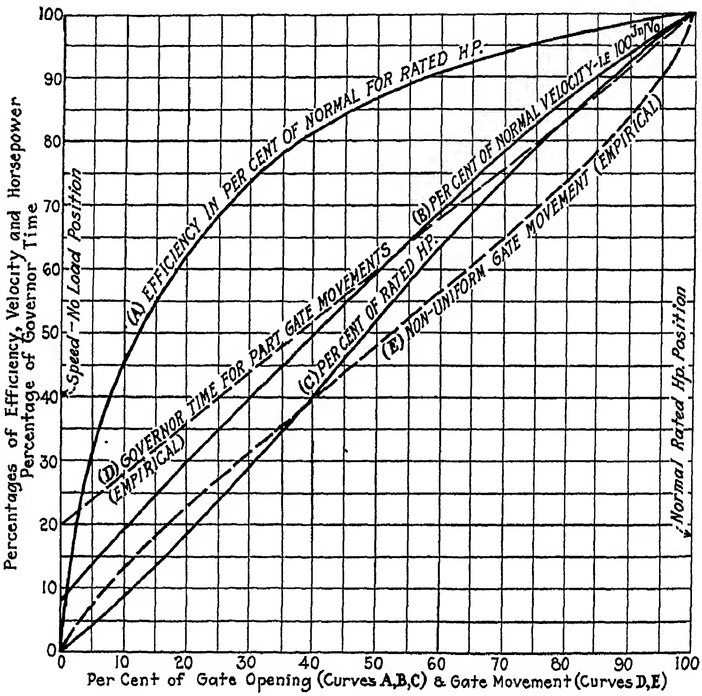


FIG. 26 PERFORMANCE CURVES OF TURBINE FROM FULL TO ZERO LOAD

closely agrees in practice with the assumption of uniform movement with respect to governor time.

As might be supposed, the characteristic curves for installations having the same model runner should be materially the same, and it seems reasonable to assume that the whole field of reaction and diagonal type runners may be covered by three or four sets of curves, particularly when it is remembered that it is not the actual efficiencies, power or velocities which are plotted but the percentages of these variables based on the values at rated horsepower under normal head.

Proceeding to the development of the graphical solution, the following nomenclature is given for reference:

G = fractional gate opening in terms of normal opening
J = velocity of water in water passages under normal head at gate opening G
E = efficiency of turbine
P = horsepower developed under normal head
V = velocity of water in water passages at gate G, allowing for pressure rise
h = pressure rise above normal head H_0
L = average length of water passages which affect pressure changes.

Subscripts zero are used to designate conditions before the load change. Subscripts 1, 2, 3, etc., are used to designate conditions at the end of the first, second, third, etc., intervals of time $\frac{2L}{a}$, from initial gate movement.

It is readily seen that the equations for the end of the first interval of time are:

$$\frac{V_1}{V_0} = \frac{J_1}{V_0}\sqrt{1+\frac{h_1}{H_0}}$$

and

$$h_1 = \frac{a}{g}\ (V_0 - V_1) \qquad \text{or} \qquad \frac{h_1}{H_0} = \frac{aV_0}{gH_0}\left(1 - \frac{V_1}{V_0}\right)$$

Here we have two simultaneous equations for solving $\frac{V_1}{V_0}$ and $\frac{h_1}{H_0}$. Coming to the end of the second interval of time, we get the equations

$$\frac{V_2}{V_0} = \frac{J_2}{V_0}\sqrt{1+\frac{h_2}{H_0}}$$

and

$$h_2 = \frac{a}{g}\ (V_1 - V_2) - h_1 \qquad \text{or} \qquad \frac{h_2}{H_0} = \frac{aV_0}{gH_0}\left(\frac{V_1}{V_0} - \frac{V_2}{V_0}\right) - \frac{h_1}{H_0}$$

For the end of the general interval n we have the equations

$$\frac{V_n}{V_0} = \frac{J_n}{V_0}\sqrt{1+\frac{h_n}{H_0}}$$

and

$$\frac{h_n}{H_0} = \frac{aV_0}{gH_0}\left(\frac{V_{n-1}}{V_0} - \frac{V_n}{V_0}\right) - \frac{h_{n-1}}{H_0}$$

The chart shown in Fig. 27 has been derived for solving these equations successively. This is a general chart for solution of the

pressure rise and pressure drop equations and is only intended to be used in an explanatory way, to help anyone desiring to plot a chart of his own to suit his own particular model runner, as

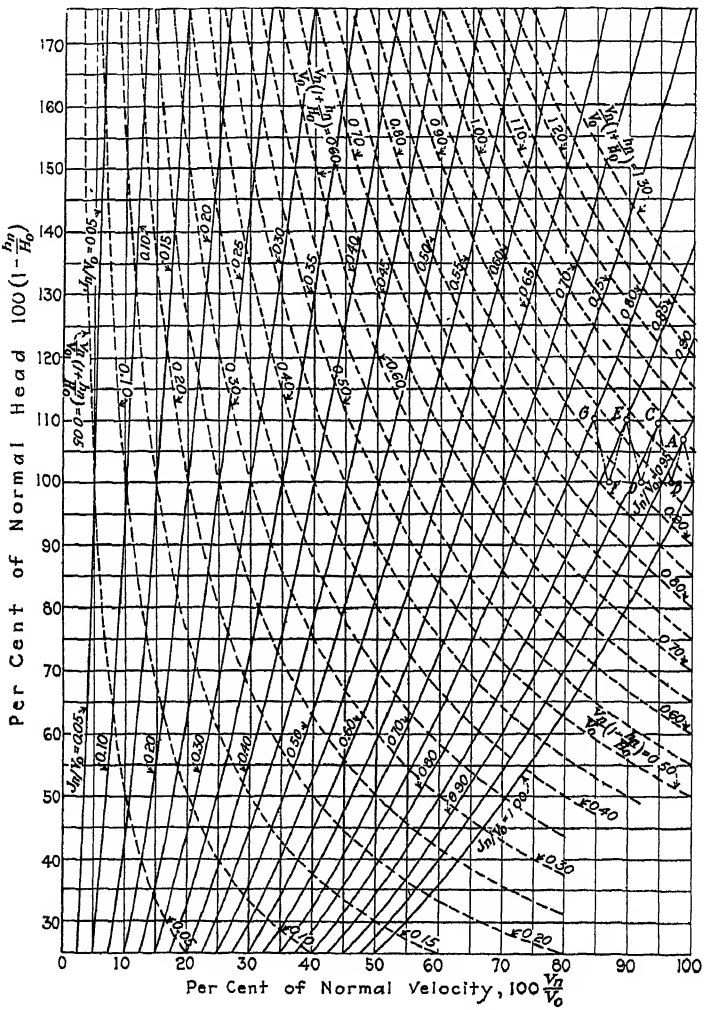


Fig. 27 Chart for Solving Equations of Relations between Velocity and Head

the writer has himself done in Fig. 28. Fig. 27 is too tedious and slow to be used in practice.

After computing the time interval $\frac{2L}{a}$, values for $\frac{J_n}{V_0}$ are found for successive intervals from Fig. 26. (Non-uniform gate movement

may be used if so desired). These values of $\frac{J_n}{V_0}$ will determine interpolated lines in Fig. 27, which may be drawn lightly in pencil as required. The value of $\frac{aV_0}{gH_0}$ is now found, and starting at the extreme right of the $\frac{V_n}{V_0}$ axis (marked 100) a line is drawn making an angle with the V axis to the left of $\tan^{-1}\frac{aV_0}{gH_0}$. The point of intersection of this line with the first $\frac{J_n}{V_0}$ line will give as its coördinates the pressure rise and percentage of normal velocity at the end of the first interval. For instance, assume for simplicity that our values for $\frac{J_n}{V_0}$ vary by 5 per cent for successive intervals, so that we can use the lines already marked on the chart. Then for a value of $\frac{aV_0}{gH_0} = 4$ we will get point A on the chart for the condition at the end of the first interval.

Now, through A draw a line with the opposite slope $\tan^{-1} 4$, giving a point B on the V axis. Starting from B, draw a line to the left parallel to the first line cutting the second $\frac{J_n}{V_0}$ line at C. This will give the conditions at end of second interval. Follow this procedure as indicated throughout the period of governor action, obtaining in this way the values of $1+\frac{h_n}{H_0}$ and $\frac{V_n}{V_0}$ for successive intervals. If these values are tabulated with their corresponding efficiencies, the latter being obtained from the characteristic curves for the runner (see Fig. 26) by referring to the values of $\frac{J_n}{V_0}$, we can now determine the horsepower input to the turbine at each period end, which will be seen to be

$$\frac{V_n}{V_0} \cdot \frac{E_n}{E_0}\left(1+\frac{h_n}{H_0}\right) \times \text{rated hp.}$$

For simplicity in using this chart, curves for $\frac{V_n}{V_0}\left(1+\frac{h_n}{H_0}\right)$ have been included so that values for this product may be read off directly at points A, C, E, etc. It remains only to multiply by $\frac{E_n}{E_0}$ in order to obtain the percentage of rated horsepower. So much for the general chart of Fig. 27, which is useful in cases where careful investigation is desired.

We can see that if it is used, each new unit will have new values of $\frac{2L}{a}$ and $\frac{aV_0}{gH_0}$, which necessitates interpolation anew for the lines $\frac{J_n}{V_0}$ and a new intersecting slope as well.

After a certain amount of thought, the writer hit upon the idea of changing the value of a, i.e., the velocity of transmission of the pressure wave, to give an interval $\frac{2L}{a}$ equal to some definite percentage of the governor time. This was decided on partly because for short lengths of water passages the number of intervals of time becomes excessive for practical use. It may be pointed out here that in installations of low head the turbine gates are usually nearer the middle of the water passages than the end and the period of pressure wave may differ considerably from the value $\frac{2L}{a}$. A particular case which had already been worked out for the theoretical period (44 intervals in a governor time of 2 sec.) was solved on the assumption of 20 intervals, and the speed regulation change for full load off was found to be 14.64 per cent as against 14.56 per cent obtained before.

An attempt was made to find the error involved in this approximation by a study of the equations for pressure rise, but without success. However, it would appear that the error is of the order of 0.1 per cent and is possibly on the safe side as far as guarantees are concerned.

Using a time interval of $\frac{2L}{a} = \frac{T}{20}$, i.e., 5 per cent of the governor time or of the gate movement, we have the relation $a = \frac{40L}{T}$, giving a revised slope for the chart of

$$\frac{aV_0}{gH_0} = \frac{40LV_0}{gH_0T} = M$$

A special chart may now be plotted for each model runner. The $\frac{J_n}{V_0}$ curves of Fig. 27 are replaced by the G_n curves of Fig. 28, the latter being in fact the interpolated $\frac{J_n}{V_0}$ curves for the gate openings at intervals of $\frac{T}{20}$.

If the time interval chosen is $\frac{T}{10}$, then the slope M becomes $\frac{20LV_0}{gH_0T}$ and in general $M = \frac{2LV_0}{gH_0 \times \text{interval}}$

Since there is a definite percentage of full load efficiency for each gate opening (neglecting slight errors due to fluctuation in the specific speed with pressure rise) it is possible to plot a set of

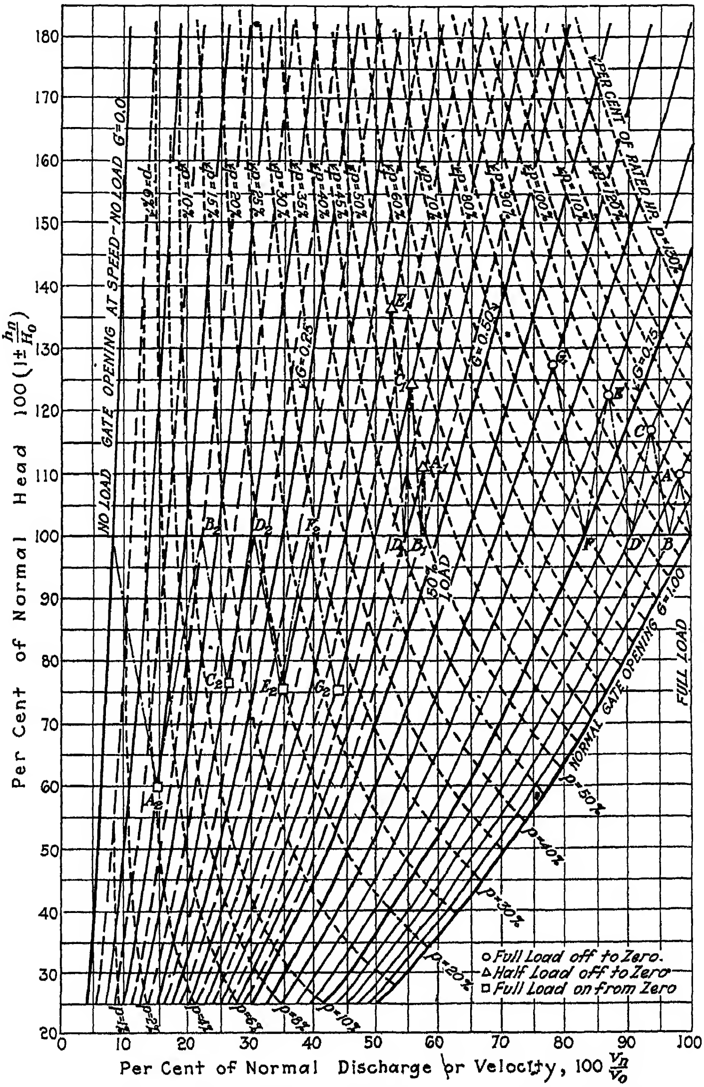


Fig. 28 Chart for Solving Equations of Relation between Velocity and Head for a Particular Model Runner
(Percentage of rated horsepower curves shown dotted)

percentages of rated horsepower curves, as shown dotted in Fig. 28, which will give directly the horsepower input in per cent of rated horsepower at the points *A, C, E,* etc.

The necessity for an assumption of uniform gate movement is apparent when we come to consider part load changes. Here similar operations are carried out as in the case of full load thrown off, the limits of $\frac{V}{V_0}$ being denoted by the initial and final horsepower. The same interval may be taken, and in the case of loads between zero and 50 per cent of rated horsepower the chart has been compiled allowing intervals of $\frac{T}{40}$. For the odd interval from the beginning of stroke to the next gate opening, a revised slope M should be calculated. Otherwise the calculations are similar to those already described.

For loads thrown on, a study of the phenomena of pressure drop along lines of thought similar to those employed by Mr. Gibson for pressure rise produces equations similar to those given above. If the absolute value of the pressure drop be denoted by h_n, we will have as our general relations

$$\frac{V_n}{V_0} = \frac{J_n}{V_0}\sqrt{1 - \frac{h_n}{H_0}}$$

and

$$\frac{h_n}{H_0} = \frac{aV_0}{gH_0}\left(\frac{V_n}{V_0} - \frac{V_{n-1}}{V_0}\right) - \frac{h_{n-1}}{H_0}$$

If these equations are solved as in the case of pressure rise, we will obtain an extension to the chart for pressure rise below the $\frac{V_n}{V_0}$ line. The writer has not had the time available for checking these equations with those developed by Mr. Kerr, although the premises are the same in both cases, but in one particular case the pressure-time curves obtained by the two methods appeared to check closely. A few examples will impress one with the fact that the pressure drop fluctuates much more than the pressure rise; consequently, an arbitrary change of the period $\frac{2L}{a}$, as made above in the case of pressure rise, is not likely to give a pressure drop curve agreeing very closely with the true pressure drop curve. Taking the average over the full range of load change, however, the approximation should give fairly correct results.

Apart from this consideration, some more definite data should be known regarding the governor characteristics in cases of load thrown on, before any reasonable values for speed change can be computed. We must have knowledge of the nature of the gate movement beyond the final stationary position, which must compensate not only for the deficiency in energy input during the load change but also for the deficiency in horsepower input at the end

of the nominal governor time due to the pressure drop and associated phenomena. However, in most modern plants a new unit is thrown on to the line gradually and the speed regulation is not of great importance.

A few examples are now given to illustrate the method of reading the charts.

Example 1.

12,000 hp. unit; full load off to zero

$$\begin{aligned} \text{Runner } Wr^2 &= 407,000 \\ \text{Generator } Wr^2 &= 9,400,000 \\ \hline \text{Total } Wr^2 &= 9,807,000 \text{ lb. ft.}^2 \end{aligned}$$

Full governor time 2 sec., $H_0 = 60$ ft., normal speed $N_1 = 125$ r.p.m.
ΣLV_0 at rated hp. = 1104

For intervals of $\frac{T}{10}$ we get $M = \frac{20V_0}{gH_0T} = 5.725$

This gives the points *A, C, E,* etc., at end of successive intervals. The percentage of rated hp. at the intervals are then read off, as follows:

Interval	0	1	2	3	4	5	6	7	8	9	10	Avg.
Per cent rated hp.	100.0	106	106	100	90 5	77	62	45	28.5	13.5	0	67.85

Therefore

$$N_2^2 - N_1^2 = \frac{0.6785 \times 12{,}000 \times 550 \times 5{,}870 \times 2}{9{,}807{,}000} = 5{,}361$$

$$N_2^2 = 5{,}361 + 15{,}625 = 20{,}986$$

and

$$N_2 = 144.9 \text{ r.p.m.}$$

giving a speed change of $\frac{144.9 - 125}{125}$, or 15.9 per cent.

Example 2.

12,000 hp. unit; half load off to zero.

From Fig. 26 we find $\frac{J_n}{V_0}$ at half load to be 0.58 and gate opening = 0.484; also assumed governor time = 2×0.60 = 1.20 sec. Therefore for intervals of $\frac{T}{20}$,

$$M = \frac{40 \times 1104}{32.15 \times 60 \times 1.20} = 19.07$$

and for the first part interval

$$19.07 \times \frac{5}{48.4 - 45.0} = 28.04$$

In this case we find the following percentages of rated horsepower:

Interval	0	0.68	1.68	2.68	3.68	4 68	5 68	6 68	7.68	8.68	9.68	Avg.
Per cent rated hp.	50.0	53.5	56 5	56.0	52.0	45.0	36.5	27.5	18.5	9.2	0	36.32

Therefore

$$N_2^2 - N_1^2 = \frac{0\ 3632 \times 12{,}000 \times 550 \times 5{,}870 \times 1.20}{9{,}807{,}000} = 1{,}722$$

$$N_2^2 = 15{,}625 + 1{,}722 = 17{,}347$$

and

$$N_2 = 131.7$$

giving a speed change of $\frac{131.7 - 125}{125}$, or 5.4 per cent.

Example 3.

12,000 hp. unit; full load on from zero; governor time 2 sec.

For interval of $\frac{T}{10}$ we get $M = 5.725$, as in Example 1.

The intermediate horsepowers are found to be as follows:

Interval	0	1	2	3	4	5	6	7	8	9	10	Avg.
Per cent rated hp.	0	4 2	12 5	19.0	26 5	33.5	42 0	49.0	57 5	66 5	74 5	34.80

Therefore, at the end of 2 sec, we have

$$N_1^2 - N_2^2 = \frac{(1-0\,3480)\ 12{,}000\times 550\times 5.870\times 2}{9{,}807{,}000} = 5{,}152$$

$$N_2^2 = 15{,}625 - 5{,}152 = 10{,}473$$

giving

$$N_2 = 102.3$$

and speed change of $\frac{125-102.3}{125}$, or 18.2 per cent.

THOMAS H. HOGG [1] and J. J. TRAILL.[2] The problems in connection with speed regulation in hydroelectric power plants have received much attention during the past thirty years. The material in this paper adds to our knowledge of these subjects in the sense that it analyzes and coördinates earlier investigations and puts their results into useful form. The authors have considered the principles governing speed regulation and pressure rise, and have applied the results of their investigation to the problems of design. To some designers of power plants it will seem that the methods of investigation and their applications are unnecessarily refined, as comparatively simple formulas based on apparently correct assumptions are available for the solution of the problems involved. The writers cannot agree with such a view. Careful investigation of all the contingencies that may arise is most important, and rapid, approximate solutions, while saving time and much laborious calculation on the part of the designer, leave him in the position of risking safety on the one hand or neglecting economy on the other. Furthermore, in actual practice the conditions assumed to apply are only realized in part. For example, the rate of closure of the turbine gates is not by any means uniform from beginning to end of stroke, but is assumed to be so in general when computations of speed rise, pressure rise, etc., are being made. It seems more reasonable to take such varying factors correctly into account, even at the expense of a considerable amount of time.

The fact that rapid approximate methods of computation give results as close to the truth as is warranted by the data available in the great majority of cases, when a project is in the design

[1] Chief Hydraulic Engineer, Hydro-Electric Power Commission of Ontario, Toronto, Canada.

[2] Engineer of Tests, Hydraulic Department, Hydro-Electric Power Commission of Ontario, Toronto, Canada.

stage, is not sufficient justification for the neglect of more reliable and more precise methods. The writers have in mind very definite economies in certain projects, resulting from hydraulic studies that, by many, would be considered unnecessarily extensive and refined.

The authors illustrate the methods of design by a number of examples in which they arrive at conclusions as to the speed rise and pressure rise that will occur under certain conditions of load rejection, or speed and pressure reduction for load increments. The writers have available the results of experimental investigations that permit a comparison to be made between results calculated by the usual methods of the design office and those actually realized on test.

The investigations were made at the Queenston Plant and the South Falls Plant of the Hydro-Electric Power Commission of Ontario. Those made at Queenston had as their object the adjustment of governor traversing time to effect the most advantageous balance between speed rise and voltage rise on the one hand, and pressure rise on the other. The nature of the load carried by the plant is such that increments of load occur so gradually that they cause no appreciable speed drop or pressure drop.

Each penstock supplies only one unit and is 425 ft. long, 16 ft in diameter for a distance of 256 ft., tapers to 14 ft. in diameter, continuing at that diameter for 106 ft., where a Johnson valve is reached, in which the diameter is further reduced to 10 ft. This last-named diameter holds for a distance of 27 ft. between the Johnson valve and the entrance to the scroll case. The penstocks have a plate thickness varying from ½ to 1¼ in., and are encased in concrete. The turbines are single-runner, vertical type, and have a rated capacity of 55,000 hp. under a head of 305 ft.

Tests were made upon Unit No. 4, by dropping loads of from 10,000 to 40,000 kilowatts. Observations were made of speed rise, pressure rise, voltage rise, and rate of gate closure. An oscillograph was used to measure the voltage variation and to record the time of load rejection. The purpose of the latter measurement was to assist in the determination of the dead time of the governor. It is regretted that observations of normal and maximum pressure only could be made; in later tests at another development the time-pressure curve was developed.

Surrounding the 10-ft.-diameter section of the penstock, just before entry to the turbine scroll case, is a ring of ¾-in. piping having four connections to the penstock on two diameters at 90 deg. to each other. This ring of piping is carried up to the turbine deck, to permit observations of penstock pressure. Exception might be taken to the considerable length of small pipe between the connection to the penstock and the gage, as the gage

was 11 ft. above the center line of the penstock. The writers do not consider that any appreciable error results from this, as readings taken in earlier tests on other gages, differently connected, checked the readings here used quite satisfactorily.

The unit upon which the observations were made had previously been subjected to very careful and complete capacity and efficiency tests. The power-discharge and turbine efficiency results have been published already in a paper by H. G. Acres.[1] Quite reliable estimates of the discharge during the pressure-rise runs were thus possible.

In the pressure-rise runs the generator output was measured by calibrated portable instruments. Pressures were read on calibrated

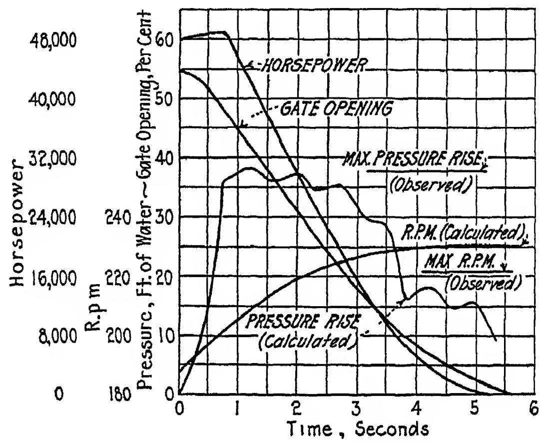


FIG. 29 REGULATION TESTS, JULY 1923, UNIT NO. 4, RUN NO 6, QUEENSTON DEVELOPMENT, HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO

(Observed and calculated speed rise and pressure rise; load rejected, 34,130 kw)

pressure gages; the gate opening and rate of closure were measured at the servomotor by a chart-drawing instrument, which gives a complete diagram of the gate movement This diagram is drawn by a pencil rigidly connected to the servomotor piston rod, which was in contact with a chart on the recording cylinder of the instrument, the cylinder being revolved at a constant, known rate. An additional pencil, controlled by a solenoid, was added to the instrument and marked on the diagram the time at which the current was impressed to open the 110,000-volt circuit breakers. This time measurement, along with the oscillograph records, permitted an estimate to be made of the dead time of the governor.

[1] Modern Hydraulic Turbines of Large Capacity. *Trans.* A.S.M.E., vol. 45 (1923), p. 52.

The speed of the unit was observed by a tachometer, and a record of maximum speed only is therefore available.

The procedure in each run was to build up the load to that at which it was desired to make the test, give signals (by electric bells) to each observer, who then took the various observations for conditions under load, signal once more, and after five seconds open the 110,000-volt circuit breakers. On the second signal, all recording instruments were operated and each observer at indicating instruments stood ready to take the necessary readings at his station.

The records of one of the runs, insofar as they are of interest in connection with this paper, are given in Fig. 29, along with calculated results, according to the methods proposed by the authors. The comparison between the calculated and observed records is quite satisfactory.

A more interesting set of results were those obtained at South Falls on the Muskoka River, Ontario. Unit No. 1 in this plant is a horizontal, double-runner, central-discharge turbine, rated at 2200 hp. under 105 ft. head, direct-connected to a 2000-kva. generator. The water supply is drawn through a 7-ft. wood-stave pipe, 977 ft. long, connecting to the turbine casing through 26 ft. of 7 ft. diameter steel penstock. There is no surge tank or relief valve.

A large pressure gage was connected to the steel section of the penstock and, beside it, a timer marking quarter seconds, and an electric bell. A moving-picture camera was used to observe the pressure rise, the individual pictures on the film having on them a record of pressure and time. The bell beside the gage was energized by the 110-volt current used to open the circuit breaker, and thus provided a record on the strip of film of the time at which load was dropped. Speed changes were recorded by a chronograph, on which one pen was actuated by a contact connected to the generator shaft. Turbine-gate closure was measured by a chart-drawing instrument connected to the servomotor piston rod, as at Queenston, and a record was made on the diagram, as before, of the instant at which the energizing current to open the 6600-volt circuit breakers was impressed. A spare pen on the chronograph gave a record of this time also, thus relating speed-rise, pressure-rise, and gate-closure diagrams correctly to each other and to the time of rejection of load.

It is scarcely necessary to do more here than give the results of some of the runs and compare them with results that would be obtained by calculations according to the methods proposed by the authors.

Nine runs were made, of which the results of two are shown. Fig. 30 from run No. 3 has more value than a simple confirmation of the authors' method of analysis with respect to pressure rise.

The observed pressure-time diagram is carried through for sixteen seconds and shows the rise and fall of pressure after gate closure.

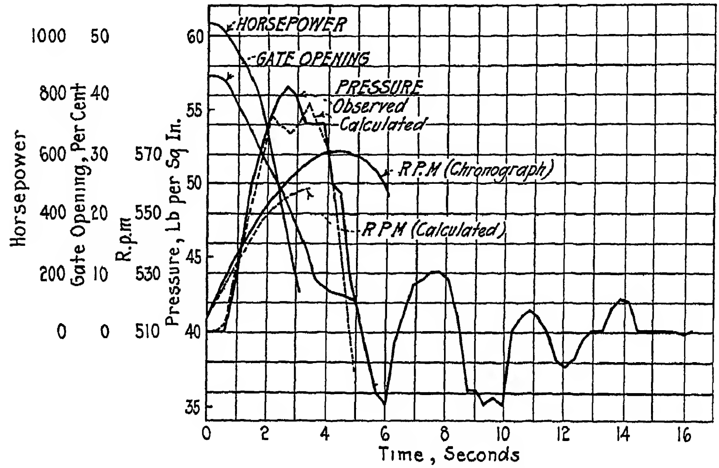


FIG 30 REGULATION TESTS, JULY 1925, RUN NO. 3, SOUTH FALLS DEVELOPMENT, HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO
(Observed and calculated speed rise and pressure rise)

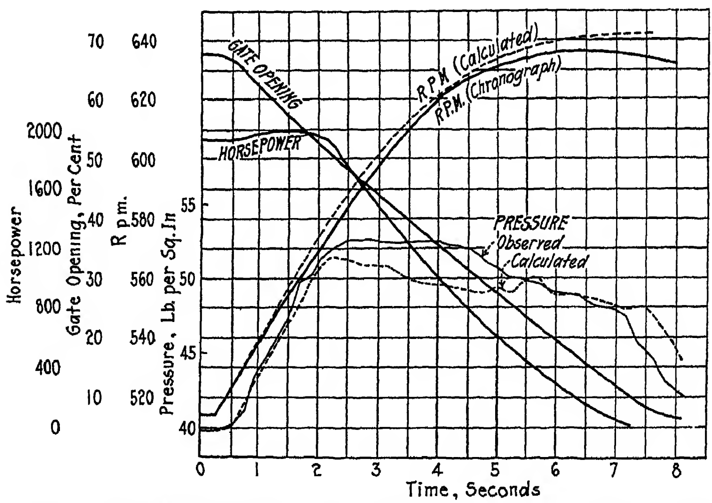


FIG 31 REGULATION TESTS, JULY 1925, RUN NO 8, SOUTH FALLS DEVELOPMENT, HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO
(Observed and calculated speed rise and pressure rise)

These, and similar waves from all nine diagrams made in the tests, permit a fairly reliable estimate to be made of the velocity of the pressure wave in the pipe. This velocity doubtless varies

along the length of the pipe, as the closer spacing of the bands at the downstream end of the pipe will cause it to have a higher value than that at the upstream end. The average value found and used was 1306 ft. per sec., and this value was used in the computed results submitted herewith.

The strikingly close agreement between calculated and observed speed rise in run No. 8 (Fig. 31) is a very satisfactory confirmation of the methods of computation suggested by the authors. Run No. 3 does not show as close agreement. Run No. 8 is fairly typical of all runs in which a large speed rise occurred.

A reference to the possibility of separation of the water column in the draft tube is perhaps permissible here, although unaccompanied by any measurements. During some of the tests at Queenston, with rejection of large blocks of power and rapid gate closure, there was separation of the water column as evidenced by a very severe shock accompanied by such a noise as would be caused by water-hammer in a large pipe. The shock was felt and heard at all points in the power house near the unit under test. With slower gate closure the shock did not occur. This, rather than excessive pressure rise, prompted the increase in governor traversing time. The experience is a practical confirmation of the conclusion of Mr. P. F. Kruse that investigation of the pressure variation in the draft tube during gate closure is important.

One other experimental result that may be of value is the measured dead time of governor. At Queenston the average of eight runs gave a dead time of 0.23 sec.

AUTHORS' CLOSURE: Mr. P. F. Kruse has contributed valuable information concerning the solution of problems in regulation and water-hammer, extending the computations to include the draft tube as well as the main water conduit. It is most interesting to note that the maximum draft head occurs very soon after the turbine gates start their stroke, indicating that this condition is found more often than would be expected. In the examples worked out in the paper showing the performance of a 70,000-hp. turbine for loads thrown off, it will be noted that the rise in pressure above normal is about the same at the end of the first second of time during the closure, regardless of the load change This is even more pronounced at the end of the first half second, the rise being between 30 and 35 ft., down to 25 per cent load change at least. As the maximum draft head tends to occur during the first second of the closure, for ordinary governor traversing times, it can be seen that only a relatively small change in load is required to produce a tendency to break the column of water in the draft tube.

While the particular case cited by Mr. Kruse may be unusual in regard to average draft-tube velocities, yet it is evident that

the effect of water-hammer in the draft tube assumes greater proportions as the total length of penstock and intake passages decreases.

The point brought out by Mr. Wilkins in regard to the determination of general Wr^2 must be qualified to some extent, particularly with respect to medium- and low-head installations. The degree of speed regulation for specified load changes as guaranteed by the turbine and governor manufacturer is always contingent upon the Wr^2 of the generator, and for this reason there is usually close coöperation between the turbine and generator manufacturers in fixing a value of flywheel effect that will not be excessive, nor yet small enough to result in instability in performance. A certain irreducible minimum Wr^2 is required for construction purposes, and it is excessive increases over this amount that become objectionable from a cost standpoint.

The effect of voltage rise on separation from the system has been studied in connection with the protection of the 70,000-hp.

TABLE 4

Case no.	Total governor time, sec.	Dead time, sec.	Net traversing time, sec.	Total speed rise, per cent above normal	Speed rise one second after load is thrown off
1	5.0	0.0	5.0	38.9	14.6
2	5.0	0.2	4.8	40.0	14.4
3	5.0	0.5	4.5	42.0	14.2
4	4.0	0.0	4.0	33.5	14.8
5	4.0	0.2	3.8	34.6	14.6
6	4.0	0.5	3.5	36.3	14.2

units noted in the paper. Detailed computations were made to determine the effect of variations in the governor timing upon the speed rise, and consequent voltage rise, on the unit and other electrical equipment. Various values of governor time were selected, together with different allowances for governor dead time and the arithmetical integration method was applied in detail to each case. The resulting curves of speed with respect to time showed a most interesting fact, namely, that the rise in speed at the end of the *first second after load was thrown off* was essentially constant for all cases considered. Table 4 will give a more comprehensive idea of this phenomenon than will a mere general statement.

The protective relays on the generator were designed to function in less than one second, and for this reason, the total rise in speed did not materially affect the problem, and the governors were set for a five-second traversing time by the authors, thus limiting the rise in pressure in the penstock to about 25 per cent of the effective head on the plant, although the original design called for a much faster rate of gate movement.

The effect of the system characteristics and the characteristics of the connected load are indeed most important elements in speed-regulation problems. These features must be studied care-

fully in connection with the adjustment of governors in the field to meet the requirements of operating companies. In fact, the character of load to be handled actually determines, to a greater degree than does the hydraulic limitations of the installation, the setting of the compensating features of the governor, and the speed variation for given load changes. For certain types of load, and combinations of load, the governor time must actually be lengthened in order to secure satisfactory regulation. In one installation supplying power to a paper mill, the combination of a large percentage of centrifugal pumping units and motor-driven pulp grinders resulted in a balanced condition in which the load variations on the grinders were absorbed by the pumping units to such an extent that good regulation could be obtained only by utilizing a very slow governor timing, and allowing a slight variation in speed to occur before the governor functioned. Other installations with opposite characteristics have been found frequently and serve to emphasize the need for a closer study of the effect of change in speed upon the load demand of various types of connected load.

The three points brought out by Mr. Quick are of interest. The effect of variation of no-load losses in the turbine and generator with speed are negligible where the speed variations are less than 50 per cent from normal. In the case of the 70,000-hp. turbine, the example given in the paper for full-load rejection would be affected to the extent of 0.2 per cent if the no-load losses were neglected in the computations.

The discharge-speed characteristic with constant gate setting becomes increasingly important as the specific speed of the turbine is increased. In one particular installation recently completed in the East, where the governor was omitted for the sake of simplicity, it was necessary to provide heavier Wr^2 in the generator to increase the time required for the unit to attain runaway speed in order to keep down the rise in pressure due to the reduction in discharge with speed. However, for normal values of governor time settings this does not assume any appreciable proportions.

The change in efficiency with changes in speed and head, as mentioned by Mr. Quick, does have a measurable effect upon the total rise in speed. With the characteristics of the runner available from the experimental tests, it is possible to determine with a fair degree of accuracy the variation of efficiency with speed and head. This variation has been computed as a correction to the efficiency curve with respect to gate opening and time as shown in Fig. 5. The result of this study for the case of full load thrown off, Fig. 6, is shown in Fig. 32. By applying these revised efficiency values to the computations in Table 1, column 9, the figures in columns 10 to 17 can be corrected and a new curve of speed with respect to time obtained. The initial portion, up to the end of

interval 4, is relatively the same as before, but from this point on a decrease in the speed change is noted, and the maximum is 35.8 per cent instead of 37.2 per cent as obtained in Table 1. This difference, amounting to 1.4 per cent of the normal speed of the unit, is relatively small, and is on the safe side. For the usual investigation work, the accuracy will not be materially affected if this refinement is neglected.

The authors are indebted to Mr. O. V. Kruse for pointing out that the speed drop shown in Fig. 21 applies only to the case of a single unit operating upon an isolated system having no connected Wr. This speed drop is of necessity utilized as a basis of comparison rather than as an actual change occurring in service. However, there are still numerous cases where the generating unit is not a portion of a large interconnected network, and hence the problem of oncoming load requires consideration. If it were possible to have available even an approximate estimate of the total connected Wr^2 of a system, which could be added to the flywheel effect of the unit under consideration, the net change in system frequency could be determined in the same manner as with the isolated unit, in which case a better idea of the problem as a whole could be obtained.

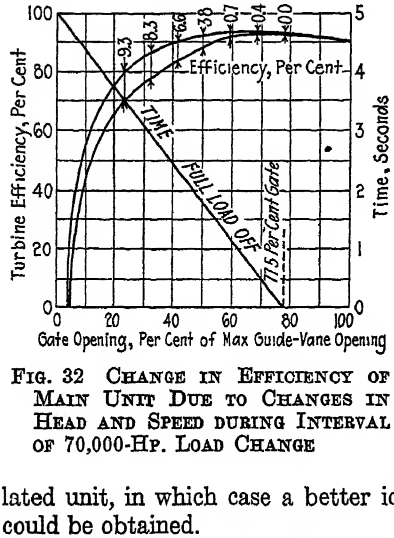


FIG. 32 CHANGE IN EFFICIENCY OF MAIN UNIT DUE TO CHANGES IN HEAD AND SPEED DURING INTERVAL OF 70,000-HP. LOAD CHANGE

The discussion by Mr. Uhl sums up in a most concise manner the unknown quantities in the present state of development of the study of regulation problems. Continued research in the field of load characteristics is needed to determine just what assumptions are justified, what commercial value can be set upon close regulation, and what "frequency tolerances" shall be allowed.

In reply to the question raised by Mr. Halmos, it is perfectly true that the water-hammer will be proportionately less as the head increases, provided that the length of conduit and the time of closure remain the same for all cases, and the velocity also remains relatively constant. However, the general tendency in hydro-electric plants is for increased length of the penstock, at least in direct ratio with the head and frequently in greater proportions. This naturally increases the effect of water-hammer as the head increases.

The use of long penstocks in low-head installations is a very serious problem, and some form of surge tank or even an open

forebay is usually necessary to secure any reasonable results. The use of very short governor times on long penstock installations with either high or low heads may give rise to return surges caused by the synchronizing of the governor characteristics with those of the pipe line. While conditions assuming proportions dangerous to the conduit may not result, yet there sometimes occurs an undesirable series of speed changes in the form of afterwaves, before steady operation is reached. These return surges can usually be eliminated by increasing the governor time one or two seconds in most cases, and even more, under severe conditions.

Mr. Wood has presented a very ingenious method for graphically solving problems of both pressure change and speed change,

TABLE 5 COMPARISON OF COMPUTATIONS OF PRESSURE RISE ON 70,000-HP. TURBINE

		Pressure rise, feet	
Interval	Time, sec	By chart	By arithmetic integration
0	0.0	0.0	0.0
1	0.25	18.2	17 5
2	0.50	30.7	29.8
3	0.75	38.2	38.0
4	1.00	42.3	43.2
5	1.25	44.8	45.8
6	1.50	47.0	47.3
7	1.75	48 0	48.0
8	2.00	48.5	48.3
9	2.25	48.5	48.5
10	2.50	48.5	48.5
11	2.75	48.5	48.6
12	3.00	48.5	48.6
13	3.25	48.5	48.6
14	3.50	48.5	48.6
15	3.75	48.5	48.6
16	4.00	48.5	48.6
17	4.25	48.5	48.6
18	4.50	48 5	48.7
19	4.75	48.5	48.6
20	5.00	48 5	48.6
	Average Rise	44 985	45.024
	Difference	0.09 of 1 per cent	

which facilitates a quick and accurate determination. Having established the charts similar to Mr. Wood's Fig. 28 for runners for various specific speeds, this graphical solution would be especially valuable where a number of regulation problems must be solved in a limited time. The authors have used Mr. Wood's chart for obtaining the pressure rise with respect to time for comparison with one of the problems given in the paper. For a full-load rejection on the 70,000-hp. unit, the comparison of results is as given in Table 5.

Problems 1, 2 and 3, given by Mr. Wood, have also been solved by arithmetic integration. The pressure rise for problems 1 and 2, as given in the table, is based upon closure to zero gate in each case. For problem No. 3 the fall in pressure is based upon an initial gate opening of 8 per cent, this being the gate opening at speed no-load.

In comparison of speed rise for problems No. 1 and 2, it was assumed that the gate closed to the speed no-load point only, which is the condition taken by Mr. Wood. Likewise in regard to problem No. 3, the decrease in speed is based upon opening the turbine gates from the speed no-load position. Uniform gate motion was assumed in all cases, to make conditions comparable with those used by Mr. Wood.

TABLE 6 COMPARISON OF PRESSURE CHANGE BY USING ARITHMETIC INTEGRATION AND BY USING CHARTS OF MR. WOOD

	Problem No 1 Pressure rise		Problem No. 2 Pressure rise		Problem No. 3 Fall in pressure	
Interval	By arithmetic integration	By chart	By arithmetic integration	By chart	By arithmetic integration	By chart
0	0.00	0.00	0.00	0.00	0.00	0 00
1	10.10	9 90	9 84	10.00	20.44	20 70
2	15.32	15 42	15.16	15.75	12.36	12.30
3	18.68	18.36	17.62	17.30	13.88	13 50
4	18 90	19 44	18 63	18.25	13.90	13.75
5	19.60	19.62	18 95	18.70	13.74	13 75
6	19.68	19.98	18 97	18.90	13.84	13.80
7	19.70	19.92	19 02	18.90	13 80	13.80
8	19 78	19.92	18.97	18.90	13.84	13.50
9	19.73	19.92	19.08	18 90	13.80	13.98
10	19.81	19.92	18.97	18.90	13.84	13 80
Average	18.13	18 24	17.52	17.45	14.34	14.29
Difference Ft.		+0 11		—0.07		—0.05
Difference Per cent		+0.06		—0.04		—0.04

Comparison of the speed change determined by arithmetic integration method and by the charts of Mr. Wood is given below for the three examples in his discussion.

	Speed change in per cent		
	Problem No. 1	Problem No. 2	Problem No. 3
By arithmetic integration................	15 23	4.57	18.95
By chart	15.9	5 4	18.2
Difference, per cent.....................	+0.67	+0.83	—0.75

The discussion presented by Messrs. Hogg and Traill includes a most interesting and valuable set of tests of the performance of hydraulic turbines under actual field conditions of speed rise for rejected loads.

In addition to the tests included in the discussion prepared by Messrs. Hogg and Traill there are several other runs on the South Falls Plant which have been investigated. The following table shows the comparisons between computed maximum speed and observed maximum speed as obtained from the chronograph.

Run No.	Computed maximum speed, r.p.m.	Observed maximum speed, r.p.m.	Percentage variation between observed and computed maximum speeds
2	539	554	+2.7
3	558	571	+2.3
4	559	565	+1.1
5	573	579	+1.1
6	568	580	+2.1
7	624	614	—1.6
8	642	636	—0.9
9	690	679	—1.6

It is most gratifying to the authors to have this added confirmation of the close agreement between the calculated and observed values of maximum speed.

No. 2010

TRANSMISSION OF POWER ON OIL-ENGINE LOCOMOTIVES

By Alphonse I Lipetz,[1] Schenectady, N. Y.
Member of the Society.

Power transmission on oil-engine locomotives depends upon the type of the oil engine for the locomotive and upon the conditions under which the locomotive is going to run.

The present paper pertains to the transmission of power only and gives a classification of power transmissions in chronological order of their appearances in the art.

Class A comprises full-power elastic-fluid transmissions in which the total output of the oil engine is transferred into elastic-fluid energy, which is, in turn, utilized for the propulsion of the locomotive. The efficiency of such a transmission is the product of the two separate efficiencies of the generation and utilization of energy. Various types of electric, hydraulic, pneumatic, compressed steam, aero-steam, and compressed exhaust-gases transmissions are described.

Class B consists of differential elastic-fluid transmissions in which the power from the oil engine is transferred directly to the driving wheels at top speeds of the locomotive with a very high efficiency, and partly directly and partly indirectly at all intermediate speeds. The theory of such transmissions is expounded and formulas for torques, speeds, and efficiencies are derived. Examples of electric, hydraulic, and pneumatic transmissions of this type are described and results so far obtained are given.

Class C pertains to mechanical (gear-clutch) and direct transmissions.

The drawing of conclusions as to the possible fields of application of various types of transmissions is postponed until the questions of the types of oil engines and different conditions of locomotive performance on railroads are discussed in a separate paper.

Introduction

One who is familiar with the progress made in the present century in the domain of heat engineering and prime movers must be exceedingly surprised at the fact that the internal-

[1] Consulting Engineer, American Locomotive Co.

The present paper contains discussion of Class B transmissions of the original in full, and abridgments of discussions of Classes A and C. The original paper was published in full in *Mechanical Engineering*, vol. 48 (1926), August, p. 797, and September, p. 929.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

combustion engine, having conquered practically all means of transportation — by air, by road, and to some extent by water — has not gained as yet any foothold in the most important line of transportation — by rail. Considering the fact that the Diesel engine is the most economical prime mover of our present age, and that, on the other hand, the reciprocating non-condensing steam engine of the present-day locomotive is far from approaching the efficiency of the latter, one cannot help but wonder why it is that the Diesel locomotive has not yet come into being, although almost thirty years have already elapsed from the time when the first marketable Diesel engine was built.

2 There are several reasons to account for this fact, but, briefly, all of them can be reduced to one, namely, that the steam engine is a very flexible machine, making the steam locomotive a wonderfully adaptable and convenient means for railroad traction, whereas the Diesel engine is the most inflexible prime mover. As is well known, the Diesel engine is a constant-torque prime mover and the torque cannot be appreciably increased; neither can it start under load. This makes direct coupling of driving wheels with the engine brake shaft either impossible, or requires special devices in order to overcome the above-mentioned handicaps of the Diesel engine. It is to these devices that the tardiness of the development of the Diesel locomotive must be attributed.

3 There are several sides to the problem of the Diesel locomotive: the Diesel engine, or broadly speaking, the oil engine, is one question; the torque-varying devices, or the transmission of power from the oil engine to the driving wheels, is a second; and the cooling of the oil engines on the locomotive is a third. We shall consider each of the three questions, but in order to facilitate the discussion of the oil engine itself, we shall take up first the various transmission systems, as the type of oil engine depends largely upon the system of transmission of power. The other two questions will be discussed in a separate paper.

Power Transmission

4 Owing to the inflexibility of the Diesel engine, the most natural thing to do is to use some sort of flexible power transmission in which a new intermediate energy would be generated (electricity, hydraulic pressure, etc.) and immediately expended, thus permitting a variation of torque and speed at will. Such a system requires, in addition to the full-power oil engine, two more full-power machines — a generator, pump, or compressor, as the case may be, and a corresponding electric, hydraulic, or pneumatic motor. Assuming that a direct transmission of power by mechanical means is not possible, the full-energy power transmission seems to be the only feasible solution. However, in speaking of the inflexibility of the oil engine, we must not forget that the latter is

not absolutely, but only relatively, inflexible, and that it can be regulated within certain limits — about 15 per cent above normal and about 75 per cent below normal. Consequently, there would be a certain range within which a direct mechanical transmission of power would seem possible. Therefore there is a certain class of power transmissions in which the power is transmitted partly mechanically and partly through an auxiliary medium (electricity, oil, etc.); these transmissions have the advantage of using smaller auxiliary generators and motors and of giving higher efficiencies within the range where the mechanical transmission of power is mostly used.

5 Further, attempts have been made to make the oil engine more flexible in order to permit the use of direct mechanical transmissions. While such attempts have not yet passed the stage of preliminary trials, nevertheless, they merit the most serious consideration as they may lead to the most desirable and promising solution.

6 Thus we have three classes of power transmissions for oil-engine locomotives:

A Full-power, elastic-fluid transmissions
B Differential elastic-fluid transmissions
C Mechanical and direct transmissions.

CLASS A — FULL-POWER, ELASTIC-FLUID TRANSMISSIONS[1]

Electric Transmission

7 The first combination of a heavy oil engine and an electric transmission is found in a Diesel-electric motor car built in 1913 by the Swedish General Electric Co. (Almänna Svenska Elektriska Aktiebolaget) in conjunction with the Diesel Engine Co. (Aktiebolaget Atlas Diesel). Later the two firms organized a new company, the Diesel Electric Car Co. (Diesel Elektriska Vogn Aktiebolaget), known as DEVA, and since 1913 about thirty cars have been built for Sweden, Denmark, France, and Tunis. The electric generator in these locomotives is in line with the engine and is connected with it by a flexible coupling. The generator is of the eight-pole, shunt-wound type, provided with a separate series winding which is used for starting the oil engine by driving the generator as a motor from a storage battery for a short time (1 to 1½ sec.). Except the 60-hp. car of the 2-2-0 type which has only one motor, all cars have two motors with a wheel arrangement 2-4-0 (150 hp.) or 2-2+2-2 for all other sizes. All motors are of the traction type, completely enclosed, with the casing in

[1] This section abridged to include descriptions of the transmission devices only. Complete paper, giving details of locomotives and performances, published in *Mechanical Engineering*, vol. 48 (1926), August, p. 797, and September, p. 929.

two parts, suspended on one end from the frame and on the other end supported by and geared to the driving axles. The speed control is obtained by varying the excitation through a controller on either end of the car, or locomotive.[1] On the large cars the motors, which are usually coupled in parallel, can be coupled also in series in order to provide greater tractive effort.

8 Very soon after the DEVA motor cars had been placed in service, several Diesel-electric motor cars were built for the Saxon and Prussian Railways by Sulzer Bros., in Winterthur, Switzerland, jointly with Brown, Boveri & Co., in Mannheim, Germany. In these locomotives, a direct-current, 140-kw., eight-pole, 300-volt generator was directly connected to the engine brake shaft. A separate 7.5-kw. exciter supplied also current for a fan and for charging a 95 ampere-hour storage battery. The latter was provided for starting the Diesel engine by reversing the generator for a short time as a motor. The six-pole, series motors were set on a jackshaft mounted on the rear four-wheel truck and connected with both driving axles by means of connecting rods. The speed control was a modified Ward-Leonard system, employing variation of the excitation of the generator and reversal of the flow of current through the armatures of the motors.[2]

9 At the end of 1923, a 300-hp., 0-4-4-0 Diesel-electric locomotive was placed in service in this country. It was built jointly by the General Electric Co. and the Ingersoll-Rand Co., and represents a development of two locomotives built by the General Electric Co. in 1917. Several more locomotives, practically duplicates of the later locomotive, have been built since 1923 by the same two companies jointly with the American Locomotive Co. The generator is a 200-kw., 600-volt, compound-wound, 6-pole, direct-current dynamo directly connected to the oil-engine brake shaft. There are four geared motors, one for each axle, mounted on the trucks. The motors are of the series wound, totally enclosed, commutating-pole, split-frame type. The four motors are subdivided in two pairs; two motors of each pair are permanently connected in multiple, and these pairs can be coupled in series usually for speeds below 5 miles per hour or in parallel for speeds above 5 miles per hour. The output of the generator is automatically adjusted to suit the varying resistances of the train. This is obtained by inserting a commutating field and a differential series field in the current line, thus modifying the excitation field of the generator so as to give the proper proportion of amperage for the tractive effort and of voltage for speed. No rheostats are used in the power circuit for speed control. The posi-

[1] Diesel-Electric Motor Cars, by E. Nilsson, *Railway Review*, August 29, 1925, pp. 301-307.

[2] Die Dieselelektrischen Triebwagen für die Kgl. Sächsische Staatseisenbahnen by H. Zenner. Elektrische Kraftbetriebe und Bahnen, 1915, Heft 26-28.

tion of the control handle determines only whether the motors are connected in series or in parallel or in reverse, and another handle affects the oil-engine throttle controlling the power generated by the oil engine. Further speed control is going on automatically with the proper division of power into tractive effort and speed.[1] This system is known as the automatic, or the Lemp, control and has been used extensively by the General Electric Co. for gas-electric motor buses, motor railcars, and locomotives. The automatic control was fully described by Hermann Lemp in his paper[2] presented at the Annual Meeting of the Society in December, 1925.

10 A great stride in the development of the oil-electric locomotive was made when the 1000-hp.[3] locomotive for Russia was built in 1924 in Germany. It is of the 2-10-2 type. The direct-current generator is connected directly with the Diesel engine. The exciter is driven from the generator shaft by means of wheels with a ratio of 1 to 2. The field of this exciter is in turn energized by another smaller exciter driven by a belt from the first exciter's shaft. The field of the latter exciter is fed by current from a storage battery through controllers placed in the engineer's cab at both ends of the locomotive. Thus a very gradual change in voltage is obtained which, as a consequence, gives a flexible control of speed and tractive effort with small resistance losses in the electric transmission. Five direct-current motors of the tramway type are geared to the driving axles, one motor to each axle.[4]

11 Quite recently the Baldwin Locomotive Works has completed a 1000-hp. oil-electric locomotive of the 2-4-4-2 type, driven by a 12-cylinder Knudsen oil engine through a Westinghouse electric transmission. The generator has a separate exciter. The speed control is by the original Ward-Leonard system with separately excited motor fields. The controller handle varies and reverses the field excitation of the generator and thereby the rotation of the motors. It is linked with a throttle regulating the oil admission to the engine, thus controlling the output of the latter.[5]

[1] New 60-ton Oil-Electric Locomotive, *Oil Engine Power*, September, 1925, p. 513.

[2] Electric Transmission for Internal-Combustion Engines, *Mechanical Engineering*, vol. 48, no. 3, March, 1926, pp. 207-208.

[3] The power of the locomotive is very often given as 1200 hp. This seems to be the maximum overload rating of the Diesel engine, as it corresponds to 86.4 lb. per sq. in. brake m.e.p. For continuous output, the author thinks it more correct to rate the Diesel engine at 1000 hp. in agreement with Dr-Ing. M. Mayer, Manager of the Maschinenfabrik Esslingen, builders of the locomotive. See his paper "Die Diesellokomotive vom Standpunkt des Lokomotivbaues," published in *Zeitschrift des Vereines Deutscher Ingenieure*, May 9, 1925, p. 637. The 1000-hp. continuous rating corresponds to 72.0 lb. per sq. in. brake m.e.p., which is more in conformity with other oil engines mentioned in this paper.

[4] *The Engineer*, November 14, 1924, p. 554. See also Die Diesel Elektrische Lokomotive, by Prof. G. Lomonossoff, Berlin, 1924.

[5] *Oil Engine Power*, September, 1925, pp. 523-527.

12 One of the latest combinations of an oil engine with electric transmission has been employed in two railcars built for the Canadian National Railways. The small unit has a 105-kw., 600-volt, direct-current, British Thomson-Houston, differentially compound-wound generator with a 6-kw., 60-volt, separate exciter, and two 150-ampere, 600-volt, General Electric motors driving the front truck axles through helical gears. The car speed is controlled by the oil-engine throttle and the Lemp automatic control system. A second handle operates the electric controller which connects the motors to the generator circuit in series or parallel, or in reverse. The large unit has a Westinghouse 200-kw., 600-volt, direct-current, differentially compound-wound generator with excitation from a 300-volt battery, four 145-ampere, 600-volt, Westinghouse motors connected permanently in parallel and driving the front and rear truck axles through helical gears. The car speed is controlled by the modified Ward-Leonard system with constant engine speed by governor and by resistances in the main generator field and by eight electro-pneumatic switches. The starting of the oil engine on either car is accomplished by driving the generator as a motor from a battery.[1]

13 A further development of the locomotive described in Par. 9 is a 100-ton, 600-hp. locomotive built by the same three companies. Several engines of this type have recently been delivered to various railroads in this country. Two oil engines and two direct-connected generators are used instead of one.

Hydraulic Transmission

14 When the Diesel locomotive built in Germany in 1913 with direct connection of the engine to the driving wheels failed to perform for reasons which will be given later, the German Railway Administration was looking for some sort of flexible transmission of power. We do not know why electric transmission was not resorted to, but it happened at the time that Hugo Lentz had just brought out a design of hydraulic transmission and the German Railway Administration decided to try it out.

15 A 30-hp. Diesel-Lentz switching locomotive was built in 1921 by the firm of A. Gmeinder & Co. in Mosbach, Germany, later reorganized into the Badische Motor-Lokomotiv-Werke A. G.

16 The Lentz hydraulic transmission, or gear, as it is usually called for the reason that it does not change the speed of the propeller shaft gradually but by steps, consists of two parts, a rotary pump attached to the prime mover and a rotary hydraulic motor connected to the driving shaft. The pump is in line with the oil engine. It has two or more revolving pistons, or disks, of different widths. The motor has only one piston. Its axis is parallel with those of the driving axles and at 90 deg. to the axis of the pump. Each piston has several very light sliding vanes which fit

[1] *Railway Age,* October 17, 1925, pp. 695 to 699.

nicely in grooves in the piston. The disks revolve between well-fitted covers made of hard bronze. The vanes are provided on both sides with pins and rollers which fit in grooves cut in the covers. The grooves are partly concentric, partly eccentric, in shape. The motor has a similar arrangement. It usually differs in that it is made of a double-acting type with double the number of vanes. The fluid, fine oil, works in a circle. It is pumped into the motor and having performed its work is led back into the suction passage of the pump. The speed of the motor is proportional to the ratio of volumes swept by the pistons of the pump and of the motor. If the pump is provided with two pistons with a ratio of volumes of 1 to 2, then three speeds can be obtained; (1) with the small piston only, the larger running idly; (2) with the large piston only, the smaller running idly; and (3) with both pistons running simultaneously. The ratio of speeds would be 1 to 2 to 3. For the idle running of one or another of the pistons, cocks should be placed in such a way as to establish communication between the pressure and the suction chambers of the pump; then the fluid passes through the pump with practically no pressure at all. By placing cocks so as to connect the pump with the motor, the fluid is forced into the motor with a pressure corresponding to the load on the driving shaft and returns into the suction chamber of the pump with practically no pressure. By reversing the position of the two cocks between the pump and the motor, the rotation of the motor can be reversed. There are two cocks for each piston, and the relative position of them can be changed independently; thus as many speeds for backward running can be obtained as there are forward speeds. If all cocks are placed in the idle position, the driving shaft remains immovable. In order that variation of speed should come forth smoothly, without shocks, the gear is provided with a throttle valve. The valve is opened every time when speed is changed. In such a way sudden rise of pressure in the gear is prevented and more uniform variation of speed is obtained. This valve also serves as a safety valve if the spring is set at a certain pressure. It also permits the use of the motor as an emergency brake by reversing the position of the cocks while the driving shaft continues to rotate in the former direction, thus pressing oil into the pump in a direction opposite to the rotation of the oil engine. The safety valve prevents breakage which otherwise would take place.[1]

17 The operating results of the 30-hp. Diesel-Lentz locomotive appeared to be so satisfactory that many builders in Germany and Austria have started construction of larger locomotives. In 1923 the firm Grazer Maschinenfabrik in Graz, Austria, built a 60-hp., 0-4-0 locomotive. At the same time the German firm which built the first 30-hp. Lentz locomotive, jointly with the Maschinenbau-

[1] *Zeitschrift des Vereines Deutscher Ingenieure,* no. 45, 1921, pp. 1160-1163.

Gesellschaft Karlsruhe, in Karlsruhe, Germany, developed a 160-hp. Diesel-Lentz locomotive. The German firm Linke-Hofmann-Lauchhammer Werke A. G. in Breslau has been active in developing Diesel-Lentz locomotives. The oil-engine firm Motorfabrik Deutz of Cöln-Deutz, Germany, built jointly with the Locomotive Company Henschel & Sohn of Cassel, Germany, an 0-6-0 Diesel locomotive with Lentz gear.

18 Two years ago, the English firm, Messrs. Hawthorn, Leslie and Co. contemplated the building of a 300-hp. gear. The design of the gear differed from the known Lentz gear in that the vane pump had been replaced by a multi-cylinder piston pump with variable stroke, thus permitting infinite variation of speeds.

19 At the time when the Lentz transmission was being developed in Germany, an 0-4-0, 150-hp. switching locomotive was built in Canada with the Universal hydraulic transmission developed by the Universal Engineering Corporation in Montreal. The transmission is also known under the name of Williams-Janney or Waterbury gear. The two parts of the transmission, primary and secondary, were separated, the primary — the pump — being attached to the prime mover, and the secondary — the motors — acting directly on the driving wheels. The pump and the motors were connected by pressure and suction pipes, the whole arrangement resembling very much an oil-electric locomotive with motors directly attached to driving axles. The gear permits an infinite change of speed from zero to maximum.[1] The Universal Engineering Corporation recently applied the variable-speed gear to a passenger railcar which has been in service on the N. Y., N. H. & H. R. R. for the last two years. A similar locomotive with a 75-hp. Diesel engine and Williams-Janney transmission was built in the Vickers plant at Barrow-in-Furness.

20 At the Exhibition of Seddin, Germany, 1924, the firm Berliner Maschinenbau A. G. vormals L. Schwartzkopff of Berlin exhibited an 0-4-0 Diesel locomotive with a new design known as the Huwiler gear. It consists of an hydraulic pump and motor, both of the vane type, similar to the Lentz gear, but differs from the latter in that the pump delivers an infinitely variable quantity of oil to the motor, thus permitting an infinite variation in speed.

21 In addition to the above-mentioned types, there are several other hydraulic transmissions, such as the Hele-Shaw gear, the Naeder drive,[2] the Lauff-Thoma transmission,[3] which have been tried with some success on small gasoline or kerosene locomotives, railcars, and automobiles.

[1] Gasoline Switching Locomotive with Hydraulic Gear, *Railway Mechanical Engineer,* September, 1922, pp. 503-506.

[2] *Revue Général des Chemins de Fer,* May, 1923, pp. 302-305.

[3] *Hanomag Nachrichten,* vol. 12, no. 140, June, 1925, pp. 94-95.

Pneumatic Transmission

22 The idea of pneumatic (air) transmission is probably one of the first which occurred to people interested in Diesel locomotives. As far back as 1909, V. A. Stuckenberg, general manager of the Tashkent Railroad in Russia, suggested rebuilding steam locomotives in the following way: to replace the tender by a unit carrying a Diesel engine and a compressor, to use the boiler as a compressed-air storage tank, and to let the air work in the existing cylinders in the same way as steam. The project was not considered practicable at that time and the idea was abandoned About the same time, James Dunlop of Glasgow, Scotland, came out with a similar proposition. As no mention is made of this locomotive in Mr. Dunlop's recent paper read before the Institution of Engineers and Shipbuilders in Scotland, December 16, 1924, it probably was never built. The German firm of Maschinenfabrik Esslingen is building at present a 1200-hp. locomotive with air transmission. The system differs, however, from the Stuckenberg-Dunlop scheme in that the compressed air is heated on its way to the locomotive by exhaust gases from the oil engine.

Steam Transmission

23 Instead of air, steam working in a closed circuit has been suggested by Cristiani of Milan, Italy. Steam from a high-pressure container is superheated by exhaust gases of the Diesel engine and expanded in ordinary steam cylinders. It is then expelled to a low-pressure receiver at 25 to 30 lb. per sq. in. which is cooled by atmospheric air circulated by a fan. A steam compressor draws the cooled steam from the receiver, compresses it to approximately 180 lb. per sq. in., and rejects it to the high-pressure container, whence it is again superheated, expanded, and so forth. The steam compressor is directly driven by a Diesel engine and the total output of the latter is turned into compressed-steam energy utilized in locomotive steam cylinders.

Aero-Steam Transmission

24 The expansion of air in working cylinders is usually accompanied by a rapid drop in temperature. This was the stumbling block in the way of the first air-transmission projects. An Italian engineer, Fausto Zarlatti, thought that he might be able to overcome this difficulty by mixing the air with steam and thus keeping the air warm during expansion due to the smaller fluctuation in temperature of the expanding steam. He suggested, therefore, an aero-steam transmission, and even went so far as to rebuild a small six-wheel locomotive. A six-cylinder gasoline engine of approximately 70 hp. drove an Ingersoll-Rand air compressor. The engine and compressor were mounted on the tender, and compressed air

was stored in the old locomotive boiler partly filled with water. The air preheated by compression before entering the working cylinders must have passed through the column of water in the boiler and thus became saturated with vapor. In addition, use was made of the heat contained in the exhaust gases of the gasoline engine, the latter being passed through the boiler and generating a certain amount of steam. The mixture of air and steam was admitted to the cylinders by means of the existing throttle and reverse levers.

25 James Dunlop, the man who was responsible for the air-transmission projects, came out lately with a new design of a main-line 1100-hp. locomotive with an aero-steam transmission. The idea is the same as that of Zarlatti, but the design is worked out on large power lines and is of great interest. Steam at 200 lb. pressure, generated in a boiler heated by the exhaust gases of the oil-engine cylinders, is superheated by the compressed air with which it is mixed. This is supposed to result from the fact that compressed air at 200 lb. pressure is more than 200 deg. fahr. higher in temperature than steam at 200 lb. pressure.[1] The water space of the boiler is connected with the water cooling jackets of the oil engine, similar to the arrangement of the Still engine.

Exhaust-Gas Transmission

26 Compressed-air transmission may provoke the danger of the ignition of lubricating oil so that special precautions must be taken to evade it. In order to eliminate the slightest possibility of an oil explosion, suggestions were made to use an inert gas instead of air. The most convenient gas to use would be the gas of complete combustion exhausted from the oil engine. An 0-4-0 locomotive was recently built by the Waggon und Maschinenbau A. G. of Görlitz, Germany, jointly with Berliner Maschinenbau A. G., vormals L. Schwartzkopff, of Berlin. The exhaust gases of the oil engine are first cooled, then compressed to about 115 to 140 lb. per sq. in. and cooled at the same time, and only afterwards, when the cooling is finished, the compressed gases are heated to approximately 660 deg. fahr. by the exhaust gases on the way to the cooler. The object of cooling the gases before and during the compression is to keep the size of the compressor and the power absorbed by compression as low as possible.[2]

[1] Internal-Combustion Locomotives, by James Dunlop, Published by the Institution of Engineers and Shipbuilders in Scotland, Glasgow, 1924, p. 11.

[2] Beschreibung der von Berliner Maschinenfabrik, A. G. während der Eisenbahntechnischen Tagung in Seddin ausgestellten Lokomotiven, pp. 17-18.

CLASS B — DIFFERENTIAL ELASTIC-FLUID TRANSMISSIONS

27 In the so-called full-power transmissions mentioned above, the total output of the prime mover is always transmitted to the driving wheels through an elastic medium with the result that, owing to unavoidable losses, only a part of the transferred energy is producing useful work. This loss of energy takes place not only at starting, or at low speeds, when a starting device or a transmission is in any event necessary, but also at speeds and torques which otherwise could be easily obtained from the oil-engine if it were directly connected with the driving wheels. A full-power transmission would seem to be an unsuitable device for speeds at which it becomes possible to employ the direct drive, as the latter would give a considerable gain in efficiency. Moreover, a main-line locomotive with infrequent stops would, most of the time, run at full speed and would require a comparatively small torque, and only occasionally at starting and on long grades would it require the help of a transmission. It would, therefore, seem a pity to use a transmission all the time for no other reason than because, for a very small portion of the time, the transmission is indispensable.

28 A. E. L. Chorlton, of William Beardmore & Co. in Glasgow, Scotland, developed a design of a 2-10-2 internal-combustion locomotive consisting of a six-cylinder V-type oil engine with a mechanical transmission and a hydraulic clutch between the engine and the driving wheels. The mechanical transmission provides the increase in torque at starting and permits a direct drive when a certain speed has been attained.[1] The details of the design, however, have never been revealed. A similar scheme has been recently devised by Prof. C. A. Norman. He uses a planetary gear with a hydraulic clutch which permits a considerable increase in torque at starting with a direct drive at certain speeds.

29 An excellent solution of the problem is offered by differential transmissions in which a certain portion of the power is transmitted by fluid and the rest is carried through mechanically, the proportion of the latter part varying from 0 to 1 with the increase of velocity from nothing to full speed. Many designs of this kind have been suggested in the last 20 to 25 years, mostly for automobiles, and some of them are now being developed for locomotives.

30 The designs of these transmissions are sometimes complicated and difficult to conceive, but the underlying idea is simple, and is helpful to the proper understanding of the most complex designs.

31 Let us first consider an ordinary friction clutch inserted between the prime mover P keyed to the primary (driving) shaft A

[1] The Internal Combustion Locomotive, by J. S. Tritton, *Journal* of the Institution of Locomotive Engineers, London, May-June, 1923, p. 367.

(Fig. 1), and the secondary (driven) shaft B connected to the driving wheels. When cone R is pressed against cone C torque T_1 is transmitted to shaft B by direct contact and practically with no loss in value. With respect to power, however, great loss is taking place during the whole period of acceleration. If we denote the speeds of shafts A and B by n_1 and n_2, the respective powers would be in certain units T_1n_1 and T_1n_2, and the loss would be equal to $T_1(n_1-n_2)$. At the moment when shaft B acquires the speed of shaft A, the loss vanishes and the whole mechanism — prime mover, shafts, and clutch — rotates as one unit. From starting to that moment, however, a certain amount of work equal to $\sum_{n_2=0}^{n_2=n_1} T_1(n-n_2)\,.\,\Delta t_1$, where Δt is the decrement of time, is wasted on destruction of the friction surfaces of the clutch.

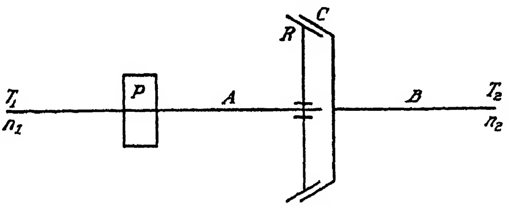


Fig. 1 Friction-Clutch Transmission

32 Imagine now a hydraulic pump instead of a friction clutch, the rotating part R (Fig. 2) of the pump being connected to the primary shaft A and the stationary part S to the secondary shaft. At starting, when S and B are at standstill, prime mover P will generate pressure in the pump until the pressure torque balances torque T_1. Then, if this pressure torque is sufficiently large to overcome the resistance torque of starting T_2, it will tend to rotate shaft B with an increasing speed until the speeds of both shafts are practically equal. During the acceleration period, however, the torque applied to S being equal to torque T_1 applied to R, speeds n_1 and n_2, however, being different, the power transmitted to shaft B is smaller than that of the prime mover applied to A. The difference $T_1(n_1-n_2)$ is the same as in the case with the friction clutch, but contrary to the latter, the loss of energy is not used on destruction of metal, but is wasted in leakage, and in heating by friction. The result of transmission is nevertheless the same, and this sort of transmission is rightly called "hydraulic clutch."

Basis of Differential-Fluid Transmission

33 Now, let us see whether it would not be possible to utilize the wasted energy $T_1(n_1-n_2)$. Suppose we take a hydraulic clutch and, in addition to it, set a motor on shaft B (Fig. 3).

Suppose further that we lead the excess fluid under pressure from pump RC into the motor, coupling rigidly casing C of the pump with rotor M, while casing S of the motor is made immovable. Passages a and b are pressure and discharge pipes between the pump and the motor. Denote the arms on which the pressures in the pump and in the motor are applied by r_1 and r_2 (if, for instance, the pump and the motor are of the cylinder-plunger type, r_1 and r_2 are the crank radii of the pump and motor respectively, multiplied by a constant). Suppose further that prime mover P

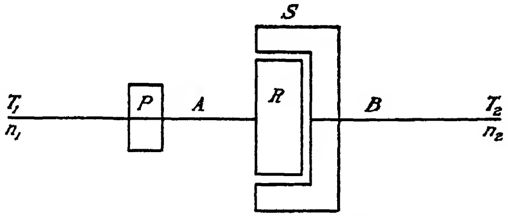


FIG. 2 HYDRAULIC-CLUTCH TRANSMISSION

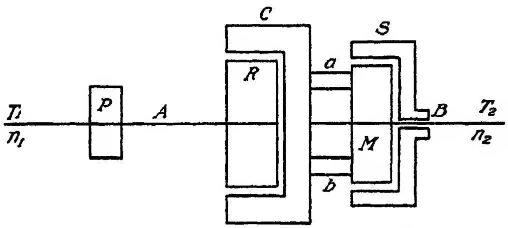


FIG. 3 DIFFERENTIAL-FLUID TRANSMISSION

is driving shaft A with torque T_1 and speed n_1, developing power N equal in certain units to

$$N = T_1 n_1$$

and that shaft B is stationary ready for starting. Then pressure p which will be established in the pump must satisfy the equation

$$T_1 = pr_1 \quad . . . . [1]$$

The existing pressure between R and C will tend to rotate C and shaft B with two torques: one applied to C which is the direct action of pressure from T_1 and is equal to it, and another applied to M equal to pr_2. The combined torque T_2 will thus be

$$T_2 = T_1 + pr_2 \quad . . . . [2]$$

which is larger than torque T_1 transmitted in the case of the friction or hydraulic clutch. If T_2 is sufficiently large to overcome the resistance of starting, shaft B will start rotating and gradually acquire a speed n_2 which will be easily found as follows: Denote

power transmitted through C by N_d; then $N_d = T_1 n_2$; denote further power transmitted through M by N_h; it equals torque pr_2 times speed n_2, then $N_h = pr_2 n_2$. If we disregard for the moment the unavoidable losses in the whole transmission, the two powers combined must be equal to N; thus

$$N = T_1 n_1 = N_d + N_h = T_1 n_2 + pr_2 n_2 \quad . \quad . \quad . \quad . \quad . \quad [3]$$

or

$$n_2 = \frac{T_1 n_1}{T_1 + pr_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [4]$$

This formula determines speed n_2 that will be maintained as long as the resistance torque equals T_2.

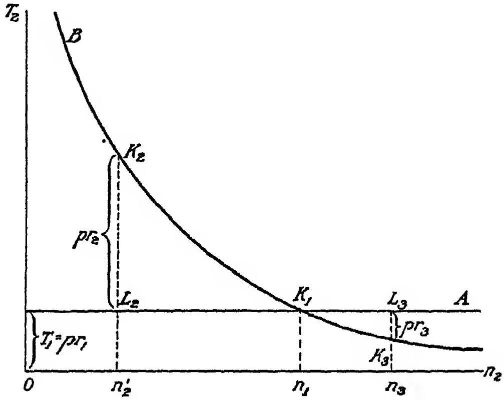


FIG. 4 TORQUE CURVES

34 Note also that the denominator in the last formula is nothing else than torque T_2; consequently

$$n_2 = \frac{T_1 n_1}{T_2}$$

or

$$T_2 n_2 = T_1 n_1 = N \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [5]$$

This is another expression for the fact that the power of the prime mover is transmitted completely, if the ordinary losses are temporarily disregarded.

35 A graphical study might better elucidate the subject treated. Assuming that T_1, n_1, and N of the prime mover are constant, we obtain a hyperbolic relation $T_2 n_2 =$ constant between torque T_2 and speed n_2 of the secondary shaft represented by curve B (Fig. 4), the ordinates of which give torques on shaft B at various speeds. With an ordinary friction or hydraulic clutch we would have a straight line A giving a constant torque T_1. The excess torque which we get for any speed n'_2 between 0 and n_1 is repre-

sented by $K_2L_2 = pr_2$. It is proportional to r_2, p being equal to T_1/r_1 and consequently constant. It follows that the eccentricity of the motor must be made variable and controllable in accordance with the speed. It can be readily derived from formulas [1], [2], and [5] that

$$r_2 = r_1\left(\frac{n_1}{n_2} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [6]$$

The higher the speed n_2 of the secondary shaft the smaller must be the eccentricity. When r_2 is made equal to zero, $n_2 = n_1$, the secondary shaft rotates with the same speed as the primary shaft. What actually happens is this: The eccentricity is zero, motor M is in no position to receive fluid under pressure (for instance, in the cylinder-piston type there is no relative movement between cylinder and piston and no room for the fluid to enter), the total quantity of fluid is locked in between R and C and both shafts with all movable parts of the pump and motor are rotating as one solid unit. This condition is known as a "hydraulic lock." If there were cocks on pipes a and b they might just as well be closed. The torque at that speed is T_1 (point K_1) and the whole power is transmitted directly from R to C, and nothing is transmitted hydraulically, that is

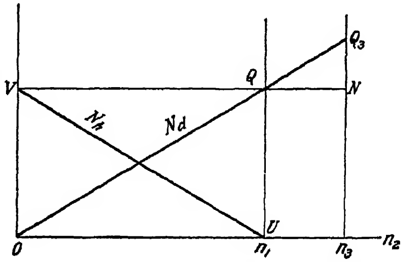


Fig. 5 Power for Various Speeds

$$N = N_d; \quad N_h = 0.$$

On a chart (Fig. 5) showing the powers N_1, N_d, and N_h for various speeds, the hydraulic-lock condition is represented by points Q and U. At starting all the power is transmitted hydraulically, and nothing is transmitted directly

$$N = N_h; \quad N_d = 0.$$

This is represented by points V and O. For all intermediate speeds the straight lines OQ and VU represent respectively the power transmitted directly, N_d, and hydraulically, N_h. It is obvious that for all speeds $N_d + N_h = N$.

The Hall Transmission

36 Before pursuing any further the theory of the differential fluid transmission, the author feels that an example of an actual design would be very elucidating. The Hall transmission will serve the purpose best. Fig. 6 represents two vertical cross-sections through the transmission. A is the primary shaft; it drives eccen-

tric R of a multi-cylinder piston pump, the cylinder block of which, C, is geared to the secondary shaft (not shown on the drawing), by means of wheel G. Cylinder block C constitutes one unit with a multi-cylinder hydraulic motor M, the pistons of which, together with their rods r and eccentric strap s, can rotate around a fixed eccentric S. The actual eccentricity of the latter in relation to the axis of the cylinder block is controlled by a wheel W keyed to a shaft T which can be turned in an eccentric bushing V. The eccentricity of the latter is equal to that of eccentric S. By turning it with shaft T in bushing V, eccentric S can be placed either in line with the prime-mover shaft and the cylinder block, or in the other extreme, or in any intermediate position.

37 The operation of the transmission is as follows: Before starting, shaft A rotates at full speed and drives eccentric R which moves pistons p in cylinder block C. Motor M with pistons q and

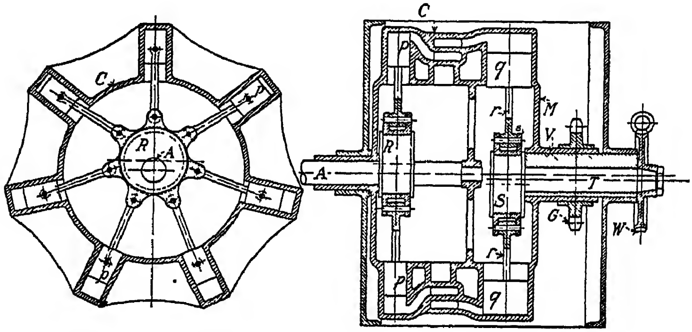


FIG. 6 HALL TRANSMISSION

gear G is at standstill. The primary part will now become a full-stroke piston pump generating pressure inside the cylinder block. As the fluid has no other means of coming out than through cylinders of the secondary part, it will push pistons q as soon as the pressure is sufficient to overcome the resistance of starting. It will then turn wheel G with everything geared to it, thus establishing a continuous flow of liquid through the pump and cylinders. On the other hand, as the speed of wheel G, due to the acceleration of the locomotive, will go up, the relative speed between shaft A and cylinder C of the pump will slow down. Let us assume that the cylinder block with wheel G attained a speed of one-sixth of that of shaft A. The relative speed of the pump will be only five-sixths of the speed of A, and only five-sixths of the full output of the pump will be discharged into the motor. The cylinders of the motor must thus absorb five-sixths of the full output of the pump, or, as wheel G rotates six times slower than shaft A, the cylinders of motor M must, during one-sixth of a revolution, sweep a volume five-sixths of the output of the pump, or, during a whole revolution,

a volume five times that of the pump. Eccentric disk S must, therefore, be placed in such a position as to provide the necessary stroke for pistons q. The torque transmitted through motor M is thus five times as large as that directly applied through cylinder block C, and the total torque on wheel G is six times that of shaft A — this is as it had to be expected, since the speeds are in a ratio of 6 to 1.

38 As the speed of wheel G goes up, the pump delivers less and less fluid. In order to keep the pressure constant and thus balance the constant torque of the engine, it is necessary to turn disk S in accordance with the reduced pump delivery. If we reduce the secondary eccentricity too much we increase the fluid pressure and the torque, and overload the oil engine; conversely, if we increase the secondary eccentricity of S, the events will be reversed and they will result in underloading the oil engine. In order to obtain the full power of the oil engine at all speeds, it is necessary to control the eccentricity of disk S in strict accordance with the variation in speed and torque. This can be arranged automatically from the pressure in the pump, and devices of this sort already have been built in connection with a gear of a certain type.

39 From the analysis given above it is clear that a differential transmission, like the full-power transmission, must have two parts, the primary and the secondary, each of which has two members, or four members altogether. For future convenience we may call them: first, second, third, and fourth members. In a full-power transmission the first is the driving member, the third is the driven member, the second and fourth are stationary. In a differential power transmission the second and third members, positively connected together (C and M on Figs. 3 and 6), constitute the driven member, the first is the driving, and the fourth is the stationary member.

Efficiency of Differential-Fluid Transmission

40 Now we go to the question of efficiency. We assumed above that the total power is transferred without any losses from the primary to the secondary shaft. In actual practice, of course, there are unavoidable losses both of the power transmitted by hydraulic action through the pump and the motor, and of the power carried over directly by static pressure of the fluid. Thus

$$N_2 = N - N' - N'' = (N_h - N') + (N_d - N'') \quad . \quad . \quad . \quad [7]$$

where N' represents the losses in the pump and motor like those in any ordinary hydraulic transmission, and N'' the losses due to static pressure, mostly leakage. If we denote the efficiency of the former by η_h and that of the latter by η_d, that is if we denote

$$\frac{N_h - N'}{N_h} = 1 - \frac{N'}{N_h} = \eta_h$$

$$\frac{N_d - N''}{N_d} = 1 - \frac{N''}{N_d} = \eta_d$$

the power obtained on the secondary shaft would be

$$N_2 = \eta_h N_h + \eta_d N_d,$$

and the total efficiency of the transmission could be expressed by the formula

$$\eta = \frac{N_2}{N} = \frac{\eta_h N_h + \eta_d N_d}{N} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [8]$$

Referring to Fig. 5 we see that

$$N_h = N\left(1 - \frac{n_2}{n_1}\right); \qquad N_d = N\frac{n_2}{n_1}$$

where n_2 is the variable speed of the secondary shaft, and n_1 is the constant speed of the primary shaft. The efficiency is thus

$$\eta = \eta_h\left(1 - \frac{n_2}{n_1}\right) + \eta_d\frac{n_2}{n_1},$$

or

$$\eta = \eta_h + (\eta_d - \eta_h)\frac{n_2}{n_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [9]$$

41 Efficiency η_d is usually very high, about 90 per cent (we shall see it later, discussing the Schneider gear); η_h, as we know it from tests with full-power hydraulic transmissions, is low, at low speeds about 65 per cent; as an average, 70 per cent; efficiency η would thus be

$$\eta = 0.7 + 0.2\frac{n_2}{n_1}$$

It is, therefore, always higher than the full-power transmission efficiency (70 per cent) attaining its maximum at $n_2 = n_1$, namely 90 per cent. Formula [9] should not, however, be used for speeds $n_2 > n_1$, as this case has not yet been investigated.

Conditions for High Driven-Shaft Speeds

42 Let us return to Fig. 4 and consider the moment when at point K_1 the actual eccentricity of S (Fig. 6) is zero and the speeds of the primary and secondary shafts are equal. If, for some reason, the speed of the secondary shaft should go up, reaching a larger value of n_3, we must, at the same time, move the eccentric in the opposite direction, if we want to avoid racing of the oil engine. Motor M on the secondary shaft will then work as a pump and will deliver oil back into the pump of the primary shaft,

actually in the space between pistons p and cylinder block C. The pressure in this space will restrain the primary shaft in its tendency to follow the secondary shaft, and will cause it to continue to rotate at constant speed. Using the same denotations as above we may write

$$T_3 = T_1 - pr_3 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2']$$

and

$$n_3 = \frac{T_1 n_1}{T_1 - pr_3} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4']$$

which follow the law of the hyperbola shown on Fig. 4, as well as

$$N = T_1 n_3 - pr_3 n_3 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3']$$

and

$$N = N_d - N_h \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3'']$$

as shown on Fig. 5, point Q_3. If we agree to consider r and N_h positive, or negative, in accordance with the position of the resultant eccentricity one way, or another, and dependent upon the work of M like a motor, or a pump, formulas [2], [3], and [4] hold good for both cases.

43 It is different, however, with the formula for efficiency. As formerly we shall have

$$N_3 = N - N' - N'' = (N_d - N'') + (N_h - N') \quad . \; . \; . \quad [7']$$

The whole power is transmitted now directly by pressure (N_d) and part of it is absorbed by working the motor as a pump (N_h being negative). We have, therefore, using the same notation for efficiencies as above,

$$\frac{N_d - N''}{N_d} = 1 - \frac{N''}{N_d} = \eta_d$$

and, N_h being negative,

$$\frac{-N_h - N'}{-N_h} = 1 + \frac{N'}{N_h} = \eta_h \quad . \; . \; . \; . \; . \; . \quad [10]$$

Formula [7′] can be transformed into

$$N_3 = \frac{N_d - N''}{N_d} \cdot N_d + \frac{N_h - N'}{N_h} \cdot N_h$$

but

$$\frac{N_d}{N} = \frac{n_3}{n_1}; \qquad \frac{N_h}{N} = 1 - \frac{n_3}{n_1}$$

and in accordance with [10],

$$\frac{N_h - N'}{N_h} = 1 - \frac{N'}{N_h} = 2 - \eta_h,$$

therefore

$$\eta = \eta_d \frac{n_3}{n_1} + (2 - \eta_h) \cdot \left(1 - \frac{n_3}{n_1}\right)$$

or $$\eta = 2-\eta_h - \frac{n_3}{n_1}(2-\eta_h-\eta_d) \quad \ldots\ldots \quad [11]$$

If we assume $n_3 = n_1$ this formula renders

$$\eta = \eta_d,$$

same as formula [9] for $n_2 = n_1$. This is quite natural.

44 We have thus two formulas for efficiency η:

for $$0 < n_2 < n_1, \qquad \eta = \eta_h + (\eta_d - \eta_h)\frac{n_2}{n_1} \quad \ldots\ldots \quad [9]$$

for $$n_3 > n_1, \qquad \eta = 2-\eta_h-(2-\eta_h-\eta_d)\frac{n_3}{n_1} \quad \ldots \quad [11]$$

Fig. 7 represents them graphically, and Fig. 8 shows efficiency

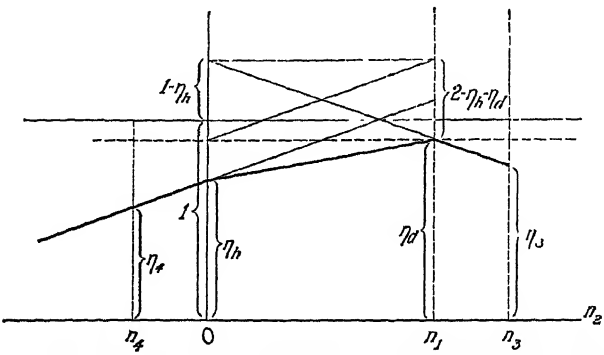


FIG. 7 EFFICIENCY CURVE, DIFFERENTIAL HYDRAULIC TRANSMISSION

curves obtained from actual tests with the Schneider gear which is a typical differential hydraulic transmission (see below). The difference in slope of the two branches of the efficiency curve, for speeds below and above the oil-engine speed, can be easily seen.

45 The efficiency drops very rapidly on the second branch of the curve. Theoretically it will become zero at

$$n_3 = \frac{2-\eta_h}{2-\eta_h-\eta_d}\,n_1.$$

If we use the numerical values above given for η_h and η_d (0.70 and 0.90), this critical speed becomes equal to 3.25 times n_1. Differential elastic-fluid transmissions are never designed for speeds above $1.25n_1$, at which speed the transmission can still render an efficiency of 75 to 80 per cent. Higher speeds should not be attempted with transmissions of this type.

46 The efficiency curves representing our formulas are straight lines, as we assumed that η_h and η_d are independent of speed. Actually they depend upon the relative piston speed, and, there-

fore, their real form is as shown on Fig. 8. The critical speed value ($3.25n_1$), at which $\eta = 0$, has no practical value and was used only as an illustration.

Reversal

47 Let us again return to Fig. 6. With r_3 negative and approaching r_1, we obtain ever increasing speeds and, theoretically speaking, when r_3 becomes in size equal to r_1, the speed becomes infinity. Let us now investigate what will happen if r_3, being still kept negative, is made larger than r_1. Imagine the starting of the transmission with the eccentric beyond its extreme negative position. Gear G and motor M, with cylinder block C, are at standstill. Rotor R will then act as a pump and deliver fluid under pressure

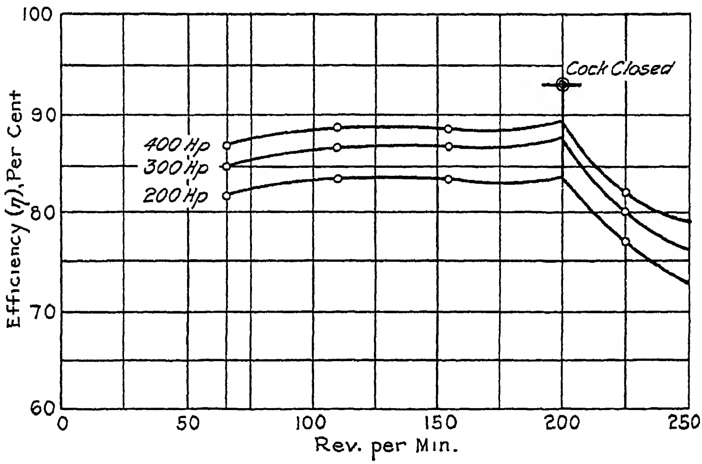


FIG. 8 ACTUAL EFFICIENCIES, DIFFERENTIAL HYDRAULIC TRANSMISSION

to motor M. Similarly to what was taking place when the eccentric was in its extreme positive position, the motor, at the moment when the pressure will be sufficiently high to overcome the resistance of starting, will begin to turn, but in the opposite direction. The rotation of C will then be opposite to R and the output of the pump will go up until a state of stabilization is established. This will take place when motor M is in a position to absorb the amount of oil delivered by the pump. The speed of the secondary shaft n_4 will be such that (r_4 being the negative eccentricity)

$$(n_1+n_4)r_1 = n_4r_4$$

or

$$-r_4 = r_1\left(\frac{n_1}{-n_4} - 1\right)$$

which is identical with formula [6], the difference being only in the signs of r_4 and n_4 which are negative.

48 The torque on the secondary shaft will be a resultant of two: first of T_1, the torque of the oil engine acting in the direction of its rotation, and second pr_4, from the hydraulic motor M acting in the opposite direction. But since r_4 is negative, we ought to write

$$T_4 = T_1 + pr_4$$

which is identical with formula [2]. We thus obtain the other

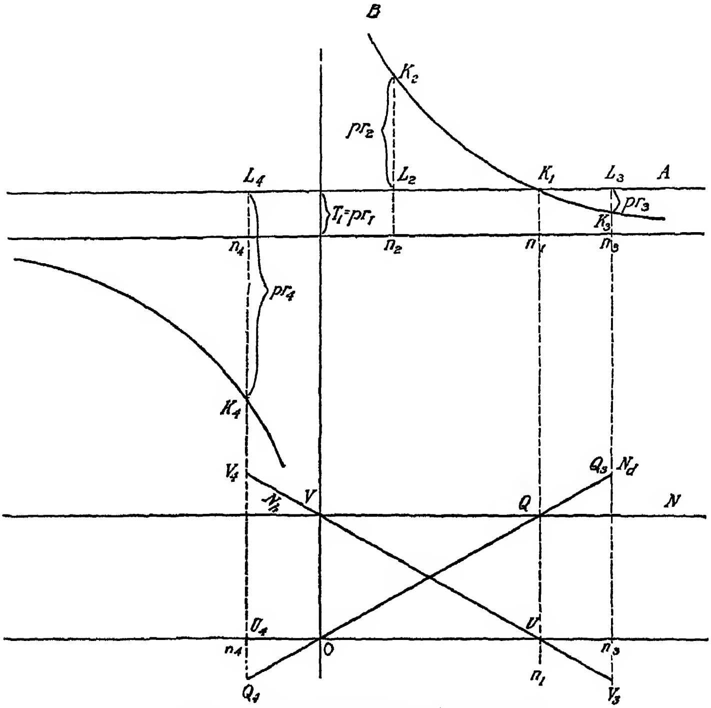


FIG. 9 REVERSED-MOTOR CONDITIONS

branch of the hyperbola, as can be seen on Fig. 9. All other formulas also hold good. For power, for instance, we have (Fig. 9)

$$OV = V_4U_4 - U_4Q_4,$$

or

$$N = N_h - (-N_d) = N_h + N_d,$$

as before.

49 But as regards efficiency, this must be again investigated separately, because losses of power are always negative. Reasoning in the same way as we did in the case $n_3 > n_1$, we obtain

$$\eta_4 = \eta_h - (2 - \eta_h - \eta_d)\frac{-n_4}{n_1} \quad \ldots\ldots \quad [12]$$

$-n_4$ being positive. At $n_4 = 0$ we obtain $\eta_4 = \eta_h$, as it should be. On Fig. 7 the efficiency for negative speeds is shown as a straight line. It is rapidly falling with the increase of the value of speed. The inclination of the line is the same as of line GH, which represents the efficiency line for speeds over n_1. This can be inferred from the respective formulas of the two lines. Theoretically speaking, the efficiency becomes zero at speed

$$n_4 = \frac{\eta_h}{2-\eta_h-\eta_d} \cdot n_1$$

If the former values should be applied ($\eta_h = 0.7$; $\eta_d = 0.9$), the efficiency is zero at $n_4 = 1.75n_1$. We thus see that the reverse speed

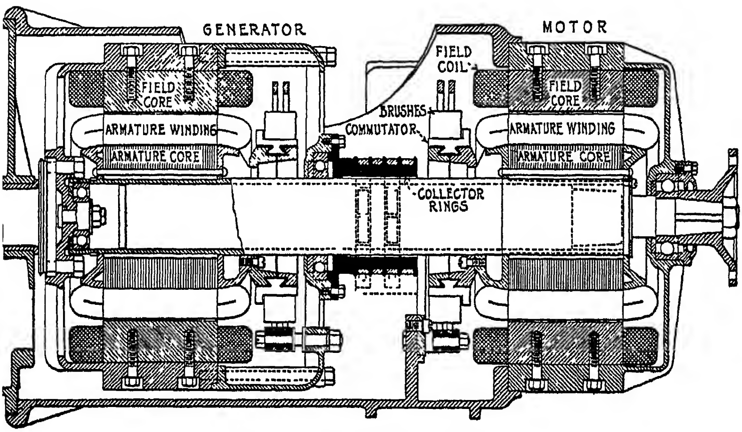


FIG. 10 ENTZ TRANSMISSION

efficiency is even worse than the efficiency at high speeds ahead. It is, therefore, not advisable to use a transmission of this type for speeds much below zero and above n_1.

50 The differential elastic-fluid transmission can be materialized with any kind of fluid, and, therefore, there can be as many types of the differential transmission as there are of the full-power transmission. So far, there have been developed electric, hydraulic, and pneumatic differential transmissions.

Differential Electric Transmission

51 *Entz Transmission.* About twenty years ago the Entz electric drive was brought forth as a speed-controlling means for self-propelled vehicles, more particularly for automobiles. It has all the characteristics of a differential transmission. A cut through an automobile Entz transmission is shown on Fig. 10. Two dynamos serve as primary and secondary units, or, in ordinary cases, as generator (on the left) and motor (on the right). The

field frame of the generator (first member) is bolted to the driving shaft, the armatures of the generator (second member) and motor (third member) are, in accordance with our scheme of differential gears, fastened to the driven shaft, and the field frame of the motor (fourth member) is stationary. At starting, when the driven shaft is also stationary, the primary dynamo (on the left) acts as a full-speed generator, and the current thus generated drives the secondary dynamo (the motor) through a suitable electric control. The transmission acts at starting as full-power electric transmission. But as the speed of the driven shaft goes up, less current is being generated and less energy is being transmitted electrically, the balance of the power being transferred from the driving shaft through the field coil directly to the driven shaft, so to speak, by a magnetic drag. At full speed, when the driven shaft has attained the speed of the driving shaft, the motor receives no more current, and the whole power is transmitted wholly magnetically, the generator acting as a very flexible clutch.

52 Simultaneously with the change in speed the motor field excitation must be gradually decreased. At full speed the generator's brushes are short-circuited. The motor, not being used at high speed, may be converted into a generator to charge the storage battery. The motor may be employed also as an electric brake. The starting of the prime mover (oil or gasoline engine) is accomplished by operating the generator as a motor from the storage battery.[1]

53 The efficiency of the transmission is the combined efficiency of the electric transmission and of the clutch generator. Formulas **[9]** and **[11]** hold good if η_h is the total efficiency of the electric transmission (generator and motor) and η_d that of the magnetic dragging effect.

54 The Entz transmission was applied to some extent in automobiles, and while the performance of the transmission was fairly good, it met with very little success. The author does not know the reasons for it, but, as far as he can see, there was very little need for such a transmission for automobiles. The efficiency, while slightly higher than that of a full-power transmission, was still less than the efficiency of the ordinary clutch and gear box. In addition, the magnetic drag is, in no way, so positive as an hydraulic lock, and considerable slippage must have taken place at high speeds. This probably explains the fact that the oil engine had to be run at a higher speed than it would have been in a geared car moving at the same speed on the road.

[1] *Electrical World,* vol. 63, no. 5, January 31, 1914, p. 281, and vol. 67, no. 2, January 8, 1916, p. 110. See also The Columbia Electrical Transmission, by V. W. Page, *New England Automobile Journal,* October 26, 1907.

55 The transmission was never tried for railcars or locomotives, although it would seem that, at least for railcars, and possibly for buses, the Entz transmission would be worth trying.

56 *Thomas Transmission.* As we have seen before, a differential transmission consists of four members, of which the first (driving) member runs at the prime mover's speed, the second and third members, rigidly connected together, are driven at a variable speed, and the fourth member is stationary. Thus it is a mechanism with two degrees of freedom. When the transmission is in opera-

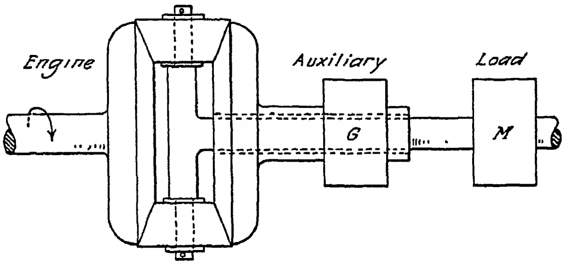


FIG. 11 PRINCIPLE OF THOMAS TRANSMISSION

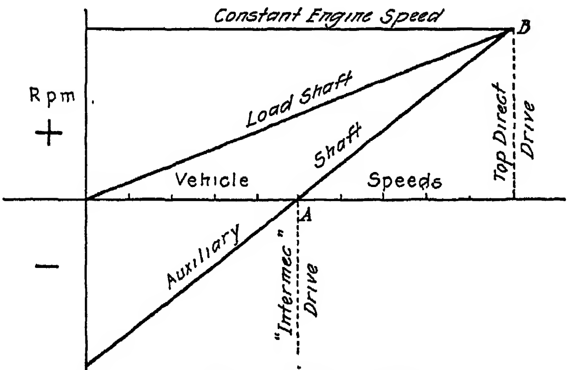


FIG. 12 SPEED DIAGRAM FOR FIG. 11

tion, the extra degree of freedom is gradually being absorbed by the resistance of the fluid until finally at top speed it is absorbed entirely.

57 The required degree of freedom can also be obtained in another way, namely, we can somewhere, between the *A* and the *B* ends of the transmission, interpose a planetary or a differential gear which, as it is known, has two degrees of freedom. Further, we can absorb the extra degree of freedom by an electric or hydraulic slippage at the *A* end which can be utilized as a booster on the *B* end. The two machines will have, however, both stators immovable. A good example of an electric drive based on this principle is the Thomas transmission.

58 A diagram of the transmission is given on Fig. 11. The engine drives the bevel wheel on the left; the arm with the two bevel pinions is connected to the driven shaft marked "load"; the right bevel wheel is attached to a hollow shaft marked "auxiliary." The speeds of the three shafts are interrelated as shown on Fig. 12. Imagine further a generator *G* on the auxiliary shaft and a motor *M* on the driven shaft (Fig. 11). At starting the resistance will keep the driven shaft at standstill, and if the engine is running forward, the generator will rotate in the opposite direction with the same speed. If we utilize the electric current for driving the motor on the load shaft we can by using proper electric control exert the necessary tractive effort and overcome the resistance. As the forward speed of the load shaft increases, the backward speed of

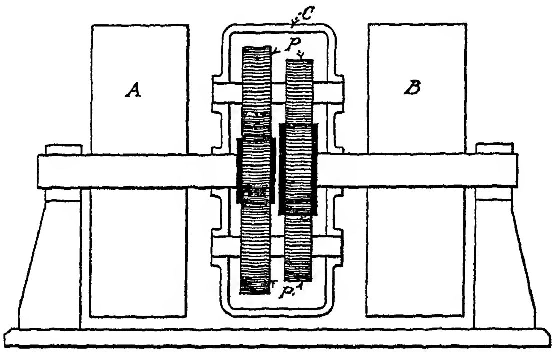


Fig. 13 Diagram of Thomas Transmission System

the auxiliary shaft decreases until the latter vanishes altogether (point *A* on Fig. 12). The load shaft is then revolving with half speed of the engine. No current is generated in *G*, and the total power of the oil engine is transmitted wholly mechanically. The torque is double that of the oil engine, disregarding, of course, the friction losses in the gear. Mr. Thomas, the inventor, calls the transmitting of power at this speed the "intermec drive." Up to that speed the power was being transmitted both mechanically and electrically.

59 If the speed of the load shaft goes up beyond the intermec speed, the auxiliary shaft begins to rotate in the forward direction and by a simple alteration of the electrical connection, *M* is converted into a generator which delivers current into *G* which becomes a motor. The speed of the auxiliary shaft goes up; so does the speed of the load shaft, although its increase is slower than that of the auxiliary shaft, until the two speeds reach the speed of the oil engine (point *B*). Thomas calls the transmitting of power at this speed the "top direct drive." Then a mechanical coupling by means of a clutch of any two of the three shafts locks the gear which continues to rotate as a unit with a direct transmission of

power from the oil engine to the wheels. The electric drive is not necessary any more and can be cut out entirely. The losses are only those of bearing friction and air resistance. Thus the efficiency is very high.

60 The bevel gear form of the Thomas differential transmission is not an essential feature of the transmission. This form has been used as an illustration in order to follow the speed relations. Actually, the Thomas transmission is built with a planetary gear, as shown on Fig. 13. Casing *C* takes place of the arm and auxiliary shaft of the differential gear; wheels *P* are the planetary wheels, the two sun wheels being fastened to the driving and to the driven shafts. The advantages of such an arrangement lie in

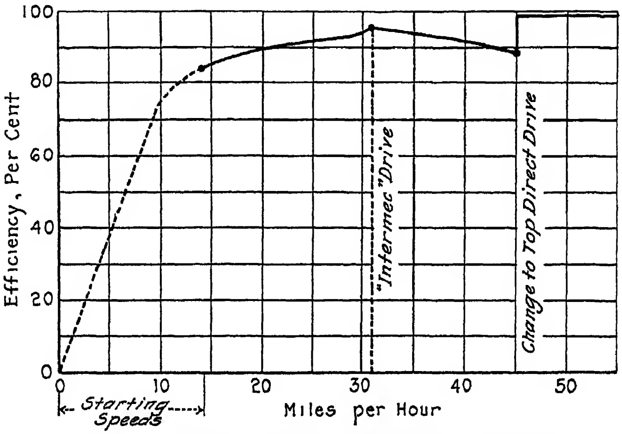


Fig. 14 Efficiency Curve of Thomas Transmission

the omission of the bevel wheels and the fact that the gear ratio is not necessarily 1 to 2. The intermec speed may be made, therefore, less or more than one-half of the top speed.

61 One difference between the Thomas and the Entz transmission is that in the former the magnetic drag of the latter is eliminated and a positive transmission through toothed wheels takes the place of the uncertain magnetic drag. The other difference is that the Thomas transmission has two speeds at which the power is transmitted purely mechanically, those of the intermec drive and of the top direct drive, whereas the Entz gear transmits the power this way only at the top speed, and then it does it, so to speak, pseudo-mechanically, by magnetic drag.

62 These two essential differences are reflected in the efficiency curves. Fig. 14 gives the calculated efficiency of the Thomas transmission. If compared with Fig. 8, showing the actual efficiency of the Schneider gear, the difference in the characters of the efficiency curves is evident. No attempt, however, should be made to com-

pare the efficiency figures of the two gears, as the Thomas transmission figures are calculated; the others are actual.

63 The Thomas transmission was applied for the first time some thirteen or fourteen years ago to a 36-hp. Leyland truck in England. The car was tested and the performance found to be satisfactory. Later a small Delahaye car weighing 3800 lb. was equipped with a Thomas transmission. Mr. Thomas reports that on a run from London to Edinburgh and back the petrol consumption was an imperial gallon per 36 miles of run.[1] Several more cars and buses were built with the Thomas transmission, and in 1914 a 160-hp. railcar weighing 26.3 short tons was built for South Africa. An official test made both ways between Pretoria and Pienarr's River on a distance of 84.6 miles (total run 169.2 miles) showed an average fuel consumption of an imperial gallon per 8.65 miles, or 228 ton-miles.[2] This corresponds to 3.5 lb. of petrol per 100 ton-miles. In 1915 another railcar with an eight-cylinder 200-hp. V-type engine and Thomas transmission was built for New Zealand, but no information as to the performance of this car is available. Nor is there any published information to be found as to the further progress made by the Thomas transmission, although designs of 1000 to 1200-hp. locomotives have been worked out.[3]

Differential Hydraulic Transmissions

64 Similarly to the full-power transmission, the differential hydraulic transmission also permits a great variety in design on account of the great number of the different types of hydraulic pumps and motors in existence. In addition, two different classes are possible, without, or with, the planetary gear, corresponding to either the Thomas or the Entz transmission. This explains possibly the fact that for the last three decades the domain of differential hydraulic transmissions, if judged by the number of patents issued, has been one of the most prolific in mechanical engineering. The majority of the transmissions were intended for automobiles but several of them were also proposed for locomotive drive.

65 *Rayburn Gear.* The Rayburn gear is the hydraulical analogy of the Entz transmission. Oil-engine shaft 14 (Fig. 15)[4] is directly attached to the first member, consisting of casing 10 with heads 11 and 12. The second member consists of a rotor 22 with blades 23. The two members constitute together a pump of the vane type. The third member consists of a similar rotor 22*a* with blades 46. Rotor 22*a* is keyed to shaft 9 to which rotor 22 is also

[1] The Thomas Transmission, by Hedley J. Thomson, p. 17; paper read before the New Zealand Society of Civil Engineers at Dunedin on March 11, 1915.
[2] *Ibid.*, p. 22.
[3] *Ibid.*, pp. 11 and 25.
[4] U. S. Patent Specification No. 1,297,733, March 18, 1919, Fig. 3.

fastened. Shaft 9 is journaled in a stationary head 2 and in heads 11 and 12. The fourth member is stationary and forms a part of casing 1. The third and fourth members represent a motor, also of the vane type, which takes the place of the booster unit of the Entz transmission. Vanes 23 and 46 can be moved toward and from the axis of shaft 9; this movement is controlled from the outside.

66 As regards the operation of the transmission one can readily see that at starting the two rotors are at standstill. The oil engine drives the outside casing of the pump at full speed, and with the blades accordingly moved, pumping action begins in the primary part (pump on the right). Fluid under pressure is passing through

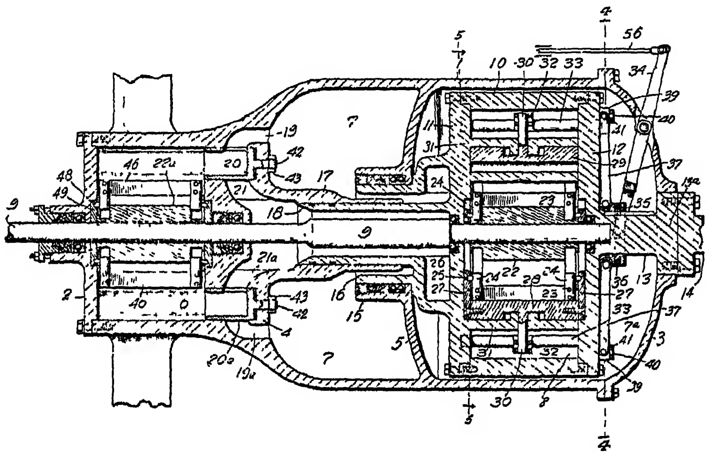


FIG. 15 RAYBURN TRANSMISSION

suitable channels and valves into the secondary part (booster unit) and tends to rotate rotor 22*a*, and with it shaft 9 and rotor 22, in the direction of rotation of casing 10. As the speed of rotor 22 goes up, the pumping action of the pump decreases, and at a certain speed it disappears altogether. This latter event corresponds to the top speed, when the hydraulic lock takes place. At that speed the total power of the oil engine is transmitted to the driven shaft 9 through the hydraulic pressure in the primary part.

67 In order to keep the pressures in conformity with the torque of the engine at all speeds the projection of the blades 23 and 46 is accordingly controlled by a handle.

68 *Hydraulic Analogy of Thomas Transmission.* A hydraulic transmission analogical to the Thomas electric transmission has been lately developed by an English firm. A planetary gear is directly connected to a gear pump, while a motor of the Williams-Janney type with a swash plate is connected to the driven shaft. The gear has not yet passed the experimental stage and, the author understands, no information is available at present.

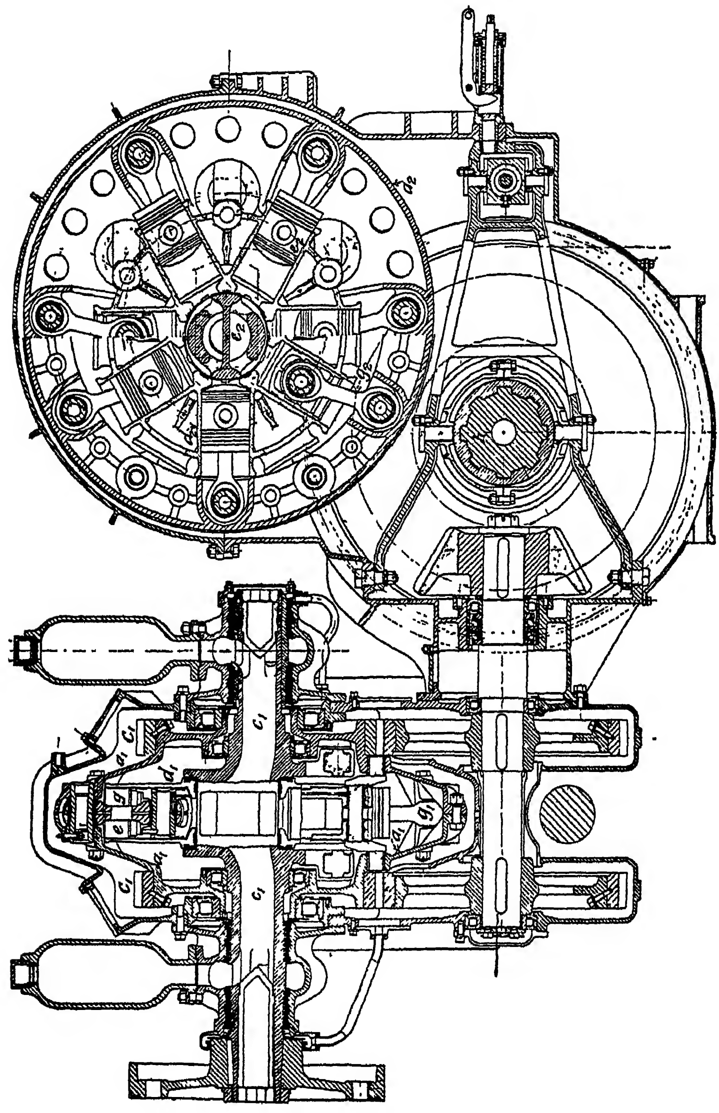


Fig. 16 Section of Schneider Transmission

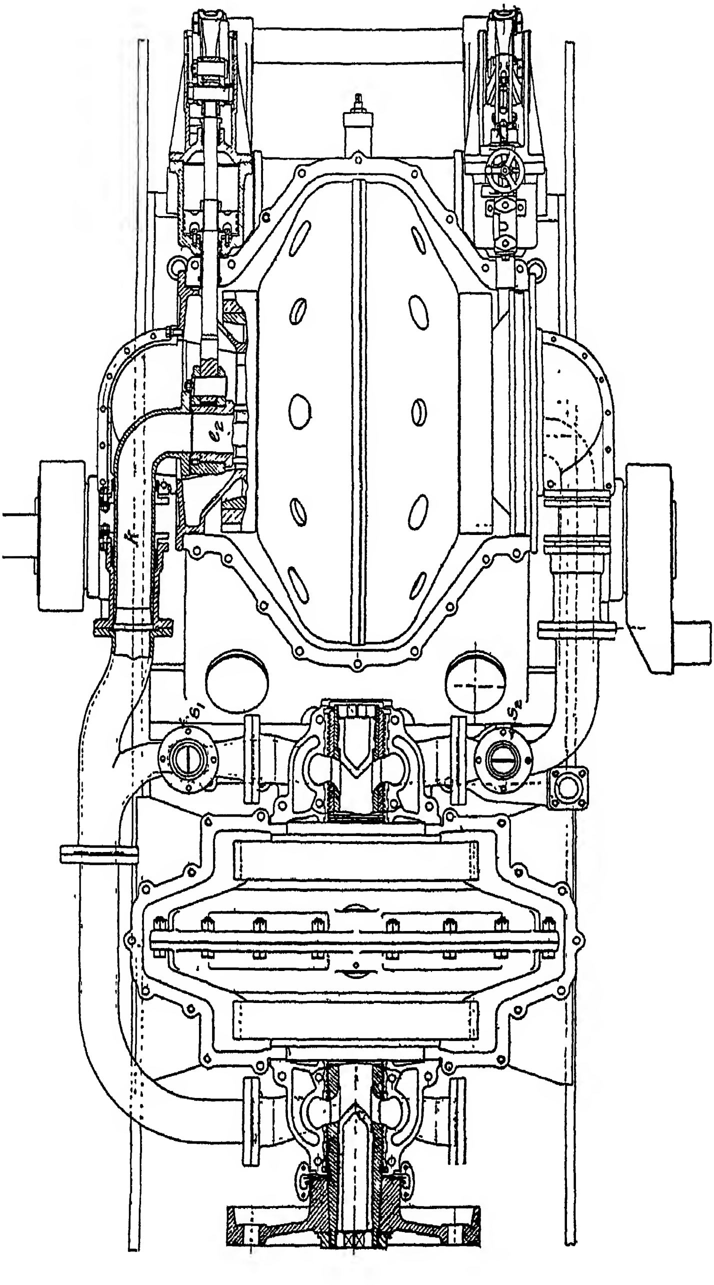


FIG. 17 PLAN VIEW OF SCHNEIDER TRANSMISSION

69 *Schneider Transmission.* A very interesting modification of the Hall principle is represented in the Schneider transmission. It also consists of a primary and secondary part, the difference being that the first and fourth members are cylinder blocks instead of pistons, whereas the second and third members are rotors with pistons fitting into the cylinders. Further, the second and third members are not made as one unit, but are rigidly connected by means of gears with a driven shaft interposed between them. Fig. 16 shows a vertical section, and Fig. 17 a top view of a 500-hp. Schneider gear built by the Swiss Locomotive Works in Winterthur, Switzerland. The oil engine drives a hollow partitioned shaft c_1 with a permanent crank operating a six-cylinder block d_1. This constitutes the first member, the constant-speed member of the primary part. The variable-speed member (second member) of the primary part consists of a rotor a_1, rods g_1 and piston e_1. The rotor has two spur-wheel rims C_1 which mesh with two spur wheels set on an intermediate shaft connected with driven shaft h by means of bevel wheels. Shaft h is in a similar way geared to rotor a_2 which, with corresponding rods and pistons, constitutes the variable speed member of the secondary part (third member). The design of the latter is similar to that of the second member, the only difference being the double number of pistons, namely, two rows of six pistons in each. The fourth (stationary) member consists of a corresponding twelve-cylinder block d_2 which can rotate around a hollow stationary shaft c_2 with variable eccentricity in relation to rotor a_2. The two shafts also perform the function of valves establishing the proper connection between suction and delivery passages. The variation of eccentricity of shaft c_2 is effected by a power reverse which can be seen on Fig. 17. Two pistons of the power reverse act directly on shaft c_2, which is guided by pipes k sliding inside stationary suction and delivery pipes.

70 From what was said before, it is easy to follow the process which takes place in a Schneider gear. Before starting, when the oil engine is idling, valve s_2 is closed and s_1 is open; shaft c_2 is placed in its extreme position, driven shaft h being at standstill, and both rotors a_1 and a_2 with pertaining bevel and spur wheels are stationary. The first member rotates at the speed of the prime mover, but no pressure is generated, as the oil circulates freely through valve s_1 from the delivery into the discharge passages of the primary part. If we gradually close valve s_1 and open valve s_2, oil is then pressed into the secondary part where it acts against the resistance of starting; the pressure thus goes up. If the eccentricity of the stationary shaft c_2 was properly chosen, the maximum torque will be sufficient to overcome the resistance torque, and rotor a_2 with shaft h and rotor a_1 will start turning, the latter in the direction of the prime mover. The relative speed

between the first and second member drops, and the amount of oil pumped by the primary part decreases, but the pressure remains constant, if the eccentricity of the secondary part is correspondingly reduced. The resulting torque varies inversely to the speed of shaft *h*, until it balances the resisting torque. The running of the locomotive will then continue at a speed corresponding to these balanced conditions.

71 When the resisting torque drops to a value in the neighborhood of the oil-engine torque, shaft c_2 is placed concentrically with rotor a_2 of the secondary part. This establishes the so-called hydraulic-lock condition at which the total output of the oil engine is transmitted directly to shaft *h* through the oil in the primary part and the gear. The secondary part delivers no power, and valve s_2 can be closed.

72 Further increase in speed of the locomotive can be obtained either by speeding up the engine itself, or by placing shaft c_2 in a negative eccentric position, as was explained in relation to the Hall transmission.

73 A Schneider transmission was built by the Swiss Locomotive Works for a 500-hp. Diesel locomotive. On May 13 and 14, 1924, the transmission was officially tested by Prof. P. Ostertag of the Winterthur Polytechnic Institute and a full report was published in March, 1925.[1] The oil engine ran at a speed of 350 r.p.m.; the driven shaft was tried at 5 different speeds: 64, 100, 150, 200, and 220 r.p.m., the 200-r.p.m. speed corresponding to the direct drive. The transmission was tested up to 400 hp. on shaft *h*, further increase not being possible on account of the limitation of the two prony brakes applied during the test. Fig. 8 gives the efficiency curves for speeds between 64 and 250 r.p.m. of the driven shaft. For 400 hp. and speeds below that of direct drive, or top speed, to use Mr. Thomas' designation, the efficiency fluctuates between 87 and 89 per cent. For smaller powers (200 and 300 b. hp.) it drops to 82 per cent, and for powers below 140 b. hp. even figures of 72 per cent were recorded. The efficiency dropped abruptly with the increase of speed beyond the top speed, this in accordance with our formulas and charts formerly given. When valve s_2 was closed at top speed the efficiency went up to 93 per cent.

74 As regards the performance of the transmission as a mechanism, it was quite satisfactory from the start, requiring very few changes or improvements. Some leakage of oil through the working parts became noticeable and a small pump was applied in order to return the oil to the suction passage of the primary part. The power absorbed by the pump did not exceed 6.5 hp., and this

[1] *Schweizerische Bauzeitung,* vol. 85, nos. 10 and 12, pp. 123-127 and 154-155. See also *Zeitschrift des Vereines Deutscher Ingenieure,* vol. 69, no. 16, April 18, 1925, pp. 499-504 and no. 18, May 2, 1925, pp. 595-600.

the Russian railways in competition with the 1000-b.hp. oil-electric locomotive formerly described. The locomotive is of the 4-10-2 type, has a reversible 1000-b.hp. Diesel engine which acts on a jackshaft by means of three gears always in mesh with three friction clutches operated magnetically. Only three speeds can be obtained at the normal speed of the Diesel engine — three forward and three backward, the engine being reversible. Intermediate speeds can be obtained within certain ranges close to the three principal speeds by changing the number of revolutions of the Diesel engine itself. In changing from one speed to another, practically one-half of the total output of the oil engine during the whole period of acceleration will be absorbed by the destruction of metal between the working parts of the respective clutches. However, the builders claim that the wear of the clutches is negligible in view of the shortness of the acceleration period. The object of the magnetical operation is to ensure a smooth change of speed, but whether this can be obtained by friction clutches, even magnetically operated, remains to be seen.

Direct Drive

85 In 1913, the Swiss firm, Sulzer Brothers, of Winterthur, built jointly with the Borsig Locomotive Works of Berlin, Germany, a 4-4-4 passenger 1000-b. hp. Diesel locomotive with a direct connection between the Diesel-engine brake shaft and the driving rods by means of side rods. Dr. Diesel himself was the designer of the locomotive, and like Watt, who did not recognize the flexibility and adaptability of the steam engine to locomotive drive, he did not seem to be fully cognizant of the inflexibility of the Diesel engine and its poor adaptability to locomotives. He tried to solve the problem by using compressed air for starting the locomotive with the train. The engine had to be run with compressed air until the speed of the locomotive reached the ignition speed, which, for this particular locomotive was 6.6 miles per hour. In order to supply enough starting and accelerating power, a separate compressed-air outfit had to be provided. The locomotive was, therefore, equipped with two Diesel engines — a main engine and an auxiliary engine. Starting with air, however, did not give good results.

86 The next example of direct connection between the Diesel engine and the driving wheels can be found in the Leroux railcar built in 1913 by the Fives-Lille Co. in France. A 150-b.hp. two-cycle, two-cylinder, vertical Diesel engine of the opposed-piston type, with cranks at 180 deg., was mounted on the front truck. Two scavenging pumps were driven from two cranks set on the engine brake shaft at 90 deg. to the oil-engine cranks. The scavenge air-pump cylinders were used for starting the oil engine by compressed air which was provided by a small separately driven air compressor

and stored in air bottles. Thus the oil-engine cylinders were relieved of the cooling effect of expanding air and the air ignition was not affected as it had been in the Diesel-Sulzer locomotive. In addition, the starting air was preheated by exhaust gases of the oil engine, which resulted in higher efficiency of the starting arrangement. The car was destroyed during the war.

87 Two direct-driven locomotives of considerable size, embodying the Still principles of starting, are at present under construction; one is being built in Leeds, England, at the plant of Kitson & Co., and another in France at the works of Schneider and Co. in Creusot. The Still engine is a combination of an internal-combustion engine with a steam engine, steam being generated in a separate boiler by waste heat of the oil engine, and expanded in the oil-engine cylinder on the under side of the piston. The boiler is placed in communication with the water jackets of the engine cylinders, and thus both the cylinder-jacket heat and the exhaust-gases heat are utilized for steam generation in the Still engine.

88 The Still engine, in addition to its low fuel consumption, offers a very good combination for starting, steam being an ideal fluid for this purpose. Of course, if the engine has not been run before starting, it becomes necessary to heat the boiler with a specially provided oil burner in order to generate steam for starting. However, in so doing, the cylinders are heated, ignition facilitated, and starting on oil is rendered much easier.

DISCUSSION

C. A. Norman.[1] The paper is an extremely timely one. We have now in this country arrived at a thorough realization of the possibilities of the Diesel engine, both from the point of view of economy and from that of dependability. In the form of the Diesel-electric drive we have also shown our willingness to help in the work of adapting the Diesel engine to locomotive drive.

However, in reading the author's paper one cannot help but be impressed by the far greater versatility of development along this line that has taken place in Europe. It is surely not desirable that we, at this early stage of the development, should confine our attention entirely to the electric drive. It is admitted that this drive is heavy, costly, and yet not startlingly efficient. Mechanical transmissions — meaning thereby all non-electric transmissions — not only proposed, but actually tried out in Europe, show that there are many other ways of solving the problem. Some of these give greater simplicity, some give greater efficiency, and almost all give lower weight and less first cost than the Diesel-electric drive.

It is certainly not for want of engineering ability that we refuse to go into these things in this country. Many ideas along this

[1] Professor of Machine Design, Ohio State University, Columbus, Ohio. Mem. A.S.M.E.

line must have come to American engineers, in addition to the author's suggestion. The difficulty seems to be in encouraging commercial concerns to spend money in experimentation. In this respect the writer believes that we have something to learn from the Europeans. It is certain that no new invention arose perfect on the first day, and very often astonishing efficiency and dependability have been attained by detail perfection of machinery which at first looked unpromising. The Diesel engine itself is an outstanding example in this line.

It is sincerely to be hoped that American industrial enterprises will furnish the engineering talent of the country sufficient support so that we may not have to look to Europe with envy for what engineers over there are allowed to try out and develop. To refuse this support is a serious waste of one of the country's most valuable mental assets.

Elmer A. Sperry.[1] This paper gives a very complete résumé of all the attempts to solve the difficult problem of producing a transmission involving all of the requirements of developing variable, including very high, torques from a prime mover having the universally recognized characteristic of developing measurably constant torque. The difficulty with all combustion engines is that when they are overloaded they stop, so that some method must be developed for preventing overloading and at the same time producing the heaviest torque at low speed, thus breaking away the load from its high journal friction coefficient of quiescence and also energizing the mass in the process of acceleration.

The paper shows the very great diversity of solutions that have been brought forward by the best minds in all industrial countries. The solution that has been most successful and has been adopted to the greatest extent in practice is of course the electrical. This has several definite disadvantages:

1 It is heavy

2 It is expensive

3 Overloading is disastrous, especially while developing high torques at low speed

4 It is sensitive, even to the point of being rendered inoperative by over-application of moisture or water

5 Its efficiency, while looked upon as commercially good, is in point of fact too low at both ends of the cycle — at low speeds and high torque and also at high speed — remembering that these losses are double; those of the generator plus the losses of the motor.

This is referred to in the last part of Par. 27, page 273, and we cannot but emphasize the statement that it is a pity to use a transmission all the time, especially after its real function of accelerating the load has been discharged. When this acceleration

[1] President, Sperry Gyroscope Co., Brooklyn, N. Y. Mem. A.S.M.E.

has been completed, practically 100 per cent of its usefulness has been realized. Then a straight-through connection should be substituted, where the losses are practically all eliminated, and the flexibility of the prime mover should be employed from that point on. This brings us to the realization that with the best service in locomotives we are bound to come to the same recognition of the value of the great feature in the automobile, namely, the prime mover of great flexibility. Another consideration is that the power developed as mechanical energy and finally utilized at the drawbar as pure mechanical energy should never be allowed to depart from the realm of mechanics. In other words, it should not be necessary to suffer all the losses consequent to transferring mechanical energy into electricity and then, on the spot, transferring it back from electrical to mechanical energy, entailing a second loss in addition to the first. Distant transmission of energy, of course, forms no part of this particular problem.

Hydraulics have been looked to as a possible means of confining the transmission to the realm of mechanics, but up to the present time this method has involved so much detail and so many expensive moving parts with their consequent wear and deterioration, at the same time operating at very low efficiencies, that while sufficient flexibility and trading of speed for torque are obtainable, the system has never reached that point where it would justify extensive and large-scale adoption. No better illustration could be asked than the ingenious Schneider transmission shown in the author's paper and discussed in Class B under the head of Differential Transmissions.

It is believed by the writer that this field of hydraulics has not yet been exhausted and whereas it has been proposed, as in the Föttinger transmission, to utilize turbo action in place of a multiplicity of plungers, pistons, cylinders, valves, etc., yet the Föttinger solution is looked upon as having characteristics not at all suiting the requirements, and as being too heavy, cumbersome, and lacking in efficiency.

For some time past a group of engineers have been engaged upon a simplification of the turbo transmission with a view to confining its action entirely to its proper sphere of usefulness, namely, that of acceleration, and have hit upon an extremely effective method of its automatic substitution for straight-through drive, eliminating practically all losses at all of the higher speeds. The interesting point has been observed in practical trials of this transmission that the change-over, which is automatically effected, may be brought in at any point desired, depending upon the load, grade, or desire of the engine driver; i.e., if his train is light or if the grade has changed he can instantly revert to straight-through connection by simple manipulation of his engine speed. Further interest centers in this new turbo method in that whenever the straight-through

drive comes in, the turbo automatically ceases all functions and is completely eliminated from the system, always standing ready to be instantly called into action as occasion arises.

This new turbo drive falls under Class B of the paper, is strictly differential, and has been found to operate at high efficiency. Since it may be interesting to note just how it compares with the electric drive, the curves of Fig. 18 have been prepared. This is an example of a 750-hp. transmission. Curve *A* is taken from a paper

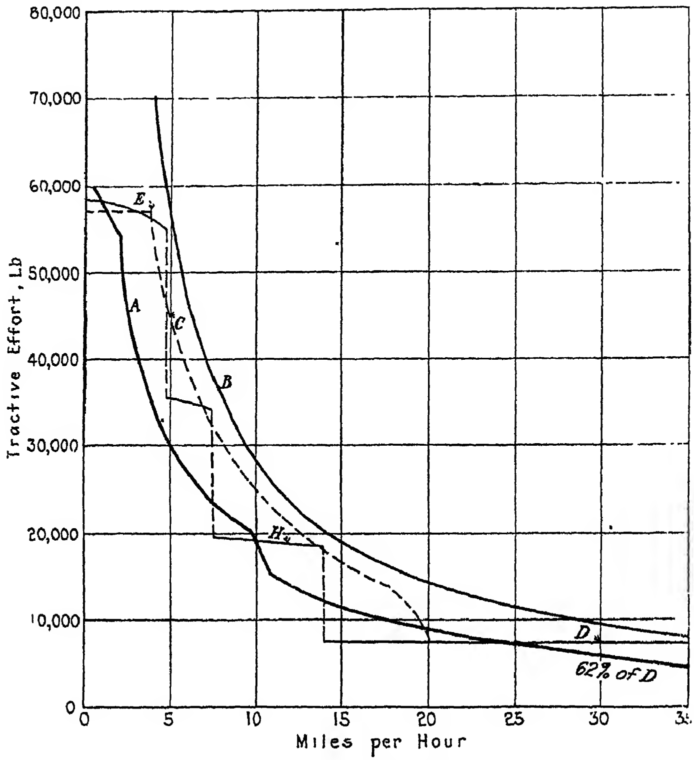


FIG. 18 TRACTIVE-EFFORT CURVE OF THE TURBO ACCELERATOR GEARED LOCOMOTIVE (750 HP.; 128 TONS)

by Mr. Katte, electrical engineer of the New York Central Railroad, submitted as a discussion at a joint meeting of the A.S.M.E. and the A.S.C.E. on February 18, 1926, and shows the electric transmission of a Diesel freight locomotive. Curve *B* shows an ideal curve with no losses. Interest centers on curve *C* which is the curve of tractive effort due to the new turbo accelerator, trailing into the line *D* which is the straight-through gear connection. It will be seen that the line *D* is very much above the electric and the same is true of curve *C*, and whereas the point *E* is not quite

as high as the peak of the electric curve, yet it has two paramount characteristics: (1) It holds on to the sustained tractive effort to a point six or eight times as far along in mileage as the electric; and (2) it will not burn up or destroy itself if run continuously at any of these high values, which is in the greatest possible contrast with the facts regarding the electric transmission in the starting zone. The smooth and continuous power-speed curve given by the accelerator gives a higher efficiency than the electric, and compared with gears, the completeness with which it is sustained from one end to the other is in marked contrast to the frequent interruptions of the power in the gear transmission at the points where the old gear ratio is dispensed with and the new ratio is taken on. See line *H,* illustrating a four-speed transmission. One outstanding advantage of the accelerator cycle of operation is, as stated, that it gives a smooth and continuous transition from the point of highest development of torque to the lowest, in this way taking the place of three out of four gear changes, including the original clutching, and eliminating all but the last interruption, at each of which points it will be remembered there is an interval during which the power plant is completely divorced from the load. The single transition at the foot of the curve to the straight-through line *D* is comparable with the series-parallel change-over seen in the electric-transmission curve to the left, the accelerator reaching the straight-through connection at a point farther along on the speed curve than either the gear or the series-parallel part of the electric. Moreover, the transition point between the lower end of this curve and the line *D* does not involve as great a percentage of change as the corresponding point on the electric. Tests are being pushed rapidly on this transmission. The curves given herewith are based on test data and are looked upon both as interesting and promising.

R. Eksergian.[1] Locomotive performance may be divided into two ranges (1) the adhesive range, wherein the tractive force is dependent upon the adhesion weight on the drivers, and (2) the horsepower range which depends upon the performance of a locomotive as a power plant. We are thus concerned in mechanically designing a mechanism that can transmit large torque loadings at low speeds and then proportioning a minimum-weight power plant to give the maximum tractive force at the higher speeds for a given axle as well as total weight of locomotive. The Diesel engine is in the nature of a constant-torque machine; i.e., the card area and mean effective pressure have fixed and constant maxima, so therefore it is evident that the transmission problem is paramount, and what we are principally concerned with is its (1) flexibility, (2)

[1] Engineer, Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

minimum weight, (3) reduction in first cost, and (4) maximum simplicity and ruggedness which go with minimum maintenance.

Directly connected with the transmission mechanism is the question of the type of drive which may be divided into (1) some form of direct axle drive, and (2) jackshaft side-rod drive. The former has certain advantages in flexibility in the arrangement of wheelbase, etc., and particularly qualifies the straight electric transmission. The side-rod drive offers certain advantages in weight reduction and compactness in wheelbase, etc., but, on the other hand, has some very definite limitations. It can be shown that with side-rod drive a change-over position occurs at the 45-degree angle with quadrant spacing of the cranks, but with unequal plays or lengths of rods of one side over the other, etc., the change-over may occur at smaller angles, with large loadings on the rods, jackshaft, and axles. This condition is more likely possible in the jackshaft drive, due to the necessity of smaller plays, than in steam-locomotive practice. Thus careful attention must be given to the design, with subsequent careful maintenance in the jackshaft drive for successful operation. Moreover, we come to a very fixed limitation in the adhesive capacity per jackshaft. In the ordinary proportions of wheel centers, it may be stated that 180,000 lb. adhesive load is the extreme limit of a single jackshaft drive, whereas comfortable proportions may be limited to 150,000 lb. Thus heavy switching service with over 240,000 adhesive capacity required would require two jackshafts and systems of transmission with corresponding weight increase, unless the jackshaft can be located between two wheelbases with corresponding spread of total rigid wheelbase.

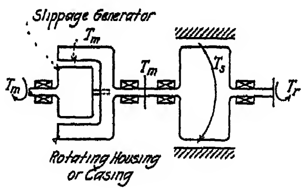


FIG. 19

The author has given a careful review of the various types of transmissions. In the writer's opinion, the differential elastic-fluid transmission offers the most interesting type for further expansion. The following simple analysis made some time ago may give a somewhat different aspect of the problem.

Referring to Fig. 19, if,

T_m = driving motor torque
T_s = torque due to auxiliary motor
T_r = resultant torque
w_m = speed of motor
w = speed of driven shaft

then, $T_r = T_m + T_s$ for the resultant torque.

That is, the motor torque is augmented by the secondary torque T_s due to the auxiliary motor and, due to the slippage of the gener-

ator $(w_m - w)$, the generator develops $T_m(w_m - w)$ ft-lb. per sec. and therefore the auxiliary motor torque, obtained from

$$E_h T_m(w_m - w) = T_s w$$

is

$$T_s = E_h T_m \left[\frac{w_m - w}{w_m}\right]$$

where E_h = efficiency of transmission.

Evidently as the drive shaft approaches the speed of the motor, T_s gradually reduces to zero, while at starting or very low speeds, $(w_m - w)$ is large and w small; therefore T_s, the secondary torque, becomes very large. This condition, of course, gives the ideal requirements for a perfect transmission.

Moreover, at any intermediate speed the efficiency with this arrangement is much higher than could be obtained by any direct system, thus:

If E is the overall efficiency, and E_m the overall mechanical efficiency of the system, then,

$$E = \frac{[T_m w + E_h T_m(w_m - w)]E_m}{T_m w_m} = \left[\frac{(1-E_h)w + E_h w_m}{w_m}\right] E_m$$

Therefore, assuming a hydraulic transmission efficiency at 0.75, the efficiencies will be as follows:

	Efficiencies				
	$w/w_m = 0$	$w/w_m = \frac{1}{4}$	$w/w_m = \frac{1}{2}$	$w/w_m = \frac{3}{4}$	$w/w_m = 1$
Slippage drive	$0.750E_m$	$0.813E_m$	$0.875E_m$	$0.938E_m$	$1.00E_m$
Direct transmission	$0.750E'_m$	$0.750E'_m$	$0.750E'_m$	$0.750E'_m$	$0.750E'_m$

It is to be noted that the equation of efficiency reduces to

$$E = E_h E_m + (E_m - E_m E_h)\frac{w}{w_m}$$

which is exactly the same form as given by the author.

The Schneider hydraulic transmission is based essentially on this principle. The torque at the generator rotor or reciprocating clutch is

$$T_m = \frac{n_m p(\pi/4) d_m^2 S_m}{2\pi} \text{ ft-lb.}$$

and the torque exerted on the secondary or auxiliary motor drive is

$$T_s = \frac{n_s p(\pi/4) d_s^2 S_s}{2\pi} \text{ ft-lb.}$$

Where n, d, and s are the number, diameter, and stroke of cylinders, and p the oil pressure used. If r_m and r_s are the respective gear ratios to the jackshaft where the total torque is combined, having angular velocity w, the resultant torque is,

$$T_r = r_m T_m + r_s T_s$$

Due to the law of continuity of the fluid, we have also,

$$n_m(\pi/4) d_m^2 S_m(w_m - w/r_m) = n_s(\pi/4) d_s^2 S_s 4/r_s$$

Therefore, as we should expect from first principles,

$$(w_m - w/r_m)T_m = T_s w/r_s$$

That is, the slippage energy is completely utilized, neglecting losses. The variation of speed and torque is effected by varying the stroke in the secondary.

A similar scheme was proposed by the writer some time ago in the form of an electromechanical transmission. Its only advantage over an electric drive would be in the reduction in weight. Two constructional difficulties, however, cannot be overlooked; they are common to the Schneider type as well and they are, first, that the bevel-gear proportions limit the drive to around 500 hp., and, second, that the rotating housing of the generator offers complications. With the introduction of two units, favorable also for the jackshaft drive, the following analysis is of interest.

The engine and rotor *A* of the magnetic clutch *A-B* are connected by the same shaft. (See Figs. 20 and 21.) The revolving casing *B* is directly connected to bevel gear *C* which transmits motion to a bevel gear *D*. Bevel gear *D* is keyed to the auxiliary jackshaft *H-H* as well as to two spur gears *E-E* at either end of shaft. Spur gears *E-E* engage with the main gears *G-G* keyed to the jackshaft. An auxiliary motor *M* drives the main gears *G-G* through the motor pinions *F-F*.

The mode of operation is as follows:

(1) At starting or low speeds the engine torque is transmitted by the clutch and increased by the gear reduction $\left(\frac{G}{H} \times \frac{D}{C}\right)$ at the jackshaft. The slippage between *A* and *B* of the clutch generates electric power which is transmitted to the motor *M*. As the slippage is maximum, at low speeds, maximum torque is exerted on the auxiliary motor, and its torque is augmented at the jackshaft by the gear ratio *G/F*. The total torque at the jackshaft is, therefore, the sum of the engine torque increased by its gear ratio, and the motor torque increased by its gear ratio.

(2) At speed, little power is transmitted by relative slippage of *AB*, the greater part of the power being transmitted mechanically directly through the clutch to the jackshaft.

Obviously the gear ratio from the casing of the clutch to the jackshaft must be designed for increasing engine torque at the jackshaft, equal to the required torque at the jackshaft corresponding to the rated speed of the locomotive.

The following data, with Fig. 22, illustrate the calculation of proportions.

1000 hp. total, 500 hp. per unit
Wheel diameter = 57 in., stroke = 24 in.
Adhesion weight = 4×35,000 or 140,000 lb.
Maximum tractive force = 30,000 lb.

$$\text{Tractive force at 30 m.p.h.} = Z = \frac{375{,}000}{30} = 12{,}500 \text{ lb.}$$

$$\text{Ratio } \frac{\text{(maximum)}}{\text{(rated)}} = \frac{30{,}000}{12{,}500} = 2.4$$

At 30 m.p.h., clutch gear ratio is designed to transmit full engine torque. Hence the auxiliary motor at starting must be designed to transmit 2.4 to 1 or 1.4 rated engine torque.

$$\text{Jackshaft torque at 30 m.p.h.} = \frac{57}{2} \times \frac{12{,}500}{12} \text{ or } 29{,}700 \text{ ft-lb.}$$

$$\text{Engine torque at 1200 r.p.m.} = \frac{1000 \times 33{,}000}{2\pi 1200} \text{ or } 4370 \text{ ft-lb.}$$

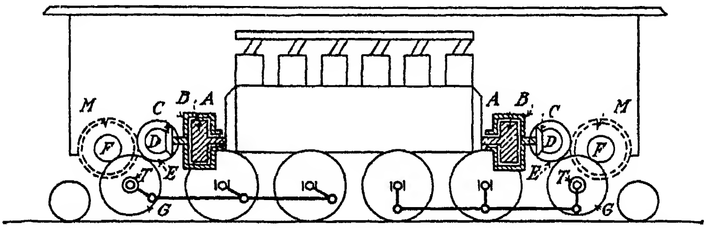


FIG. 20

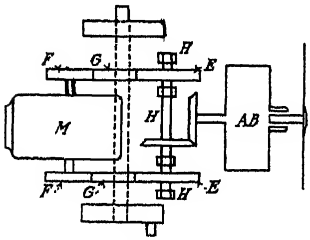


FIG. 21

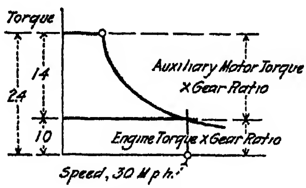


FIG. 22

$$\text{Hence the clutch gear ratio} = \frac{29{,}700}{4370} \text{ or } 6.80$$

$$\text{Tangential force bevel gear} = \frac{2185}{\frac{1}{2}} \text{ or } 4370 \text{ lb.}$$

Motor gear ratio:

Auxiliary motor speed at 30 m.p.h. = 1200 r.p.m.

$$\text{Driver speed at 30 m.p.h.} = \frac{30}{53} \times 336 \text{ or } 190 \text{ r.p.m.}$$

$$\text{Motor gear ratio} = \frac{1200}{190} \text{ or } 6.32$$

As to weights: Assume the weight of motor and generators to vary as the square root of the maximum torque (i.e., maximum current heating). That is, doubling the torque would increase

the weight by 41 per cent and with 4 times the torque, the weight would be increased twice.

Then for 500 hp. direct electrical drive (noting the gear ratios would be about the same, i.e., between 6 and 7) we have a total weight of 2 units, i.e., assuming unity for weight corresponding to motor or generator. With this drive the torque transmitted electrically is $\frac{2.4-1}{2.4}$ (see tractive-force curve) or $\frac{1.4}{2.4}$ or approximately 58 per cent of the total torque. Hence the weight ratio of electrical apparatus is roughly

$$\frac{W \text{ electro mechanical}}{W \text{ direct electric}} = \frac{\sqrt{0.58}}{\sqrt{1}} \text{ or } 0.76$$

i.e., the weight of this electric drive is equivalent to 76 per cent of the weight of the electric drive. This ratio, however, would be somewhat increased by additional gearing and combining the generators into one unit.

William Elmer.[1] In comparing the merits of Diesel and steam locomotives, we as engineers ought to face the qualifications which a Diesel locomotive must possess.

We have now reached the point with steam locomotives where we can say that it is fruitless to expend time on the smaller units on which the German and other engineers have been working. So far as the writer knows a 1000-hp. unit is the largest of the actual Diesel constructions in existence, while steam locomotives of about 4000 hp. have been built. Therefore, we should work toward the design of locomotives, of the Diesel prime-mover type, which can compete with existing steam locomotives with a 4000-hp. unit as a minimum. If such a unit cannot be produced the field for the Diesel locomotive will be very limited. Existing locomotives are useful in switching service in cities where the use of steam locomotives has been prohibited by city ordinance but for heavy open-country traffic the writer believes 4000-hp. units must be attained or the Diesel locomotive cannot compete fully with the steam locomotive.

Further, we must face the fact that we need traffic powers in the vicinity of 200,000 lb. so that any devices for reducing the power of the engine into torque at low speeds must be of the order of 200,000 lb. at the rim of the driver.

In addition to that, we must face the fact of cost. Steam locomotives of approximately the capacity named can be bought for about 17 cents a pound, so that roughly $100,000 should be the cost of the unit to be designed. Some years ago the writer attempted to work out a design but could see no possibility of reaching the

[1] Special Engineer, Pennsylvania Railroad Company, Philadelphia, Pa. Mem. A.S.M.E.

required capacity at a cost in any way competitive with the steam locomotive.

On the other hand we have the Diesel efficiencies which naturally are well worth striving for, but since modern steam locomotives are able to develop a horsepower-hour in the cylinders on about 16 lb. of steam and the efficiency of transmission is in the vicinity of 90 per cent, a serious problem is presented.

The question of availability for service should also be kept in mind. While the steam locomotive is available for service about fifty per cent of the time and the electric locomotive more than ninety per cent, we must realize that the Diesel electric may not be even as satisfactory as the steam locomotive, if the design is so complicated and the number of parts needing repair so great as to throw the engine out of service when it should be working on the road hauling trains.

The Author. Mr. Sperry's contribution supplements the paper by giving very interesting information on a transmission of the differential class. Mr. Sperry apparently does not yet see fit to give details of the actual design, and thus does not enable us to place the transmission in proper relation to other transmissions in the classification scheme which was followed in the paper, nor does he permit us to form a clear conception of its merits. However, so much can be inferred from his remarks — that the transmission is a combination of a turbo-drive with a purely mechanical transmission which at higher speeds automatically provides a straight-through drive from the prime mover to the driven shaft, the turbo-drive being entirely eliminated at these speeds. According to Mr. Sperry, the transmission is now undergoing tests, and the results are promising. It remains only to express our wish that the test results and the design should soon become known to the engineering profession.

It is very gratifying to see that Mr. Eksergian corroborates the correctness of the author's formulas and conclusions in reference to the differential elastic-fluid transmissions. The transmission of the differential electric type shown on Fig. 21 is essentially the Entz transmission with bevel and spur gearing used in the Schneider type. It may be called the electric analogy of the Schneider gear, or a "schneiderized" Entz transmission. But, whatever the classification characteristic may be, the electric features will probably render the transmission heavy and expensive. Mr. Eksergian's approximate calculation of weights takes into consideration only the motors and generators, but it is more than probable that the bevel and spur gears, the jackshaft, the additional bearings, especially the extension of locomotive frames required by the design of the transmission, the side-rod drive, etc., will offset the calculated 24 per cent saving in weight of the motors and generators.

The case, however, is different with the differential hydraulic transmission, as a hydraulic transmission is inherently light in design. A very detailed weight estimate made on the basis of a complete design of a 750-b.hp. locomotive with a differential hydraulic transmission showed a total saving of 30 per cent in weight as compared with a similar locomotive with full-power electric transmission. The differential hydraulic transmission itself weighed only about 50 per cent of the electric drive.

Mr. Elmer's remarks are very practical, but they are not exactly within the scope of the present paper which deals only with one side of the oil-engine locomotive problem, namely, with that of the transmission of power. The economic and railroad-operation aspect of the oil-engine locomotive will be discussed in a separate paper, but as the questions have here already been raised, they will be answered briefly.

As regards availability, the experience so far with the 60-ton and 100-ton oil-electric switching locomotives have proved that they are available practically all the time. In reference to price — the interest on the investment is only one item in the expense sheet, and must be considered jointly with other operation expenses of which the fuel cost is the most important one; there are no indications that the maintenance cost of oil-engine locomotives will be higher than that of steam locomotives. With respect to power, there are two reasons to account for the fact that the power of the largest oil-engine locomotive in existence does not exceed 1000 hp.: first, that the oil-engine locomotive is just in the course of its experimental development, and it so happens that we discuss the question at the time when only 1000-hp. units have been built. But progress is going on; the Schneider-Still locomotive, described in the paper, which is now under construction at the Schneider & Co.'s plant in Creusot, France, will develop at full speed about 1800 hp. Locomotive builders in this country would also be ready to build oil-engine locomotives with electric transmission of from 1500 to 2000 hp. if orders from the railroads were forthcoming. The second reason is that the 300- to 600-hp. switching locomotives seem to answer the purpose very well. The 4000 hp. of which we are apt to speak when we refer to the power of American locomotives is being developed only at high speeds, usually at from 50 to 60 m.p.h., whereas at the ordinary speeds of switching service (6 to 10 m.p.h.) the more moderate power of 600 hp., possibly 1000 hp., is very near the limit which is required for this service.

The development of the oil-engine locomotive cannot be successful without the railroads' coöperation and assistance. The progress made in Europe is mainly due to the initiative of the railroads themselves. Professor Norman's opportune appeal to American industrial enterprises should be understood as referring to railroads as well.

No. 2011

A SHORT METHOD OF CALCULATING THE PERIODIC DISPLACEMENT AND OSCILLATIONS IN SYNCHRONOUS MACHINES

By C. W. Cutler,[1] Schenectady, N. Y.
Non-Member

The paper forms a basis for the design of flywheels for engine-driven generators operating in parallel. The present rules, simple enough for commercial design, apply only in the case of operation against an infinite system. The present paper therefore develops a method whereby the power exchange, torque variations, and other phenomena can be accurately obtained with practically no greater effort than with the present approximate rule. The general solution for two engine-driven generators by means of a nomographic chart is given in an appendix which also explains the method of constructing the chart.

Introduction

THE increasing popularity of the oil engine for the development of alternating-current power has brought with it a rather difficult problem of resonance.

2 When two or more generators are placed in parallel, the combination of machines has at least one natural period of swing something like that of a pendulum. The mechanical torque variations may occur at a rate which cause the system to oscillate unduly, thus upsetting the smooth running of the units. The choice of flywheels must therefore be made by taking into account not only the requirements of the engines individually, but also the requirements when connected to the electrical bus through the generators.

3 Furthermore, this difficult electrical problem is so closely tied up with the successful operation of the units that the mechanical designer, even more than the electrical, finds a need for a quick method of properly choosing these flywheels. This need has led to an endeavor to establish and use some approximate rules to

[1] Industrial Engineering Department, General Electric Co.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

govern the design. As a result there has been adopted pretty generally throughout the industry a combination rule.[1] This rule, however, does not apply to machines operating in parallel, and there is at present no short method available for predetermining the flywheel requirements for engine-driven generators for other than the case of operation against an infinite system. The present paper, therefore, is devoted to the development of a method whereby the power exchange, torque variations, and other phenomena can be accurately obtained with practically no greater effort than with the present approximate rule.

4 In developing this method, processes are used which may reduce other problems more intricate in nature or the phenomena of which follow unknown laws, to comparably simple form.

Outline of Paper

5 For purposes of clarity this paper will be divided into the following sections:

I A discussion of the electrical transmission system involved

II Consideration of the very limited case of synchronous motors and generators operating on so-called "infinite systems"

III A study of the parallel-operation problem

IV The solution of the problem as it pertains to two Diesel-engine-driven generators in parallel, and how this applies to all types of engines

V General discussion of the accuracy required and of what the new method of calculation makes possible.

I The Electrical Transmission System

6 An engine-driven generator may be the entire source of power, or it may be only a small part of a system of electrical apparatus. This transmission system in its most complicated form consists of electrical machines, each unit of which is in constant oscillatory motion. For the first example, however, let such a system be very greatly restricted by holding all units of the system constant as regards speed, voltage, and frequency except one unit consisting of a Diesel-engine-driven generator. This makes the system one in which there is only one natural frequency. Con-

[1] A paper entitled Design of Flywheels for Reciprocating Machinery Connected to Synchronous Generators or Motors was presented by Doherty and Franklin before the December 7, 1920, meeting of the A.S.M.E. See Trans., vol. 42, p. 523. In the paper the correct solution for the equation of the synchronous machine operating from an infinite system was given. Realizing that this equation was rather cumbersome for commercial use, the authors presented a short rule which applies to this case.

siderable work[1] has already been done on a similar case under the heading of compressors driven by synchronous motors which are operating on infinite systems.

II Synchronous Motors and Generators Operating on Infinite Systems

7 Since the problem in hand, however, varies somewhat in nature from that of a compressor and since the case bears so directly upon the transmission system in hand, a moment might well be spent on it. The Diesel engine and its generator as a unit is oscillating due to the working stroke of the engine. The factors which enter to cause the unit to oscillate (in just the manner it does), may be readily set down from a knowledge of the physics[2] of the case, as follows:

Ev = variation in energy being delivered by the gas at any instant during operation. This may for a more specific definition be used as the maximum amplitude of the variation, although any other physical characteristic of the wave might be used, provided the definition remains consistent for all machines considered. Further, when used as a single factor, as in this particular treatment, it assumes a definite *shape* of torque effort delivered by the gas.

Ew = energy stored in the reciprocating parts of the engine. This includes effect of angularity of the connecting rods. It may be represented by the maximum amplitude of the energy wave at normal constant shaft speed. This energy or torque wave is the result of acceleration and deceleration of the reciprocating parts of the engine.

Ef = energy stored in the flywheel at normal uniform rotation. This is evidenced by the force which resists acceleration.

Eg = additional energy stored in the synchronous motor magnetic field due to the displacing of the rotor from its angular position under uniform motion. This is the energy evidenced by synchronizing force.

[1] In an article entitled Flywheel Requirements for Unbalanced Air and Ammonia Compressors, *Refrigerating Engineering*, vol. 12, no. 3, September, 1925, the author mentions Doherty and Franklin, Steinmetz, Berg, Astrom, Abbott, Slichter, Boucherot, Hawkins, Wallace, Janet, Schou, Putman, and particularly Stevenson, as having done work along this line.

[2] It should be kept in mind during this discussion that the problem fundamentally is one of energy storage and energy transfer. It should also be remembered that torques wherever they appear are only results of energy present and that it is the energy not the torque with which the problem has to do. It follows, however, that since this relation between torques and energy exists the problem could be solved from a torque viewpoint, although it would not be as comprehensive. These nomenclatures do not conform to those previously used by other authors but have been transposed in the appendix to this paper for convenience.

Ed = energy stored or dissipated in the damping winding for some specific wave shape of oscillation which has been chosen to measure the ability of the amortisseur winding to damp out oscillation.

V = volume of cylinders. This variable is included on the assumption that it is a measure of the size of the engine. It should be noted that the shape of indicator card working in this volume remains constant for the problem in hand, although it will become evident that other problems can be worked out for different shapes of indicator cards.

S = speed in r.p.m.

Ep = amplitude of energy pulsation at generator terminals.

8 The "dimensions" of the above factors are:

Factor	Dimensions
V	l^3
S	t^{-1}
Eg	ml^2t^{-2}
Ef	ml^2t^{-2}
Ev	ml^2t^{-2}
Ew	ml^2t^{-2}
Ed	ml^2t^{-2}
Ep	ml^2t^{-2}

9 According to dimensional analysis[1] five equations can be written, one of which is as follows:

$$V^{\alpha} \times S^{\beta} \times Eg^{\gamma} \times Ef = \text{unity}$$

Then substituting dimensions:

$$l^{3\alpha} \times t^{-\beta} \times m^{\gamma} \times l^{2\gamma} \times t^{-2\gamma} \times ml^2t^{-2} = \text{unity}$$

10 But since we know that in any complete dimensional equation the sum of exponents for each kind of quantity is zero, we write

$$\begin{aligned} 3\alpha+2\gamma+2 &= 0 \\ -\beta-2\gamma-2 &= 0 \\ \gamma+1 &= 0 \end{aligned}$$

from which

$$\begin{aligned} \gamma &= -1 \\ \beta &= 0 \\ \alpha &= 0 \end{aligned}$$

and the equation is

$$V^0S^0Eg^{-1}Ef = \text{unity}$$

which is

$$\frac{Ef}{Eg} = \text{unity}$$

[1] Considerable work has been done by Buckingham (Notes on the Method of Dimensions, *Phil. Mag.*, Nov. 1921, On Physically Similar Systems: Illustrations of the Use of Dimensional Equations, *Phy. Rev.*, vol. IV, Oct. 1914), and others with this type of mathematics.

This is reduced by substitution to

$$X = \frac{WR^2S^4}{Pof}$$

or this can be rewritten without changing its dimensional validity as

$$X_1 = \frac{Pof}{WR^2S^4}$$

11 In a similar manner the other factors can be determined, that is, by dogmatic analysis. It will be noted that S and V have dropped out, but the reason for their retention in the consideration will be seen later.

12 Another point of interest is that were the energies mentioned the only variables (which they may be in some cases), then an equation meaning that the dimensions are equal can be written

$$[Ev] = [Ew] = [Ef] = [Ed] = [Eg] = [Ep]$$

or

$$\left[\frac{Ev}{Eg}\right] = \left[\frac{Ew}{Eg}\right] = \left[\frac{Ef}{Eg}\right] = \left[\frac{Ed}{Eg}\right] = \left[\frac{Ep}{Eg}\right] = 1$$

and in the case considered in Par. 7, Equation [1] follows. It should be noted that in this latter treatment no consideration has been given to whether the energies are the same or of different kinds.

13 The former method, however, may shed some light upon the relationship of the parameter in the mathematical equation which the later method may not show. This point is brought out in the appendix.

14 After all factors have been found in the above manner, the complete dimensional equation expressing the hunting of the unit can be written as follows:

$$f\left(\frac{Ef}{Eg}, \frac{Ed}{Eg}, \frac{Ew}{Eg}, \frac{Ev}{Eg}, \frac{Ep}{Eg}\right) = 0 \quad . \; . \; . \; . \; . \quad [1]$$

$$\text{or } \frac{Ep}{Eg} = f\left(\frac{Ef}{Eg}, \frac{Ed}{Eg}, \frac{Ew}{Eg}, \frac{Ev}{Eg}\right) \quad . \; . \; . \; . \; . \; . \; . \quad [2]$$

15 Now Equation [2] was obtained by considering several quantities as variables. But if the factors

$$\frac{Ed}{Eg}, \frac{Ew}{Eg}, \text{ and } \frac{Ev}{Eg}$$

be considered constant, Equation [2] can be restricted to

$$\frac{Ep}{Eg} = f\left(\frac{Ef}{Eg}\right)$$

and the function can be plotted for this case on ordinary graph paper as shown in Fig. 1.

16 If, however, more than three of these variables enter, it is obvious that the problem cannot be plotted as shown. Fortunately

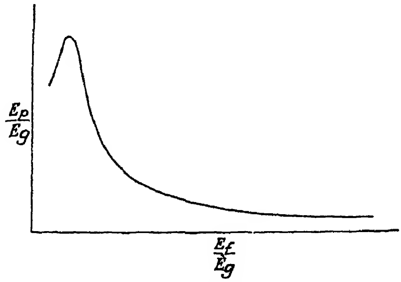


Fig. 1

(This curve is reducible to the form of " X-Y " curves used first by Stevenson and later by the author in previous works.)

in the work previously done on compressors (see footnote 1 on page 313), enough could be considered constant to handle the problem.

III The Parallel-Operation Problem

17 Now let the problem depart from the simple case above mentioned. Assume that the system is no longer constant as in Par. 6 but consists of *two* Diesel-engine-driven alternating-current generators operating in parallel. Further assume that these units are not constrained but are free to oscillate as they normally would under these conditions.[1] The variables which may enter are:

For Unit No. 1

Ev_1 = variation in energy being delivered by the gas at any instant during operation

Ew_1 = energy stored in the reciprocating parts of the engine

Ef_1 = energy stored in the flywheel which evidences itself by resistance to acceleration

Eg_1 = energy stored in the magnetic field of the generator as potential energy, which evidences itself as synchronizing force

[1] H. V. Putman, in a paper entitled Oscillation and Resonance in Systems of Parallel Connected Machines, built upon the equations of Berg, Doherty, and Stevenson the complete solution for angular variation of generators when operating in parallel and neglecting current due to the damping winding, obtains a power pulsation between units. He also gives a method of correctly calculating the natural frequencies which exist in a system of units operating in parallel. The equations become so long, however, that they are not usable in commercial work. His paper, however, is a most excellent one and in the present paper the equations there derived have been used in constructing the charts here presented.

Ed_1 = energy stored or dissipated in the damping winding of the generator during oscillation
V_1 = volume of cylinders
S_1 = speed in r.p.m.
ω_1 = rate of beats of engine
Ep_1 = energy pulsation of machine No. 1
Energy stored in electrical wires between units due to inductance, capacity, and resistance. Where the generators are in the same station, this may be neglected
Energy stored in rotating masses such as large motors normally considered as "load."

18 For unit number two there will be a similar group of variables, 20 probable variables for two machines, 32 for three machines, and so on, as more units are added to the transmission system. This seems almost unwieldy, the equation including the variables for two machines being

$$\frac{Ep_1}{Eg_1} = f\left(\frac{Ev_1}{Eg_1}, \frac{Ev_2}{Ev_1}, \frac{Ew_1}{Eg_1}, \frac{Ew_2}{Ew_1}, \frac{Ef_1}{Eg_1}, \frac{Ef_2}{Ef_1}, \frac{V_2}{V_1}, \frac{S_2}{S_1}, \frac{\omega_1}{S_1}, \frac{\omega_2}{\omega_1}, \frac{Ed_1}{Eg_1}, \frac{Ed_2}{Ed_1}, \frac{Eg_2}{Eg_1}\right) \quad . \; . \; [3]$$

Then factoring the dimensional equation, the form of which is presumably unknown,[1] the equation becomes

$$\frac{Ep_1}{Eg_1} = F\left[f_1\left(\frac{Ev_1}{Eg_1}\right), f_2\left(\frac{Ev_2}{Ev_1}\right), f_3\left(\frac{Ew_1}{Eg_1}\right), f_4\left(\frac{Ew_2}{Ew_1}\right), f_5\left(\frac{Ef_1}{Eg_1}\right), f_6\left(\frac{Ef_2}{Ef_1}\right), f_7\left(\frac{V_2}{V_1}\right), f_8\left(\frac{S_2}{S_1}\right), f_9\left(\frac{\omega_1}{S_1}\right), f_{10}\left(\frac{\omega_2}{\omega_1}\right), f_{11}\left(\frac{Ed_1}{Eg_1}\right), f_{12}\left(\frac{Ed_2}{Ed_1}\right), f_{12}\left(\frac{Eg_2}{Eg_1}\right)\right] \quad . \; [4]$$

19 Equation [4] consists of a number of functions, each representing an equation (or factor) containing only its own parameter and constants.

20 It will be recognized that Equation [4] is susceptible to solution by nomographic chart. However, not enough is known about Equation [4] alone to construct this chart. At this point, therefore, it is necessary to resort to test data[2] or to some known equation for the phenomenon represented by Equation [4].

[1] It will be assumed at this point without proof that the equation can be factored and that the mathematical tools are sufficient to factor the given equation into a form shown in [4]. A further discussion of this form of equation, with the case of two machines factored as an example, will be found in the appendix.

[2] C. A. Nickle in June, 1925, presented a paper on Oscillographic Solution of Electro Magnetic Systems before the American Institute of Electrical Engineers, which makes testing of such problems as this a simple matter.

21 But Equation [4] does tell us something useful, namely that each parameter represented within the brackets is equal to a constant. If now all except one function, such, for example, as $f_1\left(\frac{Ev_1}{Eg_1}\right)$ be kept constant, the variation of $\left(\frac{Ep_1}{Eg_1}\right)$ with $f_1\left(\frac{Ev_1}{Eg_1}\right)$

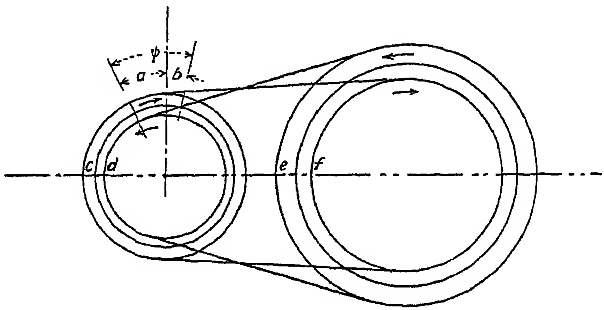


FIG. 2 DIAGRAM REPRESENTING ANGLE ψ SHOWN IN FIGS. 4, 5, AND 6

can be studied. Furthermore, this fact allows us to construct a nomographic chart and bring the problem down to a simple workable basis.

IV SOLUTION OF PROBLEM OF TWO DIESEL-ENGINE-DRIVEN GENERATORS IN PARALLEL

22 By far the most important problem of parallel operation in commercial work is the case of two units parallelling together. The reason for this is that in case the units be incorrectly installed, there is little opportunity of making adjustments to remove the difficulty without rebuilding the units. For three machines the problem still exists for Diesels, but to a lesser degree, while for four machines and more the problem is usually little more than of theoretical interest for the present sizes of flywheels, except perhaps in the case of gas engines or units where one cylinder of one machine is disconnected or misfires completely. There have been such cases of four machines which caused the user considerable inconvenience. In Diesel practice, however, only the case of two and possibly three machines in parallel need be considered. If, however, the plant consists of several units, certainly any possible combination of two machines should run in parallel, otherwise the user may find that he cannot operate the very two machines most needed. It also adds to the flexibility of the plant to be able to operate any three machines in combination.

23 The current exchange due to poor parallel operation may not cause the units to hunt apart and trip the breakers. It will

be evidenced, however, by added heating in generators and buses, higher than normal excitation, and unsteadiness of the meters, both ammeter and kilowatt meters. The author has seen units in operation where the unsteadiness of the meters was so great that it was impossible to load the engines equally by use of these meters. This is obviously undesirable.

24 The general solution for a nomographic chart has therefore been given for two engine-driven generators in the appendix to this paper. By constructing a chart in a manner explained in the appendix, Fig. 3 was derived.

25 Refer now to Fig. 4 which has been drawn to show how Fig. 3 is used. First the values of parameters are calculated from the constants of the particular units involved. Then a broken line, like the dotted one in Fig. 4, is drawn through the factors, determining at once the relative angular displacement between stator and rotor. It should be noted that $\frac{S_2}{S_1}$ and $\frac{Tv_2}{Tv_1}$ were kept equal to unity in all examples in this paper. To include these accurately would require additional scales on Fig. 3. It is not necessary to make assumptions, but where the speeds and capacities are approximately alike, the following simple relation is roughly[1] true.

$$\psi'_1 = 2\psi_1 D \frac{Tv_1}{Ts_1}\left(1 + \frac{Tv_2 S_1}{Tv_1 S_2}\right)$$

where ψ'_1 = angle for problem in hand as shown in Fig. 2
$2\psi_1$ = ordinate from curve
D = a constant of construction of the curve.

26 Reference to Fig. 2 may assist in representing the angle ψ shown in Figs. 4, 5, and 6. This figure (although not theoretically correct) will serve its purpose. Element c represents the stator field which is connected by springs to the rotor element d. Likewise e is secured to the element f by springs. Now suppose a belt placed over the elements so that any motion of f is transmitted to c. Without error these machines can be considered as stopped in space because they are synchronous machines. Then at some time during the oscillation of d and f, d will be displaced counter-clockwise through an angle a by its own hunting. At the same instant the element c will be displaced clockwise by the transmitted effect of hunting of f, through an angle b. The total angle ψ is then the sum of a and b. This angle ψ times the tension in the springs Ts_1 closely represents the torque variations.

[1] This equation is discussed more at length in the appendix.

27 With constant speed this represents kilowatts variation from the average. Fig. 3 thus becomes general for any combination

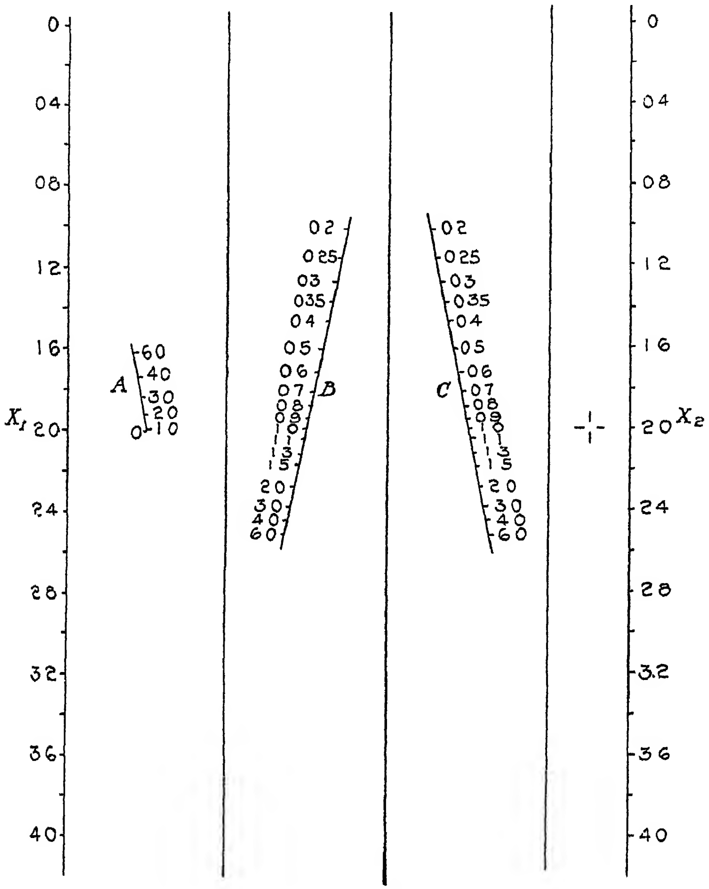


FIG. 3 "X" CHART FOR THE DETERMINATION OF ENERGY EXCHANGE BETWEEN TWO ENGINE-DRIVEN GENERATORS WHEN OPERATING IN PARALLEL

$$X_1 = \frac{Po_1 f \times 10^{10}}{WR_1^2 S_1^2} \qquad B = \frac{Po_2 S_1^2}{Po_1 S_2^2}$$

$$A = \frac{Td_2 S_2}{Td_1 S_1} \qquad C = \frac{WR_2^2 S_2^2}{WR_1^2 S_1^2}$$

$$= \text{(approx.)}\ \frac{Po_2 S_1}{Po_1 S_2}$$

of two engine-driven generators. But in order to take into account various types of engines, it is simplest to draw separate curves for different types of engines and combinations of types on Fig. 4 or on a separate sheet against X_2 as shown in Figs. 5 and 6.

Conclusions

28 Thus the torque pulsation of machine No. 1 is determined. This method of handling the problem has made it possible to reduce the time for solution down to a commercial basis.

29 It accomplishes another more important result, however. It makes completely visual the effect of each variable on the percentage of torque pulsation, which was before impossible.

30 In the choice of a flywheel for new machines the question arises as to how much pulsation is allowable in the average generator. This is to some extent a matter of judgment, as other factors, as well as the life of the electrical equipment, enter the problem.

31 A tremendous amount of experience has been obtained on

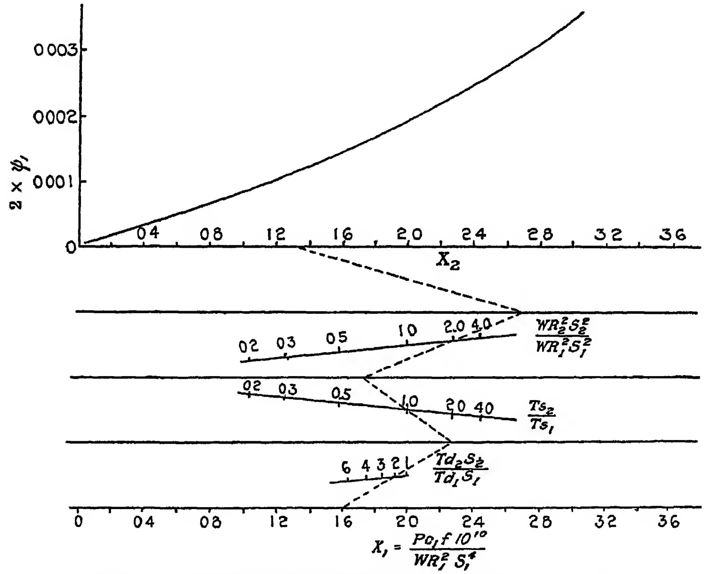


Fig. 4 Illustration of Use of Figs. 3, 5, and 6

strength of machines through the application of synchronous motors to compressors. Less experience is available on generators because the sizes of flywheels used are very much larger in proportion due to voltage stability and governor requirements. However, drawing from such experience as has become available in the synchronous field, it is believed that a similar maximum should be given generators as has been standardized for motors[1] (namely, 66 per cent pulsation expressed as a percentage of the nameplate rating). This is from the standpoint of generator protection only.

[1] American Society of Refrigerating Engineers, and Electric Power Club.

From the standpoint of smooth running and steady meters it might be advisable to keep this pulsation as low as 30 per cent.

APPENDIX

32 Equation [4] is a difficult equation to write in a general form. It might be assumed that this equation necessarily means that the differential equation of the standard form as developed by Putman must be such that it can be factored into the form of functions which when *multiplied* together give the pulsation. It is not desired to restrict the equation to this meaning alone.

33 The step from Equation [3] to Equation [4] assumes that the mathematical equation represented by Equation [3] is reducible to a definite form. This form may be stated in language as follows:

The mathematical equation expressing the phenomenon governed by a dimensional equation similar to Equation [3] may be reducible to a form in which the dimensional parameters of Equation [3] appear only as independent factors or factors which are derivatives of the respective parameters.

34 If the equation is so reducible it should be susceptible to nomographic representation. However, if it is not so reducible, then the step from Equations [3] to [4] is not justified except to the degree in which the equation is so reducible.

35 This merely means that the step remains an unjustifiable assumption until the mathematical equation (if known) has been reduced to the form known to be in accordance with the above statement which Equation [4] is supposed to represent.

36 When the mathematical equation of the phenomenon is not known, the assumption may be made that the equation, in whatever form it may be, is solvable by nomographic chart. Tests may then be conducted to construct such a chart as explained in conjunction with Fig. 7. If a chart can be so constructed and checks by test, its use is justified. Such a chart may also throw considerable light upon the form of the unknown equation by noting that the reverse of the above statement may be true.

37 The particular equation governed by the dimensional relationship of Equation [3] as given by Putman in his article mentioned in the footnote on page 316 is the solution of

$$\text{determinant } \psi''_1 = \frac{\begin{vmatrix} F_1 & 0 & -C_1 \\ F_2 & A_2 & -C_2 \\ 0 & B_2 & 0 \end{vmatrix}}{\begin{vmatrix} A_1 & 0 & -C_1 \\ 0 & A_2 & -C_2 \\ B_1 & B_2 & 0 \end{vmatrix}} = \frac{B_2(C_2F_1 - C_1F_2)}{C_1B_1A_2 + C_2B_2A_1}$$

where

$$A = Td_2 + \left(\frac{P_2}{P_1}\right)^2 Td_1$$

$$B = Ts_2 + \frac{Td_1Td_2}{I_1} + \left(\frac{P_2}{P_1}\right)^2 \left(Ts_1 + \frac{Td_1Td_2}{I_2}\right)$$

$$C = (Ts_1Td_2 + Td_1Ts_2)\left[\frac{1}{I_1} + \frac{1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]$$

$$D = Ts_1Ts_2\left[\frac{1}{I_1} + \frac{1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]$$

38 This reduces to two factors: Due to prime mover No. 1

$$\psi_1 = \frac{(Ts_2 + j\omega Td_2)\ F._1}{(D - B\omega^2) + j(C\omega - A\omega^3)}$$

and due to prime mover No. 2

$$\psi_1 = \frac{\dfrac{-P_2}{P_1}\ (Ts_2 + j\omega Td_2)\ F._2}{(D - B\omega^2) + j(C\omega - A\omega^3)}$$

write

$$\begin{aligned}(D - B\omega^2) &= Nn\\ Td_2\omega &= Rn_1\\ (C\omega - A\omega^3) &= Mn\\ Td_2\omega &= Rn_2\end{aligned}$$

39 Now as a special case assume $Td_2 = Td_1 = 0$, that is, that damping effect of both machines is zero. Then in the above equations let

$$\begin{aligned}Td_1 &= Td_2 = 0\\ A &= 0\\ B &= Ts_2 + \left(\frac{P_2}{P_1}\right)^2 Ts_1\\ C &= 0\\ D &= \frac{Ts_1 Ts_2}{I_1}\left[1 + \frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]\\ Qn_1 &= \frac{En_1}{I_1}\\ Mn_1 &= 0\\ Mn_1 &= \left[Ts_2 + \left(\frac{P_2}{P_1}\right)^2 Ts_1\right] n^2\omega_1^2 - \frac{Ts_1 Ts_2}{I_1}\left[1 + \frac{I_2}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]\\ Rn_1 &= 0\\ Hn_1 &= Ts_2\\ Gn_1 &= Mn_1\\ \psi n_1 &= \frac{Qn_1 Ts_2}{Nn_1}\ {}^*\end{aligned}$$

$$\psi n_1 = \frac{En_1 Ts_2}{I_1\left\{\left[Ts_2 + \left(\frac{P_2}{P_1}\right)^2 Ts_1\right] n^2\omega_1^2 - \frac{Ts_1 Ts_2}{I_1}\left[1 + \frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]\right\}}$$

$$= \frac{En_1 Ts_2}{\left[Ts_2 + \left(\frac{P_2}{P_1}\right)^2 Ts_1\right] I_1 n^2\omega_1^2 - Ts_1 Ts_2\left[1 + \frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}$$

$$= \frac{En_1}{\left[1 + \left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}\right] I_1 n^2\omega_1^2 - Ts_1\left[1 + \frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}$$

$$\psi n_1 Ts_1 = \frac{En_1}{\left[1 + \left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}\right] \frac{I_1 n^2\omega_1^2}{Ts_1} - \left[1 + \frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}$$

$$\psi n_1 Ts_1 = \frac{En_1}{f\left(\frac{Ts_1}{Ts_2}\right)\frac{1}{X_1} - f\left[\frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}$$

* This equation for simplicity assumes that one engine only is firing and that its torque effort consists of one harmonic only to avoid summations.

where $\frac{P_2}{P_1}$ enters the function $f\left(\frac{Ts_1}{Ts_2}\right)$ and should be considered constant.

40 The above equation is in the form

$$Bf(v) = \frac{Cf(w)}{Df(x) - Ef(y)}$$

which we know can be represented by a nomogram. But this equation can be written:

$$\frac{En_1}{\psi_1 Ts_1\left[1+\left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}\right]} = \left(\frac{I_1 n^2 \omega_1^2}{Ts_1}\right) - \left[\frac{1+\frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2}{1+\left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}}\right]$$

41 At resonance and with zero damping

$$\psi_1 Ts_1 = \infty \text{ (theoretically)}$$

Then

$$\frac{I_1 n^2 \omega_1^2}{Ts_1} = \frac{\left[1+\frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}{\left[1+\left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}\right]} \quad \ldots\ldots\ldots\ldots \quad [5]$$

$$n^2 \omega_1^2 = \frac{\left[1+\frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}{\left[1+\left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}\right]} \times \frac{Ts_1}{I_1} \quad \ldots \quad [6]$$

$$\frac{n^2 \omega_1^2}{S_1^2} = \left(\frac{Ts_1}{I_1 S_1^2}\right) \frac{\left[1+\frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]}{\left[1+\left(\frac{P_2}{P_1}\right)^2 \frac{Ts_1}{Ts_2}\right]} \quad \ldots \quad [7]$$

42 Equation [7] will be recognized as the equation for natural frequency for two machines operating in parallel

$$F = \frac{35{,}200}{S_1}\sqrt{\frac{Po_1 f}{WR_1^2}}\sqrt{\frac{1+\left[\frac{WR_1^2}{WR_2^2}\left(\frac{P_2}{P_1}\right)^2\right]}{1+\left(\frac{Po_1}{Po_2}\right)}}$$

43 This equation is in the form

$$f(u) = Kf(V) \times f(W) \times f(X) \quad \ldots\ldots\ldots \quad [8]$$

which we also know can be represented by a nomogram.

44 Now in the above case damping was neglected. Where damping does enter, the equation will not rigorously fulfil the statement as to factorability. It must therefore be treated as previously described with respect to damping.

45 Thus having shown Equation [4] to be justified, it is desirable from a practical standpoint to reduce the number of factors to the smallest possible. The number of parameters in the practical case can be restricted by stating that within the range of present flywheel requirements the effects due to reciprocating parts and connecting rods can be considered constant. Furthermore, for any specific case the rate of beat of the engines can be regarded as a fixed ratio to the revolutions of the engine, for example, all two-cylinder four-cycle machines will be

alike in this respect. Equation [4] thus simplifies down to

$$\frac{Ep_1}{Eg_1}=F\left[f_1\left(\frac{Ev_1}{Eg_1}\right),\ f_2\left(\frac{Ev_2}{Ev_1}\right),\ f_3\left(\frac{Ef_1}{Eg_1}\right),\ f_4\left(\frac{Ef_2}{Ef_1}\right),\ f_5\left(\frac{V_2}{V_1}\right),\right.$$
$$\left.f_6\left(\frac{S_2}{S_1}\right),\ f_7\left(\frac{Ed_1}{Eg_1}\right),\ f_8\left(\frac{Ed_2}{Ed_1}\right),\ f_9\left(\frac{Eg_2}{Eg_1}\right)\right]\ldots\ldots [9]$$

46 Now restrict the gas forces to a representative torque effort for the type of engine being considered. But by definition Ep_1 was variation in power of the generator and Ev_1 and Ev_2 were variations in gas forces represented by the varying terms of the Fourier's series. Under the restricted gas forces V_1 and V_2 represent the constant term from the corresponding Fourier's series. However, if Ev_1 and Ev_2 be expressed as a ratio to V_1, the factor $\frac{V_2}{V_1}$ will disappear. The Equation [9] thus becomes

$$\frac{Ep_1}{Eg_1}=F\left[f_1\left(\frac{Ev_1}{Eg_1}\right),\ f_2\left(\frac{Ev_2}{Ev_1}\right),\ f_3\left(\frac{Ef_1}{Eg_1}\right),\ f_4\left(\frac{Ef_2}{Ef_1}\right),\right.$$
$$\left.f_5\left(\frac{Ed_1}{Eg_1}\right),\ f_6\left(\frac{S_2}{S_1}\right),\ f_7\left(\frac{Ed_2}{Ed_1}\right),\ f_8\left(\frac{Eg_2}{Eg_1}\right)\right]\ldots\ldots [10]$$

47 Equation [10] can be reduced by substitution of numerically equivalent torques to

$$\frac{Tp_1}{Ts_1}=F\left[f_1\left(\frac{Tv_1}{Ts_1}\right),\ f_2\left(\frac{Tv_2}{Tv_1}\right),\ f_3\left(\frac{Ef_1}{Ts_1}\right),\ f_4\left(\frac{Ef_2}{Ef_1}\right),\right.$$
$$\left.f_5\left(\frac{Td_1S_1}{Ts_1}\right),\ f_6\left(\frac{S_2}{S_1}\right),\ f_7\left(\frac{Td_2S_2}{Td_1S_1}\right),\ f_8\left(\frac{Ts_2}{Ts_1}\right)\right]\ldots [11]$$

Here

$$Ts_1 = 4.224\times10^5\,\frac{Pof}{S^2}$$

Po = the kilowatt output of the generator when the rotor is oscillated to a position of one electrical radian displacement. This is a constant of the generator given by its manufacturer

f = electrical frequency in cycles per second.

The above parameters can be written

$$Tv_1=\frac{\sqrt{a_{n1}^2+b_{n1}^2}}{a_{01}}$$

a_{n1}, b_{n1}, and a_{01}, being factors from the Fourier's series obtained by analysis of the torque effort of machine No. 1

$$Tv_2=\frac{\sqrt{a_{n2}^2+b_{n2}^2}}{a_{01}}$$

where subscripts 2 refer to Fourier's series for machine No. 2. Tv_2 has been expressed as a ratio to the average a_{01} of machine No. 1 because the phenomenon is being referred to that machine as a base. Power pulsation will therefore be derived in per cent of the average load of machine No. 1.

$$\frac{Ef_1}{Ts_1}=\frac{WR_1^2S_1^2}{Ts_1}$$

where WR_1^2 is the total flywheel effect of machine No. 1.

48 This factor can be further reduced to

$$\frac{WR_1^2S_1^4}{Po_1f}$$

The constants have been left out of this parameter.

49 Further this can be rewritten

$$X_1 = \frac{Po_1f}{WR_1^2S_1^4}$$

$$\frac{Ef_2}{Ef_1} = \frac{WR_2^2S_2^2}{WR_1^2S_1^2}$$

$$\frac{Ts_2}{Ts_1} \text{ reduces to } \frac{Po_2S_1^2}{Po_1S_2^2}$$

Td is defined as the damping torque per mechanical radian slip per sec. For commercial generators this ranges between $0.008Ts$ and $0.012Ts$.

$$\frac{Td_1S_1}{Ts_1} \text{ is approximately } 0.01S_1$$

Then $\frac{Td_2S_2}{Td_1S_1}$ will be very closely

$$\frac{Po_2S_1}{Po_1S_2} = \frac{Ts_2S_2}{Ts_1S_1}$$

50 Equation [11] can therefore be rewritten

$$\frac{Tp_1}{Ts_1} = F\left[f_1\left(\frac{\sqrt{a_1^2+b_1^2}}{Ao_1Ts_1}\right),\ f_2\left(\frac{\sqrt{a_2^2+b_2^2}}{\sqrt{a_1^2+b_1^2}}\right),\ f_3\left(\frac{Po_1f}{WR_1^2S_1^4}\right),\right.$$
$$f_4\left(\frac{WR_2^2S_2^2}{WR_1^2S_1^2}\right),\ f_5\left(\frac{Td_1S_1}{Ts_1}\right),\ f_6\left(\frac{S_2}{S_1}\right),\ f_7\left(\frac{Po_2S_1}{Po_1S_2}\right),$$
$$\left.f_8\left(\frac{Po_2S_1^2}{Po_1S_2^2}\right)\right] \qquad \text{[12]}$$

51 But Equation [12] is

$$Tp_1 = Ts_1F(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8) \qquad \text{[13]}$$

Comparing this with the practical case and with published equations of the phenomenon, the equation will be recognized as,

$$Tp_1 = Ts_1\psi'_1 \qquad \text{[14]}$$

where ψ'_1 is the total relative angular displacement between the stator and rotor fields of machine No. 1 and the equation for which is too cumbersome for commercial use.

52 In order to justify the presence of the factor $\frac{S_2}{S_1}$, reference will here be made to the differential equations of the phenomenon. The two simultaneous equations for two machines are

$$I_1\frac{d^2\theta_1}{dt^2} + Td_1\frac{d(\theta_1-\phi_1)}{dt} + Ts_1(\theta_1-\phi_1) = f_1(t) \qquad \text{[15]}$$

$$I_2\frac{d^2\theta_2}{dt^2} + Td_2\frac{d(\theta_2-\phi_2)}{dt} + Ts_1(\theta_2-\phi_2) = f_2(t) \qquad \text{[16]}$$

53 Now from the first of these write an equation which means that the *dimensions* of the factors appearing therein are equal,

$$\left[I_1\frac{d^2\theta_1}{dt^2}\right] = \left[Td_1\frac{d(\theta_1-\phi_1)}{dt}\right] = [Ts_1(\theta_1-\phi_1)] = [f_1(t)]$$

And from the second,

$$\left[I_2 \frac{d^2\theta_2}{dt^2}\right] = \left[Td_2 \frac{d(\theta_2-\phi_2)}{dt}\right] = [Ts_2(\theta_2-\phi_2)] = [f_2(t)]$$

from which

$$\left[\frac{I_1 \frac{d^2\theta_1}{dt^2}}{Ts_1(\theta_1-\phi_1)}\right]^a = \left[\frac{Td_1 \frac{d(\theta_1-\phi_1)}{dt}}{Ts_1(\theta_1-\phi_1)}\right]^b = \left[\frac{f_1(t)}{Ts_1(\theta_1-\phi_1)}\right]^c = 1. \quad [17]$$

$$\left[\frac{I_2 \frac{d^2\theta_2}{dt^2}}{Ts_2(\theta_2-\phi_2)}\right]^d = \left[\frac{Td_2 \frac{d(\theta_2-\phi_2)}{dt}}{Ts_2(\theta_2-\phi_2)}\right]^e = \left[\frac{f_2(t)}{Ts_2(\theta_2-\phi_2)}\right]^f = 1. \quad [18]$$

54 Now keeping in mind that these are two simultaneous equations it will be seen that the parameters

$$\frac{I_1 \frac{d^2\theta_1}{dt^2}}{Ts_1(\theta_1-\phi_1)}, \quad \frac{I_2 \frac{d^2\theta_2}{dt^2}}{I_1 \frac{d^2\theta_1}{dt^2}}, \quad \frac{Ts_2(\theta_2-\phi_2)}{Ts_1(\theta_1-\phi_1)} \quad \ldots\ldots \quad [19]$$

define the relations of factors a and d. Likewise

$$\frac{Td_1 \frac{d(\theta_1-\phi_1)}{dt}}{Ts_1(\theta_1-\phi_1)}, \quad \frac{Td_2 \frac{d(\theta_2-\phi_2)}{dt}}{Td_1 \frac{d(\theta_1-\phi_1)}{dt}} \quad \ldots\ldots \quad [20]$$

are added by factors b and e. Furthermore factors c and f also introduce

$$\left[\frac{f_1(t)}{Ts_1(\theta_1-\phi_1)}\right], \quad \left[\frac{f_2(t)}{f_1(t)}\right] \quad \ldots\ldots \quad [21]$$

so that all of the dimensional factors in Equation [**12**] are obtainable in this second manner from the differential equation.

55 Now there appear factors

$$\frac{V_2}{V_1}, \quad \frac{S_2}{S_1}, \quad \frac{\omega_1}{S_1}, \quad \frac{\omega_2}{\omega_1}$$

in Equation [**12**] that do not appear in Equations [**19**] [**20**], or [**21**]. This is because all of these factors which were obtained *without the mathematical equation* are required or at least appear to explain the nature of $\frac{f_2(t)}{f_1(t)}$ and its relation to the rest of the parameters.

56 The factor $\frac{f_2(t)}{f_1(t)}$ therefore presupposes relations between these factors by a knowledge of two fixed differential equations, [**15**] and [**16**], and this presupposition acknowledges the ratios

$$\frac{V_2}{V_1}, \quad \frac{S_2}{S_1}, \quad \frac{\omega_2}{S_1}, \quad \text{and} \quad \frac{\omega_2}{\omega_1}$$

On the other hand, Equation [**12**] is not based upon the differential equations [**15**] and [**16**] and therefore defines in dimensional language the presuppositions expressed only indirectly in the dimensional factors obtained from the differential equations [**19**], [**20**], and [**21**].

57 From the equation in Par. 25,

$$\psi'_1 = 2\psi_1 D \frac{Tv_1}{Ts_1}\left(1+\frac{Tv_2 S_1}{Tv_1 S_2}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \quad [22]$$

58 This equation is not justified by the dimensional analysis of the case as given in this paper. From what has been said above it will be concluded that for any one curve drawn the ratio $\frac{S_2}{S_1}$, $\frac{\omega_2}{\omega_1}$, $\frac{Ev_2}{Ev_1}$, and hence $\frac{hp_2}{hp_1}$ of the engines should remain fixed relative to each other. This equation is based upon the assumption that the machines contribute to the pulsation in the same manner as the units used in constructing the curves. For example, Putman gives the pulsation between two machines as

$$(Y_1+Y_2) = \frac{En_1}{I_1}\left(\frac{Hn_1}{Gn_1}\right) + \frac{En_2}{I_2}\left(\frac{P_2}{P_1}\right)\left(\frac{Hn_2}{Gn_2}\right) \quad . \ . \ . \quad [23]$$

where P_2 and P_1 refer to poles or at one frequency may be substituted for r.p.m. When the machines are duplicates

$$En_1\left(\frac{Hn_1}{I_1 Gn_1}\right) = En_2\left(\frac{S_2}{S_1}\right)\left(\frac{Hn_2}{I_2 Gn_2}\right) \quad . \ . \ . \ . \ . \ . \quad [24]$$

and remembering En_1 corresponds to Tv_1 and En_2 corresponds to Tv_2. Then

$$2\psi_1 = f\left[En_1 + En_2\left(\frac{S_2}{S_1}\right)\right]$$

and

$$2\psi_1 = f\left[En_1\left(1+\frac{En_2}{En_1}\cdot\frac{S_2}{S_1}\right)\right]$$

59 But when Equation [24] no longer represents the results, Equation [22] cannot be used. This is as far as the rigorous solution can go without a complicated chart construction. These other variables can easily be included, however, by drawing other curves similar to Figs. 5 and 6 with the desired ratio of speeds or capacities as part of the characteristic.

60 It will be shown that the curves similar to Figs. 5 and 6 are derived from duplicate machines. Under these conditions (of duplicate machines), ψ_1 is proportional to $\frac{Tv_1}{Ts_1}$. Further, when $\frac{Tv_1}{Ts_1}$ is constant, the angle ψ_1 for duplicate machines is

$$\psi_1 = M(NTv_1+NTv_2) = M'\left(1+\frac{Tv_2}{Tv_1}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \quad [25]$$

but for machines not duplicates this becomes approximately

$$\psi_1 = M'\left(1+\frac{Tv_2 S_1}{Tv_1 S_2}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad [26]$$

Equation [26] is based upon the assumption that the speeds and capacities of the two machines are approximately the same. If this is not the case, the chart should be extended to include these variables.

61 Thus having given a curve similar to Fig. 5 and having found X_2, read $2\psi_1$. Then

$$\psi'_1 = 2\psi_1 D \frac{Tv_1}{Ts_1} \quad \ldots\ldots\ldots\ldots \quad [27]$$

where D is the ratio $\frac{Ts_1}{Tv_1}$ for the duplicates used in constructing the curve.

$$\text{New angle } \psi'_1 = 2\psi_1 D \frac{Tv_1}{Ts_1}\left(1+\frac{Tv_2 S_1}{Tv_1 S_2}\right) \quad \ldots\ldots \quad [28]$$

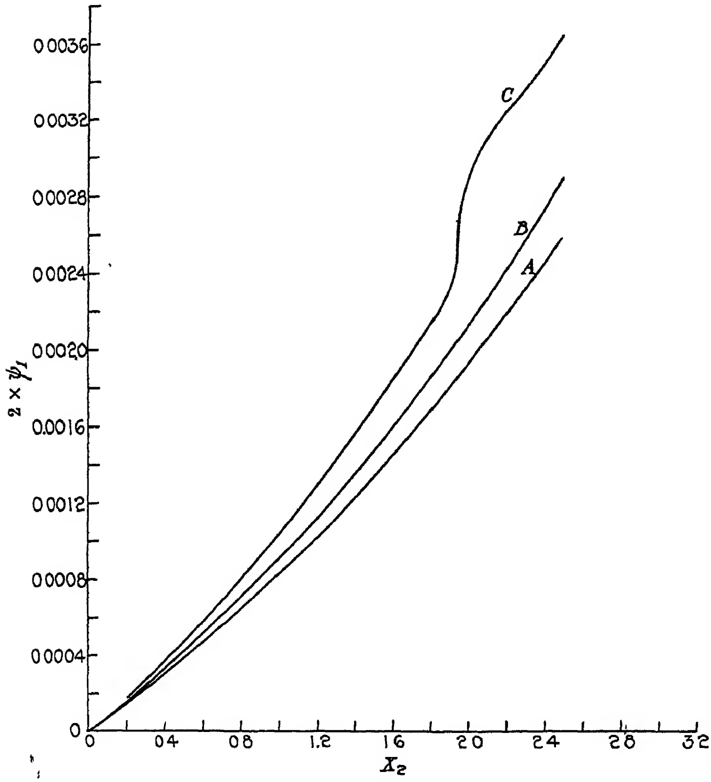


FIG. 5

A — Two 2-cylinder 4-cycle Diesel-engine-driven generators operating in parallel, Fourier's series:

$En_1 = 386$

B — Same as Curve A except Fourier's series used was

$En_1 = 414$ $\quad En_2 = 240$ $\quad En_3 = 57$ $\quad En_4 = 97$

C — Same as Curve B except one cylinder of each engine having a card 10 per cent less in area than other cylinders

Fourier's series:

$En_1 = 9.5$ $\quad En_2 = 426$ $\quad En_3 = 5.8$ $\quad En_4 = 292$

$\frac{Td_1}{Ts_1} = 0.01$ $\quad Ts_1 = 132{,}080$

$S = 277$

62 This can be further simplified by making assumptions that Tv_1 (the harmonic of the Fourier's series), divided by the constant term of the series, is constant. For machines of a given type, like steam engines or Diesels, this is closely true.

63 Assumptions might also be made that the Po is between two and three times the kilowatt rating of the generator. This is rather dangerous and is not recommended for other than most hasty estimates.

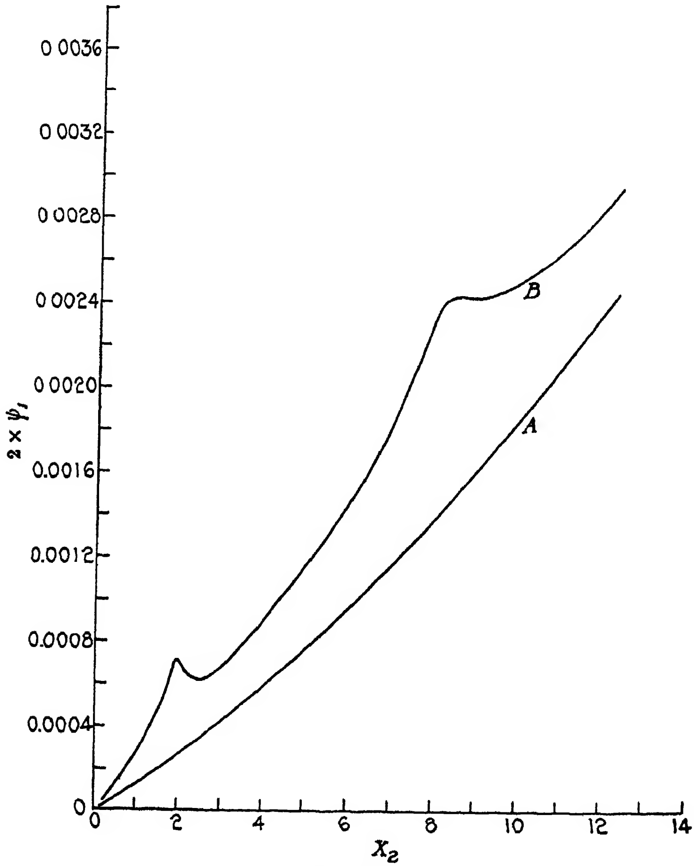


FIG. 6

A — Two 4-cylinder 4-cycle Diesel-engine-driven generators, all cylinders firing equally

Fourier's series: $An_4 = -272$ $Bn_4 = -43.29$

B — The machine of Curve *A* with one cylinder of each engine having a card 10 per cent less in area than the other cylinders

Harmonics:	1st	2d	3d	4th
An	4.31	18.57	0.51	—28.02
Bn	—1.73	0.73	2 79	—43.1

$$\left.\begin{array}{r} Ts_1 = 132{,}080 \\ Td_1 = 0.01Ts_1 \\ \omega = 14.5 \\ S_1 = S_2 = 277 \end{array}\right\}\text{Factors used}$$

It should not be used where machine No. 1 is an old generator or has overload capacity.

64 For *rough work,* therefore, the equation can be reduced by the engine builder to a form for estimation.

$$Tp_1 = 2Ts_1\psi_1 D\left[1+\frac{Kw_2S_1}{Kw_1S_2}\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [29]$$

Equation [29] is subject to the same restriction as is Equation [26].

Practical Construction of Chart

65 The construction of a nomographic chart from experimental data

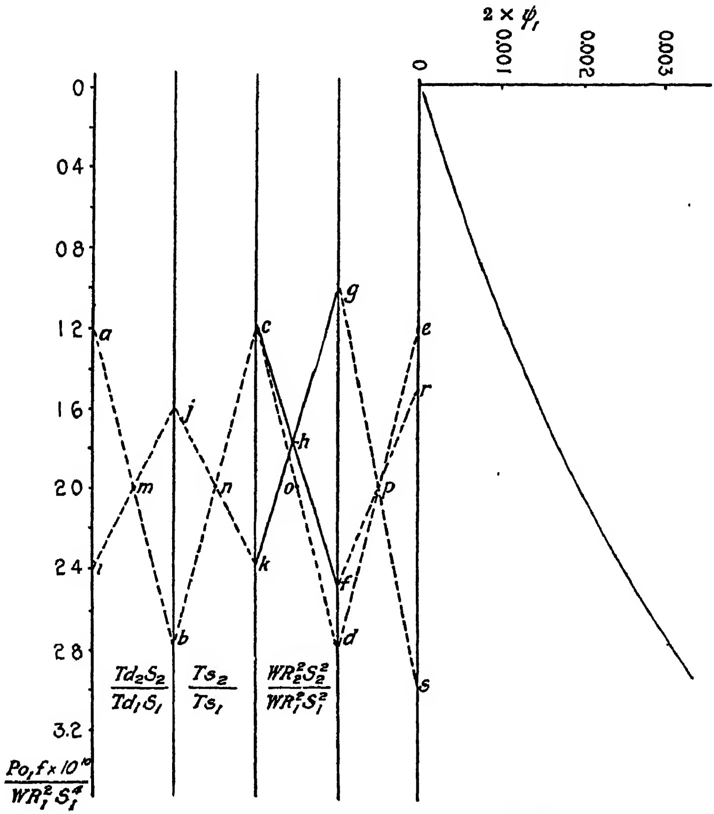


Fig. 7 Example Showing Method of Construction of a Dimensional Nomographic Chart from Test or Calculated Data as Explained in the Text

may be made as follows: Refer to Fig. 7. The first step in the construction was to lay out five parallel lines with arbitrary spacing. In this case equal spacing was used. Then four points *m, n, o,* and *p* were chosen with arbitrary positions with respect to the lines drawn. These points were labelled unity, since the dimensional parameters were unity for duplicate machines.

66 Next an arbitrary scale for the parameter

$$\frac{Po_1 f \times 10^{10}}{WR_1^2 S_1^4}$$

was established.

67 Then, calculating duplicate machines, the curve for some particular type of engine was established by keeping all other parameters unity and always going through points *m, n, o,* and *p*. This is illustrated by line *abcde* and the point shown.

68 Next, after arriving by way of line *abc* at the point *c*, WR^2 was changed, obtaining an angle and thus determining point *r*.

69 Then proceeding from *r* through *p* to *f*, the points *c* and *f* were connected by a line.

70 Likewise a similar calculation for a different value of $\frac{Po_1 f \times 10^{10}}{WR_1^2 S_1^4}$ gave the line *ijkgs* which represented the same value of the parameter

$$\frac{WR_2^2 S_2^2}{WR_1^2 S_1^2}$$

71 Thus the intersection of lines *cf* and *kg* established one point *h* both as to value and position on the curve of the parameter

$$\frac{WR_2^2 S_2^2}{WR_1^2 S_1^2}$$

72 In this way a series of points for each parameter was obtained and the chart completely determined.

DISCUSSION

H. C. Lehn.[1] This paper solves a phase of the problem, which, if heretofore given any thought by the engine builder, occasioned some perplexity by reason of its entire omission in the formula generally used. However, some study is required to discover the process by which this formula is to be made a part of the old calculation, and it would be of great value to have the process clearly set forth. Of equal value would be a general comparison of results obtained by the old formula and by the more accurate method of the author, showing wherein the former leads to serious error.

In particular, the appendix states that Equation [26] is based upon the assumption that the speeds and capacities of the two machines are approximately the same. As the rated speed of one of two units which are to operate in parallel is as likely to be half or three quarters that of the other as it is to differ by only a few per cent, it would be of value to know the limits of the approximation. Again, with regard to differences in capacities, Fig. 3 shows that for two units, one twice the capacity of the other and with synchronizing torque and WR^2 also twice as great, as would normally be the case, the value of X_2 will vary from that of X_1 by only a few per cent, the difference being caused solely by the

[1] Works Engineer, Snow-Holly Works, Worthington Pump & Machinery Corporation, Buffalo, N. Y. Assoc-Mem. A.S.M.E.

damping factor. This would indicate that for such units the old formula can be applied without serious error. This is confirmed by field experience, there being a number of units successfully operating in parallel with equal cylinder dimensions and speeds, but with different numbers of cylinders and with WR^2 and synchronizing torque on each unit proportional to the number of cylinders.

Referring now to the question of units of widely different speeds and of equal or different capacities, it would be interesting to know if (with a WR^2 on each unit of such value that the natural frequency is safely removed from the forced frequencies of both units, and also of such value that the current swing of each unit, considered as paralleling with a steady system, is sufficiently low) the result as shown by the new method would be satisfactory.

Equation [25] in the appendix states that for duplicate machines the displacement angle varies approximately as

$$\left(1+\frac{TV_2}{TV_1}\right)$$

But for duplicate machines, equally loaded, $TV_2 = TV_1$ so that the expression in the parenthesis reduces to 2, and this is the result by the old formula with two duplicate machines thrown in parallel in exactly opposite phase.

It is noticed that the displacement curves, Figs. 5 and 6, do not show any decided peaks, and it is assumed that they have not been carried through the points of coincidence of forced and natural frequencies.

It would seem that trouble might be most expected with large, slow-running, four-cycle units where it is sometimes necessary, on account of otherwise excessive weight, to make the natural frequency higher than the lowest forced frequency. New installations of this type are comparatively few at the present time, but high-speed units to parallel with existing low-speed machines are continually installed. The writer has calculated a considerable number of flywheels for both cases, and in only one was trouble experienced, this being due to the coincidence of the natural frequency of an existing unit with the forced frequency of the new machine. In all cases where all natural frequencies avoided all forced frequencies no trouble occurred.

J. H. Hansen.[1] The author has considered synchronous parallel operation of Diesel engines and generators, but has not taken into consideration the ultimate change of speed in case of load changes. For instance, in the case of the full load being thrown off, what relation would such a change have to the calculation of the flywheel as presented by the author? In other words, is it more important

[1] **President, Pacific Diesel Engine Co., San Francisco, Cal.**

to calculate the flywheel with respect to the parallel-running generator or with respect to the ultimate speed changes that may occur in the different operations of electric power plants?

J. M. DODDS.[1] The forced frequency induced by load changes probably will be so slow (that is, such a long period of time would elapse between the changes as compared with the forced frequency due to the pulsations of the crank where there may be one, two, three, or four cycles even in one revolution) that the design of a flywheel which would take care of load changes would be almost out of the question. A flywheel to take care of load changes does not enter into the problem very seriously, unless the load is pulsating at a frequency commensurate with the revolutions of the prime movers and furthermore is large relative to them. It is the forced frequency that develops due to the change of the torque delivered by the engine through one revolution that requires a special design of flywheel.

THE AUTHOR. The points brought up by Mr. Lehn and Mr. Hansen are very well taken and they will be answered in turn.

The author has taken the liberty of stating these in the form of questions, because he can by this method most effectively bring out his understanding of the points which Mr. Lehn and Mr. Hansen had in mind.

First. How is the short method presented in this paper to be made a part of the "natural frequency — angular deviation" method?

The term X_1 used in this paper can be reduced to a form which corresponds to $\left(\frac{F}{S}\right)^2$ or

$$X_1 = C\left(\frac{F}{S}\right)^2$$

where

$$F = \frac{35{,}200}{S}\sqrt{\frac{Pof}{WR^2}}$$

the familiar natural frequency formula.

But it might be pointed out that if the natural frequency for two machines be calculated according to

$$F' = \frac{35{,}200}{S_1}\sqrt{\frac{Po_1f}{WR_1^2}}\sqrt{\frac{1+\left[\frac{WR_1^2}{WR_1}\left(\frac{P_2}{P_1}\right)^2\right]}{1+\frac{Po_1}{Po_2}}}$$

as given in the appendix, and this natural frequency be sufficiently removed from all forced beats, at the same time having the

[1] Engineering Dept., General Electric Co., San Francisco, Cal.

angular deviation of both units below $3\frac{1}{2}$ electrical degrees, successful operation can be predicted. How far this natural frequency must be removed can only be determined by reference to calculations or curves as given in this paper.

Second. Give a general comparison of results by natural frequency and new method.

In general it may be said that the natural frequency method is inadequate for the solution of parallel operation. Taking into account the time required to calculate natural frequency and individual angular deviations for each engine (which must be done for each proposition), the new method takes less time to apply. This is because the X_2Y_2 curve similar to Figs. 5 and 6 are constructed once only, and no further calculations of angular deviation on the engines are necessary.

Furthermore, the new method shows the actual magnitude of pulsation and just how dangerous tuning on the mechanical beats will be, a phase of the problem which the natural frequency method does not cover.

Third. How are machines of unequal speeds handled in connection with Equation [26]?

It will be noted that the present paper factors only the left-hand member of the differential equation. The paper could be extended to factor the right-hand member which consists of $f_1(t)$ and $f_2(t)$, the two torque efforts.

The results, however, will be about as easily handled by drawing on Fig. 6, for example, curves labeled $\frac{S_2}{S_1} = \frac{1}{2}$ and $\frac{S_2}{S_1} = \frac{3}{4}$.

Fourth. Can the natural frequency formula be safely used under conditions of unequal capacities?

The natural frequency method is not reliable under these conditions.

Take the condition where two machines of equal speed and equal natural frequencies, but of which the second machine has but half the capacity of the first, are to be connected in parallel.

The circulating current flowing through the two machines will be the same in amperage. Part of it is due to the firing of engine number one and part to engine number two. The pulsation in this case will be reduced to three-fourths of what it would be were both the size of machine number one. But the size of machine number two has been reduced to one-half of number one at the same time. In percentage of machine number two, therefore, the pulsation is 150 per cent of what it would have been had they both been of equal capacity. This relation is covered in the appendix.

Fifth. When the natural frequency is "safely removed" from the forced frequencies of both units, and also if such value of current swing, considered as paralleling with a steady system, is sufficiently low, would the result by the new method be satisfactory?

This question brings out one of the greatest dangers in the use of the natural frequency method. If the individual natural frequencies of both machines are *both below* the lowest forced frequency, then the method described in the question is a guide to good operation. But if one natural frequency is above a forced beat and the other below, error will usually result.

As an example, a six-cylinder, four-cycle Diesel will operate very quietly against an infinite system with its natural frequency above half the revolution. Likewise, a two-cylinder, 4-cycle Diesel will operate successfully against an infinite system with its natural frequency below half the revolutions. But when these two machines are operated together the combination has but one natural frequency (given by the formula for F' in the appendix). This will be found in all cases to lie between the two individual frequencies, with the result that it often hits very close to half the revolutions, and unsuccessful operation results.

The natural frequency method, therefore, is not a safe or at least not a definite guide. The new method given in this paper, however, is accurate under parallel operation conditions.

Sixth. Equation [25] seems to be the same as the result obtained by the natural frequency method for two duplicate machines thrown in parallel in exactly opposite phase.

By "phase" is meant the phase angle of the cranks, so that one is hunting forward at the instant the other is hunting backward (in the case of two machines of equal speeds). This is the way the calculations have been made. All calculations in this paper pertain to the maximum phase angle which obtains during operation.

Seventh. Figs. 5 and 6 do not show any decided "peaks." Have they been carried through the points of coincidence of forced and natural frequencies?

Herein may be seen one of the advantages of the new method over the natural-frequency method. These curves have been carried over the forced frequency of half the revolutions in both cases, and Fig. 6 has been extended over the beat at the revolutions. For two machines of equal speed, X_2 of 2.0 represents half the revolutions and 8.0 represents the revolutions.

With the natural-frequency formula it would be necessary to assume that the curve of Fig. 6 went to infinity (or at least is very high), at both $X_2 = 2.0$ and $X_2 = 8.0$, which, of course, it does not do.

Eighth. Is it more important to calculate the flywheel to make it suitable for parallel operation than it is for regulation with sudden changes in load.

Experiences seems to show that in most cases the requirement of regulation demands equal attention, and that the flywheel chosen for either requirement should also fulfil the other.

No. 2012a

COMPARATIVE PERFORMANCE OF AIR PREHEATERS

By Nevin E. Funk,[1] Philadelphia, Pa.
Member of the Society

This paper gives the results of tests on a number of boiler units equipped with air preheaters and one unit using unheated air. All of the boilers are of approximately the same size, are of the same type, and have furnaces and stoker equipments which are similar. Some are equipped with economizers. Different types of air preheaters are used and the areas of the heaters are not the same. The curves which are presented show the comparative performances of the units, the relation between coal fed and flue-gas temperature, the relations between gas-temperature drop and air-temperature rise and percentage of rating, and the relation between the percentage of combustible in the ash and the coal fed.

THIS paper presents comparative data on different types of air preheaters used with boilers equipped with economizers and with boilers not equipped with economizers. The boilers in every case are of approximately the same design. However, there is no intention of drawing any specific conclusions from the data presented, in view of the danger arising from the general application of conclusions which apply only to a given set of conditions. The data will therefore be presented in comparative form and the conclusions as to the application of the different equipment to designs varying from those herein described will be left to the judgment of each engineer in solving his own individual problem.

Description of Units

2 The similarity of construction of the boiler units will be noticed in Figs. 1, 2, 3, and 4 which illustrate the complete design of the furnaces, boilers, economizers, and air preheaters in their various relations.

[1] Operating Engineer, Philadelphia Electric Company.

Contributed by the Power Division and presented at the Providence, R. I., Meeting, May 3 to 6, 1926, of The American Society of Mechanical Engineers.

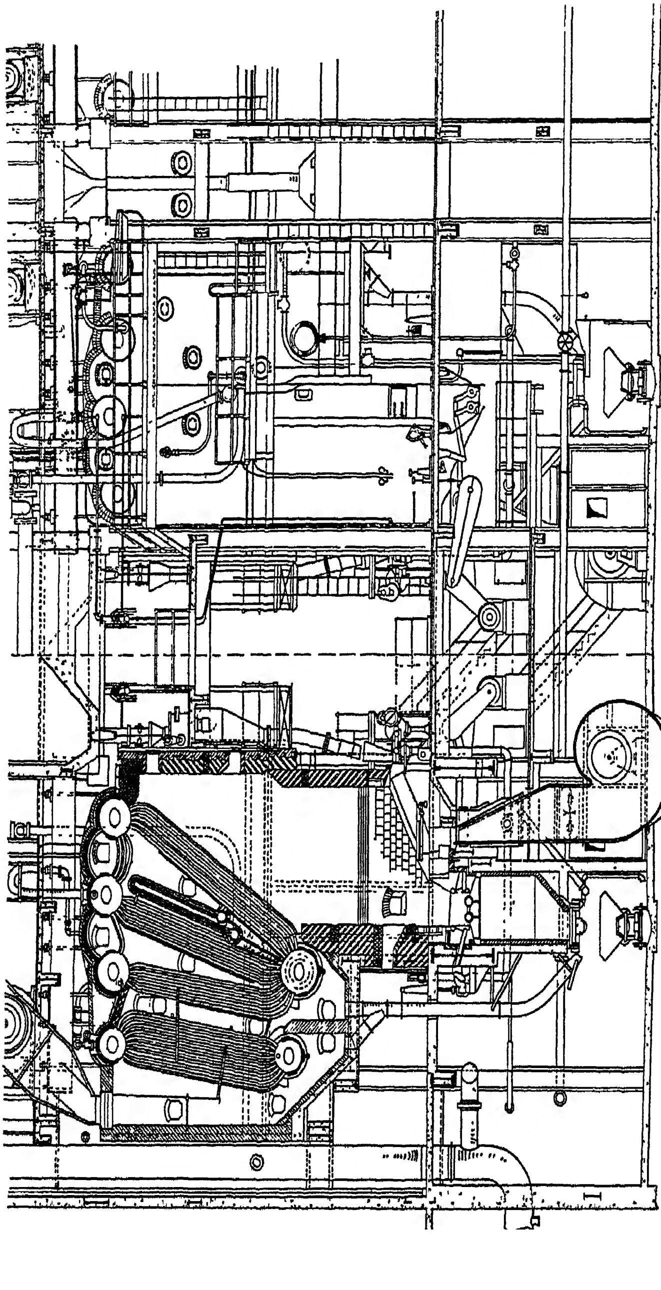


Fig. 1 Cross-Section of Unit No. 23 at the Delaware Station

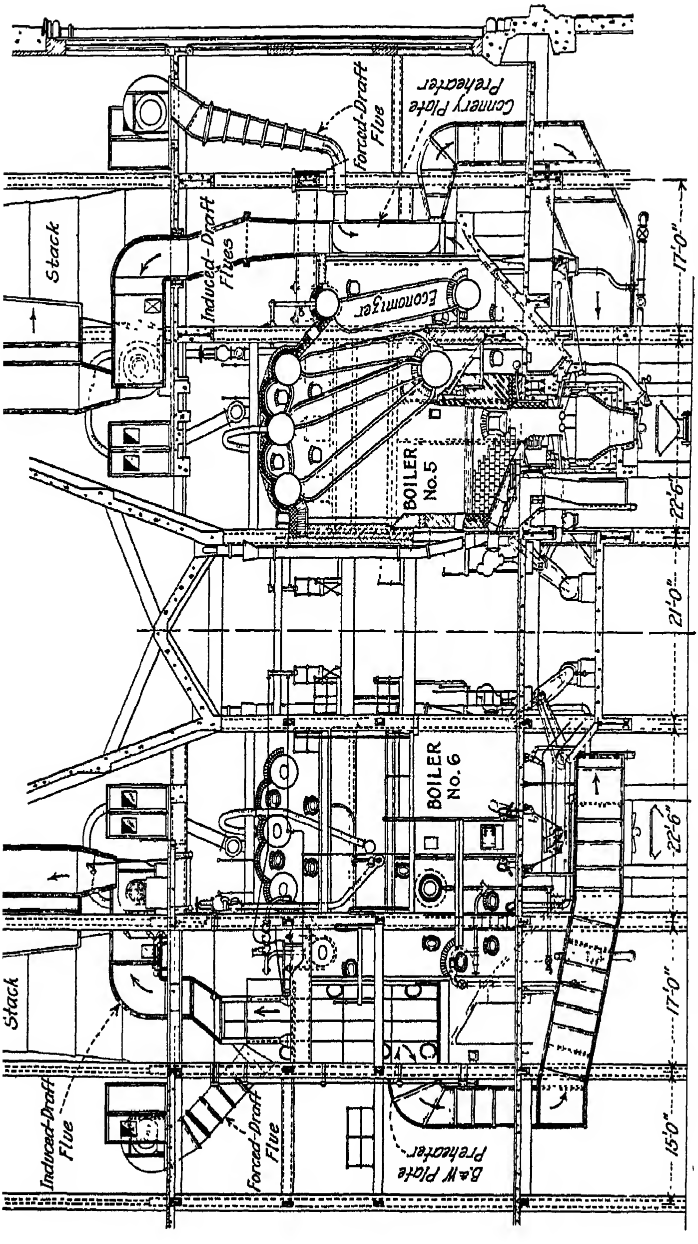


FIG. 2 CROSS-SECTION OF UNIT NO. 5 ON RIGHT. ELEVATION OF UNIT NO. 6 ON LEFT. AT THE CHESTER STATION

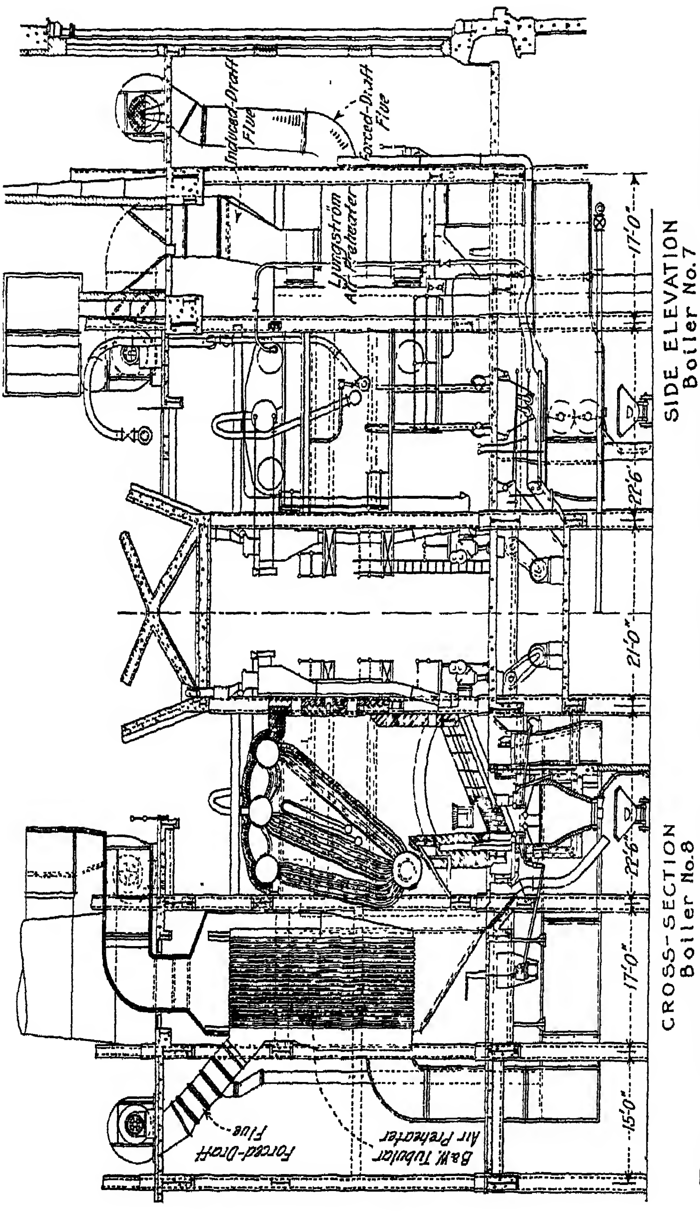


FIG. 3 ELEVATION OF UNIT NO. 7 ON THE RIGHT, CROSS-SECTION OF UNIT NO. 8 ON THE LEFT, AT THE CHESTER STATION

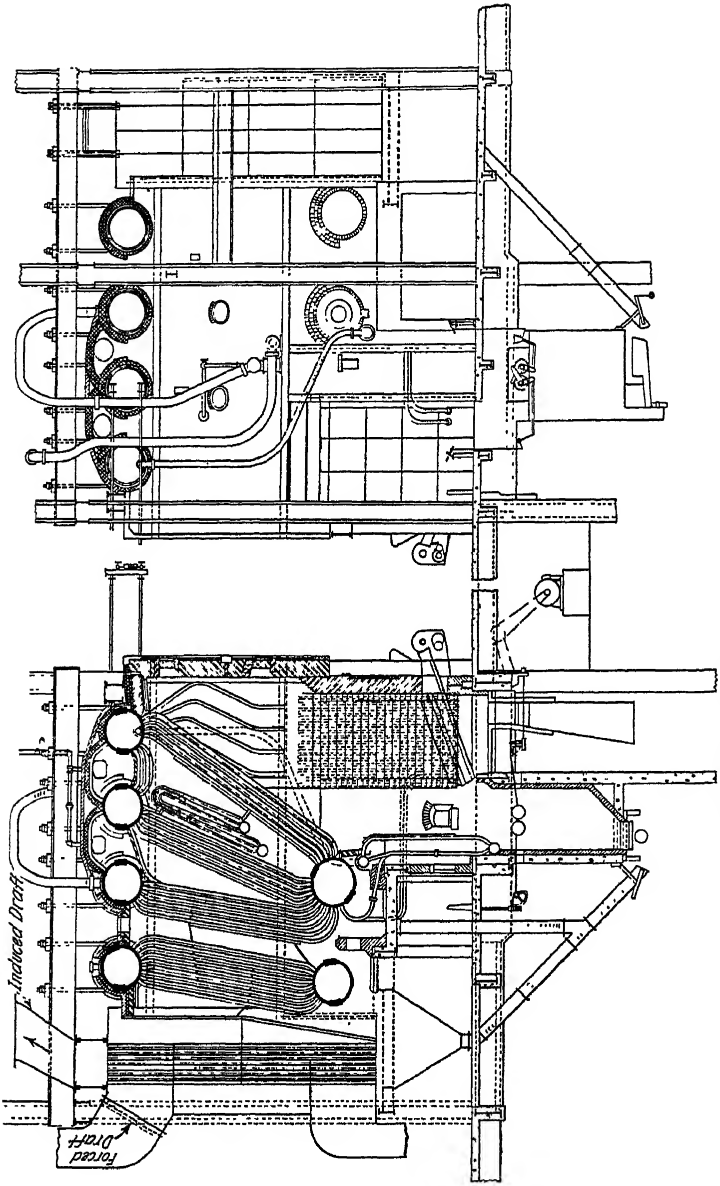


FIG. 4 CROSS-SECTION OF UNIT NO. 59 AT THE RICHMOND STATION

3 Fig. 1 is a cross-section of boiler unit No. 23 at the Delaware station. This consists of a Stirling boiler having 14,217 sq. ft. of heating surface, a superheater having 2582 sq. ft. of heating surface, and, to the left of the boiler, a two-drum, bent-tube, three-pass economizer having a heating surface of 6853 sq. ft. The stoker serving this boiler has 15 rams and is 21 tuyeres long. The ashpit is 3.8 ft. wide and 4 ft. deep, measured at the end of the extension grates.

4 On the right-hand side of Fig. 2 is a cross-section of boiler unit No. 5 at the Chester station. This consists of a Stirling boiler having 14,217 sq. ft. of heating surface, a superheater having 2582 sq. ft. of heating surface, and a two-drum, bent-tube, three-pass economizer having a heating surface of 5250 sq. ft. In the rear of the economizers is a plate air preheater designed by The Philadelphia Electric Company and developed by Connery Company, Inc. It has a heating surface of 22,500 sq. ft. On the left-hand side of Fig. 2 is an elevation of boiler unit No. 6 at the Chester station. This boiler and economizer have the same areas as those of unit No. 5. The air preheater, however, is a B. & W. plate heater having 20,719 sq. ft. of heating surface. Boiler unit No. 3 at the Chester station has exactly the same boiler, superheater, and economizer heating surface as Nos. 5 and 6, but the air preheater is of the tubular type of B. & W. design and contains 22,072 sq. ft. of heating surface. This preheater utilizes practically all the space between the rear of the boiler setting and the vertical building columns. Otherwise, the cross-section is practically the same as shown on the right-hand side of Fig. 2.

5 On the right-hand side of Fig. 3 is an elevation of boiler unit No. 7 at the Chester station. The areas of boiler and superheater heating surfaces are the same as those of the other Chester boilers. In this case, however, there is no economizer, the exit gases from the boiler passing directly into two Ljungström air preheaters in parallel, each having a manufacturers' rated heating surface area of 32,480 sq. ft. The left-hand side of Fig. 3 shows a cross-section of boiler unit No. 8 at the Chester station which is identical with No. 7 except that the heat in the waste gases is reclaimed by a B. & W. tubular air preheater having a heating surface of 50,276 sq. ft.

6 All the boilers in the Chester station are served by the same type of stokers. These stokers have 15 rams and are 21 tuyeres long and in all cases the ashpits are 3.8 ft. wide and 5.3 ft. deep.

7 Fig. 4 is a cross-section of boiler unit No. 59 at the Richmond station. The general arrangement of this boiler is the same as that of No. 3 at the Chester station. The boiler, superheater, and economizer surfaces, however, are somewhat larger. The respective heating-surface areas are 15,692, 2822, and 7515 sq. ft. The area of air-preheater surface is the same as that of boiler

unit No. 3 at Chester. This air preheater is of the tubular type. The greatest deviation in the design of the furnace of this boiler from those of the Chester boilers is in the use of Bailey water-cooled bridge and side walls. This boiler is served by a stoker with 15 rams and 25 tuyeres. The ashpit is 3.8 ft. wide and 5.3 ft. deep.

Circulation of Water in Walls

8 A description of the circulatory system of the water-cooled walls will probably be of value. Water from the lower drum of the boiler passes through the rear bent-tube connection down the circulating tubes to the bottom header of the water-cooled bridgewall. The top and bottom headers of the water-cooled bridgewall are connected by the tubes supporting the blocks which directly face the furnace and by circulating tubes back of this front row of tubes. This allows a direct circulation between the top and bottom headers of the bridgewall. The top header is connected on the outside of the boiler to the top front water drum as is indicated on the right-hand side of Fig. 4.

9 Water also passes from the bottom drum of the boiler through bent tubes outside of the boiler housing and progresses toward the front of the boiler to the bottom header of the Bailey side walls. These tubes are shown at the bottom of the right-hand side of Fig. 4. From this bottom header the water rises through the tubes, to which the blocks are attached, to the top header and thence through four bent tubes inside of the furnace to holes in the bottom of the top front drum, from which the usual tubes passing between the top and bottom drums have been omitted.

Comparison of Equipment

10 Table 1 has been compiled to permit of a rapid comparison of the characteristics of the heating surface of the boiler units. Attention should be called to the fact that all the Chester boilers and superheaters, as well as the Delaware boiler and superheater, have exactly the same amount of boiler heating-surface area. All the Chester boilers with economizers have the same amount of economizer-surface area while the Delaware boiler has a somewhat larger economizer-surface area. The small air preheaters at Chester have approximately the same amount of area of heating surface while the two large preheaters vary considerably in area, due to the wide variation in the types of these preheaters.

11 The Richmond boilers, superheaters, and economizers are somewhat larger than the other units but the ratios of their super-

TABLE 1 BOILER INSTALLATION CHARACTERISTICS
(B. & W. Stirling Boilers, 55 tubes wide)

Station	Boiler No.	Date installed	Heating surface: Boiler	Heating surface: Superheater	Heating surface: Economizer	Heating surface: Preheater	Heating surface: Water wall	Economizer feed temp deg fahr.	Approximate final steam condition: Lb. per sq. in., gage	Approximate final steam condition: Total temp., deg. fahr.
Chester	3	10/31/24	14,217 sq. ft. 100%	2,582 sq. ft. 18.15%	5,250 sq. ft. 36.95%	22,072 sq. ft. T 154.8%	....	172	257	652
	5	2/14/25	14,217 sq. ft. 100%	2,582 sq. ft. 18.15%	5,250 sq. ft. 36.95%	22,500 sq. ft. P 158.2%	....	172	257	652
	6	12/6/24	14,217 sq. ft. 100%	2,582 sq. ft. 18.15%	5,250 sq. ft. 36.95%	20,719 sq. ft. P 145.7%	....	172	257	652
	7	2/5/25	14,217 sq. ft. 100%	2,582 sq. ft. 18.15%	....	64,960 sq. ft. L 457.0%	....	172	257	652
	8	6/1/25	14,217 sq. ft. 100%	2,582 sq. ft. 18.15%	....	50,276 sq. ft. T 353.5%	....	172	257	652
Delaware	28	7/5/24	14,217 sq. ft. 100%	2,582 sq. ft. 18.15%	6,853 sq. ft. 48.2%	....	....	167	257	633
Richmond	59	10/3/25	15,692 sq. ft. 100%	2,822 sq. ft. 17.9%	7,515 sq. ft. 47.85%	22,072 sq. ft. T 140.7%	595 sq. ft. 3.79%	235	410	658

T = Tubular air heater L = Ljungström heater P = Plate type heater

heater and economizer heating-surface areas are about the same. The preheater heating-surface area of the Richmond boiler is the same as that of the tubular preheaters at Chester, but since the boiler is larger, the ratio of preheater heating-surface area to boiler heating-surface area is somewhat smaller. In addition, the Richmond boiler has 595 sq. ft. of water-cooled furnace walls, which is equivalent to 3.79 per cent of the boiler heating-surface area.

12 It will be noted that the feedwater temperature is practically the same for all of the low-pressure boilers. The Richmond boiler, however, receives feedwater at a considerably higher temperature. The steam pressure is the same at Chester and Delaware, but is approximately 150 pounds higher at Richmond. The steam temperature of all the Chester boilers is the same. It will be noted that the total steam temperature at the three stations does not vary to a great extent.

13 Table 2 shows the similarity of the stoker and furnace characteristics of the boiler settings. All stokers have 15 rams. The Richmond stoker is four tuyeres longer

than the Chester stoker. The grate surfaces of both the Delaware and Chester units are the same, and that of the Richmond unit is 27 sq. ft. greater. The furnace volume is the same for all the Chester units. The Delaware unit has a somewhat larger furnace volume because the boiler is two feet higher above the stoker. The Richmond furnace volume is larger than the Delaware furnace volume because the furnace is about one foot deeper. These changes in furnace volume will have some effect on the boiler efficiency, but the general designs are so nearly the same that a comparison of air-preheater performance is but slightly affected. The widths of all the ashpits are the same. The one at Delaware is shallower than all the rest by 1.3 ft.

TABLE 2 STOKER CHARACTERISTICS

(American Engineering Co., Taylor, Type H. C. 7)

Station	Unit No.	Grate surface, sq. ft.	Number of rams	Number of tuyeres	Furnace: Volume above ashpit, cu. ft.	Furnace: Height of mud drum from floor, ft.	Ashpit at heel of fire: Width, ft.	Ashpit at heel of fire: Depth, ft.
Chester	3	310	15	21	6650	11	3.8	5.3
Chester	5	310	15	21	6650	11	3.8	5.3
Chester	6	310	15	21	6650	11	3.8	5.3
Chester	7	310	15	21	6650	11	3.8	5.3
Chester	8	310	15	21	6650	11	3.8	5.3
Delaware	23	310	15	21	7200	13	3.8	4.0
Richmond	59	337	15	25	7780	13	3.8	5.3

Air Preheaters Compared

14 In Table 3 are given the physical characteristics of all the preheaters with the exception of the Ljungström, which cannot be compared with the others on the same basis. It will be noted that the tubular heaters are constructed of tubes of the same diameter and length, and that the increased surface in the large preheater is made up by the addition of more tubes. In event of damage to any of the tubes or to the plate headers in which they are inserted, the faulty structure may be readily removed and renewed. Due to their construction, welding of all parts of the plate heaters is necessary. It is difficult to locate leakage in such heaters, and even more difficult to correct it.

15 In comparing the characteristics of these preheaters with each other and with a boiler without any preheater, the boiler numbers have been placed on all the curves to indicate to what units they particularly apply. In reviewing the physical characteristics of the units compared, the following tabulation, which gives the economizer and air-preheater equipment of all of the boilers, may be used.

Boiler No.	
3	Economizer and small tubular air preheater
5	Economizer and small plate preheater, Connery type
6	Economizer and small plate preheater, B. & W. type
7	Ljungström air preheater only
8	Large tubular preheater only
23	Economizer only
59	Economizer, small tubular air preheater and water wall.

Comparison of Performances

16 The performances of boiler and economizer units equipped with air preheaters cannot be considered on the basis of the heat in the fuel introduced into the furnace alone, but must be compared on the basis of total heat injected into the furnace. This includes the heat in the fuel and the heat in the air above atmospheric temperature.

17 In Fig. 5 are two families of curves. Group A indicates the output of the boiler and superheater alone. Group B indicates the output of the entire unit including economizer and preheater. It is to be noted that the performance of the boiler alone without preheated air is poorer than that of any of the boilers using preheated air and that except at fairly high rates of operation there is little difference in the performances of those using preheated air at low and those using preheated air at high temperatures. With the increase in rates of operation, however, the boiler supplied with preheated air at low temperature maintains a much higher output ratio than do the two supplied with preheated air at higher temperature.

TABLE 3 PREHEATER CHARACTERISTICS

Boiler No.	3	5	6	7	8	59
Heating surface, sq. ft.	22,072	22,500	20,719	64,960	50,276	22,072
Type	Tubular, B. & W.	Plate, Connery & Co.	Plate, B & W.	Rotary, Ljungstrom	Tubular, B. & W.	Tubular, B. & W
Rows of tubes	15	......	......	......	41	15
Tubes per row	90	......	......	......	75	90
Tube diameter — or thickness of gas passage, in.	$2\frac{1}{2}$	$\frac{5}{8}$ (gas)	$1\frac{1}{4}$	......	$2\frac{1}{2}$	$2\frac{1}{2}$
Metal thickness	12 B.w.g.	$\frac{1}{16}$ in.	16 B.w.g.	......	12 B w g	12 B w g
Length of tube, ft.	25	......	......	......	25	25
Overall duct height, ft	......	15	24	......	......	......
Overall duct width	......	3 ft. 2 in.	4 ft.	......	......	......
Number of gas ducts	......	252	99	......	......	......

18 The author has no doubt concerning the accuracy of tests of units 7 and 8 in so far as their relation to the test of unit No. 3 is concerned, but he is inclined to believe that, although these were the actual performances obtained from the boilers, they are not indicative of what may be expected with highly preheated air. This belief is based on two outstanding conditions which prevent

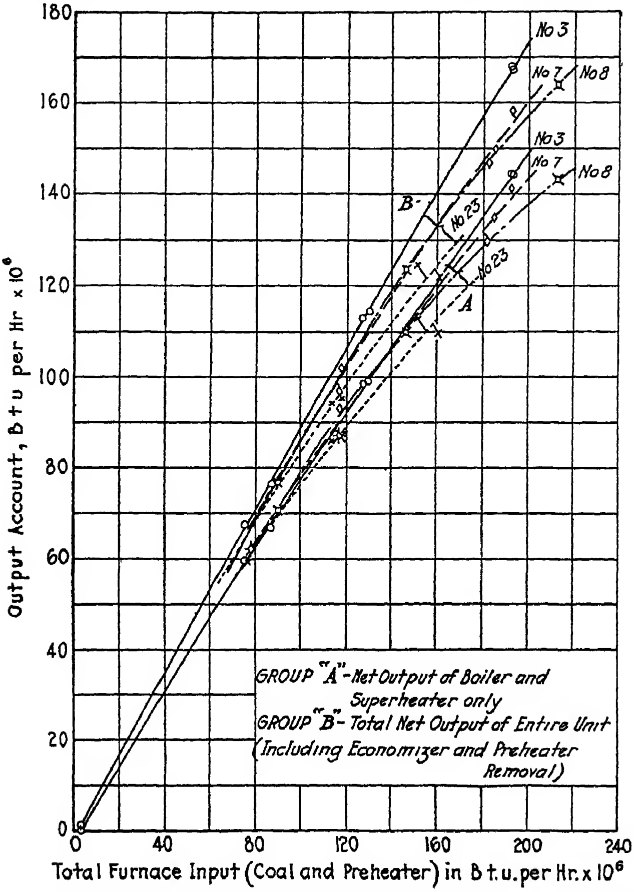


FIG. 5 RELATION OF TOTAL HEAT OUTPUT TO TOTAL HEAT INPUT

the attainment of high efficiency with these units. First, at high rates of operation, the furnace refractories will not stand up under the higher preheated-air temperatures, and it is necessary therefore to reduce the percentage of CO_2 somewhat to preserve the furnace walls. Second, with the greater quantity of heat liberated in the furnace, the boiler alone apparently does not absorb as much and the temperature of the exit gases rises rapidly. This puts an extra burden on the air preheaters.

19 The first condition can be met by water-cooled furnace construction. The second condition can be corrected by the installation of increased boiler surface or a change in boiler baffling.

20 It is desirable to bring out these two points very strongly for the reason that there might be a tendency, from an analysis of these curves, to assume that the use of a preheater, without an economizer between the preheater and the boiler, was in general productive of lower efficiency. This is true for the particular design tested, but it is felt that errors in design have been made and that the lower efficiency is not a fundamental characteristic of high preheated-air temperatures.

21 During 1926 boiler No. 7 will have a Bailey wall installed in the furnace and it is believed that future tests will show that

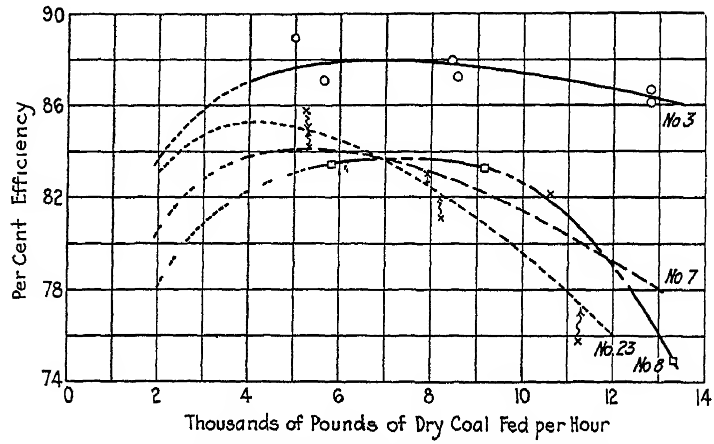


FIG. 6 RELATION OF BOILER EFFICIENCY TO RATE OF FUEL FEED

$$\text{Per cent efficiency} = \frac{\text{Net heat in steam}}{\text{Total heat in coal fired}} \times 100$$

the efficiency of this unit can be favorably compared with that of No. 3.

22 The B group of curves merely substantiate the conclusions drawn for the A group. It will be noted, however, that the curves for units 7 and 8 are much closer together than those of the A group, which indicates that the tubular preheater is performing somewhat better than the Ljungström preheater.

23 Attention is called to the fact that unit No. 3 is showing a still better performance than either No. 7 or No. 8, since the deviation of the curves for these units and that for unit No. 3 is greater in the B group than in the A group. This, however, is due largely to the fact that the economizer is reducing the flue-gas temperatures so that the air temperatures do not rise as rapidly as they do in Nos. 7 and 8.

24 Fig. 5 does not give a true comparison of the efficiencies of the boiler unit for a given amount of fuel consumption but does give the performance of the boiler surface. For this reason, curves have been plotted in Fig. 6 showing the efficiencies of these units at different rates of fuel consumption. These are overall efficiencies of boiler, superheater, economizer, and air preheater. There are some very distinct differences in the shapes of these

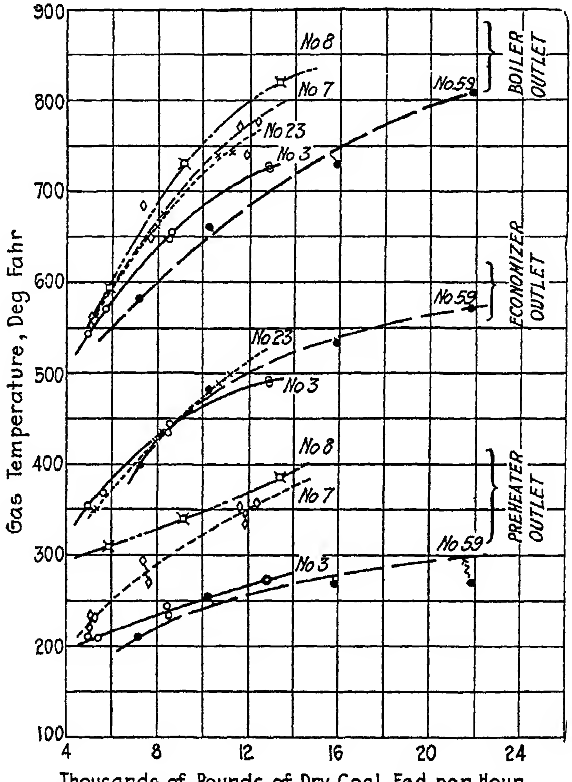


FIG. 7 RELATION OF GAS TEMPERATURES TO RATE OF FUEL FEED

efficiency curves. Unit No. 23, which has no preheater, shows a distinctly rapid drop in efficiency with increased rate of operation. Unit No. 3, which is practically the same as No. 23, except that an air preheater has been supplied, shows an unusually flat efficiency curve.

25 The efficiencies of units 7 and 8, which have the large air preheaters, are distinctly lower than the efficiency of No. 3, although they are somewhat flatter, and at heavy loads higher, than those of No. 23. Unit No. 8 shows a very rapid dropping off in efficiency, not due to the characteristics of highly preheated air,

but to the fact that the boiler furnace is not properly designed to care for the high temperatures. The rapid downward bend in the efficiency curve of unit No. 8 is accompanied with severe burning of the extension grates of the stoker. This is apparently caused by the reflected heat from the bridgewall, and if the performance of a stoker with a similar ash discharge end can be used

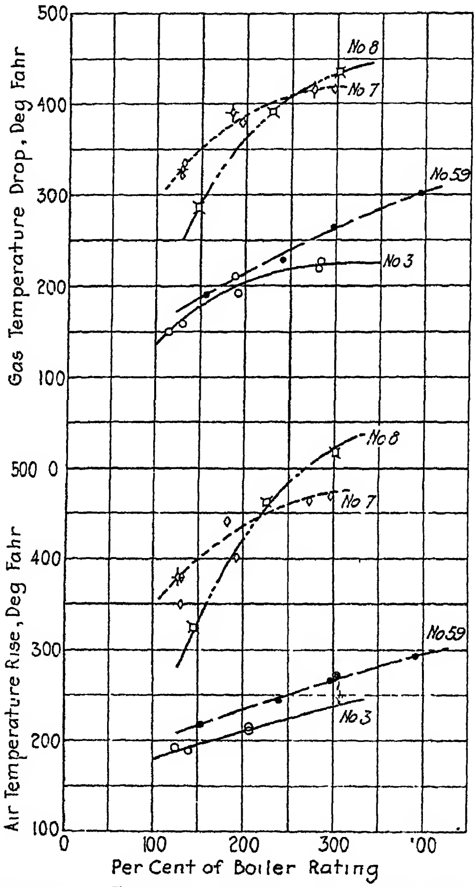


FIG. 8 TEMPERATURE CHANGES THROUGH AIR PREHEATER. RELATION OF GAS TEMPERATURE DROP AND AIR TEMPERATURE RISE TO RATE OF BOILER OPERATION

for comparison, the installation of a water-cooled wall at this point would raise by 3 or 4 per cent the lower end of the curve for unit No. 8. This is predicated on the preliminary tests of unit No. 59 at Richmond, the results of tests of which have not been shown because they are of a preliminary nature and unreliable, due to the fact that the fuel measurements could not be made accurately

at that time because of the lack of some of the weighing equipment. Accurate tests of temperature relations throughout the boiler have been made, however, and some of the succeeding curves give a fairly correct idea of what may be expected from these boilers.

Comparison of Flue-Gas Temperatures

26 In Fig. 7, comparative data are given on the boiler, economizer, and preheater outlet-gas temperatures. Attention is called to the fact that both units No. 7 and No. 8 with the highly preheated air have higher outlet temperatures than unit No. 23 with

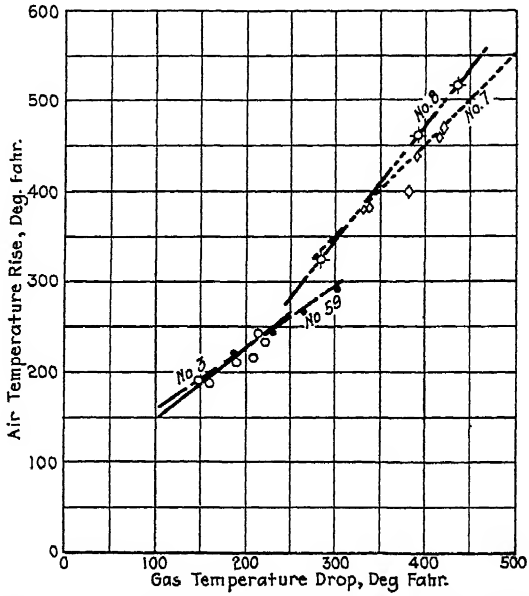


Fig. 9 Ratio of Gas Temperature Rise to Air Temperature Drop in Air Preheaters

cold air. Unit No. 3, however, with a preheated-air temperature of 300 deg. fahr., has an outlet-gas temperature considerably lower than that of unit No. 23, and unit No. 59 with the Bailey walls has a lower temperature than any of the others. The economizer outlet temperatures are approximately the same for all units having economizers, that of unit No. 3 being somewhat lower than that of No. 59. The increased temperature of economizer outlet gas in unit No. 59 is primarily due to the fact that the feedwater temperature is 63 deg. fahr. higher than that of unit No. 3.

27 The air preheaters on Nos. 3 and 59 are identical, and the exit gases from these two preheaters are at practically the same

temperature. The air preheaters on Nos. 7 and 8, however, have considerably higher exit-gas temperatures, due apparently to the higher exit temperatures of gas from the boilers. This condition is more clearly shown in Fig. 8, in which the temperature drop in the flue gas and the temperature rise in the air supplied to the stoker have been plotted against percentage of boiler rating. In this figure, as in Fig. 7, it will be noted that the air preheaters of units 3 and 59 are almost parallel in performance, No. 59 showing a somewhat better heat transfer than No. 3. These preheaters were tested under practically the same condition of cleanliness. Preheaters of units Nos. 7 and 8 reduce the gas temperature and increase the air temperature through a range from 50 to 80 per cent greater than the smaller air preheaters. This is to be expected on account of the larger area of heating surface.

Rate of Heat Transfer

28 It is interesting to note the rate of heat transfer through air preheaters of different types. The curves of Fig. 9 have been plotted to show the relation between the temperature drop of the gas and the temperature rise of the air. The temperature increment for the preheaters of units 7 and 8 is greater than that for units 3 and 59. Apparently this is caused by the rapid increase of the temperature of the outlet gases in these two boilers. This throws more work on the air preheaters, increases the average temperature through the air preheater, and permits of a high rate of heat transfer.

29 Two types of plate heaters were shown in Table 1 for units 5 and 6. In all the figures that have been shown so far no information has been given relative to these two units. This is because no efficiency data have been obtained for the boiler units as a whole. All the boilers for which tests have been shown are equipped with Richardson scales for weighing the fuel. These scales are accurately checked and balanced before, after, and during the tests. Arrangements are made to measure carefully the water to the boilers and accurate data can thus be obtained. This equipment was provided for units 7 and 8 because the preheaters were essentially different from those of the remaining installations. This test equipment was placed on unit No. 3 only, it being felt that the performance of the plate-type air preheaters relative to the small tubular air preheaters could be readily obtained from the temperature measurements of these equipments. Unit No. 23 at the Delaware station is likewise equipped with all test apparatus. Unit No. 59 at Richmond is not the test boiler and is not equipped with proper fuel-weighing devices. For this reason it is impossible to get an accurate efficiency test on unit No. 59. The temperature curves, however, have been correctly obtained from this boiler,

since it is equipped with recording instruments for operating purposes and from which the data can be readily obtained.

30 When new, the performances of the plate-type air preheaters are quite equal to those of the tubular air preheaters. It is, however, much more difficult to keep the plate-type air preheaters clean. For this reason, comparisons of these different types of air preheaters should be made after they have been in service for some time. With the desire to get accurate comparisons of the yearly performance that could be expected from the plate and the tubular preheaters, units Nos. 3, 5, and 6 were allowed to

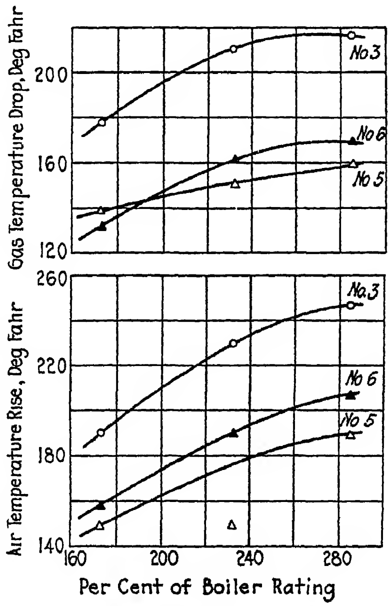


Fig. 10 Comparison of Temperatures for Plate and Tubular Heaters of Equivalent Heating Surface

run under the same load conditions for about nine months before the data included in Fig. 10 were obtained. The air preheaters, boilers, and economizers were all thoroughly cleaned so that there could be no question of discrimination in favor of one type of preheater. The boilers were then operated for approximately a month and then temperatures were taken at various rates of operation for a sufficient period of time to allow the temperature conditions to become settled. The resulting data are plotted in Fig. 10. It will be noted that there is considerably less rise in the temperature of the preheated air in the two plate heaters than in the tubular heater. Likewise, the temperature drop in the gases is less in the plate heaters than in the tubular heaters.

31 The figures of relative temperatures for units Nos. 3, 5, and 6 are given with the understanding that they are not to be compared with data from twenty-four-hour tests of clean boiler units, but are of relative value for boiler units under the usual operating conditions. They were taken from the charts of recording meters, and, while an attempt was made to operate the boilers close to

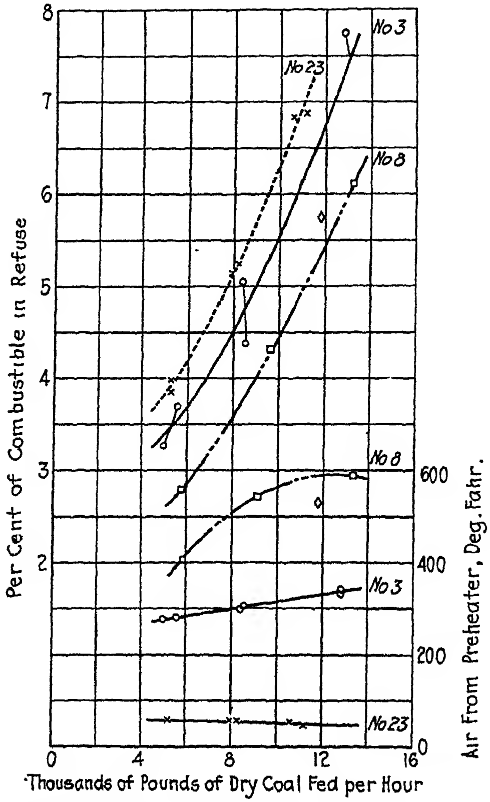


FIG. 11 A COMPARISON OF THE REDUCTION IN THE COMBUSTIBLE IN THE REFUSE WITH INCREASE IN PREHEATED-AIR TEMPERATURE

the average rate of operation as plotted, there was some variation above and below this rate due to the fluctuations in station loading. This, of course, has some effect on the performance of the thermometers. Figures substantiating the values given can be taken directly from the charts on these various boilers and always show a lower temperature of preheated air for the plate heaters than for the small tubular heaters.

Effect of Air Temperature on Combustible in Ash

32 Some figures have been given in the past showing the reduction in the amount of combustible in the ash with the use of preheated air. For this reason it is believed that the data given in Fig. 11, which show a variation in the percentage of combustible in the refuse with variation in rate of operation and with various temperatures of preheated air, will be of considerable interest, because they not only bear out data heretofore presented, but also show a very definite change with the change in temperature of preheated air.

33 In Fig. 11 the change in temperature of preheated air with the rate of coal feed and the change of the percentage of combustible in the refuse with the rate of coal feed have been plotted. The air temperature for unit No. 23 is that of the boiler room, and the fact that it has decreased slightly with the increased rate of operation merely shows that the room temperature has varied, since this air temperature has no relation to the rate of operation. It will be noted that the curve of combustible in the refuse for unit No. 23 is the highest of all. The temperature of preheated air of unit No. 3 varies from slightly below 300 deg. fahr. to about 350 deg. fahr., and the combustible in the refuse for varying rates of operation of this unit is less by about ½ per cent than the combustible in the refuse of unit No. 23. It must be noted at this time, however, that the ashpit of unit No. 3 is somewhat deeper than that of No. 23 and this in itself may be responsible to some extent for the reduction in the refuse. The air temperature for unit No. 8 varies from 400 to 600 deg. fahr., and it is extremely interesting to note that the combustible in the refuse from unit No. 8 is the lowest of all. That is, the boiler with the highest preheated-air temperature has the lowest percentage of refuse in the ash. The ashpits of units Nos. 8 and 3 are practically identical. The small diamond on Fig. 11 is a spot test for boiler No. 7, and it is worthy of note that its position between the percentage of combustible in the refuse for units 3 and 8 bears a close relation to its position between the temperatures of preheated air for units 3 and 8. This relation follows almost too closely to be a matter of chance and there is apparently a very definite relation between reduction of combustible in the refuse and increase in the temperature of preheated air.

34 The author appreciates the fact that an accurate determination of the combustible in the refuse is very difficult to obtain, and because of this fact his company makes use of a small ash grinder. All the ash from a boiler test is run through this grinder and reduced to very fine particles. The entire ash discharge from the test is then quartered to a laboratory sample by the method of the Bureau of Mines and is analyzed. In the test of unit No. 3 there are two values for approximately the same fuel rating. These are

from two separate tests which were made while the boiler was being operated, first from a low to a continually higher rate of driving, and then back to a lower rate.

35 The data given in this paper have been obtained from carefully made efficiency tests of all boilers for which efficiency curves have been given. Data for the boilers for which no efficiency tests have been plotted are not quite as accurate. An exception may be made of unit No. 59, for which the output and the temperatures have been quite accurately measured, but for which the fuel weight is not accurate and has not been given.

36 The author wishes to repeat this note of warning: that the performance of these preheaters must not be applied directly to different types of boiler installations without due allowance for changed conditions. A higher temperature of exit gas from the boiler or economizer with the same air-preheater surface area necessarily results in a higher stack-gas temperature, but it also results in a greater heat transfer through the air preheater.

37 The data on performance seem to indicate that the plate heater is not continually as effective for a given surface area as the tubular heater. This, however, may be due to the particular design of the heaters and must not be taken as a condemnation of the plate heater or as an indication that the plate heater will always perform less satisfactorily.

38 Without stating any definite conclusion, the author believes that the plate heater will always be more difficult to keep clean, and for this reason is likely to show a less satisfactory heat transfer than the tubular heater which may be more easily cleaned. The Ljungström heater is even less difficult to keep clean than the tubular heater, and for this reason will show a more uniform heat-transfer rate under continuous operation than either of the other types of heaters. The performance of the Ljungström heater in every-day operation, however, is likely to be misleading since the air leakage from the forced-draft fan into the flue gases through the rotor, due both to air entrainment and to pressure leakage, reduces the measured temperature of the flue gas below the actual temperature of exit gas from the rotor, and leads an operator erroneously to believe that the flue-gas temperatures are low.

39 The author wishes to take this opportunity publicly to thank Mr. O. H. Paxson, Jr., of The Philadelphia Electric Company, for his assistance in preparing the data contained in this paper.

DISCUSSION

John Van Brunt.[1] With the present rapidly increasing use of air preheaters any data or tests of boilers and air preheaters constitute a valuable contribution to the sum total of information made available for power-plant engineers and designers. Careful reading and study of the curves and charts included in the author's paper, however, fail to disclose the proper comparison of the various types of preheaters tested. All of the comparisons made are more properly comparisons of overall performance of the complete unit consisting of stoker, furnace, boiler, economizer, and air preheaters. Although the temperature drops through the various preheaters are given and compared, and also the relative rise of air temperature, these figures do not constitute a record of performance. From the data contained in the paper a comparison of the performances of the air heater by itself cannot be made.

At no place in the paper are the gas weights given, nor the amount of air passed through the preheaters, nor the CO_2 contained in the gas entering the preheater. Sectional areas through the gas and air passes of the preheaters are not given. Without this information preheaters cannot be properly compared.

The author mentions the necessity of lowering the furnace temperatures to avoid refractory troubles. This introduces a condition which affects the performance of the air preheater and under which condition it could not be compared with other preheaters, unless operating under identical conditions.

In Fig. 9 the curve of boiler No. 59 shows that with a gas-temperature drop of approximately 300 deg. fahr. the air-temperature rise is 290 deg. fahr. This indicates that the weight of air passed through the heater is greater than the weight of gas, a condition which obviously cannot exist unless there is excessive leakage.

With reference to the shape of the efficiency curves in Fig. 6, many conditions would account for the contour of these curves, aside from the preheated air. It does not appear that the author has demonstrated that the higher and flatter efficiency of No. 3 boiler can be attributed to the particular type of preheater used, or even to the temperature of the air supplied to this boiler. Preheated air has a marked effect on the physical characteristics of the fuel bed, on the combustible in the ash, and on the formation of clinker. It is quite possible that with some fuels preheated air of high temperature would be distinctly detrimental to the performance of the stoker, whereas preheated air of somewhat lower temperature might conceivably be very beneficial.

[1] Vice-President in Charge of Engineering, Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

Otto de Lorenzi.[1] The case against the plate-type preheater as presented by the author is misleading in that he has skipped all of the fundamental functions that enter into the design of any preheater where the heat transfer is effected directly from a hot gas through a cooler metallic surface into a cold gas. In Par. 28 the author says: "The temperature increment for the preheaters of units 7 and 8 is greater than that for units 3 and 59. Apparently this is caused by the rapid increase of the temperature of the outlet gases in these two boilers. This throws more work on the air preheaters, increases the average temperature through the air preheater, and permits of a high rate of heat transfer."

While the rate of heat transfer is dependent to a small extent on the temperature difference between the hot gas and the cooler air, this difference does not account for the variations as referred to by the author. The main factors causing the variations in heat transfer are the mean hydraulic depth of the gas and air stream, mass flow of gas and air, and length of travel. This is brought out by the formula

$$R = A+BK\left(\frac{M}{d}\right)$$

where R = rate of heat transfer
A = rate of transfer at no flow (a constant)
B = factor dependent on temperature difference, which varies from 0.00077 at 400 deg. fahr. difference to 0.00112 at 2000 deg. fahr. difference
K = constant = experience
M = mass flow in thousands of pounds per sq. ft. per hr.
d = mean hydraulic depth of gas or air stream.

Therefore, to secure a higher rise in temperature of a given quantity of gas at a constant inlet temperature, the mere addition of surface brought about by simply adding more tubes or elements will not give the desired results, because the addition of tubes or elements means increasing the free area of both the air and gas passages. This in turn means a decreased mass flow, and a resulting decrease in transfer rate. Increasing the temperature of the gas and still maintaining the same relation between the weights of air and gas will not increase the transfer rate materially, but will simply increase the outlet temperature of the air as indicated by the foregoing formula. However, the desired increase in temperature rise may be obtained by increasing the length of tubes or elements, thus securing additional area with a longer gas and air travel for the same mass flow. The desired increase in temperature rise may also be secured by maintaining the same area of heating surface and the same mass flow, but decreasing the tube

[1] Assistant Chief Engineer, Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

diameter or spacing between elements as indicated by the foregoing formula.

The writer has visited the Chester Station twice, and has observed the installation of the plate-type heater referred to by the author. In the writer's opinion, the failure of this particular heater is due, first, to heater design and, second, to improper air-inlet-duct design. As to heater design, the elements of this heater are made of two sheets welded together. The spacing of the gas and air passages is maintained by small warts bumped into the sheets to hold them apart, with the result that the air and gas may flow through the elements as they will. In the plate type of preheater as designed by the corporation with which the writer is connected, both the gas and air spacing are subdivided by spacer angles, giving several distinct paths for both the air and gas, and the inlet areas of these various paths are proportioned to the length of travel of gas and air through them. The proper temperature rise is thus secured by means of the proper distribution of gas and air in each individual element.

As to the duct design, the fan for supplying the air to the preheater in question is located on the induced-draft fan floor, above the boilers. The air is blown down through a relatively narrow duct which does not flare to the full preheater width until it reaches a point quite close to the preheater inlet. The result is that while there is a relatively good gas distribution over the heater width, the air flow is greatest through the center sections of the heater, leaving the end sections practically worthless. This has the effect of cooling the gas through the center of the heater considerably, probably down to the dew point at the cold end of the center section of this heater; and causes an accumulation of moisture and a rapid deposit of dust or soot, which in time blocks the gas passage at the center, thus causing the gas to flow through the outside elements while the air still flows through the center elements. A relatively small temperature rise in the air as well as a relatively small temperature drop in the gas results, which indicates that the heat-transfer rate in this particular heater is lower than would be expected if the duct design were properly executed. This reasoning was pointed out to one of the engineers at the Chester Station, who answered that this blocking up of the center section and low preheating of the air was actually occurring as outlined above. The writer's corporation had one similar experience, but after redesigning the ducts to secure the proper air distribution, the guaranteed temperature rise was exceeded by about 20 deg. fahr., indicating that the question of air distribution is all-important.

The author states that it is rather difficult to clean the plate type of preheater. The writer's corporation installed a plate-type preheater in the Northeast Station of the Kansas City Power &

Light Company about three years ago. The fuel in this plant comes from Illinois, Kansas, or Missouri. It is high in sulphur and its ash has a low fusing temperature. The preheater itself is located after an economizer, with the result that the gases to the preheater are very seldom over 400 deg. fahr. in temperature. For a time this preheater was operated with a steam jet in the lower section of the preheater for cleaning. Later it was decided to eliminate the use of this steam jet, with the result that over a period of 30 days the temperature of the air out of the preheater dropped 10 deg. and remained constant. On opening the heater it was found that there was a small accumulation of soot which had built up to approximately $\frac{1}{16}$ in. thick. Continued operation of the heater did not increase the thickness of the soot film. It was therefore decided that it would only be necessary to clean this heater whenever the boiler came out of service. For the last year and a half the preheater has been cleaned by washing, and recently one of the elements was removed after having been in service for three years. Holes drilled through various parts of the plate showed that its thickness was the same as that of the original mill stock. The conclusions, therefore, are that if the proper exit-gas temperature is maintained from the heater, corrosion need not be feared, and that if proper precautions are taken the heater may be readily cleaned, even while in service. By precautions is meant the installation of soot blowers in the gas passage at the point where the gas enters the heater as well as where it leaves it. In a number of other installations, particularly where pulverized fuel is used, no provisions have been made for cleaning the heater, and these heaters have been inspected from time to time and show no deposits of soot other than a very light dust which may readily be rubbed off with the fingers. Thus it may be easily seen that the plate type of heater does possess many advantages over either the regenerative type or the tubular type. The main disadvantage in the regenerative type of heater is that the regenerative element is a moving element, and if it stops all preheat is lost. There is also a considerable leakage in this type of heater, which appears in a heat balance through an increased radiation and unaccounted-for loss, showing that the efficiency of this type of heater is somewhat lower than indicated by the rising temperature of air.

Both the tubular and plate types of heater are static heaters, in that there are no moving parts. The advantage of the plate type over the tubular is that due to the smaller spacing in the former, greater temperature rise may be secured for the same length of element. It is necessary in the tubular heater to maintain tubes usually not less than 2 in. in diameter, and due to this diameter the transfer rate is materially lower than that of the plate type of heater, where the spacing on both the gas and air sides is only $\frac{1}{2}$ in. The writer also desires to state here that in

practically every case where his corporation has installed a plate type of heater, guarantees have been exceeded at practically all rates of operation, indicating that their methods of calculation and the data upon which they are based are conservative.

C. F. Hirshfeld.[1] The air preheater naturally reduces the temperature of the gases after they leave the boiler. The writer understood the author to say that the use of preheated air increases the furnace temperature and also the temperature of the flue gases as they reach the boiler damper. If this understanding is correct there appears to be a discrepancy that requires explanation, because these observations do not appear to be consistent with the well-known diminution of temperature of flue gases with increasing percentage of CO_2 and corresponding increasing furnace temperature. Undoubtedly the two apparently contradictory sets of observations can be shown to be consistent when we have accumulated sufficient knowledge; at present the explanation is not apparent to the writer.

The use of preheated air has been shown to result in more nearly complete combustion of the carbon which finds its way into the grinder pit. Is there any evidence to show that the use of water-cooled walls, with their natural tendency to reduce the furnace temperature, has an appreciable counteracting effect? Several possibilities appear to exist, and information on this phase of the subject appears to be highly desirable.

Irving E. Moultrop.[2] Several points in the author's paper should be emphasized. The first is the study of the effect of the use of preheated air on the combustible in the ash. Heretofore little attention has been paid to this phase of the subject. The information presented by the author is both interesting and valuable.

Another point to be emphasized is the comparison of figures on the performance of air preheaters. Figures presented in the past, mainly in advertising literature, were of such a character as to arouse skepticism. The present paper furnishes data with which to measure and compare performance.

A question that might be suggested in connection with the author's data is the point to which furnace temperatures and CO_2 can be carried without affecting adversely the overall efficiency. How much higher can the furnace temperature and CO_2 be maintained if water-cooled walls are used as compared with solid refractory walls? How much higher if air-cooled refractory walls

[1] Chief of Research Department, Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

[2] Chief Engineer, The Edison Electric Illuminating Company of Boston, Boston, Mass. Mem. A.S.M.E.

are used? Also, what will be the effect of preheated air in burning fuels of high ash content?

These are important questions that must be answered before the problem of preheated air is fully solved.

Vincent M. Frost.[1] We have had the same difficulties as the author in determining the carbon loss in the ashpit on account of the increasing size of the clinker-grinder pits.

Calculations show that the clinker pit will contain the refuse for from 12 to 15 hours' operation of the boiler, depending upon the rate of operation. For this reason our method of conducting these tests is to operate the boiler at the rate at which the test is to be made, for at least a corresponding period before the start of the test, and reject the refuse from the hopper for the same period after the beginning of the test, before beginning to collect a sample to determine the carbon content. In this way many of the difficulties of starting and stopping the tests are eliminated and more consistent results are obtained.

Jos. S. Bennett, 3rd.[2] A careful study of a large number of tests on Taylor rotary-ash-discharge stokers with cold air shows

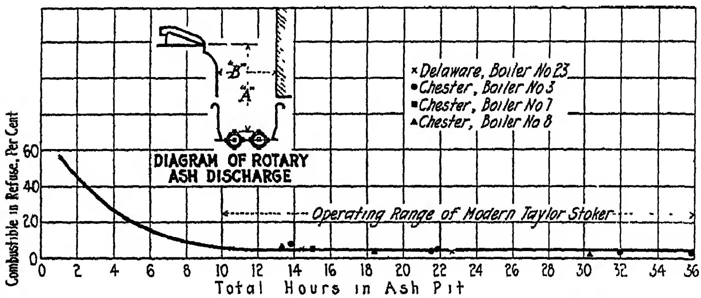


Fig. 12 Chart Showing Effect on "Combustible in Refuse" by Varying Time that Refuse Remains in Ashpit

A, depth of ashpit
B, Average width of ashpit
Weight of refuse, 40 lb. per cu. ft.; time in ash pocket based on weight of refuse discharged by rolls.

that there is a very definite relation between the length of time that the ash particle remains in the ashpit and the amount of combustible in the refuse. The curve derived from such a study is shown in Fig. 12.

It is interesting to plot the points given in the author's paper on the curve previously established. It will be noted that the

[1] Assistant to General Superintendent of Generation, Public Service Elec. & Gas Co., Newark, N. J. Mem. A.S.M.E.

[2] Assistant Engineer, American Engineering Company, Philadelphia, Pa. Assoc-Mem. A.S.M.E.

points obtained with preheated air fall somewhat below the curve established with cold air in the majority of cases, showing a greater recovery of the combustible in the refuse. In these tests as well as in most of those reported in the paper on Hell Gate results,[1] the loss due to the combustible in the refuse amounts to less than one-half of one per cent of the heating value of the coal burned.

In the case of the data presented in the author's paper, the conditions are very similar, making the comparison of much greater value than such comparisons ordinarily are.

Emmett B. Carter.[2] The writer knows of a case where an air preheater may be used, in which the fuel is wet tanbark chips, with a high percentage of moisture. Possibly 10 to 15 per cent of the net heating value of the fuel is used to evaporate the water out of it. The question that arises is whether or not the presence of so much moisture, of a corrosive character, will affect adversely the operation of the preheater. It is quite possible that the effect will be similar to the burning of coal high in sulphur and moisture. The writer feels that the temperature of the gases must be so controlled that there may not be a stream of water pouring through the preheater, resulting in corrosion or the formation of scale.

C. A. Jacobsson.[3] Regarding the statement on air leakage in the Ljungström air preheater, the writer refers to Fig. 9 in Mr. Funk's paper, which shows the relation between increase in air temperature and reduction in gas temperature for the different types of air preheaters.

Any leakage would consist of air leaking into the flue gases, as the air side is under pressure and the gas side under suction. A large infiltration of air into the flue gases would result in a greater reduction in flue-gas temperature than that corresponding to the heat transfer from the flue gases to the air. However, the curves in Fig. 9 do not show any greater reduction in gas temperature in relation to the increase in air temperature for the Ljungström air preheater than for the tube-type preheaters, and this proves that no excessive air leakage takes place in the Ljungström preheater.

Examination of curve No. 3 in Fig. 9 discloses that in this preheater the reduction in gas temperature is greater than the increase in air temperature. As the weight of the flue gases always is more than the weight of the air in any boiler plant, the results shown by this curve are difficult to explain.

[1] Boiler and Stoker Performance at Hell Gate Power Station, by H. W. Leitch. See p. 119.

[2] Consulting Engineer, Tannin Corporation, New York, N. Y. Mem. A.S.M.E.

[3] Sales Manager, The Air Preheater Corporation, New York, N. Y.

With reference to Mr. Carter's discussion, the writer calls attention to the advantages of the regenerative-type preheater in regard to an even distribution of the air and gases over the heating surface. In the Ljungström air preheater the surface is in continual rotation, so that the same temperature applies equally to all parts of the heating surface. This prevents low temperature and condensation in certain sections of the preheater, such as occur in other types of preheaters.

George W. Stetson.[1] It would be of interest if the author would give us information on the relative space occupied by the larger tubular preheater and the Ljungström preheater, and also the comparative cost of preheaters of each type, both to give approximately the same results. The relative amount of power required by the fans for the two types would also be of interest.

W. E. Caldwell.[2] It would be interesting to compare the performance of an air heater with a good economizer if the author has any information available on the coefficient of heat transmission of his air heater. Mr. Leitch in his paper[3] quoted a coefficient of about 7½ B.t.u. per degree difference per square foot of economizer surface. The fact that preheated air reduced the combustible in the refuse is indicated by Mr. Funk's paper and undoubtedly applies to the type of stoking equipment which was employed. With the operating conditions and type of equipment employed at Hell Gate Station, it is doubtful if preheated air could account for a saving of more than 0.25 per cent due to the possible reduction of combustible in the refuse, because with the present equipment and method of operation at this station the loss due to combustible in the refuse is almost insignificant.

We have found that when the economic capacity of a stoker is exceeded and excessive blast pressure is employed, large quantities of coke may be blown off the grate and into the clinker pit, in which instance it is doubtful whether any material improvement could be expected due to the use of preheated air, since under such conditions the combustible has reached a point where the mere elevation of furnace temperature has but slight effect upon it. A large clinker pit is a great disadvantage in reducing the combustible in the refuse, but it is not a good substitute for grate surface. Where the capacity of the active grate surface of a stoker is exceeded and large quantities of coke are blown into the clinker pit this loss cannot be recovered, even if the refuse is left in the clinker pit for a week.

[1] New England Air Preheater Corporation, Boston, Mass.

[2] Assistant to General Superintendent of Power Plants, The United Electric Light & Power Co., New York, N. Y. Assoc-Mem. A.S.M.E.

[3] See p. 119.

In the type of stoker reported by the author the active length is somewhat shorter than in the type employed at Hell Gate, and it is probable that the former requires somewhat greater air pressure for the same coal-burning rate. It is not stated in the paper whether air temperatures were measured at the stoker or when leaving the preheater. The area exposed to radiation between the point of heat recovery and point of application is much greater with an air preheater than with an economizer. As a consequence the heat lost by radiation in the case of the former would be many times greater than in the case of the latter.

J. M. Duncan.[1] The writer would like to ask Mr. Funk whether he has any data on the effect which the fouling of the flue-gas surface has in increasing the draft loss through the preheater. Also, would he anticipate serious corrosion in air preheaters if used in a station operating principally in the winter, and only very intermittently at other seasons?

The Author. The furnace conditions during the tests of the respective boilers outlined in the paper were kept as nearly constant as possible, the CO_2 being maintained at the highest continuous value consistent with satisfactory fuel bed and furnace conditions.

Data on CO_2 are plotted in Fig. 13 against thousands of pounds of dry coal per hour, and in Fig. 14 against per cent boiler rating. The data in Fig. 13 were obtained from accurate boiler tests. The data in Fig. 14 are average values taken from CO_2 meters installed on these boilers, the meters having been checked with Orsat apparatus at frequent intervals during the period that the data were obtained. In all cases the CO_2 was taken directly at the boiler outlet and the sampling pipe was arranged so as to get as nearly correct gas sample as possible.

The CO_2 data in Fig. 13 are to be used in conjunction with the data given in Figs. 5, 6, 7, 8, 9, and 11 of the paper. The data given in Fig. 14 are to be used in conjunction with Fig. 10.

All the boilers are equipped with perforated air blocks in the bridgewalls and side walls and no correct measurement of the air supplied to these walls could be obtained. The information on CO_2 permits obtaining gas volume at the boiler outlet as accurately as can be done by the author. With these data anyone so desiring can make more detailed analyses of the various air preheaters.

In reply to Mr. Van Brunt's discussion the author believes that the CO_2 values supplied in this closure will give sufficient data to enable a complete comparison of air preheater performance. In addition to these, Table 3 gives the complete information on the

[1] Mechanical Engineer, Hydro Electric Power Commission of Ontario, Toronto, Canada. Assoc-Mem. A.S.M.E.

gas duct areas with the exception of the Ljungström preheater, which is so different in type that direct comparisons can scarcely be made on this basis. Admittedly, the air passages in these various preheaters have not been given and can scarcely be supplied without submitting a complete set of shop drawings for each heater, which is scarcely within the scope of this paper.

The temperature measurements plotted in the various curves were given as they were obtained without any attempt to obtain temperatures that could be theoretically justified and the author admits that the readings of No. 3 boiler do not seem to be consistent. This, however, has no effect whatever on the efficiency curve which was obtained directly from the coal input and the steam output.

Mr. Van Brunt further states that it does not appear that the author has demonstrated that the higher and flatter efficiency of

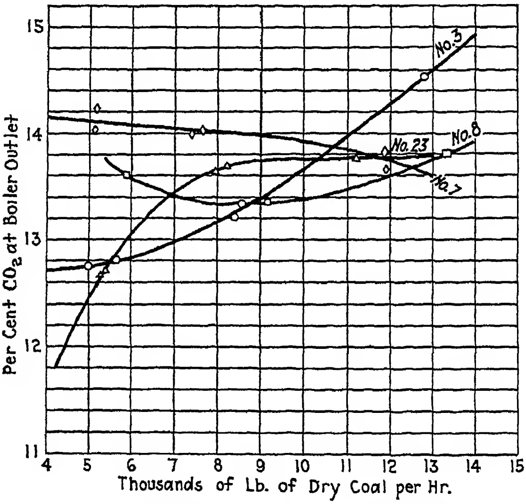


FIG. 13 RELATION OF CO_2 TO COAL FEED

No. 3 boiler can be attributed to the particular type of heater used. There was no attempt to credit the performance of No. 3 boiler to the particular type of air preheater used, since the shape of curve for No. 5 and No. 6 boilers with plate preheaters is quite similar to that of No. 3, although these have not been given for the reason explained in the paper. The statement to which Mr. Van Brunt refers merely points to the fact that the air preheater has flattened out the curve of a boiler without an air preheater. Comparison is made of No. 23 and No. 3 boilers. Both boilers are identical so far as design and surface are concerned. Both boilers have the same amount of superheater surface. No. 23 boiler has somewhat larger economizers than No. 3, which should make it more efficient than No. 3 boiler and economizer alone. No. 3 boiler

has in addition 22,072 sq. ft. of heating surface in the air preheater. Since these units are so nearly identical except for the air preheater, is it not fair to credit the better performance of the unit to the fact that an air preheater has been added to it? The author can see no logical reason for arguing otherwise. The author is sorry that Mr. Van Brunt feels that a case particularly unfavorable to the plate heater has been shown; in Par. 37 may be found a statement which the author believes points clearly to the fact that the design of these particular plate heaters may be responsible for their performance.

Referring to Mr. de Lorenzi's discussion, the author must point to the first paragraph in this paper in which he states that the object of the paper is merely to present the data obtainable from these installations and not to draw any design comparisons between the types of installations. Since there was no intention of doing this, there was no object in presenting a more or less complicated

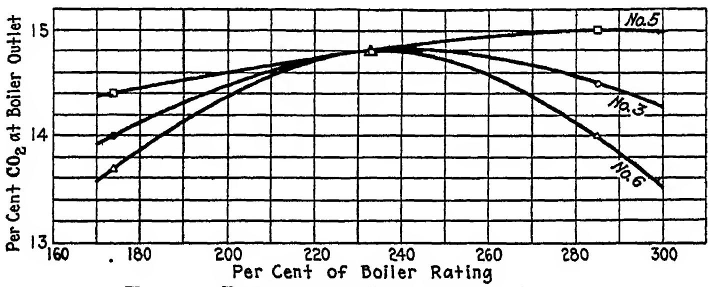


Fig. 14 Relation of CO_2 to Boiler Rating

discussion of fundamental functions of preheaters, and the author believes that the CO_2 readings included in this discussion will probably give Mr. de Lorenzi the information necessary to make any comparisons desired.

Mr. de Lorenzi further in his discussion calls attention to his visit to the Chester station and criticizes the plate heater installation on No. 5 boiler. He fails, however, to say anything about the plate heater on No. 6 boiler that is very similar to the one designed by his own company and which did show a somewhat better performance than No. 5 preheater.

In criticizing the improper air duct inlet design, Mr. de Lorenzi has probably overlooked the fact that when the station was originally built in 1918 there was no thought of installing air preheaters. The necessity of air-duct design was to some extent forced by the building limitations. This air-duct design, however, applies to both the plate and the tubular heaters, which must work under similar conditions so far as air supply is concerned. It was recognized at the time that there would probably be trouble with improper air distribution, the center part of the heaters receiving most of the air and thus allowing dead space in the heater. With

this in mind deflecting vanes were designed which were later to be installed in the air ducts in event of necessity. It was found upon starting the station that improper air distribution was obtained, necessitating the installation of these deflecting vanes which were put in place before the tests given in the paper were obtained. The author believes that this fully covers the points made by Mr. de Lorenzi and desires further to call attention to the fact that Mr. de Lorenzi's statements apply to both types of heater, although he made the point concerning only the plate type heater in his discussion.

Relative to the ability to keep the tubular and the plate-type heaters clean, it would seem to be almost axiomatic that the smaller the space available for cleaning the less readily may an object be cleaned. Experience with the plate-type heaters at Chester has indicated that it is more difficult to keep clean these particular types of heaters than the tubular-type heaters installed in this station.

Referring to Mr. Hirshfeld's discussion, relative to the statement concerning increased furnace temperatures and flue-gas temperatures at the boiler damper, the author admits that this apparently is a discrepancy in what has been expected in the past. However, if Mr. Hirshfeld will refer to Fig. 13 in this closure he will note that the CO_2 is higher for both No. 8 and No. 7 boilers than for No. 3 at 8000 lb. of dry coal per hour, and he will likewise note that at the same coal rate No. 7 and No. 8 boilers have higher flue-gas temperatures than No. 3. Admittedly, there is some possibility of error in measurements, but, remembering that the boiler and stoker equipment are identical in every respect, the data presented would seem to indicate that this condition persisted in the test in question.

Mr. Hirshfeld asks if there is any evidence to show that the use of water walls would have an appreciable counteracting effect on the carbon in the refuse. As yet we have no data on boilers with an installation of water walls so that this question cannot be correctly answered.

In reply to Mr. Moultrop's question in the third paragraph of his discussion relative to the degree to which air can be preheated before there is the possibility of an adverse effect on boiler performance the author desires to state that unfortunately data on this point are not available. As stated in the paper the author believes that the furnace designs for No. 7 and No. 8 boilers with the highly preheated air are not satisfactory and that it is possible to obtain better results by some changes in the furnace design. A water-cooled bridgewall and side walls are being installed in No. 7 boiler at the Chester station and information obtained from tests made on this unit after the installation of these walls will in all probability give an indication of what may be expected from this installation when the furnace is not a limiting condition.

Mr. Jacobsson indicates, using Fig. 9 as an example, that leakage is not shown to be greater in the Ljungström heater than in the other types of heaters. If Fig. 9 is referred to it will be noted that for 470 deg. fahr. rise in air temperature the tubular heater, No. 8, causes a drop in gas temperature of 400 deg. fahr., while the Ljungström causes a drop in gas temperature of 425 deg. fahr., and this would seem on the surface to indicate a possibility of a greater air leakage in the Ljungström than in the tubular heaters.

Mr. Caldwell asks the location of air-temperature measurements. These temperatures were measured at the outlet of the air preheater. All the ducts, however, have been thoroughly covered with insulation so that the radiation is reduced to a very small quantity, which is evidenced by the low temperature near the air ducts and the fact that the measurements taken on one boiler directly under the stoker did not indicate any difference in temperature between the stoker and the air preheater that was measurable with the thermometers used.

In reply to Mr. Duncan's question regarding fouling of the flue-gas surface in affecting draft losses, the author desires to state that no data have been obtained which indicate an appreciable increase in draft losses due to fouling of the passages. When it is recalled that these measurements must be made over a period of operation of the plant under varying load conditions it is quite possible that there is an increase in drop with the fouling of the preheaters, but since the preheater loss is only a small portion of the total drop through the boiler, economizer, and preheater, the increased resistance does not show up as an appreciable amount at the suction of the induced draft fan. No readings of drop across the air preheater alone have been obtained to check on this loss. From an operating point of view, if the total power required of the induced draft fan is not appreciably increased, the result of this fouling is a minor quantity in the boiler losses.

Mr. Duncan further asks whether serious corrosion will be anticipated in air preheaters if used principally in the winter time and only intermittently at other seasons. This is a very difficult question to answer from an experience point of view because we have no air preheaters operating under these conditions. It would seem, however, that there should be no very great difficulty, since without any air moving in the air preheater and only sufficient gas to clear the furnace, the air in the preheater reaches a high temperature, which would have a tendency to rapidly dry the heater and prevent the deposit of moisture in it. This condition is experienced on banked boilers, but that it is effective in preventing corrosion under the conditions stated cannot, of course, be predicted. It would appear that if the heaters are thoroughly cleaned at the beginning of the period of intermittent operation serious trouble would not be experienced.

No. 2012b

THE EFFECT OF WATER-COOLED WALLS ON PREHEATER PERFORMANCE

By Nevin E. Funk,[1] Philadelphia, Pa.
Member of the Society

THE data presented in this paper supplement those in the author's paper Comparative Performance of Air Preheaters,[2] read before the Society at its Providence, R. I., meeting, May 3 to 6, 1926. These data were not available at the time the above-mentioned paper was read.

2 In May information was presented on boiler unit No. 59 at the Richmond station relative to temperatures only, since the efficiency tests had not been completed. Particulars are now given regarding boiler No. 49 which is identical in every respect with No. 59, with the single exception of a steam-separating apparatus for measuring the amount of steam delivered by the Bailey water-cooled walls. In making comparisons with the previous paper, No. 49 can be substituted in relation to any of the other boilers for boiler No. 59. However, the temperatures obtained are quite different since increased experience in handling the preheated-air boilers with the Bailey wall installation permitted operation at higher CO_2 than was thought advisable during the tests when temperature readings were taken on boiler No. 59. This affects the temperature distribution throughout the boiler.

3 The cross-section and side elevation of boiler No. 49 at Richmond are the same as shown in Fig. 4 of the previous paper for boiler No. 59 at Richmond, the steam separator for the Bailey walls having no effect on the boiler performance.

4 The elevation of unit No. 7 at Chester was given in Fig. 3 of the previous paper. The performance of this boiler was unsatisfactory from an operating point of view, due to serious trouble from burn-outs at the ashpit end of the stoker and the destructive effect of the furnace temperature on the brickwork. To correct this trouble Bailey water-cooled walls were installed in the bridge-wall and the side walls as shown in Fig. 1 of this paper. No change, other than these walls, has been made in boiler No. 7 at Chester.

[1] Operating Engineer, Philadelphia Electric Company.
[2] See p. 337.

Presented at a meeting of the Philadelphia Section of The American Society of Mechanical Engineers, Philadelphia, Pa. November 23, 1926.

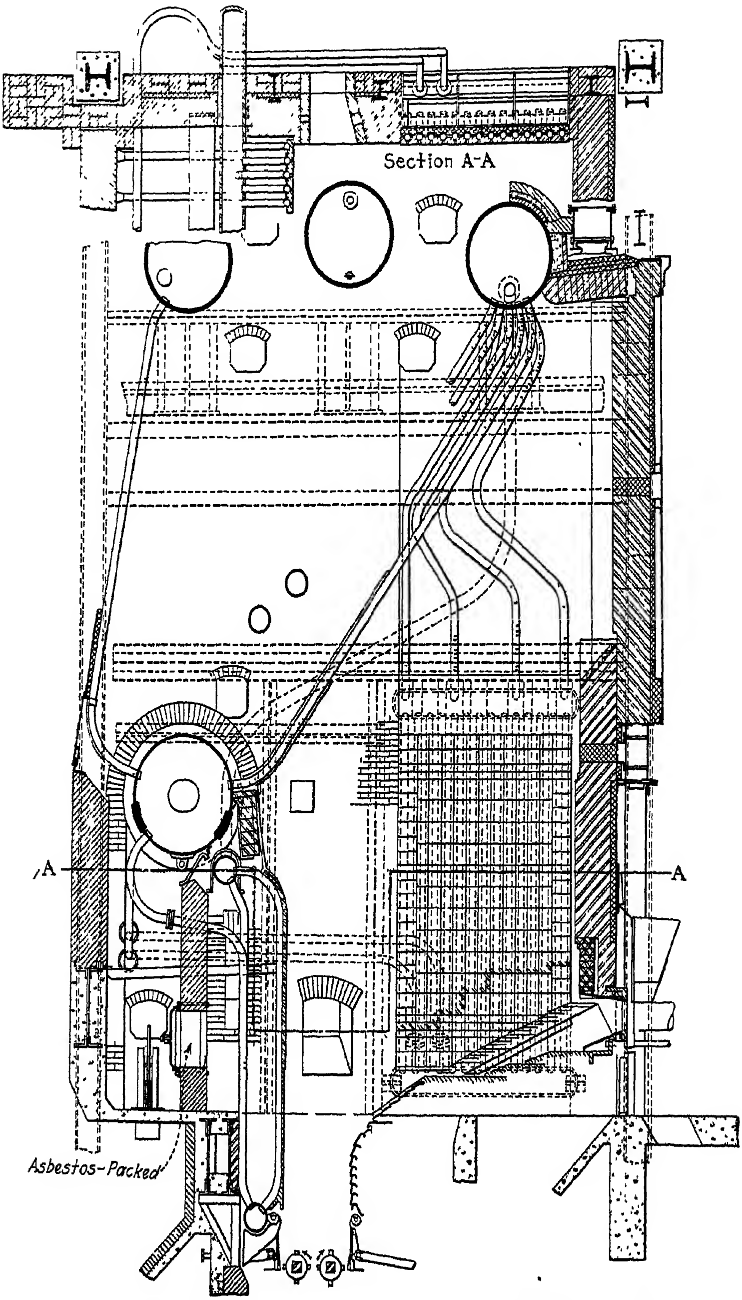


Fig. 1 Cross-Section of Boiler No. 7 at Chester Station after Installation of Water Wall

All the succeeding data on boiler No. 7 with the Bailey wall installation will be referred to as No. 7-A tests. The data given for all the boilers in Table 1 of the previous paper remain unchanged, those for boiler No. 49 being the same as those for No. 59 there given, and No. 7 having 645 sq. ft. of surface in Bailey water walls in addition to the surfaces given in the table. Briefly, they are as follows:

		Heating surface, sq. ft.				
Station	Boiler No.	Boiler	Superheater	Economizer	Preheater	Water wall
Chester	3	14,217	2,582	5,250	22,072	...
	7	14,217	2,582	....	64,960	645(a)
Richmond	49	15,692	2,822	7,515	22,072	595

(a) In the 7-A tests.

5 There are no changes in the physical characteristics of the stoker or air preheater from those given in the May paper.

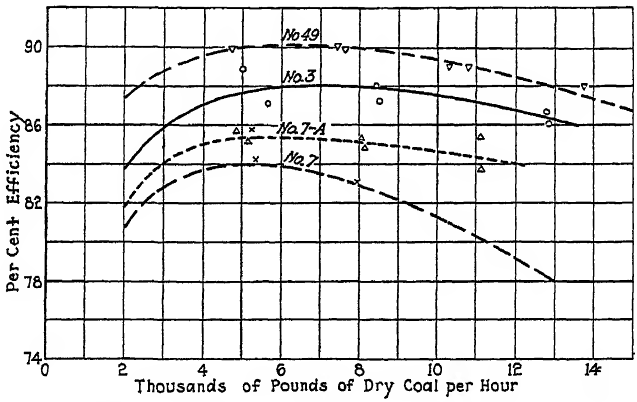


FIG. 2 EFFICIENCIES PLOTTED AGAINST THOUSANDS OF POUNDS OF DRY COAL BURNED PER HOUR—BOILERS NOS. 49, 3, AND 7

$$\left(\text{Efficiency in per cent} = \frac{\text{Heat input to steam}}{\text{Heat in coal fired}} \times 100.\right)$$

6 In Fig. 2 the author has plotted the efficiencies of boilers Nos. 49, 3, and 7 against thousands of pounds of dry coal burned per hour. The curves for the No. 3 and No. 7 boilers were given in Fig. 6 of the previous paper, but are duplicated in Fig. 2 for the purpose of comparing No. 49 with No. 3, and test No. 7-A with No. 7.

7 The air preheaters on boilers No. 3 and No. 49 are both of the B. & W. tubular type and have the same heating surface.

8 The economizer surface on boiler No. 49 is 43 per cent greater than on boiler No. 3. The superheater surface is 9 per cent greater on boiler No. 49 than on boiler No. 3, and the boiler surface of No. 49 is 10 per cent greater than that of No. 3. The boilers are of practically the same design with the exception of the Bailey

wall installation on the bridgewalls and side walls of No. 49 which replace the firebrick walls and perforated carborundum air blocks installed in boiler No. 3. These differences in characteristics were given in Table 1 of the previous paper. The stoker serving boiler No. 49 is slightly longer than that serving boiler No. 3.

9 It will be noted that boiler No. 49 is practically 2 per cent higher in efficiency than boiler No. 3 throughout the range of the tests. The effect of the water-wall installation can be better deter-

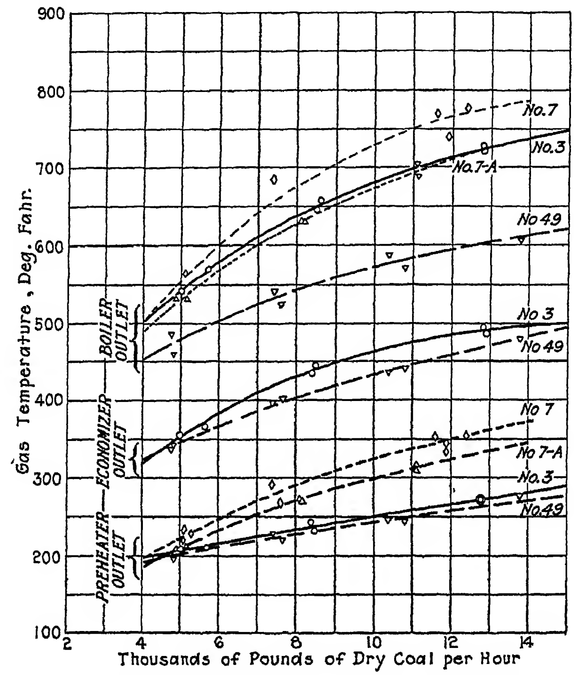


Fig. 3 Flue-Gas Temperatures at Boiler, Economizer, and Preheater Outlets Plotted against Thousands of Pounds of Dry Coal Burned per Hour

mined, however, from the temperature changes throughout the boiler unit which are given in succeeding curves.

10 The tests listed as No. 7-A in Fig. 2 were made on boiler No. 7 after the brick and perforated carborundum air blocks had been removed and Bailey water-cooled walls installed in the bridge and side walls. It will be noted that while the increase in efficiency was only slightly over 1 per cent at low rates of operation, the efficiency differential was increased to about 5 per cent at a rate of 12,000 lb. of dry coal per hour.

11 In order to follow the performance of these boiler units as far as subdivision of the heat absorption is concerned, the flue-gas

temperatures at the boiler outlet, economizer outlet, and preheater outlet have been plotted in Fig. 3. At a rate of 12,000 lb. of dry coal per hour it will be noted that the boiler outlet temperature for boiler No. 49 was 120 deg. below that for boiler No. 3. This reduction is due to increased boiler surface and the installation of Bailey walls on boiler No. 49. A definite allocation of the temperature reduction due to these two factors cannot be readily made, due to the better combustion obtained in the furnace with the Bailey wall installation in which it was possible to obtain higher CO_2 without the danger of CO loss.

Effect of Water Walls

12 The effect of the Bailey walls is more definitely shown in the difference between tests No. 7 and No. 7-A in Fig.3. In this case there was no other disturbing factor in making the comparison, and it is therefore safe to credit the Bailey wall installation with the reduction of 54 deg. in the boiler outlet temperature at the rate of 12,000 lb. of dry coal per hour.

13 As would be expected, the reduction in the outlet temperature of boiler No. 49 lowered the gas temperature at the outlet of the economizer. This reduction, however, was only a small percentage of that in the boiler outlet temperature, the heat absorbed by the economizer being greatly reduced due to the decrease in the temperature of the gases passing to the economizer.

14 The reduction in the economizer outlet temperature likewise affected the air-preheater outlet temperature to some extent, although the difference between the preheater outlet temperatures for boilers No. 49 and No. 3 is quite small. The decrease in stack losses is practically all accredited to the reduction in gas volume due to higher CO_2.

15 The reduction in the boiler outlet temperatures for test No. 7-A is reflected in the reduction in the preheater outlet temperature for the same test. The reduction in the preheater outlet temperature, however, is not as great as the reduction in the boiler outlet temperature because the preheater did not absorb as large a quantity of heat due to the lower initial temperature of the gases. Since the CO_2 was practically the same in both No. 7 and No. 7-A tests, better efficiency can be credited to the better heat absorption of the boiler and the Bailey walls as a unit.

16 The temperature change of the gas and air through the air preheaters has been plotted in Fig. 4 against the rate of boiler operation. It will be noted that the performances of air preheaters Nos. 3 and 49 are practically identical. The apparent performance of the air preheater on boiler No. 7 with and without the Bailey walls is quite different. There is some uncertainty in accounting for this difference. It is known that the leakage of cold air into the flue gas in test No. 7-A was somewhat greater

than in test No. 7. This would reduce the measured flue-gas temperatures below the correct gas discharge temperature from the heater, which would, by subtraction, give an increased gas drop from the correct value. It is therefore possible that the difference in the actual gas temperature drop between No. 7 and No. 7-A should be greater than shown in Fig. 4, being more of the order of the difference plotted for the air temperature rise.

17 While there is the possibility of an error in the flue-gas temperature measurements, the air temperature measurements are

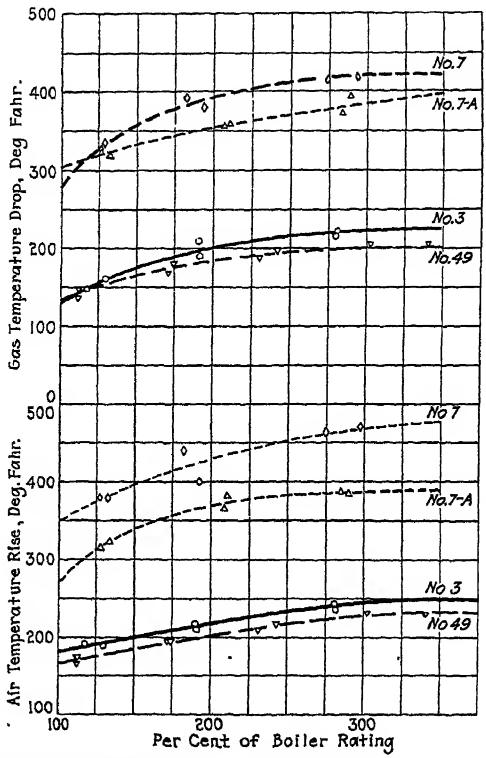


Fig. 4 Temperature Changes of Gas and Air through the Air Preheater

correct, and for this reason the amount of heat returned to the boiler by the air preheater is in the direct ratio of the temperature rises plotted in Fig. 4. This reduction in preheated-air temperature is due to the fact that there is less heat supplied to the air preheater because of the better heat absorption of the boiler and Bailey wall unit.

Effect of Preheated Air on Amount of Combustible in Refuse

18 The effect of preheated air on the amount of combustible in the refuse has been plotted in Fig. 5. Boilers Nos. 49 and 7-A, both with Bailey walls, show a marked reduction over No. 3 and No. 7 without water-cooled walls, indicating that the installation of these walls has effected a decided improvement in the combustion throughout the fuel bed. There might be some doubt as to the walls being directly responsible for this reduction in the No. 7 and

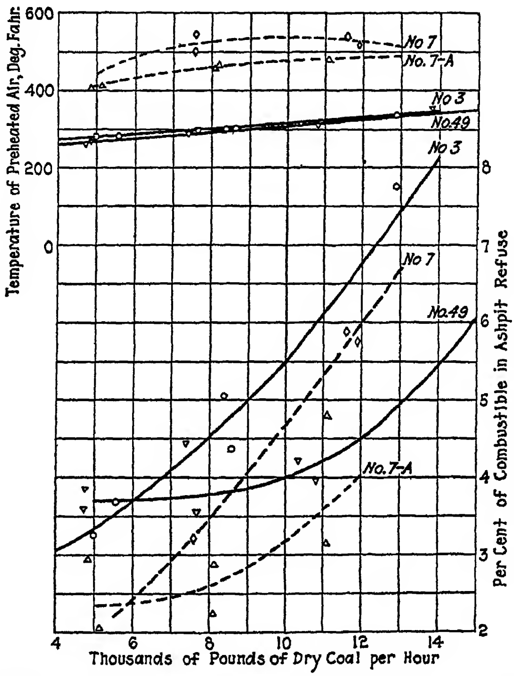


Fig. 5 Effect of Preheated Air on the Amount of Combustible in the Refuse

No. 7-A tests, since the preheated-air temperature in No. 7 was higher than in No. 7-A, and it was shown in the author's May paper on preheaters that an increased preheated-air temperature reduces the combustible in the refuse. This factor, however, is not present in the test of boilers Nos. 3 and 49 since the preheated-air temperatures were practically the same, and therefore the reduction in combustible in the refuse must be credited to the effect that the walls had on combustion.

19 The comparison of the combustible in the refuse, however, does not give a true picture of the loss due to unburned combustible matter. In Fig. 6 the author has plotted thousands of

pounds of dry coal per hour burned as abscissas, and as ordinates the percentage of combustible in the refuse from the ashpit alone, with both the ashpit refuse as the 100 per cent base and the percentage of combustible in the ashpit plus the soot collected from the boiler with the ashpit refuse as the 100 per cent base. Attention is called to the fact that while the combustible in the refuse from the ashpit alone is very low, the combined combustible loss is as high as 20 per cent of the ash withdrawn from the ashpit.

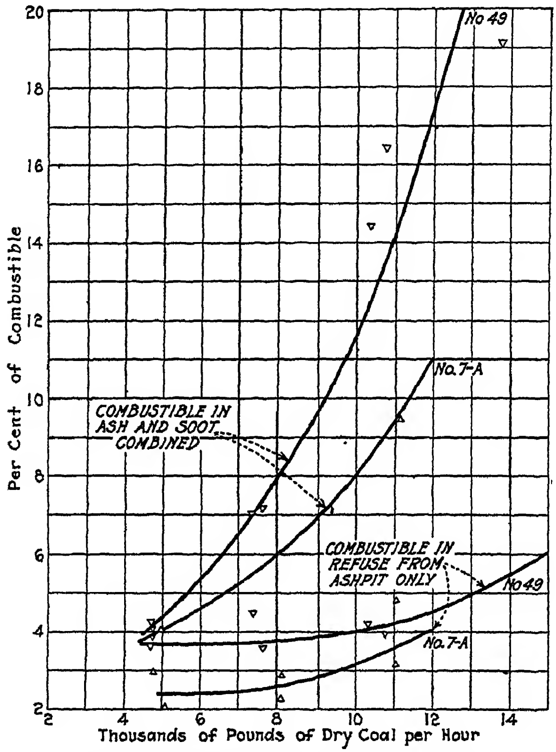


Fig. 6 Percentage of Combustible in Ashpit Refuse and Soot Plotted against Thousands of Pounds of Coal Burned per Hour

20 Unfortunately in the test made without water-cooled walls this combustible loss in the soot in the boiler has been thrown into the unaccounted-for losses in accordance with the A.S.M.E. Boiler Test Code, in which will be found a definite statement of the manner in which the unconsumed combustible in the refuse is to be calculated.

21 In all probability this matter of soot with the water-cooled walls would have been neglected in these tests had it not been discovered that large quantities of soot were made when the boilers were first started with the walls unslagged. These quantities were

so large as to require daily cleaning from the soot chambers, and of course drew immediate attention to the amount of soot produced. As the walls became coated with slag the amount of soot production was reduced tremendously. It was found that after there were three inches or more of slag on the walls, the soot production reached a minimum. All tests that were made prior to the formation of three inches of slag were discarded and have not been used in this discussion, so that the information presented is on a uniform basis. The author has arranged, due to this experience, to have a complete record of the soot formation obtained on all future boiler tests both with and without water-cooled walls.

22 That increased preheated-air temperature reduces the amount of loss in unburned combustible matter is interestingly

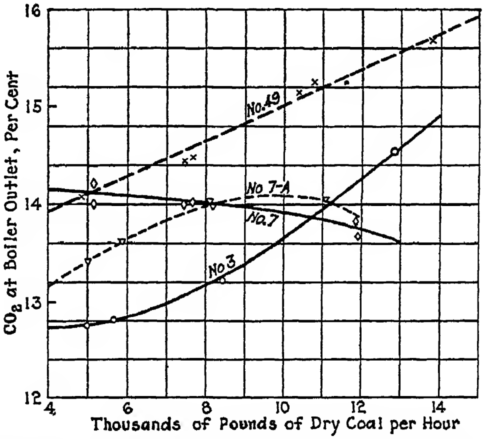


FIG. 7 CHANGE IN CO_2 AT BOILER OUTLET WITH CHANGE IN RATE OF DRIVING

corroborated by the fact that the preheated-air temperature of boiler No. 7-A averaged 150 deg. higher than that of boiler No. 49, and the total combustible loss in the soot and ash combined at high rates of operation was approximately 6 per cent less on boiler No. 7 than on No. 49.

23 In the previous paper a description was given of the care taken in obtaining accurate ash samples by grinding the entire quantity of refuse from the pit. However, when the quantities of combustible in the refuse become as small as 3 or 4 per cent, it is very difficult to obtain sufficient accuracy with variations in rating to justify an accurate curve of performance. For this reason the curves plotted for No. 49 and No. 7-A to show the combustible in the refuse alone must be considered as mere trends and not as definitely accurate quantities, although they were the accurate analyses made of the ash samples obtained; the difficulty lies in obtaining an accurate ash sample. The effect of a 20 per cent

error in the combustible in the refuse at these low values would cause no error in the boiler efficiency, but would affect the unaccounted-for losses by approximately 1½ per cent of those losses, and is not a serious factor in the heat balance of the boiler test.

24 To complete the information on these later tests for comparisons that the reader may desire to make with the tests given in the previous paper, the author has plotted in Fig. 7 the CO_2 at the boiler outlet against thousands of pounds of dry coal per hour.

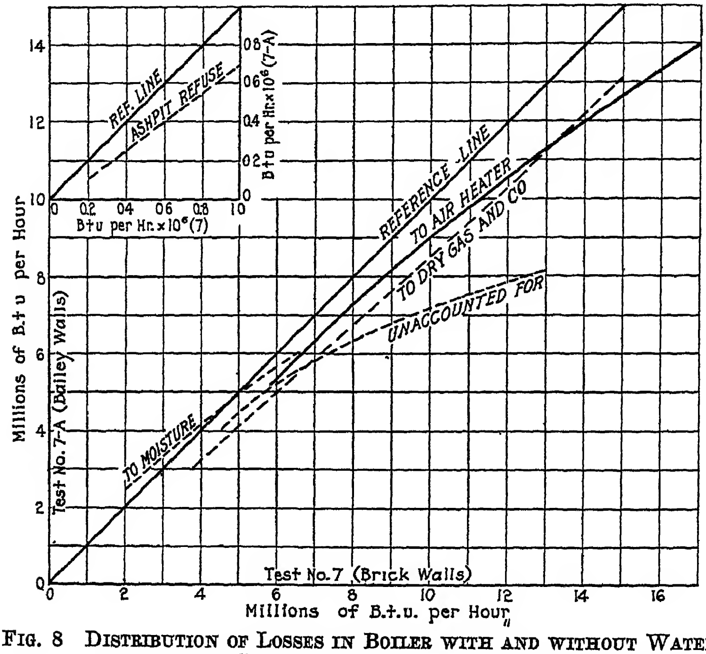


FIG. 8 DISTRIBUTION OF LOSSES IN BOILER WITH AND WITHOUT WATER-COOLED FURNACE WALLS

25 An interesting comparison of the distribution of losses on boiler No. 7 with and without water-cooled furnace walls is given in Fig. 8. The losses in B.t.u. per hour for boiler No. 7 without the Bailey walls have been plotted as abscissas against the B.t.u. losses in boiler No. 7-A, with the Bailey walls, as ordinates for the same input to each boiler. The inputs were varied from low to high rates of operation. The 45-deg. reference line would indicate that the distribution of losses was the same for boilers with and without the Bailey walls. Where any of the curves lie below this 45-deg. reference line they show a decrease in these losses.

26 The losses have all been plotted as dotted lines. The heat transferred to the air by the air preheater has been shown as a solid line. The ratio of loss in the ashpit refuse has been plotted

to a larger scale in the upper left-hand corner of the figure. Attention is called to the fact that the loss in the refuse with the Bailey walls is lower than before the walls were installed.

27 Operating experience would lead to the belief that a larger amount of soot is produced in a boiler with water-cooled walls than in one without them. This loss in unburned carbon in the soot is contained in the unaccounted-for loss, and in spite of this possible increase in soot loss, the unaccounted-for losses, after the water-cooled walls have been installed, are less than prior to the installation, showing that if there is an increased loss in the soot it is more than counterbalanced by the reduction in radiation losses.

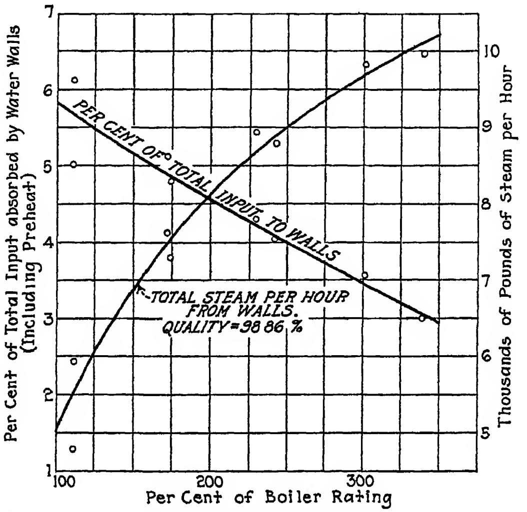


FIG. 9 TOTAL AMOUNT OF STEAM GENERATED IN THE BAILEY WALLS AND THE PERCENTAGE OF TOTAL HEAT INPUT TO THE FURNACE IT REPRESENTS, PLOTTED AGAINST PER CENT OF BOILER RATING

28 The loss in the dry gas and CO has been reduced by increase in CO_2 as shown in Fig. 7. The loss due to moisture is probably the same in both cases, and the variation shown is due to the slight variation in moisture and hydrogen in the fuel. It will be noted that the energy transferred to the air by the preheater is less over the entire range of operation for the unit after the Bailey walls were installed. This is to be expected on account of the increased absorption in the boiler and furnace as shown from the gas temperatures plotted in Fig. 3.

29 Measurements were made of the total amount of steam generated in the Bailey walls. This quantity has been plotted against per cent of boiler rating in Fig. 9, together with the percentage of the total heat input to the furnace which this steam output represents. The curves in Fig. 9 were obtained with a formation of three inches or a somewhat greater depth of slag on the water-

cooled walls, the limitation of slag thickness being between three and four inches.

30 Heat-transfer rates have been plotted in Fig. 10. Curves *B* and *C* show the rate of B.t.u. transfer per hour per square foot of water-wall surface for variation in rate of boiler operation. Curve *A*, which was obtained with the unslagged walls, has been plotted as an indication of the increased heat transfer with this condition. However, the measurements are somewhat in doubt since the slag formation was varying, and this curve of wall performance is given merely to show the trend of the heat absorption in the walls from the point of initial operation until the normal amount of slag formation has occurred.

31 It is believed that the data presented herewith may be of some value in comparing the design of water-wall surface to boiler

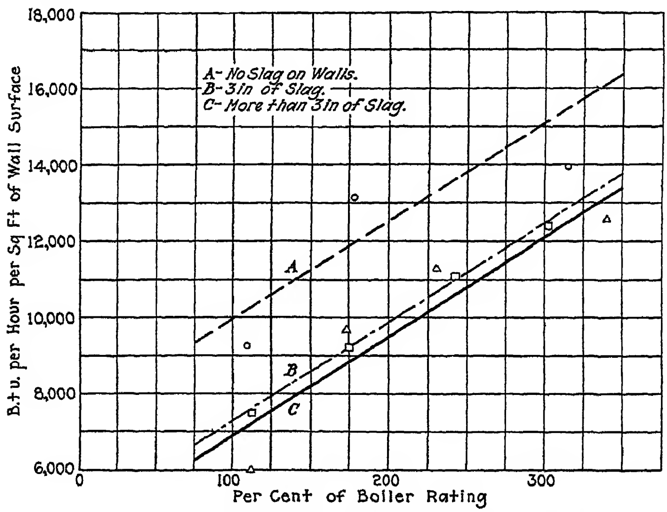


Fig. 10 Heat Transfer through Bailey Walls

surface, and air-preheater surface to boiler surface. It must be remembered that the exact values given apply to the units as intsalled and that there are so many variables affecting the individual operation of each element of the unit that extreme care must be taken in applying the information. The data are of value in indicating the general trend in the performance of these various pieces of equipment. The absolute values for each individual piece might be considerably changed by merely baffling the boiler differently, although in all probability this would have less effect on the performance of the Bailey wall than on the remainder of the boiler unit.

32 The author desires to take this opportunity to publicly thank Mr. H. W. Phillips of The Philadelphia Electric Company for his able assistance in preparing the data contained in the paper.

No. 2013

THE CHANGING RELATIONS BETWEEN EMPLOYER AND EMPLOYEE

W. L. Abbott, Chicago, Ill.
President of the Society

THE great civilizations of the past, with their achievements of wealth, learning and culture, were for the favored few and rested on the shoulders of the enslaved masses, whose unrequited toil, however poorly directed, piled up great wealth for the master class, leaving the masters' time free for pleasure and for war or for the pursuit of learning and the fine arts; and be it said in passing that the accomplishments of the masters in those fields of endeavor remain, after the lapse of hundreds and of thousands of years, the classics of the world.

The ratio of the numbers of slaves, serfs, thralls, peons, or bondmen of other names to the numbers in the master class naturally varied with the age and with the country, but may be taken as ranging from one to six — for the most part sullen or broken-spirited slaves who grudgingly wrought under the threat of the lash, without hope of present or future reward.

Compare the condition of that patrician of the olden days, surrounded by his slaves, with the standing of the American workman in this day of steam and electric power. This modern patrician, with his mechanical aids before him, has at his hand and command an amount of power equal to the strength of scores of tireless slaves of the greatest skill, willingness and discipline. As the slave of the past brought to his master wealth and, in its train, ease, learning and culture, the slaves of the present-day American toiler are making him the richest workman the world has ever seen, and these riches, too, are bringing their gifts of pleasure, comforts, education and culture. This condition of the American workman is the product of two principal factors, the first being the introduction of power-using and labor-saving machinery, designed to enable the worker to vastly increase his production; the second, and equally important, factor being the desire and will of the worker to use this improved equipment to the limit of his physical strength and endurance, in attaining the greatest possible result.

Presidential Address at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

The Family of Craftsmen

This situation is a comparatively recent outcome of a long series of slow changes in the relations between employer and employee, which have been going on for one hundred or two hundred years, or since the time when the manufacturer wrought with members of his family in his home or in his little shop near by. Sons and daughters were trained in the arts of the parents, and all working together as a family group, spun and wove or fashioned metals or clay by main strength. If it were found mutually advantageous for a neighbor's son or daughter to be admitted to that home circle of industry to learn and to produce, that apprentice shared not only in the long, hard work and small pay, but in the sentiment and society of the home as one of the family; and what was true of the apprentice remained true of him after he became a journeyman, as long as he cast his lot with that family and employer. Here began the voluntary relations of employer and employee, master and man, capital and labor, and with that relation went the personal touch, accord, and complete understanding born of the intimacy developed during the long hours of labor and afterward. Nor did that intimate relation suffer any material change when the household prospered and, one after another, workers were added to the circle and the detached shop definitely succeeded to the home; the master was there among them and of them. He directed their efforts, perfected their technique, provided material, and found markets. They, in turn, brought him skill, diligence and devotion, and rejoiced and shared in his success.

However, increasing prosperity and volume of business must in time change the status which was of the home shop. The master's head outweighed his hands. More and more of his time was spent in buying and selling, in accounting and financing, so that the workers got to see him less and less. Because of their numbers they were no longer members of his household, nor could he keep in touch with their personal affairs as formerly. True, indeed, the master still had a cheery greeting for his old-time shopmates whenever he met them, and their loyalty to him was unchanged, but with the newer men he had little in common aside from the work on which they were engaged.

The Growth of the Corporation

The shop continued to grow until it surpassed the ability of the master and of his friends to finance; steam was harnessed, and machinery perfected. Then slowly came the inevitable corporation, which separated by miles master and men, who already had drifted far apart. Executive cares engrossed their old-time friend and companion. Foremen and superintendents intervened, and while the company and its officials were visibly prosperous, the men had no seeming part in that prosperity. Long hours, hard

work, small pay, irritations, and disagreeable conditions, which formerly were taken as a matter of course from the master, would not be endured from the corporation. Unable now to pour their troubles into the sympathetic ear of their old counsellor and thus find solace, they discussed their grievances with each other and so multiplied their woes and became originators and disseminators of ill will.

This change did not come about in a decade nor in a generation, but over a long period of time, covering the period when workmen were being employed in greater numbers and in larger shops, made possible by the introduction of power and power-using machines.

In shop, mill, mine and forest, on land and on sea, this change was taking place, the better-managed companies growing larger and absorbing the smaller. Stock-holders, represented by boards of directors, in intimate touch with and superior to the management, considered large dividends the objective to which all else should be subordinated, involving the cutting of costs wherever possible, including the payroll and rates of pay.

Periods of depression were weeks and months of unemployment, which the men sought to forestall by slowing down to make the work in hand last longer, and any reduction in payroll and rates of pay was countered by a reduction in rate of output.

Under such circumstances human nature were less than human if the workmen did not come to feel that they were abandoned — worse, exploited by the owners and reduced to pawns in the enterprise — that they were of one class and the owners of another; that to obtain a better wage and to correct senseless and irksome conditions and to share in the company's prosperity, mass action and war were necessary. And so it came to that, and war — as ever, cruel, stupid, blind — was waged for generations. The mass, in the exhilaration of seeming triumph, imposed onerous conditions upon the employer and the industry, ignoring the injury to itself done by these acts. It was being avenged for ancient wrongs, and for the present that was enough. The employers fought fire with fire and vengeance with vengeance, and as the tide rolled back and forth across the industry, both lost and both won. Both lost in the chaos of the business on which they mutually depended for a livelihood. Both won in that out of the moil came a better understanding that each was indispensable to the other, that employees could not prosper unless the employers did, and that to get the best out of human beings their human instincts should be considered and their human interests enlisted.

The Newer Understanding between Employer and Employee

The new condition was essentially a return to the sympathetic relations which existed in the family group in the workshop of a previous century. The intimate social contact is, of course, no

longer possible and no longer desired, but the right of a workman to be respected as a man and not treated as a machine, and to have the privilege of making suggestions not only for the advancement of his own interests but for the betterment of the work of the shop and for the general good of the company, is not only granted but desired by his employer.

Gone is the hostile array of sullen workmen and arrogant superintendents, the inclination to skimp wages wherever possible, and the counter policy of restricting output and doing as little as possible for the highest wages that can be extorted. The mere creation of an impartial court for the hearing of grievances has reduced the number of such complaints to the vanishing point. Different groups of workmen vie with each other in efficiency and in loyalty, while the individuals are given all possible aid in expediting their work, and their pay increases with their increased efficiency, a liberal amount of these earnings being invested in the stock of the company for which they work. Due to his financial and friendly interest in his employing company, the workman's former policy of doing as little as possible is now changed to one of doing all he physically can, and while his earnings mount to heights that are the amazement of workmen of the world, the employer enjoys an unprecedented prosperity.

The Millennium Has Not Arrived

I would not leave the impression that the millennium has arrived and that the academic question of the relations of capital and labor has been settled finally and for all, with nothing else to be done. Progress is being made daily, and all hope that progress will continue indefinitely, but the results already achieved are so far in advance of the situation twenty-five years ago, or even just before the World War, that there is little possibility of a return to the former status.

London Trade-Union Mission Found Coöperation and High Wages

A few months ago the industries of this country and of Great Britain listened with marked attention to the report of a delegation of eight British trade unionists, selected by their respective unions and sent to this country at the expense of a London newspaper to investigate at first hand and to discover and report upon the secret of the prosperity of American industries and the high wages paid to American workmen, at a time when English industries were depressed and there was much unemployment among their workmen. A study of the reports of the members of the mission shows that they were singularly open-minded, of a high order of intelligence and keen observers; yet, as might be expected, their trade-union bias occasionally crops out.

The Mission, as it was called, spent six weeks in America, visited several cities, and examined carefully into the workings of forty-two shops, talking freely with superintendents and workmen, and upon their return home each member of the mission made an independent report of his findings and impressions.

Six of them say directly that the cause of American prosperity is high wages.

All comment on the cordial, even friendly, relations existing between management and men, and the absence of lines of demarcation between the two groups.

All comment on the high rate of production of the workmen, with earnings in proportion, and the eagerness on both sides to speed production to still higher rates with still higher pay.

Following are a few excerpts from these reports, which are more illuminating than any general discussion of them would be:

> The feeling between employers and workmen appears excellent. A feeling of confidence in each other, which is sadly lacking in the old country, was evident everywhere.
>
> The men have an instinctive knowledge that no more can be taken from industry than they and their employers put into it.
>
> The workmen on their part seem to be convinced that to maintain high wages they must maintain high production, and they seem to be just as ready to talk of high productive methods as are the employers.
>
> Everywhere . . . they not only pay more but pay it eagerly, and on all sides have expressed a desire to pay still higher wages, provided, of course, they receive higher production in return.
>
> I have approached this question of mass production with all the prejudice of a craftsman and have been driven to the conclusion that work on mass production is neither soul-destroying nor nerve-wracking.
>
> The members of the mission were much interested to learn of the facilities granted by the larger firms to their workmen to purchase stock on somewhat more favorable terms than were available to outsiders.
>
> Employees are everywhere encouraged to purchase shares in the company which employs them . . . . That the idea is popular and taken advantage of is proved by the fact that in some of the works visited 100 per cent of the employees are share holders and in no case where the system was in operation was the percentage less than 15.
>
> With regard to the position of trade unions in America, I think it must be frankly admitted that they are without influence in the engineering industry. Employers everywhere stand for the open shop.

Lord Buckmaster, former chancellor of England, summed up the reports of the eight union-labor investigators in a chapter on good will, in which he said:

> No one who has ever been in the United States can have failed to realize that this point (the spirited coöperation and fellowship) upon which such emphasis was placed, lies at the center of their commercial success. The next proceeds from the first. It is the willingness and eagerness of everybody to increase output. Without the common fellowship and common purpose of all, this result is impossible.
>
> In the last resort there is opportunity on the land, both in Canada and the United States, for any man of vigor . . . and that fact . . . prevents workmen from being faced with the abyss of unemployment,

which over here too often stares them in the face. It is their dread of this catastrophe to which are due all the influences that limit output. If a man lays 1000 bricks a day, he is, according to this view, doing the work of two men, each of whom could lay 500, and this fact obscures the greater fact that the larger the output the more work there is to do.

After this review of the findings of the British Union Labor Mission of comparative conditions in the manufacturing industry in the two countries, we are disposed to wonder at the short-sighted policy of British workers in following their time-honored policy of demarcation and restriction of production, but lest we of America become "puffed up" over our assumed superior wisdom and good sense in such matters, let us in all humility take a brief glance at conditions in some other of our industries: building trades and coal mining, for instance.

Conditions in the Building Trades

The building trades, from their very nature, easily lend themselves to unionization. Their work must largely be done locally. It is done but once for a single owner. The added cost due to union domination and union conditions is much less than the cost of union opposition and strike delays. Hence the inclination to proceed with the work, in full realization that often the labor costs will be from two to three times as high as they would be with non-union labor at the same hourly rate, all due to the policy of making more days' work for working men out of any given job.

Brick masons, usually the most reasonable of the building trades, are satisfied to lay 500 bricks a day, when two or three times that number would not be excessive.

Painters have a motto and admonition reading: "We work too hard when we work. That is why the jobs don't last. Slow up." Plumbers used to have an unholy alliance with the manufacturers and master plumbers to keep outsiders from doing plumbing work or even getting plumbing fixtures, and around these rules and practices, restricting a day's work, cluster many ancient jokes. And so on down the list. Some unions are not so bad and some are worse, but in the pronouncements of none is found anything approaching a declaration expressed or implied for a full-day's work for a full-day's pay.

The policies and practices of the building trades often constitute the most glaring of union oppression, and yet so difficult it is for architects and builders to maintain a consistent policy in opposition to those oppressions that in the building trades will probably be found the last stand of labor unions.

The Coal Industry

Whatever may be the policies of coal operators at the present time and however just and liberal may they be in their dealings

with their employees, they cannot in one, two or three decades erase the memories of the wrongs and indignities perpetrated by grasping mine operators upon ignorant and unorganized miners, who were at last aroused to rebellion against their oppressors, and following victory the imposition of humiliating conditions upon their employers, some of which unwise conditions contained the germs of elements which would in the end cause the disintegration of the very organization under whose banner they obtained their freedom.

Following the conquest of the bituminous-coal mines, mining rates were advanced, mine discipline upset, and mine management forced to endure humiliating and arbitrary rulings from all-powerful pit committees. This was of no great concern to the public, and the operators received scant sympathy from it. On the contrary, they became still more unpopular because of the necessary advance in the price of coal as a result of new labor rates and conditions. The miners greatly profited and the public paid the bill.

About this time undercutting mining machines were introduced, materially reducing the time and labor required to get out a ton of coal, but the miner claimed and got the same rate per ton for his work as he had received when he was obliged to lie on his side and laboriously undercut the coal seam with his pickax. As the undercutter and mine electric haulage which came along about the same time materially increased the output and reduced overhead per ton, the miner was allowed all of the direct savings resulting from the introduction of the undercutter. Now, however, when the miner's remaining duties consist principally in shoveling coal into pit cars and it is proposed to introduce an expensive time- and labor-saving machine to do this work, and there is prospect of fewer days' work for miners or perhaps a lower cost per ton for coal, there is serious objection to the introduction of the loading machine. It is said that in England the union has solved this problem by forbidding the miners to load more coal by machine than by hand. In America the miners propose to take all of the savings themselves, leaving none for the public and none to repay the operator for his investment. Little wonder that coal loaders are not being developed and installed.

An equitable distribution of the savings to be realized from the installation of labor-saving apparatus would be a substantial increase in the employee's pay envelope and a decided reduction in price to the consuming public, while the employer, after receiving an amount sufficient to warrant the additional investment, would benefit principally by the increased business which would result from the reduction in cost of the product. To give the entire saving to either employer or to workman or to divide it between them would be unfair to the public, and to give it all to the public would discourage further improvement.

As a result of union policies, there has been a rapid development of non-union mines and mining districts, so that whereas three years ago union mines produced two-thirds of the soft coal of the country and non-union mines one-third, these proportions are now reversed, the non-union mines now producing two-thirds of the country's soft coal and the union mines one-third.

The opening and operating of many small mines in non-union fields while the large and well-equipped mines in union fields stand idle is, of course, economically wrong, but it appears to be the only way to correct some of the recognized faults in the coal-mining industry, which are necessary to pave the way for higher returns for miner and operator and lower prices and more dependable supply to the public.

It appears that in the long run the Golden Rule works itself out in business as elsewhere, and where employee or employer is by chance able to exercise tyrannous monopoly over a public necessity, he will in the end be deposed by results flowing from his own acts. But meanwhile the payment of the tribute which he extorts is passed on from hand to hand down the line, until it reaches the burdened and bent shoulders of the country's ultimate producer and consumer — the farmer.

It is unnecessary here to add other examples of absurdities of the workings of rules intended to benefit the working man at the expense of the public, by restricting output and increasing the cost of the product. Every engineer has had his own difficulties of this kind to contend with.

The Dawn of a New Order

But there is great promise that a new order of things is about to dawn, and as already millions of workers are employed under conditions which provide for frank interchange of views and discussion of differences between men and bosses, and all cordially unite to deliver to the public the best product at the lowest consistent cost, it may, with some assurance, be said that the new order of things is here — the reconciliation of capital and labor — a consummation which has been devoutly wished for since one man first worked for another.

The coming of the new order was not arranged by law or in convention or by following doctrinnaire theories of repression or of confiscation. It came almost overnight when it was realized that, in any discussion of the relative rights of employer and employee, no settlement can be enduring or permanently beneficial which is not just to the public; that those serve themselves most who serve the public best, and that there is no end to the market so long as the product is being constantly improved and its cost reduced.

That this is true is shown by the stability of our business conditions while there is so much depression elsewhere, and by the admiration and envy with which the world seeks to achieve a prosperity similar to ours.

Our present condition of labor not having been brought about through the intervention of those who professionally pose as the champions of workingmen, is regarded by them as a menace to the institutions and theories which they represent, and already threats are heard that this new cordial direct relation between workmen and employer shall be disrupted, that they and their influence may intervene.

What is the outcome to be?

The answer is largely in the hands of operating and superintending engineers.

The Part of the Engineer

You have long and carefully studied the characteristics of the materials you employ, that your treatment of those materials may best adapt them to the service of mankind. Have you been as considerate of the characteristics of the human bodies and human souls that enter so largely and vitally into your products? Herein lies the hope not only of employer and employee, but of society itself.

With such knowledge it should be possible to build a perfect house of industry, wherein are combined all essential features in proper proportion and place, in beauty, harmony, strength, and permanence. For stability such a structure should have its three widespread supporting columns firmly anchored to the foundation stones: Justice to the employer, justice to the employee, and justice to the public. If there be coddling of any one or a denial of equity to any one, the structure becomes unstable and liable to fall, and in that crash will go the interests of all.

Engineers and employers, the long-sought prize of ideal industrial relations is already within your grasp, but only by eternal vigilance can it be retained, for in this changing world, "Time makes ancient good uncouth."

When we fully realize that industrial relations are a major and not an incidental problem of production and that the employee's coöperation, welfare, self-respect, ways of thinking, and humors are major ingredients of success, and when we respect and use them accordingly, then we may visualize on our shores another statute of gigantic proportions, typifying America lighting the way to industrial freedom, prosperity, and peace.

No. 2014

LAWS OF MANUFACTURING MANAGEMENT

By L. P. Alford,[1] New York, N. Y.
Member of the Society

The paper is divided into two parts, the first of which discusses the laws of management in general. It is stated that the principles of management have been discussed but not generally accepted, a situation which is in contrast with the older sciences. Early attempts to formulate these principles sought for a few statements, although certain facts indicate that there are many management fundamentals. The possible advantages of formulating the principles are pointed out and the questions are asked: What is their origin? What is their nature? What are they?

The investigation is limited to manufacturing management, inasmuch as management originated in manufacturing.

It is shown that manufacturing developed to satisfy human needs and that accepted ways for getting work done existed before manufacturing. Three such anciently accepted ways are: division of labor, leadership, incentive reward. Other accepted ways are: authority with responsibility, transfer of skill, task work. These accepted ways are the result of societal development. Such accepted ways from societal development are resistive to change, universal in application, and imperative to success, and therefore such management fundamentals are regarded as laws. The investigation shows the discovery of these laws and the determining quantitative factors for them.

Part II sets forth forty-three laws of manufacturing management with supporting citations.

PART I ON THE LAWS OF MANUFACTURING MANAGEMENT

PRINCIPLES of management were discussed at great length during the early years of the modern movement for improved manufacturing operation and control; to such an extent, in fact,

[1] V. P., Ronald Press Co. and Management Engrg. Corp.

Contributed by the Management Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

as to indicate the existence of deep interest in the discovery and declaration of the underlying rules.

2 But this widespread discussion and evident interest did not yield a body of fundamentals to become universally or even commonly accepted and applied. In this respect, indeed, management is still at the starting point, although the codification of methods and practice, which has taken place in recent years, points anew to the existence of ascertainable fundamentals.

3 This situation is in sharp and unfavorable contrast with that of the older engineering sciences, such as mathematics, physics, and chemistry, each of which has an established body of laws. These fundamentals of the older sciences are recognized as having always existed, as being natural, or God-made. They have been formulated for use by putting together the discoveries and results of the work of many scientists, investigators and engineers. They are taught to all engineering students, and are universally applied in engineering work. The result of conformity to the truth they express has been mastery over the forces and resources of nature.

4 Amid the pioneer work in management of the past generation, the attempts to formulate general principles or "regulatives" seem to have been directed, in most instances, not so much toward completeness of survey as toward expressing such knowledge as was at hand in a minimum number of statements.[1] Such "principles," "underlying principles," "regulative principles," and

[1] Harrington Emerson gives the twelve principles of efficiency in his book by that title (1913) as:

1 Clearly defined ideals
2 Common sense
3 Competent counsel
4 Discipline
5 The fair deal
6 Reliable, immediate, and adequate records
7 Dispatching
8 Standards and schedules
9 Standardized conditions
10 Standardized operation
11 Written standard-practice instructions
12 Efficiency reward.

Frederick W. Taylor's four underlying principles of management are (The Principles of Scientific Management, p. 69):

1 The development of a true science
2 The scientific selection of the workman
3 His scientific education and development
4 Intimate, friendly coöperation between the management and men.

Church and Alford reduced the regulative principles of management to three. (*American Machinist,* vol. 36, p. 857):

1 The systematic use of experience
2 The economic control of effort
3 The promotion of personal effectiveness.

Charles W. Gremple gives a single axiom or aphorism of management, called Gremple's Law. (Manual of Organon, p. 16): "Like conditions plus like causes necessitates like effects."

"law" as were thus declared, lacked the outstanding characteristics of natural law, that is, the results of their operation could not be closely predicted nor precisely measured. However, the writings of the pioneers yield an extensive and valuable amount of discussion of rules and regulatives in addition to the few principles they seem to have considered of most importance.

5 The steady accumulation of evidence points toward the establishment of management as a science, and the very complexity of its applications indicates the existence of many rather than a few fundamentals. This possibility is strengthened by the extent of the recent codification of methods and practice.

6 If, then, management fundamentals do exist in some number: What are they? What is their origin? How unvarying is their operation? How universal is their application? How far do they deserve our adherence?

7 The principal advantages to be derived from such a body of management regulatives, if and when it is developed, are obvious. When once reduced to codified form,

1 These laws may be taught to all students in engineering schools who wish to prepare for executive and managerial responsibilities

2 They may be consciously applied in everyday managerial and operating activities

3 They will form one of the foundations of the material progress of succeeding generations, and result in vastly outstripping the achievements of the present.

8 W. A. Shewhart,[2] in presenting some of the applications of the laws of probability in manufacturing operation, hints at this development in the future.

> Let us look at the industrial development based upon applied scientific laws, making use of only a fraction of past experience, and then think of the future based upon applied laws and applied laws of chance, thus making best use of all our data.

9 This paper is an investigation of the general question. It advances a theory as to the origin of management fundamentals, formulates a number of them in uniform language, and supports their importance and the extent of their application by citations from recognized leaders in management and other scientific pursuits. It is the hope of the author that further investigation will be stimulated which, in time, will establish the laws of management as securely as the laws of mathematics, physics, and chemistry are fixed today.

10 The term management is so broad in meaning and implication that for the purpose of this investigation it must be restricted so that the scope of this paper may be held within reasonable limits. I shall, therefore, deal only with *manufacturing manage-*

[2] *Manufacturing Industries,* February, 1926, p. 128.

ment. This limitation is natural, for management, as the term is understood today, originated in manufacturing. The pioneers in management were concerned with manufacturing operations, and they are still the principal interest of the members of the Society.

Origin of Management Fundamentals

11 As to the origin of management fundamentals, modern manufacturing, like every other human institution, is an outgrowth of human customs. We bow to the custom of wearing clothes, hence the garment-manufacturing industry; we are accustomed to eating animal food, hence meat packing; we also eat cereals and fruits, hence we have milling and canning; for our comfort we have the custom of creating artificial climate in our houses, hence the industry of building construction and all others that contribute to the erection, maintenance, and operation of our dwelling places. Without multiplying such examples it is evident that the institution of manufacturing industry permits us to adopt easy, or easier, ways and means to meet the conditions and limitations under which we must live, and this adjustment is none the less real because of the difference in conditions in arctics or tropics, high altitudes or sea level, accessibility or isolation, city or country, plenty or scarcity, peace or war, and like contrasts.

12 As manufacturing has organized itself and been disengaged from other customs, it has come to have the foremost position in meeting living conditions and satisfying civilized needs, and is now considered indispensable to human welfare. It has been developed, however, and is still operated without much reasoned purpose. Changes and modifications have usually been put into effect to meet particular conditions and circumstances; the cases of long-time pre-planning are few indeed.

13 When modern manufacturing began there were ready at hand traditional ways or procedures for doing work and getting work done. They represented a concurrence of belief or willingly accepted arrangements on the part of peoples. They had been proved by experience to be good practice or successful expedients. In short, they were known to be "best ways." They, and many others like them which have to do with the maintenance of the race, are called "folkways" or "mores." In these folkways we find the origin of manufacturing fundamentals.

14 W. G. Sumner[3] defines the mores thus: "They are ways of doing things which are current in a society to satisfy human needs and desires." And he goes on to say, "Institutions and laws are produced out of the mores." A. G. Keller[4] says of them: "No one planned them, but they grew up and are practiced unquestioningly, unconsciously, and automatically. They correspond

[3] Folkways, pp. 53 and 59.
[4] The Evolution of Man, p. 137.

to habits in the individual. Taken all together, they constitute the code of behavior in a society. Keller also points out that division of labor is one of the most typical of these expedient ways Two others are the exercise and response to leadership and the incentive reward.

15 A brief examination of the three typical "best ways" in organizing, directing, and doing work, noted in the preceding paragraph and commented upon extensively by students of societal development, will show more clearly their origin and nature.

16 In the first place, the division of labor: Work has always been divided among the individuals of a family, tribe, city or nation, so far as we know or can imagine. Some fought, others made arms; some hunted and fished, others sailed the seas and traded; some raised crops and tended flocks and herds, others made brick, hewed wood, cut stone and erected structures. Many references to this practice in ancient affairs can be cited, but one is sufficient here. Xenophon,[5] writing about 370-375 B. C., describes the division of labor among Greek citizens:

> It is impossible for a man who is a jack-of-many-trades to do all things well. In large cities, because of the fact that many persons need each commodity, a single trade suffices for making a living, and often not even one complete trade; but one workman makes men's sandals, another women's. One person makes his living exclusively by stitching sandals, another by cutting them out. One man is exclusively a cutter of garments. Another takes no part in this work, but merely puts the pieces together.

17 In the second place, leadership: Leadership in work, as in all other human activities, has always been exercised and followed. A few have given orders, many have taken and followed them. The successes of leadership have always been praised. History is filled with such examples, particularly of military and religious leaders, and to a lesser extent, of builders and constructors.

18 In the third place, incentive reward: Promise of reward for achievement is another folkway for getting something done. The loot and captives of a conquered city have spurred many a soldier, or pirate, to do his utmost in battle. The favor of prince or emperor, the smile of a fair lady, the prospect of wealth, power or position, all these have called forth exertion and effort far beyond the usual, whether in war or in work. We can imagine no condition of society where such incentives have not been effectively used.

19 Other accepted ways of getting work done, which might be mentioned as equally typical, are fixing authority and responsibility, the transfer of human skill to tools and mechanisms, and the setting of task work. In fact, there are many such usages which exert a coercion on the individual to conform to them in doing work or getting work done.

[5] Cyropedia, viii, p. 2.

20 These expedients are prominent among those practices which have developed modern manufacturing. But they antedated the industrial revolution by thousands of years. When that event came, they were established and ready to be used in building and separating out another great human institution.

21 This reasoning leads to the conclusion that fundamentals of manufacturing management, of which those considered are typical, are the outgrowth of the customs of mankind in doing work, or of common physical capabilities of mankind; in short, their origin is in societal or organic evolution. But the evolutionary process is still going on, and as new ways develop to meet new circumstances and conditions, these also can be credited to the same origin.

22 This theory of the origin of these basic principles has a significant bearing on our view of them. It increases our belief in their practical importance and value. It shows that they have withstood the test of centuries of application and use in human affairs amid changing conditions of every conceivable kind. It indicates that they deserve our adherence, and are entitled to adoption and use in the code of operation of manufacturing concerns.

Franklin's Dicta

23 B. A. Franklin[6] has reached a similar conclusion by somewhat different reasoning.

Mankind, even though it may be most unconscious of it, is ruled by laws.

First, there are the laws of nature. Man had nothing to do with the making of these laws. They operate the seasons, the natural phenomena, such as electricity, and all of life. Physical science discovered them.

Understood and used, operated with and obeyed, they afford great comfort and health. The laws of health come under this group.

Disobey them and you court eventual disaster.

Then there are the laws of government. These are made by man to regulate our relations, one with another. Some of them are based on the experience of ages, some on new conditions, and some on mistaken ideas.

Legislatures or rulers make them.

Disobey them and you court the courts.

Finally, there are the laws of human relations and a principal group of these are the laws of economics.

These laws are neither natural nor are they consciously made by man nor set down in codes.

They grow out of the built-up relations which men have developed with each other in their affairs with one another. They grow out of customs, the creeds, the habits, the methods of thinking and dealing with each other which mankind has built.

They may be broken, they may be taken advantage of, but usually not often, nor for long.

24 Additional evidence of the evolutionary origin of these fundamentals of manufacturing management is found by examining certain of them as already formulated. They are found to be

[6] *Open Shop Review*, May, 1926, p. 185.

statistical laws, indicating trends and tendencies, and are stated in terms of probability. Furthermore, they have the characteristics of resistance to change, universality of application and being imperative to the highest achievement and success.

25 The origin of some at least of the fundamentals of manufacturing management is thus found to be in organic and societal evolution; and they have the force of law.

26 The task of formulating these laws in uniform language now confronts us, realizing that the barriers between the divisions of science are rapidly breaking down, and that management—the science of getting work done—must draw fundamentals from other branches of science wherever they are applicable. Also, to be of the greatest practical value, laws of this kind need to have established for them experiential factors or statistical quantities for the relationships of cause and effect which they indicate. Very few such factors or quantities have been determined.

27 The second part of this paper is an attempt at formulating some of these laws of manufacturing management. It presents upwards of forty arranged under some twenty-five headings. In practically every case numerous citations support the validity and importance of the statement offered. To all those engineers, managers, and scientists, whose experience and researches have been so freely drawn upon, the author acknowledges a deep debt of gratitude and extends earnest appreciation. Without their labors stretching back so many years into the past, this paper would not have been possible.

PART II CERTAIN LAWS OF MANUFACTURING MANAGEMENT

Laws of Specialization

28 The laws of specialization are as follows:

1 Law of Division of Work or Specialization of the Job

Subdividing work so that one or a very few manual or mental operations can be assigned to a worker tends greatly to improve the quality and increase the quantity of output.

2 Law of Division of Effort or Specialization of the Individual

Assigning to each worker one or a very few manual or mental operations which he is particularly adapted to perform greatly improves the quality and increases the quantity of output.

Corollary: Law of Functional Management (Functional Foremanship) or Specialization of the Management.

The highest managerial efficiency is obtained by functionalizing the duties of the executives.

3 Law of Transfer of Skill or Specialization of Tools and Machines

The attention and skill required to use a tool or operate a machine is inversely as the skill transferred into its mechanism.

4 Law of Simplification or Specialization of Product

Concentrating upon the manufacture of a single or a few types and sizes of product tends to improve the quality and lower the production cost.

29 The first three laws of specialization have been commonly confused or combined in the law of division of labor. However, they are distinct. The practice of division of work seems to be as ancient as the performance of useful tasks, but the development of labor-saving machinery has come within the past 150 years, while the scientific selection of the worker and his adaptation for a particular task have had their rise in the present century.

30 Adam Smith[7] in discussing the division of labor recognized three reasons why it increases production.

> This great increase in the quantity of work, which, in consequence of the division of labor, the same number of people are capable of performing, is owing to three different circumstances, first, to the increase of dexterity in every particular workman; secondly, to the saving of the time, which is commonly lost in passing from one species of work to another; and, lastly, to the invention of a great number of machines which facilitate and abridge labor, and enable one man to do the work of many.

31 Babbage[8] believed that Smith's analysis was incomplete and added a further explanation which is a statement from the employer's point of view of the law of specialization of the individual:

> That the master manufacturer, by dividing work to be executed into different processes, each requiring different degrees of skill and force, can purchase exactly that precise quantity of both which is necessary for each process; whereas, if the whole work were executed by one workman that person must possess sufficient skill to perform the most difficult, and sufficient strength to execute the most laborious, of the operations into which the art is divided.[9]

32 Early considerations of the law of division of labor restrict it to manual work. However, both Babbage and Kimball have shown that mental work and effort should be included, that the fundamentals hold for professions as well as trades.

> The division of labor can be applied with equal success to mental operations, and . . . . ensures by its adoption the same economy of time.[10]

[7] Wealth of Nations, p. 11.
[8] On the Economy of Machinery and Manufactures, p. 137.
[9] Babbage gives credit to a simultaneous though independent recognition of these further advantages by Gioja in a work published in 1815.
[10] On the Economy of Machinery and Manufactures, p. 153.

It is human experience that as a man concentrates his efforts, either mental or manual, his skill in his chosen specialty and the quantity of his product increases.[11]

33 The corollary to the law of division of effort is a consequence of the specialization of the individual. Taylor[12] explains functional management by saying that it "consists in so dividing the work of management that each man from the assistant superintendent down shall have as few functions as possible to perform. If practicable the work of each man in the management should be confined to the performance of a single leading function."

34 The third law of specialization is based on the concept that skill, thought, and intelligence—the ability to do—are transferred into and become fixed in mechanism so that a tool or machine through its motions can do productive work. Roland[13] has clearly described the process.

Intelligence may be transferred from the brain of the machine maker to the machine itself, to such an extent and with such permanence that the machine which the tool-maker has thus imbued with a portion of his own wits can subsequently, so long as it is in working order, supplement with this endowment the small mechanical skill of its tender.

35 Kimball,[14] who has written more understandingly on this law than anyone else, says:

Transfer of skill is in fact the basic idea in all tool construction. It is embodied in the first stone ax, and all succeeding improvements in tools of production were, essentially, advances in transfer of skill.

It is evident that for a given operation the more skill that is transferred to the machine the less is required in the operation. When nearly all the skill has been so transferred, but the machine still needs an attendant, it is called a *semi-automatic* machine.

. . . . it is possible to make machines to which all the required skill and thought have been transferred and the machine does not even require an attendant. Such machines are known as *full automatic* machines.

The true significance of the industrial revolution, therefore, is that prior to that time the tool was always an adjunct to the skill of the worker. The great inventions carried transfer of skill to the point where the skill of the worker became an adjunct to the tool or machine.

36 The fourth law of specialization, simplification, is being applied extensively in American manufacturing. Hudson[15] explains the practice as:

When a manufacturing company reduces its types and sizes to the least number possible, it is simplification. If a group of manufacturers of a certain line of products should agree to concentrate their produc-

[11] Principles of Industrial Organization, p. 46.
[12] Shop Management, p. 99.
[13] *Engineering Magazine,* October, 1899.
[14] Principles of Industrial Organization, pp. 12 and 13.
[15] Management's Handbook, p. 989.

tion and sales on a specific group of common products, and on specific sizes of each article in the group, then simplification is carried out on a broader scale. But the articles themselves may not have been standardized in any sense.

Law of Standardization

37 The law of standardization is:

Fixing the types, sizes, and characteristics of product reduces the cost of its manufacture.

Corollary: Interchangeable manufacture reduces manufacturing cost and, all other characteristics being equal, produces a product of maximum serviceability.

38 Kimball's[16] explanation of standardization is:

One of the essentials of cheap production is *quantity,* and for a given *total* output the greatest number of each element entering into the product is secured when the numbers of types and sizes is a minimum. By *standardization* is meant the reduction of any one line to fixed *types, sizes,* and characteristics.

39 Peck[17] offers this definition of standardization:

Selection of a small number of types or sizes which are most suitable, and giving specifications of them in measurable terms so that large quantities can be made which will be uniform.

He also says:[18]

Standardization has for years been recognized as fundamental for mass production.

40 Interchangeability, the "principle of the American system of manufacture," is outlined by Simeon North,[19] one of its discoverers, in a contract for 20,000 pistols entered into with the Commissary General of the United States on April 16, 1813, as:

The component parts of pistols are to correspond so exactly that any limb or part of one pistol may be fitted to any other pistol of the 20,000.

41 In a contract entered into 15 years later for 5000 rifles the requirement of interchangeability is even more explicit:[20]

And it is further agreed, that the said rifles shall have that perfect uniformity of their respective component parts, that any one part, or all parts of either, or any one of the rifles, may be exchanged for its corresponding part or parts, in either or any other rifle, made or to be made under this agreement. And also, that the component parts may be exchanged in a similar manner, with the rifles made, or making at the National Armory. The said exchanges to be made without impairing in the least the efficiency of perfection of the arms, which are thus composed of exchanging parts.

[16] Principles of Industrial Organization, p. 41.
[17] Management's Handbook, p. 1001.
[18] *American Machinist,* December 6, 1923.
[19] Simeon North, A Memoir, p. 80.
[20] Simeon North, A Memoir, p. 160.

Law of Responsibility and Authority

42 The law of responsibility and authority may be stated as follows:

Responsibility for the execution of work must be accompanied by the authority to control and direct the means for doing the work.

43 The authority to issue an order involves the responsibility to see that it is properly executed. (Gantt.[21])

44 Authority and responsibility for performance must be centered in the same individual. (Gantt.[22])

45 This law governs organization whether in the static form of functions, or in the dynamic form of personnel.

46 Gantt declared that this law more than any other is influential in promoting the success of an organization. The system of management which he advocated was based upon it, for a man can only assume the responsibility for doing a thing properly when he not only knows how to do it, but can also teach somebody else to do it.

47 Radford[23] has said:

It is a well-accepted principle that responsibility should be reinforced by adequate authority.

48 This law declares, on the one hand, the simple justice of accompanying responsibility with delegated authority and control, and on the other, of unavoidable responsibility for the proper execution of an order once the authority for its issue has been assumed.

Law of Leadership

49 The following is the law of leadership:

Wise leadership is more essential to successful operation than extensive organization or perfect equipment.

50 This law has long been recognized in military affairs, having been succinctly stated by one of Napoleon's historians as "a wise direction is of more avail than overwhelming numbers, sound strategy than the most perfect armament." Gantt[24] paraphrased this for industry as "a wise policy is of more avail than a large plant; good management than perfect equipment."

51 Napoleon said of military leadership:

In war men are nothing; it is the man who is everything. The general is the head, the whole of any army. It was not the Roman army

[21] Industrial Leadership, p. 8.
[22] Organizing for Work, p. 78.
[23] Management's Handbook, p. 710.
[24] Industrial Leadership, p. 12.

that conquered Gaul, but Cæsar; it was not the Carthagenian army that made Rome tremble in her gates, but Hannibal; it was not the Macedonian army that reached the Indus, but Alexander; it was not the French army that carried the war to the Weser and the Inn, but Turenne; it was not the Prussian army which, for seven years, defended Prussia against the three greatest powers of Europe, but Frederick the Great.

The history of famous armies is the history of great generals, for no army has ever achieved great things unless it has been well commanded. If the general is second-rate, the army also will be second-rate.

52 These facts so plainly recognized in military history have their exact counterpart in manufacturing. The factory and office invariably reflect the manager. His policy and direction are the deciding factors for success or failure in operation.

53 The application of this law replaces the autocratic management methods of driving and forcing by the democratic methods of teaching and training.

Law of Exceptions

54 The law of exceptions is:

Managerial efficiency is greatly increased by concentrating managerial attention solely upon those executive matters which are variations from routine, plan, or standard.

55 Taylor[25] declared the importance of the "exception principle," saying that it should extend throughout the entire field of business operation. He explained it in this way:

It is not an uncommon sight, though a sad one, to see the manager of a large business fairly swamped at his desk with an ocean of letters and reports, on each of which he thinks that he should put his initial or stamp. He feels that by having this mass of detail pass over his desk he is keeping in close touch with the entire business. The exception principle is directly the reverse of this. Under it the manager should receive only condensed, summarized, and *invariably* comparative reports, covering, however, all of the elements entering into the management, and even these summaries should all be carefully gone over by an assistant before they reach the manager, and have all of the exceptions to the past averages or to the standards pointed out, both the especially good and especially bad exceptions, thus giving him in a few minutes a full view of progress that is being made, or the reverse, and leaving him free to consider the broader lines of policy and to study the character and fitness of the important men under him.

56 Harrison[26] recognized its "supreme importance" in disclosing "variations between actual and standard costs" when he developed his cost variation formulas.

Having determined that a certain article should cost a certain amount to manufacture, in other words, having set a standard cost for it, the manufacturer should be able to dismiss the question of this

[25] Shop Management, p. 126.
[26] Management's Handbook, p. 1382.

cost from his mind until such time as there is a discrepancy between the actual and the standard cost. It is the prime function of the cost accountant to draw the attention of the operating executive promptly to the existence of such a discrepancy. In brief, this method of cost accounting is based upon the principle of exceptions.

57 Graphs (curves, charts, and diagrams) have had an extensive application in manufacturing operation as a means to bring exceptions quickly and forcefully to the attention of the management.

Laws of the Task and the Wage Incentive

58 These laws are stated as follows:

1 *The average worker accomplishes the most when assigned a definite amount of work to be done in a given time.* (Taylor.)

2 *An adequate wage incentive for the accomplishment of a definite task influences a workman to maintain his maximum output.*

59 Taylor[27] stated the first law as:

There is no question that the average individual accomplishes the most when he either gives himself, or some one else assigns him, a definite task, namely, a given amount of work which he must do within a given time; and the more elementary the mind and character of the individual the more necessary does it become that each task shall extend over a short period only.

60 And again:[28]

Perhaps the most important law belonging to this class (those that influence workers) in its relation to scientific management, is the effect which the task idea has upon the efficiency of the workman. This, in fact, has become such an important element of the mechanism of scientific management, that by a great number of people scientific management has come to be known as "task management."

61 Taylor[29] also gave factors for the amount of the adequate wage incentive:

To get maximum output for ordinary shop work requiring neither especial brains, very close application, skill, nor extra hard work, such, for instance, as the more ordinary kinds of routine machine-shop work, it is necessary to pay about 30 per cent more than the average. For ordinary day labor requiring little brains or special skill, but calling for strength, severe bodily exertion, and fatigue, it is necessary to pay from 50 per cent to 60 above the average. For work requiring especial skill or brains, coupled with close application, but without severe bodily exertion, such as the more difficult and delicate machinist's work, from 70 per cent to 80 per cent beyond the average. And for work requiring skill, brains, close application, strength, and severe bodily exertion, such for instance, as that involved in operating a well-run steam hammer doing miscellaneous work, from 80 per cent to 100 per cent beyond the average.

[27] Shop Management, p. 69.
[28] Principles of Scientific Management, p. 63.
[29] Shop Management, p. 26.

Law of Individual Productivity

62 The law of individual productivity is:

The highest individual productivity is possible only when the worker is given the highest class of work for which his natural abilities fit him.

63 Taylor repeatedly stressed this law, saying its application should be a principal aim in every manufacturing establishment. Quoting,[30]

> It would seem to be the duty of employers, therefore, both in their own interest and in that of their employees, to see that each workman is given as far as possible the highest class of work for which his brains and physique fit him.

Laws of Economic Production

64 The laws of economic production are:

(1) Production at Increasing Relative Rate

The unit cost of production decreases when the rate of increase in output increases faster than the rate of input or use of the production factors

(2) Production at Decreasing Relative Rate

The unit cost of production increases when the rate of increase in output increases at a lower rate than the rate of input or use of the production factors

and

The unit cost of production increases when the rate of output decreases and the rate of input or use of the production factors increases.

65 Church[31] defines a production factor as "any expense that has a *definite* relation to the cost of production."

66 These laws govern the results in manufacturing of efforts to increase at an economic cost the volume of goods needed to satisfy an ever-increasing demand. Different production policies result differently in respect to quantity and quality of the product. The total output of any given period can be increased over a former period by methods and policies which are perfectly well known, provided in a former period the methods and policies were less efficient than those now adopted. However, not all combination and manipulation of the production factors yield a proportional increase, and in fact an actual decrease may ensue. Thus any one of three conditions may obtain: (1) Output may increase more than in proportion to the increase in the productive

[30] Shop Management, p. 28.
[31] Production Factors, p. 15.

factors; (2) Output may increase but less than in proportion to the increase in production factors; (3) Output may decrease. These conditions operate under the laws of economic production.

67 Kimball[32] has stated the law of increasing productivity as, "The unit cost can, in general, be decreased as the quantity to be produced increases." The law of diminishing productivity (diminishing returns) is stated by Taylor[33] (F. M.) as,

> If attempts are made to increase the output of any factor of production by increasing the quantity of auxiliary factors used, a time will come, before the absolute limit is reached, when though there continues to be an increase in output, that increase is less than proportional to the increase in the quantity of assisting factors added.

Laws of Mass Production

68 The laws of mass production are:

(1) *Large-scale production tends to increase operating efficiency and competitive power.*

(2) *In large-scale production the unit time of production tends to approach the actual operating time as a limit.*

69 A remarkable fact in the development of manufacturing is the increase in the size of factories and industrial enterprises. The advantages of large size are: reduced cost of production, relatively smaller reserves to meet contingencies, and a stronger position in competition. The first of these is due to the greater opportunity to apply fully the laws of specialization; the second to the greater fluidity of stores of materials and tools and stocks of goods; the third to enlarged prestige, influence, and personnel ability.

70 Kimball[34] says of it:

> The prestige and influence of a large factory assist materially in selling product, largely because of the apparently greater stability and permanency which it suggests. A large organization can afford to hire a better class of men, especially for the higher positions of administration and design. A large enterprise also is in a better position to acquire patents and trade secrets. The advantages were, no doubt, appreciated before the present industrial era by the masters of the handicraft factories, as records clearly show.

71 Kimball[35] also states and explains the second law:

> With increasing numbers the unit time tends to approach the actual operation time as a limit. This principle, which holds true whether these values are measured in time or in dollars is, in fact, the basic principle of mass production; and the larger the number that are to be made, the greater the time and money that can be economically expended upon preparation and planning; provided, of course, that the product can be disposed of as fast as produced.

[32] Management's Handbook, p. 1210.
[33] Principles of Economics, p. 136.
[34] Principles of Industrial Organization, p. 32.
[35] Ibid., p. 268.

Law of Production Control

72 The law of production control is:

The highest efficiency in production is obtained by producing the required quantity of product, of the required quality, at the required time, by the best and cheapest method.

73 Kimball [36] in discussing the coördinating mechanism of production control writes:

In certain production enterprises, however, there is, in addition, a much more difficult problem, namely, that of *controlling production* so that the required product shall be produced in the best and cheapest method; that it shall be of the required quality, and that it shall be produced in the required time.

Law of Planning or Law of Mental Labor of Production

74 The law of planning may be stated as follows:

The mental labor of production is reduced to a minimum by planning before work is started, what work shall be done, how the work shall be done, where the work shall be done, and when the work shall be done.

75 Kimball [37] writes:

They (management engineers) aim to do the mental labor of production in a separate planning department and to predict the results of productive processes in a manner analogous to that in which the engineering department conducts the scientific part of machine design.

The planning of industrial operations involves four considerations, namely: *what* work shall be done; *how* the work shall be done; *where* the work shall be done; and, lastly, *when* the work shall be done.[38]

Laws of Material Control

76 The laws of material control are:

1 *The highest efficiency in the utilization of materials is obtained by providing the required quantity, of the required quality and condition, at the required time and place.*

2 *The highest efficiency in the storage of materials, tools and supplies is obtained by providing a definite place to store every item, keeping every item in its assigned place, and keeping an adequate record thereof.*

77 Gantt forcefully emphasized the importance of the first law by giving as his opinion that two-thirds of all the gain possible through the most efficient management (of production) could be realized by having all the material ready *when* you want it, *where* you want it, and *in the condition* you want it.[39]

[36] Ibid., p. 136.
[37] Ibid., p. 48.
[38] Ibid., p. 143.
[39] Materials and Their Handling, by Joseph W. Roe, p. 4.

78 The second law is a restatement of the folk maxim: Have a place for everything and keep everything in its place.

Laws of Quality Control and Inspection

79 The laws of quality control and inspection are:

(1) *The quality of manufactured goods is a variable with an upward trend under conditions of competitive manufacture.*

(2) *Control of quality increases output of salable goods, decreases costs of production and distribution, and makes economic mass production possible.*

(3) *The inspection function in manufacturing (measuring and judging production) for highest efficiency must be independent of but coördinate with the functions of engineering, production and sales.*

80 Radford[40] presents the fundamental of the first law in these words:

> There is a general upward trend in quality standards, amounting almost to an unformulated law of economics. When production outstrips consumption, competition enters to elevate the standard; when once the purchaser has become familiar with the better grade, his subsequent demand is for at least as good as before. This movement toward higher quality is not entirely continuous, as there are recurring cycles when consumption is greater than production, and quality is allowed to drop to the level which will be accepted. But even these retrograde movements do not carry the standard back to the previous low level as will be evident from comparing nearly any product of a generation ago with current production.

81 On the second law Radford[41] is equally explicit and positive:

> It becomes insidiously easy to reason that quality, of necessity, costs more to produce. From this false premise the manufacturer may, and often does, conclude that quantity, rather than quality of output is his first aim as a producer.
>
> As a matter of fact, the reverse is true: *When quality is controlled, quantity takes care of itself*—and output of salable goods increases, costs of production and selling decrease; and quality establishes the market which makes possible quantity production with its attendant advantages.

82 Radford[42] explains the third law in this way:

> He (the inspector) enforces quality by refusing to accept poor work, but this act of rejection is passive as regards enforcing the production of good work. Quality or lack of it must necessarily be worked into the material by the production department which controls production processes. This principle may be summarized by stating that the inspector does not make goods, consequently the responsibility for the quality produced rightly rests with the manufacturing department.

[40] Management's Handbook, p. 700.
[41] Ibid., p. 701.
[42] Ibid., p. 709.

And further[43]:

Of equal importance is the principle that inspection must be conducted as an independent function. . . . . As applied to the factory proper this means that inspection must be independent of production, and in fact, on a par with the two other main functions of manufacturing—engineering and production.

Laws of Wages

83 Two laws of wages are:

1 Law of Relative Wages.

Wages tend to lower when the supply of labor exceeds the demand; wages tend to rise when the supply of labor is insufficient to satisfy the demand.

2 Law of Wage Level.

The normal wage level of each country depends upon and corresponds to that country's general average productivity of labor.

84 Cox[44] explains the first law:

If there are many men capable of doing a given class of work, and not much demand for their services, then wages or salaries for that work will be comparatively low. But if there are very few men who can perform the work competently, and a great deal depends on its being done well, then the salaries paid will be proportionately high.
In a free country like the United States this is practically the only consideration influencing relative wages (that is, the wages or salary of one individual in relation to that of others).

85 In regard to the second law, Cox[45] states that the high wage level of the United States is "because our average *per capita* production is very high and we consequently are able to pay these high wages and still sell our products in competition with the products from other countries. The only way to secure them (high wages) is first to raise the average productivity of labor, through better coöperation between workers and employers, and through better industrial organization, and then allow the higher wages to come of themselves."

Law of Wage Rates

86 The law of wage rates is:

Wage rates on standardized jobs should never be changed except a material change has previously been made in conditions, methods or equipment.

87 This is a law of good faith between employer and employee. Once accurate job standardization has been done and time and

[43] Ibid., p. 710.
[44] *Manufacturing Industries*, April, 1926, p. 245
[45] Ibid., p. 247.

piece rates set therefrom, these rates cannot be changed without a previous change in working conditions, operations, tools, machinery, or equipment. Otherwise the morale, influence of incentives, and industrial relations are destroyed.

Laws of Hours of Work

88 Concerning the hours of work the law reads:

All other factors influencing production being constant, a decrease in the hours of work increases the leisure of the workers, and an increase in the hours of work increases the comfort of the workers. (Cox.)

The hours to be worked are always a compromise between greater leisure and greater comfort for the workers. This is not generally realized but is nevertheless true. Whether the schedule of working hours should be shortened or lengthened depends on whether increased leisure, or an increased consumption of the good things of life, is most desirable in the opinion of the general average.[46]

Laws of Acquiring Skill

89 The three laws relating to the aquisition of skill are:

(1) Law of Speed (or Facilitation)

As the newly acquired nerve path is strengthened, the new response tends to proceed more rapidly.

(2) Law of Accuracy (or Elimination)

As the new connections between impressions and memories improve, there are fewer useless and erroneous movements, the response becomes more precise and more accurate.

(3) Law of Learning

Under usual conditions an average worker acquires skill rapidly during the first half of the training period, then more slowly for a time if at all, and finally at a rapid rate until average proficiency is attained.

90 The first two laws are called the laws of habit formation. Bigelow[47] recognized the third, after analyzing several thousand cases of training in manufacturing operation, and has stated:

We believe that we have discovered an absolute fact that the average worker when he or she has covered about one-half normal training period, that there must be expected a period of time, depending upon the nature of the work, where progress or increase in skill practically ceases. Then after a short while, he attains balance of average proficiency very rapidly.

91 If the progress is represented by a curve, the period of no or little progress is represented by a flat (plateau) between two slopes.

[46] *Manufacturing Industries*, July, 1926, p. 17.
[47] Management's Handbook, p. 962.

92 Piacitelli[48] has presented curves with these characteristics both for individuals and groups.

Laws of Hand Motions in Doing Work (Gilbreth)

93 Five laws of hand motions are as follows:

(1) *Both hands should work and rest at the same time.*

(2) *Both hands should begin and complete their "therbligs" at the same instant.*

(3) *The arms should move in opposite and symmetrical directions.*

(4) *The paths of fast motions should be taught and learned.*

(5) *The sequence of fewest therbligs is usually the best way of doing work.*

94 These laws were disclosed in a paper[49] by Frank B. and Lillian M. Gilbreth, based upon the researches of these two investigators. A therblig is an element of a cycle of motions. Seventeen are recognized:

1 Search
2 Find
3 Select
4 Grasp
5 Position
6 Assemble
7 Use
8 Disassemble
9 Inspect
10 Transport loaded
11 Pre-position
12 Release load
13 Transport empty
14 Wait (unavoidable delays)
15 Wait (avoidable delays)
16 Rest (to overcome fatigue)
17 Plan

Law of Motion Time (Segur[50])

95 The law of motion time is:

Within practical limits the times required by all expert workers to perform true fundamental motions are constant.

96 This law was discovered by A. B. Segur and is an outgrowth of motion-study investigations. It provides means to determine the base or standard time for performing operations from the

[48] *Manufacturing Industries,* June, 1926, p. 438.
[49] Bulletin of Society of Industrial Engineers, September, 1923.
[50] *Manufacturing Industries,* November, 1926, p. 357.

fundamental motions required, and without using either time-studies or motion-picture studies. Its application within practical limits is independent of sex, job, place or condition. By " practical limits " is meant those that usually surround the doing of work, and motions taking longer than 0.0001 minute to complete. It is not claimed that the law applies to mental work. By " time fundamental motions " is meant the therbligs classified by Gilbreth.

Law of Delay Allowances (Barth [51])

97 Barth expresses the law of delay allowances as:

$$P = 20 + \frac{49.5 - 0.326C}{\sqrt{0.376 - 0.0000216C^2 + h}}$$

98 This formula was developed by Barth from the results of several thousand observations made by Merrick, largely in the metal-working industries. It applies to fatigue and variation allowances for cycles of short duration and in connection with good time-study work. In this formula

h = estimated minimum time study handling time for an entire cycle of operations on a job

C = the percentage that h is of the entire cycle of operations

P = the percentage by which h is to be increased (the allowance percentage) to give the handling portion of the total task time for the entire cycle.

Law of Manufacturing Cost

99 The law of manufacturing cost is:

The manufacturing cost of an article includes only those expenses actually necessary for its production.

Corollary: The indirect expense chargeable to the output of a factory should bear the same ratio to the indirect expense necessary to run the factory at normal capacity as the output in question bears to the normal output of the factory. (Gantt.[52])

100 The three items of manufacturing cost are: material, direct labor, and indirect expense. The law of manufacturing cost limits charges to a given product to:

(1) Sufficient material for its production

(2) Sufficient direct labor for its production

(3) Only that part of indirect expense which contributes to its production.

101 This law is in conflict with a common theory that the cost of an article must include all of the expenses incurred while producing it, whether such expenses actually contributed to its production or not.

[51] Time Study for Rate Setting by Dwight V. Merrick, p. 64.
[52] Organizing for Work, p. 34.

102 The determination of material and labor items is not difficult, but indirect expense is a convenient blanket to hide idleness and waste.

103 One application of this law of manufacturing cost is the effort to reduce idleness and eliminate waste. Methods of determining idleness, both of machines and men, and as to amount and cause, are highly developed, and the favorable effect of idleness reduction on costs is generally appreciated.

Law of Profit

104 The law of profit states:

A steady and reasonable profit can only come as the reward for rendering essential service.

105 Franklin,[53] one of our clearest thinkers on the question of profit, has written:

Every industrial executive and every owner in an industrial enterprise, whether in whole or in part, strives for and desires a profit. This is the essential reason for ownership. But ownership is no longer, if it ever was, a warrant for profit, for profit must come from the public, and can come from nowhere else. The public is only willing to pay a profit for service, and, with the wider spread of industry, for the best service; therefore, essentially and fundamentally, industry is a service.

If industry is conducted as a service, then certainly the public must expect to reward that service, and the renderer of that service is entitled to expect a steady and reasonable profit, for that is the essential urge to begin and continue rendering service.

106 The National Machine Tool Builders Association[54] has declared:

The reward of business for service rendered is a fair profit plus a safe reserve, commensurate with risks involved and foresight exercised.

107 Swope[55] forcefully supports this law.

The reason for the existence of industrial organizations is not first and primarily for profit, but to furnish the community with something the community desires to have. If an organization furnishes something that the community wants, of good quality and at a fair price, it will always be rewarded for that service by an adequate profit.

Law of Plant Maintenance

108 The law of plant maintenance is:

Anticipating repairs and replacements prevents interruptions due to bad-order or broken-down equipment.

[53] The Industrial Executive, p. 100.

[54] Guide for Applying Code of Ethics.

[55] From address before Illinois Manufacturers' Association, December 9, 1925.

109 Waldron[56] has expressed the concept of this law as:

Purpose of plant maintenance is to anticipate and prevent to greatest possible extent, interruptions in operation and loss of output due to bad-order or broken-down machinery and equipment in a manufacturing concern.

The folk maxim, "A stitch in time saves nine," has the same thought.

Law of Flow of Work

110 The law of flow of work reads:

The greatest economy in progressing materials through a manufacturing plant is secured when the materials move a minimum distance in passing from operation to operation.

111 Kimball[57] discusses this fundamental under the heading "Plant Planning Flow":

It will be obvious that greatest economy in transporting the material through the plant will result when the several departments and buildings are so managed with reference to each other that the material shall be moved along with a minimum amount of traveling and handling and so that the factory shall work smoothly as a whole.

Law of Discrimination

112 The law of discrimination is as follows:

Sensations increase in arithmetical progression as the stimuli increase in geometrical progression. (Weber.)

113 This law is the basis of the theory of preferred numbers and the application of geometric series in determining proportions, sizes, dimensions, and industrial ratings. It has been proved experimentally and the least perceptible difference of intensity determined for sensations as, visual, auditory, olfactory, gustatory, tactile, kinesthetic, warmth and cold. It is believed to hold for size and duration intensities.

114 In management work Barth[58] has used geometric progressions in standardizing the feeds and speeds of machine tools and for establishing wage scales. For the first use he has adopted the ratio $\sqrt[4]{2}$, for the second the ratio 1.15, approximately $\sqrt[5]{2}$.

115 The French preferred number series (Série de Renard) has three series, the ratios being $\sqrt[10]{10}$, $\sqrt[20]{10}$, and $\sqrt[40]{10}$, respectively.

116 In Germany where preferred numbers are extensively used in design, the adopted system has five series, the ratios being $\sqrt[5]{10}$, $\sqrt[10]{10}$, $\sqrt[20]{10}$, $\sqrt[40]{10}$, and $\sqrt[80]{10}$, respectively.

[56] Management's Handbook, p. 1037.

[57] Principles of Industrial Organization, p. 76.

[58] *Manufacturing Industries*, December, 1925, p. 357, and May, 1926, p. 373.

Law of Economic Lot Size (Davis[59])

117 The law of economic lot size is:

The quantity of product that can be manufactured at the lowest unit cost varies directly as the square root of the preparation costs and inversely as the square root of the interest charge, and storage charge.

That is

$$Q = \sqrt{\frac{A}{K+H}}$$

where

Q = the most economic number of pieces to manufacture
A = total cost of preparation for manufacturing
K = a constant where values depend upon the ratio F of the minimum ordering point quantity to the theoretical minimum quantity withdrawn from stock during the manufacturing period
H = storage factor (influence of storage charges on economic size of lot).

K (for F less than unity) =

$$\left[\frac{M + MS(2F-I) + FS^2(F-I)}{2M^2S}\right] C'I$$

K (for F greater than unity) =

$$\left[\frac{M + MS(2F-I) + 2S^2(F-I)}{2M^2S}\right] C'I$$

H (for F less than unity) =

$$\frac{(M+SF)\,[M+S(F-1)]}{M\,S} BE$$

H (for F greater than unity) =

$$\frac{(M+S)\,[M+S(F-1)]}{M\,S} BE$$

where

B = bulk factor, square feet of net storage space per unit
C' = unit cost of quantity to be manufactured, in dollars
E = storage charge expressed in dollars per square foot of net storage space per year.
I = current rate of interest
M = yearly manufacturing rate
S = yearly consumption rate.

[59] *Manufacturing Industries,* April, 1925, p. 353, and August, 1926, p. 129.

Laws of Economy of Labor-Saving Equipment (A.S.M.E.)[60]

118 The laws of economy of labor-saving equipment may be expressed as follows:

$$Z = \left[\frac{(S+T_a+U-E)X-T_b}{A+B+C+D}\right]-K$$

$$Y = I(A+B+C+D)$$

$$V = [(S+T_a+U-E)X+T_b]-[Y+(KA)]$$

$$P = \frac{V}{I}+A$$

$$H = \frac{100}{P+D}$$

119 These formulas were developed for and adopted by the Materials Handling Division of The American Society of Mechanical Engineers in 1925. They provide means for economic analysis of manufacturing to show the savings that have accrued or may be secured from improved mechanical equipment. The factors involved are:

Debit factors

- A = percentage allowance on investment
- B = percentage allowance to provide for insurance, taxes, etc
- C = percentage allowance to provide for upkeep
- D = percentage allowance to provide for depreciation and obsolescence
- E = yearly cost of power, supplies, and other items which are consumed, in dollars

Credit factors

- S = yearly saving in direct cost of labor, in dollars
- T_a = yearly saving in labor burden, in dollars
- T_b = yearly fixed charges, on mechanical equipment employed as a standard of comparison or which will be displaced, in dollars
- U = yearly saving or earning through increased production, in dollars

Other factors

- X = percentage of year during which equipment will be operated
- I = initial cost of mechanical equipment, in dollars
- K = unamortized value of equipment displaced, less its resale or scrap value, in dollars

[60] Formulas for Computing the Economies of Labor-Saving Equipment, by James A. Shepard and George E. Hagemann. Presented, May, 1925.

Results

Z = maximum investment which will earn simple interest, in dollars

Y = yearly cost to maintain mechanical equipment ready for operation (fixed charges), in dollars

V = yearly profit, in excess of simple interest, from operation of mechanical equipment, in dollars

P = yearly profit from operation, in per cent on investment

H = years required for complete amortization of investment out of earnings.

DISCUSSION

John Gaillard[1] and F. J. Schlink.[2] It is suggested that consideration be given to an elaboration of the Law of Standardization along the following lines:

1 *A standard, in order that it shall raise rather than lower the standard of products, should deal with those elements or principles which are sufficiently understood and recognized to be stable. It should be concurred in by the most competent experts, so that their consensus as crystallized in the standard can be expected to control production for a reasonable period. However, a standard should deviate to the least possible extent from economical current practice, and where practices vary over a large range, effort should be made to follow the main path of common practice, if thereby economy and utility do not have to be sacrificed.*

Evidently, if a standard were "designed" like a new machine, that is, by establishing it anew from the ground up, the one best way would be to choose the most efficient, economical solution for each of its elements. However, standardization naturally takes place only after a certain progress in an art or industry has been made, and special customs ("unrecorded standards") have been formed. Therefore, when the art has crystallized sufficiently to enable the setting-up of standards, a compromise will generally have to be reached between different solutions already obtained for the same problem. It is often a difficult and delicate work to decide on the best compromise. It will thus be seen that a standard, when analyzed, need not absolutely give the most perfect solution for each of the elementary problems involved, as it may happen

[1] Mechanical Engineer, American Engineering Standards Committee, New York, N. Y.

[2] Assistant Secretary, American Engineering Standards Committee, New York. Mem. A.S.M.E.

that a less valuable solution of one part of the problem has been adopted in order to allow the application of a more important and a more valuable one in respect to another item.

2 *The value of a standard increases rapidly with the extent to which its provisions are based upon, and are therefore verifiable by, measurement. Therefore, those elements of a standard which depend upon, or are decided by personal judgment unaided by instrumental measurement, should be reduced to a minimum.*

Corollary: The successful operation of production under standards requires the establishment of concrete means of measurement and responsibility for their maintenance, including such items as systems of testing and inspection (testing devices, master and working gages, etc.)

Setting a standard means, in principle, the establishment of a level or reference-line from which to measure quality or quantity. It is necessary that the relation between each object falling within the standard, and the standard itself, be expressed by measurement in some definite unit. Therefore, in places where a standard refers to terms such as "reasonable," "in accordance with the best workmanship," etc., it presents weak spots allowing discussion between interested parties or even litigation, whereas one of the purposes of setting a standard is to avoid such discussion.

On the other hand, it should always be borne in mind that the accuracy of all measurement is relative, depending, among other things, on the size of the unit of measurement. Consequently, it is desirable and often required that any essential item stated in terms of measurement in a standard, be accompanied by tolerances allowing for slight and non-essential deviations from the exact nominal value, as such deviations cannot be avoided in human work.

3 *Difficulty of arriving at a standard is increased by the number of departments of an industry or conditions of application which it must serve and satisfy.*

It would sometimes appear that the difficulty in setting up a standard increases at a faster rate than the number of groups involved in the establishment of the standard. In fact, standardization work must be undertaken with a definite desire to arrive at a solution, and with the willingness to sacrifice part of one's own interest to the problem under consideration. Here again, but in a different way than mentioned under (1), the statement holds good that the standard ultimately set up may not be the one best for a particular group; but on the other hand, the fact that a standard exists at all, including the stability which such condition brings about, is usually well worth the partial sacrifice of direct interest of each individual group.

4 *Every standard should be subject to revision from time to time in order that it shall continue to reflect the available and applicable knowledge of the art.*

Standardization does not mean the establishment of something rigid for all time; it means moving forward together in a coördinated manner. Therefore, the desirable procedure is to have the groups which set up a standard get together periodically in order to check up whether any change is necessary, and if so, to have them make that change in the same way as the original standard was set up, that is, by consensus of opinion.

5 *Standardization focuses the attention upon the essential elements of the object under consideration and thereby transfers competition to a field where it has a positive social value.*

After a standard has been set, all parties interested in the subject, even those who are not experts, will know what to look for, and even to a certain extent how to measure the value of the object under consideration. This creates a new situation, as the standard thus lays a sound basis for rational judgment of things by those who, before standardization, had been more or less subject to indefinite and unverified claims made for a product, method, etc.

Jos. W. Roe.[1] We all join with the author in hoping that the Laws of Management may be established as firmly as those of mathematics, physics, and chemistry. This paper will help toward that end.

Science has been defined as knowledge based on exact observation, related by correct thinking. Exact observations in science are obtained only through measurements, that is, by comparison with fixed and accepted standards. Exact observations alone do not constitute a science — not until the interrelations of the data, i.e., the laws, are established do we have a science by means of which we can predict results. These laws also must be verifiable by measurement. Until they are so verified they are considered only hypotheses. An example is Einstein's Law of Relativity. The elaborate attempts to verify it are justified because it cannot be accepted until it has been verified by exact measurement. Laws which cannot stand the test of measurement are either not correctly thought out or the fundamental data are incorrect or incomplete.

Much of Frederick W. Taylor's work is scientific because it is based on exact observations through the use of measurement, and the laws he deduced from them are verifiable by measurement.

The applied sciences have developed and grown in value directly with the improvements in measurements. L. W. Wallace has said

[1] Professor of Industrial Engineering, New York University, New York, N. Y. Mem. A.S.M.E.

"a science always relates to something known, an art to something done." Management is both a science and an art. In so far as it is a science its underlying data and laws are verifiable by measurement. In so far as it is an art it must be evaluated by judgment rather than measurement. In so far as measurement can be and is applied to management it will be as helpful as it has been in other fields of sciences. By measurement is meant the comparison with agreed-upon standards, which may be ratios, quantities, or other bases. The essential thing is that they be generally accepted, unchangeable, and mean the same thing at all times to all concerned. The financial ratios which are finding increasing use are examples.

In a paper before the Society some years ago, the writer attempted to give some of the limitations and principles which apply to the use of measurement in operations of management. The writer believes that it can be used to test the laws of management and the operations of executives using them. He suggests, therefore, that we might add another to the principles suggested by the author, namely:

Since exact observation underlies all science, measurement, i.e., comparison with uniform standards, can be applied to such phases of management as have a scientific basis and in so far as this is done it will aid in predicting results and in the execution of work.

Wallace Clark.[1] The author deserves the congratulations of engineers and all others interested in management for his work in collecting and codifying these laws of management. This is an important step in the development of the science of management. There is, of course, room for difference of opinion as to which of the laws as stated should be generally accepted and which should not.

The first two laws presented in Par. 28 read as follows:

1 Law of Division of Work or Specialization of the Job

Subdividing work so that one or a very few manual or mental operations can be assigned to a worker tends greatly to improve the quality and increase the quantity of output.

2 Law of Division of Effort or Specialization of the Individual

Assigning to each worker one or a very few manual or mental operations which he is particularly adapted to perform greatly improves the quality and increases the quantity of output.

[1] Consulting Management Engineer, New York, N. Y. Mem. A.S.M.E.

The splitting of the law of division of labor into two parts, i.e., the specialization of the job and the specialization of the individual, is confusing. The first seems to be only a paraphrase of the second. The expression used in the first, "tends greatly to improve," and that in the second, "greatly improves" do not convey the impression that the consequences are inevitable.

Kimball seems to have expressed this more compactly when he said: "As the worker restricts the field of his endeavor, his product rises in quality and quantity."[1] Expressed in this way, the law is susceptible of measurement and meets the requirements set forth by Professor Roe in his discussion of this paper. It is therefore suggested that in place of the first two laws presented by the author we have one, reading as follows:

Law of Division of Labor

As the worker restricts his field of endeavor, his product rises in quality and quantity. (Kimball).

The corollary to the two laws cited (Par. 28) reads as follows:

Law of Functional Management (Functional Foremanship) or Specialization of the Management

The highest managerial efficiency is obtained by functionalizing the duties of the executives.

It has been the writer's experience that functionalized management cannot equal in efficiency non-functionalized management, unless the scattering of authority due to splitting up executive responsibilities is balanced by stronger coördination of the organization as a whole.

I would therefore suggest the substitution of the following corollary:

As the scope of his responsibility is narrowed, the efficiency of an executive increases.

The efficiency of an organization is increased by subdividing the responsibilities of its executives, if there is a corresponding improvement in the technic of coördination and an increase in the energy and wisdom of the chief executive.

Carle M. Bigelow.[2] The author has rendered a real service to management in the preparation of this paper. For the first time, the real principles of scientific management application are brought together in a limited space. This is particularly valuable, not only because it will enable all consistently to review the funda-

[1] Article in *Management and Administration*, July, 1923.

[2] President, Bigelow, Kent, Willard & Co., Boston, Mass. Mem. A.S.M.E.

mentals which are often lost sight of in the whirl of detail in their application, but also since it should be of particular value to students who, wading through the vast mass of management literature of the present time, fail to discern the fundamentals.

Regarding the Laws of Material Control, Pars. 76 and 77, it is the writer's belief that the original Taylor and Gantt concept of material control has been gradually misconstrued from the proper definition of "required quantity" meaning "the necessary amount and no more," to a popular conception of "more than enough." During the last thirty years it was fairly safe most of the time to have some excess of everything on hand, inasmuch as material prices were gradually rising. At the present time, however, with falling prices, it is necessary to limit all inventories to their absolute minimum.

With this in view, possibly another corollary to the Laws of Material Control should be added, as follows:

3 *Material inventories should be maintained at exact requirements during periods of falling prices, while speculative advantage may be taken in excess stock during periods of rising prices.*

The writer well realizes the distinction between the "Art of Application" and the "Science" of management. However, one of the outstanding reasons why scientific management has not been properly applied, even when its laws are understood and observed, has been the failure to consider the effect of each law, not upon the phase of management it primarily affects, but upon the entire management concept. Production increase beyond possibility of sales absorption, while thoroughly grounded on the Laws of Labor Economic Production, may easily ruin a business. It would therefore seem that, after all the other laws have been defined, we should have one final law which governs their application. The following is suggested:

The application of every individual law of management must be so made that it contributes its proper share to the final satisfactory composite result, and the reaction of each law on every other law should be given proper consideration.

It is realized that this is possibly invading the field of the art of application rather than the science, but without this consideration of the final composite result, confusion is almost sure to result.

The laws of management must necessarily deal with the recognizable and definite phases of management. As experience is gained in the application of scientific management, it seems to become more and more evident that with or without its laws, and no matter what the method of application, there is an underlying something above and beyond laws, principles, or methods, which can be

described only as the spirit of scientific management. If, in the application of scientific management, the belief is inculcated that improvement is possible, it would seem as if 98 per cent of scientific management had been applied. Be the application good or bad, if a manufacturing organization is once thoroughly imbued with a real conscious belief that things can be improved, the success of that organization is assured.

When the laws of management are finally agreed upon and placed in definite form, someone should be given the task of preparing the proper preamble, maintaining above all else that there is a spirit or soul of management which is greater than all methods or even laws.

RALPH G. WELLS.[1] One handicap to the management movement has been the over-emphasis of system and routine rather than underlying principles. The writer is fully in accord with the majority of the laws set forth by Mr. Alford, and desires to see them extended. He suggests certain amplifications and additions as follows:

The author's laws of specialization, taken together with one of Taylor's laws, suggest the following corollary:

(*a*) *Clearly defined tasks, duties, responsibilities, and authority tend to increase individual and coöperative effectiveness of both management and worker.*

To the laws regarding training the writer suggests as an addition:

(*b*) *Clear, definite, precise, adequate instructions, based on sound, standard practice, improve both quality and quantity of output, reduce the probability of error, and promote harmony.*

(*c*) *Thorough training of employees increases their interest and effectiveness. It tends to reduce the amount of supervision required.*

The law of specialization and individual productivity suggests another corollary along the following lines:

(*d*) *Highest quality and greatest output from a given job or production center is secured when the employees are selected carefully and accurately, in accordance with predetermined job specifications.*

To the laws of planning and management effectiveness the following might be added:

(*e*) *Coördination of departmental activities through the development of comprehensive annual or seasonal manufac-*

[1] Management Engineer, Boston, Mass.

turing and sales programs, enforced through some form of budgetary control, secures the highest managerial efficiency.

Add also:

Wise policies definitely established and thoroughly understood by the entire organization, if consistently adhered to, promote stability and permanence of management.

Somewhere in this summary of management principles the writer would like to see a statement emphasizing the fact that continued success and progress in a business is dependent on frequent overhauling and checking up of all operating, management, and control methods, with a view to making possible improvements.

It is apparent that the author has opened the way to an interesting and somewhat unexplored field. It is the writer's hope that as a result of Mr. Alford's paper The American Society of Mechanical Engineers will appoint a committee, with Mr. Alford as chairman, to consider further this matter and make a systematic study of the entire subject which he has opened up in such an interesting manner.

G. Charter Harrison.[1] J. Arthur Thompson in his well-known Introduction to Science states: "There is no doubt that all the sciences — not excepting psychology and sociology — sprang from concrete experience. Mathematics is abstract enough, but what does its history show? Man began arithmetic with experience of the number of his fingers and toes, and geometry with experience of the magnitude of his hands, feet, and arms. He went on to use these concrete bodies as standards to measure other bodies . . . Mathematics, in short, began with concrete bodies, such as could only be reached by means of experience, and only gradually receded from the concrete to the abstract, to the units of abstract arithmetic, and the points of abstract geometry."

The science of management has followed the same course as other sciences — its history is a story of experiment — of trial and error. Gradually out of the mass of experience there have developed certain basic laws, many of which the author has incorporated in a paper which represents an outstanding contribution to the literature of management science.

No intelligent student of management science can have failed to realize and deplore its incompleteness and the lack of agreed-upon statements of fundamental laws. As the author points out, when the laws of management science have once been reduced to codified form it will be possible for these laws to be taught to students in engineering schools who wish to prepare for executive

[1] Stevenson, Harrison & Jordon, Management Engineers, Chicago, Ill.

and managerial responsibilities. It may be added that when this result is accomplished it will be possible for the members of the management engineering profession to build up their organizations with men who have a definite recognized standard of knowledge of fundamental principles. At present a management engineer can hire a qualified mechanical engineer with the definite assurance that he will at least be properly grounded in fundamental principles, but this is not always the case when employing a management engineer.

In stating the law of wage rates the author says: "Wage rates on standardized jobs should never be changed except a material change has previously been made in conditions, methods, or equipment." Apparently he restricts the word "conditions" to mean working conditions in the shop, whereas it would seem that the law should be read to include as a change in conditions an upward or downward change in the general wage scale, reflecting a change in the cost of living.

In setting forth his forty-three laws of manufacturing management the author makes no claim that all of the laws of management are included in his paper. An important law which the writer believes should be added is the law of diminishing returns. Kemper Simpson says: "As the production is increased, and as new units of labor and capital are added, the unit cost of production is decreased. However, in almost any type of business, there is a limit beyond which additional units of labor and capital goods can be added economically." Mr. Ford, in his latest book, Today and Tomorrow, states that when he reached a production of 1000 cars a day and jammed the freight facilities of Detroit he began seriously to examine into the wisdom of having so large a plant. Two serious objections were discovered: The first, that with the payroll concentrated in a single locality there was a tendency to profiteer upon the workmen. Secondly, there arose the difficulty of transporting the help. Mr. Ford sums up his conclusions in these words: "It is better to avoid difficulties than to overcome them, and not only do we find it easier to manage smaller plants, but also — which is most important — *the costs of production in the smaller plants are lower.*"

The law of diminishing returns also restricts the application of the laws of specialization, of planning, and of quality control and inspection. As J. P. Jordan pointed out in the discussion on George D. Babcock's paper[1] there are distinct limits to the extent to which highly detailed methods of production control can be successfully and economically adopted. There is certainly a point of diminishing returns in inspection work.

[1] Production Control. Trans. A.S.M.E., vol. 46 (1924), p. 685.

Professor Henry Ludwell Moore, in his Laws of Wages, states two laws of wages which might be added to those given by the author, as follows:

1 *That the more rapid the increase of capital in the industry, the more rapidly do wages increase.*

2 *That the fluctuations of wages about their general trend are inversely correlated with the machine-power with which the laborers work.*

Professor Moore says regarding the above, "It is true that these points were established only with reference to the one industry for which we could obtain adequate data, but there is the great satisfaction of knowing that the inductive findings with regard to this one industry are in complete accord with the conclusions of *a priori* reasoning."

Lillian M. Gilbreth.[1] Two things in this paper might well be emphasized. The first is the need for accurate measurement. Whether we call these "laws" statements, fundamentals, or accepted practice, we must have accurate measurement if we are to prove or disprove the findings that they cover. Its application will be one means of tying together the various fields of investigation and types of investigators, and doubtless this is one of the things that the author had in mind.

For example, take Par. 94. If one begins to study hand grasp, and to analyze it, he may find that what he really needs is tool grasp. Then perhaps from tool grasp he may proceed to machine grasp and finally to a complete re-study of the resulting machine, to determine how the worker grasps the handle to run it, which will begin the cycle from hand grasp again.

In the second place, there is need to realize the broad scope of possible application. The author has been careful to say that while he believes the laws are as stated, there is room always for improvement. For example, take Par. 89, where, speaking of the learning process, he says, "Under usual conditions, an average worker acquires skill rapidly during the first half of the training period, then more slowly for a time if at all, and finally at a rapid rate until average proficiency is attained." It should be said that under unusual conditions even an average worker may avoid a plateau, and under usual conditions a worker above the average may avoid one. The upper boundary is not set, and the room for improvement is practically limitless.

It is to be hoped that these "laws of manufacturing management" may be extended into papers on laws of office procedure, selling, etc. In such papers will be included a new field for management, that of the home. We have made a start in the application

[1] **Chief of Staff, Gilbreth, Inc., Montclair, N. J. Mem. A.S.M.E.**

of management to this field, and, while American engineers so far do not seem to take the work seriously, our foreign guests tell us that Europe feels that the application of management must be extended into the home if waste elimination is to be properly carried out.

Industry itself will profit by this work, for we shall at last have control of knowledge of the entire situation and of the twenty-four-hour day.

Carl G. Barth.[1] The author deserves much credit for his bold attempt to formulate a set of laws for the guidance of managers. However, the writer does not share his belief that the laws of management may be established as securely as are the laws of mathematics, physics, and chemistry, except, perhaps, in that field of management which deals with materials and machinery. In the field which deals with human problems numerous helpful rules and principles may undoubtedly be established and made into a code for general guidance; but these can never possess that rigidity which would properly make the term law applicable to them.

On the other hand, some of the "laws" formulated by Mr. Alford are of such a self-evident nature that they should be designated axioms rather than laws of management, while some of them are indeed debatable. Thus the Law of Responsibility and Authority credited to Gantt, though he only repeated what Taylor impressed upon all his disciples, is surely an axiom. Nobody possibly can dispute the truth of that proposition the moment it is announced, even though we still have men in managerial positions who do not always act in accordance with it. The same is true of the Law of Economic Production. The Law of Wage Rates has been misstated in the usual way. It is not the wage rate that should never be changed, as stated, but the production time on which the wage rate is based. Wage rates on standardized jobs are subject to the same fluctuation as are day wages. Another "law of wage rates," in the sense used by the author, is:

Wage rates must be based on such simple mathematical rules that the worker will have no difficulty in determining, unaided, the money he earns in doing a job.

Under the Law of Delay Allowances the author gives the empirical formula developed by the writer from data submitted by Dwight V. Merrick. While this formula has been used quite successfully by Mr. Merrick and others in connection with the general run of machine-shop work, and also in the more limited field of a clothing manufacturing establishment, it certainly is not of a nature that justifies its being elevated to the dignity of

[1] New Haven, Conn. Life Mem. A.S.M.E.

a law. It should be given, if at all, in a possible special group of empirical or tentative laws. Neither is the Law of Manufacturing Cost as formulated by Gantt a law, but merely a choice of policy, the propriety of which the writer, for one, is always ready to dispute.

Under Law of Discrimination the author refers to so-called preferred numbers; but the mere fact that such numbers are preferred implies a choice and not a law, even though preferred numbers, as generally understood, are numbers following some simple mathematical law.

It would therefore seem that a better title for that which the author had in mind when he undertook the preparation of his paper would be, say, A Code of Axioms, Laws, and Principles for the Guidance of Industrial Managers.

C. W. Beese.[1] The mention of interchangeable manufacture opens a field that will bear some expansion. Perhaps the statement under discussion would be more widely acceptable if the factor of quantity were introduced to qualify the statement that "interchangeable manufacturing reduces manufacturing cost." The methods of interchangeable manufacturing introduce certain economies, particularly with reference to two aspects of direct labor charges. When based on proper tools, such methods not only involve a smaller expenditure of time by direct labor, but permit the efficient use of a less expensive type of workman. But the tool equipment necessary to attain interchangeability and to check this attainment during manufacture is costly, and is justified only if it promotes ultimate economy.

In any manufacturing problem, there is a point of maximum economy, the location of which may be calculated with respect to the quantity of product to be produced. At this point, the predominance of the effects on cost of these two opposing factors transfers from one to the other. The production of a smaller quantity of goods by methods of interchangeable manufacturing would be less economical, owing to the fact that the saving in direct labor charges would not offset the increased costs of special tools. At a higher rate of production the reverse is true. It should be clear that the aim of the manufacturer is the ultimate economy, and that considerations of cost and convenience to the consumer of a product may warrant an increased cost of manufacture in order to attain the advantages of interchangeability.

In a discussion of interchangeable manufacturing, Buckingham states that the principal elements of economical production are:

1 A thorough knowledge of the object (function) of the article and of all the conditions essential in attaining it.

[1] Head of Department of Industrial Engineering, Pennsylvania State College, State College, Pa. Assoc-Mem. A.S.M.E.

2 The development of manufacturing methods and facilities that will most economically produce a satisfactory product.

3 The development of testing methods and apparatus to determine in an economical manner, at any stage, whether or not the desired results are being achieved.

The second and third items are the ones which introduce elements of expense and which promote economy only if such expense is distributed over a sufficient quantity of product.

One other point is that it is a little dangerous to include among a list of principles to be applied in a more or less general fashion, a statement in the form of Mr. Barth's equation for delay allowances. It is doubtful if we can go farther than a general statement that the data at hand indicate that the delay allowance varies inversely as some functions of the length of the cycle time, and inversely also as some function of the percentage of the cycle time consumed in handling operations.

Numerical values of the constants in such an equation are determined by the values on which the mathematical analysis is made and on the judgment of the person making the analysis, first, as to the form of the equation, and, second, as to the fit of a curve drawn to plotted data. Obviously, data taken under different conditions or analyzed by different persons to show the relationships of the variables involved will produce at least an equation having different values for the constant terms and probably one of an entirely different form. This is a point which might warrant sufficient study to evolve a law of general application.

Leonard M. Kuvin.[1] Probably the greatest single fact about this paper is its capacity to force people to think. As most of the laws stated in the paper are not quantitative, they are open to criticism, which is, similarly, not quantitative.

The laws that the writer proposes to take up are the laws of: (*a*) Hours of Work, Par. 88; (*b*) Task and Wage Incentive, Par. 58; and (*c*) Profits, Par. 105.

The number of hours of work in industry is taken to be the number at which there is a balance between comfort and leisure, when all other factors influencing production are constant. This law silently assumes, first, that comfort and leisure are independent of each other, and, second, that all other factors can be constant. The general average of industrial workers think neither of comfort nor leisure nor the product of the two when their work day is too long for their endurance; they are concerned merely with animal rest.

The length of work day in any industry at any time is a function of: (*a*) The nature of the work, monotony, noise, etc., and

physical wear of the body of the worker; (*b*) competition within the industry at home and abroad; and (*c*) the relative productivity of the industry and others in the country.

The laws of task and wage incentive as given in the paper deal only with the wage incentive. When we consider that non-financial incentives are as great as, if not greater, than wage incentives, it seems wrong not to include them in a body of laws on management. Just how much more effective non-financial incentives are than financial incentives is a difficult thing to measure; but the experiences of Wolff and others give testimony to their importance. However, discussing wage incentives, the factors suggested by Taylor deal only with the possible additional yield of the individual. Wage incentives should be apportioned to the burden and productivity of the machine when skill has been transferred from the operator to it.

As regards the law of profit, we may agree with the authorities cited, and with Gantt, that profit should fundamentally be a reward for service rendered and not for ownership. That such is the case, in practice, there are few facts to indicate. If public utility rates, for example, were based not on property owned, but on actual productivity, then profits could be said to be based on service rendered. Until costs are relieved of the increment due to idle machinery, profits will continue to contain that element due to the cost of their ownership.

J. A. Brown.[1] The writer believes that the matter in Par. 118 entitled Laws of Economy of Labor-Saving Equipment should be subjected to more critical scrutiny both as to its scope and subject matter.

Just what are the so-called "laws" when stated in words? John Maurice Clark, in speaking of the general law of mechanical improvement, says: "The general rule governing all such questions of policy may be put in this form: Most labor-saving devices of a mechanical sort call for an investment in some sort of machinery or equipment and by means of this investment the labor costs of the operation can be reduced. The quantity and quality of equipment which it pays to install depends on the amount of use that will be made of it."

J. P. Jordon, in Cost Accounting, says: "Department heads and foremen may ask for the purchase of time-saving and labor-saving machines without considering fully the increased capital and running costs of the machine which may offset the savings in time and labor. In other words, not enough attention is given in such cases to the operation of the economic principle of selection or substitution."

[1] Mechanical Engineer, Brooklyn, N. Y. Mem. A.S.M.E.

John M. Williams several years ago before the Taylor Society covered a wide scope in fundamentals with exceptional clarity and precision. Under Elements of Cost these statements are made: "There are two conflicting elements of cost, the confusion of which is at the bottom of a large part of our business troubles: Variable cost, that part of cost which varies in proportion to the volume of business done; and fixed cost, that part of cost that exists irrespective of the volume of business done. If we are consistent in dividing *all* our costs into the foregoing elements, we find the whole problem of administrative accounting wonderfully simplified."

Is there any better basis for comparing costs than on that of the total annual cost? Is there any better functional grouping of cost elements than as variable cost and fixed cost, or as some writers prefer, controllable and non-controllable expense?

Robert T. Kent.[1] The author has presented a paper that is a milestone in the progress of Scientific Management. All sciences in their development go through a more or less definite cycle, viz: The collection of a mass of somewhat unrelated facts and data; the correlation of these data, and the derivation from them of some general principles; the application of these principles to concrete cases; and finally, the formulation of laws based on the results of application of the principles. The modern management movement, although barely a quarter of a century old, has, with the presentation of this paper, passed through all of these phases. It now remains only to test these laws to determine their truth, and to add to them, to finally establish management as a true science.

The paper gives a fairly complete answer to the question, which in the early days of the management movement was often asked but seldom answered, "What is scientific management?" Scientific management, fifteen years ago or less, was passing through the second and third stages, noted above, in the development of a science. As a science it was exemplified only in the practices of a few manufacturing companies. The question as to what it was could be answered only by referring to the practice of these companies. It is small wonder, therefore, that it was regarded generally as only a system of routine on the part of employers, and a device for the exploitation of the workers on the part of organized labor. Being thus misunderstood on both sides, it is not remarkable that it encountered the violent opposition that it did, which opposition even extended to the prohibition of some of its features by Congress in connection with work for the United States Government. Had there been in existence, at the time, a body of

[1] Superintendent of Prison Industries, State of New York. Mem. A.S.M.E.

laws of management, whose truth or falsity could have been demonstrated, it is probable that much of the opposition would have been absent. Happily, the day has passed when the science of management must fight for its very existence, and the things that were then regarded as revolutionary in industry are today accepted as commonplaces. This, however, does not detract in the slightest from the service that the author has done for industry in formulating these laws.

There may be some difference of opinion regarding some of the laws that the author has set before us. Some of them admittedly are of controversial character. Without question, they are incomplete. Nevertheless, taken as a whole, they are a distinct addition to the literature of management, and as such are bound to have widespread effect. It is with one or two of these effects, rather than with the laws themselves, that the writer wishes to deal.

The formulation of these laws should have a profound effect on the teaching of management in the colleges. A comparison of the courses offered by several schools a few years ago revealed a rather imperfect understanding on the part of those laying out the courses of what management really was. They emphasized the system and the routine, as exemplified in those institutions that had come under observation, and neglected to a considerable extent the basic principles of management. Most of them laid great stress on time study, whereas time study is one of the minor phases of management. They seemed to have no common objective, nor a common starting point. It would be unfair to criticize the schools for this, because up to this time there has been no clearly defined basis from which to start. The only possible way to teach management has been to teach the practice, for the science had not yet been developed. With the presentation of laws that can be tested and verified in the same manner that the laws of physics and chemistry are tested, management can be taught in the same manner as any other science. This does not mean that the courses in different schools will necessarily be uniform, or that the teachers will have no opportunity to express their individuality. It does mean that all courses can be made to start from a common point and will have a common basis of comparison. If these laws, and those that will follow them, are made the basis of courses in management engineering, the schools will be on a solid foundation and will be able to render even greater service to industry than they are now rendering.

Another field in which these laws should have a marked effect is in industry itself. The laws form a most convenient foot-rule by means of which the manager can measure the work of any department, and, what is more important, his own work. Admitting for the sake of argument that all the laws as stated by the author are true, the manager of any industry can learn many things about his work by examining each law in relation to that work. If, after

reading each law, he asks himself whether or not that law is being applied by him, and if so, how effectively it is being applied, he will get a view of his work that will amaze him. There will be revealed to him many shortcomings, both of himself and of his subordinates, that will require correction. Leaks that should be stopped will become apparent. Departments that are weak and need strengthening will stand out. The entire business will benefit by such a survey. Thus used, the laws will become a most useful tool in industry, and not a mere statement of abstract principles.

B. A. Franklin.[1] It would seem that the author's codification is not only a definite step forward in the gradual outlining and setting down in understandable form of principles operating in management, but it is a very definite proof of the progress that the science of management is making. This paper promises the possibility of teaching management eventually precisely as geometry, for example, is taught, with its axioms, propositions, and eventually with its problems. Undoubtedly the author himself looks forward to the fact that this paper is only a beginning of the codification of the laws, each one of which will certainly produce other divisions into more detailed phases.

It may be suggested that one of the most important elements in management is organization, and a very important element in organization is promotion. Eventually there will be made certain laws of organization, and certain laws of promotion within the organization.

This paper opens up a field within which there may well develop much speculation, discussion, and development which will put the science of management in the next decade materially ahead. It will take it still further out of the field of mere shrewdness — and this will always be an important element in management — into the field of science.

Harold V. Coes.[2] The author has advanced the cause of intelligent management in manufacturing by codifying the laws pertaining to and underlying the present nebulous science of management. To the best of the writer's knowledge and belief, laws as derived by competent authority, tried and tested under critical conditions, have never before been assembled, classified, and presented in one composite whole in the present manner.

Many skilled managers and executives, and also many students and professors, have been cognizant of the existence of the material from which the laws were abstracted or derived, but few indeed

[1] Vice-President, Strathmore Paper Company, Mittineague, Mass. Mem. A.S.M.E.

[2] Vice-President and General Manager, Belden Manufacturing Co., Chicago, Ill. Mem. A.S.M.E.

of all those interested, either academically or practically, in the science and art of management have had the benefit of such complete classification of the underlying laws and principles as here presented. This classification serves to show very succinctly the tools with which we have to work, and their extent and variety. We doubtless will find that certain tools will need to be modified to enable us to do a given job, and again we will find that entirely new tools must be devised. This situation, however, differs in no way from that existing in other sciences, in which a body of working principles and laws have been or are being formulated and built up. The prime reason tending to prevent the formulation of the science of management is the fact that management, in a broad sense, utilizes something from nearly all the sciences. This has tended to confusion and loose thinking, with a distortion of values and warped perspective.

Let us make certain that what we term a law of management or a fundamental principle is, in effect, peculiar to the science of management, and not a law that has been deftly borrowed from some other science and dressed up for the occasion. We all recognize frankly that there has been some discreditable practice in this respect, at least in the past, that has not redounded to the credit of the scientific management movement.

If we can proceed still further from the admirable ground-work that the author has set up, by classifying still further or perhaps in another way the law and fundamental principles we recognize as essential in management operations, as to their origin — that is, is a given law derived from the basic law of medicine, psychology, chemistry, physics, or mathematics — it will be decidedly helpful. It would enable investigators and research workers to keep their balance and perspective if they knew from what basic sources the laws originated with which they are working or attempting to secure a reaction. It would furthermore tend to bring home the fact that the science of management can be visualized as a pyramid, with all the other basic sciences forming the base and body of the pyramid, and management forming the pinnacle or apex.

The writer lays stress upon the desirability of doing this, for the reason that one frequently finds among able managers and executives a surprising supreme contempt for some of the things that have been brought into management from some of the other sciences, such as applied psychology, and again, some of the things that have been brought to our attention by medical science. Hence, the writer feels that the cause of management would be decidedly advanced if some one, properly qualified and competent, could show the relationship of management to the other sciences and the actual indebtedness of management to these sciences; some one with the proper sense of proportion and perspective who would be willing to give credit where credit is due, and to break down some

of the existing misconceptions and prejudices which tend to prevent progress at the present time.

JOHN YOUNGER.[1] The writer suggests that management be divided into its two functions, human and mechanical. The latter term is used for the want of a better one. We have gone far in our understanding of the mechanical phase, the side that the manager uses, but we are still on the threshold as regards the understanding of the human side.

Par. 28, law 4, states, "concentrating upon the manufacture of a single or a few types and sizes of product tends to improve the quality and lower the production cost." This is open for discussion. In the automotive industry we find that manufacturers who have become established in a firm way have been concentrating on a number of products with, of course, the final opportunity in view of completing an automobile. It was formerly regarded that it would be better to have certain functions done by specialists — for example, the rear axle to be built by rear-axle specialists; the engine built by engine specialists; the transmission by transmission specialists, etc. There is still a diversity of opinion on this subject. Is it better to have specialists make the detailed sub-assembly of an automobile and have the assembling looked upon as a specialty itself, or is it better to have the complete car regarded as a single product? Today, for example, we see that the electrical units, such as the generator, self-starter, distributor, etc., are still regarded as specialties. The transmission, however, is not so regarded. In many cases the axle is looked upon as a specialty, as is the steering gear. In other cases, manufacturers add these products to their own completion of the car. In general, however, the writer regards the law as correctly stated, subject, however, to the above discussion.

In Par. 62, the law of individual productivity is stated. It would seem that there should be a further discussion of this law. There is a growing tendency to add more and more unskilled workers to operating forces. As converse of this law, therefore, it might be said that *it is advisable in a particular job to use the least skilled workmen that are capable of doing the job.*

Par. 82 states that the responsibility for the quality produced rightly rests with the manufacturing department. The writer has found in this connection that the slogan "Quality starts with the man at the machine" has been of great benefit in coming to a complete understanding of this phase of manufacturing.

The principle that inspection must be independent of production is also open for further discussion. Preventive inspection has become such a force today that it should be recognized. Preventive

[1] Professor of Industrial Engineering, The Ohio State University, Columbus, Ohio. Mem. A.S.M.E.

inspection is distinctly a part of production and should not be separated from it.

G. S. Radford.[1] In accordance with scientific method, the various parts of management have been subjected to searching analysis. This process has required the efforts of many men over a period of years, but has now reached a point where it is in order to summarize results and draw conclusions. Hence this paper is of most timely interest, and is another milestone in the literature of management. It is appropriate also that the author should be the man to present it. His expressed hope that further investigation of this subject will be stimulated should be translated into action by the Management Division.

It would be of far-reaching value to industry if there were available for the guidance of all those concerned with the practice of management a concise exposition of the basic principles involved. Every failure to obtain the best economy of manufacture can be traced to the violation in some degree of one or more of these fundamentals. Yet how few industrial operators have a complete grasp of them — a situation that is doubtless traceable to the fact that hitherto there has been no convenient place to find them collected for use without considerable hard reading.

In Par. 22, the author suggests their "use in the code of operation of manufacturing concerns." Would it not be a suitable piece of constructive work for the Management Division to prepare such a code to be issued with the Society's endorsement and approval? Each principle in such a code necessarily would be accomplished by a clear explanation of its meaning and practical application. Needless to say there should be a warning to the effect that these basic principles are interrelated and must be used as guides rather than rules — therefore judgment always must enter to determine to what degree any one law can be given scope without interfering with the proper application of the others.

Ralph G. Macy.[2] Has the author stated these forty odd laws in the order of their importance? Did he mean to infer that the laws stated first were most important? The laws are divided as follows: Sixteen on material, eleven on the worker, ten on management, and three or four on wages. We should know whether or not any of them are axiomatic and what their relative importance is.

The Author. The spirit in which this paper has been received and discussed is a source of sincere gratification. Seemingly there

[1] Consulting Engineer, New Canaan, Conn. Mem. A.S.M.E.
[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

is complete agreement with the thought that a group of management fundamentals, such as the paper suggests, can be formulated and declared. Indeed, seven of those who have taken part in the discussion have offered eighteen other laws to be included in a more complete code than the one presented in the paper.

Several suggestions are offered by way of modification and changes of the laws as given. The most of these are elaborate and a conclusion can only be reached by consensus of opinion. However, the objections offered to the Law of Wage Rates by Messrs. Harrison and Barth are well founded. A better phrasing is:

> *Base time on standardized jobs should never be changed except a material change has previously been made in conditions, methods, or equipment.*

In this form it might be considered a corollary to the Law of Motion Time.

The author heartily concurs with these comments and suggestions: The need of a preamble to declare above all else that there is a spirit or soul of management which is greater than all methods or even laws (Bigelow); the axiomatic nature of some of the fundamentals which raises the question as to whether they are laws (Barth); the need of accurate measurement to prove or disprove the findings (Mrs. Gilbreth); the possible effects of the laws on the teaching of management and in the operation of industry (Kent); the relationships of these laws to the fundamentals of other sciences (Coes); the interrelation of these laws in any scheme of management and the need of exercising judgment in their application (Radford); the preparation of a "code of operation of manufacturing concerns" by the Management Division to be issued with the Society's endorsement and approval (Radford).

Mr. Macy asks if the laws as presented are in the order of their importance. No attempt was made to establish such an order; rather, they were roughly grouped according to topic as Mr. Macy points out.

No. 2015

BALANCING FACTORS IN THE USE AND OBLIGATIONS COVERING OWNERSHIP OF FREIGHT-TRAIN CARS

By L. K. Sillcox,[1] Chicago, Ill.
Member of the Society

In this paper the author discusses the development of railroad equipment to date and points out some of the principles to be observed if railroads would continue to expand and become more efficient transportation agents.

The functions of transportation are first defined, following which is a general discussion of the limitations of railroad capacity because of terminal facilities. Terminal expenses also are treated at some length. There is a discussion of the slow-freight problem and recommendations for speeding up traffic are made. The matter of attaining the greatest utilization of equipment and materials is emphasized and recommendations made. Under this part of the paper the latest methods of car and locomotive repair are mentioned and their effect on the movement of freight emphasized. A very important part of the paper deals with the depreciation and retirement factors. A discussion of the aspects of design shows the great opportunity of engineers to assist in the solution of the problem of transportation by providing better equipment that will not require frequent repairs and which will, consequently, spend more time in actual service. In this connection the importance of standardization is emphasized.

Function of Transportation

CARRIERS are concerned with the use of freight train cars because they must assume the obligation of ownership in such a manner as to insure service and suitable return in operating revenues, which will properly assist in carrying the operating expenses on the one hand and other non-operating burdens on the

[1] General Superintendent of Motive Power, Chicago, Milwaukee and St. Paul Ry.

Contributed by the Railroad Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

other hand, so as to have a proper net return at the close of the year. The obligation carries with it the exercise of judgment as to the amount of equipment required for a given service. A given service cannot be stated as a specific or constant quantity, but there is within the control of management the possibility for operation and use of cars which may, to an ever increasing extent, tend to take care of the constant increase in the demand with a minimum number of units.

2 It has been said that the function of transportation is to overcome the economic handicap of distance. Because we have created in this country the most extensive system of railroads and have the most efficient transportation at the lowest cost, we have been able to develop and utilize our natural resources and man-power to better advantage than any other country of the world. We have within our boundaries about two-fifths of the railroad plants in the world and about four-fifths of all the world's motor cars. From the viewpoint of a miller, the distance from Montana wheat fields to Minneapolis is not measured in miles but in the *cost* of getting a ton of grain from the point of origin to the unloading track at the mill, on the one hand, and from the mill to the consumer of the finished product on the other hand; and the efficiency of our railroad transportation has consequently enabled the milling industries to avail themselves of the advantages which such centers as Minneapolis present with respect to labor supply and the location of markets. It has also made possible the economies of centralized manufacturing. As the growth of our national needs for transportation seems to be unlimited, the situation becomes more complicated, and also more worthy of our best effort.

Limitation of Railroad Capacity Because of Terminals

3 In order to assist in bringing about a maximum use of freight train cars, improvements must be made in freight terminal layouts, particularly freight car yards where cars are shifted into classification position for road movement. In addition to improvement in layouts, consideration should be given to frequency of train tonnage revision, which should be reduced as far as possible and which perhaps can be done more readily when better freight yards are installed. An undue frequency of train tonnage revision is a costly and serious retarding factor in car movement.

4 A railroad as a transportation machine is limited in its capacity by the ability of its terminals and its connections to handle and promptly dispose of its traffic. The provision of sufficient railway capacity at all times to meet maximum prospective demands of traffic appears, from the standpoint of the public interest and good transportation management, to be a sound policy, even though fixed charges, and to some extent expenses

for maintaining equipment and other facilities, are increased through employing such a policy, because it represents substantial surplus capacity most of the time and some reserve at all times. It should be kept in mind also that nothing is gained by having greater facilities for classification and transportation than to provide freedom of continuous uninterrupted movement and facilitate the interchange and delivery of business. It is lack of suitable and adequate terminal facilities generally, and the permitting of cars that should be held out to block yards and delay normal delivery, that causes equipment to stand still more than it should in order to cover a satisfactory number of miles per car per day in the direction of its required movement. In our efforts to expedite movement and to reduce cost, we have increased the size and weight of locomotives and cars very considerably. Even under the present circumstances the possible axle load on the existing standard gage probably will not exceed 80,000 lb., while on some of the trackage of the country, especially in terminals, the permissible load per axle now hardly exceeds more than 53,000 lb., and, at present, the heaviest A.R.A. standard journal provides a maximum load per axle of only approximately 63,000 lb.

Terminal Expense

5 It is generally agreed that the problem of terminal expense is one of the most vital questions before the railroads today because more money is lost in terminal operation than in any other single branch of the service. While some may be considering the increase of track capacity to meet the very evident demand for quicker handling of less-than-carload freight, this can only result, in most cases, in increasing the storage capacity available through the use of freight cars.

6 Improvement can be further attained by developing a proper supply of cars and locomotives as needed. There is now a movement toward increasing the utilization of locomotives, particularly by increasing the length of runs so as to reduce the number actually in service and increase the miles per day of those used. The need of replacing smaller with larger power still exists, however, but with proper maintenance and rate of replacement there does not seem to be need for any particularly large increase in the number of locomotives or freight cars. It should be possible to develop a proper anticipation of requirements for prompt movement, not only in equipment units, but roadway facilities to reduce the number of unnecessary stops. Aside from peak load periods, the tonnage does not increase rapidly. There remains, however, the necessity of expediting movement the moment tonnage is offered, not only to reduce the cost of handling, but to reduce the time from initial to final destination and assist the present trend of merchandising small stocks by means of rapid delivery.

7 There is a great possibility of movement of so-called less-than-car-load lots by motor trucks. This should not seriously affect carriers, inasmuch as short hauls involve such a high terminal expense that they are not profitable. Railways will continue to carry long-haul products. Terminal movement is very expensive. It has been said that the cost of getting a car loaded out of a large terminal is equivalent to the cost of a 150-mile road haul. To what extent carriers will go into the field of motor truck transportation is not yet well established, but the motor truck traffic has been encouraged by the fact that terminal handling of freight cars, together with short hauls on the railways, is not profitable. Other roads are using interchangeable shipping containers which are handled on flat cars in regular train movement.

8 Capital costs begin in any industry when labor-saving tools take the place of actual labor, and the proportion which they represent of the aggregate of all costs depends upon the extent of the use of labor-saving devices. When a machine, operated by one man, is introduced to displace ten men whose work it is able to do, the cost of its service is not in the wages of the operative alone, but in that sum plus the maintenance of the machine in conjunction with the fixed charges covering interest upon capital invested, depreciation, taxes, insurance, etc. If, subsequently, this machine is replaced by another costing twice the amount paid for the original, but capable of doing three times as much work, the new cost may be determined according to the same formula. Such changes as these are not made unless there is reason to believe that they will reduce the aggregate cost, and no thoughtful executive claims for one moment that the whole saving in labor is the net reduction in cost.

9 Reduction in grades, elimination of curvature, the introduction of heavier rails, the use of larger freight train cars and more powerful or more economical locomotives have all required vast outlay of capital. These expenditures have been incurred because there was ample reason to believe that they would add to the earning power of the properties in behalf of which they were made, but no executive ever intimated that they would not add to capital costs. On the other hand, however, it is believed that they would reduce operating expenses by a larger sum than that which they would add to interest and other necessary fixed charges. That they have somewhat reduced the proportion of operating costs to aggregate revenue is a matter of general record; but the definite relation of the savings in this direction to the actual addition to fixed charges is still a problem. That every substantial improvement in railway facilities ought to reduce the ratio of operating expenses and to a somewhat small degree increase the fixed charges is perfectly clear. It is equally evident that changes which have this effect should be encouraged in every possible way. Capital is the great labor-saving device and the means of all

labor-saving additions. It is desirable that it should be utilized to the utmost practicable extent, especially where this is possible currently out of earnings at a rate such as to retire a reasonable amount of the capital annually and still carry the fixed charges in addition to the current operating expenses involved.

Problem of Slow Movement

10 One of the leading tasks confronting the traffic and operating departments today is the scarcity of cars and their low average daily movement. Statistics show that a freight car travels about 30 miles in 24 hours. It is known, from actual experience, that loaded freight cars are moved, while they are in transit, with very few exceptions, at a speed of not less than 20 miles per hour and, quite often, at 40 miles per hour. If a car runs over 20 miles of track in an hour, it will cover 30 miles in one hour and thirty minutes. We then must ask ourselves what the car is doing the other 22 hours and 30 minutes of the 24 hours. It is evidently standing still, or being switched, or running empty, and a portion of each of the above items is unavoidable. We sometimes hear the statement that no means of improvement are known, under present conditions, and a suggestion that more unloading tracks at terminals be provided. We also hear demands for more cars and engines. When all is said and done we need to emphasize the fact that the best of systematic work on the part of the seven most important subordinate officials must be obtained, for without the intelligent and earnest coöperation of these officials the work of the most exacting and energetic executive is sure to fail. They are the roundhouse foreman, the car foreman, the chief dispatcher, the trainmaster, the traveling engineer, the yardmaster and the freight-house foreman. To bring this about, we need to have the most intelligent and best inspection, lubrication and light repair work on locomotives and cars before each trip, and the latter should contemplate a maximum uninterrupted mileage. A breakdown of one engine or one car will delay the whole train and a large number of disastrous derailments can be traced to careless or indifferent work on the part of the inspectors. Careful work by inspectors will also diminish terminal delays and the attendant lost motion. All employees connected in any way with the movement of a train should perform their duties strictly on schedule time, that is, engines should be put through the house for return trip as quickly as possible, ways to eliminate extraordinary delays between terminals should be discovered, and delays to trains or road engines by switch engines should be avoided. Successful car movement can only be accomplished by careful, painstaking work by those who are out in the field of action and in positions enabling them to watch and note every move that is made. This kind of

supervision cannot be accomplished over long-distance telephone or by telegraph. The local officers need to detect the apparently insignificant things which impair the efficiency of the system, because railroad employees, as a class, while doing the great bulk of their work conscientiously and well, are prone to slight the little things unless closely watched by their superiors. These so-called trivial things or circumstances, too numerous to list, are known to the practical man by intuition and actual acquaintance. If taken advantage of, they mean much in the proper dispatchment of cars and locomotives with a view toward maximum utilization and a proper minimum necessary investment for facilities and equipment to handle a given unit of business.

11 The unit "average car miles per car day" was used by railroads before it was adopted by the Interstate Commerce Commission in 1920. The Railroad War Board's monthly statements, prepared by the Bureau of Railway Economics, used it in 1917 and 1918, and it was continued by the U. S. Railroad Administration in 1918 and 1919. The method of determining this performance is to take all cars, both serviceable and unserviceable, divide them into the number of car miles for the month and divide the result by the number of calendar days in the month. This does not express the utilization of freight train cars as accurately as may be desired. If the unserviceable cars, which are not making mileage, were excluded from the calculation, it would increase the car miles per day by about 4 per cent. Even counting the serviceable cars in this calculation would not give the actual result because so many cars which are standing at terminals and loading and unloading platforms are not actually making mileage. It would be reasonable to assume that cars actually running make from 75 to 80 miles per day, or at least a distance commensurate with the average daily train miles. The data as called for by the Interstate Commerce Commission, however, do express the use of cars on the line in relation to the number handled and are, therefore, of value in expressing a high or low number in relation to the business handled. For the country as a whole, the average per car day for 1920 was 24.4 miles, and for 1926, about 30 miles, representing an increase of about 22 per cent. It indicates that the ownership of units has not increased at the same rate as the average miles per car day. This is a favorable situation, in that the distribution of cars throughout the country and the movement of tonnage has greatly improved. The average distance traveled per car day as a whole has always been surprisingly low as expressed by the Interstate Commerce Commission's formula and constitutes one of the principal problems of ownership.

Intensive Use of Cars

12 No greater responsibility confronts the administrations of the various railroads than that of attaining greater utilization of equipment, materials, manpower and facilities. When considering the latter, it can be quickly appreciated that this feature not only involves more mileage and more tonnage with fewer units of equipment, but also takes into account the problem of advance in design to attain greater service. It may be assumed that design and utilization are relative in that the production of satisfactory construction must naturally keep pace in the development of the use of facilities. In order that these features may be advanced in proper relation to each other, there should be a thorough knowledge of the rate at which obsolescence accrues or exists in any given property, knowing that it will generally require a term of years in order to satisfactorily dispose of practices and establishments which may have been in force for some time.

13 Full utilization of freight train cars is not entirely a question of mechanical administration; it depends very much upon other factors, such as road, yard, shop and terminal facilities, as well as prompt handling of empty cars, to say nothing of the necessity for uninterrupted road movement of trains through the provision of passing tracks sufficiently long to meet modern tonnage requirements, to have trains properly blocked so that they may run between two terminals a maximum distance apart without interruption, to provide road and grade conditions of such a character that freight-train movement at a proper rate of speed, without disturbance, can be carried on for a maximum economical distance, efficiency of train loading, dispatchment, etc. In general, freight train cars are capable of performing more hours of service and attaining greater mileage per day than are now accomplished. Every carrier is confronted with the serious problem of proper design and spacing of engine and car terminals in relation to obtaining maximum utilization per day for all classes of equipment, with consequent speeding up of average train movement. By utilization is meant the miles run per car per day, the days serviceable per year, and the tons hauled per day. If new and overhauled cars are not of modern design, the object to be attained is not accomplished in the full sense of the word. In order to arrive at a maximum utilization with an economical maintenance cost, the officers in charge should have full knowledge of performance of such equipment, and especially should obsolete freight car maintenance, shop and terminal facilities be studied and corrected if possible.

14 During 1925 the Car Service Division of the American Railway Association completed a study of the average loading of freight cars with various commodities carried by Class I rail-

TABLE 1

District	Products of agriculture			Animals and products			Products of mines			Products of forests			Manufactures and miscellaneous			Grand total, all carload traffic		
	1925	1924	1923	1925	1924	1923	1925	1924	1923	1925	1924	1923	1925	1924	1923	1925	1924	1923
Eastern	22.0	21.8	21.9	12.8	12.8	12.7	48.9	48.6	48.0	24.2	24.5	23.8	24.2	24.1	23.9	32.7	33.2	33.3
Allegheny	22.0	21.2	21.4	13.4	13.5	13.3	52.8	52.0	51.0	25.9	25.8	26.2	28.4	28.3	28.8	38.6	38.5	39.3
Pocahontas	16.6	16.5	17.7	12.0	11.6	11.8	57.4	57.4	56.7	27.3	27.3	27.6	26.1	26.5	27.6	51.8	50.7	49.5
Southern	16.4	16.5	16.9	11.7	11.8	11.6	48.1	47.6	47.0	25.4	25.5	26.1	24.4	24.1	24.1	31.5	31.1	31.5
Northwestern ...	30.9	31.6	30.9	11.3	11.2	11.3	50.5	49.2	52.4	32.4	32.5	32.3	27.7	27.2	27.5	35.2	34.2	35.3
Central West ...	25.7	27.6	26.6	11.5	11.5	11.4	46.9	46.7	46.1	30.1	30.2	30.7	26.7	26.7	26.1	30.7	30.7	30.5
Southwestern	20.8	22.1	21.8	11.8	11.5	11.7	43.2	43.3	43.0	26.2	26.4	27.3	26.4	26.3	26.7	28.3	28.4	28.4
Total all Districts	23.4	24.4	24.1	11.8	11.7	11.7	50.3	49.7	49.1	28.0	28.1	28.4	26.1	26.0	26.0	34.4	34.1	34.5

roads in the United States, with a comparison for 1923 and 1924. The information indicates what use is being made of the freight cars and provides a means of determining to what extent the carrying capacity is employed by the various lines of industry and by the shipping public in the various districts of the country. The average of all cars loaded is influenced very greatly by the character of traffic, which changes not only from year to year, but also during different seasons, so that to attempt a study of the utilization of freight cars, a number of factors must be considered, requiring careful study of the detailed data. The average freight car load for 1925 was 27.0 tons, the same as in 1924, compared with 27.9 tons in 1923 and the high average of 29.3 tons in 1920. Table 1 gives the number of tons in cars for carload freight originated (not including less-than-carload merchandise).

15 Table 2 is a commodity loading statement which shows some increases and some decreases and develops a number of large differences in the car tons.

16 A detailed study of this information will show wide differences in the car loading as between railroads serving the same territory and covering specific commodities. For example, the loading of wheat in the section west of Chicago is 35 tons to the car on some roads, while others are obtaining as high as 44.9 tons, a difference of 9.9 tons to the car, or 28 per cent, accounted for almost entirely from a difference in car carrying capacity. For lines serving the eastern section of the country out of Chicago, the average loading fluctuates from 35.4 tons to 46.4 tons or 31 per cent. Conditions of like nature can be observed with regard to practically every other commodity. Therefore, a study of the statement prepared by the A.R.A. Car Ser-

vice Division and an investigation by all railroads of the conditions on their line, should, through obtaining full coöperation on the part of the shippers, encourage greater use of the individual freight car as well as suggest the most economical carrying capacity for freight cars and eliminate any possible wastefulness in practice, thereby resulting in considerable benefit in the direction of economical and efficient service.

17 The performance of freight equipment may be measured roughly in terms of average miles per car day, average load per car, and the relationship between loaded and empty car mileage. The reason for the increase in percentage of empty car mileage does not result from the lack of a car unit that can be loaded in both directions with various classes of commodities, but is practically a result of the better performance on the part of the

TABLE 2

	1925 Average, tons		1924 Average, tons		1923 Average, tons	
Commodity	Low	High	Low	High	Low	High
Wheat	27.6	47.8	30 0	50 1	31.5	44 2
Corn	16 0	44.3	15 0	45.1	13 8	42 3
Oats	14.0	37.5	13 0	36 6	12.6	35 0
Other grain	13 8	40.6	16.9	41.5	17 7	40.8
Flour and meal	9.8	31.7	9.7	30 5	9 8	30 4
Other mill products	10 8	26.2	10.4	27 2	9.6	26 1
Hay, straw and alfalfa	8 5	14.3	8 5	14 2	8.4	13 7
Tobacco	8.9	15.6	8 9	15.7	9.5	14.7
Cotton	8.1	18.4	8.1	20 2	7 6	21.1
Cotton seed and prod. (exc. oil)	19.4	28.2	19 7	27.8	16 2	25 6
Citrus fruits	13 3	17.5	13 2	17.6	12 8	16 0
Other fresh fruits	9 8	19 0	10 1	18.0	10.6	18 7
Potatoes	13.3	20.5	9.7	21 5	13.4	20.7
Other fresh vegetables	8 8	21 6	9.2	24 0	9.0	23 0
Dried fruits and vegetables	15.1	29.0	14.9	29 5	14 8	30.5
Other agricultural prod.	12 1	47 1	12.6	48.0	12.6	46.9
Total products of agriculture	11.7	36.9	11 7	40 5	12.0	35 9

carriers. It is quite generally acknowledged that just so long as the railroads continue to make an improvement in movement, they will show an increase in the percentage of empty mileage compared with periods when the movement has not been so prompt, because when low percentages of empty mileage were experienced, the empty cars were standing still and there was a shortage of cars for loading.

18 On page 32 of the Interstate Commerce Commission's statistical report for 1920 it will be found that the average haul of all roads of Classes 1 and 2 considered as one system was 307.51 miles. The preliminary abstract for 1925 does not give a comparable figure but from the basic data it may be worked out for Class 1 roads. The total revenue ton miles were 413,823,173,484, and the originated tons were 1,247,243,183; thus by dividing we obtain an average haul of 331.79 miles. To be sure, the average haul per ton per railroad has declined in that period from 181.55 to 179.38, but on the whole, the unit first referred to is the most

reliable for general purposes. Furthermore, the average load per car has not increased in the past five years.

19 In considering the tons carried per loaded car one year, compared with another, it should be realized that it means nothing unless all of the factors are taken into account. The present year (1926) will probably show the greatest number of tons per loaded car, for all Class 1 railroads as a whole, of any year up to date, but this will be influenced very largely by the great increase in the production of mines, including sand, stone, and gravel. The average load per car, therefore, is a matter which is nearly but not entirely beyond the control of the carrier, but the train performance is within such control, to a large extent. It is in this particular feature that much improvement has been made.

20 The effective performance of freight cars is disclosed by the relation between loaded and empty car mileage. The ratio of empty to loaded car miles has run about 30 per cent in the past few years. It is necessarily high because of prompt handling of empties and to maintain a satisfactory car supply. The predominating movement of tonnage eastward and also, as long as this condition exists, the empty car mileage, will be considerably affected, as empty cars must be forwarded to loading points. Unnecessary car mileage has been reduced very greatly by the accomplishment of the Car Service Division of the American Railway Association, and this work involves the supervision, distribution, and movement of about 2,500,000 freight cars each day. The matter has been so handled that car shortages have practically disappeared. Car shortages have usually occurred because of:

1 Unusual and rapid increases in freight tonnage offered
2 Unexpected high loading peaks
3 Inequalities in volume of traffic
4 Inadequacy of terminals to take care of unusual heavy movements
5 Tendency to use cars as warehouses in spite of demurrage charges
6 Lack of industrial facilities for prompt loading and unloading of cars
7 Reconsignment abuses
8 Inadequacy of motive power
9 Individual carriers failing to equip themselves with a reasonable number of modern cars or an adequate number of cars of special type needed in certain industries
10 Unwillingness of shippers to load available cars to capacity
11 Weather conditions.

This is a brief summary of some of the retarding factors in car movement.

21 The performance of cars, locomotives and trains is a matter of sufficient importance to justify much study. Train runs are

on a more or less arbitrary and specific basis, but car runs intermingle. Cars are picked up en route between terminals, handled at terminals, set off between terminals, etc. The handling of cars may be divided into two groups, that is, on the road and in terminals. Handling on the road shows that the average speed of freight trains was 11.8 miles per hour in 1925 and the average mileage per car per day was 28.3. If all cars could be handled from terminal to terminal without interruption at 20 miles per hour, in 8-hour runs this would mean 160 miles per day. The train speed of 11.8 miles per hour expresses delays en route for intermediate switching, meeting trains, held by signals, etc. It would seem that much attention should be given to roadway dispatching and facilities to increase train speed by eliminating movement interruptions, because the investment involved would not be in proportion to the investment necessary in equipment if train tonnage were to be further increased. For every mile per hour increase in train speed (terminal to terminal time), the gross ton-miles per train-hour factor is increased by the train tonnage. A 1200-ton train at 8 miles per hour will produce 9600 gross ton miles per train hour; at 9 miles per hour this will increase the gross ton miles per train hour by 1×1200, or 1200 added to 9600 G.T.M.P.T.H., making 10,800 G.T.M.P.T.H. At 10 miles per hour this would be 9600 G.T.M.P.T.H. plus 2×1200 or 2400, making 12,000 G.T.M.P.T.H. (see Fig. 1). Expressed in formulas, where a = gross tons per train, b = speed in M.P.H., and c = G.T.M. P.T.H.

$$ab = C$$
$$a(b+1) = C+1a$$
$$a(b+2) = C+2a, \text{etc.}$$

22 Therefore, when train tonnage cannot be controlled definitely because of fluctuation in traffic volume, it still remains within the control of a railroad to increase train speed, as every factor of speed increase is an equivalent of increase in gross ton miles per train hour. The lowest speed recorded for all Class I railroads in 1925 was 10.6 miles per hour, and the highest 15.9 miles per hour, or a difference of 50 per cent. This is a wide range and doubtless the lower figures can be improved with no increase in the cost. Train speed should not be construed to mean actual running speed, but the average rate at which the mileage is produced from initial to final terminal. It is improved by eliminating elements which delay train movements so that the hours consumed from initial to final terminal may be reduced. The elements which retard train movement are:

(*a*) Too great a frequency of tonnage revision due to improper assignment of power.

(*b*) Unbalanced regulation of tonnage movement in through and way freight trains, which should be corrected to eliminate, as far as possible, delays due to set-outs and pick-ups, road switching, etc.

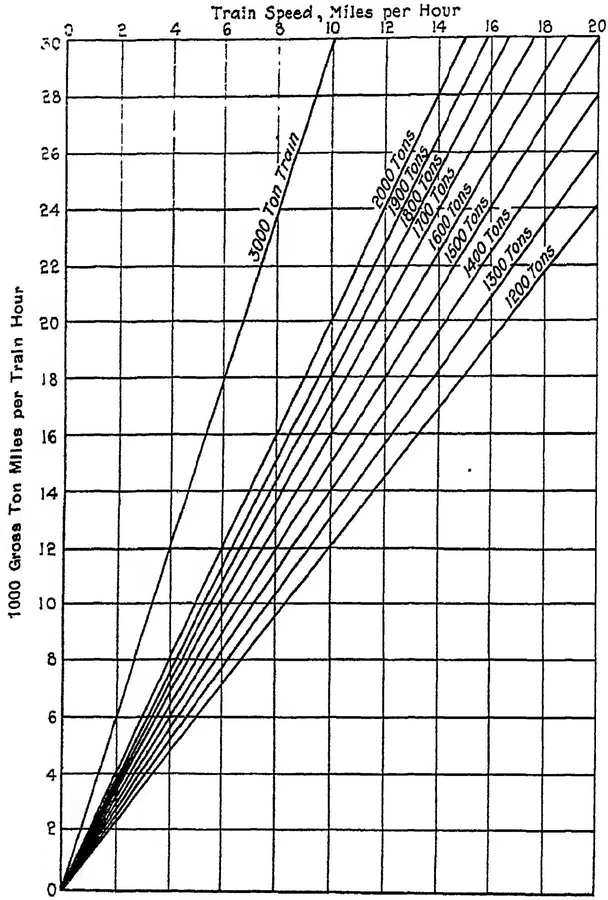


FIG. 1 RESULTANT GROSS TON MILES PER TRAIN HOUR
(Train miles per hour and gross tons per train as factors.)

(*c*) Dispatching of trains to eliminate delays for train meets, standby losses of time on sidings, etc.

(*d*) Siding and passing track arrangements to prevent complete stops.

(*e*) Spacing of trains, etc.

23 Unfortunately, there are no specific data to show terminal movement, but it may be considered as a retarding factor under

present operation. It is essential that yard lay-outs be improved so as to permit cars to be worked through more rapidly. In order to overcome the present retarding factors, longer locomotive and train runs have been instituted with the hope of eliminating terminal delays as far as possible. This is a move in the right direction, but should be further improved by proper terminal facilities, in conjunction with appropriate placement of sanding, coaling and watering stations and suitable volume and quality of water supply where required.

24 Improvements have been made in locomotive performance. Fuel consumption per thousand gross ton miles has been reduced from 197 pounds in 1920 to 159 pounds in 1925 or 18 per cent. Trains have been more carefully handled because of better maintenance of air brake equipment and education of trainmen, so that payments made for loss and damage to freight decreased 69 per cent in the same period; this being assisted somewhat by a better condition of freight cars. Gross ton miles per train-hour have improved likewise by reason of a slight increase in train speed and a marked increase in train tonnage, the latter being brought about by better train assembling and the use of larger power units. Car shortages have disappeared for some time because of better distribution and movement. Empty car mileage, on account of characteristics of traffic, has remained about the same. The physical condition of freight cars and locomotives has improved as a result of better maintenance and at the same time the cost per unit of such work has been reduced.

25 While freight car miles per day have increased from 25.1 in 1920 to 28.3 in 1925, yet this has not been an expression of ultimate utilization, because the character of maintenance accorded has aided in the matter of availability. Freight car maintenance may be divided into two groups, one pertaining to heavy repairs and the other to light or running repairs. Heavy repairs become due in periods of about once every 8 or 10 years, whereas running repairs and inspection are a continuous necessity. While terminal handling may be considered the greatest retarding factor of freight-car movement, repairs may be considered as a somewhat important element. The bad order car situation has improved materially in the last few years and this applies to cars held out of service for heavy repairs. Cars requiring running repairs are usually not held more than a few days. Table 3 shows a comparison, on 100,000 individual units which were specially tabulated for a wide range of territory, of the car parts causing delays for repairs other than periodical overhauling, and which must be taken out of service and placed on repair tracks for that purpose.

26 This indicates that defective air brakes and bodies represent the largest cause for taking cars out of service. Next in order are draft gears, underframes, trucks and wheels. Much has been

said about the design of the body underframe, and the importance of a proper design is indicated by this statement. The trouble is usually experienced with cars which were designed and built from 8 to 10 years ago or longer. The character of materials in manufacture of wheels is a vital matter, as their renewal period is probably more frequent than should appear necessary, considering that they have a life of about 60,000 miles only. The betterment of designs now being worked out is with a view toward reducing the frequency of attention to vital parts, and while this may add to the initial cost of the car, this increase is very small in proportion to the resultant maintenance expense and serviceability of the car. Where 5 per cent or less of the cars are out of service for repairs the condition is considered somewhat normal and it may be generally understood that this factor is not as great in the retardation of car movement as terminal handling. Expressed in another way, carriers having a long haul per ton are, as a general rule, more prosperous than those having a short haul per ton. The frequency of terminal handling is, therefore, to be considered more seriously than maintenance at this time. A standard form is used by some administrations to record the time out of service because of repairs, and is a means of bringing to the attention of local officers the delays encountered and possibilities of improvement.

TABLE 3 DELAYS CAUSED BY CAR PARTS, PERCENTAGE

Air brakes	26.00
Body work	17.20
Door work	2.90
Roof	1.50
Safety appliances	5.00
Trucks	12.00
Draft gears and underframes.	14 00
Wheels, general	13 40
Wheels, slid flat	1.50
Operating conditions	6.50
	100.00

Depreciation and Retirement Factors

27 It is of vital importance that more tonnage be carried per car and more miles traveled per car in so far as is possible for practical purposes with a view toward carrying the total tonnage with the least possible number of units necessary therefor.

28 Investment in equipment is a heavy burden since carriers must make suitable charges for depreciation, which should take care of the cost of the equipment during its life. Features of interest on the investment also have to be regarded covering the cars themselves as well as the cost for carrying investment in facilities for maintaining the cars. So far as operating expenses are concerned, the maintenance of the cars and the facilities needed for their up-keep is a feature deserving of close and careful analysis. A study of this situation shows that the cost per day

per car to cover such features is about $1.00 or more. Therefore, a carrier owning 100,000 cars has a burden of about $100,000 per car day. In consequence an increased use of freight cars is, therefore, highly desirable so as to further minimize this burden. The question of the number of freight car units necessary to handle the business has never been solved specifically, and whether the individual carrier is to own more or less cars than the average on the line is a problem involving the elements which go to make up the per diem cost. It would seem that the number of freight cars now in use is sufficient to handle an increase in business, provided there is an increase in the utilization of cars. However, there are many older and lighter cars still in existence of a design not fitted for heavy train movement and this means that there must be continued a proper rate of retirement of old and acquisition of new cars. It is natural by this process to increase the average carrying capacity per car, and if the character of loading can be improved so as to increase the average load per car and at the same time increase the average miles per car day, this should take care of a normal increase in traffic without a specific increase in freight carrying units.

29 The problem of proper freight car utilization and maintenance is not solved merely by low unit cost for repairs or even by a large car supply. For instance, if 50-ton box cars will carry the prevailing grain tonnage, and if that represents a majority of the business offered, such class of equipment should be employed to save track room as well as detailed maintenance and make it possible to retire two old cars for one modern unit. If an economical unit cost is to be attained constantly, it is necessary to consider the rate at which obsolete cars are retired and new ones acquired to maintain a proper complement of equipment. Take for instance the case of an administration applying the following rates of depreciation to their freight train cars:

Period	Rate of depreciation
Prior to July 1, 1907	(Charged profit and loss)
July 1, 1907, to Dec. 31, 1912	1½ per cent
Jan. 1, 1913, to Dec. 31, 1915	1 per cent
Jan. 1, 1916, to Dec. 31, 1920	2 per cent

30 In such a case these rates would be used merely to charge a certain amount to operating expense each month, based on the above percentages applied to the total amount in the investment account for freight train cars, and a corresponding credit would be made to "reserve for accrued depreciation," which is built up month by month. Under these circumstances, even where the highest rate of depreciation was charged at 2 per cent it would involve a total life expectancy of 50 years (neglecting salvage)

and the manner in which this would affect a railroad, if it was desired to retire a car at 20 years of age, would be:

Original cost	$1000
Salvage	200
	$800

From July 1, 1900 (date built) to July 1, 1907, or 84 months, the amount chargeable to profit and loss would be

$$\frac{84}{\text{Total, or 240 mo. (20 yr.)}} \times \text{(cost to date less salvage}$$

equals profit and loss	$186.00
From July 1, 1907, to Dec. 31, 1912, or $5\frac{1}{2}$ years, at $1\frac{1}{2}$ per cent on $1000 less salvage	66.00
From Jan. 1, 1913, to Dec. 31, 1914, or 2 years, at 1 per cent on $1000 less salvage	16.00
From Jan. 1, 1915, to Dec. 31, 1920, or 6 years, at 2 per cent on $1000 less salvage	96.00
Total	$364.00

31 This means that at time of retirement a road so situated would have remaining to charge to operating expenses under the retirement account the following:

Investment	$1000
Salvage recovered	200
Total	$800
Depreciation accounted for	364
Remaining to charge to operating expense	$436

32 Contrast this with another administration using a 6 per cent rate of depreciation; a parallel case would be as follows:

Original cost	$1000
Salvage	200
Profit and loss	$800
$(84 \div 240 \times \$800)$	$186
6 per cent for 13.5 years	648
Accumulated	$834
To be written out	800
Over depreciated	$34

33 This makes a credit of $34 to operating expense and it might result in an administration following such a practice to permit the only factors taking equipment out of service to be physical condition and obsolescence, and under such circumstances the maintenance expense on a property so situated might run 25 per cent below that of the first administration unless retirements were made regularly. In one case all charges would be made to depreciation; in the other case they would be split between depreciation and retirements, but both are in operating expense. In the first instance, in checking up the average age of cars held by various administrations, we find it ranging from 18 years to 35 years. It is important, therefore, to observe the policy of more frequent renewal and fewer heavy repairs or rebuilding, as it would seem to be more economical than extended life, and the accounting rate of depreciation, which is usually a straight-line rate, has a serious indirect bearing on the policy of maintenance which is employed. Proper turnover is the easiest and the most economical means of reducing repairs and overcoming obsolescence, because it is, in the end, very costly to extend the life of equipment to avoid the burden on operating expenses of writing out the equipment.

Aspects of Design

34 In 1901 the Pennsylvania Railroad built the first 50-ton box cars, because of their greater earning capacity over that of the 40-ton cars. An analysis of the real benefit in operating 50-ton box cars must take into consideration two important points. Can the greater loads they can carry be obtained? If so, what is the greater carrying capacity during a given period over and above the additional first cost, and the extra cost of hauling the extra dead load of the heavier car?

35 In view of the fact that many thousands of 50-ton box cars have been added to the equipment of the railroads in recent years and large numbers are now under construction for future delivery, it seems not inappropriate to raise at this particular time the question as to whether the results obtained from the use of the 50-ton car in actual service will justify its extended employment in most localities as a standard car in place of the 40-ton car, or whether these results, when properly analyzed, will not show that the 50-ton car, from a practical or commercial standpoint, can only be regarded as a specialty confined to a particular class of traffic in a territory where the limits of its usefulness are governed by well-determined lines of demarcation. A proper and intelligent analysis of this subject necessarily involves a collection or compilation of itemized data on the essential factors that have a bearing or controlling effect on the relative value of this car

compared with the others above mentioned. Among these most important factors are the following:

(*a*) First cost of car
(*b*) Capacity of car, cubic contents and car weight
(*c*) Average load carried in tons
(*d*) Conditions which militate against full load
(*e*) Cost of maintenance
(*f*) Extra cost of hauling additional dead weight when moved with less than full capacity
(*g*) Extra cost for maintenance of permanent way, bridges, etc.

36 An essentially fair comparison can be attained only by actual experience with a large capacity car under operating conditions and it should be borne in mind that such only tends to show the value of the car in the particular territory where it is in service and where the comparisons are being made. Therefore, it would be highly interesting and very valuable if a thorough and complete report could be had of the results obtained by the use of a given number of 50-ton cars in comparison with others, covering a period of two or three years, furnishing the exact figures and facts, that others who may contemplate the addition of new and heavier equipment may be better able to reach a correct conclusion in the premises as to the type of car best adapted for the locality through which their lines may run.

37 Some observations of a general character may serve to emphasize certain points of more or less interest. The Union Pacific and Southern Pacific, two of the most prominent railroad systems in the West, have at present in service thousands of 50-ton cars, and it is interesting to observe that they have had in service cars of this capacity for more than 20 years. Such equipment, as a rule, is loaded to a point reasonably near its capacity, for the entire route over the owners lines and this, of course, is what might be termed an unusually favorable condition of service for the 50-ton cars and gives them the place at once among the improved facilities that count for increased net earnings. As further evidence of this, an enumeration of some of the various commodities handled at different points on the line and the quantity in percentage of the car capacity which enter into the general average attained are as follows:

Wheat	107.1 per cent
Corn	81.7 per cent
Barley	85.1 per cent
Other grain	100.4 per cent
Ore and bullion	114.6 per cent
Coal (largely in box cars)	84.0 per cent
Coal (open top cars)	106.0 per cent
Gravel	109.0 per cent
Beets	101.4 per cent

38 This affords an excellent insight, on the other hand, into the contention made by many that there is a special field for the

40-ton car which the 50-ton car cannot invade without a positive loss, either direct or indirect, to the operating companies; and the field is the one where such commodities as hay, merchandise, mill stuff and miscellaneous products predominate and where conditions necessitate their movement in a manner to suit the shipper, regardless of the wish of the carrier. Furthermore, it is not within the range of possibilities for the carriers to educate their shippers so as to secure the delivery of their traffic for movement at a time and in a manner that will permit a prompt and full car-load movement, at a specified time, of commodities of this character which will compare with the handling of ore, coal, grain and similar commodities which are regulated by the train load rather than by the car load, and which are offered for shipment at a time and in such quantities as not to require movements of less than the maximum train load.

39 The freight car mileage for 1925 was 26,729,831,000 miles; of this, 35 per cent was empty car mileage, which would indicate that many lines were operated under conditions where it was almost impossible to secure loading in both directions for their cars. While this is doubtless largely the case with roads that are essentially coal or ore lines, yet much of it is in the agricultural districts where the empty mileage is box car equipment, handling an average of about 24 tons per loaded revenue car. This would seem to warrant the suggestion, if not the conclusion, that a general utility car substantially built to meet all the physical conditions resulting from interchange service, with minimum dimensions of the American Railway Association's standard box car, of a capacity not exceeding 80,000 lb., is from some points of view not only the more typical, but more desirable commercially, physically and financially, as a common standard for American railroads than the 50-ton car; although the latter has been adopted on some of the most important trunk lines and by virtue of the interchange arrangements between various roads is not only in evidence on, but in some localities forms a no small percentage of the cars on the line, which would otherwise favor 80,000 lb. capacity equipment.

40 It seems rather inconsistent, from a business and commercial standpoint, that large trunk lines should spend enormous sums of money in perfecting their permanent way, and other facilities for handling their business, and also equally large sums in the purchase of motive power and equipment which is peculiarly adapted to their line and meets with the view of their officers, and then a large portion of this equipment should be diverted into a class of traffic and on to lines where the conditions are at considerable variance to those which hold directly opposite views as to its commercial value as an operating unit and as to the practicability of its use. While much of this modern heavy equipment is found on roads whose officers question its

adaptability to the conditions which they have to meet, at the same time a no less conspicuous feature of what might be termed unbusinesslike conditions that result from the interchange of cars is the presence on the large trunk lines, which have adopted the modern heavy car, of a great number of small antiquated cars belonging to the lines which do not favor the use of the large car. Many of these small cars were built years ago when the tractive power of freight engines averaged about 35,000 lb. and the original construction of the cars was light. This, together with their age, renders their physical condition such that in some cases they are scarcely safe for service in light trains on local runs and are absolutely dangerous when placed in modern heavy trains handled by large types of heavy freight engines; dangerous not only to the cars themselves, but in case of accident, as a rule they are not only badly damaged but are the direct or indirect cause of destruction of the modern heavier cars with which they are intermixed in train service. The high cost of repairs to freight cars, on some lines, can be traced to the retention in service in some cases of light antiquated equipment that should have been retired from service immediately following the advent of modern car construction.

41 The more highly developed a railroad becomes in providing fast passenger service and regular and profitable freight service, the greater is the necessity for the use of good materials to prevent accidents and provide reliability of movement. When such services are offered to the public, there is imposed a moral responsibility to furnish materials which shall be equal to the service requirements.

42 Generally, coal cars returning empty to mines make substantially the same number of empty miles as is made on the loaded haul. There should be a decided saving in operation where larger cars are used. Furthermore, if there is a continued shortage of coal cars at mines in competitive territory during peak business, there would be a further gain in net revenue as a net result of whatever percentage of increased load obtained. Assuming the net earnings per day, over and above all cost to handle, is one dollar on a smaller car, then with a larger car representing a 25 per cent increase in carrying capacity, this net would be increased to more than $1.25 a day. If the car requires ten days to make the round trip, the increased revenue would, at least, be more than $2.50 (cost does not vary directly with capacity and the net saving will be more) for the round trip and would probably result in a substantial net return from the additional net revenue on the additional investment required to be made for the larger cars, providing the peak movement under these circumstances existed during 40 per cent of the yearly period.

43 In designing a car, the service in which it is going to run must be considered, not the service in which it may run. One of the advantages given for all steel, steel underframe, and steel framed cars, is that train resistance is less than with wooden cars. The principal reasons for expecting a lower resistance with a steel car are: First, on account of its greater rigidity there is less deflection in the bolsters, and the side bearings are not in contact under the maximum load; hence, there is less flange friction and a lower tractive resistance. Second, the capacity of steel cars is so much greater than that of wooden cars that it is possible to haul the same tonnage in one-half or two-thirds the number of cars, and it has been found that the increase in train resistance for a given total tonnage increases with the number of cars required to equal that given tonnage. This is to be expected from the greater area exposed to atmospheric resistance and because of the large number of wheels producing flange friction. With equal coupler clearance on straight track, there should be no difference in the resistance of the wooden and steel cars, so far as the drawbars are concerned, but on the curves the rigidity of the steel draft rigging prevents the coupler from accommodating itself on the normal line of the pull, and if the clearance is too small the wheel flanges are forced against the rail, thus increasing the train resistance. With the wooden car, however, the draft timbers are easily displaced or compressed and the bolts in the wooden draft attachment work to one side and the whole rigging gives enough to provide a movement of the coupler equivalent to greater clearance and is sufficient to prevent undue flange friction.

44 The foundation of economic efficiency in freight-train service is the utilization of each car to its fullest capacity, and the first step in this direction must be taken at the origin of the traffic. The latter points, for the most part, when considered west of Chicago, are in isolated sections, not lending themselves easily to intensified supervision. It is essential that cars be in proper condition with respect to wheels and air brakes before loading, because far too many refrigerator and oil-tank cars are unnecessarily delayed under load for wheel changes, and for defects which should have been discovered and corrected at the last repair point before loading.

45 Freight cars are used in common under the A.R.A. car service rules, the per diem agreement and the interchange agreement. They are repaired (with certain exceptions) on the road where the need for the repairs develops. The average freight car of an individual road is at home not much more than one-half of the time. Obviously, if there is a common standard for the types of car used for the great bulk of the interchanged traffic, each road will be required to carry a much smaller stock of repair

parts, and there will be a reduction in the time now lost by cars which are held while the repairing road is obtaining parts of special design.

46 Dependable cars have always been regarded as the most vital requirement upon car department executives. The consequences of the failure on the road are such that in car construction and subsequent maintenance, operating reliability is the first consideration. No feature in design, however efficient, and no maintenance practice, however economical, will ever be tolerated on our railroads, if it impairs the ability of the car to function successfully in regular train service. No saving in the cost of operating car shops and terminals will justify car failures resulting from too rigid economy in their maintenance. Nor can the time that cars are available for service be increased at the expense of the time actually required for maintaining these cars in a dependable operating condition. Next to reliability in train operation comes the efficiency of design with respect to weight, cost and structural stability.

47 Net tons per car load seem to remain stationary in the face of improvement in practically all other factors such as net tons per car day, car miles per car day, freight locomotive miles per locomotive day, freight cars per train, gross tons and net tons per train, and average speed of freight trains. Empty car miles seem to increase by reason of better performance on the part of carriers. This situation indicates that where an improvement in other directions obtains, a decided increase in the percentage of empty mileage will result, and the sole reason for having an increase in the percentage of empty mileage comes from the fact that administrations are at present moving empties promptly, whereas when they had a low percentage of empty mileage the empties were standing still and there was a shortage of cars. It is felt by the best authorities that as long as we continue to make an improvement in the movement of business we are going to show an increase in the percentage of empty mileage compared with periods when the movement was not so prompt.

48 The argument is often made in favor of an all-service car for various combinations of traffic. It can be said that there is no such thing in existence as a common-service box car, because the only way this could be provided would be to maintain all box cars at all times fit for the handling of the highest class traffic that moves, regardless of the fact that they may be used for that traffic only a small percentage of the time. Many administrations find it economical and proper to confine new and rebuilt box cars to the higher class traffic and not permit them to get into the moving of stone, brick, tile, cement, lime, tar and a lot of other commodities that make a car unfit for high-class traffic. In the case of cars used for automobile loading, it only necessitates a

few trips to make such equipment unfit for flour, sugar and other high class loading, for the simple reason that the floors are stained with oil, cars are filled with nails, spikes, cleats, etc., and while it does not restrict their use for automobile and similar loading, it does for such other commodities as referred to.

49 An inference as to the weaker points in car construction may be derived from the reports of the Division of Safety of the I.C.C. as to derailments, caused by defective equipment. Aside from the substantial progress made and the earnest activities of the American Railway Association, many of the larger railroad systems have been following the principle of standardization for years. Some of the freight car standards of these roads are the result of a very careful study of experts over a series of years. It has been necessary, therefore, for the A.R.A., in formulating standards for general adoption, to give special thought to eliminating excess weight not justified by traffic and engineering requirements, because such added weight means not only a needless expense in first cost, but also in train operating expenses as a result of increase in the dead weight of the train. To whatever extent the unproductive weight of the train can be diminished, to that extent the productive weight may be increased by the absorption of the same amount of energy of locomotive tractive power without reduction in rate of speed. It is, therefore, an element of efficient use of tractive power, from a business point of view, that the dead-weight proportion of a train shall be maintained at the lowest point consistent with reliability in performance. If we were to consider the cost of hauling 800 pounds (avoidable) additional weight per car, because of using a certain feature of design instead of another of equal engineering value, and in order to save the extra weight it was necessary to pay $60 extra per car in first cost of the car, and the maintenance features were not affected, the problem would work out as follows:

Average miles per car owned per day (includes bad orders)......	28.3
Cost of hauling one ton of dead weight one mile by Class I Railroads varies largely from $0.004 to $0.007 so that by using the mean figure, it should be considered acceptable..	$0 0055
Considering 6 per cent bad order cars would mean 94 per cent of 365 days in service or...	343

Therefore, $800 \div 2000 \times 343 \times 28.3 \times 0.0055 = \$21.36$ per year, or years to pay $= 60 \div 21.36 = 2\frac{3}{4}$.

50 It should be understood that any figure used, such as $21.36, is more or less theoretical, since the cost of hauling per ton of weight is not directly proportional to the weight carried but rather the relation of full tonnage possibilities to actual tonnage hauled. A carrier having full tonnage trains will have a lower cost per ton-mile than one with variable loads of less than full tonnage.

51 Notwithstanding the claims made as to the indestructibility of steel cars, the question of normal deterioration and attention

required for equipment encountering yard and wreck damage and unfair usage is still a very important design consideration, in conjunction with the problem of repairs and methods and facilities needed to economically dispose of the work. Considerations of strength required need to be constantly reviewed, because of changing operating characteristics of service required from freight cars.

52 The proportioning of the longitudinal sills in the underframe for vertical load and end shocks is a very important question in freight-car design. In the past there has been a wide difference of opinion as to the proper distribution of weight to each individual member, but the Car Construction Committee of the American Railway Association has made some exhaustive studies and cleared up this question, in an endeavor to lead to a sound understanding.

53 In rare cases, for example, in designing flat cars, concentrated load in the center of car must be considered. The present American Railway Association requirement for this class of equipment is very severe and an exceptionally heavy structure is required to meet the situation. The ballast car and coal hopper car also present a special problem as to load distribution, but in other types of freight cars a uniformly distributed load can safely be assumed, provided that the design is based upon maximum loading conditions.

54 It is also an open question as to how sills should be proportioned in relation to each other; that is, as to whether the center sills should carry all of the load, or whether the side and center should each carry their portion, and, in the case of cars requiring sides for containing the lading, whether these sides should also act as girders and carry the greater portion of the load. In the case of flat cars, for instance, a common method employed is to design them with deep center sills, proportioned to carry most of the load, with rolled section for side sills, figured as a continuous beam supported at the bolsters and cross bearers, and carrying a small portion of the load distributed along the sides of the car. Other administrations figure that the side sills should be made sufficiently strong to carry their portion of the load, thus necessitating fish-belly sills throughout. Such a requirement, while not universal, is sufficiently evident in normal interchange of cars, designed to haul machinery, blocks of stone, tractors, etc., to make it a problem requiring thought and study at times when selection in design is made. For steel gondola and such cars as require sides for containing lading, and which usually consist of a web plate with top and bottom flange members, it is, of course, the most economical construction to figure them as complete girders, and so proportioned to carry the greater portion of load, keeping in mind the necessity of providing for lateral stiffeners to prevent bulging and the collapse of the girder. The carrying of the load

on the side construction, because of its depth, makes an economical girder and usually works out the cheapest, but of course would not apply to house car equipment fitted with wooden superstructure. In this case the framing should not be depended upon to carry the load; on account of insufficient strength of wooden connections, framing becoming loose as a result of wood drying out and tie rods imbedding themselves into the framework, or otherwise losing their tension. It would apply, however, where steel side sill at the bottom and steel side plate construction at the top form an open truss between the bolsters. In proportioning the side framing to carry a portion of the vertical load, account should be taken of the bulging pressures encountered in service under certain conditions of lading.

55 The automatic coupler now used has eliminated one of the most prolific causes of personal injuries. It also provides a construction sufficiently heavy and strong to take care of the most severe service, insure safety and prevent break-in-two. Often enough consideration is not given to the problems involved in providing and maintaining a suitable draft gear at the back of the coupler. Impact blows must be absorbed in some manner. These should be taken by the draft gear, and gradually and equally transmitted to the component parts of the underframe, delaying the full effect of the blow until it can be partially dissipated by the movement of the car. With the rapid disappearance of wooden cars a great amount of train flexibility, due to the resilience of the wooden underframing, has been lost.

56 The splicing of the center sills in front of the bolster and the use of separate draft sills to facilitate repairs was until recent years a detail which has been the source of many arguments. One view is that if the center sills are made proportionally stronger at and between the bolsters than they are between the bolster and the end sills, any stress great enough to damage the draft sills could be easily reached and repaired. On the other hand, if the center sills were made in one continuous length from end to end gear, the cost of repairs, in a limited number of cases, would not be nearly as great as the additional expense of making the splice on all the cars at the time of building. With the use of a combination rear draft lug and bolster center casting there is no reason left for employing separate draft sills. In common with the practice followed by the American Railway Association, the Chicago, Milwaukee & St. Paul Railway employs combination rear draft lug and bolster center casting of the type shown in Fig. 2. Such construction makes it possible to deliver a maximum buffing shock with the least destruction to the car, principally because of the possibility of proper distribution to the center sills, bolster, and other longitudinal members of the car. Latest practice calls for integral center-sill construction from end to end of car, some

administrations using a 5-ft. distance between the center line of the bolster and the face of the striking casting, in order to minimize the swing of the coupler on sharp curves and permit the shortest possible overall length in the case of hopper cars. Others prefer a $5\frac{1}{2}$-ft. distance in order to provide an ideal application of the side sill step and prevent it from interfering with the opening of the journal box lid when the sill step is placed centrally under the side ladder; also to take advantage of the reduction of stresses in the car structure, which is possible by reason of the greater overhang on the ends of the car.

57 The proportioning of the bolsters depends, of course, to a large extent on the manner in which the vertical load is distributed and carried by the longitudinal members, thereby determining how

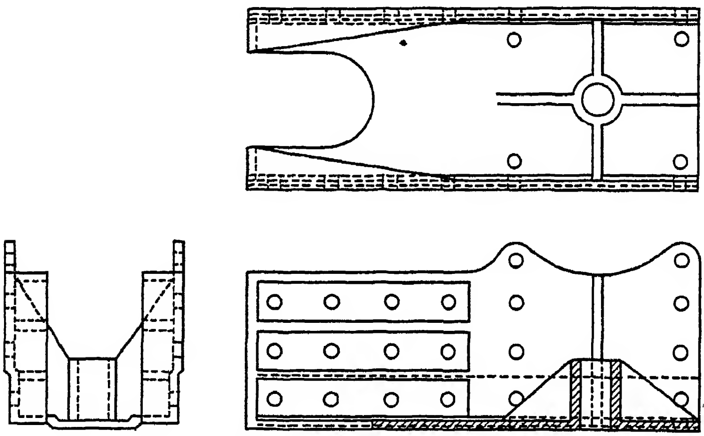


FIG. 2 REAR DRAFT LUG AND BOLSTER CENTER CASTING

much load is to be transmitted to the center plate through the bolster. The need for taking into account the raising of the car while under load when jacks are used at the extreme end of the body bolsters should not be overlooked. One of the most important details in bolster construction is the center filler between the center sills, shown in Fig. 3, located directly over the center plate. This detail must be designed to protect against fracture of the center plate, and the possible tearing or bending of the bolster cover plate, thus allowing the side bearing clearance to be taken up, resulting in derailments of cars, if the defect is not detected in time.

58 The improvement of couplers, draft gear and attachments promoting speedier and safer connecting and disconnecting of cars, as well as in securing them more firmly while in train service, has been accompanied by an even more remarkable development in brake apparatus for arresting the motion of the cars, either singly

or in trains. Draft gears and brakes need to be systematically maintained and known to be in good operating condition, with complete stencilled identification on each car showing date and place of last inspection and repair. While this is a mandatory practice with respect to air brakes, it has not, as yet, been extended to the draft gears, except by individual roads. Wherever it has been practiced, the operation of the cars, their maintenance, and condition of lading carried have shown marked improvement End construction has been particularly benefited in such cases.

59 Another feature respecting freight train car design, which is often not sufficiently considered, is that of the permissible center of gravity of cars under load. Experience has shown that $6\frac{1}{2}$ ft. from top of rail is about all that is safe for ordinary service; yet,

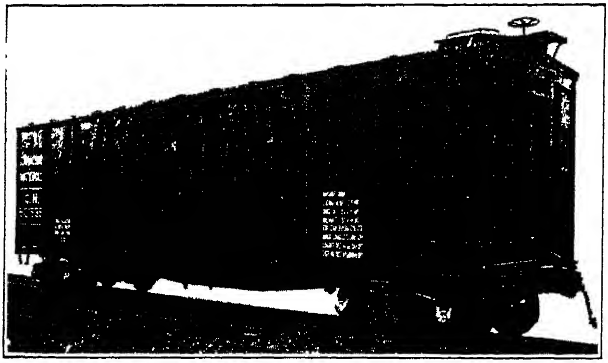


Fig. 3 Canadian National Box Car of Composite Construction. Capacity, 122,800 Lb.

many of the large-sized hopper, automobile and furniture cars are operating under load with centers of gravity in excess of 7 ft. This is the main reason why so many derailments and wrecks occur with cars 10 ft. high inside and loaded to the roof. The strictest of care is necessary in train handling and track maintenance on curves in order to minimize that element of danger.

60 The diagonal brace, which is usually placed in all underframes at the corners, is a detail which varies according to the ideas of the designer. Some advocate extending it from the ends of the center sill to the intersection of the side sill and bolster. Others prefer to have the brace extended from corner to center sill at the bolster. The former arrangement relieves the center sill of a portion of the buffing shocks, transmitting them to the sides; also any cornering of the car sufficient to damage it to a considerable extent would be great enough to damage the brace if it extended from the corner of the car to the intersection of the center sill and bolster, and without the brace at the corner the under-

frame is much more easily repaired. However, on cars where push pole pockets are used to a large extent, it is quite important that the brace be provided in the latter location to transmit the thrust to the center sill. The employment of roping staples located at or near the bolster has largely done away with push poles and more cars are being pulled instead of pushed in spotting them. This is particularly true at mines, logging camps or industrial tracks.

61 Much difficulty is being experienced at this time with truck coil-springs under 50- and 70-ton capacity cars as a result of existing designs going solid under load and causing breakage. A freight car truck spring has necessarily to work through a tremendous range of carrying capacity, namely, that of the light weight of the car body as a minimum and this weight plus the full carrying capacity of the car, which is generally equal to 5 or 6 times the light weight. While space restrictions are significant with the conventional design of truck, the provision of a safe spring, namely one that (1) will allow the car to remain on the track and (2) will not permit the load to punish the track unnecessarily, are important considerations. Some roads are resorting to the addition of springs in order to give greater cushioning capacity, others are employing improved material and still others are modifying the shape of the bar forming the coils in order to attain greater capacity and more travel. The bar of steel forming the spring should not be any larger than absolutely necessary in order to allow proper heat treatment. It is quite possible that, with the increase in freight train speeds being demanded at this time, new designs of trucks will be required to provide greater safety, and certainly to relieve the truck springs of the tremendous side thrust action they are now subject to. Lastly, railroad spring shop practice may be largely benefited from a study and application of methods employed in the automobile industry.

62 The peculiar feature of the rolling stock of American railways is the center-bearing truck. Its superiority to the rigid wheel base is obvious in its facility for continuous adjustment to the track, whether that be due to curvature or to defective maintenance. It has also the merit of neutralizing the horizontal and vertical oscillations of a car in motion. The introduction of the center-bearing truck in the construction of car equipment brought about the transfer of the journal bearings from the underframe of the car body to the truck frame. In this change of structure the load from the superimposed weight in most classes of cars is transmitted through the side sills to the center bearing by the introduction of the body bolster, which therefore becomes an important member of the underframe. For this reason it is important to keep in mind that the body bolster should be sufficiently rigid to transmit this load without sharing any part of the superimposed weight with the side bearings. The latter are intended simply to

TABLE 4 PERCENTAGES OF THE DIFFERENT FEATURES OF CONSTRUCTION USED IN CARS BUILT IN THE U S DURING THE PERIOD 1921 TO 1926 INCLUSIVE

Type of car	Box cars		Automobile cars		Gondola cars		Hopper cars	
Capacity in tons	40	50, 55 & 60	40	50 & 55	50 & 55	70 & 75	50 & 55	70
Total number of cars analyzed	61822	51029	33941	17850	43428	9655	39829	32083
Kind of center sills:								
Channel	65	79	23	75	85	28	100	100
Fish belly	31	21	77	25	14	72	.....	.....
Wood	4	.....	.....	.....	.....	.....	.....	.....
Miscellaneous	.....	.....	.....	.....	1	.....	.....	.....
Total	100	100	100	100	100	100	100	100
Kind of trucks:								
Arch bar	16	16	10	.....	8	.....	27 3	.....
Cast steel	75	82	90	87	90	100	62.4	100
Pressed steel	4	.....	.....	2	5	.....	1.0	.....
Miscellaneous	5	2	.....	11	1 5	.....	9.3	.....
Total	100	100	100	100	100	100	100	100
Kind of wheels:								
Cast iron	99	99 5	100	88	7 4	.....	78.4	1 0
Steel	1	.5	.....	.....	91 0	100	12.3	94.4
Miscellaneous	.....	.....	.....	12	1 6	.....	9 3	4 6
Total	100	100	100	100	100	100	100	100
Width of side doors:								
5 ft., 0 in.	1.8	.....	.....	.....	.....	.....	.....	.....
5 ft, 5½ in.	1.3	.....	.....	.....	.....	.....	.....	.....
5 ft., 6 in.	7.0	.....	.....	.....	.....	.....	.....	.....
5 ft., 11 in.	2.4	.....	.....	.....	.....	.....	.....	.....
6 ft, 0 in.	83.6	86 0	.....	.....	.....	.....	.....	.....
6 ft., 2 in.	.....	14 0	.....	.....	.....	.....	.....	.....
7 ft, 0 in	2.2	.....	.....	.....	.....	.....	.....	.....
7 ft., 6 in.	.....	.....	3 0	.....	.....	.....	.....	.....
9 ft., 8½ in.	.....	.....	.....	2 8	.....	.....	.....	.....
9 ft., 9¾ in.	.....	.....	.....	5 8	.....	.....	.....	.....
10 ft., 0 in.	1.7	.....	81.0	80.2	.....	.....	.....	.....
10 ft., 0⅜ in.	.....	.....	4 0	.....	.....	.....	.....	.....
10 ft., 1$\frac{7}{16}$ in.	.....	.....	.....	1 2	.....	.....	.....	.....
10 ft., 1¾ in.	.....	.....	2.0	.....	.....	.....	.....	.....
10 ft, 2 in.	.....	.....	.....	1.5	.....	.....	.....	.....
10 ft, 3⅜ in.	.....	.....	2.0	.....	.....	.....	.....	.....
10 ft., 5¾ in.	.....	.....	5.0	2.7	.....	.....	.....	.....
10 ft, 6 in.	.....	.....	3.0	5.8	.....	.....	.....	.....
None	.....	.....	.....	.....	100	100	100	100
Total	100	100	100	100	100	100	100	100
Size of end door:								
23⅝″ wide × 28″ high	.....	8	.....	.....	.....	.....	.....	.....
None	100	92	100	100	100	100	100	100
Total	100	100	100	100	100	100	100	100
Average light weight per car (lb.)	43753	45508	46402	48982	44351	51326	41262	50864
Average light weight per cu. ft. capacity (lb.)	15.82	14.76	14.31	14.09	26.06	31.36	22.15	20.43
Style of construction:								
All steel					42.5	82.0	83.9	100
S. U. & S. F.					14.5	2 0	9.5	.....
Steel frame					17.0	13.0	.....	.....
S. U. & S. S.					1.0	2 0	.....	.....
Steel und.					25.0	.....	3.8	.....
Steel center sill					.....	.....	.3	.....
Comp. S. F. & S S					.....	.....	2.5	.....
Total					100	100	100	100

limit the lateral swaying of the car body in rounding curves, or in passing over low joints, frogs or crossings. There is no tendency in the truck itself to resume its normal position after leaving a curve, except from the reaction of the flanges against the rail. If this reaction is neutralized by friction of the side bearings, the flanges may grind against the rail long after the curve has been passed. Reduction in flange wear reduces the amount of wear on the rail as well as train resistance. It is therefore important that, when a car is at rest, the body bolster should sustain the entire weight without assistance from the side bearings.

63 The introduction of all-steel or steel-framed cars began when the first of the modern steel cars were built about 1897, yet we have record of steel cars having been built, to a limited extent, as far back as 1853, and in much larger numbers a few years later in Europe The Eastern Railway of France, alone, in 1905 had over 20,000 cars with metal framing or all-metal construction in service. Taking the year 1907 as a base and one of normal operation in the car-building industry, the returns from thirty-six car building concerns indicate 284,188 freight cars built, of which 72 per cent were of all steel or steel underframe construction, and the proportion of steel cars to wooden cars built emphasizes the rapidity with which the introduction of metal into freight train cars took place from this period on.

64 The percentages of different features of construction used in cars built by six of the leading car building companies in the country over the period 1921 to 1926 inclusive, are shown in Table 4. Cars of the following classes are represented: 40-, 50-, 55- and 60-ton box cars, 40-, 50- and 55-ton automobile cars, 50-55-, 70- and 75-ton gondola cars, 50-, 55- and 70-ton hopper cars. Typical examples of these cars are shown in Figs. 4 to 10, inclusive.

65 Of the number of cars shown, 31.8 per cent were of all-steel construction; 15.3 per cent were of steel-framed construction with wooden lining; 52.9 per cent were built with steel underframe and double-sheathed wooden superstructure; 100 per cent had steel center sills. So far as trucks are concerned, 89.2 per cent were equipped with cast-steel trucks and 10.8 per cent with other than cast-steel trucks; also 81.2 per cent were fitted with cast-iron wheels and 18.8 per cent with steel wheels. The average light weight of all cars per cubic foot of carrying capacity was 19.85 lb , ranging from a minimum of 12.24 lb. to a maximum of 34 27 lb.

66 The designing, up to the year 1912, of the modern all-steel and steel-framed car had been more or less left to the car builder, in conjunction with, and often under the supervision of, railway mechanical officers, because of the fact that this type of construction was at that time quite new, and it was necessary for the manufacturers to design and present the many advantages to be gained in substituting steel for wood in car construction. This

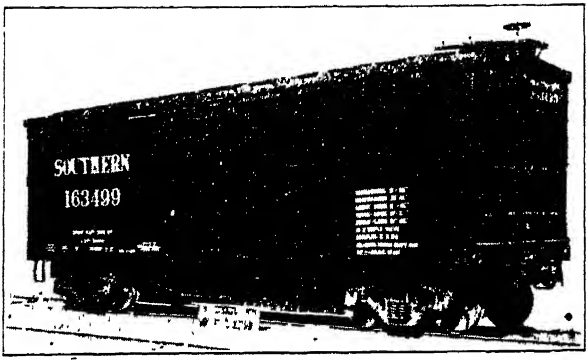


Fig. 4 Box Car for the Southern Equipped with Cast Steel Trucks. Capacity, 80,000 Lb.

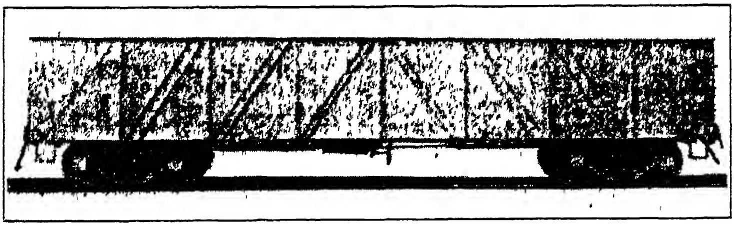


Fig. 5 Gondola Car of All-Steel Construction

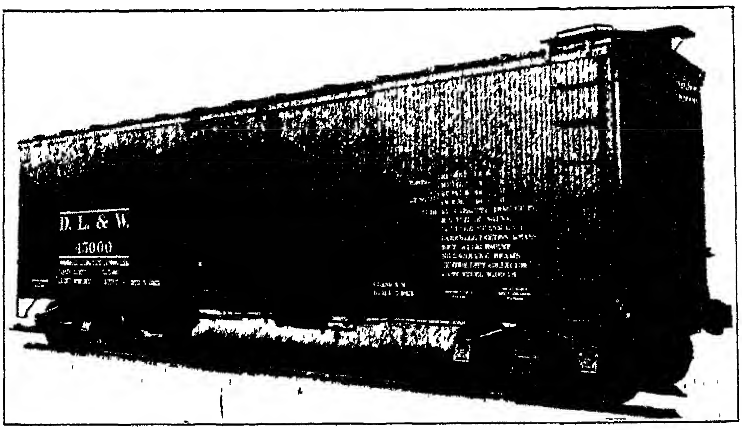


Fig. 6 Delaware, Lackawanna & Western Box Car Equipped with Cast Steel Trucks. Capacity, 110,000 Lb.

period was followed by the larger systems taking up the designing of their steel-car equipment on quite an extensive scale. They consequently not only furnished builders with complete specifications covering the construction of the car, but also engineering

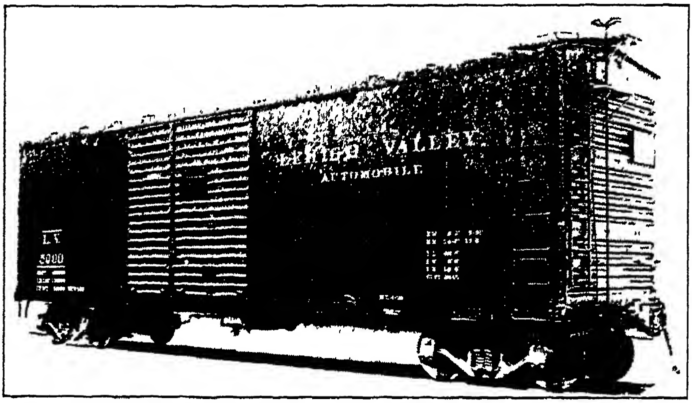


FIG. 7 AUTOMOBILE CAR EQUIPPED WITH STEEL DOORS AND ENDS. CAPACITY, 100,000 LB.

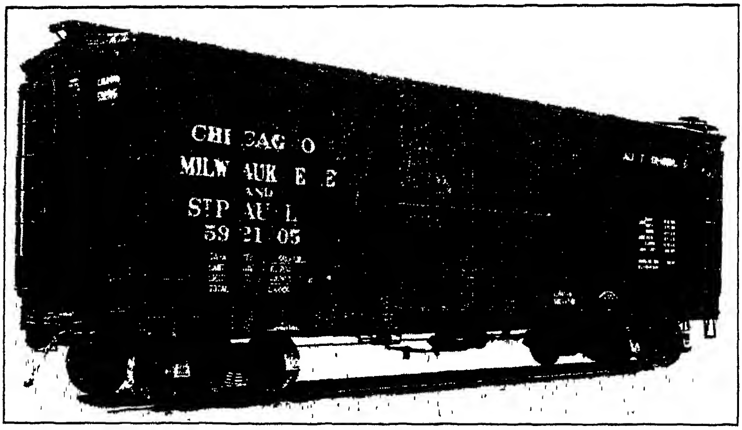


FIG. 8 CHICAGO, MILWAUKEE & ST. PAUL AUTOMOBILE CAR OF COMPOSITE CONSTRUCTION. CAPACITY, 80,000 LB.

analyses of all parts together with detailed drawings, including bills of material. Both arrangements have their advantages and disadvantages. In the case of a railroad designing its own cars, it is possible to standardize its castings, forgings, and miscellaneous parts to a considerable extent, and, by keeping in mind its standard cars of different classes, it is often able to design them in such a

manner as to have several parts common to all classes, thereby assisting in the reduction of the investment for repair parts, as well as making substitutions conveniently. This, however, often works out to the disadvantage of the car builder, as he, too, has standardized patterns, etc., which, of course, are to his advantage to employ, but which may differ from the railroad company's

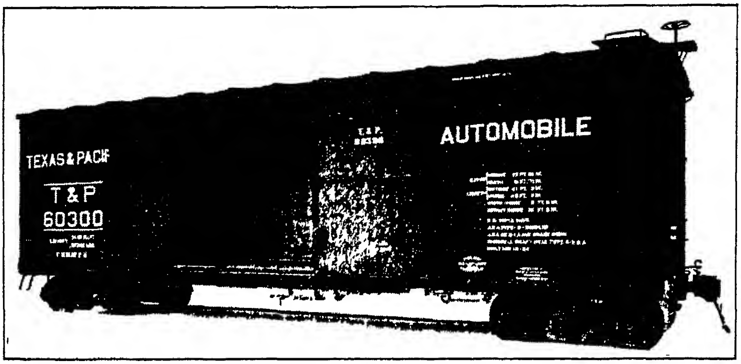


FIG. 9 COMPOSITE AUTOMOBILE CAR FOR THE TEXAS & PACIFIC. CAPACITY, 80,000 LB.

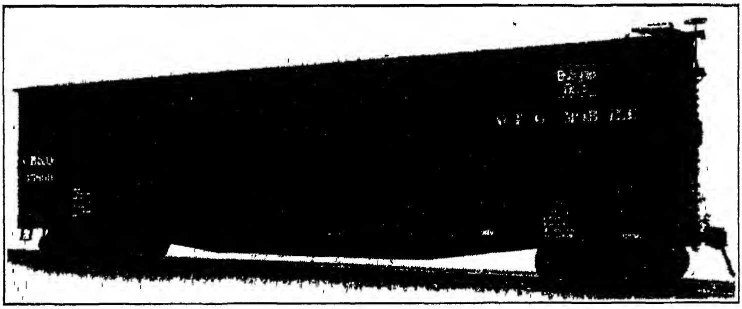


FIG. 10 AUTOMOBILE CAR FOR THE CHICAGO, BURLINGTON & QUINCY WITH FISH-BELLY CENTER SILLS. CAPACITY, 100,000 LB.

standard just enough to necessitate new dies and patterns to such an extent that the saving gained in having the drawings, specifications and bills of material furnished is more than offset by die and pattern expense. These dies may never be employed on later orders, which means, of course, that the entire die and pattern cost must be borne by this particular order of cars. In the case of large assignments this cost, distributed over the entire lot, would, of course, not represent as important a charge per car as

in the case of a smaller order, but it is a serious factor to be taken into consideration in estimating and designing. The number of complicated shapes should, therefore, be kept as low as possible. It is essential that a maximum standardization of all castings and such forgings as might be termed running repair parts, such as brake gear, draft gear, safety appliances, etc., should be standardized as far as possible, in order to attain economical manufacturing shop practice and road running repair maintenance. The need for standard construction, where practicable, should be emphasized constantly, and while the American Railway Association has accomplished admirable results in the way of standardizing car designs, truck parts, draft-gear parts, etc., this endeavor cannot be made successful unless taken seriously by the various large administrations in the purchase of new equipment, first, on account of the obvious reasons mentioned, and second, in order to make possible a minimum purchase price for such construction, by reason of volume required. The question of limiting stresses to be used in the design of cars needs to be thoroughly understood and carefully applied. The following is offered as a recommendation and is based upon exhaustive studies made with respect to A.R.A. standard car designs now being developed to a conclusion:

67 *Unit Stresses.*

Structural Steel

Tension ....16,000 lb. per sq. in.

Compression for columns whose ratio of length to least radius of gyration is less than 40......................16,000 lb. per sq. in.

For columns whose ratio of length to least radius of gyration exceeds 40, determine the maximum unit compressive stress at the center of the column due to a direct load by means of the Rankine formula, fixed ends, transposed as follows:

$$S = \frac{PC}{A}$$

where

$S =$ Maximum unit stress in lb. per sq. in. in a long column due to a direct load

$P =$ Direct compressive stress in lb.

$A =$ Area of section in sq. in.

$C = 1 + \frac{1}{25{,}000}\left(\frac{L}{r}\right)^2$

$L =$ Length of column in inches

$r =$ Least radius of gyration in inches.

For ends of columns, where the bending movement due to slenderness ratio is neglected ....16,000 lb. per sq. in.
Shear ....12,800 lb. per sq. in.

Rivets

Shear ....10,000 lb. per sq. in.
Bearing ....20,000 lb. per sq. in.

Wood

Tension .... 600 lb. per sq. in.

68 The car body should be designed to carry under service conditions the maximum rail load limit prescribed for capacity of car intended.

69 *Center Sills* should be designed to withstand
(*a*) An end load of 250,000 lb. applied on the back stop at the center line of draft
(*b*) Weight of center sill and attachments
(*c*) Vertical load reactions of floor planks.

70 *Bolsters and Crossbearers* should be designed to carry
(*a*) Total reactions from side sill for bolster, and center sills for crossbearer
(*b*) Weight of bolster or crossbearer attachments. Only the cover plates of bolsters and crossbearers should be considered as effective in resisting bending moment.

71 All connections should be designed for the maximum load to which the member connected may be subject and so detailed as to eliminate, in so far as possible, eccentricity effects within the members. Where it is not possible to eliminate secondary stresses in any members caused by eccentric loads these should be combined with the direct stresses in such members.

Conclusions

72 The function of mechanical budget allowances is very often controlled from decisions made by officers other than those who are actually held personally accountable for the results. These decisions, respecting maintenance procedure, are based upon available funds or immediate earnings or prospective needs, to cope successfully with the present or future business and quite often neglect adequate consideration for economies from steady employment. Those who immediately direct should be cautious of the programs governing the handling of the work, and the time rate of output, so that the facilities may be, in so far as possible, loaded to an economical range of working. This is not as simple as it might at first appear. The budget of requirements should be

worked out by forecast, plus experience and adjustments. A yearly allotment should be worked out, the same being based on a policy of procedure covering a term of years, as this is an absolute essential to intelligent and businesslike management. Estimates should be made in advance for each month, and the various items tabulated on a budget sheet which can be laid out to accommodate as many divisions of the work as are required. The big underlying principle of budgeting has regard for the needed advice which must be given the purchasing and stores department in advance so they can make the material supply coincide with a policy of steady employment of man-power in so far as possible, all with a view toward a businesslike and economical procedure.

73 The following topical suggestions are offered in closing: Railroads are going concerns not subject to being closed down at a moment's notice, consequently—

(*a*) They should systematically consider the current requirements of the service as they become increasingly apparent from year to year in view of enlargements in traffic and the ever changing conditions of operation.

(*b*) Maintenance policies need to be formulated far in advance, in order to currently maintain and turn over a proper number of cars by classes, also to employ a least complement of facilities and labor force regularly to keep the entire plant structure working efficiently and uniformly to the greatest reasonable degree and, yet, with a maximum economical output.

(*c*) Adhere to a 20- to 25-year life for freight-train cars with an annual depreciation rate to coincide with whatever figure is used and provide for suitable and current replacement.

(*d*) They should procure freight train cars, when built new, embodying design fundamentals recommended by the A.R.A. and of minimum weight consistent with traffic and engineering considerations.

(*e*) Give constant thought to the fact that freight-train cars should be built interchangeably between roads in so far as possible in order to lessen first cost and improve current maintenance when cars are off line, and also to avoid delay when needing repairs, which may be occasioned when special material is needed.

(*f*) Special classes of cars should not be built unless they are really justified from the standpoint of actual traffic and operating experience factors.

DISCUSSION

W. E. Symons.[1] To emphasize, if possible, the great importance of cordial coöperation in the solution of the problem outlined by the author, certain fundamental features are presented in Table 5 for the benefit of those who may not fully appreciate the magnitude of the problem, or the benefits to be derived from its partial or complete solution.

TABLE 5 RAILWAYS AS A FACTOR IN NATIONAL LIFE

Total investment in U. S. railways (approx.)	$25,000,000,000
Income from operation (1925)	$6,239,353,447
Income from freight (1925)	$4,596,952,895
Average freight income per day	$12,594,386
Average freight income per hour	$524,764
Total number of freight cars	2,407,000
Total number of freight cars (less work cars and cars under repair, 275,000)	2,132,000
Average earning per car per year	$2,156
Cost of repairs (approx.)	$385,000,000
Average per unit (approx.)	$160
Number of security holders in U. S. railways (approx.)	2,000,000
Held by banks and insurance companies (approx.)	$3,000,000,000
Citizens either served, or affected by, and who should be interested in, this problem (approx.)	117,000,000

While the author has presented statistics or figures pertaining to each of the factors embraced, amplification may serve as an aid in solution.

Depreciation as a Factor in Operating Economy. While the concrete examples by the author on depreciation and retirement factors very clearly illustrate the points mentioned, it is thought that a tabulated display of present practice on a number of leading railways may be of value in urging greater uniformity or standardization, and thus add to the reliability of monthly and annual reports of operation. Such a tabulation is given in Table 6,

TABLE 6 RATES OF DEPRECIATION

Railway company	Locomotives, per cent	Passenger cars, per cent	Freight cars, per cent	Work equipment, per cent
One	4.5	4.5	4 5	4.5
Two	3	2	3	2
Three	3	3	4	5
Four	2.5	2 5	3	2.5
Five	2	2	2	2
Six	4.5	2.5	5	5
Seven	3	3	4	4
Eight	2	1.5	2	2
Nine	2 5	2.5	2 5	2.5
Ten	3.5	3.5	4	4
Eleven	4	4	4	4
Twelve	4	4	4	4
Thirteen	3	3	3	4
Fourteen	2	2	2.75	2.75
Fifteen	3.44	3.73	3.31	4.75
Average	3.13	2.88	3.54	3.4

from which it will be seen that on fifteen of the largest railway systems in this country the theoretical life of equipment is as

[1] Consulting Engineer and Associate Editor, ***Railway and Locomotive Engineering***, New York, N. Y. Mem. A.S.M.E.

follows: Locomotives from 22 to 50 years; passenger cars, from 22 to 66 years; freight cars, 22 to 50 years; work equipment, from 20 to 50 years.

The Transportation Act of 1920 directed the Interstate Commerce Commission to fix uniform rates of equipment depreciation to be used by all carriers, and investigations were at once begun to determine a fair set of rates. During Federal control the Director-General issued an order in May, 1919, that all carriers during the period of control should depreciate equipment placed in service after December 31, 1917, at the rate of 4½ per cent per year. A considerable number of lines have combined to use this rate since the termination of Federal control, and to that extent depreciation charges are now more uniform or comparable than in pre-war years.

Depreciation charges with respect to any equipment shall cease when the difference between the ledger value and the estimated scrap value shall have been credited to the accrued depreciation account.

With the foregoing facts in mind it is not difficult to understand why there is such a wide range of difference in the apparent operating efficiency of two lines, fairly comparable throughout, except as to the amount of depreciation charges taken up and charged to operating expenses each month. This one feature alone has much effect on the integrity of operating statistics, particularly when used for comparative purposes, as in considering values.

It may be said that on lines other than those shown in the tabulation fluctuations have ranged from 1 to 6 per cent, and until the Commission shall specify an exact rate to be charged on account of depreciation of equipment, desired results will not be attained. Increased mileage per day, and increased tons per car, as well as improved train movement, are all important features, and improvements in these factors will tend to greater efficiency, while the subdivision of car parts causing delays for repairs, given in per cent of contributing cause, should be given careful consideration.

The question of the relative value of the 40- or 50-ton car seems to be open to much discussion, as each capacity and design has strong champions among the brightest minds of leading executive, operating, traffic, and engineering officers.

During the past few years railways have greatly increased the efficiency of operation. A marked change in their relations with the public has also taken place. While these two features combined have resulted in a wonderful improvement, there remain many directions in which still greater improvement can be made. One of these is the *use* and *abuse* of freight cars by their owners and users, and also by shippers.

As a result of the splendid work of the American Railway Association and other public relations bodies, and due also to the great help rendered by railway officers and employees, there has been

almost a complete reversal of the former antagonistic feeling toward railways. Today the public in general, and the shipper in particular, not only concede that the railways have some rights, but are disposed to give them a reasonably fair deal in almost all matters except the use of freight cars. Thousands of shippers have been for many years, and now are, using loaded freight cars consigned to them, as auxiliaries to their warehouses or for storage purposes. A majority of these offenders have no intention whatever of providing themselves with proper terminal or warehouse facilities, or of promptly making empty, and returning to service, cars consigned to them under load. Thousands of these people actually do a retail business, not from their store or warehouse, but from a freight car which is not their property, and which they should not be allowed either to use in this manner, or to hold under load longer than it can be made empty with proper facilities.

In the absence of any accurate check as to the extent of this evil, both of omission and commission, it has been estimated that from 150,000 to 200,000 freight cars are thus either being used for purposes other than intended, or held under full load awaiting a change in market price of some commodity, or for some other reason entirely foreign to transportation, when other shippers badly need cars of this kind or class.

The suggestions in Par. 55 are not only appropriate and timely, but if given proper consideration should result in marked improvement in one of the badly neglected features of car design.

The author's conclusions in Pars. 72 and 73 contain excellent suggestions combined with good sound business logic, the adoption of which will result in marked improvement in the freight car situation in general.

The point that the writer wishes to stress in this whole matter, however, as paramount to all others at this time, is the *misuse* and *abuse* of freight cars. A freight car is an integral part of the physical property of a complete transportation unit, regardless of all questions of design. The railway in its entirety belongs to the shareholders, whole official representatives should see to it that all property intended for, and essential to proper transportation service be used for this purpose only, and not diverted to other uses.

The prohibition as to *misuse* of freight cars is particularly directed against the pernicious, expensive, and unnecessary custom of allowing great numbers of these cars to be regularly used as auxiliaries to storehouses, warehouses, etc., when they should be promptly made empty and returned to transportation service.

No. 2016

INTERNAL FRICTION IN SOLIDS

By A. L. Kimball,[1] Schenectady, N. Y.,
Associate-Member of the Society

and

D. E. Lovell,[1] Schenectady, N. Y.,
Non-Member

In connection with a test devised to determine quantitatively the amount of friction within the metal of a steel shaft, it was found that the internal frictional forces were totally unlike those of a viscous fluid, as assumed by many investigators, where the forces are greater the more rapid the deformation. Instead of this, the dissipative forces were found to be the same whatever the speed of deformation. The apparatus with which the tests were conducted is described, and the results are tabulated. Equations relating the variables involved are derived in the analysis of the problem.

THE present investigation is the outcome of a study by one of the authors of the effect of the internal friction within the metal of a steam-turbine wheel in dissipating vibration(1)[2] and also of the effect of internal friction within a rotor and shaft in producing shaft whipping(2). In connection with a test devised to determine quantitatively the amount of friction within the metal of a steel shaft(3), it was found unexpectedly that the internal frictional forces were totally unlike those of a viscous fluid, as assumed by most investigators(4), where the forces are greater the more rapid the deformation. Instead of this, the dissipative forces were found to be the same whatever the speed of deformation, in somewhat the same way as the frictional resistance to the motion of a weight on a table is about the same, whether the weight be moved rapidly or slowly across the table's surface.

[1] Research Engineer, General Electric Company.
[2] These, and similar numbers in parentheses, refer to the bibliography at the end of the paper.

Presented at the Annual Meeting, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

2 A study of the literature of internal friction shows that, although it is not generally known, a few investigators have appreciated this characteristic of internal friction (5). This characteristic was found to be general for all solids thus far tested by means of a special experimental test to be described later in this paper.

Method of Test

3 The method of test is similar to that used by Wöhler in fatigue investigations (7). Figs. 1 and 2 show a photograph and drawings of the apparatus used. The material to be tested was made in the

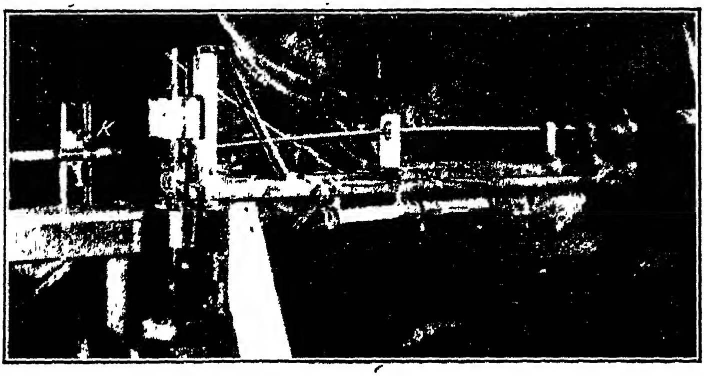


Fig. 1 Photograph of Apparatus

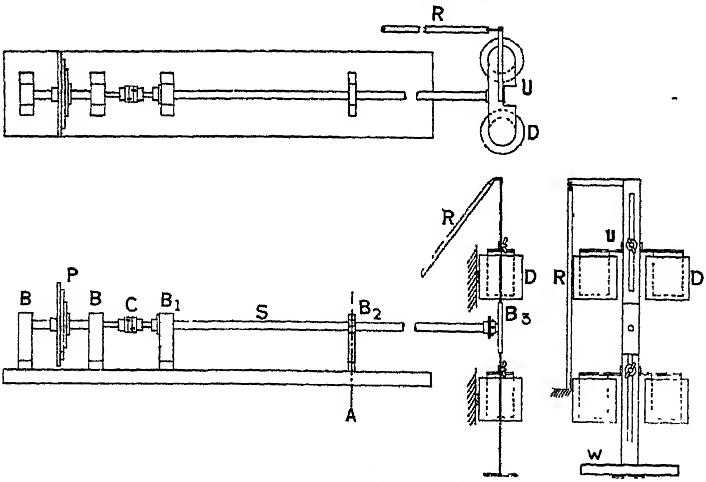
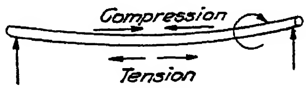


Fig. 2 Plan and Elevations of Apparatus

form of a rod, usually $\frac{1}{2}$ in. in diameter, and about 36 in. long. The rod was supported in two ball bearings, B_1 and B_2, and driven by an electric motor through the pulley P which was supported in the

bearings marked *B*, the torque being transmitted to the rod by means of a small Oldham coupling *C*. The bearings B_1 and B_2 were so situated that more than half of the rod overhung. On the projecting end of the rod was another ball bearing B_3, on which was hung the frame *U*, which carried a weight *W* on the pan at its bottom. The rod was thus deflected downwards, as shown in the photograph of Fig. 1, and at the same time it was free to revolve. The frame *U* carries four plungers which dip



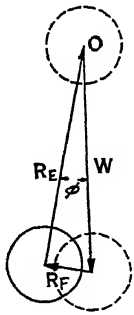


FIG. 3 VIEW OF WEIGHTED END OF SHAFT SHOWING SIDEWAYS DEFLECTION DUE TO INTERNAL FRICTION

into four rigidly supported cups containing a suitable damping fluid. This proved an effective means of steadying the end of the rod during revolution, so that the amount of its deflection could be observed.

4 Fig. 3 shows the way the end of the rod was deflected in these tests. The deflection was not exactly downwards but was displaced by the angle ϕ from the vertical, due to the internal friction of the revolving rod. As the rod revolves, the fibers parallel to its axis are carried around its axis, alternately, up one side and down on the other. As they move up, they stretch, and as they move down, they shorten. Any frictional resistance to the process of lengthening results in tension, and resistance to the process of shortening results in compression. Consequently, in this case the upward-moving fibers are in frictional tension (T_F, Fig. 4), and

the downward moving ones are in frictional compression, C_F. For the direction of rotation shown in Fig. 4, the left-hand fibers are therefore in tension due to this cause. These frictional stresses are superimposed upon the elastic stresses, T_E and C_E, and just as the elastic stresses produce an upward reaction R_E, so the frictional stresses produce a transverse reaction. The forces R_E and R_F are components of the force which balances W exerted by the weight on the end of the shaft (Fig. 3). If the weight is removed, R_E and R_F vanish (neglecting the weight of the rod), so that both the elastic and frictional couples must vanish. If $\phi = 0$, $R_F = 0$, that is, no frictional force is present. This is the case when the rod has the weight on its end but is not revolving. When it is revolved, with the weight on it, a frictional

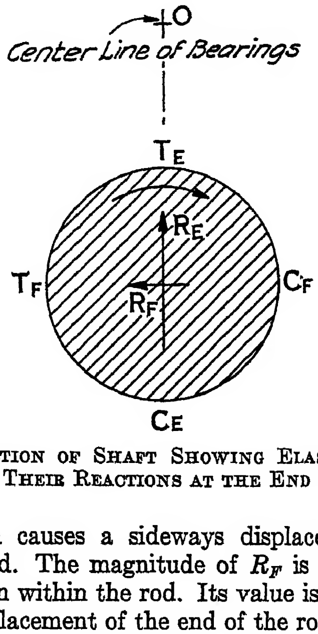


Fig. 4 Cross-Section of Shaft Showing Elastic and Viscous Stresses and Their Reactions at the End of the Shaft

force arises which causes a sideways displacement of W until R_F is just balanced. The magnitude of R_F is thus a measure of the internal friction within the rod. Its value is easily found when the amount of displacement of the end of the rod from the vertical is known, together with the weight W. This sideways displacement of the rod was measured by means of a cathetometer (K, Fig. 1). This method of test has a peculiar advantage over those previously used for the determination of internal friction in solids, in that it gives the internal friction directly as a function of the frequency of the strain cycles. The frequency of the strain cycles in the revolving deflected rod is equal to its speed of rotation, because, for each revolution, every longitudinal fiber of the rod (except those along its axis) undergoes a complete cycle of extension and compression. Thus the frequency of the strain cycles is varied by varying the rotational speed of the rod.

Internal Friction Independent of Speed of Performance of Strain Cycle

5 According to the classical theory of viscosity, which is often applied to solids(4), the transverse deflection of the revolving rod should be proportional to the speed of rotation. The first test was made on a rod of $3\frac{1}{2}$ per cent nickel steel with a view to

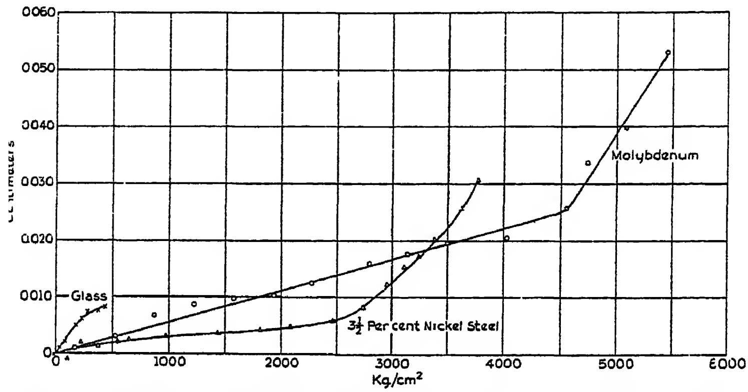


Fig. 5 Curves of Sideways Deflection at End of Shaft Plotted Against Downward Deflection and Maximum Stress Intensity

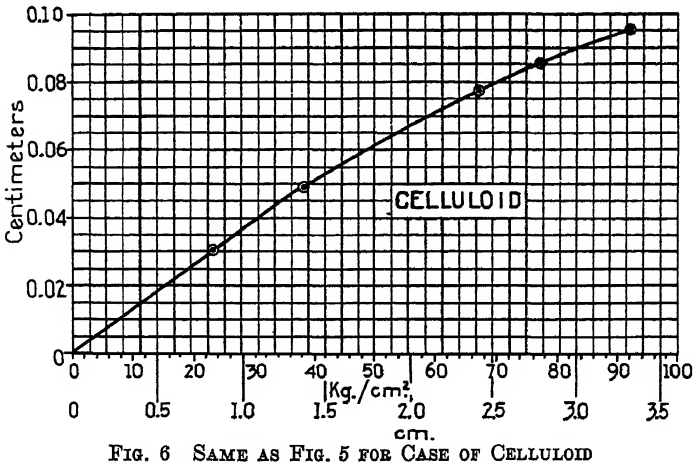


Fig. 6 Same as Fig. 5 for Case of Celluloid

observing the increase of the transverse deflection with speed, but no such change was observed over a speed range of from 2 or 3 up to 200 cycles a second. A definite deflection was present, however, as was found by reversing the direction of rotation of the rod. The amount of the transverse deflection could be increased only by increasing the load on the end of the rod, but was found for every load tried to be entirely independent of the rotational

speed of the rod. This showed that for nickel steel the forces produced by friction within the rod are independent of the velocity of strain and that, for a given amplitude of strain, the frictional loss per cycle is independent of the speed of performance of the strain cycle.

6 A considerable number of metals, alloys, and other solids have been tested by the revolving-rod method and every solid thus far examined exhibits an internal friction which is independent of the speed of performance of the strain cycles. This characteristic of internal friction in solids thus appears to be universal.

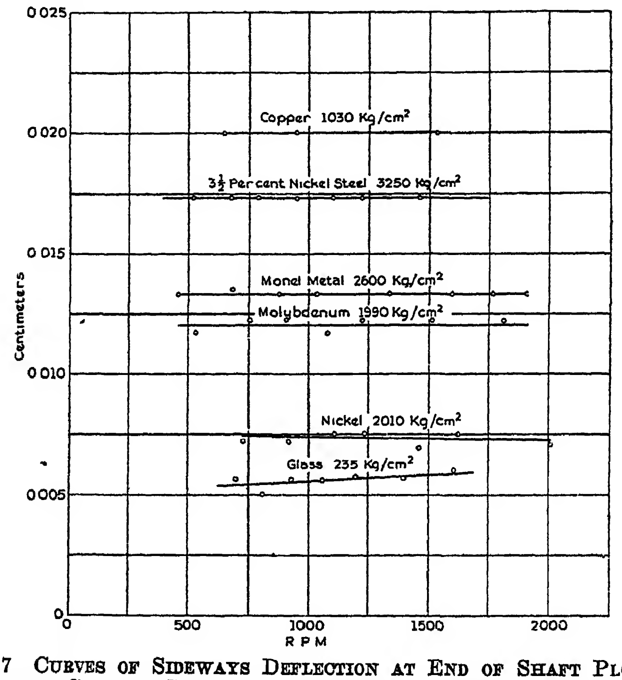


FIG. 7 CURVES OF SIDEWAYS DEFLECTION AT END OF SHAFT PLOTTED AGAINST SPEED OF ROTATION FOR SEVERAL DIFFERENT LOADS ON THE END OF THE SHAFT

7 Figs. 5 and 6 show typical curves of internal friction for molybdenum, 3½ per cent nickel-steel, glass, and celluloid for varying loads on the rods; that is, for varying bending stresses in the rods, the speed in each case being constant. Fig. 7 shows the same thing but with fixed loads on the rods and varying speed.

8 Other materials which exhibit a linear relation between internal friction and maximum stress are: swaged quarter-inch-diameter tungsten (up to its breaking point), maple wood, zinc, commercial rolled nickel, and copper (up to 7000 lb. per sq. in.). In the case of the other materials listed, a straight line will lie

among the points below the limit of proportionality only as an approximation. However, such a straight line represents the materials much better than any other single type of curve and affords the most useful comparison. Many investigators have recognized the instability of the stress-strain state and for different purposes have attempted to get the material into a stable state. This unstable state, due to past thermal and mechanical history, has been eliminated in varying degrees, and these variations are not large, as compared with variations between metals.

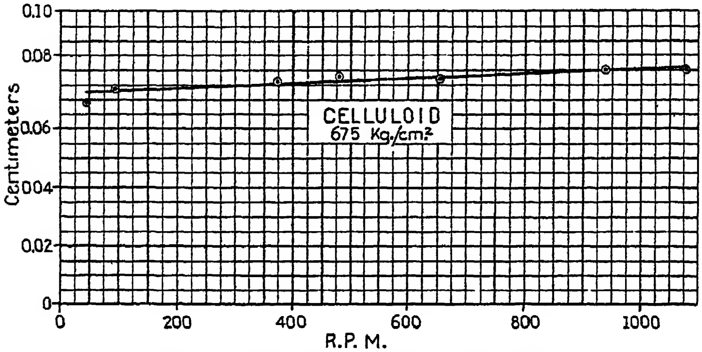


Fig. 8 Same as Fig. 7 for Celluloid

9 The effect of bearing friction upon the deflection in the rotating-rod test was negligible. Theoretical considerations show that the couples produced by friction are directly balanced by the torque applied. The application of exaggerated bearing friction produced no observable change in the deflection ϕ from which the internal friction was measured.

Definition of Coefficient of Internal Friction

10 A question which arises at this point in the discussion is how the coefficient of internal friction of a solid is best defined. To have definite meaning the coefficient of internal friction must be based upon a definite law of internal friction. From a study of a large number of curves of the revolving-rod test and also a study of the literature on this subject, the law of internal friction which most generally fits the facts and which at the same time is a simple one is

$$F = \xi f_m^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [1]$$

where F = frictional loss per unit volume per cycle at a particular point

f_m = amplitude of the stress cycle above and below zero stress at the point in question

ξ = internal friction constant = internal friction loss per unit volume per cycle per unit stress amplitude. In the c.g.s. system of units this constant equals internal friction per cu. cm. per cycle per dyne of stress amplitude. In terms of inches and pounds, it equals internal friction per cu. in. per cycle per pound of stress amplitude. Table 1 gives values of this constant expressed in both systems of units for a series of solids.

11 This table gives a complete list of the solids whose internal frictions have been measured thus far, no exception to the law of Equation [1] being found. All materials in this table, with three exceptions, are seen to be metals and alloys. But even for the three other materials, celluloid, glass, and maple wood, the same law holds. Glass is somewhat erratic, but there is no doubt as to

TABLE 1

Material	Tan ϕ ($\times 10^{-3}$)	Elastic modulus, lb per in.[2] ($\times 10^{6}$)	Internal-friction constant: Inch-lb. units ($\times 10^{-10}$)	Internal-friction constant: C. g. s. units ($\times 10^{-15}$)
Celluloid	14.4	0.31	14500	21000
Tin, swaged	40.7	4.5	278	402
Maple wood	6.69	1.9	122	172
Zinc, swaged	6.30	13.7	14.4	20.8
Glass	2.05	9.1	7.1	10.2
Aluminum, rolled	1.07	8.4	4.0	5.7
Brass, cold-rolled	1.54	12.3	3.9	5.7
Copper, rolled	1.59	14.6	3.42	4.95
Tungsten, swaged	5.24	56.2	2.93	4.26
Swedish iron, annealed	2.50	27.4	2.88	4.16
Annealed phosphor bronze	1.01	17.2	1.85	2.67
Mild steel, cold-rolled	1.57	30.3	1.55	2.33
Molybdenum, swaged	2.19	50.2	1.37	1.95
Nickel, rolled	1.02	30.0	1.07	1.55
Nickel steel, 3½%, swaged	0.73	30.0	0.76	1.10
Monel, rolled	0.454	25.8	0.55	0.79
Hard-rolled phosphor bronze	0.117	16.8	0.204	0.306

the frictional loss per cycle being practically independent of the velocity of performance of the strain cycles, under the conditions of the experiment. Of the metals and alloys tested, tin and zinc are seen to have much higher coefficients of internal friction than the others. Monel metal and nickel, though very ductile, have little frictional loss.

Discussion of the Law of Internal Friction: Equation [1]

12 It should be clearly understood that the internal friction studied in this paper is for small stress amplitudes, such as those which arise in a vibrating body where the stress amplitudes at all points in the body are comparatively low. If the stress range is so large that the elastic limit of the solid is approached, the internal friction begins to rise rapidly, and the law of Equation [1] no longer holds. The plastic yielding characteristic of many solids above the yield point belongs to a class of internal-friction phe-

nomena completely outside of the range of, and governed by different laws from, the phenomena studied by the authors of this paper.

13 The first column of Table 1 gives the value of $\tan \phi$. This is useful because $\tan \phi$ is a direct measure of the ratio of the frictional force to the maximum stress for any given point in the rod. It gives a direct means of calculating ξ, because it can be shown that

$$\xi = \frac{\pi}{E} \tan \phi \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad [2]$$

where E is the elastic modulus. For the derivation of this equation see Appendix No. 1. These latter statements are true only when the law of internal friction expressed by Equation [1] holds. With this law the ratio of the frictional force to the amplitude of the stress cycle is the same for every point in any vibrating body, because with this law the frictional force is a linear function of the stress amplitude, so the ratio is a constant for all stresses up to the limit of proportionality. Furthermore, this law gives a logarithmic decrement of vibration amplitudes, because the frictional loss per cycle is proportional to the square of the amplitude. Thus this law of internal friction and the old viscosity law give the same law of decrement of vibration amplitude. The values of ξ obtained from Equation [1] give a very useful means of comparing the internal frictions of solids, whereas constants based on the old idea that the viscosity of a solid is like that of a liquid are completely misleading.

14 These statements are proved only within the range of the tests outlined in this paper, but the authors are of the opinion that a law of the type represented by Equation [1] will be found to hold over a very wide range of frequencies.

15 The only case, to the authors' knowledge, where measurements have been made for very high frequencies is that of the experiments of Quimby(6). He uses the old liquid-viscosity law in interpreting his results and obtains constants smaller than those of previous investigators in the same ratio as his frequencies are higher than theirs. On the basis of the authors' law, however, Quimby's results and those of previous investigators give constants of the same order of magnitude.

16 Attention should be called to the fact that for frequencies of the order of those in bells and in bodies which respond with a prolonged ring when struck the law of Equation [1] shows that such a ring is perfectly possible, whereas on the basis of the old liquid-viscosity law, using the constants of previous investigators for that law (except Quimby's constants), no ring is possible. A bell would be aperiodic, as if made of putty. This is mentioned by Quimby in his paper.

Vibration Tests

17 In order to be certain that forces other than those of internal friction were not responsible for part of the deflection ϕ, several of the same rods which were given a rotation test were subjected to a vibration test, as shown in Fig. 9. The heavy iron bars *B*, each three feet long, were clamped to the end of the rod to be tested, by means of the clamping device shown enlarged in the figure. The system was then suspended by the two flexible

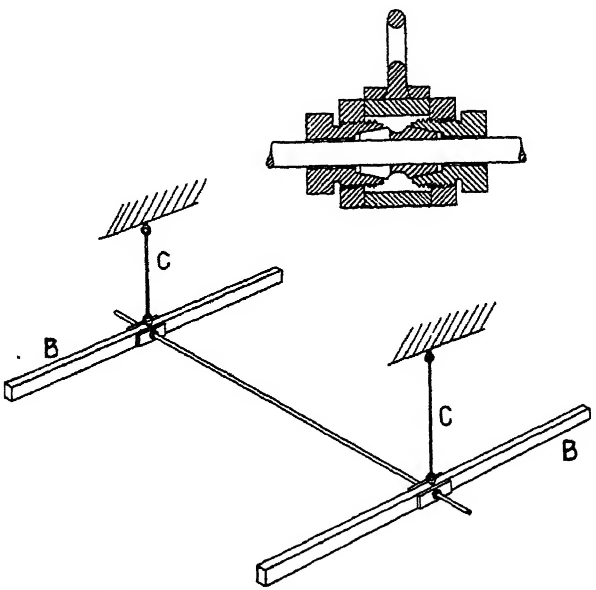


Fig. 9 Apparatus for Vibration Test Including Section of Device for Clamping Rod

and practically frictionless cords, *C*. The ends of the heavy bars were vibrated towards and away from each other so that the rod was deflected to and fro in a horizontal plane at the natural period of vibration of this system.

18 This arrangement was used because the strain cycles of individual fibers were longitudinal like those in the rotation test, although the stress distribution was somewhat different. The internal friction was approximately determined for different vibration amplitudes by decrement observations. Some results of these observations are shown for steel, brass, and copper (see Figs. 10, 11, and 12), which show the decrease of vibration amplitude with time.

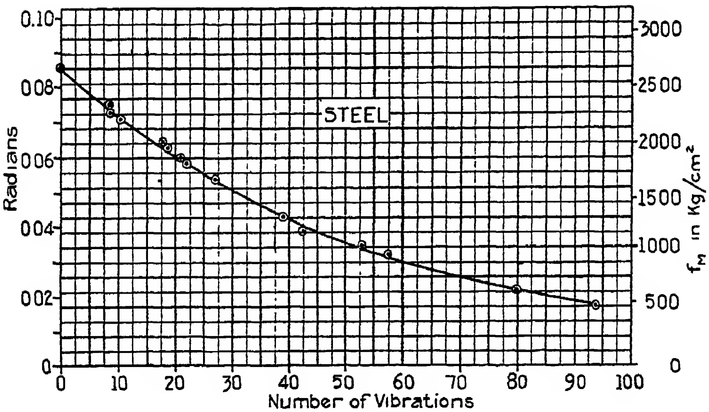


FIG. 10 CURVE SHOWING RATE OF DECREASE OF AMPLITUDE FOR STEEL ROD OBTAINED FROM APPARATUS OF FIG. 9

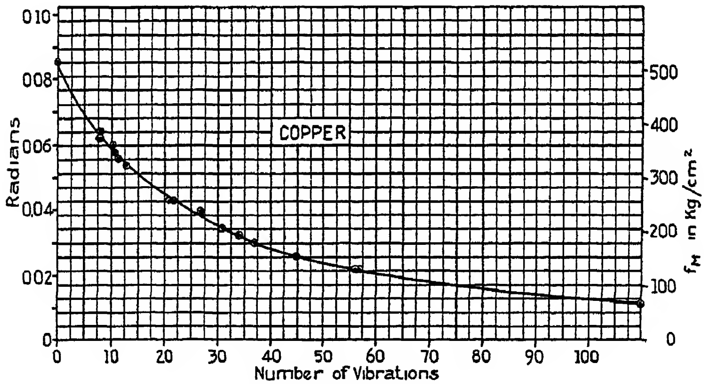


FIG. 11 SAME AS FIG. 10 FOR COPPER

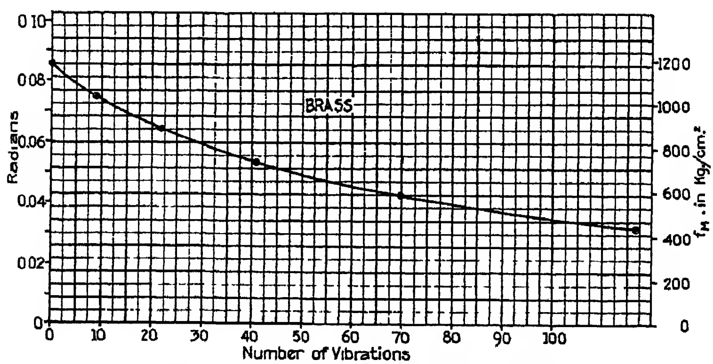


FIG. 12 SAME AS FIG. 10 FOR BRASS

19 It is easily shown, on the basis of the internal-friction law of Equation [1], that the internal-friction constant ξ is given by the expression

$$\xi = \frac{1}{Ef}\left(\frac{dy}{dt} \quad y\right) \qquad [3]$$

where
E = Young's modulus
f = frequency of vibration
y = ordinate of curve of amplitude decrease
$\frac{dy}{dt}$ = slope of this curve at the point y.

For the derivation of Equation [3] see Appendix No. 2. The ratio $\frac{dy}{dt} \quad y$ should be a constant along the entire curve for a true logarithmic decrement. This is not exactly true, though very nearly so for the case of steel.

20 Table 2 shows values of ξ for these materials determined by the rotational test compared with values determined from these curves. The check is as close as expected. The constants for the vibrating rods are higher than those for the same rods when revolving, but bear approximately the same relation to each other. Considerable energy was dissipated by surface friction in the clamps.

TABLE 2

Material	Internal friction constant in inch-pound units	
	Revolving rod ($\times 10^{-10}$)	Vibrating rod ($\times 10^{-10}$)
Brass	3.93	7.04
Copper	3.42	6.97
Mild steel	1.62	5 3

Application to Fatigue Tests

21 A method similar to this has been used in the study of the fatigue of metals (7). It is possible that an apparatus of this type might prove of considerable value in determining the fatigue limit of materials because of the ease with which the sudden rise of internal friction at the yield point is located.

APPENDIX No. 1

INTERNAL-FRICTION CONSTANT IN ROTATING-ROD TESTS

22 Proof is offered in the following paragraphs that in the rotating-rod tests the internal-friction constant $\xi = \frac{\pi\eta}{E}$, where $\eta = \tan\phi$ and E = Young's modulus. According to the definition of Equation [1] the frictional loss per cycle per unit volume at a given point in the solid is

$$\xi f_m^2$$

where f_m = maximum value of the stress at that point.
If ϵ_m = strain per unit length produced by the stress

$$f_m = E\epsilon_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [4]$$

or

$$\xi f_m^2 = \xi E^2 \epsilon_m^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [5]$$

The frictional loss per cycle per unit volume also equals

$$4\epsilon_m \times \text{frictional force} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [6]$$

23 This assumes that the frictional forces reverse instantaneously when the direction of the strain reverses at the extremes of the strain cycle, and that the frictional force remains constant between these points of reversal. This results in a rectangular "hysteresis" loop of stress vs. strain, and hence the factor 4 in Equation [6]. If the value of ξ can be found on this assumption, it is completely established for the law expressed by Equations [1] and [5], because according to this law the internal-friction loss per cycle depends only upon the maximum value of the stress and is independent of the shape of the stress-strain loop.

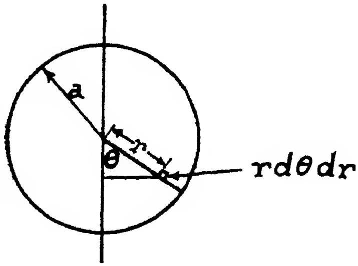


FIG 13

24 Evidently the frictional force is proportional to $\tan\phi = \eta$. But

$$\eta = \frac{\text{Frictional moment}}{\text{Elastic moment}} = \frac{M_F}{M_E}$$

for any section of the rod. The elastic moment which balances the weight on the end of the rod is well known to be

$$M_E = \frac{F_E I}{a} = \frac{\pi}{4} F_E a^3$$

where I = moment of inertia of cross-section
a = radius of rod
F_E = maximum fiber stress in this section.

25 The value of M_F has to be found by integration. The frictional moment about the vertical axis of a cross-section due to an element of area $rd\theta dr$ (see Fig. 13) is

$$dM_F = frd\theta dr\,(r \sin\theta)$$

where r = the radius to the element $rd\theta dr$
$r\sin\theta$ = the lever arm

$$f = \text{intensity of frictional stress} = F_F \frac{r}{a}$$

for the law assumed where F_F = frictional stress intensity on the outside fibers of the rod at the radius a.

26 Substituting and integrating over the whole cross-section:

$$M_F = F_F a^3$$

Therefore,

$$\eta = \frac{M_F}{M_E} = \frac{4F_F}{\pi F_E} = \frac{4f_F}{\pi f_m} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad [7]$$

where the small letters signify the frictional and elastic stresses for any point in the rod. F_E and f_m are maximum values of the elastic stress intensity during a cycle.

27 Solving Equation [6] for the frictional-force intensity and substituting in Equation [7] and eliminating f_m by means of Equation [4]:

$$\text{Frictional loss} = \pi\eta E\epsilon_m^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad [8]$$

28 Equating [8] to [5] and solving for ξ:

$$\xi = \pi\frac{\eta}{E}$$

APPENDIX No. 2

INTERNAL-FRICTION CONSTANT FOR A VIBRATING BODY

29 Proof is offered in the following paragraphs that for a vibrating body the internal-friction constant

$$\xi = \frac{1}{Ef}\left(\frac{dy}{dt} \Big/ y\right)$$

where E = Young's modulus

f = frequency of vibration

$\frac{dy}{dt} \Big/ y$ = ratio of rate of decrease of vibration amplitude to the amplitude during the vibration.

The latter quantity is a constant for all cases of logarithmic amplitude decrement, such as that produced by the law of internal friction expressed by Equation [1].

30 The potential energy per unit volume of a strained body is

$$P = \tfrac{1}{2}E\epsilon_m^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad [9]$$

where the symbols have the same meaning as in Appendix No. 1. Differentiating with respect to time,

$$\frac{dP}{dt} = E\epsilon_m \frac{d\epsilon_m}{dt} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad [10]$$

But from Appendix No. 1 the loss per unit volume per cycle $= \pi\eta E\epsilon_m^2$ which gives a loss per unit volume per sec. of

$$\pi\eta E\epsilon_m^2 f \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad [11]$$

This must be equal to the rate of change of the potential energy per second as given in [10]. Thus equating [11] to [10]:

$$\pi\eta f = \frac{\frac{d\epsilon_m}{dt}}{\epsilon_m}$$

or since

$$\xi = \frac{\pi\eta}{E}$$

then

$$\xi E f = \frac{\frac{d\epsilon_m}{dt}}{\epsilon_m}$$

which gives

$$\xi = \frac{1}{Ef}\left(\frac{\frac{d\epsilon_m}{dt}}{\epsilon_m}\right) \quad \ldots\ldots\ldots\ldots \quad [12]$$

but

$$\frac{\frac{dy}{dt}}{y} \quad \frac{\frac{d\epsilon_m}{dt}}{\epsilon_m}$$

evidently, so that

$$\xi = \frac{1}{Ef}\left(\frac{\frac{dy}{dt}}{y\cdot}\right) \quad \ldots\ldots\ldots\ldots \quad [13]$$

BIBLIOGRAPHY

(1) The Protection of Steam-Turbine Disk Wheels from Axial Vibration. W. Campbell. Trans., A.S.M.E., 1924, pp. 79-80.

(2) A. L. Kimball, Internal Friction as a Cause of Shaft Whirling. *Phil. Mag.*, vol. 49, Apr. 1925, pp. 724-727. Internal Friction Theory of Shaft Whirling. *G. E. Rev.*, vol. 27, Apr. 1924, pp. 244-251.

(3) A. L. Kimball, Measurement of Internal Friction in a Revolving Deflected Shaft. *G. E. Rev.*, vol. 28, Aug. 1925, pp. 554-558.

(4) See Honda and Konno, *Phil. Mag.*, vol. 42, 1921, p. 115; Iokibe and Kakai, *Phil. Mag.*, vol. 42, 1921, p. 397; Lesch, Zeit. angew. Math. und Mech., vol. 4, 1924, p. 124; C. E. Guye, *Journal de Phys.*, 1912, p. 620. See also Ibbetson, Mathematical Theory of Elasticity, p. 175 (Macmillan, 1887); Prescott, Applied Elasticity, p. 44 (Longmans, 1924).

(5) See, for instance, Hopkinson and Williams, The Elastic Hysteresis of Steel, Roy. Soc. Proc. A, vol. 87, (1912), pp. 502-511; F. E. Rowett, Elastic Hysteresis in Steel, Roy. Soc. Proc. A, vol. 89 (1914), pp. 528-543; K. Bennewitz, *Phys. Zeit.*, vol. 21 (1920), p. 703; K. Bennewitz, *Phys. Zeit.*, vol. 25 (1924), pp. 417-431; H. Jordan, Deutsch. Phys., vol. 18 (1915), p. 423. See also W. E. Dalby, Phil. Trans., Roy Soc., Series A, vol. 221 (1920), pp. 117-138.

(6) S. L. Quimby, *Phys. Rev.*, vol. 25, April 1925, pp. 528-573.

(7) A similar method was used by Wöhler in investigations of fatigue failures, but not for the study of internal friction arising from strains much below the yield point of the material. See article by W. Mason in *Engineering*, vol. 115 (1923), pp. 698-699. Professor Mason gives an interesting mathematical discussion of the Wöhler bar test in this article.

DISCUSSION

H. F. Moore[1] and N. P. Inglis.[2] The authors have proposed a form of equation which seems to be readily applicable to the study of the dying out of vibrations in a member. Reference (7) in the paper is evidently to the work of Professor Mason at the University of Liverpool. In his tests of energy loss in a specimen

[1] Research Professor of Engineering Materials, University of Illinois, Urbana, Ill. Mem. A.S.M.E.

[2] Walker Engineering Laboratories, University of Liverpool, England. Holder of Robert Blair Traveling Fellowship in Applied Science and Technology.

under cycles of stress, Mason secured sensitivity, not by the use of a long, slender specimen, but by the use of the optical lever principle to measure deflections, horizontal and vertical. He used a rotating specimen subjected to a constant bending moment over a considerable portion of its length, and took extreme precautions to insure freedom from vibration while the specimen was running.

The materials tested by Mason included 0.37 per cent carbon steel and 0.13 per cent carbon steel. He was primarily interested in the *energy* lost in mechanical hysteresis, and made no attempt to derive any expression for the coefficient of internal friction. He did not find evidence of hysteresis loss at low stresses, but other English investigators, especially Guest and Lea,[1] did find evidence of such loss at very low stresses. Probably there is some loss under low stresses, but its detection is a matter of sensitivity of apparatus, and its magnitude may change after many thousands of cycles of stress.

An interesting question of terminology is raised by the study of mechanical hysteresis under stress. There may be hysteresis developed during a cycle of stress, after which there is no appreciable permanent set. This is illustrated strikingly by tension tests on rubber. By the usual definition, such a cycle involves no inelastic action — no permanent change of form — yet there is loss of energy and production of heat. Should we revise our definition of elastic action?

The statement in Par. 5 "that forces produced by friction within the rod are independent of the velocity of strain, and, for a given amplitude of strain, the frictional loss per cycle is independent of the speed of performance of the stress cycle" is, as the authors note, a confirmation of the work of other investigators. Especially is this true of the work of Rowett,[2] who found that the area of the hysteresis loop (i.e., the work lost per cycle depends only on the range of stress and not on the speed of reversal. The authors' conclusion that monel metal and nickel, although very ductile, show little "frictional loss," raises the intriguing question of the part ductility plays in these and in other similar tests.

To the writers, the note in Par. 21 on the significance of internal friction in connection with fatigue failures of metals is of chief interest. In the tests of Mason, and of others, an attempt was made to locate the fatigue limit by means of a sudden increase in hysteresis or internal friction which was supposed to occur at the fatigue limit. The apparatus used in Mason's tests was designed to make "running-deflection" tests, i.e., tests in which rotating-beam specimens were run for several thousand cycles at each of a series of loads, the deflection of the running specimen under each load being carefully measured. The stress at which the

[1] Proc. Royal Society, A, vol. 93 (1916-1917).
[2] See reference (5) in the bibliography.

load-deflection diagram departed from a straight line has been found by some investigators to agree fairly well with the fatigue limit for a number of ferrous metals.[1] Later tests, especially those on cold-drawn non-ferrous metals,[2] throw doubt on the general reliability of this "running deflection" test for endurance limit. In the tests by Mason for both the 0.37 per cent carbon steel and the 0.13 per cent carbon steel the limit of proportionality under static load was found to be appreciably higher than the fatigue

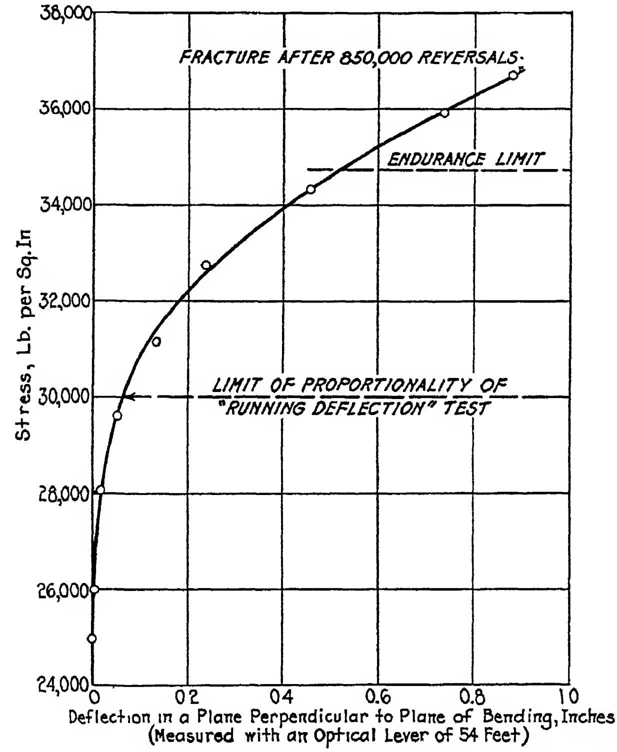


FIG. 14 RUNNING DEFLECTION TEST ON 0.13 CARBON STEEL

limit as determined by tests to destruction, and the departure of the "load — running deflection" diagram was found to occur at a stress lower than the fatigue limit. The appearance of lateral deflection, indicating appreciable internal friction, was at even a slightly lower stress. Fig. 14 gives a plot of values of this lateral

[1] H. J. Gough, Improved Methods of Fatigue Testing, *Engineer* (London), Aug. 12, 1921.

[2] Bulletin 152, Engineering Experiment Station, University of Illinois, p. 61.

deflection for the 0.13 per cent carbon steel, and it is seen that there is no definite " break " in the curve at the fatigue limit as determined by tests to fracture.[1] The rise-of-temperature test used at Illinois is another test, which, like the " running deflection " test, shows the occurrence of marked slip in the specimen. This test while giving fairly good results for the fatigue limit of rolled ferrous metals, was not found satisfactory for non-ferrous metals. For cold-worked non-ferrous metals the rise-of-temperature test gave values *above* the values determined for fatigue limit by tests to fracture.

As the authors suggest, there may be, for some materials, a good correlation between *yield point* of a metal, and the " break " of the lateral deflection curve, but in several laboratories it has been found that there is little correlation between elastic limit or yield point on the one hand, and fatigue limit on the other. Elastic limit or yield point denotes the beginning of widely distributed *slip* in a metal; fatigue limit denotes the beginning of a spreading *fracture* — perhaps a highly localized fracture. While present knowledge of fatigue phenomena in metals is not sufficient to justify dogmatic statements, yet the writers see little probability of an accurate determination of fatigue strength from any kind of a load-deflection diagram for a specimen.

T. McLean Jasper.[2] The authors touch on a subject which, the writer believes, is closely allied to rate of stress propagation, internal stress relief, and to the gyroscopic action of bodies under bending stresses. From a close reading of references (2) and (3) in the bibliography, the writer has been unable to satisfy himself that the methods used result in the measuring of the internal friction of the various metals tested. The writer suggests that with rods of the identical materials but of much thicker diameter, with the same length and loading conditions, the so-called internal friction constants might be materially changed.

This reasoning is partially based upon some static results obtained by the writer on the elastic constants of materials, as compared to the same elastic constants obtained dynamically and also upon the report of Professor Jenkins[3] on fatigue limits of steel, using a very high frequency of stress alternations. The endurance

[1] At high stresses the lateral deflection tends to increase with an increase of the number of cycles of stress, but usually attains a constant value after about 2,000,000 cycles of stress — if the specimen lasts that long. The total amount of work absorbed during a fatigue test may be calculated from these measurements and has been found to increase as the range of stress is reduced and apparently may be infinite when the range is lower than the fatigue limit.

[2] Director of Research, A. O. Smith Corpn., Milwaukee, Wis. Mem. A.S.M.E.

[3] Of the University of Oxford. Discussed at the last Toronto meeting of the British Association.

limits in this case were very much higher than those found at low frequencies. The reasoning also is based partially on the fact that elastic hysteresis under repeated stress varies materially from the first cycle up to the hundred millions cycle. Much of this variation has been attributed to the relief of internal stress. Fatigue hysteresis much below the endurance limit shows conclusively after the hundred million cycle that the value of the energy of the loop becomes almost negligible as compared to this value when such a test is started. Whether or not such a condition indicates internal friction or gradual relief of internal stress, the writer is not prepared to suggest, but due to the fact that internal stresses are unquestionably relieved by repeated stressing below the endurance limit he has attributed this phenomenon largely to stress relief.

The writer also believes that rate of stress propagation, internal stress relief, and gyroscopic effects on bent specimens may have some bearing on the authors' method of measuring internal friction within the elastic range of materials. This, it is suggested, can be partially determined by changing the relative volume of the specimens used.

In the determination of fatigue limits in various steels by the rise-of-temperature method the point of disproportional increase of the temperature seemed to mark the endurance limit. The point at which internal friction (although below the elastic limit of the steel) became important in this case indicated the endurance limit, as obtained in the fatigue machine, with a good degree of precision.

It is suggested that the authors' vibration tests representing the number of vibrations for a given amplitude in Figs. 10, 11, and 12 should give values, when considered in conjunction with the appropriate elastic constants, which bear some relation to the internal friction constants shown in Table 1, if the internal friction constants are correct. This does not seem to be the case. This, of course, assumes specimens of comparable size. It is noted that all of the materials represented in these figures are cold-rolled, which suggests fairly high initial internal stresses.

J. B. Kommers.[1] The writer wishes to call the attention of the authors to a paper by Prof. W. Mason entitled, The Mechanics of the Wöhler Rotating Bar Fatigue Test.[2] This paper discusses the hysteresis effect in producing lateral deflection, and suggests

[1] Associate Professor of Mechanics, University of Wisconsin, Madison, Wis.

[2] Reports of the British Advisory Committee for Aeronautics, vol. 2, 1922-1923, p. 421, and *Engineering*, June 1, 1923, p. 698.

the experiment of measuring the lateral deflection, which the present authors have actually carried out.

For those to whom the explanation in Par. 4 in the present paper is not very clear, the following picture of what is going on in a rotating specimen is suggested. Due to mechanical hysteresis in an axial stress experiment, the deformation in tension is not zero even when the tensile load has been reduced to zero, but becomes zero only after a certain amount of compressive load has been applied. A consideration of a tension-compression hysteresis loop will make this obvious. In a similar manner, in Fig. 4 of the present paper, when a tension fiber has reached the point of C_F it still has tensile deformation in it and will continue to have it until part of the quadrant from *CF* to *CE* has been traversed. For the same reason when a compression fiber reaches T_F it will still have some compression deformation in it and will continue to have it until part of the quadrant from T_F to T_E has been traversed. For this reason the neutral axis, instead of being horizontal in Fig. 4, makes an angle with the horizontal, so that the axis falls in the second and fourth quadrants. It is bending about this neutral axis which causes a vertical and also a lateral deflection.

The authors are to be congratulated on the evidence which they have presented tending to show that the internal friction forces in a material do not depend upon the speed of application. This also helps to explain why the effects of speed in a fatigue test seem to be negligible.

S. TIMOSHENKO.[1] The phenomenon of lateral deflection was explained by Dr. Mason in his paper published in *Engineering* in 1923, wherein he connected lateral deflection with internal friction or hysteresis. Other experimenters also have been interested in this question. In Germany, Bennewitz published a paper in 1920, basing his conclusions on the same assumption as the authors, that the hysteresis loops for various loads are geometrically similar. In such a case the loss of energy will be proportional to the square of the load. Schenck also adopted the same method, employing a special machine which uses the measurement of lost energy for predicting the endurance limit. Schenck has compiled[2] special tables for the lost energy of various materials. It would be interesting to compare his results with those of the authors.

THE AUTHORS. The paper by Guest and Lea, mentioned in the discussion by Professors Moore and Inglis, was read by the authors

[1] Research Engineer, Westinghouse Electric & Mfg Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[2] Published by Lerthe in Stuttgart.

and is one of the very large number of papers relating to this subject which were not included in the bibliography. The authors are familiar with the work of Mason which is mentioned, both by Professors Moore and Inglis and by Professor Kommers. This work was first called to the writer's attention at the British Association Meeting at the University of Toronto in the summer of 1924, and it was at once recognized that his method was similar to that used by the authors of this paper, although the measurements were not made with the purpose of determining internal friction below the elastic limit.

It should be mentioned again that this method was independently arrived at by one of the authors through experiments on vibrating shafts, as stated in the first paragraph of the paper, and the characteristic of frictional force there mentioned was revealed by such revolving rod experiments in the year 1922, some four years before the publication of this paper.

Professor Jasper has found difficulty in conceiving how a transverse deflection will give a quantitative measurement of internal friction. This is a point which is difficult to see at first. A good way to study this phenomenon is to make an experimental shaft which has an excessive amount of internal viscosity introduced artificially; for instance, a rod may be made up of a bundle of wires which rub against each other when the rod is deflected. It will at once be seen when the rod is revolving with one end overhanging, that a transverse deflection results which depends upon the amount of friction. As to Mr. Jasper's suggestion about using a larger rod, that is a very good one and we are now planning to make such experiments. The authors are aware that hysteresis diminishes with the number of cycles when the stress amplitude is below the endurance limit, but in our experiments this change took place in the first few cycles and remained constant for 100,000 cycles in several materials. Regarding the vibration tests mentioned in the latter part of the paper, we do not regard these as particularly useful because of the fact that there were necessarily considerable losses and it would perhaps have been better to omit these few paragraphs.

Since the writing of this paper, it has been found that Equation [**13**] reduces to δ/E where δ equals the logarithmic decrement of the vibration; that is, the $\log_e$ of the ratio of the larger to the smaller of any two successive amplitudes during the decreasing vibration on the basis of the assumed law of Equation [1] which requires a logarithmic decrement. This ratio must necessarily be constant.

Professor Kommers' way of explaining why the deflection angle ϕ is a measure of the internal friction loss is very interesting.

Dr. Timoshenko mentioned the fact that a similar law of internal friction loss is proposed by Bennewitz. This work of Bennewitz is referred to in the bibliography of our paper as representing one of the few which show appreciation of the true nature of hysteresis in solids. His mathematical analysis is interesting, but the authors have been unable to make it check experimentally for such a substance as celluloid, although it does appear to work for many other materials.

No. 2017

WORM-WHEEL CONTACT

PRELIMINARY REPORT OF A.S.M.E. SPECIAL RESEARCH COMMITTEE ON WORM GEARS[1]

By Earle Buckingham,[2] Cambridge, Mass.
Member of the Society

The object of this paper is to show how any worm-wheel contact condition can be determined by analysis and to point out in particular the probable influence of the nature of the contact lines between a worm and a worm wheel upon the lubrication conditions, efficiency, and load-carrying ability. Analyses of three helicoids are made and their equations given. The first of these, a convolute helicoid, had its generatrix tangent to a cylinder of any diameter concentric with the axis of the helicoid. The second, a screw helicoid, had its generatrix passing through the axis of the helicoid, and was one limiting case of the first, with the diameter of the base cylinder reduced to zero. The third, an involute helicoid, had its generatrix tangent to a concentric cylinder of such diameter that the helix angle at this diameter was the same as the angle of the generatrix with a plane perpendicular to the axis. The conjugate action of racks is discussed and equations given. An analysis of worm contact is made. Contact lines of screw helicoids used as worms and those of involute helicoids used as worms are discussed. Contact lines of screw helicoids with large helix angles, and involute helicoids with large helix angles also are treated.

[1] The organization of an A.S.M.E. Special Research Committee on Worm Gears was authorized by the Council in September, 1926, and the A.S.M.E. Main Research Committee has appointed Professor Buckingham as its Chairman. While Chairman Buckingham has used the method described in his report with success on two or three critical drives, he presents it not as a recommended practice to be followed in design, but as a subject for investigation. It is hoped that the discussion of the report will bring to light a large amount of comparative data which can be used by the Committee in its further study of this subject.

[2] Associate Professor, Standardization and Measurement, Massachusetts Institute of Technology.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

THE problem of worm drives is a very perplexing one. Despite their widespread use for a long period of years, data in regard to their efficiencies and load-carrying abilities are very incomplete and often contradictory. Even in the best text books on machine elements, this subject is disposed of very briefly. The periodic literature on this subject offers but little useful information, as the contributions are usually brief and seem to bear little relation to each other.

2 Very little study of this problem makes apparent the difficulties that exist in analyzing the contact conditions between a worm and a worm wheel. The warped surfaces of the worm are not the simplest to deal with. In addition to this, the surfaces of the teeth of the worm wheel, which must be conjugate to the warped surface of the worm, introduce further complications.

3 Practically all previous analyses of these surfaces have considered only the conjugate action between parallel sections of the worm, treating them as racks driving a spur gear. Furthermore, the forms of these sections have usually been established by descriptive geometry, which requires careful and painstaking draftsmanship to obtain accurate results. Notable among such contributions have been Ernst's Treatise on Worm Drives, published in Berlin in 1901, and Robert A. Bruce's paper on Worm Contact, presented before the Institution of Mechanical Engineers in London in 1906. In addition, several manufacturers of worm gears, mostly in England, have conducted extensive experimental work and have developed special designs for such gears, yet very little fundamental information on this subject has been made public.

4 The American Society of Mechanical Engineers has recently organized a Special Research Committee on Worm Gears to study this problem and to have tests made under its supervision to obtain reliable experimental data if possible. The first problem confronting this committee is the selection of a rational program for such study and tests. The purpose of this paper is to present what is believed to be a somewhat novel viewpoint of this problem in the hope of concentrating attention upon what is believed to be one of the most vital factors involved, and of arousing sufficient interest in the whole project to insure the necessary support, including coöperation and engineering assistance, exchange of existing data, financial assistance, and samples for test from the industries which are directly interested. In other words, it may be considered as a prospectus or preliminary report of the A.S.M.E. Special Committee on Worm Gears.

5 In essence, a worm drive is nothing more than a thrust bearing, complicated by the helicoidal surface of the worm, and having line contact instead of surface contact, thus introducing very high pressures and stresses between the contacting lines or minute surfaces.

6 Worm drives are very often inefficient. The loss of power is caused by friction. The effects of friction are reduced by lubrication. Therefore, it seems logical to attack this problem of worm drives as a problem of lubrication. The first step toward this end

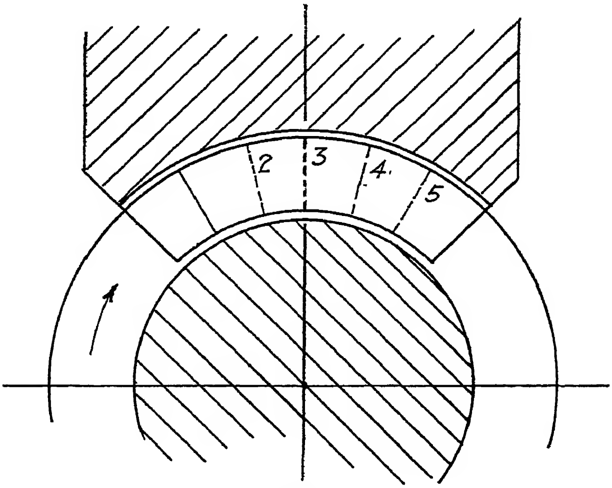


Fig. 1

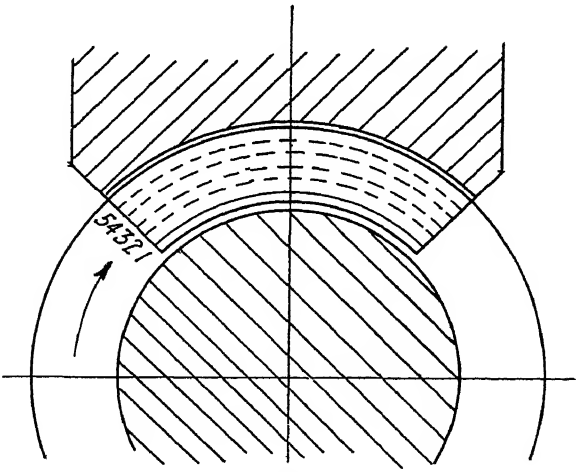


Fig. 2

is to determine the nature of the contact between a worm and a worm wheel as it affects the lubrication. Most of the sliding action in these drives is caused by the rotation of the worm. This sliding is so much in excess of the sliding caused by the conjugate action on the tooth profiles that it would seem as though this conjugate action could be safely neglected, for the present at least.

7 As regards lubrication, the ideal contact conditions would be to have the contact lines between the worm and the worm wheel as radial lines on the worm, as shown in Fig. 1. This would result in a condition similar to that on a Kingsbury thrust bearing, where a wedge of lubricant is carried along just ahead of the contact line, and this contact line would extend across the worm tooth or thread from top to bottom, thus insuring practically uniform wear over the entire tooth profiles.

8 As regards lubrication, the poorest contact conditions would be to have the contact lines between the worm and the worm wheel as arcs of circles concentric with the worm, as shown in Fig. 2. Here the lubricant would have but little chance of being renewed as the contact progressed from the bottom to the top of the worm tooth. In addition, the worm tooth would be subject to local wear, and thus tend to lose its form.

9 As regards the specific stresses in the material set up by such line contact, the conditions shown in Fig. 1 would be much more favorable than those set up by the conditions shown in Fig. 2. In Fig. 1, the line contact would be along a profile with a relatively small radius of curvature, while the point contact in a section perpendicular to the contact line would be between surfaces whose radii of curvature would be relatively large, depending upon the thread angle and the helix angle, often amounting to as much as sixteen or twenty times as large as the average radius of curvature of the tooth profiles. In Fig. 2, on the other hand, the line contact would be along a large radius of curvature, while the point contact would be between small radii of curvature. Thus with equal lengths of contact lines in both cases, a worm drive with the conditions shown in Fig. 1 would have a load-carrying ability about sixteen times as great as a worm drive with the conditions shown in Fig. 2. The length of the contact line under the conditions shown in Fig. 1, however, would only be about one-quarter the length of the contact line under the conditions shown in Fig. 2. Even so, its load-carrying ability would still be about four times as great, together with better conditions of lubrication. We shall therefore assume the contact conditions shown in Fig. 1 to be the most desirable ones, and use them as a basis of comparison.

10 The next step is to find some method of determining the forms and positions of these contact lines between a worm and a worm wheel. We must now direct our attention to the warped surfaces of the worm, or its helicoidal surfaces. These surfaces are, in effect, warped cones.

Helicoidal Sections

11 The general mathematical properties of most regular solids have been long established. The helicoidal, or screw form, has received but little attention, however. This has been due, no doubt, to the slight relative importance of these surfaces mathematically, as well as to the difficulties of analyzing such warped surfaces.

12 In mechanical work, however, these forms are met in many places. Screws must be made and measured. Helical gears consist of helicoidal surfaces, as do also the hobs often used to produce them. The tooth surfaces of worms also are such surfaces. The relieved surfaces of any backed-off milling cutter also will be found to be helicoidal surfaces. Examples could be multiplied indefinitely.

13 Some years ago, as a matter of "mental gymnastics," the author worked out the equations of the intersection curves and their tangents of certain helicoidal surfaces with planes in various positions, as has been done with a cone in conic sections. All of these helicoids had straight-line generatrices and all of the intersection curves were spirals, corresponding in many respects to the equivalent sections on a cone. The spiral hyperbolas had their asymptotes, etc.

14 Three helicoids were analyzed. The first had its generatrix tangent to a cylinder of any diameter concentric with the axis of the helicoid. Such a surface would be produced by setting a threading tool with straight cutting edges above or below center, or by tipping the tool off center. This, for want of a better name, will be called a convolute helicoid.

15 The second had its generatrix passing through the axis of the helicoid, and was one limiting case of the first, with the diameter of the base cylinder reduced to zero. This is the common screw thread form and will be called the screw helicoid.

16 The third had its generatrix tangent to a concentric cylinder of such a diameter that the helix angle at this diameter was the same as the angle of the generatrix with a plane perpendicular to the axis. Or, in other words, the generatrix was a continuation of the helix on the base cylinder. This is another specific form of the first. It is also the form of an involute helical gear and will therefore be called an involute helicoid.

17 *Convolute Helicoid.* The following notation will be used throughout, with additional symbols listed as required:

L = lead of the generatrix, or the distance it advances along the axis of the helicoid in one revolution

r = any radius of the helicoidal surface; also the length of the radius vector in all polar equations

γ = angle between the generatrix and a plane perpendicular to the axis of the helicoid

δ = angle between the tangent to the helix at radius r and a plane perpendicular to the axis — often called the helix angle or lead angle.
θ = vectorial angle
ε = angle of rotation of generatrix
a = radius of base cylinder to which generatrix is tangent
D = distance from axis of helicoid to intersecting plane.

18 The inclination of the generatrix may be in the same direction as the helix on the base cylinder or it may be in the opposite direction. Here we shall consider only the case where this inclination is in the same direction as the helix on the base cylinder.

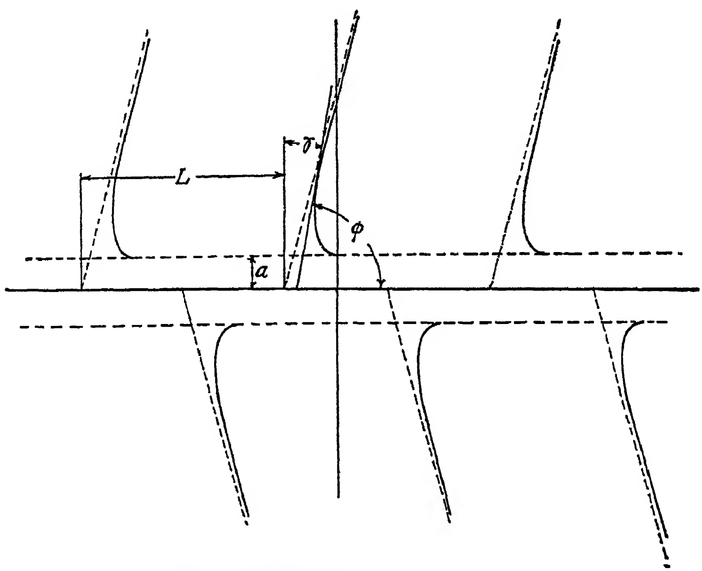


Fig. 3 Axial Intersection of Convolute Helicoid

19 The equation of the intersection curve of the convolute helicoid with a plane that contains the axis is

$$x = \tan\gamma\sqrt{y^2-a^2} - \frac{L}{2\pi}\arctan\sqrt{\frac{y^2-a^2}{a^2}} \quad . \; . \; . \; . \quad [1]$$

The equation of the tangent to this intersection curve is

$$\tan\phi = \frac{dy}{dx} = \frac{2\pi y\sqrt{y^2-a^2}}{2\pi y^2\tan\gamma - aL} \quad . \; . \; . \; . \; . \; . \quad [2]$$

The form of this intersection curve is shown in Fig. 3.

20 The equation of the intersection curve of the convolute helicoid with a plane perpendicular to its axis is as follows:

$$\theta = \frac{2\pi \tan \gamma}{L} \sqrt{r^2 - a^2} - \arctan \sqrt{\frac{r^2 - a^2}{a^2}} \quad . . . [3]$$

The equation of the tangent to this intersection curve is

$$\tan \psi = \frac{r d\theta}{dr} = \frac{2\pi r^2 \tan \gamma - aL}{L\sqrt{r^2 - a^2}} \quad . . . . [4]$$

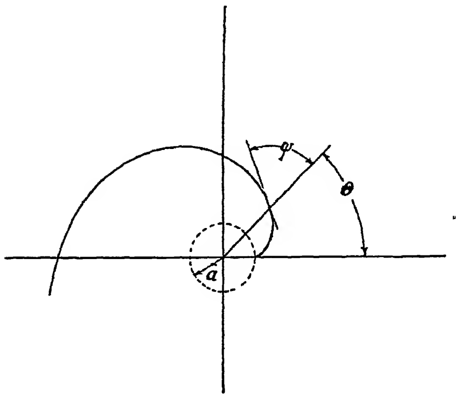


FIG. 4 INTERSECTION OF CONVOLUTE HELICOID WITH PLANE PERPENDICULAR TO AXIS

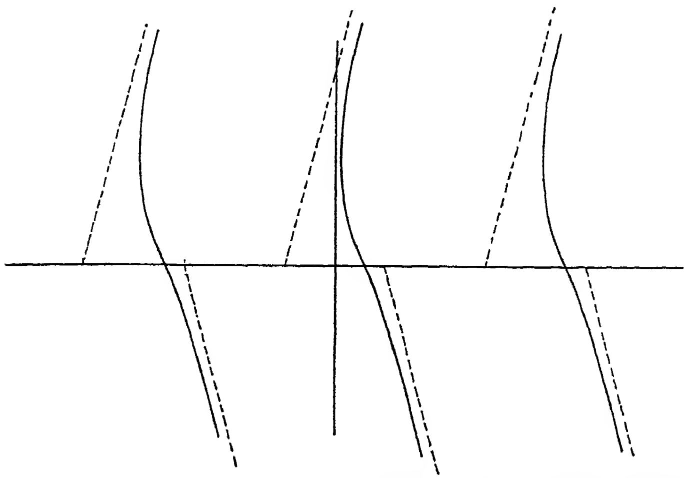


FIG. 5 INTERSECTION OF CONVOLUTE HELICOID WITH PLANE PARALLEL TO AXIS

The form of this intersection curve is shown in Fig. 4.

21 The equation of the intersection curve of the convolute helicoid with a plane parallel to the axis and at a distance D from the axis is

$$x = \tan\gamma\sqrt{D^2-a^2+y^2} - \frac{L}{2\pi}\arccos\frac{ay \pm D\sqrt{D^2-a^2+y^2}}{D^2+y^2} \quad [5]$$

The equation of the tangent to this intersection curve is

$$\tan\phi = \frac{dy}{dx} = \frac{2\pi(D^2+y^2)\sqrt{D^2-a^2+y^2}}{2\pi y\tan\gamma(D^2+y^2)-L(ay\pm D\sqrt{D^2-a^2+y^2})} \quad [6]$$

The form of this intersection curve is shown in Fig 5.

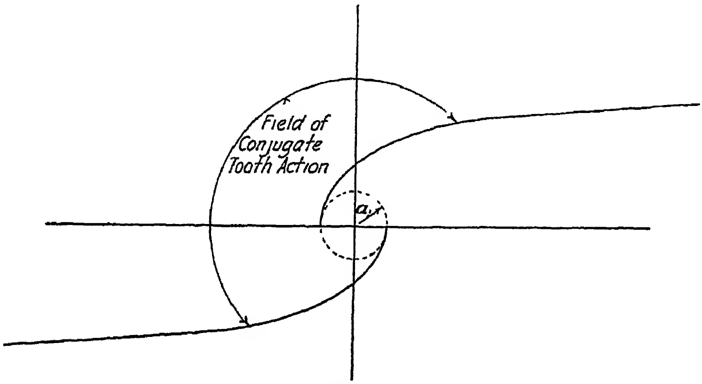


Fig. 6 Limits of Conjugate Tooth Action on Convolute Helicoid

22 The limit of conjugate tooth action on these helicoidal surfaces is reached when the tangent to the intersection curve of the helicoid with a plane parallel to the axis is equal to infinity. This would give a rack form with a pressure angle of zero degrees. This limit of conjugate tooth action would be established by the projection of the points where $\frac{dy}{dx} = \infty$ on a plane perpendicular to the axis of the helicoid. The projection of these points may be determined by the following equations:

$$\sin\varepsilon' = \frac{-\pi D\tan\gamma \pm \sqrt{(\pi D\tan\gamma)^2-2\pi aL\tan\gamma+L^2}}{L} \quad [7]$$

$$x = D \quad [8]$$

$$y = \frac{D\sin\varepsilon'+a}{\cos\varepsilon'} = \frac{x\sin\varepsilon'+a}{\cos\varepsilon'} \quad [9]$$

This curve is plotted in Fig. 6.

23 *Screw Helicoid.* The screw helicoid, as defined before, has a generatrix which passes through the axis of the helicoid. Thus it is

one limiting example of the convolute helicoid with the size of the base cylinder reduced to zero.

24 The intersection curve of the screw helicoid with a plane that contains the axis will represent the successive positions of the generatrix 180 degrees apart. Substituting the value of $a = 0$ in Equation [1], we get the equation of this intersection curve as follows:

$$x = y \tan \gamma - \frac{L}{2\pi} \arctan \infty \quad \ldots \ldots \quad [10]$$

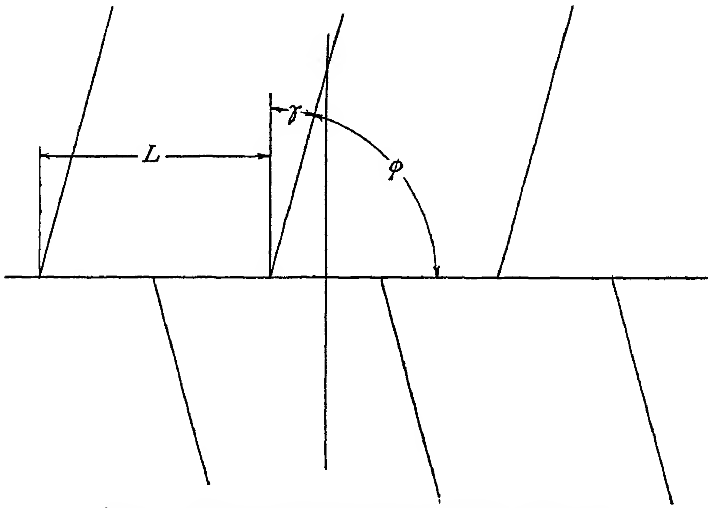


FIG. 7 AXIAL INTERSECTION OF SCREW HELICOID

25 The arc whose tangent is infinity may be $\pi/2$, $3\pi/2$, $5\pi/2$, etc. This curve is composed of a series of straight lines whose tangent is given by the following equation:

$$\tan \phi = \frac{dy}{dx} = \frac{1}{\tan \gamma} \quad \ldots \ldots \quad [11]$$

This intersection curve is shown in Fig. 7.

26 The equation of the intersection curve of the screw helicoid with a plane perpendicular to the axis is

$$\theta = \frac{2\pi r \tan \gamma}{L} \quad \ldots \ldots \quad [12]$$

This curve is an Archimedes spiral. The equation of its tangent is

$$\tan \psi = \frac{r d\theta}{dr} = \frac{2\pi r \tan \gamma}{L} \quad \ldots \ldots \quad [13]$$

But $\frac{2\pi r}{L}$ = the cotangent of the helix angle at radius r or $\frac{1}{\tan \delta}$,

Whence

$$\tan\psi = \frac{\tan\gamma}{\tan\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [14]$$

The form of this intersection curve is shown in Fig. 8.

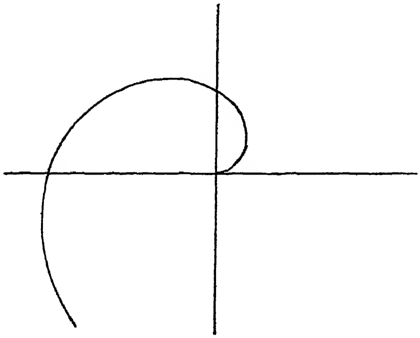


FIG. 8 INTERSECTION OF SCREW HELICOID WITH PLANE PERPENDICULAR TO AXIS

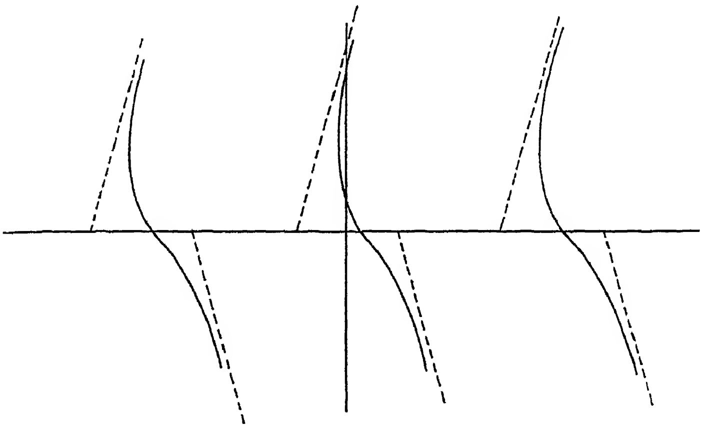


FIG. 9 INTERSECTION OF SCREW HELICOID WITH PLANE PARALLEL TO AXIS

27 The equation of the intersection curve of the screw helicoid with a plane parallel to the axis at a distance D from the axis is

$$x = \tan\gamma\sqrt{D^2+y^2} - \frac{L}{2\pi}\arctan\frac{y}{D} \quad . \quad . \quad . \quad . \quad [15]$$

The equation of the tangent to this intersection curve is

$$\tan\phi = \frac{dy}{dx} = \frac{2\pi(D^2+y^2)}{2\pi y\tan\gamma\sqrt{D^2+y^2}-LD} \quad . \quad . \quad . \quad . \quad [16]$$

The form of this intersection curve is shown in Fig. 9.

28 The projection of the points where $\frac{dy}{dx} = \infty$, or the limit of conjugate action, on a screw helicoid, on a plane perpendicular to the axis may be plotted by means of the following equations:

$$\sin \varepsilon' = \frac{-\pi D \tan \gamma \pm \sqrt{(\pi D \tan \gamma)^2 + L^2}}{L} \quad . \quad . \quad . \quad [17]$$

$$x = D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [18]$$

$$y = D \tan \varepsilon' = x \tan \varepsilon' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [19]$$

This curve is plotted in Fig. 10.

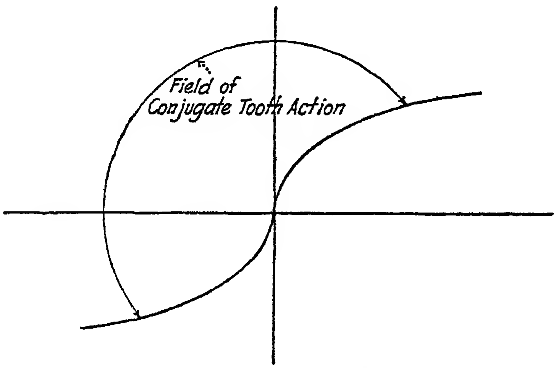


FIG. 10 LIMITS OF CONJUGATE TOOTH ACTION ON SCREW HELICOID

29 *Involute Helicoid.* The involute helicoid, as defined before, is one whose base cylinder is of such diameter that the helix angle there is equal to the angle of inclination of the generatrix. This is another limited example of the convolute helicoid where $\tan \gamma = \frac{L}{2\pi a}$. The equations of the intersection curves of the involute helicoid may be determined by substituting this value of $\tan \gamma$ into the various equations of the convolute helicoid.

30 The equation of the intersection curve of the involute helicoid with a plane that contains the axis is as follows:

$$x = \frac{L}{2\pi}\left[\frac{\sqrt{y^2-a^2}}{a} - \arctan \frac{\sqrt{y^2-a^2}}{a}\right] \quad . \quad . \quad . \quad [20]$$

The equation of the tangent to this intersection curve is

$$\tan \phi = \frac{dy}{dx} = \frac{2\pi a y}{L\sqrt{y^2-a^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad [21]$$

The form of this intersection curve is shown in Fig. 11.

31 The equation of the intersection curve of the involute helicoid with a plane perpendicular to the axis is

$$\theta = \frac{\sqrt{r^2-a^2}}{a} - \arctan \frac{\sqrt{r^2-a^2}}{a} \quad . \; . \; . \; . \; . \quad [22]$$

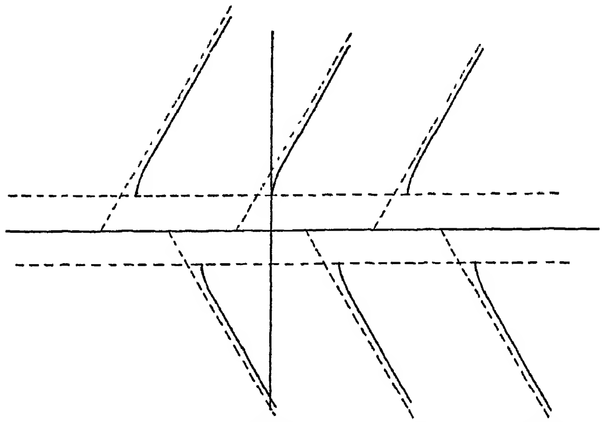


FIG. 11 AXIAL INTERSECTION OF INVOLUTE HELICOID

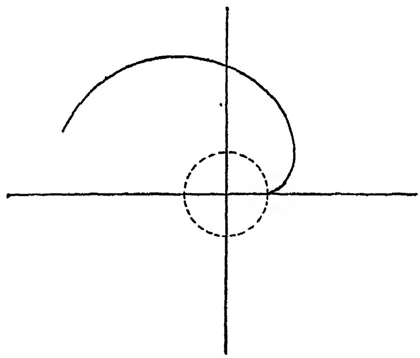


FIG. 12 INTERSECTION OF INVOLUTE HELICOID WITH PLANE PERPENDICULAR TO AXIS

This is the polar equation of the involute which is a uniform rise spiral. The equation of the tangent to this curve is

$$\tan \psi = \frac{r d\theta}{dr} = \frac{\sqrt{r^2-a^2}}{a} \quad . \; . \; . \; . \; . \; . \quad [23]$$

The form of this intersection curve is shown in Fig. 12.

32 The equation of the intersection curve of the involute helicoid with a plane parallel to the axis and at a distance D from the axis is as follows:

$$x = \frac{L}{2\pi}\left[\frac{\sqrt{D^2-a^2+y^2}}{a} - \arccos\frac{ay \pm D\sqrt{D^2-a^2+y^2}}{D^2+y^2}\right]. \quad [24]$$

The equation of the tangent to this intersection curve is

$$\tan\phi = \frac{dy}{dx} = \frac{2\pi a(D^2+y^2)}{L(-aD \pm y\sqrt{D^2-a^2+y^2})} \quad . \; . \; . \quad [25]$$

This intersection curve is shown in Fig. 13.

33 The projection of the points where $\frac{dy}{dx} = \infty$, or the limit of conjugate tooth action on an involute, on a plane perpendicular

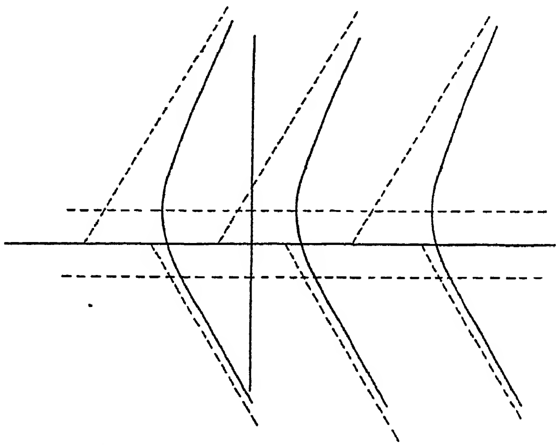


Fig. 13 Intersection of Involute Helicoid with Plane Parallel to Axis

to the axis may be plotted by means of the following equations:

$$\sin \varepsilon' = 0$$

$$x = D$$

$$y = \frac{D \sin \varepsilon' + a}{\cos \varepsilon'} = a \quad . \; . \; . \; . \; . \; . \; . \; . \quad [26]$$

This curve, which is a straight-line tangent to the base cylinder, is shown in Fig. 14.

Conjugate Action of Racks

34 As noted in the introduction, the most common method of analyzing the action between a worm and worm wheel is to study the conjugate tooth action of several parallel sections of the worm with corresponding sections of the worm wheel. The determination

of both the resulting rack profiles on the sections of the worm and also their lines of conjugate action is ordinarily accomplished by geometric layouts.

35 The preceding equations of helicoidal sections enable these rack profiles to be determined to any required degree of accuracy by analysis without the necessity of making enlarged and accurate layouts. The equation for the line of conjugate action is readily determined and may also be used instead of layouts when the equation of the rack profile is known. These equations for the line of conjugate action are as follows:

When x = abscissa of profile
y = ordinate of profile measured from axis of helicoid
R = distance from axis of helicoid to pitch plane of worm (or pitch radius of worm)
ϕ = tangent to profile measured from the axis of x
x' = abscissa of line of conjugate action
y' = ordinate of line of action measured from the pitch line

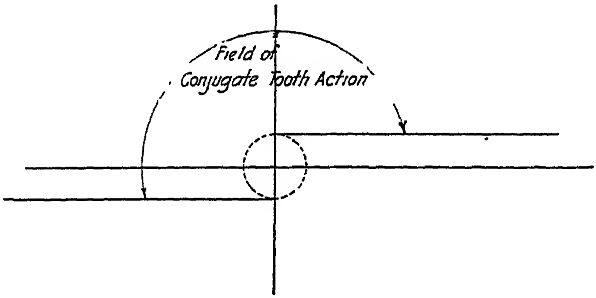


Fig. 14 Limits of Conjugate Tooth Action on Involute Helicoid

We know that the normal to the tooth profiles at the point of contact must pass through the pitch point. Thus when the pitch point is the origin of the coödinate system, we have

$$y' = y - R \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [27]$$

$$x' = -y' \tan \phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [28]$$

Analysis of Worm Contact

36 As noted in the introduction, in many respects the conjugate tooth action between a worm and worm wheel is of secondary importance. Such action must exist, of course, but its exact nature has but little influence on the conditions of lubrication. It would seem to be of more importance to determine the positions of the actual contact lines between the worm and worm wheel at various positions to see whether these lines approach the conditions shown in Fig. 1 or those in Fig. 2.

37 There are three ways in which this may be accomplished: First, by determining the profiles of several parallel sections of the worm and their lines of conjugate action by geometric layouts, then using these layouts to determine the projection of the actual contact line on an end section of the worm. Second, by plotting the profiles and their lines of conjugate action in the several parallel sections, using the preceding equations to determine the several coördinates, and then using these graphs or layouts as before to establish the projection of the actual contact line on an end section of the worm. Third, determining directly, by analytical means, the coördinates of the projection of the contact line on the end section of the worm.

38 Until one has thoroughly assimilated the characteristics of the various sections of different helicoids, either of the first two methods gives a clearer understanding, or rather mental picture, of the conditions. After these characteristics have become familiar, however, the third method is by far the most direct.

39 We shall, therefore, give the equations necessary to establish the form and position of the actual contact line on an end section of the worm.

Contact Lines of Screw Helicoid Used as a Worm

40 We shall consider first the contact lines of a screw helicoid used as a worm. In order to simplify the problem, we shall consider this worm as meshing with a worm wheel of infinite diameter, ignoring for the present the questions of interference and undercutting, and other similar limitations to the field of contact. Eventually these questions must be answered in the same manner here as they are for other conjugate surfaces, such as spur gears, etc. The diameter of the worm wheel has no influence on the forms or positions of these contact lines, the larger worm wheel simply using more of them.

41 The contact lines of a worm with its worm wheel can be determined in the same manner as the contact lines of a rack of varying profile with a gear. At any moment, an infinitely small part of the motion can be considered as a turning motion about the contact line of the pitch cylinder of the worm wheel and the pitch plane of the worm. This contact line, when considered as the axis of this turning motion, will therefore be called the momentary center of the motion.

42 In order for the worm to be able to turn about this momentary center relative to the worm wheel, the normal line to any contact point between the worm surface and the worm wheel surface must pass through this momentary axis. Otherwise it would be impossible for the worm to turn, or rock, slightly about the momentary axis in both directions. This is another way of expressing the fact that, for conjugate tooth action, the normal to

the tooth profile at the point of contact must pass through the pitch point.

43 This discussion does not include the contact conditions between the hour-glass-shaped or globoid worms with their worm wheels. In these cases there is neither conjugate tooth action nor pitch planes nor pitch cylinders. The action here is more of the nature of a screw in a nut. It should be apparent that, under such conditions, even an infinitely small turning or rocking movement is not possible.

44 The contact line for any position of the worm can be determined, therefore, by drawing through any point of the momentary axis a line normal to the surface of the worm, and by determining the intersection point of this normal line with the worm surface. Every such intersection point will be a contact point.

45 The normal to the surface of a helicoid must be perpendicular to its generatrix, also perpendicular to the helix angle at the point of intersection, and also perpendicular to the tangent of any intersection curve at its point of intersection.

46 Referring to Fig. 15, the position of the screw helicoid relative to the drawing plane can be determined from the location of its intersection profile with the drawing plane. In this case, the drawing plane is perpendicular to the axis of the helicoid and the intersection curve is the Archimedes spiral given by Equation [12], as follows:

$$\theta = \frac{2\pi r \tan \gamma}{L}$$

47 This intersection curve can be located in any position. For convenience, Equation [12] can be plotted directly. Referring to Fig. 15, let

L = lead of generatrix
γ = angle of generatrix
θ = angle of rotation of generatrix
h = length along radial line from momentary axis to intersection curve of screw helicoid
k = distance of projection of contact point along radial line from momentary axis
R = distance of pitch plane from axis of screw helicoid.

Then

$$h = r - \frac{R}{\sin \theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [29]$$

$$k = (h-k)\left[\tan^2 \gamma - \frac{L \tan \gamma \cos \theta}{2\pi(k \sin \theta + R)}\right] \quad . \quad . \quad . \quad [30]$$

48 Solving Equation [30] for k, this develops into a very long equation. For simplification in handling, therefore, we shall let

$$A = 2\pi \sin \theta$$
$$B = 2\pi R - L \sin \gamma \cos \gamma \cos \theta - 2\pi h \sin^2 \gamma \sin \theta$$
$$C = h(2\pi R \sin^2 \gamma - L \sin \gamma \cos \gamma \cos \theta)$$

Then

$$k = \frac{-B + \sqrt{B^2 + 4AC}}{2A} \quad \ldots\ldots\ldots\ldots\ldots \quad [31]$$

When $\theta = 90$ deg., Equation [30] reduces to the form

$$k = h \sin^2 \gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [32]$$

49 When h is plus, the value of k should also be plus, and the point of contact is above the pitch line; when h is minus, k should also be minus, and the contact will be below the pitch line. If the

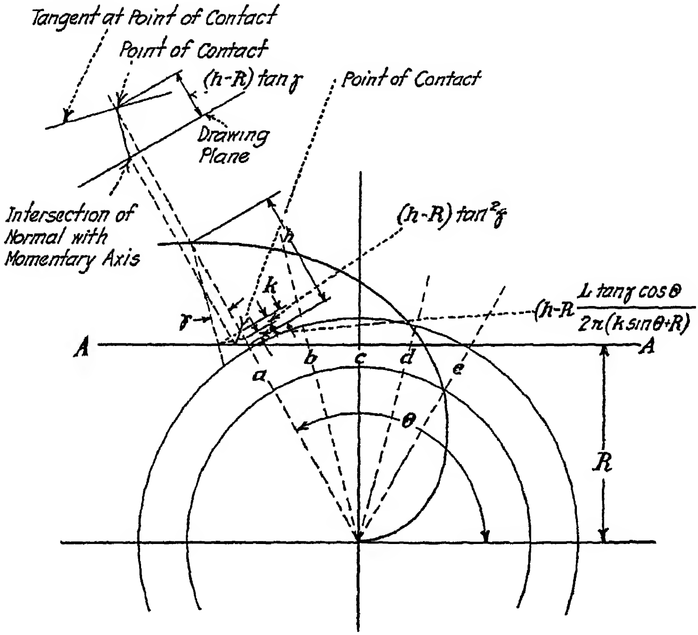


FIG. 15 CONTACT ON SCREW HELICOID

solution of Equation [31] gives a plus value of k for a minus value of h, it indicates that the contact point in question is outside of the field of conjugate tooth action.

50 Although the solution of Equation [31] is quite involved, it will be noted that in large measure the intermediate values contain many constants, so that the actual labor of solving for a series of values is very much reduced after the first set is solved.

51 Study of Fig. 15 will make it apparent that the contact line will always lie between the momentary axis and the intersection curve of the helicoid, its position depending upon the lead of the

helicoid, the angle of the generatrix, and the position of the pitch plane. The smaller this lead and angle, the closer the contact line will be to the momentary axis, or intersection of the pitch plane with the drawing plane.

52 To obtain the contour of any contact line for any given position of the helicoid in relation to the momentary axis, the preceding equations are solved for several positions of the radial planes, or different values of θ.

53 In order to determine the change in position of the contact line as the worm is revolved, these equations are solved for successive positions of the momentary axis in relation to the helicoid. This is accomplished by determining a new series of values for h, corresponding to the changed positions of the helicoid. For example, if the helicoid is revolved one-quarter of a revolution, all of the original values of h will be altered equally an amount equal to $\frac{L}{4} \cot \gamma$. The several successive contact lines can then be plotted, which will show the nature of the action as it affects the conditions of lubrication. The actual duration of contact can be determined by establishing the turning angle of the helicoid which carries the contact lines across the face of the worm wheel. This, of course, will also depend upon the diameter of the worm wheel, the duration of contact being greater with worm wheels of larger diameter.

54 If projections of these contact lines are desired in any other plane, they can be obtained readily by projecting the contact points from the given plane and locating them at a distance equal to $(h-k) \tan \gamma$ from the projection of the momentary axis.

Contact Lines of Involute Helicoid Used as a Worm

55 The contact lines of an involute helicoid used as a worm are much easier to determine. The nature of an involute helicoid is such that every perpendicular to its surface is tangent to the base cylinder. Thus if we consider a series of intersecting planes tangent to the base cylinder, these planes will contain both the generatrix and the normal to the helicoidal surface. This permits a very simple solution of the position of the contact points. Referring to Fig. 16, let

L = lead of generatrix
γ = angle of generatrix
ε = angle of rotation of generatrix
a = radius of base cylinder
R = distance of pitch plane from axis of helicoid
h = length along line tangent to base cylinder from momentary axis to intersection profile
k = distance of projection of contact point along line tangent to base cylinder from momentary axis.

$$h = a\varepsilon - \frac{R - a\cos\varepsilon}{\sin\varepsilon} \quad \text{[33]}$$

$$k = h\sin^2\gamma \quad \text{[34]}$$

Equation [34] holds true for all positions of the planes tangent to the base cylinder.

56 In order to determine the change in position of the contact line as the worm is revolved, the new length of h is multiplied by the constant $\sin^2\gamma$. This new length of h increases uniformly on all tangent planes. For a turning movement of one revolution it

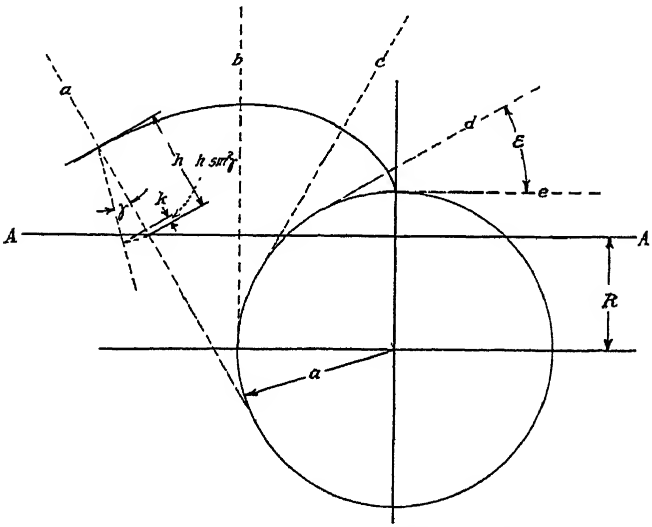


FIG. 16 CONTACT ON INVOLUTE HELICOID

increases an amount equal to the circumference of the base cylinder, or $2\pi a$, and for all other movements in proportion.

57 If the projection of the contact line is desired in any other plane, it can be projected in the same manner as in the case of the screw helicoid, and the contact points would be located at a distance of $(h-k)\tan\gamma$ from the projection of the momentary axis as before.

58 We are now in a position to determine the position of the contact lines between a worm and a worm wheel to see whether they approach the conditions of film lubrication, as shown in Fig. 1, or conditions of semi-lubricated surfaces, as shown in Fig. 2. Space here is too limited to do more than examine a very small part of the possible field. We shall, therefore, confine our attention to a worm of large helix angle, about 45 deg. We shall use as our first example a worm of the form of a screw helicoid.

Contact Lines on Screw Helicoid with Large Helix Angle

59 We shall use as this first example a worm of 3 in. pitch diameter, 8 in. lead with 8 threads or starts, pitch 1 in., and with an included angle of thread of 60 deg. This worm will be of the form of a screw helicoid. This gives us the following values:

$$L = 8.000$$
$$\gamma = 30 \text{ deg.}$$
$$R = 1.500$$

Transposing Equation [12], we have

$$r = \frac{\theta L}{2\pi \tan \gamma}$$

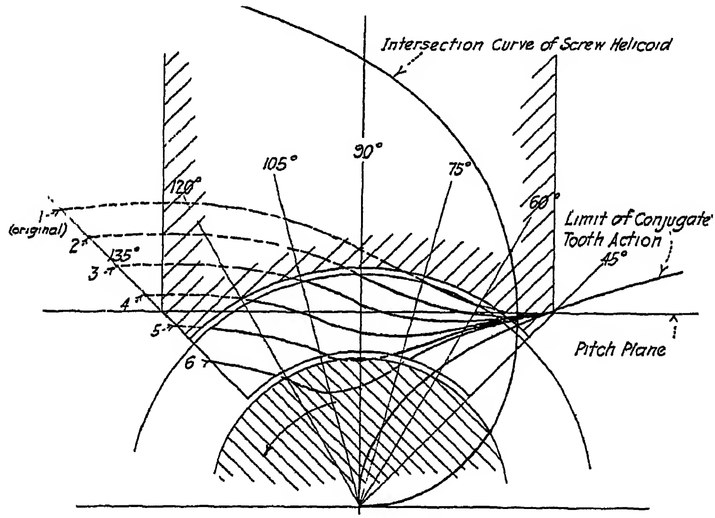


Fig. 17 Contact Lines on Screw Helicoid of Large Helix Angle

60 With this transposed equation and Equations [29] to [32], and values of θ from 45 to 135 deg., we get the following tabulation:

Original positior of worm

$\theta =$	45 deg.	60 deg.	75 deg.	90 deg.	105 deg.	120 deg.	135 deg.
$r =$	1.73205	2.30940	2.88675	3.46410	4 04145	4.61880	5.19615
$h =$	−0.38927	+0.57735	+1 33384	+1.96410	+2.48854	+2.88675	+3.07483
$k =$	+0.00489	+0.05018	+0.24348	+0.49102	+0.73532	+0.95128	+1.10525

61 This contact line is plotted as the first contact line in Fig. 17, together with the intersection curve of the screw helicoid in this original position. When $\theta = 45$ deg., the sign for k is different from that for h, which indicates that this contact is outside of the field of conjugate tooth action. This first contact line

is just at the edge of the field of contact between the worm and the worm wheel. We shall therefore turn the worm back in the direction shown by the arrow in Fig. 17 to several successive positions, and establish the forms of other contact lines. This worm has 8 starts, so that a turning movement of 45 deg. would move the thread form the distance of one pitch or tooth interval along the axis of the helicoid. We shall use successive turning movements of 22½ deg., which will correspond to a linear advance of one-half a tooth interval, or ½ in. in this case. Proceeding as before, we obtain the following tabulations:

	Second position of worm (turned back 22½ deg.)					
θ =	60 deg.	75 deg.	90 deg.	105 deg.	120 deg.	135 deg.
h =	−0.28867	+0.46782	+1.09808	+1 62252	+2.02073	+2 20881
k =	−0.02264	+0.08206	+0.27452	+0 48777	+0.68175	+0 81414
	Third position of worm (turned back 45 deg.)					
h =	−1.15470	−0.39821	+0 23205	+0.75649	+1.15470	+1 34278
k =	−0.08172	−0.06659	+0.05801	+0 23248	+0.40112	+0.51014
	Fourth position of worm (turned back 67½ deg)					
h =	−2.02072	−1.26423	−0.63397	−0.10953	+0.28868	+0.47676
k =	−0.12953	−0.19985	−0.15849	−0.03467	+0.10416	+0.18810
	Fifth position of worm (turned back 90 deg)					
h =	−2.88675	−2.13026	−1.50000	−0 97556	−0.57735	−0.38927
k =	−0.16833	−0.31589	−0.37500	−0.32231	−0 21903	−0.16141
	Sixth position of worm (turned back 112½ deg)					
h =	−3.75278	−2.99629	−2.36603	−1 84159	−1 44338	−1 25530
k =	−0.20003	−0.41411	−0.59151	−0 65565	−0.59812	−0.55945

62 All of these contact lines are also plotted in Fig. 17. We shall also determine the limit of conjugate tooth action by means of Equations [17], [18], and [19]. These equations give us the following tabulation:

x =	0.5000	1.0000	1.5000	2.0000
y =	0.99234	1 32718	1 53920	1.68614

These values are also plotted in Fig. 17, and show that when $\theta = 45$ deg. there can be no conjugate tooth action along this plane on the working face of this combination.

63 A study of Fig. 17 shows that contact lines 5 and 6 are fairly good as regards conditions of lubrication, because they extend from top to bottom of the worm thread at an angle to a concentric circle on the worm. The remaining contact lines tend to follow such concentric circles, however. It is also apparent that the pitch plane seems to have a bad influence on these contact lines, all of them tending to converge toward this pitch plane. Under these circumstances, it should be of interest and value to determine the results of shifting this pitch plane. We shall therefore determine the contact lines on this same worm when the pitch plane is shifted nearer to its axis. Shifting this pitch plane an amount equivalent to adding two more teeth in the worm wheel

without changing the center distance, we shall proceed as before. This gives us the following values:

$$L = 8.000$$
$$\gamma = 30 \text{ deg.}$$
$$R = 1.18169$$

Original position of worm (third contact line)

$\theta =$	60 deg.	75 deg.	90 deg.	105 deg.	120 deg	135 deg.
$r =$	2 30940	2.88675	3 46410	4.04145	4.61880	5.19615
$h =$	0.94490	1.66337	2.28241	2 81807	3.25430	3.52499
$k =$	0 02579	0 27988	0.57060	0.84539	1 09342	1.29556

64 This contact line is plotted in Fig. 18. In order to cover the face of the worm wheel, it will be necessary to turn the worm back in the direction shown by the arrow in Fig. 18 about four increments as before and also to turn it ahead in the opposite direction about two of these increments. When turned ahead, it will be necessary to determine the position of the contact points only on the planes where $\theta = 60$ deg., 75 deg., and 90 deg. Thus we get the following values:

$\theta =$	60 deg.	75 deg	90 deg.	105 deg.	120 deg.	135 deg.
	First position of worm (turned ahead 45 deg.)					
$h =$	2 67695	3.39542	4.01446			
$k =$	0 12232	0.62804	1.00362			
	Second position of worm (turned ahead 22½ deg)					
$h =$	1 81093	2.52940	3 14844			
$k =$	0.06294	0.44847	0.78711			
	Fourth position of worm (turned back 22½ deg)					
$h =$	0 07887	0.79734	1.41638	1.95204	2.38827	2.65896
$k =$	0 00175	0.12582	0.35410	0.59785	0.82447	1.00536
	Fifth position of worm (turned back 45 deg)					
$h =$	−0.78816	−0.06869	+0.55036	+1.08602	+1.52225	+1 79294
$k =$	−0.01462	−0.01003	+0.13759	+0.34175	+0.54375	+0.71241
	Sixth position of worm (turned back 67½ deg)					
$h =$	−1.65418	−0.93471	−0.31566	+0 21999	+0.65622	+0.92691
$k =$	−0 02632	−0.12477	−0.07892	+0.07189	+0.24531	+0.37905
	Seventh position of worm (turned back 90 deg.)					
$h =$	−2.52021	−1.80074	−1.18169	−0.64604	−0 20981	+0.06089
$k =$	−0.03503	−0.21765	−0.29542	−0.22390	−0.08377	+0.02642

65 A comparison of the contact lines in Figs. 17 and 18 shows that the conditions of lubrication in Fig. 18 are very much improved over those shown in Fig. 17. It is hoped that definite tests can be made to prove or disprove the truth of such an analysis as this. According to such a paper analysis, the efficiency of the drive shown in Fig. 18 should be greater than that of the drive shown in Fig. 17, and also its load carrying ability should be materially increased. Samples for such a test would consist of one-worm and two-worm wheels, both hobbed with the same hob and running at the same center distance; the first worm wheel

would be of standard proportions, while the second would have two more teeth.

66 The foregoing analysis gives an indication of the change in the character of the contact lines when the pitch plane is shifted. This change is much more pronounced with large helix angles than with small ones. We shall now direct our attention to the effect of a change in the form of the helicoid on these contact lines, examining for this purpose the contact lines of an involute helicoid

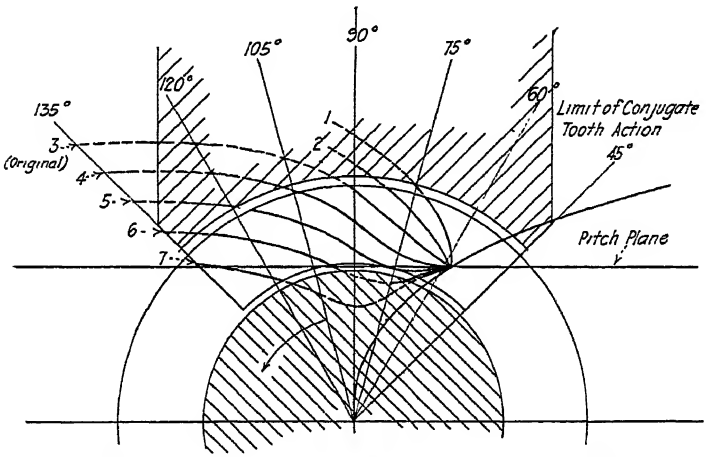


FIG. 18 CONTACT LINES ON SCREW HELICOID OF LARGE HELIX ANGLE

used as a worm. We shall use the same lead and diameter as before, and also select an involute helicoid whose tangent to the axial intersection curve at a diameter of 3 in. is the same as that of the screw helicoid just examined.

CONTACT LINES ON INVOLUTE HELICOID WITH LARGE HELIX ANGLE

67 Referring to Equation [21], we have

$$\tan \phi = \frac{2\pi a y}{L\sqrt{y^2 - a^2}}$$

Transposing this equation to solve for a, we have

$$a = \frac{yL \tan \phi}{\sqrt{4\pi^2 y^2 + L^2 \tan^2 \phi}}$$

We have the following values:

$$\begin{aligned} y &= R = 1.500 \\ L &= 8.000 \\ \tan \phi &= \tan 60 \text{ deg.} = 1.73205 \\ a &= 1.24029 \end{aligned}$$

Whence

$$\tan \gamma = \frac{L}{2\pi a} = 1.02657$$

$$\gamma = 45 \text{ deg.} - 45 \text{ min.} - 4 \text{ sec.}$$

68 Equations [33] and [34] enable the contact points to be determined. Using values of ε from 0 to 90 deg., we get the following for the original position of the worm:

$\epsilon =$	15 deg.	30 deg.	45 deg.	60 deg	75 deg.	90 deg.
$h =$	−0.84202	−0 20234	+0.09309	+0 28286	+0.40296	+0.44824
$k =$	−0 43205	−0.10382	+0.04777	+0.14514	+0.20676	+0.23000

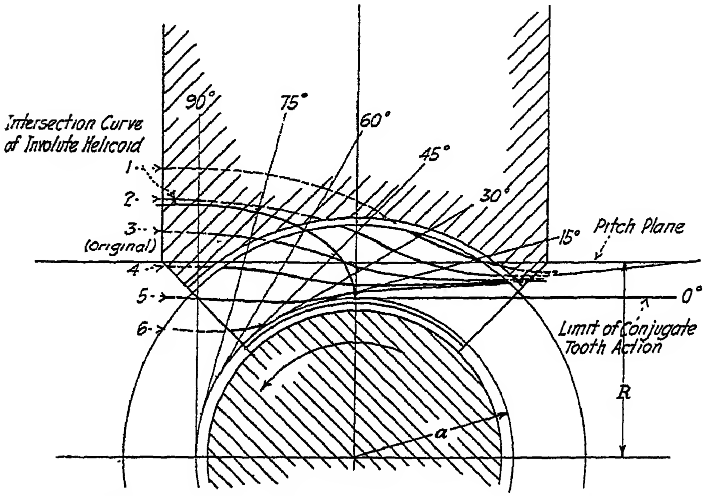


Fig. 19 Contact Lines on Involute Helicoid of Large Helix Angle

69 These points are plotted in Fig. 19 as position 3, together with the line where $y = a$, which is the limit of conjugate tooth action.

70 Additional contact points for successive positions of the worm equal to one-half pitch advance, or $22\frac{1}{2}$ deg. turning movements of the worm, are tabulated below:

$\epsilon =$	15 deg.	30 deg.	45 deg.	60 deg.	75 deg.	90 deg.
$k_1 =$	+0.06777	+0.39600	+0.54759	+0.64496	+0.70658	+0.72982
$k_2 =$	−0.18214	+0.14609	+0.29768	+0.39505	+0.45667	+0.47991
$k_3 =$	−0.43205	−0.10382	+0.04777	+0.14514	+0.20676	+0.23000
$k_4 =$	−0.68196	−0.35373	−0.20214	−0.10477	−0.04315	−0.01991
$k_5 =$	−0.93187	−0.60364	−0.45205	−0.35468	−0.29306	−0.26982
$k_6 =$	....	....	−0.70196	−0.60459	−0.54297	−0.51973

71 These values are also plotted in Fig. 19. A comparison of these contact lines with those of Fig. 17 shows very similar characteristics, although their shapes are not exactly alike. This change in the form of the helicoid has had but little influence in this case on the character of these contact lines.

72 We shall now drop the pitch plane to a distance of 1.18169 in. from the axis of the helicoid as before, and determine the new contact points. This gives us the following values for the original position of the worm:

ϵ =	15 deg.	30 deg.	45 deg.	60 deg	75 deg.	90 deg.
h =	+0.38784	+0 43427	+0 54325	+0.65041	+0.73250	+0 76655
k =	+0.19900	+0 22283	+0.27875	+0.33373	+0.37585	+0.39332

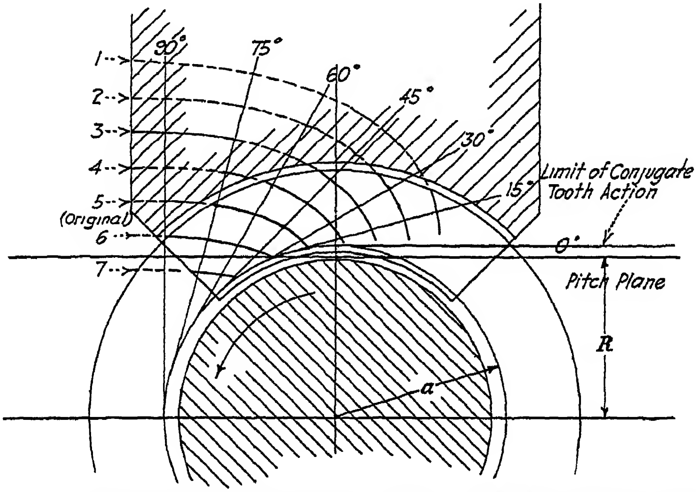


FIG. 20 CONTACT LINES ON INVOLUTE HELICOID OF LARGE HELIX ANGLE

73 These points are plotted in Fig. 20 as the fifth position of the worm. Additional contact points for successive positions of the worm are determined as before and are tabulated below:

ϵ =	15 deg.	30 deg.	45 deg.	60 deg.	75 deg.	90 deg.
k_1 =	+1 19864	+1.22247	+1.27839	+1.33337	+1.37549	+1.39296
k_2 =	+0.94873	+0.97256	+1.02848	+1 08346	+1.12558	+1.14305
k_3 =	+0.69882	+0.72265	+0 77857	+0.83355	+0.87567	+0.89314
k_4 =	+0.44891	+0.47274	+0.52866	+0 58364	+0.62576	+0 64323
k_5 =	+0.19900	+0.22283	+0.27875	+0 33373	+0.37585	+0.39332
k_6 =	.. ..	—0.02708	+0.02884	+0 08382	+0.12594	+0.14341
k_7 =	....	....	—0.22107	—0.16609	—0.12397	—0.10650

74 These values are also plotted in Fig. 20. The nature of these contact lines is very similar to those in Fig. 18, which indicates that the position of the pitch plane has a greater influence on the characteristics of the contact lines than has the exact form of the helicoid.

75 It is of interest to note that the FJ worm drive developed by the David Brown and Sons Company of England is an involute helicoidal worm with the pitch plane tangent to the base cylinder and also tangent to the outside diameter of the worm wheel.

76 The subject of worm wheel contact is too large to be covered fully in a single paper. This paper has touched on but one small phase of the subject. The study of the most favorable combinations of thread angles and helix angles, for example, is one that would require a treatise in itself. The object of this paper has been to attempt to find a suitable "yard-stick" to use in comparing data obtained from tests on various designs of worm drives. It may prove possible in this way to reconcile seemingly contradictory data. If this "yard-stick" should prove to be a reliable one, there are great possibilities of improvement in the design of many of such drives. It would be futile, or rather premature, however, to elaborate on this factor until it has proved itself to be a controlling one.

DISCUSSION

Joseph C. O'Brien.[1] The author has set forth a novel method of attacking a complicated and important problem, exhibiting as he does extreme good and bad conditions for contact lines. However, it is of vital importance that proper ideals be adopted at the outset to avoid both erroneous conclusions or conclusions that do not fit the entire range of conditions. It is from this viewpoint that the following criticism is offered.

In Fig. 1, on a line of contact such as line 3, there would be no conjugate action between the worm thread and wheel tooth. Instead, there would be pure sliding from top to bottom along an inclined plane. There would be a similar, but lesser, condition of sliding on lines 2 and 4, and 5 and 1. That is, the conjugate action tends to increase as the distance of the lines from the worm axial plane increases.

The writer believes that the contact lines in Figs. 18 and 20 are superior to those selected as ideal. Suppose, in Fig. 20, that the worm turns one increment in a direction contrary to that shown by the arrow and that contact is at line 7. Transpose line 7 circumferentially 22½ deg. in the direction of the arrow. Then the length of worm thread from this point to line 6, measured along the helix, comes into action during this rotation. The length of wheel tooth from lines 7 to 6, measured along the helix, also comes into action, indicating about 33⅓ per cent rolling action between the helices. This action, as regards lubrication, would be superior to that selected as ideal. The contact lines of Fig. 20 are the best of the four conditions exhibited, not on account of their similarity to the ideal selected, but rather on account of the uniformity of spacing across the face of the wheel. Such spacing means uniform progression of contact, proportional to the rotation of the worm. Also, the lines of contact are further apart, which

[1] Pittsburgh Gear & Machine Co., Pittsburgh, Pa.

implies for the same increment of rotation a greater percentage of rolling action.

With reference to the best position of the contact lines with respect to the axis, it is not wise to neglect entirely the conjugate action of the teeth, particularly when the lead angle approaches 45 deg. and a large number of threads are used. Gear ratios as low as 3 to 1 may be encountered, and lines of contact that are good with respect to the worm axis, may be bad with respect to the gear axis. The mutual effect of the contact lines should be considered, and also the path of the lines. This may be made clear by reference to Fig. 20. The contact lines at the extreme left approach a position parallel to the wheel axis, which has been characterized as bad in the case of the worm, although the worm turning action is at an angle of 45 deg. Conjugate sliding action is at its maximum at this point. Now consider contact line 1, in which the contact approaches a plane perpendicular to the wheel axis. There is but little conjugate action, the worm turning action is 22½ deg., and worm rotation sliding predominates. Whether or not the position of the contact lines can be shifted to improve the general condition is problematic. To obtain the best action in all parts of the contact, it is possible that the ideal lines may vary across the face of the wheel, being affected possibly by the diameter of the worm, the diameter of the wheel, and by other factors.

Referring again to Fig. 20, each contact line represents a movement of the worm through 22½ deg. Therefore, three contact lines in planes of rotation of the wheel, on the working surfaces of the teeth, represent two teeth in contact. Continuity of tooth action exists only in a small area at the 22½-deg. point to the left of the worm axial plane. The action to the right of the axial plane has barely one-half pitch path of contact. Such a worm apparently would have continuity of tooth action in planes of rotation of the wheel in only about 25 per cent of the width of the wheel face.

From the data in the paper, contact diagrams of single, double, triple, and even quadruple worm threads, would appear very poorly in comparison with the diagrams used as illustrations. They apparently would be so similar as to render this method of analysis unsuitable for worms of very low lead angles; that is, unsuitable as a method of estimating the capacity of the worm drive.

H. E. Merritt.[1] This paper is the more valuable owing to the absence of published data on the geometrical aspects of worm gearing. The object of the writer is to bring forward the fact that although very little has been published on the subject considered by the author, it is a question which has been very thoroughly analyzed by the firm of David Brown & Sons, Ltd., of England.

[1] Research Engineer, David Brown & Sons, Ltd., Huddersfield, England.

We are able to confirm the author's surmise that the method of determining the location of the lines of contact is an essential step in worm-gear design. Although in the paper it is suggested that such a detailed geometrical analysis should only be used to obtain information of a general kind on worm-gear design, it is our practice to determine the "zone of contact" for every worm gear put into production. This applies particularly to the type of worm gear referred to in the paper.

In determining the lines of contact we would observe that it is essential to determine the location of these lines, not merely on the projected end section of the worm, but also their position axially with regard to the worm and their circular projection on the wheel tooth surfaces. This is necessary in order to determine the total length of the lines of contact and also the curvature of the "zone of contact" in planes parallel to the worm and normal to the wheel axis. In order to determine the location of the paths of contact we use a geometrical method which does not involve constructing either the worm or wheel tooth profiles, but gives the line of contact directly. This construction takes about 20 minutes to complete.

In order to reduce the amount of labor required in designing worm drives, however, the David Brown & Sons' standard system of worm-gear design makes use of the principle of geometrical similarity. This system is based upon the use of 40 standard forms of worm thread, each of which has been completely analyzed for "zone of contact." These worms range from 1 to 10 threads, and the pitch diameter is a nominal dimension which is made equal to a whole number of modules. This number is denoted by the symbol q and ranges from 6 to 10.

The zone of contact, however, is only a preliminary stage in determining the load-carrying capacity of worm gearing. Having found the length of the lines of contact the relative radius of curvature is determined for what is regarded as a mean position, and from this a "zone factor" is deduced by means of which, given the module, the number of teeth in the wheel, and the tooth load, the surface stress can be computed. It has been found that surface stresses calculated in this way serve to form a reasonably accurate basis of design. In cases where failure of the tooth surface by "pitting" has occurred, calculation of the surface stresses, based on analysis of the zone of contact, has explained the failure.

L. R. Buckendale.[1] The author's mathematical analysis of the worm wheel contact conditions substantiates the results of graphical investigations of these same conditions which have been made in the past. It also substantiates the hypothetical reason assigned for the evident tendency to pit, which is shown by most applica-

[1] Sales Engineer, Timken-Detroit Axle Company, Detroit, Mich.

tions of the straight involute type gearing, figured with the conventional addendum and dedendum. The author also confirms mathematically the radical difference in contact lines obtained by means of the FJ system. It may also be of interest to state that these contact lines, as shown, have been confirmed by placing worms and wheels together and etching to show the lines of contact obtaining at each instant of tooth phase. As stated by the author, this subject of worm wheel tooth contact is an extremely large one and will bear a great deal of investigation and study.

George H. Acker.[1] The writer believes that the immediate problem has a slightly different nature from the one suggested by the author, namely, lubrication. In the present state of the art, the real loading limitation is determined by the safe fatigue limit of the bronze, of which it is at present thought that the gear must be composed, rather than any safe lubrication load. The author's work, however, is even more useful in that connection than in the matter of lubrication.

The specific examples of tooth contact in the paper cover only the two extreme cases of tooth form, whereas the actual practice in tooth form may range between these two forms, without actually meeting either. That being so, the writer will confine his remarks to the type of gear described by the author as " convolute helicoid " gearing. The writer has seen, on this gearing, evidences of wear due to lubrication failure, but this wear, in almost every case, has been accompanied by pitting of the bronze. More often, however, the pitting is to be found in some degree, without any indication of undue wear. He has therefore concluded that, with this convolute gearing, with the ordinary pitch plane placement, the ability of the lubricant to carry load exceeds that of the bronze. It should be emphasized that such a conclusion is predicated entirely on the use of the correct lubricant. This conclusion has been justified many times in the experimental department of the company with which the writer is connected, as well as in the field. On one installation of industrial gears, which, through some error, were made to carry a considerable overload, rapid pitting and wear were observed. Various lubricants were tried, and one found that was successful in eliminating wear, but only by changing the material in the gear itself could the pitting be checked.

The author has brought so clearly before us the means of scientifically attacking this very problem by gaging the extent of contact and relative worth of contact of varying designs of the gearing, that some sure step toward improvement must result. However, a mathematical siege of a problem of such intensity is often shirked, and the writer believes that some short cut can be

[1] Chief Engineer, Cleveland Worm and Gear Company, Cleveland, Ohio.

found to make a physical investigation take the place of the mathematics. The author's contact lines are the intersection of the plane of action with the helicoidal worm surface. Cannot a broad, thin beam of light be substituted for the contact plane; a highly polished worm for the formula; and a photographic plate, or the eye, for the plot? No time has been available for any great investigation of this matter on the writer's part, but a desk lamp, two slotted pieces of cardboard, and a worm served to indicate that the matter was at least worth investigating. If such a method can be developed, it will have one distinct advantage over a mathematical plot, in that, for a fixed thickness of light beam, the relative load-carrying ability of varying surfaces can be roughly gaged by the width of the reflection from the worm.

Thomas W. H. Jeacock.[1] The failure of the gear is laid, at times, to the bronze, at times to the formation of the tooth, and at still other times to the lubricant. We have been working for a considerable period with the Timken-Detroit Axle Company, the Cleveland Worm & Gear Company, and the Bosch Machine Tool Company in an endeavor to better the old 89-11 bronze alloy, or stone metal, as it was called some years ago. We found that with the increased load that the gears were asked to carry, because of the development of motor trucks, the teeth were pulling out. In other words, the gear bronze was asked to do things far beyond its physical limitations. In working with different alloys, some improvements were accomplished with aluminum bronze. We then found, by experience, that aluminum bronze under certain pressures, where the temperatures ran up, failed us. We then had to go back to the old gear-bronze formula and work from that. We have had a great deal of success with the introduction of nickel in varying quantities, and this possibly will be the solution of this phase of the problem.

The Author. Mr. O'Brien in his discussion lays too much stress on conjugate tooth action. Conjugate tooth action is not a necessity on worm drives. All of the globoid worms are free from it and many of them have proved satisfactory. Conjugate tooth action is a necessary evil that exists when the standard forms of helicoids are used as worms. This action must be considered when determining the field of contact, and plays its part in the determination of effective combinations of helix angles and thread or pressure angles, but its nature is a secondary matter, as most of the sliding between the worm and wheel is caused by the turning movement of the worm.

He is correct in stating that the contact diagrams of single, double, triple, and even quadruple worm threads of conventional

[1] President, Buffalo Bronze Die Cast Corporation, Buffalo, N. Y.

designs and proportions show up very poorly. As a matter of fact the contact conditions in these cases, as regards lubrication, are very poor. The author believes that this is one of the most important features of worm and worm-wheel design, and if it should prove true in practice, and by actual experiment, it is possible to modify the design of any of these drives in a simple manner to obtain contact lines of almost any nature that may prove to be most desirable. The A.S.M.E. Special Research Committee on Worm Gears hopes to be able to have tests conducted to determine the influence of the nature of these contact lines on the efficiency and load-carrying ability of worm drives. With such data as a foundation, the next step would be to work up design data so that the most favorable conditions as proved by actual tests would be secured.

Mr. Merritt brings out the fact that it is also necessary to establish the zone of contact as well as the nature of the contact lines. The author's analysis gives the basic equations necessary to establish this zone of contact, but his discussion has been limited by the space available to but a single feature of this involved subject — that of the influence of the contact lines on the lubrication conditions. As Mr. Merritt states, once a specific type of general design has been adopted, it is possible to work up general data in such form that each individual worm drive may be rapidly analyzed in detail. There are also certain simple approximations which can be used to give an accurate qualitative analysis in a very short and simple manner.

Mr. Acker and Mr. Jeacock call attention to the limitations of the load-carrying ability of the materials and express the opinion that this is a more immediate problem to be attacked than that of lubrication. The author cannot take any exception to this, but would call attention again to the fact that an improvement in the nature of the contact lines as regards lubrication also reduces the maximum stresses that are set up in the materials under load, so that a series of experiments along the lines suggested in the paper would be attacking both problems at the same time. Thus if experiments should point the way to a worm and worm-wheel design of higher efficiency and greater load-carrying ability by the same material, such experiments would seem to be worthy of serious consideration. A paper analysis shows that this improvement is possible, and it is hoped that the interested industries will give substantial support to the work of the committee.

No. 2018

ROUGH TURNING WITH PARTICULAR REFERENCE TO THE STEEL CUT[1]

By H. J. French [2] and T. G. Digges,[3] Washington, D. C.
Non-Members

Tests described in this report extend portions of Taylor's original investigations in rough turning carbon steels to current commercial high-speed tool and structural alloy steels. They were made primarily to show the effect upon tool performance of variation in chemical composition and mechanical properties of the steel cut, and accordingly include lathe tests on carbon, nickel, low- and high-chromium, chromium-vanadium, chromium-molybdenum, and nickel-chromium steels having tensile strengths between 65,000 *and* 195,000 *lb. per sq. in. However, many of the tests with the customary high-tungsten-low-vanadium steel tools were duplicated with low-tungsten-high-vanadium or cobalt high-speed steel tools, and there is included a partial study of the effects of cutting speed, feed, depth of cut, and coolants upon tool life and the power required in cutting. Graphical representation of results is employed, wherever possible, to show the laws of cutting.*

Introduction

During the past several years, the authors have studied so-called "lathe-breakdown" and "Taylor" (1)[4] tests and the effects of certain major changes in chemical composition and heat treatment upon lathe-tool performance, dimensional changes, and other characteristics of commercial high-speed steels (2) (3). In general, the tools were tested "dry" at fixed feed and depth of cut on 3 to 4 per cent nickel-steel forgings heat-treated to show

[1] Published by permission of the Director of the Bureau of Standards.
[2] Senior Metallurgist, Bureau of Standards.
[3] Assistant Scientist, Bureau of Standards.
[4] The numbered references refer to the bibliography appended to this report.

Contributed by Machine Shop Practice Division and the Research Sub-Committee on Cutting and Forming of Metals, and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

tensile strengths in the neighborhood of 100,000 lb. per sq in. In one case lathe tests were made on harder steels, but little or no attention was paid to the effects upon tool performance of variations in the chemical composition and treatment of the steels cut.

2 The need for further study of this question was indicated in Taylor's presidential address before The American Society of Mechanical Engineers (4) in the statement — "one of the greatest needs in the art of cutting metals is a more accurate standard by which to foretell the cutting speed of forgings and castings, and this should form a subject for future experiments."

3 Taylor's well-known lathe tests were made on carbon steels, representing the principal types then commercially machined, and with tools having a chemical composition somewhat different from that of current commercial high-speed steels. Today a variety of alloy steels are regularly rough turned and subjected to many other cutting operations subsequent to heat treatments producing very high tensile strengths which in some cases reach 190,000 lb. per sq. in. or more. No extended and systematic investigation of this subject has come to light, and since there are differences of opinion regarding the "machining properties" of various structural alloy steels, it is important to study the effects of the metal cut upon the performance of modern high-speed-steel tools as well as to determine whether the fundamental relations developed by Taylor in cutting carbon steels apply also in rough turning alloy steels of high tensile strength and hardness.

4 The tests to be described were made on carbon and various alloy steels having definite tensile strengths within the range 65,000 to 195,000 lb. per sq. in. (approximately 130 to 400 Brinell hardness). The investigation was primarily a study of tool performance as affected by the conditions of cutting (variations in material cut, cutting speed, feed, and depth of cut); the form of the tool and its heat treatment were not varied. However, many of the tests with the customary high-tungsten, low-vanadium high-speed steels were repeated with other commercial types, thus permitting comparisons under variable cutting conditions.

5 The tests, made intermittently throughout a period of about three years, included more than a thousand tools and the cutting of over 30 tons of carbon and structural alloy-steel forgings. It was only possible to complete the investigation through the coöperation of a number of manufacturers who donated practically all of the large amount of metal cut. Chief among these was the Central Steel Company, Massillon, Ohio, and the authors here acknowledge in particular the assistance and suggestions of E. C. Smith, Chief Metallurgical Engineer. Other contributors of materials include the U.S. Naval Gun Factory, Washington, D. C., Interstate Iron and Steel Company, Chicago, Ill., United

Alloy Steel Company, Canton, Ohio, and the Climax Molybdenum Company, New York, N. Y. The constructive suggestions of Dr. L. B. Tuckerman of the Bureau of Standards in relation to the mathematical treatment of some of the data are also gratefully acknowledged.

Previous Investigations

6 Much of the abundant literature relating to rough turning, published since Taylor's original work (4), has dealt primarily with tool form, heat treatment, the theories of hardening of high-speed steels, and other phases not directly related to the field to be covered. Important publications which throw light upon the subjects to be discussed will be referred to in the individual sections of this report.

MATERIALS AND EXPERIMENTAL METHODS EMPLOYED

The Tools

7 The original plans for the investigation, prepared jointly with Jerome Strauss, Materials Engineer of the Naval Gun Factory, Washington, D. C., consisted mainly of a study of tool performance in cutting 3½ per cent nickel, nickel-chromium, chromium-molybdenum, and some carbon-steel forgings having tensile strengths between about 80,000 and 150,000 lb. per sq. in. However, as progress was made in these tests, reduction in available funds necessitated withdrawal of the active coöperation of the metallurgical division of the Naval Gun Factory; at the same time, suggestions were received from representatives of interested manufacturers materially to extend the investigation to include tests on steels of higher hardnesses and other types, such as the carbon-chromium, chromium-vanadium, and other nickel-chromium steels. Upon their signifying willingness to supply the necessary materials, arrangements were made to carry out the additional tests.

8 An ample supply of high-speed steels had been obtained for the work originally in view, but this appreciable extension of the program of tests made it necessary to secure for the tools additional steels from different heats. While a disadvantage when attempting comparisons of the materials cut, this condition has certain desirable aspects, for it will bring to light the order of difference in machining properties of various structural steels in relation to changes arising from differences in the quality of the tool steels. Furthermore, it should overcome the objection which might be raised to comparisons of tools based on tests with but one brand of each type of high-speed steel, that the results are characteristic of particular lots of steel rather than typical of the types which they represent.

9 The chemical compositions and heat treatments of the high-speed-steel tools used are reported in detail in Table 1. The equipment and procedure employed in all phases of preparation of the tools and in carrying out the lathe tests were similar to those described in a previous report (2). All tools had a cross-section in the body of ½ in. by ¼ in. and when first prepared were about 4½ in. long; some were hot-finished to the desired size, while others were cut from larger bars, either ⅞ in. or 1 in. by 1½ in. In such

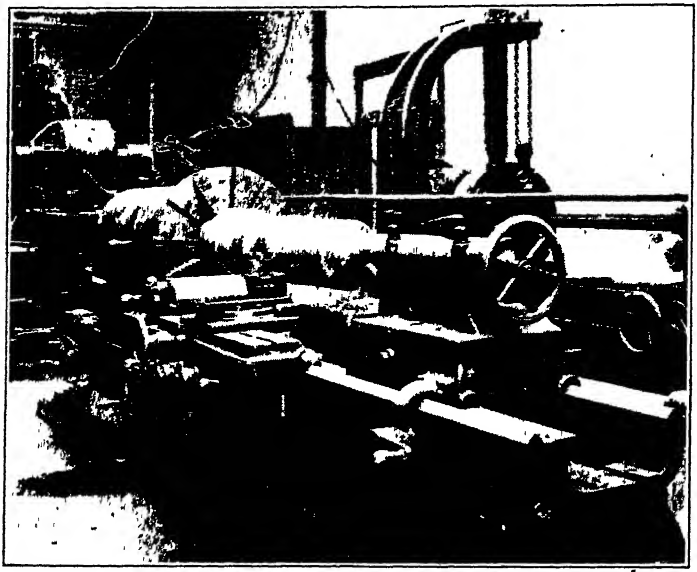


Fig. 1 Type of Lathe and Tool Holder Used in Tool Tests Described

cases, care was taken to grind the top of the tool near the surface of the hot-finished bars to avoid introducing variables from segregation and porosity. The tool form was the same as in tests previously reported; it was not varied throughout the investigation, and consisted of front and side clearances of 6 deg., side slope of 14 deg., back slope of 8 deg., and a nose radius of ⅛ in. Except in special cases in which the effects of coolants were studied, the tools were run dry. Cutting speed, feed, and depth of cut were varied, depending upon the purpose of each set of tests. A photograph of the test lathe and tool holder is reproduced in Fig. 1.

The "Logs" and Test Methods

10 The pieces upon which the cuts were taken, commonly called "logs," varied from about 2 to 14 in. in diameter and from 2 to 6 ft. in length. The largest, and solid, forgings were used for the development of low tensile strengths or in cases in which a

TABLE 1 CHEMICAL COMPOSITIONS AND HEAT TREATMENTS OF THE LATHE TOOLS USED IN THE INVESTIGATION

Steel No.	Chemical composition, per cent: C	Mn	P	S	Si	Cr	W	V	Co	Ni	Mo	Annealing[b]: Temp., deg. fahr.	Annealing[b]: Time, hr.	Pre-heating for hardening	High-heat[c]: Temp., deg. fahr.	High-heat[c]: Time, min.	Tempering
							HIGH-TUNGSTEN–LOW-VANADIUM										
E	0.62	0.32	0.020	0.014	0.32	3.94	17.86	0.68	...	0.14	...	1600	3	1600 deg fahr., 20 minutes	2400	1½	1100 deg fahr., 30 minutes and cooled in air
I	0.63	0.29	0.012	0.024	0.25	3.97	18.88	0.81	....	0.15	....	1600	3	"	2400	1½	"
M[a]	0.69	0.29	0.025	0.022	0.24	3.78	18.12	1.14	....	..	..	....	..	"	2400	1½	"
N[a]	0.70	0.29	0.034	0.015	0.24	3.74	18.68	1.15	...	...	...	....	..	"	2400	1½	"
D	0.64	0.39	0.017	0.017	0.06	3.93	18.33	0.85	....	..	...	1600	3	"	2400	1½	"
							LOW-TUNGSTEN–HIGH-VANADIUM										
H	0.66	0.32	0.029	0.025	0.42	2.99	12.67	1.74	....	0.18	....	1600	2	"	2350	1½	"
L	0.71	0.31	0.029	0.026	0.24	3.60	12.75	1.99	....	...	....	1600	3	"	2350	1½	"
C	0.68	0.42	0.022	0.018	0.16	4.37	13.91	1.64	0.09	0.12	..	1600	2	"	2350	1½	"
							HIGH-TUNGSTEN–COBALT										
G	0.59	0.41	0.021	0.008	0.15	3.25	18.51	1.17	3.45	....	....	1600	3	"	2450	1½	"
K	0.60	0.12	0.008	0.026	0.12	2.95	18.23	.99	3.50	0.09	...	1600	3	"	2450	1½	"
P[a]	0.70	0.30	0.028	trace	0.36	3.90	17.92	1.08	4.83	....	0.41	1600	3	"	2450	1½	"
B	0.68	0.42	0.013	0.016	0.15	3.67	17.97	.73	3.06	0.04	...	1600	3	"	2450	1½	"

[a] Manufacturer's analysis.
[b] Tools were pack-annealed in a mixture of dry sand plus 10 per cent wood charcoal.
[c] All tools were quenched in a proprietary quenching oil

TABLE 2 CHEMICAL COMPOSITION, HEAT TREATMENT, AND OTHER DETAILS OF THE MATERIALS CUT IN THE LATHE TEST

Test Log No.	Approx. dimensions of test logs: Outside diam., in.	Diam. of hole,[a] in.	Supplier	Supplier's melt number	Chemical composition, per cent[b]: C	Mn	P	S	Si	Ni	Cr	V	Mo	Cu	Heat treatment[c] (all temperatures given in deg. fahr.)
1	14	5½	D	P983-10	0.26	0.57	0.041	0.035	0.15	3.81	....	....	....	0.11	1650 emulsion; 1560 water; 1200.
2	14	5½	D	P983-8	0.26	0.57	0.041	0.035	0.15	3.81	...	..	....	0.11	1650 emulsion; 1500 water; 1340.
3	7¼	..	D	S	0.44	0.62	0.014	0.032	0.14	0.27	....	....	...	trace	Tested as received; exact treatment not known.
5	7¼	..	D	53560	0.44	0.62	0.014	0.032	0.14	0.27	....	trace	....	trace	1560 emulsion; 1020.
6	13	..	A	8403	0.30	0.81	0.018	0.022	0.16	....	0.89	....	0.14	...	1580 emulsion; 1060.
7	8¼	..	A	7846	0.53	0.73	0.017	0.020	0.20	....	0.80	....	0.36	....	1500 emulsion; 1210
8	9¾	..	C	6387	0.28	0.45	0.016	0.027	0.11	....	0.51	....	0.17	....	1600 emulsion; 1180.
9	12¾	..	A	5378	0.29	0.62	0.018	0.020	0.23	....	0.85	0.17	....	....	1680 water; 1230.
10	10¼	..	B	15248	0.37	0.60	.....	.....	....	....	1.00	....	0.35	....	1540 emulsion; 1010
10A	5¾	..	B	15248	0.37	0.60	.....	.....	....	....	1.00	....	0.35	....	1530 water; 1000.
11	7½	1¾	C	5379	0.32	0.68	0.017	0.031	0.20	....	1.02	0.15	....	....	1720 water; 940.
11A	5¾	1¾	C	5379	0.32	0.68	0.017	0.031	0.20	....	1.02	0 15	....	....	1800 water; 450.
12	9¾	4¼	D	R70-4	0.28	0.62	0.040	0.019	0.25	3.11	....	....	....	0.28	1540 water; 960.
12A	7	4¼	D	R70-4	0.28	0.62	0.040	0 019	0.25	3.11	....	....	....	0 28	1560 water; 900
13	10½	..	A	1603	0.32	0.61	0.017	0.020	0.18	3.27	....	....	0.36	....	1560 emulsion, 1120.
14	7	..	D	1320	0.53	0.54	0.017	0.014	0.26	2.26	1.78	....	....	0 13	1650 air; 1000.
14A	5¼	..	D	1320	0.53	0.54	0.017	0.014	0.26	2.26	1.78	....	...	0.13	1520 emulsion; 1140
15	12½	..	A	XXX	0.49	0.78	0.017	0.029	0.21	....	0.93	..	....	...	1550 furnace cool to 1275; soak and slow cool.
15A	5¼	..	A	XXX	0.49	0.78	0.017	0.029	0.21	....	0.93	..	.	....	1040 water; 920
16	8	..	H	......	0.18	0.50	0.010	0.040	0.04	....	..	..	....	....	Tested as received; exact treatment not known.
17	12	4	D	R564-1	0.31	0.56	0.040	0.027	0.31	3.27	....	....	....	0 26	1460 furnace cool.
18	12	4	D	R564-1	0.31	0.56	0.040	0.027	0.31	3.27	....	....	....	0 26	1460 furnace cool.
19	5½	2¾	E	......	0.26	0.46	.....	.....	....	....	0 53	...	0.19	...	Water quenched; not tempered; exact treatment not known
20	2	.	H	SS33	0.28	0.55	0.026	0.038	0.11	3.33	0.12	....	..	0 15	1560 water; 660.

[a] If hollow bored.

[b] With exception of logs 16, 27A, and 29, analyzed at the Bureau of Standards, compositions are those reported by the respective producers.

[c] Where the heating times are not given they are not known exactly. The emulsion referred to consisted of 5 per cent rapeseed oil, 15 per cent paraffin oil, and 80 per cent water containing 1 per cent sodium carbonate.

TABLE 2 CHEMICAL COMPOSITION, HEAT TREATMENT, AND OTHER DETAILS OF THE MATERIALS CUT IN THE LATHE TESTS—CONTINUED

Test Log No.	Approx. dimensions of test logs: Outside diam., in.	Diam. of hole,[a] in.	Supplier	Supplier's melt number	Chemical composition, per cent[b]: C	Mn	P	S	Si	Ni	Cr	V	Mo	Cu	Heat treatment[c] (all temperatures given in deg. fahr.)
21	9	2½	F	E517	1.05	0.30	0.024	0.012	0.20	nil	trace	...		.	1500—12 hr.; cool in 1½ hr. to 1110; slow cool in furnace
22	9	2½	F	E517	1.05	0.30	0.024	0.012	0.20	nil	trace	. .	. .	.	1345—18 hr.; cool in 5 hr to 1185; slow cool in furnace.
23	9	2½	F	E517	1.05	0.30	0.024	0.012	0.20	nil	trace	....	....	....	1500—12 hr. furnace cool, 1400—2 hr. oil quench to 1095; air cool.
23A	9	2½	F	E517	1.05	0.30	0.024	0.012	0.20	nil	trace	....	.		1400—2 hr. slow cool (40 hr. to reach 1095).
24	9	2½	F	E517	1.05	0.30	0.024	0.012	0.20	nil	trace	..	...	....	1500—12 hr. furnace cool, 1560—3 hr. water; 1300—3 hr air cool.
25	6¾	2½	A	5011	0.52	0.40	0.009	0.016	0.27	1.64	0.92	...	....	.	1575—3½ hr oil; 850—3½ hr.
25A	4¾	2½	A	5011	0.52	0.40	0.009	0.016	0.27	1.64	0.92	..		...	1575—3½ hr. oil; 1325—2 hr
26	6¾	2½	A	7892	0.48	0.67	0.016	0.016	0.21	3.42	0.15	...	..	...	1575—3½ hr. oil; 750—3 hr.
27A	5¼	2½	A	5103	0.31	0.55	0.016	0.022	0.18	...	1.00	....	0.21	....	1575—1 hr. water; 730—2 hr
29	7	..	H	......	0.33	0.66	0.016	0.060	0.19	....	....	....	.	.	Tested as received; exact treatment not known.
30	2¼	..	H	SS30A	0.29	0.50	0.024	0.032	0.21	0.28	1.03	0.21	...	0.13	1650—1 hr water; 670—1 hr
30A	2¼		H	SS30A	0.29	0.50	0.024	0.032	0.21	0.25	1.03	0.21	....	0.13	1650—1 hr. water; 670—1 hr
31	6¾	2½	A	5011	0.52	0.40	0.009	0.016	0.27	1.64	0.92	....	....	. .	1575—3½ hr. oil; 850—3½ hr
32	3¾	..	D	53560	0.44	0.62	0.014	0.032	0.14	0.27	...	trace	...	.	1560—2 hr. water; 550—3 hr
33	4	..	D	53560	0.44	0.62	0.014	0.032	0.14	0.27	...	trace	....	.	1560—2 hr. water; 550—3 hr
34	3	..	K	......	0.15	0.42	0.013	0.030	0.28	0.35	24.4	....	.		Rolled at about 1850, water quench from rolling heat (about 1650); slightly warmed for straightening.
35	3	..	J	...	0.09	0.48	0.011	0.011	0.24	....	12.3	...	..	. .	1290—1 hr. air cool.
36	14	..	F	7/275	0.40	0.74	0.028	0.039	0.24	3.32	nil	.	. .	.	1600—12 hr. air cool to 1250; held 10 hr furnace cool.

[a] If hollow bored.
[b] With exception of logs 16, 27A, and 29, analyzed at the Bureau of Standards, compositions are those reported by the respective producers.
[c] Where the heating times are not given they are not known exactly. The emulsion referred to consisted of 5 per cent rapeseed oil, 15 per cent paraffin oil, and 80 per cent water containing 1 per cent sodium carbonate.

TABLE 3 AVERAGE TENSILE PROPERTIES OF THE TEST LOGS AS DETERMINED FROM SPECIMENS CUT LONGITUDINALLY AT DIFFERENT DIAMETERS TO REPRESENT THE METAL REMOVED IN THE CUTTING TESTS

Log No.	Type of steel	No. of tests made	Prop. limit, 1000 lb. per sq. in.	Yield point, 1000 lb. per sq. in.	Tensile strength, 1000 lb. per sq. in.	Breaking strength, 1000 lb per sq. in.	Elong. in 2 in., per cent	Red. of area, per cent	Area under stress-strain curve [b]: Sq in	Area under stress-strain curve [b]: Compared to log 1, per cent	Energy required to break the specimen, in-lb. per cu in.	Maximum variation from the mean value of tensile strength, per cent (+)	Maximum variation from the mean value of tensile strength, per cent (−)
1	3½% Ni	4	65.8	69.5	94.6	169.8	25.9	60.9	1.41	100	1730	+2.5	−1.8
2	3½% Ni	6	41.1	49.2	85.6	144.2	27.3	55.1	1.46	101	1750	+1.6	−3.7
3	0.44% C	5	51.7	56.4	92.8	167.5	25.8	55.2	1.52	106	1820	+5.6	−3.0
5	0.44% C	6	62.3	66.3	107.0	162.8	21.6	47.6	1.36	94	1630	+8.4	−4.7
6	Cr-Mo	4	62.2	73.6	96.4	182.6	23.1	64.9	1.30	90	1560	+5.3	−5.1
7	Cr-Mo	4	83.5	95.3	125.0	211.9	21.4	58.8	1.62	112	1940	+6.0	−8.4
8	Cr-Mo	5	50.7	53.8	83.7	157.2	26.7	57.7	1.37	95	1640	+2.8	−1.4
9	Cr-V	5	58.3	64.1	93.8	180.7	24.4	65.1	1.32	92	1580	+6.8	−1.7
10	Cr-Mo	4	97.0	110.0	138.5	217.8	18.2	57.3	1.47	102	1760	+4.9	−4.9
10A	Cr-Mo	4	79.8	117.1	135.5	206.4	15.4	48.3	1.37	95	1640	+14.4	−12.2
11	Cr-V	4	88.5	105.0	127.0	201.2	18.0	54.1	1.39	97	1670	+2.8	−2.6
11A	Cr-V	4	63.0	117.0	134.0	195.2	18.1	48.5	1.15	80	1380	+1.1	−1.1
12	3½% Ni	6	86.9	106.1	128.7	171.1	13.2	35.5	1.28	89	1540	+8.5	−5.2
12A	3½% Ni	2	129.5	138.2	148.8	176.0	10.2	23.0	0.96	67	1150	+0.5	−0.5
13	Ni-Mo	6	59.3	93.0	127.7	175.4	16.2	37.6	1.57	109	1880	+1.0	−1.3
14	Ni-Cr	4	59.2	70.3	128.0	175.4	18.9	39.4	1.63	113	1960	+4.7	−2.4
14A	Ni-Cr	2	127.0	135.5	150.2	226.2	18.5	49.8	1.75	122	2100	+0.2	−0.2
15	C-Cr	4	33.5	37.9	91.9	124.4	19.6	30.2	1.28	89	1540	+1.2	−1.0
15A	C-Cr	2	95.0	118.8	148.4	180.9	12.8	26.3	1.26	88	1510	+0.9	−0.9
16	0.18% C	4	24.5	31.6	66.4	110.0	34.9	51.8	1.40	97	1680	+4.2	−5.9
17	3½% Ni	2	51.5	53.2	87.3	149.6	29.2	54.2	1.62	112	1940	0	0
18	3½% Ni	2	51.5	53.2	87.3	149.6	29.2	54.2	1.62	112	1940	0	0
19	Cr-Mo	2	45.0	88.8	107.3	184.4	18.0	56.5	1.15	80	1380	+0.3	−0.3
20	3½% Ni	4	115.0	162.9	176.4	191.8	8.8	10.1	0.50	35	600	+5.2	−5.7
21	1.05% C	1	42.5	45.2	92.9	92.9	...	....	....	...	....	.....	.....
22	1.05% C	1	81.0	42.0	87.5	118.5	23.5	25.9	1.52	106	1820	.....	.....
23	1.05% C	1[a]	67.5	80.0	182.0	186.7	8.0	3.5	.38	26	460	.....	.....
23A	1.05% C	1	50.0	51.0	99.0	106.4	6.0	6.7	.	...	....	.....	.....
24	1.05% C	1	65.0	66.0	101.0	149.5	28.5	41.4	1.60	111	1920	.....	.....
25	Ni-Cr	4	128.5	162.8	177.5	234.8	11.0	37.5	1.27	88	1520	+0.9	−5.9
25A	Ni-Cr	2	76.0	81.2	114.5	202.8	23.0	59.9	1.55	108	1860	+0.9	−0.9
26	3½% Ni	4	86.5	120.6	141.3	208.4	17.5	46.4	1.28	89	1540	+3.2	−4.1
27A	Cr-Mo	3	127.3	181.5	194.7	250.3	10.0	38.0	1.22	85	1460	+3.0	−2.4
29	0.38% C	2	31.8	35.2	69.8	121.8	25.0	52.3				+0.8	−0.8
30A	Cr-V	3	94.7	141.8	157.0	236.8	12.8	49.9	1.22	85	1460	+5.8	−4.7
31	Half of log No. 25; refer to tests above.												
32, 33	0.44% C	4	74.6	87.4	124.8	161.8	17.0	33.4	1.33	92	1600	+1.4	−1.4
34	Chrome-iron	2	49.5	55.3	77.5	143.7	31.5	60.9	1.45	101	1740	0	0
35	Stainless iron	2	61.3	75.2	92.8	198.3	26.0	74.4	1.26	88	1510	+0.8	−0.8
36, 37	3½% Ni	2	49.8	58.4	99.2	143.0	24.5	39.8	1.48	103	1780	+0.3	−0.3

[a] This specimen cut at 90 deg. to the long axis of the forging.
[b] Area in square inches was obtained from charts in which 1 in = 1200-lb load and 1 in. = 0.4-in. strain.

high tempering temperature could be used subsequent to hardening to produce the desired physical properties. Hollow-bored forgings were prepared when tensile strengths were desired in the neighborhood of 135,000 to 175,000 lb. per sq. in., but despite the increase in speed of cooling and an increase in depth of hardening obtained by removing metal at the axis of large rounds prior to heat treatment, it was not possible to secure the highest tensile-strength values or a reasonable degree of uniformity in physical properties with certain of the steels. In such cases, the lathe tests were made on small rounds about 2 to 4 in. in diameter and 3 ft. long. The difficulties mentioned were largely restricted to the chromium-vanadium and carbon-chromium steels heat-treated to show tensile strengths around 150,000 to 175,000 lb. per sq. in.; no such difficulties were encountered with the 3½ per cent nickel, nickel-chromium, or chromium-molybdenum steels.

11 Care was taken that a reasonable degree of uniformity in physical properties was obtained at least throughout the portions of the metal removed in the cutting tests, but an additional precaution was taken to secure representative results by carrying out, as far as possible, all tests in "sequence." For example, in comparing the performance of high-tungsten–low-vanadium and high-tungsten–cobalt-steel tools, a test was first made with one type, then the other; the third test was made with the first type of steel and the fourth with the second, etc. The average tool life was referred to the average properties of the test log as determined upon specimens cut from metal at the starting and finishing diameters of the cutting tests. An appreciable amount of metal was also removed from the surface of the forgings before starting the lathe tests to eliminate scale and the large hardness gradients usually found next to the surface when large sections are quenched and tempered to show high hardnesses. With these precautions and care in the preparation of the tools, consistent results were obtained, as will be evident from data given in succeeding sections of this report.

12 The details of the chemical compositions, heat treatments, and tensile properties of the various test logs are summarized in Tables 2 and 3.

EXPERIMENTAL RESULTS

Relation between Cutting Speed and Tool Life in Dry Rough Turning

13 It is not practicable to select a single cutting speed for tool tests on steels varying from 65,000 to 195,000 lb. per sq. in. tensile strength when using fixed feed and depth of cut. Direct experimental determination of the "Taylor speed"[1] would give a

[1] This is the cutting speed which causes the tool to fail in exactly 20 minutes. The term originates from Taylor's recommendation that tool comparisons be made on this basis.

satisfactory basis for comparison of both tool performance and the metals cut but makes necessary the use of more tools, takes more time, requires that more material be cut, and is generally more expensive than a so-called "breakdown" or life test.[1] Obviously, what is needed is the relation between cutting speed and tool life so that from a breakdown test at one speed the life at other speeds may be computed (other conditions being the same).

14. Taylor (4) found that the change in life of a high-speed-steel tool with changing cutting speed could be represented approximately by an empirical equation:

$$VT^n = c \quad \text{. . . . . [1]}$$

in which V = the cutting speed in feet per minute
T = the tool life in minutes
c = a constant which is dependent for its numerical value upon the exact cutting conditions (other than speed). It will vary with the form and size of tools, material cut, steel (and treatments) from which the tools are prepared, feed, and depth of cut.

Working with carbon steels having a tensile strength in the neighborhood of 70,000 lb. per sq. in. with tools of definite size and shape made from steel containing about 1.9 per cent carbon, 8.5 per cent tungsten, and 2 per cent chromium, Taylor found $n = \frac{1}{8}$ and $c = 90$, but stated "we have made a number of experiments . . . . with different qualities of steel, and find that approximately the same relation exists between the duration of cut and cutting speed for steels of different degrees of hardness. This statement, however, does not apply to cast iron."

15 This empirical relationship between tool life and cutting speed was recently confirmed by the Lathe Tools Research Committee (5). In tests reported by the committee the numerical value of the constant c varied between about 120 and 150, but of greater importance is the fact that the law originally developed by Taylor was confirmed for durations of cut between about 3 and 100 minutes under conditions differing widely from those of Taylor's experiments.

16 However, not all investigators have confirmed Taylor's value of the exponent. Ripper and Burley (6) reported the value of $n = \frac{1}{12}$. Examination of the data published by the Lathe Tools Research Committee showed that it fitted as well the value $n = \frac{1}{7}$ as the value $\frac{1}{8}$ which they chose. The value $\frac{1}{7}$ also fits more closely some of the results previously published by the authors (2), as shown graphically in Fig. 2. Additional tests were run with the same shape of tool and same feed and depth of cut. The tests

[1] In this test the basis of comparison is the tool life (time for breakdown) under fixed cutting speed.

cover in all four different types of high-speed steel, cutting steels differing either in heat treatment or chemical composition.

17 The results given in Table 4 all agree sufficiently closely with $n = \frac{1}{7}$. It was decided, therefore, to use the equation

$$VT^{\frac{1}{7}} = c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

for all computations involving the relations between cutting speed and tool life.

18 The question whether this value of n would fit best a wider range of conditions is not important. The tool lives observed in

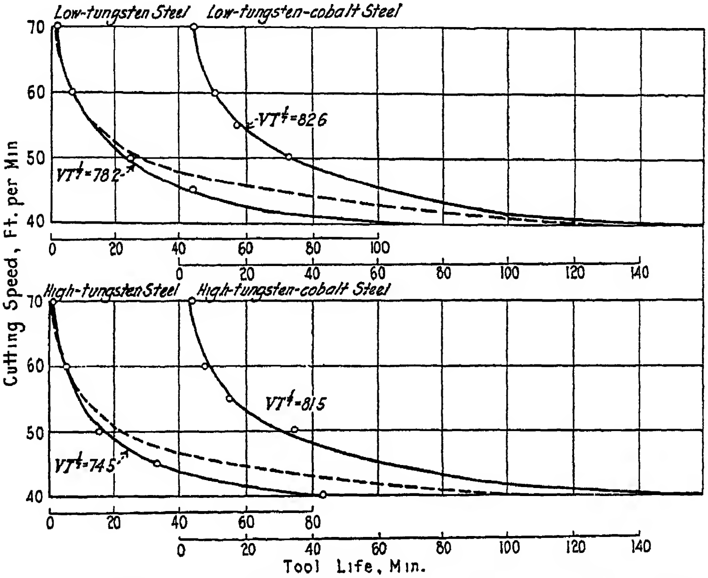


FIG. 2 SUMMARY OF SOME LATHE TESTS SHOWING THAT CUTTING SPEED OF HIGH-SPEED STEEL TOOLS VARIES INVERSELY AS $\frac{1}{7}$ POWER OF THEIR LIFE

Tests made at 0.028-in. feed and $\frac{3}{16}$-in. depth of cut

Experimental results indicated by circles. Solid lines represent equation $VT^{\frac{1}{7}}$ = constant; dotted lines, $VT^{\frac{1}{8}}$ = constant.

All tests made on $3\frac{1}{2}$ per cent nickel steel of about 100,000 lb. per sq. in. tensile strength, with $\frac{1}{2}$ by $\frac{1}{2}$ in. tools having nose radius of $\frac{1}{8}$ in., front and side clearance of 6 deg., side slope of 14 deg., and back slope of 8 deg.

Tools were tested "dry," with nose at dead center and 90 deg. cutting angle.

the main body of the investigation ranged between 2 and 20 minutes. The difference in computed Taylor speeds from any of these observations caused by a change from $n = \frac{1}{7}$ (the value adopted) to $n = \frac{1}{8}$ (Taylor's original value) would in no case be greater than 5 per cent, which is within the limits of reproducibility of results.

19 For convenience in computing the Taylor speeds or the life of tools at various speeds, Fig. 3 was prepared. With the logarithmic coördinates used, Equation [2] is represented by

straight lines, which makes it easier to interpolate with accuracy between values of the constant c shown on the chart than would be the case with the curves obtained with cartesian coördinates (compare with Fig. 2).

Machinability of Carbon and Special Steels from the Standpoint of Lathe Roughing Tools

20 The economic advantages derived from an improvement in machining qualities of structural materials are generally well recognized, as is also the fact that in an era of mass production

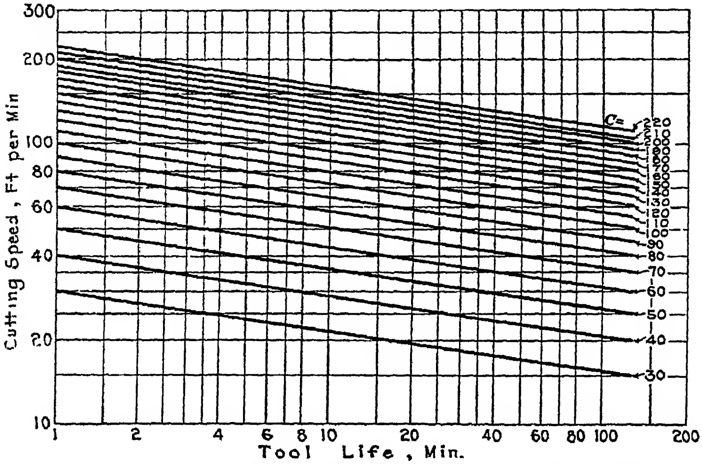


Fig. 3 Chart Giving the Relation Between Cutting Speed and Tool Life

This applies only to rough turning of steels with high-speed steel tools. Lines drawn according to equation

$$VT^{\frac{1}{8}} = c$$

in which

V = cutting speed
T = tool life
c = constant

small gains in this direction may result in exceedingly large savings and increased output. It is therefore hardly necessary to mention again the importance of a study of machinability of different structural steels now widely used industrially and subjected to many cutting and forming operations.

21 The subject is a very large one, and it may be well to emphasize at this point the fact that the experiments to be described are restricted to a type of machining commonly referred to as "hogging" or rough turning. No attempt should be made, without additional data, to apply conclusions to other forms or conditions of cutting. The service to which the test tools were subjected is similar to that found in Taylor's well-known studies, and is more

directly related to the preliminary machining of large sections, such as gun forgings, heavy machine parts, etc., than to the finer cuts taken on small sections.

22 The terms, "machinability" and "machining qualities," have undoubtedly been loosely employed and to some have meant the finish produced on the work, to others the power required in removing metal and in some cases the speed with which machine tools may be operated or the length of time before tools must be dressed or reground, etc. Any or all of these factors may be important under certain conditions, and can properly be considered to contribute to what has been called machinability. However, the primary object in rough turning is to remove the maximum amount of metal in minimum time at the least cost. Under such conditions the most important factor is tool life, and steels which, under otherwise fixed conditions, permit the longest cuts without regrinding of the tools may be said to be the most readily machinable or to have the highest degree of machinability, because the power required and the finish produced are of secondary importance. It is on the basis that "machinability" is proportional to tool life (and conversely to cutting speed permitting a definite tool life) that this term is used in subsequent sections of this report; it cannot be used in the same sense for types of cutting in which other factors are of as great if not greater importance.

23 Taylor (4) pointed out that "the physical properties of steel constitute a fairly accurate guide to its cutting speed (speed which causes the tool to fail in a fixed time); and these properties are best indicated by the tensile strength and percentage of stretch and contraction of area obtained from standard tensile test bars cut from such a position in the body of the forging as to represent its average quality and then broken in a testing machine."

24 However, "no absolute and accurate relation exists between the cutting speeds of forgings whose chemical, and even physical, properties very closely agree."

25 Despite this very definite statement from Taylor, which was based on extended experience, it appears to be common practice to use Brinell hardness as a criterion of cutting qualities [refer for example to the work of the Lathe Tools Research Committee, reference (5)]. While justified under certain conditions, such relations are at best only a general indication, and many exceptions can be cited even for a restricted class of metals such as structural carbon or alloy steels.

26 Allcut and Miller (7), in their book on engineering materials, give a tabular comparison of Brinell hardness, tensile strength, and approximate cutting properties of carbon and alloy steels, a partial summary of which is shown graphically in Fig. 4. The assigned values are stated to be based upon general observations

in commercial machining and are expressed as a proportion of the cutting properties of 0.20 to 0.25 per cent carbon steel having a tensile strength of about 56,000 lb. per sq. in. (25 tons per sq. in.) which has been given a value of 100 for turning with a single-point tool. The limit of commercial machining is stated to be about 30, which corresponds approximately to a Brinell hardness of 207 for drilling and 225 to 250 for turning, planing and milling.

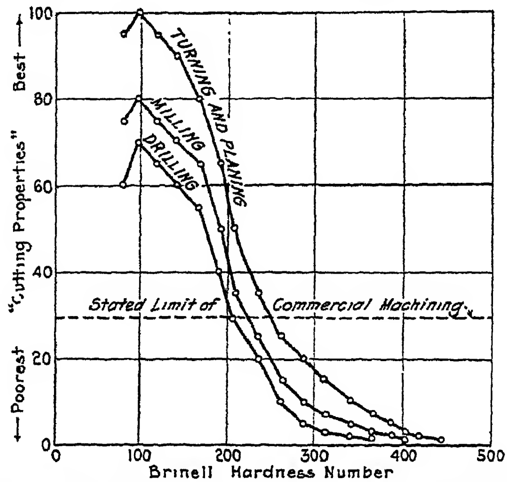


FIG. 4 RELATIVE CUTTING PROPERTIES OF STEELS AS REPORTED BY ALLCUT AND MILLER

Note that steels are designated only by Brinell hardness. For discussion of these data refer to text.

27 These values are exceedingly low for some types of cutting and quite different from limits found in commercial practice in the United States.

28 The comparisons of Allcut and Miller show that large increases in Brinell hardness adversely affect the machining properties of steels, a fact which is very well known, but in other respects they are liable to be misleading, for, as has previously been indicated, Brinell hardness is no certain criterion of the behavior of steels in machining.

29 As an example, there may be cited the investigations of Kessner (8) who has taken as a measure of machinability the penetration of a drill in 100 revolutions under fixed speed, pressure, etc. The results of a few of his experiments are summarized in Table 5 and show that even in one type of metal there is no quantitative relation between drill penetration and Brinell hardness or ball indentation tests. Emmons (9) has discussed in considerable detail the structure of carbon tool steels in relation to different cutting and forming operations, and showed that the best structures for one type of cutting were not necessarily the best or even good for others.

TABLE 4 SUMMARY OF TESTS SHOWING THAT THE CUTTING SPEED IN DRY ROUGH TURNING IS INVERSELY PROPORTIONAL TO THE 1/7 POWER OF TOOL LIFE

(All tests made dry with 0.028-in. feed, $\frac{3}{16}$-in depth, and the form of tool described in the text)

Test log			Tools used		Cutting speed, ft. per min.	Average tool life in minutes		Assigned value of the constant c in $VT^{\frac{1}{7}} = c$
No.	Type of steel	Tensile strength, lb per sq. in.	Type of high speed steel	Lot		Determined experimentally	Computed [c]	
2	3½% Ni	85,600	High-tungsten–low-vanadium	E	80	45.9	44.5	137.6
					90	19.5	19.5	
					105	7.1	6.6	
1	3½% Ni	94,600	High-tungsten–low-vanadium	E	70	43.6	42.2	119.5
					80	16.6	16.6	
					90	7.1	7.3	
6	Cr-Mo	96,400	Low-tungsten–high-vanadium	H	70	16.7	16.9	104.8
					80	7.1	6.6	
					90	2.8	2.9	
9	Cr-V	93,600	High-tungsten–low-vanadium	E	70	31.5	25.2	111.0
					80	9.9	9.9	
					90	4.6	4.3	
5	0.44% C	107,000	High-tungsten–low-vanadium,	E	70	7.3	7.3	93.0
					80	3.7	2.9	
			High-tungsten–cobalt	G	70	11.6	11.6	99.4
					80	6.6	4.6	
14A	Ni-Cr	150,200	High-tungsten–low-vanadium	I	45	6.5	6.5	58.8
					53	2.5	2.1	
[a]	3½% Ni	115/125,000	Low-tungsten–high-vanadium	C	45	43.4	47.7	78.2
					50	24.7	22.8	
					60	6.4	6.4	
					70	2.2	2.2	
[a]	3½% Ni	115/125,000	Low-tungsten–cobalt	A	50	32.4	33.4	82.6
					55	16.1	17.3	
					60	9.9	9.3	
					70	3.3	3.2	
[a]	3½% Ni	115/125,000	High-tungsten–low-vanadium	D	40	(83.2)[b]	77.6	71.5
					45	32.8	34.1	
					50	15.0	16.3	
					60	5.3	4.6	
					70	1.4	1.5	
[a]	3½% Ni	115/125,000	High-tungsten–cobalt	B	50	34.2	30.4	81.5
					55	14.6	15.6	
					60	7.6	8.5	
					70	3.1	2.9	

[a] These tests, taken from a previous investigation [see reference (2)], were made on several logs, the average values of tool life shown in the table are based on a like number of tests at each cutting speed on each test log.

[b] Where tool life is given in brackets, it is approximate, as some of the tools lasted too long to run to failure.

[c] Computed from $VT^{\frac{1}{7}} = c$ = values shown in next column

TABLE 5 SUMMARY OF SOME DRILL TESTS REPORTED BY KESSNER [a]

Material cut	Chemical composition, per cent						Ball penetration test Pressure to give a depth of impression of 0 05 mm , kg.	Machinability as compared with that of hollow-drill steel [b]	Brinell hardness number
	C	Mn	P	S	Si	Ni			
Hollow-drill steel	0.85	0.26	0.085	trace	0.05	....	..	100	207
Punch steel	0.70	0.51	0.022	0.026	0 21	0.25	...	104	144
Soft steel for use in turret lathe	0.22	0.54	0.025	0.085	0.20	....	...	95	120
Steel for crane hooks	0.16	0.53	0.041	0.029	0 18	....	..	165	159
Wrought iron	0.07	0.16	0.147	0.016	0.10	....	..	118	113
Cast iron	3.11	0.50	0.96	0 068	1.76	....	..	316	141
								as compared with copper 100	
Machine steel	....	....	.....	.....	....	....	250	298	..
Machine steel	....	....	....	....	....	....	205	252	..
Machine steel	....	....	.....	.....	....	....	190	204	..
Machine steel	....	....	.....	.....	....	....	174	268	...
Machine steel	....	....	.....	.....	....	....	143	162	...
Machine steel	....	....	.....	.....	....	....	128	111	...
								as compared with brass without lead = 100	
Brass (60 : 40 Cu : Zn type) no lead	....	...	.....	.....	...	....	20	100	...
0.5% lead	....	....	.....	.....	...	....	22	255	...
1% lead	....	....	.....	.....	..	....	24	495	...
2% lead	....	....	.....	.....	....	....	26	790	...
4% lead	....	....	.....	.....	...	....	25	1150	...
8% lead	....	...	.....	.....	....	....	22	1270	...
12% lead	..	....	...	.....	...	....	21	1300	...

[a] Refer to *Testing*, I, p. 270 (1924) for details of these experiments.
[b] Machinability is measured by the penetration of a drill in 100 revolutions under fixed conditions of cutting

30 Haughton of the National Physical Laboratory in England was recently credited with the statement (10) that of the "high-tensile steels," that with 3 per cent chromium had machined better than the nickel-chromium structural steels and much better than the customary nickel steels of equal hardness.

31 Of late, considerable emphasis has been placed upon the desirable machining qualities of the structural alloy steels containing molybdenum in comparison with nickel-chromium and $3\frac{1}{2}$ per cent nickel steels when heat-treated to show comparable tensile strength or hardness. The chromium-molybdenum steels containing from about 0.5 to 1 per cent chromium and from 0.15 to about 0.35 per cent molybdenum have been the ones generally used as the basis for such comparisons, but little detailed information has been published to substantiate the superiority claimed. In a comparatively recent report relating to this question Pierce (11) claimed an increase in production for certain drilling, boring, reaming, and tapping operations, amounting to about 10 to 25 per cent when the nickel-chromium steel being machined was replaced by chromium-molybdenum steels of the customary commercial types.[1]

32 While these reports contain much useful information, they refer in most cases to types of cutting quite different from that associated with the original development of high-speed cutting tools. Likewise, they deal largely with the machining of particular types of steel or with comparisons of a few steels (and other metals) within relatively narrow ranges of hardness. They do not all contain sufficient detail to permit a check upon the validity of the conclusions and so are not always convincing.

33 Three factors are recognized as affecting the machining properties of steel forgings: their chemical composition, the heat treatment to which they are subjected subsequent to hot working, and the quality of the metal.[2] While most attention will be given to the first two of these variables, the experiments which were carried out throw some light upon the third.

[1] Through the courtesy of J. D. Cutter, Vice-President and Metallurgist of the Climax Molybdenum Co., New York, the authors have had for examination similar but more extensive production records which were stated to have been obtained at different plants.

[2] The term "quality" refers to those details of melting, casting, and working not defined by ordinary chemical analysis and a statement of the heat treatment, which combine to make the metal especially well suited for particular service. As so-called high quality for one purpose may be inferior for different service, the term is general in nature as used in this report. It simply implies recognition that there are other factors beside chemical composition (as now determined) and the heat treatment which contribute to the final properties of the metal.

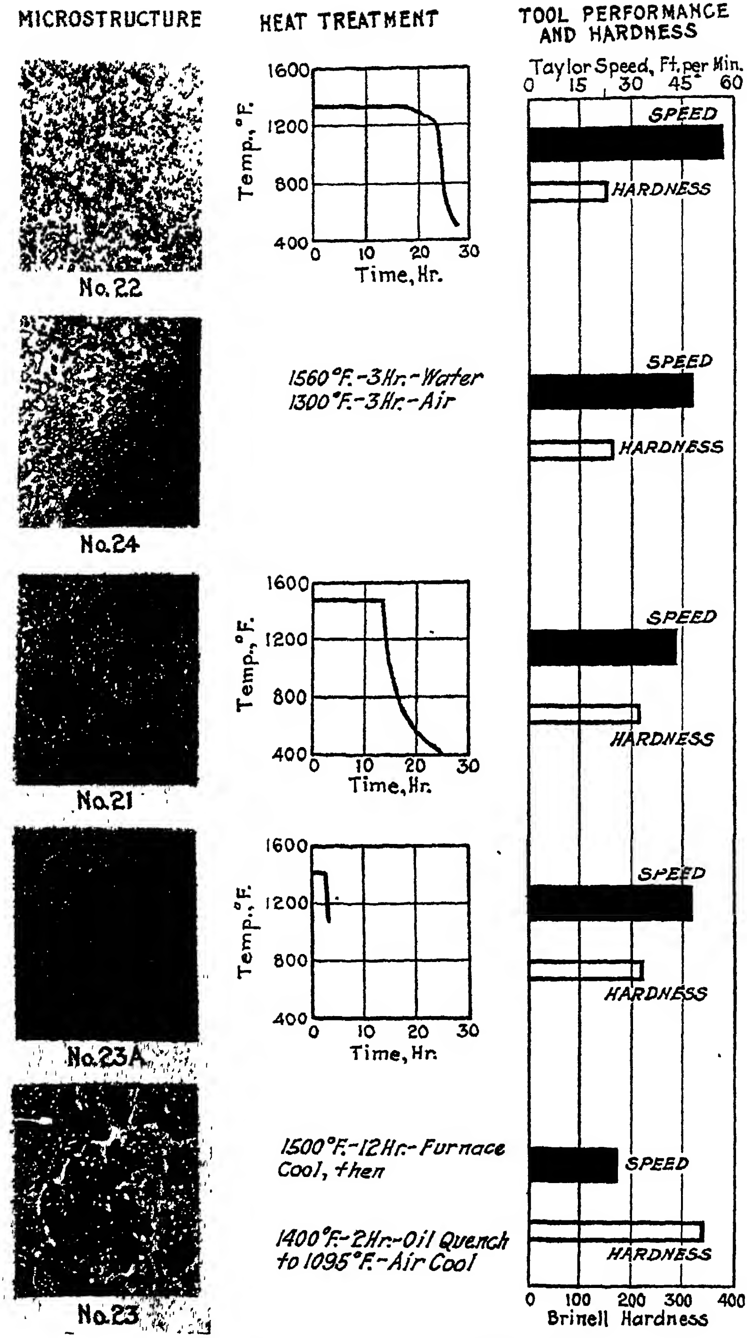


Fig. 6 See caption at foot of following page

Carbon Steels

34 No new tests were made to show the effect of carbon upon the cutting speeds of annealed or hot-finished carbon steel forgings, as this subject formed the basis of many of Taylor's experiments. His results are reproduced in part in Fig. 5, as they have a bearing upon some of the tests made in this investigation; they show that the cutting speeds vary for different forgings whose chemical and physical properties very closely agree but that appreciable increases in carbon tend to reduce the cutting speeds. For example, under the conditions of Taylor's experiments an

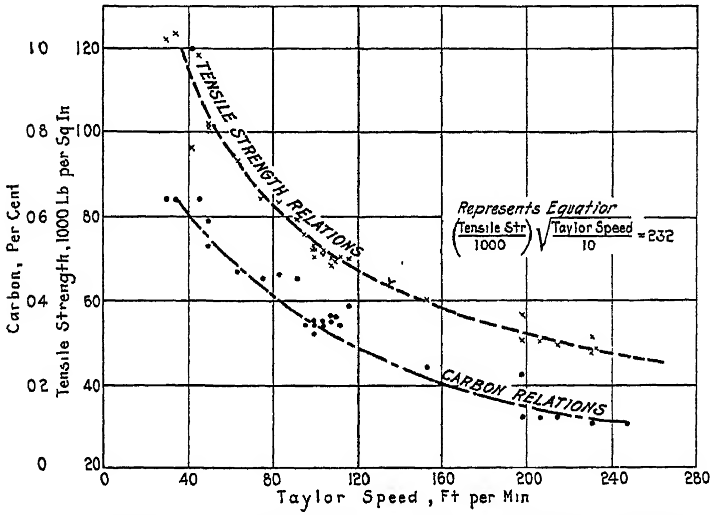


Fig. 5 Summary of Taylor's Lathe Tests Showing Effect of Carbon Content and Tensile Strength on Cutting Speed of Annealed or Hot-Worked, Carbon-Steel Forgings

Tests made dry with ⅞-in. tools containing 0.68% C, 5.75% Cr, 18.0% W, 0.3% V, a 3/16-in. depth and 1/16-in. feed.

increase in carbon from about 0.15 to 0.35 per cent, which corresponds to an increase from 50,000 to 70,000 lb. per sq. in. in tensile strength, has lowered the cutting speed from around 200 to 100 ft. per min.; likewise an increase from 0.35 to 0.55 per cent carbon has lowered the speed from about 100 to 50 ft. per min.

35 Taylor did not so fully follow the effects of variations in annealing practice, particularly with respect to the cutting of high carbon or so-called hypereutectoid steels, and therefore this

---

Fig. 6 Correlation of Lathe-Tool Performance with Heat Treatment, Brinell Hardness, and Microstructure of a 1.05 Per Cent Carbon Steel

Lathe tests were made with high-speed steel I (Table 1) and tools of form described in text, run dry with a depth of cut of 3/16 in. and a feed of 0.028 in. The photomicrographs are at a magnification of 500 diameters. Samples etched with 2 per cent nitric acid in alcohol.

subject was the basis of a set of experiments which are summarized graphically in Figs. 6 and 7. Their purpose was not only to show differences in tool performance arising from changes in the condition of the carbon in the steel cut, but also to correlate with structures recognized industrially as good and bad for different types of machining the results of the lathe breakdown tests used as a basis for comparisons in this report.

36 The experiments were made on four forgings from one melt of 1.05 per cent carbon steel to show the effect of different methods and degrees of spheroidization upon the cutting speed in comparison with the pearlitic steel. Test log 24 (Fig. 6) represents spheroidization from an initial structure of sorbite; test

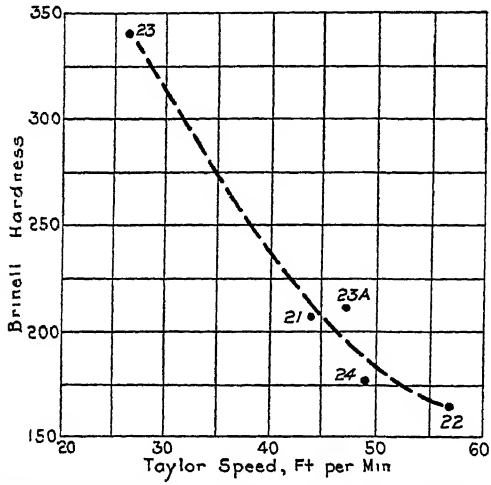


FIG. 7 RELATION BETWEEN CUTTING SPEED AND BRINELL HARDNESS OF 1.05 PER CENT CARBON STEEL

Based on data included in Fig. 6.

log 23 A, the spheroidization from an initial structure of pearlite; test log 21, nearly 100 per cent pearlite; and test log 22, a commercially spheroidized product. Through an error of the supplier one of the four forgings was oil quenched in such a manner as to produce sorbite surrounded by envelopes of cementite and the lathe tests made prior to retreatment give the fifth condition shown as test log 23, in Fig. 6.

37 With the exception of test log 23, which is not to be classed with the annealed steels, the different forgings vary in two respects; first, the amount of cementite brought out of solution, and secondly, the form in which it exists, i.e., globules as in the spheroidized samples, or plates as in pearlite.

38 The data given in Figs. 6 and 7 show that both factors have an influence upon the cutting speed. For example, the form and distribution of the cementite in test log 22 is quite similar

to that in test log 24, yet the latter, which appears to have less free cementite corresponding to the somewhat higher Brinell hardness, has an appreciably lower cutting speed. On the other hand, the Brinell hardness of test log 21 consisting of nearly 100 per cent lamellar pearlite is practically the same as that of test log 23 A, which has a mixed structure of granular and lamellar pearlite, and these structural variations likewise correspond to a measurable difference in cutting speeds.

39 The most desirable structure from the standpoint of rough turning is that produced by the prolonged annealing (commercial spheroidizing) applied to test log 22, in which a large cementite divorce is associated with its spheroidization. The mere absence of plates of cementite does not insure a high cutting speed (compare the speeds for test logs 23 A and 24 with 22), but even partial spheroidization is better than the wholly pearlitic steel.

40 The differences in cutting speed just referred to, and shown in Figs. 6 and 7, are not large and it may therefore be well to throw further light upon their relation to commercial rough turning. Comparisons may be made on the basis of the weight of the steel removed per hour when taking a $\frac{3}{16}$-in. depth of cut at 0.028-in. feed and a speed which in each case permits a 1-hour tool life. Under such conditions it is possible to remove only about 90 per cent as much metal from the rapidly spheroidized steel (test log 24), and about 80 per cent from the pearlitic sample (test log 21), as from the steel subjected to the long-time spheroidizing treatment (test log 22).

41 As has already been pointed out for other types of cutting (9), the rounded globules of the hard cementite undoubtedly have a smaller tendency to cut away the tool than do the sharp or angular particles of cementite in pearlitic steels. However, from the standpoint of rough turning, where there is less danger than for other types of machining of getting the steel too soft, and where "tearing" and the character of the finish produced, etc., need not be considered, it is also important to get a large amount of the carbon out of solution. This increases the number of the cementite globules, gives more hard particles to cut away the tool and hence tends to lower the cutting speeds; at the same time it reduces the carbon in the matrix, and according to the results of Taylor, reproduced in Fig. 5, this should tend appreciably to increase the cutting speed. In rough turning, as is indicated by data in Figs. 6 and 7, the reduction of carbon in the matrix seems to more than counterbalance the undesirable effects from more free cementite, provided the latter is in small globules.

Special Steels

42 The most attention was given to the rough turning of the special steels such as are used in automotive construction, for

ordnance, railroad equipment, special machine parts, etc. The group studied consisted of the following:

1 Carbon-chromium steel containing about 0.3 per cent C and 1 per cent Cr
2 Chromium-vanadium steel containing about 0.3 per cent C, 1 per cent Cr, and 0.2 per cent V
3 High-chromium "stain-resisting" steels (perhaps better called irons) containing not more than 0.15 per cent C and more than 11 per cent Cr
4 3½ per cent nickel steel containing about 0.3 per cent C
5 Nickel-molybdenum steel containing about 0.3 per cent C, 3½ per cent Ni, and 0.35 per cent Mo
6 Chromium-molybdenum steels containing 0.3 per cent C, 0.5 to 1 per cent Cr, and 0.15 to 0.35 per cent Mo
7 Nickel-chromium steels containing about 0.5 per cent C, 2 to 2½ per cent Ni, and 1 to 1¾ per cent Cr.

Lathe tests were also made on some 0.45 per cent carbon-steel forgings for comparisons with the special steels.

43 The tool tests were made after most of the test logs had been heat-treated to show equal tensile strengths throughout the range 80,000 to 175,000 lb. per sq. in. and with a few exceptions the desired properties were obtained by quenching and tempering (refer to Table 2).

44 The majority of the steels were heat-treated by the suppliers who were apparently not always able to develop the exact tensile strengths required. Re-treatment by the authors brought desired results in one or two cases, but difficulties arising from the limited equipment available for handling large masses made it advisable to use some of the test logs with tensile strengths either higher or lower than the desired values. However, this does not, in general, limit the comparisons which may be made, as sufficient tests were carried out to establish the relations between the cutting speed and the tensile strength for most of the types of steel cut.

45 The high-chromium stain-resisting steels were used as received from the manufacturers and represent the conditions in which these metals are ordinarily applied industrially.

46 Comparisons of the different steels cut may conveniently be made on the basis of equal tensile strengths and the Taylor speed taken as a measure of machinability in rough turning.

47 To simplify discussion, all Taylor speeds are given in terms of the performance of tool steel E (Table 1), except in Tables 6 and 7 containing a tabular summary of all the lathe tests. The basis of conversion of tests with other brands into the equivalent performance of high-speed steel E is described later in the report.

Structural Alloy Steels

48 As shown in Fig. 8, a steel which permits a higher cutting speed than another at some one tensile strength does not neces-

sarily do so at another tensile strength. Contrary to conclusions reached in previous investigations relating to other types or conditions of cutting, the tests here reported show that steel containing 0.3 per cent carbon and 3½ per cent nickel have better machinability at any selected tensile strength up to about 160,000 or

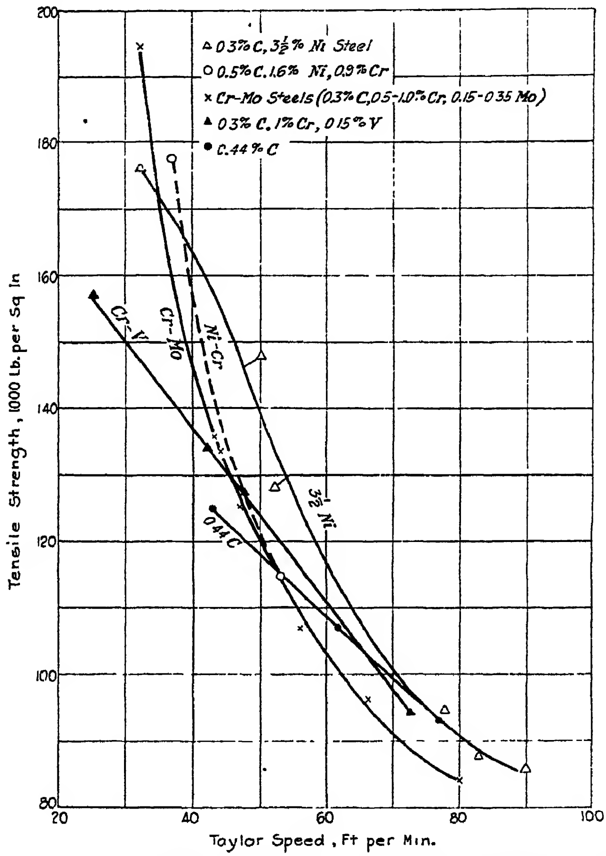


Fig. 8 Relation of Taylor Speed to Tensile Strength in Rough Turning Quenched and Tempered Special Steels

For details of chemical compositions and heat treatments of steels cut and high-speed tools used in these tests refer to Tables 1 and 2. The lathe tests were made with the form of tool described in text, cutting "dry," with $\frac{3}{16}$-in. depth of cut and a feed of 0.028 in. per revolution, and all results are given in terms of tool steel E, Table 1.

170,000 lb. per sq. in. than steels of similar or higher carbon content containing 1 per cent chromium with or without vanadium or molybdenum. The same applies to a comparison between the 3½ per cent nickel and the somewhat higher-carbon nickel-chromium steels. However, above about 170,000 lb. per sq. in. tensile strength the order of superiority is reversed and the chromium-

molybdenum and nickel-chromium steels permit higher cutting speeds.

49 A noteworthy feature is the very rapid drop in cutting speed with increase in tensile strength of the chromium-vanadium steel. Tensile strengths over about 140,000 lb. per sq. in. are only obtained at the expense of machinability. For example, under the conditions of the experiments the Taylor speed for the chromium-vanadium steel having a tensile strength of 140,000 lb. per sq. in. was about 37 ft. per min.; it dropped to 26 ft. per min. when the tensile strength was raised to about 155,000 lb. per sq. in. Contrast this with the change for a corresponding increase in tensile strength in the chromium-molybdenum steel for which the speeds were respectively about 42 and 38 ft. per min.

50 While the chromium-vanadium steel shows the most rapid decrease in cutting speed with increase in tensile strength of the different special steels, the loss is not so marked as in the plain carbon steel containing 0.44 per cent carbon (Fig. 8).

51 These results indicate that the effect of changes in the chemical composition of steel forgings upon their cutting speeds is dependent upon the tensile strength (or hardness) at which comparisons are made. Since the highest tensile strength values considered in these tests are only produced by quenching, with or without subsequent tempering, and represent martensitic structures (refer to Fig. 11), changes in chemical composition may act in opposite directions, depending upon whether the steel cut is close to the fully hardened (martensitic) or the annealed or "natural" (pearlitic) condition.

52 In other words, the character and slope of tensile-strength–Taylor-speed curves varies for different structural steels. The curves for any two compositions may cross and so reverse the order of superiority. At the same time if the slopes are not very different there will be an appreciable range in tensile strength (or hardness) at which for all practical purposes the cutting speeds are the same.

53 From the summaries of results given in Figs. 8 and 9, and those in Fig. 5, it will be seen that, when approaching the annealed or natural condition, increase in carbon or the addition of any of the special elements considered has a tendency to lower the cutting speed.

54 For example, Fig. 8 shows that at a tensile strength of about 95,000 lb. per sq. in. the chromium-molybdenum steel has a lower cutting speed than that of the chromium-vanadium steel. Since both types have similar proportions of carbon and chromium (refer to Table 2 and Fig. 9), the lower cutting speed of the chromium-molybdenum steel may be ascribed to the molybdenum.

55 The chromium-vanadium steel, in turn, has a somewhat lower cutting speed than the 3½ per cent nickel steel. No direct

comparisons are available between the latter and plain carbon steel of similar carbon content, but an idea may be obtained of the effect of nickel by comparison of results in cutting test logs

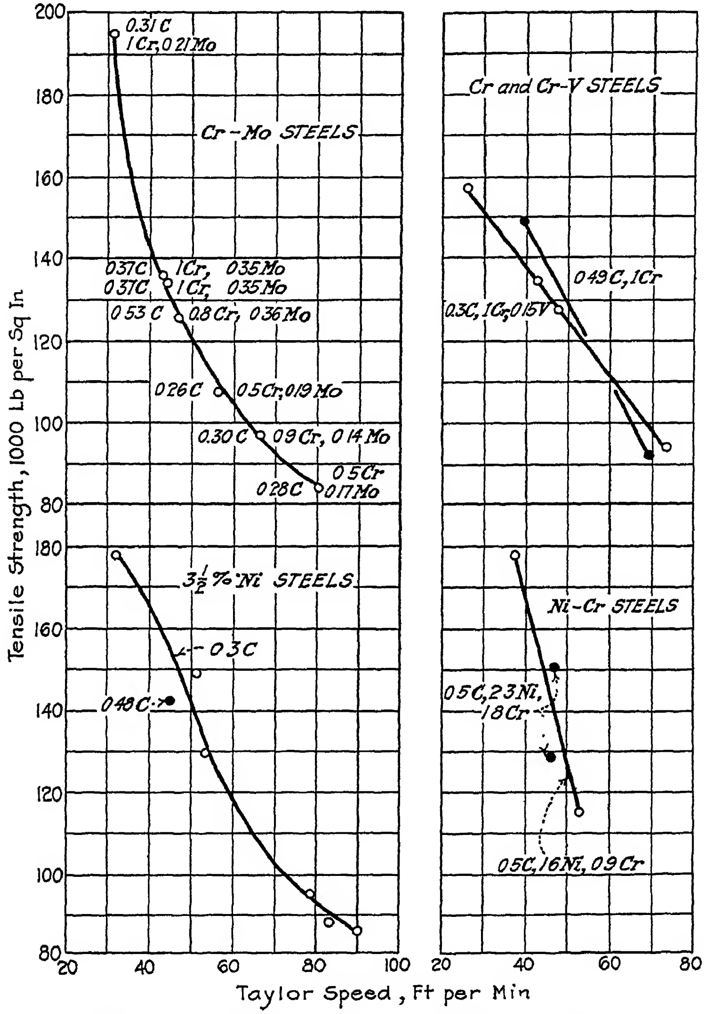


FIG. 9 EFFECT OF MODIFICATIONS IN CHEMICAL COMPOSITION OF VARIOUS TYPES OF STRUCTURAL ALLOY STEELS ON RELATION OF CUTTING SPEED TO TENSILE STRENGTH

All results given in terms of tool steel E, Table 1. The lathe tests were made with form of tool described in text, cutting dry with $\frac{3}{16}$-in. depth of cut and feed of 0.028 in. per revolution. For the heat treatments of test logs refer to Table 2.

3 and 1. These have practically the same tensile strength (respectively 92,800 and 94,600 lb. per sq. in. as shown in Table 3) and practically identical cutting speeds (given in Table 7 as respec-

tively 77 and 78 ft. per min.). Test log 3 contains 0.44 per cent carbon, while test log 1 contains 0.26 per cent carbon and 3.8 per cent nickel. Since increase in carbon has been shown to reduce the cutting speed, the equality in these two steels means that the addition of 3½ per cent of nickel has had about the same effect as an increase from 0.26 to 0.44 per cent of carbon. This is not intended as a quantitative comparison but merely to demonstrate that, like the other elements considered, nickel has a tendency to reduce the cutting speeds. However, the reduction produced by 3½ per cent of nickel is less than that from 1 per cent of chromium. (Refer to results in both Figs. 8 and 9.)

56 In steels approaching the fully hardened (martensitic) condition the cutting speeds depend upon how the chemical composition affects the characteristics of the martensite or other products produced by heat treatment. The structures produced by quenching steel containing 0.3 per cent carbon and 1 per cent chromium have a higher ratio of cutting speed to tensile strength than those in 0.44 per cent carbon steel but not so high as those in the 3½ per cent nickel steel. The ratios for the latter are likewise not so high as those in the chromium-molybdenum or the nickel-chromium steels.

57 Elements which cause a lowering in the cutting speeds of the annealed steels may improve the cutting speeds when considering higher tensile strengths approaching those of the martensitic condition. The most effective of the special elements in the different steels cut are the combinations of nickel and chromium or chromium and molybdenum, but it should be recognized that carbon increase may also be beneficial. This is shown by comparison of the chromium and chromium-vanadium steels in Fig. 9; the low carbon chromium-vanadium steel has the highest cutting speed at low tensile strengths but the lowest cutting speed at high tensile strengths.

58 The reasons why certain alloys, which tend to reduce the cutting speeds in annealed steels, permit the highest cutting speeds in the quenched metals are obscure. Favorable machining properties previously reported for chromium-molybdenum steels have been ascribed to decided air-hardening tendencies and the fact that these steels resist tempering. The result is that rather high tempering temperatures may be used subsequent to hardening in comparison with those required to retain equally high tensile strengths in other steels. These features are associated in some cases with superiority in cutting speeds but cannot be accepted as an explanation.

Effect of Method of Heat Treatment on the Cutting Speed

59 If high cutting speeds are dependent upon tempering temperature it should be possible by varying the dimensions of the

TABLE 6 SUMMARY OF LATHE TESTS ON DIFFERENT STEELS AT 0 028 IN FEED AND $\frac{3}{16}$-IN. DEPTH OF CUT [a]

Test log				Average tool life in minutes for type of high-speed steel indicated [b]						Power required in cutting, Kw		
				High-W. Low-V		Low-W. High-V		High-W. Co				
Cutting speed, ft. per min.	No.	Steel type	Tensile strength, lb per sq in	Mark	Life, min	Mark	Life, min.	Mark	Life, min.	High-W. low-V tools	Low-W. High-V tools	High-W. Co tools
70	1	Ni	94,600	E	43 6	..	....	..	...	2 8	...	...
80	1	.....	......	E	16 6	..	....	G	20.8	3.2	...	3.4
90	1	......	......	E	7.1	..	.. .	..	....	3.5	...	...
80	2	Ni	85,600	E	45 9	..	....	..	....	3.0	...	.
90	2	......	......	E	19.5	H	12 8	G	27.6	3.5	3 4	3 4
105	2	......	......	E	7.1	..	....	..	....	3.9	...	.
90	3	0.44% C	92,800	E	7.1	H	6.7	G	10.0	2.9	3 0	3 0
70	5	0.44% C	107,000	E	7.3	H	8 0	G	11.6	2.6	2 6	2 5
80	5	......	......	E	3.7	..	....	G	6 6	2.8	...	3.0
70	6	Cr-Mo	96,400	.	. ..	H	16.7	..	...	...	2 7	.
80	6	......	......	E	5.6	H	7.1	G	12 7	3.1	3 1	2 9
90	6	......	......	..	.. .	H	2.8	..	...	...	3 4	.
55	7	Cr-Mo	125,000	E	7.3	H	15.0	G	10.9	2.5	2 4	2.5
90	8	Cr-Mo	83,700	E	8.8	H	11.0	G	13 6	3.1	3.2	3 3
70	9	Cr-V	93,600	E	31.5	..	....	..	....	2.9	...	..
80	9	......	......	E	9 9	H	15 4	G	13.0	3.2	3 4	3 5
90	9	......	......	E	4.6	..	....	..	....	3.6	...	..
55	10	Cr-Mo	133,500	E	4.8	H	5.1	G	9.4	2.5	2.6	2 5
35	10A	Cr-Mo	135,500	I	15.0	..	....	..	....	1.8	...	...
45	10A	......	......	..	....	..	....	K	9.8	...	...	2.3
55	11	Cr-V	127,000	E	6.7	H	6 4	G	17.4	2.3	2.4	2.4
40	11A	Cr-V	134,000	I	3.9	..	....	..	....	2.2	...	. .
45	11A	.... .	......	..	....	..	....	K	11.0	...	...	2.2
55	12	Ni	128,700	E	14.6	H	13 4	G	22.9	2.3	2.3	2.2
45	12A	Ni	148,800	I	9.8	..	....	..	....	1.9	...	..
50	12A	......	......	.	....	..	.. .	K	10.7	...	...	2.2
55	13	Ni-Mo	127,700	E	10.7	H	7 4	G	14.5	2.5	2.5	2.5
55	14	Cr-Ni	128,000	E	6 2	H	5.8	G	13.9	2.3	2.3	2.3
45	14A	Cr-Ni	150,200	I	6.5	..	....	..	....	2.3	...	.
50	14A	......	......	..	....	..	....	K	6.1	...	...	2.8
53	14A	......	......	I	2.5	..	....	..	....	2.8	...	...
80	15	C-Cr	91,900	E	7.8	H	6.2	G	11.4	3.0	3.1	3.1
35	15A	C-Cr	148,400	I	6.0	..	....	..	....	1.8	...	.
40	15A	. ....	......	....	....	..	....	K	9.6	...	...	2.0
140	16	0.18% C	66,400	M	3.4	L	3 2	..	....	5.9	6.4	..
85	17 & 18	Ni	87,300	I E 21	7.4 6.3	L ..	11.0	.. ..	....	3.6 3.6	3.7 ...	... .
55	19	Cr-Mo	107,300	I	6.3	..	....	..	....	3.0	...	..
30	20	Ni	176,400	I	2.5	..	....	..	....	1.9	...	..
55	21	1.05% C	92,900	I	4.5	..	....	..	....	2.6	...	...
65	22	1.05% C	87,500	I	7.7	..	..	..	....	2.9	...	...
35	23	1.05% C	132,000	I	3.1	..	....	..	....	2.0	...	...
55	23A	1.05% C	99,000	I	5.8	.	.. .	..	....	2.6	...	..
55	24	1.05% C	101,000	I	8.9	..	.	..	....	2.8	...	..
65	25A	Cr-Ni	114,500	E	5.1	..	..	..	....	3.2	...	...
33	25	Cr-Ni	177,500	I	5.0	.	..	K	10.7	2.2	...	2.1
40	26	Ni	141,300	I	7.5	.	...	..	....	2.2	...	..
45	26	......	......	..	....	..	.. .	K	9.2	...	...	2.3
25	27A	Cr-Mo	194,700	I	9.9	..	.. .	..	....	1.6	...	..
125	29	0.33% C	69,800	N	1.9	..	....	..	....	6.1	...	...
20	30A	Cr-V	157,000	I	3 0	..	....	..	....	1.7	...	..
55	32 & 33	0.44% C	124,800	E	3 4	..	....	..	....	2.5	...	...
80	34	Chrome iron	77,500	E	8.5	..	....	..	....	3.8	...	.
75	35	Stainless iron	92,800	E	10.6	..	....	..	....	3.5	...	..
75	36	Ni	99,200	..	....	L	11.0	..	....	...	3.4	. .

[a] All tests were made dry with the selected form and size of tool described in the text.
[b] In general these values were obtained from at least 8 tests representing 4 tools each tested after two grindings.

test log and the methods of heat treatment to produce in one steel structures which have different cutting speeds but equal tensile strengths. As shown in Table 8 the cutting speed at a given tensile strength is not appreciably affected by the method of heat

TABLE 7 TAYLOR SPEEDS FOR VARIOUS STRUCTURAL STEELS

Test log			Taylor speed for the high-speed steels indicated [a]					
		Tensile strength, 1000 lb. per sq. in.	High-tungsten		Low-tungsten		High-tungsten, Cobalt	
No.	Steel type		Lot	Ft. per min.	Lot	Ft. per min.	Lot	Ft. per min.
				Carbon Steels				
16	0.18% C	66.4	M	108	L	107	..	..
29	0.33% C	69.8	N	90	..	...	..	.
3	0.44% C	92.8	E	77	H	76	G	81
5	0.44% C	107.0	E	62	H	61	G	66
32 & 33	0.44% C	124.8	E	43	..	...	..	..
21	1.05% C	92.9	I	44		...	..	..
22	1.05% C	87.5	I	57	..	...	..	..
23	1.05% C	132.0	I	27	..	...	..	..
23A	1.05% C	99.0	I	46		...	..	..
24	1.05% C	101.0	I	49	..	..	..	..
				Chromium Steels				
15	1% Cr	91.9	E	69	H	67	G	74
15A	1% Cr	148.4	I	29	..	...	K	36
34	Chrome-iron	77.5	E	71	..	...	..	..
35	Stainless-iron	92.8	E	69	..	...	..	..
				Chromium-Vanadium Steels				
9	Cr-V	93.6	E	73	H	76	G	75
11	Cr-V	127.0	E	47	H	46	G	54
11A	Cr-V	134.0	I	32	..	..	K	41
30A	Cr-V	157.0	I	15+	..	...	..	..
				Nickel Steels				
2	3½% Ni	85.6	E	90	H	85	G	94
17 & 18	3½% Ni	87.3	I	73	L	78	.	..
1	3½% Ni	94.6	E	78	..	...	G	81
36	3½% Ni	99.2	..	..	L	69	P	57
..	3½% Ni	120.0	D	49	C	51	B	53
12	3½% Ni	128.7	E	52+	H	52	G	56
26	3½% Ni	141.3	I	35	..	...	..	..
12A	3½% Ni	148.8	I	41	..	...	K	46
20	3½% Ni	176.4	I	22	..	...	..	..
				Nickel-Chromium Steels				
25A	1.6 Ni, 0.9 Cr	114.5	E	53	..	...	..	..
14	2.3 Ni, 1.8 Cr	128.0	E	46	H	46	G	52
14A	2.3 Ni, 1.8 Cr	150.2	I	38	..	...	K	42
25	1.6 Ni, 0.9 Cr	177.5	I	27	..	...	K	30
				Nickel-Molybdenum Steels				
13	3½ Ni, 0.35 Mo	127.7	E	50	H	46+	G	52
				Chromium-Molybdenum Steels				
8	0.5 Cr, 0.17 Mo	83.7	E	80	H	83	G	85
6	0.9 Cr, 0.14 Mo	96.4	E	66+	H	68	G	75
19	0.5 Cr, 0.19 Mo	107.3	I	46	..	...	..	..
7	0.8 Cr, 0.36 Mo	125.0	E	47+	H	53	G	50
10	1.0 Cr, 0.35 Mo	133.5	E	44+	H	45	G	49
10A	1.0 Cr, 0.35 Mo	135.5	I	33+	..	...	K	41
27A	1.0 Cr, 0.21 Mo	194.7	I	22+	..	...	..	..

[a] For dry-cutting at $\frac{3}{16}$-in. depth and a feed of 0.028 in. per revolution with the form of tool described in the text. Calculated from life tests by means of Equation [2] of the text or obtained from Fig. 3.

treatment by which this strength is produced. The six sets of experiments were made on several types of steel having tensile strengths between 85,000 and 135,000 lb. per sq. in., and consideration is given to variations in quenching and tempering temperatures, coolants used in hardening, etc. While no comparisons are available for steels in the martensitic condition, with a tensile strength around 175,000 lb. per sq. in. or more, there is no reason to believe that with such structures results different from those given in Table 8 would be obtained.

TABLE 8 EFFECT OF METHOD OF HEAT TREATMENT ON THE CUTTING SPEEDS OF STEEL FORGINGS HAVING EQUAL TENSILE STRENGTHS

(Tests made dry at $\frac{3}{16}$-in. depth and 0.028-in. feed)

Steel cut: Test Log No.	Steel cut: Type composition, per cent	Heat treatment (all temperatures given in deg. fahr.)	Tensile strength, 1000 lb. per sq. in.	Taylor speed, ft. per min., when using tool steel: E	Taylor speed, ft. per min., when using tool steel: G
		COMPARISON OF DIFFERENT METHODS OF QUENCHING AND TEMPERING			
10	0.37 C, 1 Cr, 0.35 Mo	1540 emulsion; 1040	133 5	44	49
10A	0.37 C, 1 Cr, 0.35 Mo	1530 water; 1000	135.5	44[a]	51[b]
11	0.32 C, 1 Cr, 0.15 V	1720 water; 940	134.0[c]	45[c]	54
11A	0.32 C, 1 Cr, 0.15 V	1800 water; 450	134.0	43[a]	51[b]
6	0.30 C, 0.8 Cr, 0.14 Mo	1580 emulsion; tempered around 1100	107 0	58[f]	....
19	0.26 C, 0.5 Cr, 0.19 Mo	Water quenched; not tempered	107 3	57[a]	...
		COMPARISON OF QUENCHED AND TEMPERED STEELS WITH ANNEALED STEELS			
2	0.26 C, 3.8 Ni	1650 emulsion; 1340	85 6	90	94
17	0.31 C, 3.3 Ni	1460 furnace cool	87 3	84[a] [d]	...
9	0.29 C, 0.85 Cr, 0.17 V	1680 water; 1230	93 6	73	75
15	0 49 C, 0 93 Cr	1550 furnace cool to 1275; soak and slow cool	91.9	69	74
..	0.3 C, 3.1 to 3.8 Ni	Quenched and tempered	100.0	73[e]	...
36	0.4 C, 3.3 Ni	1600 air cool to 1250; soak and furnace cool	99.2	75[d]	...

[a] Estimated from the known difference in cutting speeds of tool steels E and I Refer to Fig. 9.
[b] Estimated from the known difference in cutting speeds of tool steels G and K. Refer to Fig. 9.
[c] Estimated from tests at a tensile strength of 127,000 lb. per sq in
[d] Estimated from the known difference in cutting speeds of tool steels E and L.
[e] Estimated from tests at tensile strengths of 94,600 and 128,700 lb. per sq. in.
[f] Estimated from tests at tensile strengths of 96,400 lb. per sq. in.

60 Perhaps the most interesting tests are those which show that a single cooling, as for example, accelerated furnace or air cooling, gives substantially the same cutting speed as quenching followed by tempering, provided these two methods give the same tensile strength. As shown in Table 8 this applies not only to comparisons of annealing practice with quenching and tempering but also to comparisons of high-temperature tempering with none at all subsequent to hardening (made possible by variations in size of test log and coolant used in hardening).

61 Thus it makes no great difference by what method a given tensile strength is produced in a steel in so far as the cutting speed is concerned. This should not be taken as approval of

quenching without tempering, as such procedure, even in small sections, leaves hardness gradients which are undesirable from a practical viewpoint. These were eliminated in the experiments by removing to an appreciable depth metal at the surface of the quenched forgings before making the lathe tests.

Stain-Resisting Iron Alloys

62 Some interesting comparisons may be made between the high-chromium irons and the different carbon and low-alloy-content steels. At a tensile strength of about 93,000 lb. per sq. in. stainless iron, containing about 0.1 per cent carbon and 12½ per cent chromium, has practically the same cutting speed as steel containing 0.5 per cent carbon and 1 per cent chromium (test log 15, Table 7), and a steel containing 0.3 per cent carbon, 1 per cent chromium and 0.2 per cent vanadium (test log 9, Table 7), provided the two latter are quenched and tempered to produce the same tensile strength.

63 The fact that the method of treatment by which a given strength is produced has been shown to have no appreciable effect upon the cutting speed indicates that large amounts of chromium do not, in the absence of carbon, seriously affect the rough-turning properties of steels. Chromium in large amounts is supposed to be partly in solid solution in the ferrite and in this condition probably has a small effect upon the cutting speed. In addition there is only a very small amount of carbon present in this stainless iron with which chromium can form the very hard carbides which tend to wear away the cutting tools and thus reduce the cutting speeds.

64 The "chrome-iron," containing about 0.15 per cent carbon and 24½ per cent chromium, with a tensile strength of 78,000 lb. per sq. in., has about the same cutting speed as the stainless iron with a tensile strength of 93,000 lb. per sq. in. or the 0.44 per cent carbon steel with a tensile strength of 100,000 lb. per sq. in. (Refer to Table 7.) Thus the higher proportions of both chromium and carbon in the "chrome iron," as compared with the stainless iron, have reduced the cutting speeds (compared on the basis of equal tensile strength), but not to a very marked degree.

65 Aside from demonstrating that these high-chromium iron alloys can quite readily be rough turned these tests show that the reduction in cutting speeds produced by large additions of chromium to iron is dependent very largely upon the carbon present. This is consistent with what is known for other properties and for other types and conditions of cutting.

66 Attention should perhaps be called to some observations made during the progress of these tests. For cutting conditions under which blue chips are obtained with carbon and the ordinary low-alloy-content steels, those of the stainless iron were colored

a light straw and the chips of the "chrome iron" were not colored at all. This gives interesting confirmation of the superior resistance to oxidation in air of the 25 per cent chromium-iron alloys in comparison with iron containing 15 per cent or less of chromium.

Interchangeability of the Special Steels

67 An important feature brought out by the described tests relates to the interchangeability of the special steels. As is well known, it is entirely practicable to duplicate definite combinations of tensile strength and ductility or other mechanical properties with various combinations of alloying elements, provided only the heat treatment is modified to suit the particular compositions selected. The described tests show that some steels have better rough-turning properties than others and that interchangeability with respect to mechanical test values does not at the same time imply equal interchangeability with respect to rough turning or, for that matter, other types of machining.

68 It has, of course, been shown that different combinations of composition and heat treatment may be used to produce at the same time similar mechanical test values and cutting speeds, but interchangeability is somewhat more restricted under such conditions than when considering only such factors as tensile strength, endurance limits, etc., and is dependent to a greater degree upon the relation of the heat treatment to the desired properties. Interchangeability in one range of tensile strength or hardness does not mean interchangeability in other ranges.

69 While certain well-defined differences in cutting speed have been found for the various structural alloy steels heat-treated to show equal tensile strengths, these are no greater, in most cases, than differences which may ordinarily arise from variations in the quality of the steel from which the tools are made. High-speed steel I, which is practically identical in chemical composition with steel E (Table 1), in so far as the elements ordinarily determined are concerned, and which when made into tools was subjected to the same heat treatments, has a consistently lower cutting speed than tool steel E. As shown by comparison of Figs. 8 and 10 this difference is as great as most of the differences in cutting speed arising from variations in the materials cut. Since both high-speed steels represent commercial products obtained in the open market from reputable manufacturers, it is clear that any benefits derived from selection of the steel machined may be completely counteracted by neglect to guard the quality (and likewise the treatment) of the high-speed steel used. This is probably the most important single feature developed and deserves special emphasis.

70 Another interesting and significant feature is shown in the lathe tests described. So far all comparisons of the steels cut have

been based upon their chemical composition. As has already been pointed out the quality factor in structural steels is important in determining their cutting speeds, yet there is exceptionally good agreement in the results obtained on a large number of test logs made by different manufacturers under different conditions in various parts of the country.

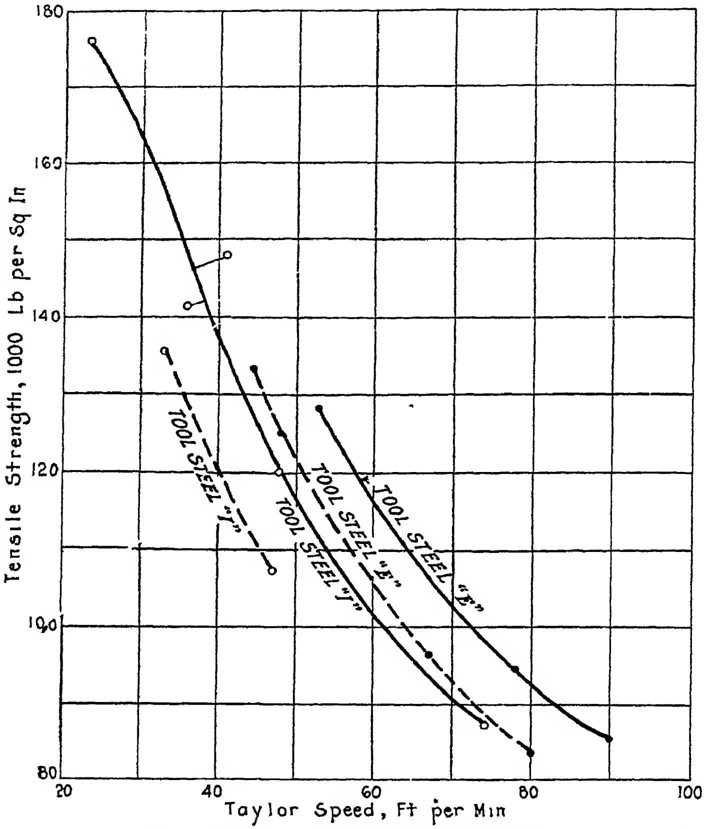


FIG. 10 COMPARISON OF CUTTING SPEEDS OF TWO LOTS OF HIGH-SPEED STEEL OF SIMILAR CHEMICAL COMPOSITION WHEN SUBJECTED TO SAME HEAT TREATMENTS

Solid lines denote tool tests on nickel steels. Dotted lines, tool tests on chromium-molybdenum steels.

Note that cutting speed of steel E is equal approximately to that of steel I plus 10, when cutting both hard and soft steels.

71 Consistent results could not, of course, have been obtained without careful experimental work, but the relatively small deviations of the experimental values from smooth tensile strength-Taylor speed curves (Figs. 8 and 9) is striking evidence of closely comparable quality in the structural steels ordinarily produced by different manufacturers, at least in so far as cutting speeds in

rough turning are concerned. (Refer to Table 2 for the sources of the various test logs.)

Effect of Feed and Depth of Cut on the Cutting Speed for Different Steels

72 The preceding lathe tests under fixed feed and depth of cut (respectively 0.028 and $\frac{3}{16}$ in.) give useful and important comparisons of tool performance when cutting different steels but are not generally applicable to industrial work until extended to cuts at various feeds and depths. Taylor (4) determined the effects of feed and depth upon the cutting speed but called attention to the need for confirmatory tests, stating, "We attempted to make what we believed to be typical experiments but we did not feel satisfied with either the extent or the thoroughness of this work. Therefore it seems of great importance to more carefully verify the various laws obtained by us through direct experiments with the latest high-speed tools."

73 Other investigators have given attention to this question but for the most part confined their work to cutting steels of relatively low hardness and tensile strength in comparison with some of the steels cut in this investigation so that additional tests on higher strength steels seemed desirable. In addition, the present series of tests were made with tools smaller than Taylor had considered desirable for tests of this kind, because of the difficulty of obtaining consistent results.

74 A complete study of the effect of feed and depth of cut upon the cutting speed is accompanied by many practical difficulties. Large amounts of uniform metal are required upon which the tools may be tested. The heavy cuts and feeds which may be taken at relatively high speeds with modern high-speed steels, even with small tools, require that the test lathe have a powerful driving unit and be equipped with close speed control and that the test logs be rigid and therefore of large dimensions. While these and other factors are troublesome in lathe tests on soft metals they introduce serious difficulties when cutting steels of very high tensile strength. For example, the heat treatments which must be used to develop the required hardness or strength introduce sharp gradients in both hardness and cutting properties in large masses and so reduce the zones in which consistent and valid results may be secured.

75 Because of such features no attempt was made to cover the complete range of practical cutting conditions, but instead, to cover in separate series of tests a wide range of each of the variables. In this way it was possible with relatively few tests and a minimum of metal cut to check the validity of applying Taylor's empirical equation in cutting a variety of steels with widely different combinations of depth of cut and feed.

76 Taylor found that the relation between the cutting speed

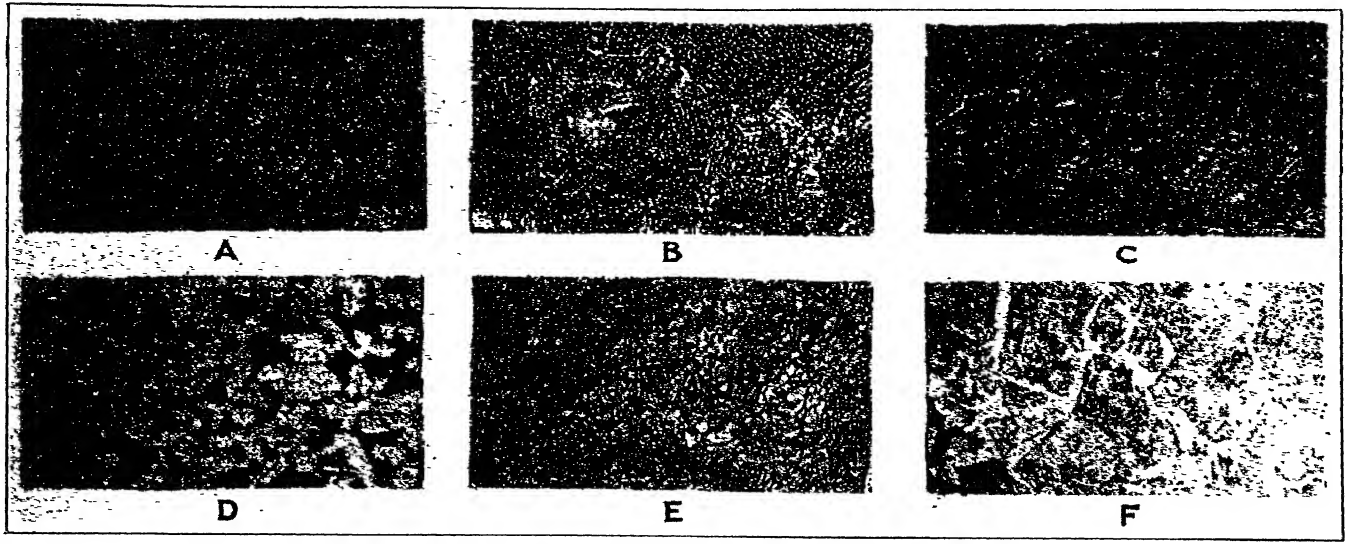


FIG. 11 MICROSTRUCTURES OF SOME STEELS CUT IN LATHE TESTS ($\times 250$)

For chemical compositions and heat treatments applied to these steels refer to Table 2. Etched with 2 per cent nitric acid in alcohol.

	Photomicro-graph	Test log No.	Steels cut	Tensile strength	Cutting speed
"I" Tools:	A	27A	Chromium-molybdenum	194700	22
	B	20	3½% Nickel	176400	22
	C	30A	Chromium-vanadium	157000	15
"E" Tools:	D	8	Chromium-molybdenum	83700	80
	E	2	3½% Nickel	85600	90
	F	3	0 44 Carbon	92800	77

giving a fixed tool life, the feed, depth of cut, and radius of nose of the tools could be approximately represented by the empirical equation

$$V = \frac{\text{constant}\left[1 - \frac{8}{7(32r)}\right]}{F^{\frac{2}{5}+\frac{2.12}{5+32r}}\left(\frac{48}{32r}D\right)^{\frac{2}{15}+0.06\sqrt{32r}+\frac{0.8(32r)}{6(32r)+48D}}} \quad . \; . \; . \quad [3]$$

in which V = cutting speed for selected tool life, ft. per min.
F = feed in inches per revolution
D = depth of cut, in inches
r = nose radius of the tool, in inches.

77 As already pointed out, only one form of tool was used ($\frac{1}{4}$ by $\frac{1}{2}$ in. with $\frac{1}{8}$-in. nose radius) and no check was made on the validity of this equation in expressing the change of tool life with change of nose radius when cutting high strength steels.

78 For the tool radius used Equation [3] reduces to:

$$V = \frac{\text{constant } (0.714)}{F^{0.6356}(12D)^{0.2538+\frac{3.2}{24+48D}}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

79 The cutting speed V, as has already been shown and as is well known, is dependent upon the quality composition and heat treatment of the tools and the properties of the metal cut as well as upon the tool form, feed, and depth of cut. As the changes due to the tool form, feed, and depth of cut are supposed to be explicitly represented in the form of the equation, the value of the constant in Equation [4] should depend only upon the properties of the metal in the tool and of the metal cut.

80 For the $\frac{3}{16}$-in. depth of cut and the 0.028-in. feed used in the experiments described previously, Equation [4] reduces to

$$\text{constant} = 0.1922V \quad . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

This equation was used to calculate the constant in Equation [4] for all these tests.

81 Two series of tests were then made on test log 36 ($3\frac{1}{2}$ per cent nickel steel, tensile strength approximately 100,000 lb. per sq. in.), one at a constant feed of 0.028 in. per revolution and depths of cut ranging from $\frac{1}{8}$ to $\frac{7}{16}$ in., and the other at a constant depth of cut of $\frac{3}{16}$ in. and feeds ranging from 0.015 to 0.092 in. per revolution. The results are shown in Table 9 and Fig. 12. On this same chart (Fig. 12) are drawn curves computed from Taylor's Equation [3] with the constant computed from the previous tests. The agreement is as close as could be expected, showing that Taylor's equation represents these tests with good accuracy. The slope of the observed points on the Taylor speed-depth of cut curve is slightly greater than that curve computed from Taylor's equation, but the difference is not of practical importance.

82 Additional tests were then made upon four different test logs covering steels from 66,000 to 177,000 lb. per sq. in. tensile strength, but covering in each case only one or two variations

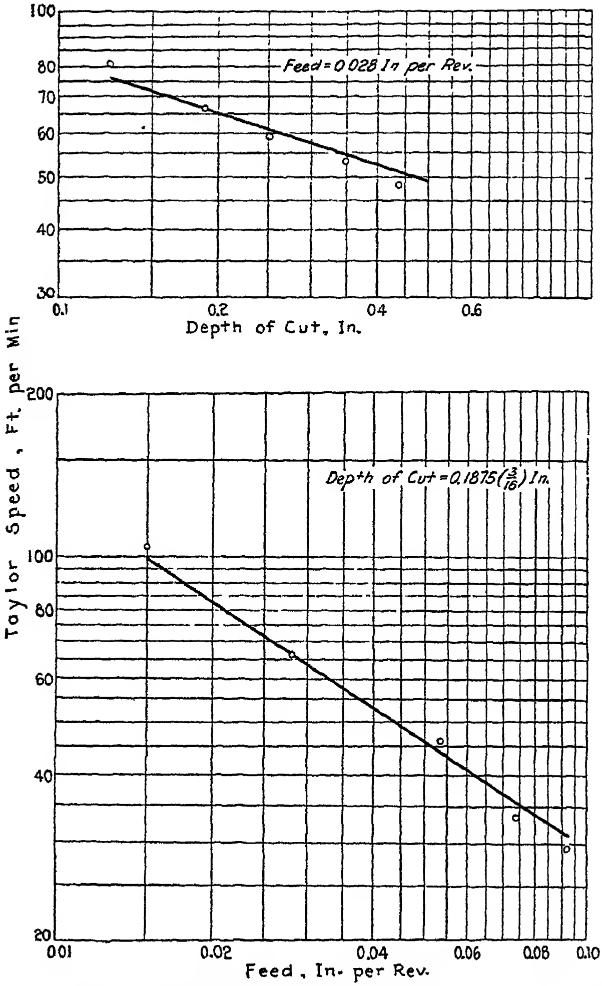


FIG. 12 EFFECT OF FEED AND DEPTH OF CUT ON CUTTING SPEED

Test made with tool steel P on test log 36. Solid lines represent Equation [6]; circles represent results of lathe tests. Note that logarithmic coördinates are used.

in speed or depth of cut. The results of these tests are also given in Table 9.

83 In each case the cutting speed for 1-hour tool life has been computed in two ways; first, directly from the observations by means of Equation [2], and secondly, from the observations at standard feed (0.028 in.) and depth of cut ($\frac{3}{16}$ in.) by means of

Equation [2] and Taylor's Equation [4]. The agreement in all cases is as close as could be expected.

84 These tests then give good reason to believe that Taylor's equation, so far as it concerns the effect of feed and depth of cut, can be safely used with high-tensile-strength steels, to estimate the relations between tool life, cutting speed, feed, and depth of cut.

Graphic Representation of the Relation Between Feed, Depth, Steel Cut, and the Speed Giving 1-Hour Tool Life

85 The tests just described check Taylor's Equation [3] sufficiently well in cutting high-tensile-strength steel to warrant using it as a basis for computing for the form of tool described, the speeds giving a selected tool life (for example, 1 hour) under varying conditions. Provided the tool life is known at some one speed, feed and depth of cut, these computations only require the use of Equations [2] and [4]. They are, however, tedious because of the form of Equation [4] and for this reason the chart reproduced in Fig. 13 was constructed, embodying the application of Taylor's equation (in the form of Equation [4]) to the results of the present series of experiments. This gives in a form adapted for quick use the cutting speeds permitting a 1-hour tool life, when taking different cuts on various steels with the $\frac{1}{4}$ by $\frac{1}{2}$-in. lathe tools used in this investigation. By using logarithmic rectangular coördinates for depth of cut and feed, the equal speed lines are nearly straight, and the effect of the tools and the material cut is provided for by simply shifting identical scales of speed up or down. Speed scales, properly located, are provided for many of the materials tested. Those applying to cutting steels having tensile strengths of 135,000 lb. per sq. in. or over are based on the results obtained with tool steel I, Table 1, while those applying to cutting lower strength steels are based on the results obtained with tool steel E, (Table 1). In addition, a sliding speed scale is provided (at the left). This may be cut out and when slid up or down to such a position that it gives correctly the speed for 1-hour tool life at one depth of cut and feed, it will then indicate corresponding speeds for other cuts. The proper location for a given "quality" of tool and metal cut requires only a knowledge of the speed for some one feed and depth.

86 In using the chart, Fig. 13, first find the point corresponding to the selected feed and depth of cut; follow the corresponding curved speed line to the left edge of the feed-depth chart, and then follow the horizontal to the speed scale applying to the particular steel to be cut.

87 These detailed comments have been given to illustrate methods of application but too great a degree of accuracy in predictions should not be expected. The chart in Fig. 13 is a useful

guide to the selection of cutting speeds and aside from what is believed to be a new method of presentation includes extension and slight modifications of the laws originally developed by Taylor in cutting carbon steels to alloy steels varying widely in tensile properties.

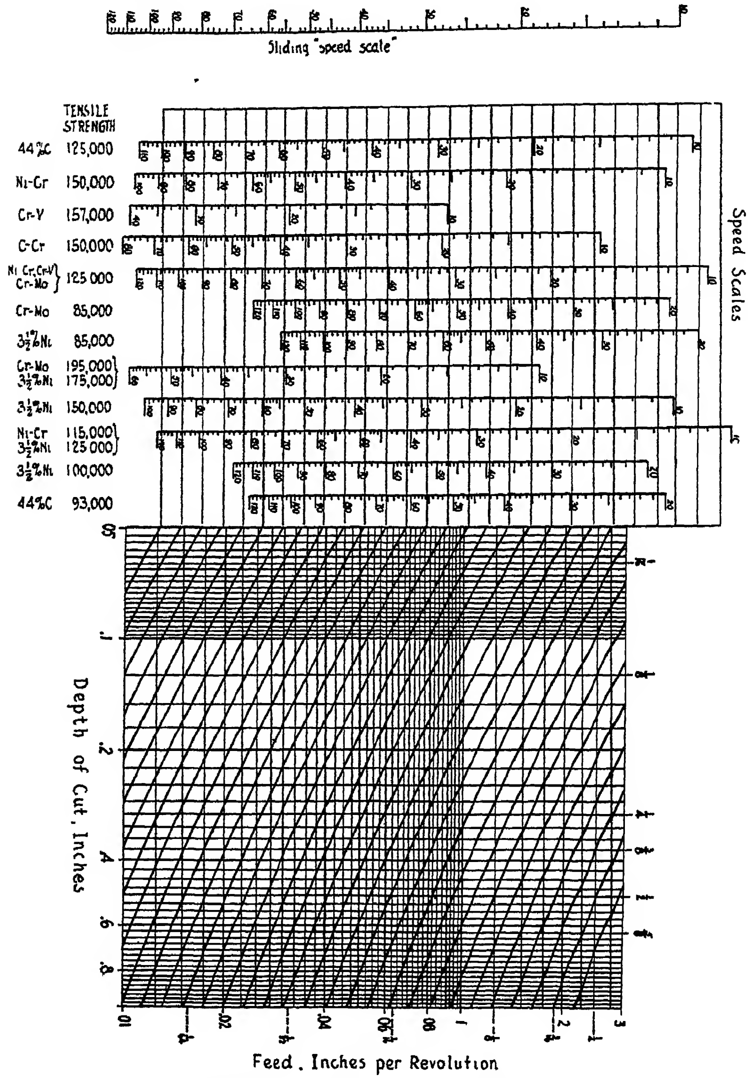


Fig. 13 Cutting-Speed Chart for Selected Size and Form of Tools Used in Experiments

This gives cutting speed permitting a tool life of one hour in dry turning various steels at different feeds and depths. Below each speed scale will be found type of steel cut and its tensile strength. Refer to text for a discussion of use of chart.

88 While it is not possible at this time to specify definite limits within which Fig. 13 may be applied, it is advisable to restrict applications to severe service, i.e., conditions truly characterized as rough turning. However, it should be kept in mind that what are light cuts on a soft steel may represent severe service and rough turning for a much harder steel; for this reason the chart in Fig. 13 has included a wide range in feeds and depths, but it should not be inferred that the limiting values correspond to the limits within which the empirical laws apply.

Factors Ordinarily Determined in Customary Hardness and Tension Tests of Steels Cut Not Quantitative Criteria of Their Cutting Speeds

89 The question has often been raised whether hardness or other simple and convenient mechanical tests of the steel cut bear any direct relation to its machining properties including, as in this investigation, the cutting speeds in rough turning. The problem is not one of obtaining a general indication but a quantitative relation between some mechanical test and the cutting speed for steels varying widely in chemical composition and physical properties.

90 Taylor's investigation in cutting annealed or hot-finished carbon-steel forgings led to the conclusion that the cutting speed was best indicated by "the tensile strength and percentage of stretch and contraction of area obtained from standard tensile test bars cut from such a position in the body of the forging as to represent its average quality and then broken in a testing machine." However, it should be recognized that "a falling off in stretch due to increased hardness in metal will cause a slower cutting speed in the tool, while a falling off in the percentage of stretch due to an inferior quality of metal will not necessarily cause a slower cutting speed."

91 The following equation represents the empirical relations derived by Taylor between the cutting speed and the tensile properties of the steel cut:

$$V = \frac{125\left(1 - \dfrac{215}{(15+E)^2}\right)}{\sqrt{\left(\dfrac{S}{10{,}000} - 3\right) - 0.9}} \qquad [6]$$

in which V = the speed in ft. per min. which causes the tool to fail in exactly 20 min.

S = tensile strength in lb. per sq. in.

E = per cent elongation in 2 in.

This is characterized as only a partial guide which does not, for either very hard or very soft steels, give values very close to the experimental observations.

92 There is no reason why any one of the factors determined in the customary hardness or tension tests should be a quantitative criterion of the cutting speed when considering different steels. Roughing lathe tools of high-speed steel fail due to the continued rubbing or wearing action of the chips passing over the tool and this finally results in a fairly sudden spalling of the nose.

Fig. 14 Typical Failures of Roughing Lathe Tools in Breakdown Tests

Note the rubbing off of the nose and the "gutter" or groove worn on the top surface of the tools.

(Refer to Fig. 14.) As is well known, hardness as customarily determined is not a criterion of wear and since wear causes the tools to fail, hardness would not be expected to give a quantitative indication of tool life (or the cutting speed).

93 Apart from this, however, the tools and chips are both at high temperatures in rough turning and it is known that the performance and properties of metals under such conditions are not indicated by, nor necessarily related to, tests at atmospheric temperatures.

94 While it is not yet possible to isolate all the important factors contributing to wear under any stated set of conditions, the problem is known to be complex and not tied up solely with the strength and hardness of the metals concerned. From this angle, as well, it seems improbable that any general quantitative relations exist between hardness or other single mechanical tests and the cutting speed.

95 Experimental facts are more convincing than general theories and since the tests previously described offered an exceptionally good opportunity to make comparisons between cutting speeds and the mechanical properties of the steels cut, a complete set of hardness tests were made and are summarized in Table 10 and Fig. 15. Included are determinations with the Brinell, Shore, Rockwell, Bierbaum scratch, and Herbert pendulum hardness testers.

96 As was the case with the tension tests which are summarized in Table 3 and Fig. 16, the determinations of hardness were made on samples taken from the test logs at the starting and finishing diameters of the cutting tests and only average values used for comparisons. Over 4000 hardness determinations were made on about 35 test logs to obtain representative numerical values with the different instruments. In general, a number of measurements were made on each specimen as follows: two Brinell impressions, eight to ten Rockwell impressions with the diamond cone penetrator and a similar number with the steel ball, five readings on each of two scratches made with the Bierbaum scratch hardness tester, two time-hardness and five or six scale-hardness readings with the Herbert pendulum tester. Additional determinations were made when unusual or erratic results were obtained but in no case did these materially change the values originally secured.

97 As shown in Fig. 15 none of the hardness tests is a quantitative criterion of the cutting speed but some give a better general indication than others. The scratch test in its present form appears to give absolutely no indication of the cutting speed and shows the widest scatter of points, but this is undoubtedly due in part to the susceptibility of the method to errors in measurement.

98 There does not seem to be much choice between the Brinell, Rockwell B scale, Shore, or Herbert time hardness from the standpoint of a correlation with cutting speed, but certain other factors give to the Brinell test the most favorable rating. The two separate Rockwell scales required for steels varying so widely in properties as those cut in the reported lathe tests is a decided disadvantage. The Shore scleroscope, as is well known, must be used with considerable care to secure consistent results, and the Herbert hardness tester is primarily a laboratory instrument whose readings depend largely upon the manipulative technique. Because of these features the Brinell test is at present considered the most

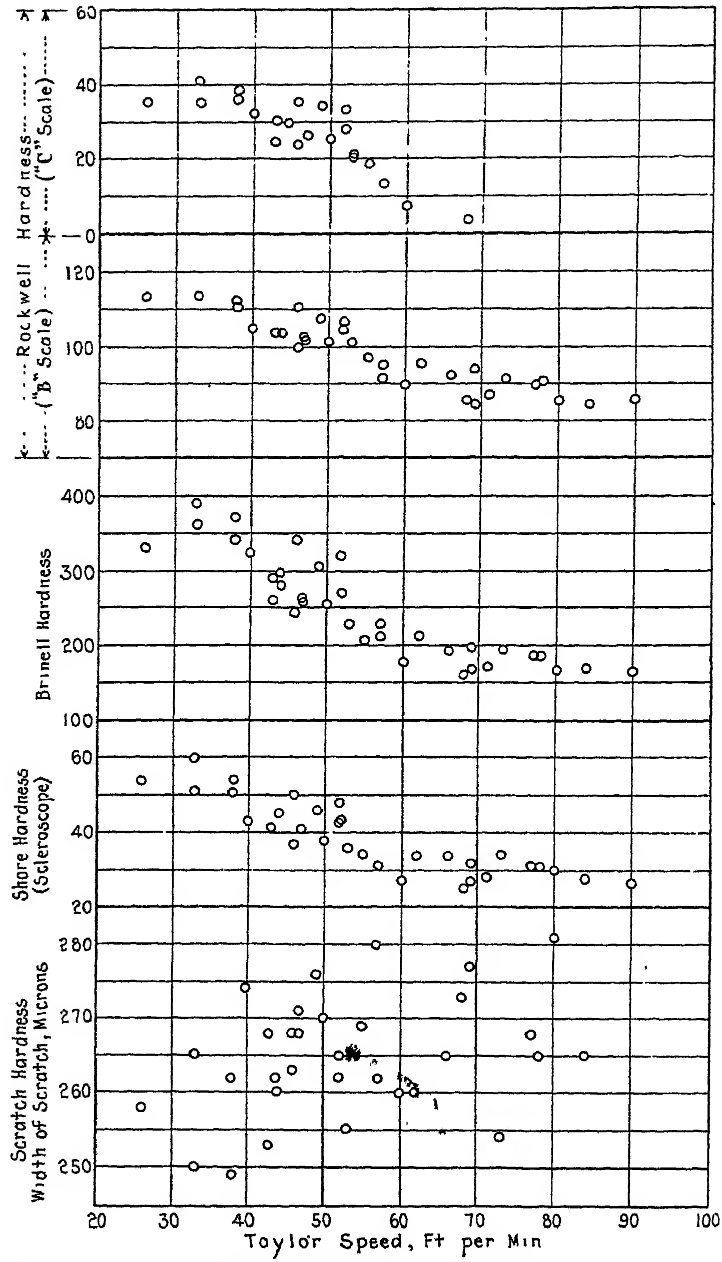


Fig. 15 Cutting Speed in Relation to Hardness of Steels Cut

Cutting speeds used are for tool steel E, Table 1, when dry turning at $\frac{3}{16}$-in. depth and a feed of 0.028 in. per revolution. Practically all steels listed in Table 2 are included in comparisons above

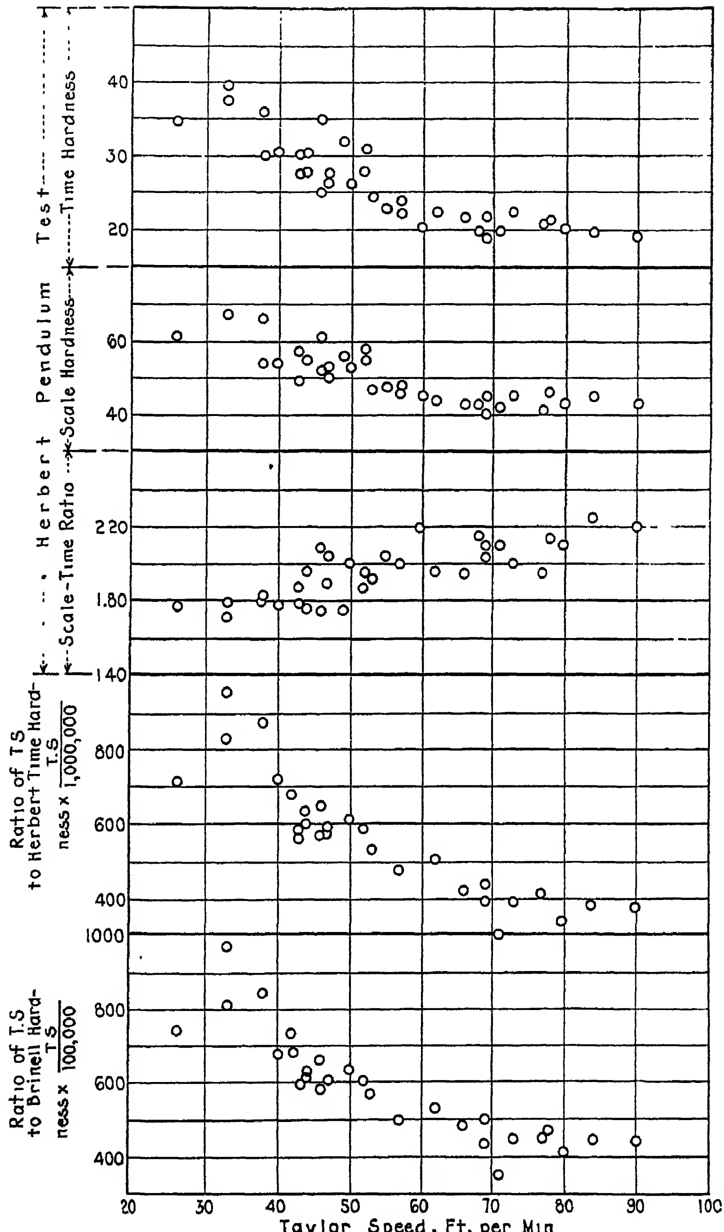


FIG. 15 Continued

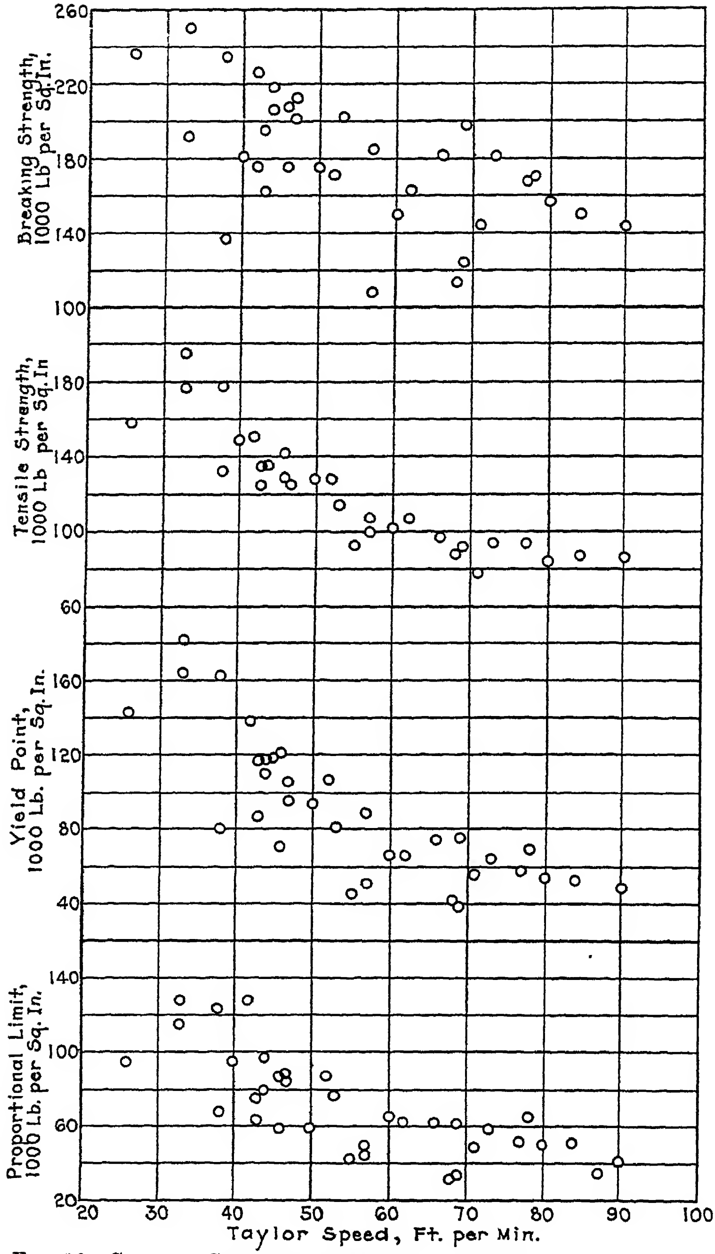


FIG. 16 CUTTING SPEED IN RELATION TO TENSILE PROPERTIES OF STEELS CUT

Cutting speeds used are for tool steel E, Table 1, when dry turning at $\frac{3}{16}$-in. depth and a feed of 0.028 in. per revolution. Practically all steels listed in Table 2 are included in comparisons above.

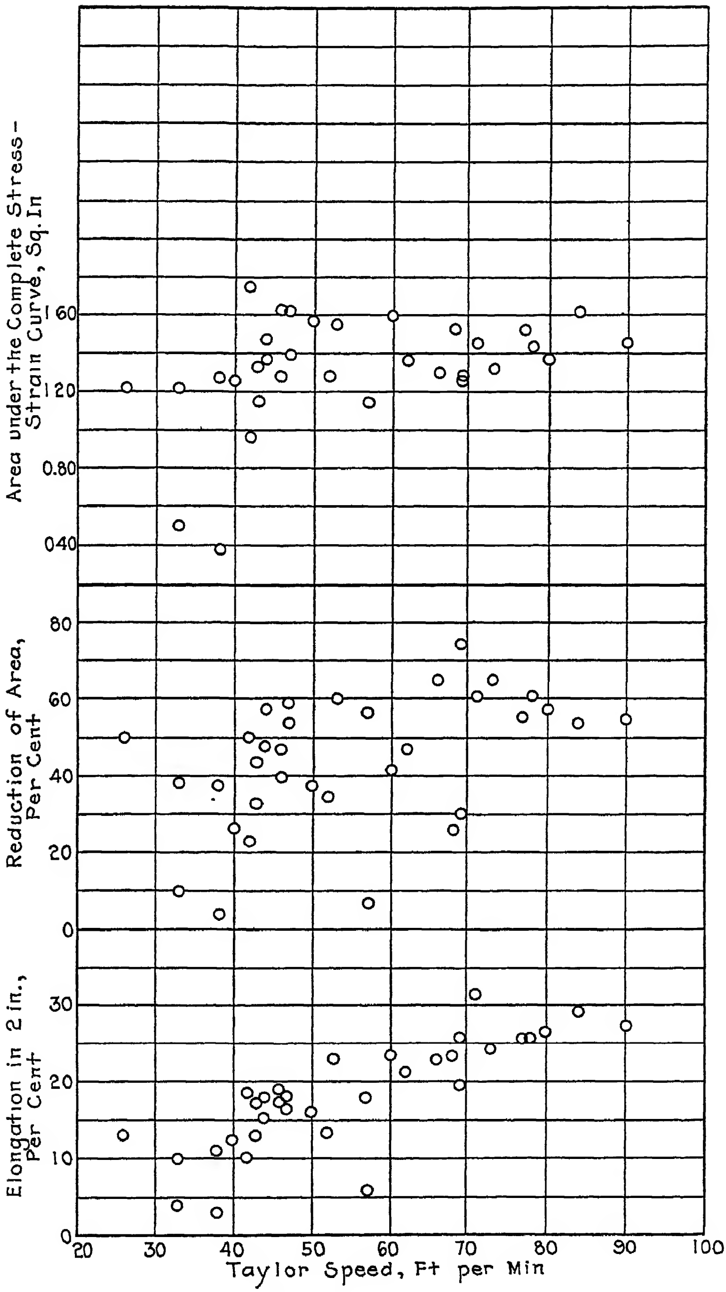


FIG. 16 Continued

useful of the group in giving a qualitative, but by no means a quantitative, indication of the cutting speed.

99 Correlation of cutting speed with tension tests is given in Fig. 16. In general the cutting speed for a fixed tool life increases with decrease in tensile strength, proportional limit, yield point, and breaking strength and increase in elongation and reduction of area, but on the whole there is a wide scatter of points for each of these factors. The best general indication of the cutting speed is given by the tensile strength. Likewise, there is a smaller scatter in points representing elongations than in those representing the reduction of area, but the authors do not believe that these factors will ever be generally useful in the development of an accurate criterion of the cutting speed as they are too greatly influenced by variables which do not materially affect the cutting properties. The same may, of course, be said of the proportional limit and yield point.

100 It is of interest to note that the cutting speed is in no way even generally related to the energy required to break the steel in tension, as represented by the area under the stress-strain curves. For example, an area of 1.45 sq. in. corresponds at the same time to cutting speeds of about 45 and 90 ft. per min.; there are likewise variations in the area from 0.96 to 1.74 sq. in. for a cutting speed of about 45 ft. per min. (For detailed data refer to Tables 3 and 7.)

101 Many combinations of mechanical test values were compared with the cutting speed but in no case was an especially close correlation found for all of the steels cut. For example, there is given in Fig. 17 for a majority of the steels cut a comparison of the experimentally determined cutting speeds with those obtained from Equation [6] representing the empirical relations developed by Taylor. The computed cutting speeds are based entirely on the results of the tension tests and while they clearly follow the variations observed in the experimental results, do not closely check the numerical values except in a relatively few cases. A close relation can hardly be expected when it is recalled that variations in heat treatments, which are known materially to affect the elongation and reduction of area obtained with a particular tensile strength, were previously shown to have no measurable effect upon the cutting speeds.

102 Another set of comparisons which may be mentioned involves both the tensile strength and Brinell hardness. The ratio of Brinell hardness to tensile strength is not exactly the same for different steels or for one steel which has been heat-treated to show widely different degrees of hardness. On the basis that some function of both of these factors might tend to smooth out the variations observed in a correlation of tensile strength and cutting speed, comparisons were made with the product of the

tensile strength and the tensile-strength–Brinell ratio These are included in Fig. 15 but do not in general give a better indication of cutting speed than the tensile strength alone.

103 Fig. 15 also shows that the Brinell numbers in the foregoing comparisons may be replaced by Herbert time hardness numbers without appreciably changing the scatter of points.

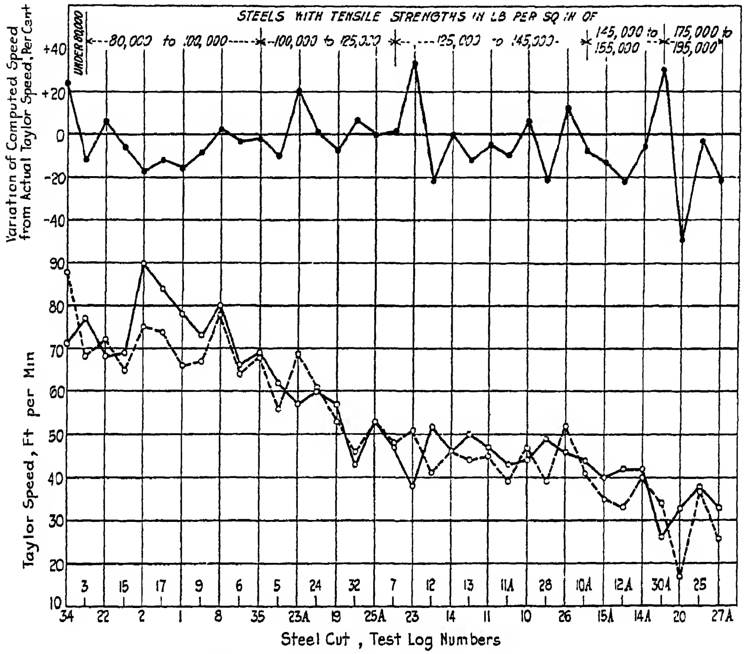


Fig. 17 Experimentally Determined Cutting Speeds in Relation to the Speeds Computed from the Results of Tension Tests of the Metal Cut by Means of Taylor's Equation

Solid line denotes experimentally determined cutting speeds. Broken line denotes cutting speeds computed from the equation

$$V = \frac{125\left(1 - \frac{215}{(15 + El)^2}\right)}{\sqrt{\left(\frac{T.S.}{10,000} - 3\right)} - 0.9}$$

in which (El = percentage of elongation in 2 in , $T. S.$ = tensile strength in lb per sq. in. The chemical compositions and tensile properties of the different test logs are given in Tables 2 and 3 The cutting speeds are for tool steel E, Table 1, when dry turning at $\frac{3}{16}$-in. depth and a feed of 0.028 in. per revolution)

104 Tests were also made on an Amsler wear-testing machine in which metal specimens in the form of disks, 2 in. in diameter and 0.4 in. thick, are driven simultaneously at two different speeds in contact with each other. The two specimens, representing in this case respectively the hardened tool and the metal cut, rub over each other in much the same way as the chip passes over a lathe tool, although under very much lower pressures and temperatures.

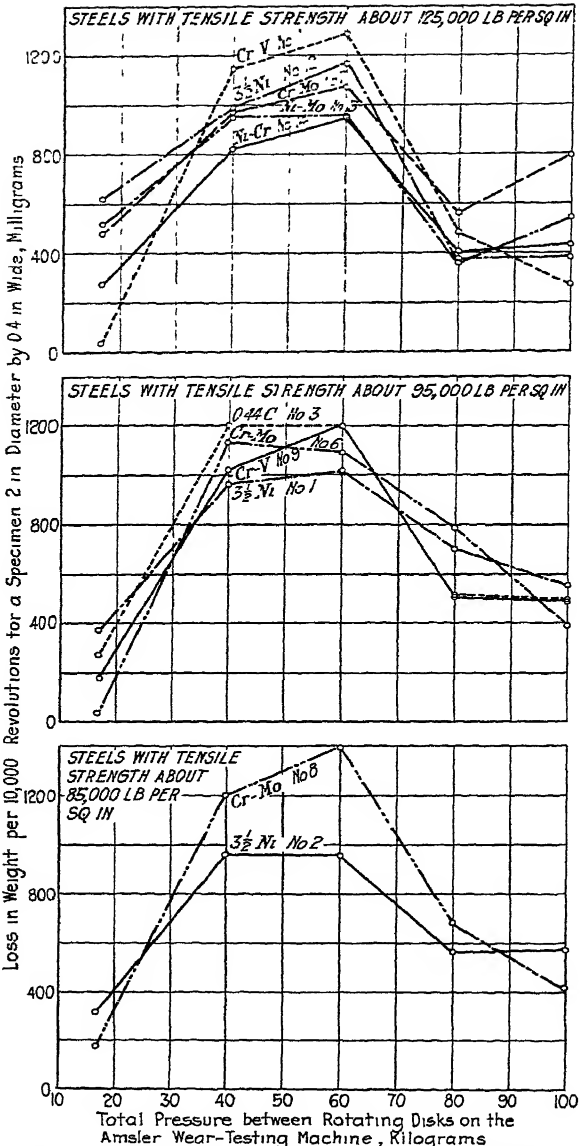


Fig. 18 Wear in Various Structural Steels in Contact With Hardened and Tempered High-Speed Steel as Determined in the Amsler Wear-Testing Machine

Steels referred to represent the steels cut in lathe tests. Different types and test log numbers are given on each curve. Chemical compositions and tensile properties are given in Tables 2 and 3.

105 The results obtained were disappointing, for, although no quantitative relation was expected between the Amsler wear test results and the cutting speed, there was a possibility, despite the wide difference in conditions in the two types of test, that the steels which had the highest cutting speeds might also lose weight the most rapidly in the wear test.

106 The most striking feature of the results summarized in Fig. 18 is the complexity of the relations governing the wear of the different steels which makes it impossible to interpret results from the present Amsler test in terms of tool performance. As shown, the order of wear resistance may be reversed by relatively small changes in one variable such as the contact pressure.

107 Considerable attention and space have been given to a correlation of cutting speeds with different mechanical tests of the steels cut, not because the subject is one of great importance but to present experimental proof of the basis for the opinion that factors determined in the customary mechanical tests cannot be taken as quantitative criteria of cutting speed when a wide range of steels is considered. After all, it is not difficult actually to determine tool life or the cutting speeds for a given tool life, and even if a quantitative relation had been found in these experiments for roughing cuts it would be almost certainly inapplicable to other types or conditions of cutting and would therefore be of limited interest.

Comparison of High-Speed Steels With and Without Cobalt

108 It has already been shown that when one high-speed steel has a higher Taylor speed than another in cutting relatively soft steel, it will show approximately the same superiority in feet per minute when cutting harder steels. (Refer to Fig. 10.) Under such conditions, the percentage of increase in output, whether based on the rate at which metal may be removed or the amount of metal cut before the tools must be dressed, is greater the lower the cutting speed.

109 Since the cutting speeds which may be used decrease with increase in hardness of the steel cut and with increase in feed and depth of cut, it follows that the greatest gain from superior tools is made in cutting the hardest steels or in taking the heaviest cuts (large depths and feeds).

110 To illustrate the order of magnitude of these effects, tool steels E and I may be considered. While comparable in respect to their chemical compositions, as ordinarily determined, and their heat treatment, they differ by about 10 ft. per min. in Taylor speed when taking a $\frac{3}{16}$-in. cut at 0.028-in. feed. This represents a superiority in cutting speed of E over I of 10 per cent when cutting at 100 ft. per min. but a difference of 50 per cent when

cutting at 20 ft. per min. When translated into conditions more nearly approximating practical service — for example, speeds which permit a 1-hour tool life — these differences become somewhat smaller numerically but are still of considerable magnitude, as will be evident from Fig. 19. It is therefore of interest to consider the differences in cutting speed of the several lots of high-speed steel used in this investigation and the reasons for the superiority of some in comparison with the others.

111 The three main factors which influence the performance of high-speed steels are: the quality[1] of the tool steel, the heat treatment to which it is subjected, and its chemical composition.

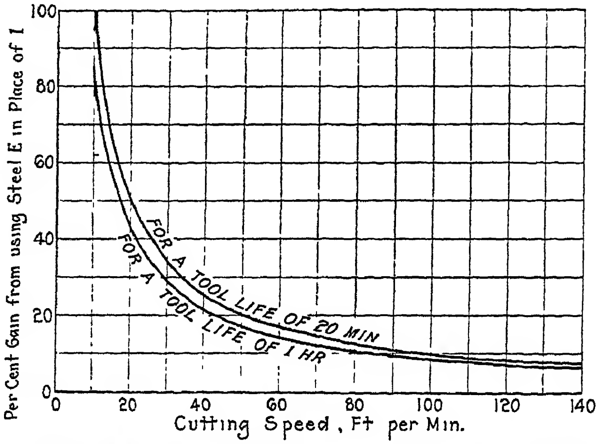


Fig. 19 Percentage of Gain Resulting from Use of Steel E in Place of I at Different Cutting Speeds

112 The heat treatment of the tools was not varied in this investigation as this subject was quite fully covered for lathe roughing tools in a previous report (2). The selected heat treatments (reported in Table 1) represent those known to give the best combination of properties to each of the types of high-speed steel used, and this phase need not again be discussed in detail. The tests made in this investigation throw further light upon the two remaining factors which contribute largely to tool performance and can to advantage be given further brief consideration.

113 It is well known that two lots of tool steel containing practically identical proportions of carbon and special elements such as chromium, tungsten, vanadium, etc., may have appreciably different cutting speeds, and this is emphasized in a comparison of tool steels E and I, Table 1. When taking a cut $\frac{3}{16}$ in. deep at a feed of 0.028 in. per revolution steel E has a cutting speed

[1] Refer to second footnote on page 549 for the meaning of the word "quality" as here used.

which, on the average, is about 10 ft. per min. higher than that of steel I. Similarly, tool steel G has a cutting speed which is 4 ft. per min. higher than that of steel E and about 14 ft. per min. higher than that of steel I. The superiority of steel K over I is about 6 ft. per min.

114 Steels G and K are so-called cobalt high-speed steels and for reasons already discussed (2) were heated to somewhat higher temperatures for hardening than similar steels without cobalt. The first of these cobalt steels, G, is superior to the best of the high-tungsten steels, lot E, while the second, K, is somewhat inferior. However, both are superior to the high-tungsten steel I. While this confirms previous results to the effect that cobalt additions assist in improving the performance of roughing lathe tools, it is evident that any superiority from such a source may be completely counteracted by inferiority in quality, as for example, in steel K.

115 The low-tungsten–high-vanadium steel H has a cutting speed which is, on the average, about equal to that of the best high-tungsten steel E; at the same time, steel H and the low-tungsten steel L are better than the high-tungsten steel I.

116 These comparisons indicate that the quality factor is more important than the modifications in chemical composition which are considered, but at the same time confirm the generally accepted fact that the addition of about 4 per cent cobalt to the customary 18 per cent tungsten high-speed steel, combined with a modification in heat treatment, tends to improve performance in roughing lathe tools.

117 It should be recognized, however, that the gains from superior quality of tool steel, cobalt additions, and modifications in heat treatments are greater the harder the steel cut or the heavier the feeds and cuts. For what may be called moderate service the differences found in the described tests are small.

Approximate Determination of the Power Required in Cutting Different Steels

118 The power required in cutting was determined in practically all of the lathe tests referred to in previous sections of this report. These and the additional determinations, made to fill gaps in the information originally secured, permit an approximate determination of the power required in rough turning different steels with the selected form and size of tool under a variety of speeds, feeds, and depths of cut.

119 While indicating wattmeters were used in some of these tests, most of the results were obtained from ammeter and voltmeter readings which were taken at more or less regular intervals throughout the life of the lathe tools. Since the lathe was driven by a direct-current motor the average of the product of these

TABLE 9 COMPARISONS OF EXPERIMENTALLY DETERMINED CUTTING SPEEDS AT DIFFERENT DEPTHS AND FEEDS WITH THOSE DERIVED FROM TAYLOR'S LAWS OF CUTTING

Metal cut				Cutting conditions				Speed for 1 hr. tool life.	
Log No.	Type of steel	Tensile strength, lb. per sq. in.	Tool steel used	Speed, ft. per min.	Feed, in. per rev.	Depth of cut, in	Average tool life, min.	From experiment,[a] ft. per min.	Calculated from tests at $\frac{3}{16}$-in. depth and 0.028-in. feed,[b] ft. per min.
36	8½% nickel	99,200	P	96	0.028	$\frac{1}{8}$	6.4	70	67
			P	82	0.028	$\frac{3}{16}$	4.6	57	57[c]
			P	74	0.028	$\frac{1}{4}$	4.2	51	52
			P	64	0.028	$\frac{11}{32}$	5.5	46	48
			P	59	0.028	$\frac{7}{16}$	4.7	41	41
36	3½% nickel	99,200	P	125	0.015	$\frac{3}{16}$	5.6	89	86
			P	53	0.053	$\frac{3}{16}$	6.5	39	38
			P	44	0.074	$\frac{3}{16}$	2.7	28	30
			P	37	0.092	$\frac{3}{16}$	3.9	25	27
16	0.18% carbon	66,400	L	63	0.074	$\frac{7}{32}$	9.6	48	49
			L	140	0.028	$\frac{3}{16}$	3.2	92	92[c]
15A	C-Cr	148,400	I	65	0.015	$\frac{3}{16}$	2.2	40	38
			I	35	0.028	$\frac{7}{16}$	6.0	25	25[c]
25	Ni-Cr	177,500	I	33	0.028	$\frac{3}{16}$	5.0	23	23[c]
31	Ni-Cr	177,500	I	47	0.020	$\frac{1}{4}$	6.9	34	33
			I	23	0.053	$\frac{3}{16}$	11.7	18	16

[a] Converted from actual experiment by the equation $VT^{\frac{1}{7}} = \text{constant}$.
[b] Calculated from the laws of cutting as developed by Taylor and as described in the text of this report.
[c] Standard tests from which the other speeds in each group were computed.

voltmeter and ammeter readings represents the average power required for cutting plus that required to turn the test log in the lathe. By subtracting from this value the average product of voltmeter and ammeter readings taken while the test log was turning freely in the lathe, both before and after each tool test,

TABLE 10 SUMMARY OF HARDNESS TESTS ON THE DIFFERENT METALS CUT IN THE LATHE TESTS

Steel cut			Hardness							
				Rockwell			Scratch	Herbert Pendulum		
Test Log No.	Type	Tensile strength, lb. per sq. in.	Brinell	*B* Scale	*C* Scale	Shore	Width in microns	Time	Scale	Scale-time ratio
16	0 18% C	66,400	127	70.7	...	23	2.80	16	30	1.88
29	0.33% C	69,800	133	76.7	....	20	2.77	17.5	29	1.66
3	0.44% C	92,800	187	89.8	....	31	2.68	21	41	1.95
5	0 44% C	107,000	214	95.5	...	34	2.60	22.5	44	1.96
32, 33	0.44% C	124,800	260	101.5	24.4	41	2 68	27.5	49	1.78
21	1.05% C	92,900	207	97 0	18.5	34	2 69	23	47	2.04
22	1.05% C	87,500	163	85.4	3.7	25	2.73	20	43	2.15
23	1.05% C	132,000	341	110.8	36.4	51	2.49	30	54	1.80
23A	1.05% C	99,000	211	91.6	13.8	31	2.62	22.5	46	2.04
24	1.05% C	101,000	177	89 3	7.9	27	2.60	20.5	45	2.20
15	C-Cr	91,900	169	84.5	....	27	2.77	19	40	2.10
15A	C-Cr	148,400	323	104.8	32.2	43	2.74	30.5	54	1.77
2	3½% Ni	85,600	166	86.0	....	27	2.77	19 5	43	2.20
17, 18	3½% Ni	87,300	169	84 2	....	28	2 65	20	45	2 25
1	3½% Ni	94,600	187	90.8	....	31	2.65	21 5	46	2.14
36, 37	3½% Ni	99,200	198	91.4	....	32	2.59	22	46	2.09
12	3½% Ni	128,700	271	104.1	28 1	43	2 65	28	55	1 96
26	3½% Ni	141,300	341	110.5	35 7	50	2.63	35	61	1.74
12A	3½% Ni	148,800	320	106.1	33 5	48	2.62	31	58	1.87
20	3½% Ni	176,400	361	113 4	35 2	51	2.50	37 5	67	1.79
25A	Ni-Cr	114,500	228	101.0	20.6	36	2.55	24.5	47	1.92
14	Ni-Cr	128,000	245	99.5	24 0	37	2 68	25	52	2.08
14A	Ni-Cr	150,200	305	106.0	34.5	46	2.76	32	56	1.75
25	Ni-Cr	177,500	372	112.2	38.5	54	2 62	36	66	1.83
13	Ni-Mo	127,700	255	101 1	25.7	38	2.70	26 5	53	2.00
9	Cr-V	93,600	195	91 6	...	34	2.54	22.5	45	2.00
11	Cr-V	127,000	264	102.2	26 8	41	2.71	28	53	1.89
11A	Cr-V	134,000	290	103.6	30.2	41	2.53	30.5	57	1 87
30A	Cr-V	157,000	331	113.3	35.8	54	2.58	34.5	61	1.77
8	Cr-Mo	83,700	167	85 9	....	30	2.81	20.5	43	2.10
6	Cr-Mo	96,400	193	92.3	....	34	2 65	22	43	1.95
19	Cr-Mo	107,300	229	95.0	....	30	2.80	24	48	2.00
7	Cr-Mo	125,000	257	101.5	26.5	41	2.68	26.5	50	1.89
10	Cr-Mo	133,500	281	103.8	29.7	44	2.60	28	55	1 96
10A	Cr-Mo	135,500	297	103.6	29.5	45	2.62	30 5	54	1.77
27A	Cr-Mo	194,700	390	114.0	40 9	60	2.65	39.5	67	1.70
34	Chrome iron	77,500	171	87.0	....	28	2.97	20	42	2.10
35	Stainless iron	92,800	198	94.0	....	32	3.40	22	45	2.04

a numerical value is obtained which represents the average power required in cutting. (Refer to Tables 6 and 11.)

120 This method is not sufficiently sensitive for comparisons of individual tools but served well for the purpose in view, namely, to secure data upon which to base computations of the power required for rough turning. The results are chiefly of interest in connection with power-transmission problems but it should be recognized that only average values were obtained, and that the equipment available did not permit determination of the fluctua-

TABLE 11 POWER REQUIRED IN CUTTING DIFFERENT STEELS WITH THE SELECTED FORM OF TOOL—CONTINUED ON PAGE 591

Test log								Power required in cutting, kw.		
No.	Steel type	Tensile strength, 1000 lb per sq. in.	Tests made with tool steel	Cutting speed, ft. per min.	Depth of cut, in.		Feed, in. per revolution	Determined experimentally	Computed from Equation [9]	Value of K of Equation [7]
16	0.18% C	66.4	I	20	3/16	0.188	0.028	0.8	0.7	...
16	0.18% C	66.4	I	40	3/16	0.188	0.028	1.7	1.4	...
16	0.18% C	66.4	I	60	3/16	0.188	0.028	2.5	2.1	...
16	0.18% C	66.4	I	80	3/16	0.188	0.028	3.0	2.8	...
16	0.18% C	66.4	I	100	3/16	0.188	0.028	3.6	3.5	...
16	0.18% C	66.4	I	120	3/16	0.188	0.028	5.1	4.2	...
16	0.18% C	66.4	L	140	3/16	0.188	0.028	6.4	4.9	...
16	0.18% C	66.4	L	63	7/32	0.219	0.074	7.5	6.9	...
36	3.5% Ni	99.2	P	125	3/16	0.188	0.015	3.2	2.9	9.1
36	3.5% Ni	99.2	P	96	1/8	0.125	0.028	2.6	2.8	7.8
36	3.5% Ni	99.2	P	82	3/16	0.188	0.028	3.9	3.6	9.0
36	3.5% Ni	99.2	P	74	1/4	0.250	0.028	5.4[a]	4.3	10.4
36	3.5% Ni	99.2	P	64	11/32	0.344	0.028	7.4[a]	5.1	12.0
36	3.5% Ni	99.2	P	59	7/16	0.438	0.028	8.6[a]	6.0	11.9
36	3.5% Ni	99.2	P	53	3/16	0.188	0.053	3.9	4.4	7.4
36	3.5% Ni	99.2	P	44	3/16	0.188	0.074	5.0	5.1	8.2
36	3.5% Ni	99.2	P	37	3/16	0.188	0.092	4.6	5.3	7.2
37	3.5% Ni	99.2	K	20	3/16	0.188	0.015	0.5	0.5	8.9
37	3.5% Ni	99.2	K	30	3/16	0.188	0.015	0.7	0.7	8.3
37	3.5% Ni	99.2	P	30	3/16	0.188	0.015	0.8	0.7	9.5
37	3.5% Ni	99.2	P	30	3/16	0.188	0.015	0.7	0.7	8.8
37	3.5% Ni	99.2	P	50	3/16	0.188	0.015	1.5	1.2	10.6
37	3.5% Ni	99.2	K	50	3/16	0.188	0.015	1.5	1.2	10.6
37	3.5% Ni	99.2	K	50	3/16	0.188	0.015	1.2	1.2	8.5
37	3.5% Ni	99.2	K	80	3/16	0.188	0.015	2.1	1.9	9.3
37	3.5% Ni	99.2	K	80	3/16	0.188	0.015	1.9	1.9	8.4
37	3.5% Ni	99.2	K	100	3/16	0.188	0.015	2.7	2.3	9.6
37	3.5% Ni	99.2	K	100	3/16	0.188	0.015	2.6	2.3	9.2
37	3.5% Ni	99.2	K	20	1/8	0.125	0.028	0.5	0.6	7.2
37	3.5% Ni	99.2	P	30	1/8	0.125	0.028	0.8	0.9	7.6
37	3.5% Ni	99.2	K	33	1/8	0.125	0.028	1.0	1.0	8.6
37	3.5% Ni	99.2	K	50	1/8	0.125	0.028	1.4	1.5	8.0
37	3.5% Ni	99.2	P	50	1/8	0.125	0.028	1.5	1.5	8.6
37	3.5% Ni	99.2	P	50	1/8	0.125	0.028	1.4	1.5	8.0
37	3.5% Ni	99.2	K	55	1/8	0.125	0.028	1.6	1.6	8.3
37	3.5% Ni	99.2	K	80	1/8	0.125	0.028	2.2	2.3	7.9
37	3.5% Ni	99.2	K	88	1/8	0.125	0.028	2.3	2.6	7.5
37	3.5% Ni	99.2	K	110	1/8	0.125	0.028	3.0	3.2	7.8
37	3.5% Ni	99.2	K	20	3/16	0.188	0.028	0.8	0.9	7.6
37	3.5% Ni	99.2	P	30	3/16	0.188	0.028	1.4	1.3	8.9
37	3.5% Ni	99.2	P	30	3/16	0.188	0.028	1.4	1.3	8.9
37	3.5% Ni	99.2	K	33	3/16	0.188	0.028	1.6	1.4	9.2
37	3.5% Ni	99.2	P	50	3/16	0.188	0.028	2.1	2.2	8.0
37	3.5% Ni	99.2	K	55	3/16	0.188	0.028	2.4	2.4	8.3
37	3.5% Ni	99.2	P	60	3/16	0.188	0.028	2.8	2.6	8.9
37	3.5% Ni	99.2	P	70	3/16	0.188	0.028	3.3	3.1	9.0
37	3.5% Ni	99.2	P	75	3/16	0.188	0.028	3.4	3.3	8.6
37	3.5% Ni	99.2	K	80	3/16	0.188	0.028	3.2	3.5	7.6
37	3.5% Ni	99.2	K	88	3/16	0.188	0.028	3.4	3.9	7.3
37	3.5% Ni	99.2	P	90	3/16	0.188	0.028	3.7	3.9	7.8
37	3.5% Ni	99.2	P	100	3/16	0.188	0.028	4.1	4.4	7.8
37	3.5% Ni	99.2	K	110	3/16	0.188	0.028	4.7	4.8	8.1
37	3.5% Ni	99.2	K	20	1/4	0.250	0.028	1.0	1.2	7.1
37	3.5% Ni	99.2	P	30	1/4	0.250	0.028	2.0	1.7	9.5
37	3.5% Ni	99.2	K	33	1/4	0.250	0.028	2.0	1.9	8.7
37	3.5% Ni	99.2	K	50	1/4	0.250	0.028	2.6	2.9	7.4
37	3.5% Ni	99.2	P	50	1/4	0.250	0.028	3.1	2.9	8.9
37	3.5% Ni	99.2	K	65	1/4	0.250	0.028	3.7	3.8	8.1
37	3.5% Ni	99.2	K	80	1/4	0.250	0.028	4.3	4.6	7.7
37	3.5% Ni	99.2	K	20	5/16	0.313	0.028	1.5	1.4	8.6

[a] These values are rather far from the averages shown in Figs. 20 and 21, due probably to some uncontrolled variable in the individual experiments. This accounts for the discrepancy between the experimental and computed power values. It will be noted that in these cases the value of K of Equation [7] is either considerably higher or lower than the mean value of 8.8.

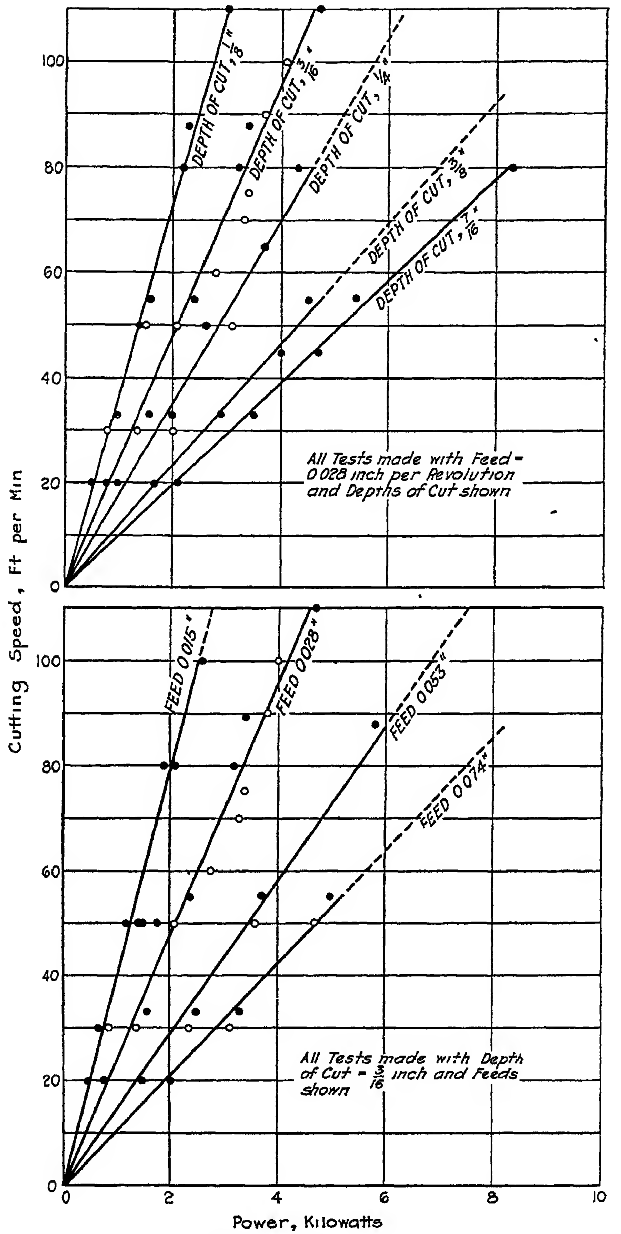


FIG. 20 EFFECT OF CUTTING SPEED, FEED, AND DEPTH OF CUT ON POWER REQUIRED IN CUTTING $3\frac{1}{2}$ PER CENT NICKEL STEEL WITH TENSILE STRENGTH OF ABOUT 100,000 LB. PER SQ. IN.

Tests made on test log 37, with tool steels K (represented by heavy dots) or P (represented by circles).

tions or the maximum momentary power consumption, both of which must ordinarily be considered in power transmission. However, the experiments tie together the effects of variations in speed, feed, depth, and material cut upon the power required in cutting with the selected form and size of tool.

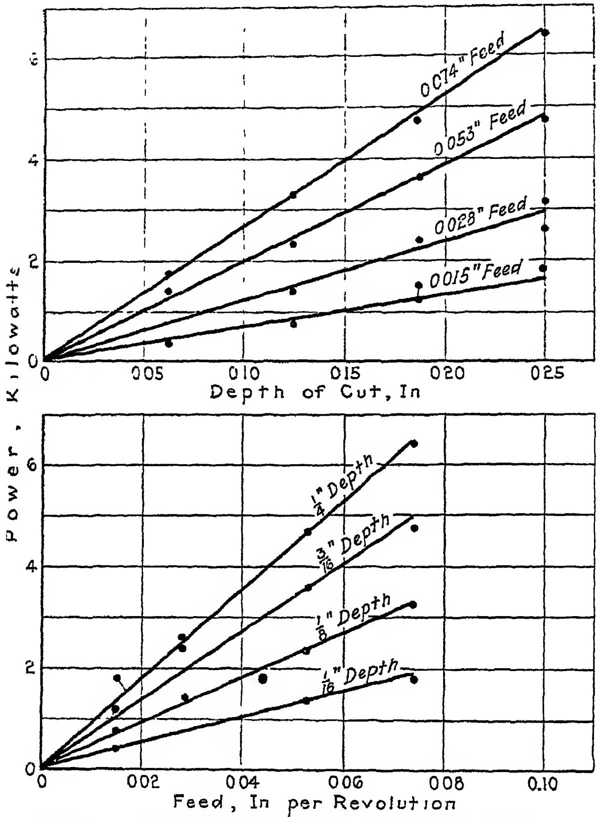


Fig. 21 Effect of Feed and Depth on Power Required in Cutting 3½ Per Cent Nickel Steel Having a Tensile Strength of About 100,000 Lb. Per Sq. In.

All tests made dry at a cutting speed of 50 ft. per min. with high-speed steel K and selected size and form of tool used throughout investigation

121 Within the limits of accuracy attained in the experiments there was no difference in the power required when cutting with the high-tungsten–cobalt, low-tungsten–high-vanadium, or high-tungsten–low-vanadium, high-speed steels. Likewise there was no detectable difference in the power required in cutting steels of different compositions when these showed equal tensile strengths.

122 The results obtained in cutting 3½ per cent nickel steel having a tensile strength close to 100,000 lb. per sq. in. are shown

in Figs. 20 and 21. As can be seen, at constant feed and depth of cut, the power is approximately proportional to the speed. Similarly at constant speed and feed it is approximately proportional to the depth of cut and at constant speed and depth of cut approximately proportional to the feed.

123 If the constants of proportionality were consistent these relations could all be represented by the equation:

$$P = KFDS = KAS \quad . \; . \; . \; . \; . \; . \; . \; . \quad [7]$$

in which K is a constant for the particular size and form of tool used and properties of the steel cut, F the feed, D the depth of cut, S the speed, and $A (= FD)$ the area of cut.[1] A closer examination, however, shows that the constants are only roughly consistent with this equation. In Table 11 are summarized the results of the power determinations. The constant K determined from Equation [7] is given for all the determinations on 3.5 per cent nickel steel. As can be seen, they vary from 6.5 to 12.0 with an average of 8.3. The Equation [7] can be relied upon to estimate the power roughly with a maximum error of about 40 per cent.

124 From the consistency with which the slopes of the lines in Figs. 20 and 21 depart from the linear relation it seems possible that a more thorough analysis of the data would allow an empirical formula to be written more closely representing the data.

125 However, since the measurements themselves are subject to considerable fluctuation, duplicate runs sometimes differing by 10 or 15 per cent in power consumption, a high degree of accuracy in predicting power requirements cannot be expected.

126 Equation [7] applies only to cutting the steel containing 0.3 per cent carbon and $3\frac{1}{2}$ per cent nickel when heat treated to show a tensile strength of about 100,000 lb. per sq. in. To gain an idea of the variation in power when cutting harder or softer steels the results obtained in cutting four different steels (at 0.028-in. feed, $\frac{3}{16}$-in. depth of cut) were plotted in Fig. 22. The ratio m of the slopes of the power-cutting speed lines to the slope for the nickel steel having about 100,000 lb. per sq. in. tensile strength were then plotted as ordinates with the tensile strength as abscissas. The points lie close to the straight line.

$$m = 1 + \frac{4.5}{100{,}000}\,(T - 100{,}000) = \frac{4.5T - 350{,}000}{100{,}000} \quad . \; . \quad [8]$$

where T is the tensile strength of the material cut. If the same variation with tensile strength be assumed to hold over the whole range of cutting conditions we can write

$$P = mKAS = mKFDS \quad . \; . \; . \; . \; . \; . \; . \quad [9]$$

[1] That this equation gives approximate values for the power has previously been shown by Chas. Robbins, Power Required to Cut Metal. Trans. A.S.M.E., vol. 32, 1910, pp. 199-209.

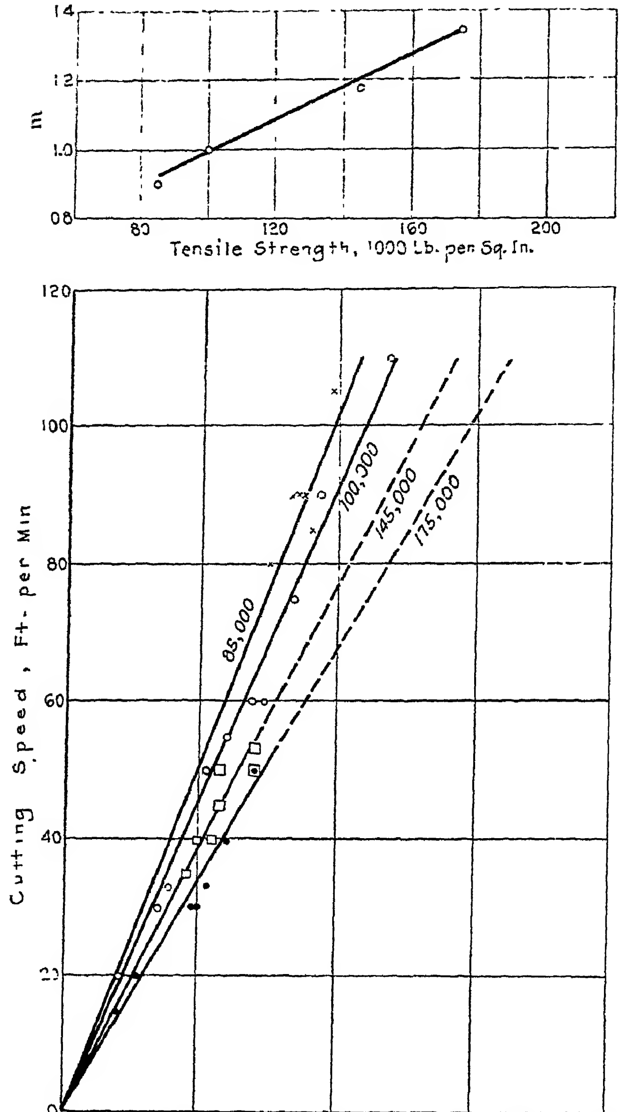


FIG. 22 EFFECT OF MATERIAL CUT ON POWER REQUIRED IN CUTTING STEELS OF DIFFERENT TENSILE STRENGTHS WITH SELECTED FORM AND SIZE OF TOOL

Values of power given in Fig. 23 when cutting 3½ per cent nickel steel with a tensile strength of 100,000 lb. per sq. in. may be converted into the power required to cut harder or softer steels by multiplying by a material constant m whose values are given in upper part of chart.

Tests made dry with the selected form of tool described in the text at a feed of 0.028 in. per revolution and a depth of cut of $\frac{3}{16}$ in.

TABLE 11 POWER REQUIRED IN CUTTING DIFFERENT STEELS WITH THE SELECTED FORM OF TOOL—CONTINUED FROM PAGE 586

Test log								Power required in cutting, kw		
No.	Steel type	Tensile strength, 1000 lb. per sq in	Tests made with tool steel	Cutting speed, ft per min.	Depth of cut, in.		Feed, in per revolution	Determined experimentally	Computed from Equation [9]	Value of K of Equation [7]
37	3.5% Ni	99.2	K	33	$\frac{5}{16}$	0.313	0.028	3.0[a]	2.4	10.4
37	3.5% Ni	99.2	K	45	$\frac{5}{16}$	0.313	0.028	3.6	3.3	9.1
37	3.5% Ni	99.2	K	55	$\frac{5}{16}$	0.313	0.028	4.4	4.0	9.1
37	3.5% Ni	99.2	P	30	$\frac{11}{32}$	0.344	0.028	3.0[a]	2.4	9.6
37	3.5% Ni	99.2	P	30	$\frac{11}{32}$	0.344	0.028	3.1[a]	2.4	10.7
37	3.5% Ni	99.2	P	50	$\frac{11}{32}$	0.344	0.028	4.6	4.0	9.5
37	3.5% Ni	99.2	K	20	$\frac{3}{8}$	0.375	0.028	1.7	1.7	8.1
37	3.5% Ni	99.2	K	33	$\frac{3}{8}$	0.375	0.028	2.9	2.9	8.4
37	3.5% Ni	99.2	K	45	$\frac{3}{8}$	0.375	0.028	4.0	3.9	8.5
37	3.5% Ni	99.2	K	55	$\frac{3}{8}$	0.375	0.028	4.5	4.8	7.8
37	3.5% Ni	99.2	K	20	$\frac{7}{16}$	0.438	0.028	2.1	2.0	8.5
37	3.5% Ni	99.2	K	33	$\frac{7}{16}$	0.438	0.028	3.5	3.4	8.6
37	3.5% Ni	99.2	K	45	$\frac{7}{16}$	0.438	0.028	4.7	4.6	8.5
37	3.5% Ni	99.2	K	55	$\frac{7}{16}$	0.438	0.028	5.4	5.6	8.0
37	3.5% Ni	99.2	K	80	$\frac{7}{16}$	0.438	0.028	8.3	8.2	8.5
37	3.5% Ni	99.2	K	20	$\frac{3}{16}$	0.188	0.053	1.5	1.6	7.5
37	3.5% Ni	99.2	P	30	$\frac{3}{16}$	0.188	0.053	2.4	2.5	8.0
37	3.5% Ni	99.2	P	30	$\frac{3}{16}$	0.188	0.053	2.4	2.5	8.0
37	3.5% Ni	99.2	K	33	$\frac{3}{16}$	0.188	0.053	2.5	2.7	7.6
37	3.5% Ni	99.2	P	50	$\frac{3}{16}$	0.188	0.053	3.6	4.1	7.2
37	3.5% Ni	99.2	K	55	$\frac{3}{16}$	0.188	0.053	3.7[a]	4.5	6.8
37	3.5% Ni	99.2	K	88	$\frac{3}{16}$	0.188	0.053	5.8[a]	7.2	6.6
37	3.5% Ni	99.2	K	20	$\frac{3}{16}$	0.188	0.074	2.0	2.3	7.2
37	3.5% Ni	99.2	P	30	$\frac{3}{16}$	0.188	0.074	3.1	3.5	7.4
37	3.5% Ni	99.2	P	30	$\frac{3}{16}$	0.188	0.074	3.0	3.5	7.2
37	3.5% Ni	99.2	K	33	$\frac{3}{16}$	0.188	0.074	3.3	3.8	7.2
37	3.5% Ni	99.2	P	50	$\frac{3}{16}$	0.188	0.074	4.7[a]	5.7	6.9
37	3.5% Ni	99.2	K	55	$\frac{3}{16}$	0.188	0.074	5.0[a]	6.3	6.5
37	3.5% Ni	99.2	P	30	$\frac{3}{16}$	0.188	0.092	3.5[a]	4.3	6.7
37	3.5% Ni	99.2	P	30	$\frac{3}{16}$	0.188	0.092	3.5[a]	4.3	6.7
25A	Ni-Cr	114.5	I	20	$\frac{3}{16}$	0.188	0.028	1.5	1.0	...
25A	Ni-Cr	114.5	I	40	$\frac{3}{16}$	0.188	0.028	2.3	1.9	...
25A	Ni-Cr	114.5	I	60	$\frac{3}{16}$	0.188	0.028	3.4	2.8	...
25A	Ni-Cr	114.5	I	70	$\frac{3}{16}$	0.188	0.028	3.6	3.4	...
15A	C-Cr	148.4	I	65	$\frac{3}{16}$	0.188	0.015	1.9	1.8	...
15A	C-Cr	148.4	I	35	$\frac{3}{16}$	0.188	0.028	1.8	1.8	...
25	Ni-Cr	177.5	I	10	$\frac{3}{16}$	0.188	0.028	0.5	0.6	
25	Ni-Cr	177.5	I	20	$\frac{3}{16}$	0.188	0.028	1.4	1.2	...
25	Ni-Cr	177.5	I	30	$\frac{3}{16}$	0.188	0.028	2.2	1.8	
25	Ni-Cr	177.5	I	33	$\frac{3}{16}$	0.188	0.028	2.2	2.0	...
25	Ni-Cr	177.5	I	40	$\frac{3}{16}$	0.188	0.028	2.4	2.4	...
25	Ni-Cr	177.5	I	50	$\frac{3}{16}$	0.188	0.028	2.8	3.0	...
25	Ni-Cr	177.5	I	60	$\frac{3}{16}$	0.188	0.028	2.9	3.5	...
31	Ni-Cr	177.5	I	63		0.08	0.015	0.9	0.9	...
31	Ni-Cr	177.5	I	47	$\frac{1}{8}$	0.125	0.020	1.2	1.3	...
31	Ni-Cr	177.5	I	15	$\frac{3}{16}$	0.188	0.028	0.8	0.9	...
31	Ni-Cr	177.5	I	20	$\frac{3}{16}$	0.188	0.028	1.1	1.2	...
31	Ni-Cr	177.5	I	30	$\frac{3}{16}$	0.188	0.028	2.0	1.8	...
31	Ni-Cr	177.5	I	40	$\frac{3}{16}$	0.188	0.028	2.6	2.4	...
31	Ni-Cr	177.5	I	23	$\frac{3}{16}$	0.188	0.053	2.2	2.7	...
37	3.5% Ni	99.2	K	50	$\frac{1}{16}$	0.063	0.015	0.4	0.4	8.5
37	3.5% Ni	99.2	K	50	$\frac{1}{16}$	0.063	0.053	1.4	1.4	8.5
37	3.5% Ni	99.2	K	50	$\frac{1}{16}$	0.063	0.074	1.8	1.9	7.8
37	3.5% Ni	99.2	K	50	$\frac{1}{8}$	0.125	0.015	0.8	0.8	8.5
37	3.5% Ni	99.2	K	50	$\frac{1}{8}$	0.125	0.053	2.3	2.7	7.0
37	3.5% Ni	99.2	K	50	$\frac{1}{8}$	0.125	0.074	3.3	3.8	7.1
37	3.5% Ni	99.2	K	50	$\frac{1}{4}$	0.250	0.015	1.8	1.6	9.0
37	3.5% Ni	99.2	K	50	$\frac{1}{4}$	0.250	0.053	4.7	5.5	7.1
37	3.5% Ni	99.2	K	50	$\frac{1}{4}$	0.250	0.074	6.4[a]	7.7	6.9

[a] These values are rather far from the averages shown in Figs. 20 and 21, due probably to some uncontrolled variable in the individual experiments. This accounts for the discrepancy between the experimental and computed power values. It will be noted that in these cases the value of K of Equation [7] is either considerably higher or lower than the mean value of 8.3.

127 This shows that (Fig. 22) the power required in cutting steel having a tensile strength of about 175,000 lb. per sq. in. is roughly 35 per cent more than that used in cutting the $3\frac{1}{2}$ per

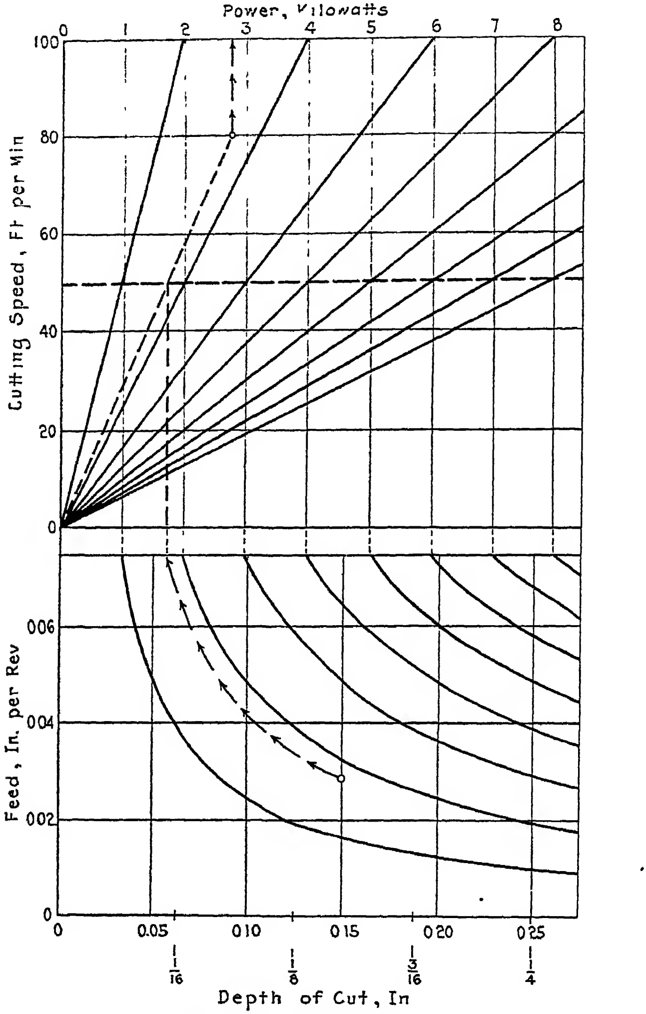


Fig. 23 Power Chart

This gives an approximate determination of power required in cutting, with selected size and form of tool at different speeds, feeds, and depths, $3\frac{1}{2}$ per cent nickel steel with tensile strength of about 100,000 lb. per sq. in. Chart is drawn to represent Equation [7]. To obtain power required in cutting harder or softer steels under otherwise comparable conditions multiply numerical values obtained from above chart by a material constant m whose values are given in Fig. 22.

cent nickel steel with a tensile strength of about 100,000 lb. per sq. in.; that in cutting steel with a tensile strength of 85,000 lb. per sq. in. is about 10 per cent less.

128 A graphical summary of the approximate relations between power, feed, depth, and speed, as represented by Equation [7], is given in Fig. 23. This chart is divided into two parts; in the lower half feed is the ordinate and depth of cut the abscissa and the heavy curved lines represent combinations of feed and depth which give equal areas of cut and which likewise require equal power in cutting at any selected cutting speed. In the upper half of Fig. 23 speed is the ordinate and power the abscissa. The numerical values of the power required in cutting the 3½ per cent nickel steel having a tensile strength of about 100,000 lb. per sq. in. (test log 37) at 50 ft. per min. are placed at the top of Fig. 23.

129 To find the power required in cutting at, say, 80 ft. per min. at 0.028-in. feed and 0.15-in. depth of cut, find the point in the lower half of the power chart, Fig. 23, representing the selected feed and depth and project upward as shown by the arrows to the base line of the upper half of the chart. At this intersection project vertically to the 50-ft. speed line. Connect this point by a straight edge with the origin of the speed-power lines in the upper half of Fig. 23, and the point of intersection between this line and the chosen speed of 80 ft. per min. lies directly under the power required in cutting. By multiplying this value by the proper value of *m* given in Fig. 22 the power required in cutting harder or softer steels than the selected 3½ per cent nickel steel can be obtained.

Improvement in Tool Performance from the Cooling of the Tools with Water or Other Liquids

130 The tests so far described were all made dry, while most industrial machining is done wet. It was therefore considered important to supplement the "dry" lathe tests by experiments in which different liquids were thrown on the tool.

131 It is, of course, well known that increased tool life results from the use of water or other coolants in rough turning, but the magnitude of these effects is known to vary, depending upon the cutting conditions. For example, by throwing a heavy stream of water directly upon the chip where it is being removed from the work, in order to cool the tool, Taylor (4) obtained a gain in cutting speed of about 40 per cent with his original high-speed steel tools. However, with his improved steels, the compositions of which more nearly resembled current commercial types, the gain in cutting speed was only about 15 per cent. But even this is important for, as pointed out by Taylor, elaborate studies of the exact angles to which the tools should be ground resulted in many instances in "the very small gain in cutting speed amounting perhaps to from 2 to 5 per cent."

132 Several sets of experiments were made with high-tungsten–low-vanadium and high-tungsten–cobalt steel tools cooled with

oil,[1] with an oil-water emulsion,[2] and with water to which about $1\frac{1}{2}$ per cent by weight of soda was added to reduce corrosion of the machine-tool equipment. Care was taken to direct a copious

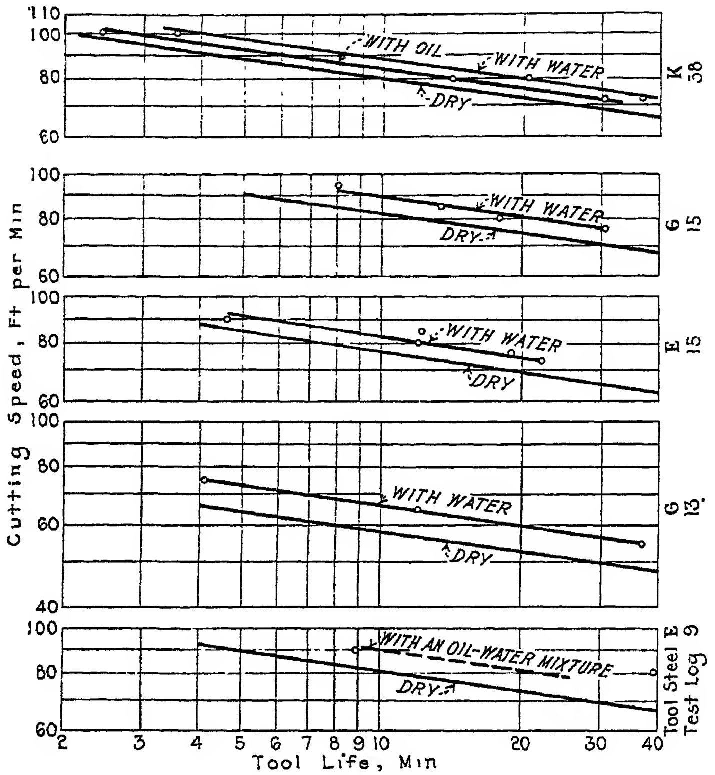


FIG. 24 EFFECT OF COOLANTS ON PERFORMANCE OF HIGH-SPEED LATHE TOOLS

Tests were made at $\frac{3}{16}$-in. depth of cut, 0.028-in feed per revolution and selected form and size of tool used throughout investigation. For other details concerning conditions of experiments refer to text.

stream of the chosen liquid upon the chip and tool, but the exact rate of flow was not measured except in tests on test log 38. A circulating system was installed on the test lathe in which the coolant was fed by a small pump through a half-inch nozzle so

[1] A paraffin oil was used, having a flash point of 320 deg. fahr., a fire point of 340 deg. fahr., specific gravity of 0.884, and a viscosity at 100 deg. fahr. of 75 Saybolt seconds, at 130 deg. fahr. of 53, and at 210 deg. fahr. of 34.

[2] The oil-water emulsion consisted of 1 part oil and 10 parts by volume of water. The oil consisted of a mixture of mineral oil and sulphonated saponifiable oils commercially known as soluble cutting oil. Sap. No. = 8.56, free organic acid (computed as oleic) = 1.46 per cent, sulphur = 0.53 per cent.

placed that the stream of liquid was thrown in the direction of the feed and down at an angle of about 45 deg. upon the top surface of the nose of the tool. By this method both the chip and the working portions of the tool were cooled.

133 In tests on log 38 the coolant was handled in the manner described except that the rate of flow was under closer control. This was kept at $\frac{3}{4}$ gal. per min. and for the water was fed through a $\frac{1}{4}$-in. nozzle.

134 The results summarized in Table 12 and Fig. 24 show that the gain in tool performance is somewhat greater with water than with the oil-water cutting compound and likewise somewhat greater with the latter than with the paraffin oil. The effectiveness of these liquids in increasing tool life increases with the amount of water present and indicates that a large part of the benefits derived from their use is due to their cooling action rather than lubrication. Were lubrication largely responsible, oil would be expected to show an appreciable superiority, compared with water. This agrees with the results recently reported by Herbert (12) who has measured tool temperatures in rough turning. The lowest working temperatures, under otherwise comparable conditions, were obtained with water, intermediate temperatures with oil, and the highest working temperatures when cutting dry.

135 While there appear to be measurable differences in the effectiveness of the three liquids used, they are not large, as will be evident by examination of Fig. 24. This chart also shows that the equation representing the relation between the cutting speed and tool life in dry rough turning is applicable to wet turning, for when plotted to logarithmic coördinates the slope of the cutting speed-endurance line in dry cutting is parallel to the lines representing tools cooled with water, the paraffin oil, or the emulsion. The lines are shifted to higher speeds by coolants but their slopes, for all practical purposes, are the same.

136 The relatively small increase in cutting speed produced by the several coolants, amounting in the described experiments to from 5 to 15 per cent, is consistent with the relatively small improvement found by Taylor when cooling his best high-speed steel tools.

137 Current types of high-speed steel for rough turning are unquestionably superior to and have a higher degree of what may be called "red-hardness" than Taylor's original high-speed steel tools. Since the benefits derived from the use of water or other liquids appear to be due largely to reduction in working temperatures of the tools, it is not strange that a relatively small gain in cutting speed is shown in the experiments just described in comparison with the large gain reported by Taylor with the older types of steel. The latter were not so well fitted to withstand high operating temperatures and very severe service.

TABLE 12 EFFECT OF DIFFERENT COOLANTS ON THE PERFORMANCE OF HIGH-SPEED STEEL LATHE TOOLS

Coolant [a] used in lathe tests	Material cut: Test log no	Material cut: Type of steel	Material cut: Tensile strength, lb. per sq. in.	Tool steel used	Cutting speed, ft. per min.	Tool life, min. First grind 1	First grind 2	First grind 3	First grind 4	First grind 5	Regrind 1	Regrind 2	Regrind 3	Regrind 4	Regrind 5	Average all tests	Average power required in cutting, Kw.
None	15	C-Cr	91,900	E	80	5.9	7.5	7.7	4.7	. ..	8 5	6 3	9.5	8 3	..	7.3	3 0
Water	15	C-Cr	91,900	E	73	24.3	25.0	B	20.8	....	18 7	21.8	....	. .	....	22 1	2.5
Water	15	C-Cr	91,900	E	76	20 0	....	25.0	16.0	....	13.7	13.3	28.8	21.2	13 9	19 0	2.7
Water	15	C-Cr	91,900	E	80	10.9	12.8	10.1	13 3	....	12 6	13 5	....	....	11.0	12.0	3.0
Water	15	C-Cr	91,900	E	85	10.8	12.4	15.3	11 8	12.3	11 2	11 9	11 9	..	....	12 2	3.1
Water	15	C-Cr	91,900	E	90	4.3	4 5	....	....	.. .	3 6	..	5 9	....	....	4.6	3 3
None	15	C-Cr	91,900	G	80	11 2	9.4	7.1	B	12 0	15.1	12.7	12 3	11.1	. ..	11.4	3 1
Water	15	C-Cr	91,900	G	76	36 9	15 8	41 8	27 9	...	23 5	14 3	18 2	19 3	22 6	24 3	2.6
Water	15	C-Cr	91,900	G	80	18.6	18.2	17.8	16 6	....	18.6	B	..	17 9	. .	18.0	2 7
Water	15	C-Cr	91,900	G	85	19.1	14 5	14.2	16 2	....	11.7	10.3	11.2	10.4	....	13.4	2 9
Water	15	C-Cr	91,900	G	95	10.0	B	..	7 6	9 0	6 4	10.1	6.5	7.0	....	8 1	3.3
None	9	Cr-V	93,600	E	70	....	21.3	42.3	....	32.0	23.7	34.0	33 9	33.3	....	31 5	2 9
None	9	Cr-V	93,600	E	80	13 8	10.2	8.6	....	....	7.7	9.2	9 8	9.2	11.0	9.9	3 2
None	9	Cr-V	93,600	E	90	6.4	4.1	3.8	4.5	....	4.7	4.9	3.8	4.4	....	4.6	3.6
Water	9	Cr-V	93,600	E	90	B	9.3	13.0	14.1	7.6	7.3	....	14.5	16.2	....	11 2	3 5
Oil-water ...	. 9	Cr-V	93,600	E	77	42 0	42 2B	....	....	....	(90)	(90)	. ..	....	...		2.8
Oil-water ...	. 9	Cr-V	93,600	E	80	30.5	40.9	....	27 3	B	36 3	....	41.5	47.3	49.7	39.1	3 0
Oil-water	9	Cr-V	93,600	E	90	7.7	7.9	12 8	8.2	....	10 8	7.2	7.6	....	....	8 9	3.3
Oil	9	Cr-V	93,600	E	90	9.1	6 7	5 1	11.0	6.5	7.0	11.3	....	8 4	.. .	8 1	3 4
None	38	3½% Ni	99,200	K	80	13.1	7.1	8 0	9 0	8 4	11.2	....	....	....	....	9 5	3 6
Water	38	3½% Ni	99,200	K	72	30 8	35.8	.. .	. ..	....	....	....	43.6	35 6	....	36 5	3.1
Water	38	3½% Ni	99,200	K	80	18 6	22 8	20.4	17 0	24 2	....	....	....	....	....	20 4	3 3
Water	38	3½% Ni	99,200	K	100	3.1	3.8	....	....	....	.. .	4.9	2 8	2 3	4 2	3 5	4 1
Oil	38	3½% Ni	99,200	K	72	35.7	15 7	34.5	. ..	....	....	....	21.3	35.0	38.3	30 1	2 9
Oil	38	3½% Ni	99,200	K	80	11.3	11.8	14.8	. ..	....	....	....	14.8	14 4	16 7	11.0	3 0
Oil	38	3½% Ni	99,200	K	100	2 2	3.0	2.2	3.0	. ..	. ..	....	..	2.1	2 0	2 4	4 0
None	13	Ni-Mo	127,700	E	55	13.3	10.1	13.0	. ..	7 0	7 5	10.1	10 0	14 6	....	10 7	2 5
Water	13	Ni-Mo	127,700	E	55	59 7	30.4	32.4	B	29 2	B	24.7	29.3	31.0	....	33.8	2 4
None	13	Ni-Mo	127,700	G	55	20.3	19.8	12.4	16 4	....	13.8	10.6	9.6	14.7	13.2	14.5	2 5
Water	13	Ni-Mo	127,700	G	55	25.6	36.6	27.8	38.0	....	....	39.7	....	54.6	....	37.0	2 5
Water	13	Ni-Mo	127,700	G	65	5.9	....	13 9	8 1	8 7	. .	15.8	....	15 3	16.7	12 1	2 7
Water	13	Ni-Mo	127,700	G	75	2.6	2 3	3.7	4.1	....	5.5	....	....	5.5	5.2	4.1	3 0

B = tool broke.

[a] For details concerning the several coolants and the manner in which they were used refer to the text of the report.

Summary and Conclusions

138 As stated at the outset, the primary purpose of the described tests was to extend to current commercial alloy steels some of the empirical laws originally worked out by Taylor in rough-turning carbon steels. In so far as this phase of the work is concerned, the various charts and equations given throughout the report themselves constitute a summary of important features.

139 Since in all cases investigated the results agree with the empirical equations originally derived by Taylor, the numerical values of the different constants required for their application to modern high-speed and high-tensile alloy structural steels become of particular interest. These also are best summarized in the various charts given throughout this report.

140 Measurement of the average power required in cutting with the selected form and size of lathe tool resulted in the following approximate empirical relations:

$$P = mKFDS$$

in which P = power required in cutting, kilowatts
F = feed, inches per revolution
D = depth of cut, inches
S = cutting speed, feet per minute
m = a "material constant"
K = a constant for the particular form and size of tool used.

141 From the standpoint of the steel cut and the performance of the several types of modern high-speed tool steels the following are considered of special interest.

142 If machinability in rough turning is measured by tool life or the cutting speed permitting the tools to last a definite time, then measurable differences are observed between the various structural alloy steels cut in the lathe tests. However, steel which, when heat treated to show low tensile strengths around 80,000 to 100,000 lb. per sq. in., permit the tools to cut for the longest periods do not show similar superiority when treated to show high values of tensile strength in the neighborhood of 175,000 to 195,000 lb. per sq. in. In other words, the character and slope of the cutting-speed–tensile-strength curves vary for steels of different compositions.

143 At low tensile strengths (for the steel cut) increase in carbon or the addition of special elements such as nickel and chromium reduces the cutting speed. However, $3\frac{1}{2}$ per cent nickel steel containing 0.3 per cent carbon is more machinable than the steels containing similar carbon and 1 per cent chromium, with or without the addition of the elements, molybdenum, vanadium, or nickel.

144 At very high tensile strengths in the neighborhood of 180,000 lb. per sq. in. the order of superiority is reversed and the nickel-chromium and chromium-molybdenum steels permitted the highest cutting speeds.

145 With the exception of the chromium-vanadium steel, for which the cutting speed dropped very rapidly with increase in tensile strength, these differences in machinability were generally no greater than differences in tool performance, arising from variations in the quality and heat treatment of the high-speed steels from which the tools were made.

146 The tests described show that when one high-speed steel, either through superior quality, modification in chemical composition, heat treatment, or for other reasons, has a higher cutting speed than another in cutting relatively soft steel, it will show approximately the same superiority in feet per minute when cutting harder steels. Under such conditions the percentage of increase in output, whether based on the rate at which metal may be removed or the amount of metal cut before the tools must be dressed, is greater the lower the cutting speed. Since the cutting speed which may be used decreases with increase in the hardness of the steel cut and with increase in feed and depth of cut, it follows that the greatest gain from superior tools is made in cutting the hardest steels or in taking the heaviest cuts.

147 The addition of about 4 per cent cobalt to the customary high-tungsten–low-vanadium type of high-speed steel, combined with a modification in heat treatment, was again found to improve tool performance. However, it was shown that any gain from these sources may readily be more than counteracted by inferiority in the quality of the steel.

148 The improvement in performance produced by cooling the tools with a heavy stream of water, or other liquids, was found to be relatively small, amounting in the described experiments to from 5 to 15 per cent increase in cutting speed. The effect of such liquids is largely to reduce the working temperatures, for water gave a greater gain in cutting speed than an oil-water emulsion which in turn was more effective than oil.

149 Hardness tests of the steel cut, with the Brinell, Rockwell, Shore, Herbert, and Bierbaum hardness testers, as well as factors ordinarily determined in tension tests, did not give quantitative criteria of the cutting speeds of the different steel forgings. The best general guide to the cutting speed was given by the Brinell hardness and the tensile strength of the steel cut, but quantitative relations were only given by actual tool tests.

SELECTED BIBLIOGRAPHY

1 H. J. French and Jerome Strauss. Lathe Breakdown Tests of Some Modern High Speed Tool Steels. Trans., American Society for Steel Treating, vol. 21 (1922), p. 1125. Technologic Papers of the Bureau of Standards, No. 228.

2 H. J. French, Jerome Strauss and T. G. Digges, Effect of Heat Treatment on Lathe Tool Performance and Some Other Properties of High Speed Steels. Trans., American Society for Steel Treating, vol. 4 (1923), p. 353.

3 H. J. French and T. G. Digges, Experiments with Nickel, Tantalum, Cobalt and Molybdenum in High Speed Steels. Trans., American Society for Steel Treating, vol. 8 (1925), p. 681.

4 F. W. Taylor, On the Art of Cutting Metals. Trans., A.S.M.E., vol. 28 (1907), p. 31.

5 Department of Scientific and Industrial Research, Manchester Association of Engineers. The 1922 Report of the Lathe Tools Research Committee, published by His Majesty's Stationery Office, London.

6 W. Ripper and G. W. Burley, Cutting Power of Lathe Turning Tools. Proceedings, Instn. Mech. Engrs., 1913, parts 3 and 4, p. 1967.

7 E. A. Allcut and E. Miller, Materials and Their Application to Engineering Design, 1923, p. 206, J. B. Lippincott Co., Philadelphia, Pa.

8 A. Kessner, A Method for Determining the Resistance of Metals to Drilling and Its Application to the Investigation of the Machinability of Metals. *Testing*, vol. 1 (1924), p. 270. The Pullman Publishing Co., New York.

9 J. V. Emmons, The Effect of Structure upon the Machining of Tool Steel. Trans., American Society for Steel Treating, vol. 2, (1922), p. 1100.

10 *Engineering* (London), vol. 117 (June 13, 1924), p. 762.

11 E. W. Pierce, Molybdenum Steel — Some Production Data. Trans., American Society for Steel Treating, vol. 5 (1924), p. 571.

12 E. G. Herbert, The Measurement of Cutting Temperatures. Paper presented at Feb. 19, 1926, meeting of the Institution of Mechanical Engineers.

DISCUSSION

R. Poliakoff.[1] The chemical compositions of the high-speed steels used by the authors are given in Table 1. These, we assume, are representative steels of American manufacture. The writer had to do with a large number of high-speed steels of European manufacture and Table 13 gives the compositions of some of these steels, which may be of interest for the purpose of comparison. It will be noticed that as a rule all the brands listed contain more tungsten, more molybdenum, less sulphur, less phosphorus, less silicon, more chromium and a little more nickel, where the latter element shows up, than did the steels used by the authors. A British investigator stated as far back as 1904 and 1905 that nickel is an important factor increasing the elastic limit and breaking stress of the tool and adding to its toughness and ductility.

Since the authors refer frequently to F. W. Taylor and his On the Art of Cutting Metals, it is of interest to see to what extent

[1] Mechanical Engineer, New York, N. Y. Mem. A.S.M.E.

Taylor's recommendations have been accepted by the different steel manufacturers. According to Taylor, the best high-speed steel must have a carbon content not above 0.68 per cent and not below 0.50, in order that the steel may be easily forgeable, not be brittle, but sufficiently hard. The steels, as shown in Table 1, come well within Taylor's recommendation, but some of the European ones have a still lower carbon content. The European steels show practically no manganese; the older high-speed steels had a manganese content up to 0.30 per cent. The steels used by the authors show in some cases even a higher content. Taylor recommended that the manganese content should be as low as

TABLE 13 CHEMICAL COMPOSITION OF EUROPEAN HIGH-SPEED STEELS

Steel No.	C	Si	Mn	S	P	Cr	W	Mo	V	Ni
1	0.60 0 62	0 15	trace	0 004	0.035	4.28	16.44	...	...	0.31
2	0 44 0 44	0.12	...	0.006	0.026	4.26	14.56	...	...	...
3	0 51 0 52	0.08	...	0 004	0.030	4.99	18.18	...	...	traces
4	0 61 0 62	0.10	...	0.010	0.023	4.49	14.45	1.15	...	0.31
5	0 48 0.50	0.15	...	0.004	0.023	4.64	10.30	...	...	traces
6	0 42 0.44	0 08	...	0 005	0.025	4.64	14.53	...	...	...
7	0.59 0.57	0.12	...	0 004	0.026	4.99	16.67	...	...	...
8	0.50 0.51	0.10	...	0.300	0.020	5.04	18.54	1.56	0.82	traces
9	0.45 0.47	0.15	...	0.003	0.026	4.24	14.53	...	...	...
10	0.59	0.28	...	0 011	0.010	4.31	16.29	...	0.35	...
11	0.51	0.14	...	0.009	0.046	4.53	17.0	...	...	...
12	0.56	0.14	...	0.009	0 046	4.53	17.0	...	...	...
13	0.61	0.19	...	...	...	3.60	19.44	1.39	...	...
14	0.61	0.19	...	...	...	3.60	19.44	1.39	...	...
15	0.59	0.28	...	0.011	0.010	4.31	16.29	...	0.35	...
16	0.51	0.14	...	0.009	0.046	4.53	17.0	...	...	...
17	0.53	0.19	...	0.007	0.012	6.18	20.64	1.63	1.23	...
18	0.84	0.21	...	0.020	0.015	6.18	17.62	1.73	1.23	...
19	0.59	0.28	...	0.011	0.010	4.31	16.29	...	0.35	...
20	0.52	0.11	0.07	0.007	0.029	4.05	17.78	1.77	0.44	...
21	0.51	0.14	...	0.009	0.046	4.53	17.00	...	...	...
22	0.51	0.14	...	0.009	0.046	4.53	17.00	...	...	...
23	0.50	0.10	*trace	0.015	0.026	3.40	18.65	1.77	0.28	...

possible, because with a low manganese the steel becomes more ductile, less brittle, less liable to fire cracks, and more forgeable. It would seem that with regard to manganese the European manufacturers follow Taylor's recommendations more closely.

As to silicon: Taylor's limit is "not over 0.15 per cent." The European steels are within this limit; the American steels overstep it (with one exception). It would be interesting to learn from the authors their opinion as to the influence of such a larger silicon content on the durability of the tool, i e., whether it affects it favorably or adversely. The writer ventures his opinion that with a lower silicon content the tungsten content can also be decreased without affecting, to a certain extent, durability.

In the matter of chromium both American and practically all European steels fail to follow Taylor's recommendations, his figure being 5.5 to 6 per cent, particularly in the case of high-tungsten steels.

The authors devote — and rightly so — a considerable part of their paper to the formulas expressing the relation between the life of the tool and cutting speed. Taylor's formula for this relationship is

$$VT^{\frac{1}{8}} = \text{constant} \quad \ldots\ldots\ldots\ldots \quad [1]$$

The authors quote also the formula of Ripper and Burley which is

$$VT^{\frac{1}{12}} = \text{constant} \quad \ldots\ldots\ldots\ldots \quad [2]$$

and the formula of the British Lathe Tools Research Committee which is

$$VT^{\frac{1}{7}} = \text{constant} \quad \ldots\ldots\ldots\ldots \quad [3]$$

In the authors' opinion the different values of the exponent in the Formulas [1] and [3] are so close as to cause differences in computations not greater than 5 per cent — which is a quite permissible difference. To this the writer would add that the difference would not be over 5 per cent for the lower limits of T (time), up to, say, 20 minutes, but would amount to 8 or 9 per cent for T values nearer the higher limits (90 minutes), which is still not so bad. However, should we adopt the Ripper and Burley Formula [2], with the exponent value of $\frac{1}{12}$, the difference in computations would amount to over 30 per cent, which is clearly not permissible.

The authors do not offer any explanation as to the reasons for the discrepancy in the values of the exponent in Formula [1], on the one hand, and Formulas [2] and [3], on the other; neither has the writer found any explanation of it in the original report of the Lathe Tools Research Committee. As his own explanation, he advances the thought that the reason has to be looked for probably in the different values of the constants adopted or assigned by the different experimenters.

Carl G. Barth.[1] It is to be regretted that the great amount of patient and painstaking work done by the authors has apparently resulted in very little of positive value to the practical metal cutter; though, if the negative results obtained in trying to establish definite relations between machinability hardness, of which there are two kinds, and the many other kinds of hardness of materials, can be made sufficiently known to dissuade the shop experimenter from wasting his time in this direction, then something practically useful will nevertheless have been accomplished.

There are some things in the paper to which the writer must

[1] New Haven, Conn. Life Mem. A.S.M.E.

take exception. There is not enough evidence in the experiments made to redetermine the speed-time relation, to justify substituting the exponent $\frac{1}{7}$ for $\frac{1}{8}$ in the formula given in Taylor's On the Art of Cutting Metals[1] for that relation, namely $V = c/T^{\frac{1}{8}}$.

When the experiments were made from which that formula was deduced the precautions taken to insure uniformity of both tool and material cut were so exceptional, that much more than appears in this paper is necessary to convince the writer of the justification of this modification. For carbon tools Mr. Taylor, personally and early, established the relations $V = c/T^{\frac{1}{5}}$; and as the later formula was based on experiments with the early type of Taylor-White high-speed tools, there is reason to believe that the further improved tools would actually call for a still smaller exponent, rather than an intermediate one. For this, as stated in the paper, there has also been evidence put forth. On the strength of a few indications in the writer's experiments on cast iron at the works of Wm. Sellers and Co., the formula $V = c/T^{\frac{1}{12}}$ was adopted for the cast-iron slide rule, and it is interesting to note that this formula has the same exponent as that arrived at by Messrs. Ripper and Burley.

The rather casual remark in defining machinability that the power required is of secondary importance is noted with particular pleasure. Experimenters should realize this and follow Taylor's advice not to waste their efforts on power and pressure experiments under the false impression that such are of primary importance. The experiments under discussion are admitted to point to substantial agreement with the Taylor and Barth empirical formulas. Those, the authors imply, need only to have their constants modified to suit both the slightly better tools of today and the various new steel alloys. Nevertheless, the writer must also take exception to the too simple form of the power formula $P = mKFDS$ in Par. 126; for as the power is directly proportional to the tangential pressure on the tool, and as this varies with the feed raised to a power whose exponent is less than unity, so does also the power. This exponent has been indicated to be even as low as 0.7, in experiments made some forty years ago by Wilfred Lewis, which were found in Taylor's records, and formulated for comparison with the results obtained from the writer's own experiments at the works of Wm. Sellers and Co. For cast iron the pressure found by the writer varies even with an exponent of the depth of cut, though this exponent was much nearer to unity than that for the feed.

The authors have paid some attention to the Taylor and Barth formula [6] that was worked out to give some slight indication, by means of the tensile strength and percentage of elongation, of the machinability of various grades of carbon steels. Since the

[1] Trans. A.S.M.E., vol. 28 (1907), p. 31.

authors have introduced this formula, it may be stated that the writer, some years after it was developed, went over the records and made the following much simpler formula that covers the ground somewhat better. It is given only for what it is worth, i.e., a means for a first and scientific guess, when in practical work such a guess at times becomes necessary. For convenience, the formula has been embodied on a slide rule.

$$V = C\,\frac{1-\dfrac{7}{10+E}}{S-33{,}000} \quad \ldots\ldots\ldots \quad [10]$$

as against the original

$$V = C\,\frac{1-\dfrac{215}{(15+E)^2}}{\sqrt{\left(\dfrac{S}{10{,}000}-3\right)}-0.9} \quad \ldots\ldots \quad [11]$$

in both of which

V = cutting speed for $\frac{3}{16}$-in. depth of cut, $\frac{1}{16}$-in. feed, 20-minute life
S = tensile strength, lb. per sq. in., for carbon steels only
E = elongation in 2-in standard specimen
C = constant depending on the efficiency of tool used

The authors have attempted to establish a relation of V with S, without at the same time taking into account the corresponding value of E.

The authors seem to have the idea that Taylor conceded that he had found that the better high-speed tools did no more than 15 per cent better when cooled with water, even when used on the softer steels. In fact, when the writer told him that he could get no more than this, when running on a very hard piece of steel at the works of the Link-Belt Co., Taylor would not believe it until the experiments had been repeated with consistent results several times in his presence. He died probably with the idea which the writer still retains, namely, that a gain of some 35 per cent may be obtained when cutting soft grades of steel, on which the tools invariably "burn out," while on hard grades they wear off, as they also do when cutting cast iron. But a very heavy stream of water is necessary. At Bethlehem up to 3 gal. per min. were used. Besides, Mr. De Leeuw's remarkable experiments with high-speed milling cutters demonstrated that the life of these when cutting low carbon steels was merely a question of the quantity of cooling water supplied.

Thomas H. Wickenden.[1] Some broad claims have been made regarding the superior machining properties of certain types of

[1] The International Nickel Company, New York, N. Y.

alloy steels. We have been interested in investigating some of these claims and have been unable to convince ourselves of their justification. In one set of tests the power consumption under certain standardized conditions of feed and speed was taken as an index of the relative machinability of different alloy steels. Steels tested came within the following S.A.E. steel specifications:

2340 — 3½ per cent nickel steel
3140 — chrome-nickel steel
4140 — chrome-molybdenum steel
5140 — chrome steel
6140 — chrome-vanadium steel

These steels were all heat-treated to secure approximately an equal Brinell hardness of around 310. Samples of the various steels were turned with the same tool and the power required in each case was determined, after which the order of machining the samples was reversed. The average result of 49 tests showed that machining the 3½ per cent nickel steel consumed the least amount of power, while the chrome-molybdenum steel consumed the greatest amount of power, amounting to about 10 per cent more power than the 3½ per cent nickel steel. No data were secured on tool life in these tests. The carbon content of the chrome-molybdenum steel was toward the upper limit of the specifications, which might account for some of the difference, although the steels were so heat-treated that the tensile strengths were almost identical.

On a great deal of production work the amount of finish left on forgings does not permit removing the metal at the maximum rate which the tool will stand. Hence it is often desirable to secure a combination between the best possible finish and the amount of metal removed in a single machining operation. In cases of this kind, we have found that the machining speeds to be used are often left to the judgment of the operator, and that he is apt to judge the machinability of the metal by the type of finish he secures. If he secures a rough, torn finish, he is of the opinion that he is machining at the highest practical speed.

Several instances of this nature were brought to our attention and led to further investigation. Apparently there is a critical rate of metal removal which tends to produce a rough, torn finish, above or below which rate a smooth finish will be obtained. A typical example will probably best explain this condition.

An 0.19 carbon, 3½ per cent nickel steel was normalized at 1600 deg. fahr. and showed a Brinell hardness of 166. When machined with a depth of cut of 0.045 in. and a feed of 0.030 in. a smooth finish was produced at a cutting speed of between 0 and 25 ft. per min.; a poor finish was produced between the speeds of 25 and 35 ft. per min. and a smooth finish was again produced at from 35 ft. to 110 ft. per min., the maximum speed tested. We

have known cases where the machinist, getting this rough finish, believed that he was operating at the maximum practical speed of machining, while if he had increased his cutting speed 5 or 10 ft per min. he would have secured a smooth finish. It was further found that the cutting speed producing a rough finish changed when the depth of cut and feed relation were altered; that is, by using a depth of cut of 0.030 in. and a feed of 0.030 in. a poor finish was produced between the speeds of 0 and 60 ft. per min. and a smooth finish was secured from 60 ft. to 285 ft. per min, which was the maximum speed tested.

Heat-treating the material to a higher hardness was found to lower the cutting speed, producing a rough finish. At a Brinell hardness of 195 and a cut of 0.030 in. deep and 0.030 in. feed, a rough finish was produced at speeds between 0 and 45 ft. per min, while speeds of from 45 to 155 ft. per min. produced a smooth finish. When the steel was annealed, producing a Brinell hardness of 156, a cut of 0.030 in. deep and 0 030 in. feed produced a smooth finish at speeds of from 0 to 45 ft. per min., a rough finish at from 45 to 115 ft. per min., and a smooth finish again from 115 to 285 ft per min It was also found that a sharper cutting angle on the tool lowered the speed at which a rough finish was produced.

Hence, if a rough, torn finish is being secured it may be corrected in several ways:

a With the same tool conditions by either lowering or preferably increasing the cutting speed until outside of the speed range producing a rough finish

b With the same cutting speed and tool, by changing the depth of cut and feed relation

c By heat-treating the material to a different hardness

d By changing the cutting angle of the tool.

H. W. Graham.[1] This study of rough turning is a valuable contribution to our all too elementary knowledge of the factors that govern the cutting of steel. The very title of the paper, however, makes clear that the authors have dealt only with rough turning. A still greater variety of problems is presented by the finer finish turning.

For years rough turning has received considerable attention, generally from the standpoint of tools and cutting conditions in the lathe. The writer does not decry the usefulness of such work, but if we consider the part that machine work plays in the manufacture of automobiles and accessories, motor and electrical machinery, typewriters and office equipment, washing machines and household utilities, and all that host of articles which is part and parcel of our daily life, it is immediately apparent that the prob-

[1] Chief Inspector, Jones & Laughlin Steel Corporation, Pittsburgh, Pa.

lems associated with the light cutting and oftentimes delicate machining of the integral parts of such equipment is of tremendous importance. The economic importance of light cuts and finish cuts would appear to overbalance that of rough turning. The possibilities of profit to the manufacturer and of saving to the consumer grow to vast proportions when the huge total volume of work is realized in which light and finish cuts are so large a part. It is therefore to be sincerely hoped that the studies of the authors may be continued into the field of finish cuts.

A. L. De Leeuw.[1] The writer is in complete agreement with the statement that a coolant increases materially the speed of the tool. In fact, two cases should be distinguished: First, when the tool is embedded in the work all the time, as in a lathe tool; and second, when it comes out of the work, as with a milling cutter. It is possible with a milling cutter to have almost unlimited speed. For instance, the writer has taken cuts as heavy as 5 in. wide and $\frac{1}{4}$ in. deep at a speed of 835 ft. per minute, but such conditions cannot be attempted when the tool is embedded in the work.

A. L. Davis.[2] The writer agrees with the suggestion by Mr. Graham that the work described by the authors should be extended to the consideration of finishing tools. If this should be done it would be valuable to conduct the experiments with a cutting time of not only 1 hour, but 4 or 5 hours as a base. This latter time is more nearly what practical men in the shops need in their work. If a tool setter is required to operate half a dozen different machines he may only get around to each machine once in an hour or two, and the tools should last a little longer than this interval.

The Authors. In reference to Mr. Barth's discussion, the determination of power in the authors' experiments was wholly a secondary matter. Furthermore, we regard the value of the exponent of VT in Equation [2] as of minor importance. However, Mr. Taylor's On the Art of Cutting Metals was searched very carefully without revealing as much direct experimental evidence to justify the exponent $\frac{1}{8}$ as the authors find in their paper to indicate that the exponent $\frac{1}{7}$ will fit the results.

We have called attention to the fact that the constants for the straight-line relation between power and the various cutting variables were not entirely consistent, the variation being short of plus or minus 20 per cent. A closer determination of power would have given exponents which were not equal to 1. As stated

[1] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.
[2] Research Engineer, Scovill Mfg. Co., Waterbury, Conn. Mem A.S.M.E.

in the paper, however, the selected exponents give a fair approximation, which is believed to be consistent with the accuracy or lack of accuracy of the power measurements Attention is also called to the fact that comparisons between the cutting speed and the various properties of the material cut, such as, hardness, combinations of tensile strength and ductility, etc., which were used or mentioned by Taylor or other experimenters, were included only for the sake of completeness. The authors do not believe that a quantitative relation between the cutting speed and any one or two, or perhaps more, of the physical properties of the metal cut could ever be obtained Such relations, however, are generally indicative in a given class of material, such as steels, as to what may be expected. For example, the cutting speed of a steel of Brinell hardness 400 is lower than that for a steel of Brinell hardness 200.

Mr. Poliakoff has compared American and European tool steels. The authors believe that the amounts of silicon and manganese which are normally found in the American steels have only a minor effect on the durability of lathe roughing tools. This opinion is based on a large number of experiments in which various elements have been varied intentionally in otherwise standard compositions of high-speed steel.

We realize that finishing cuts are now perhaps of wider interest than roughing cuts and experiments are now under way in that field. It is probable, however, that few realize the size of the problem that is involved when it is recommended that the same things be done for finishing cuts in turning as have already been done for roughing cuts. Methods of test are available in rough turning which are not available in finished turning, and progress, therefore, must necessarily be slow, even though the investigators have the genius of F. W. Taylor and the facilities of wealthy organizations back of them.

No. 2019

THE PLASTIC BEHAVIOR OF METAL IN DRAWING

By C. L Ekserglan,[1] Philadelphia, Pa.
Junior Member of the Society

Metal drawing, unlike other classes of engineering work, has proceeded mostly as an art rather than a science. The empirical method has of necessity been the basis of advancement, due to the limited knowledge of the attending phenomena.

The purpose of this study is to foster the development of analysis in drawing as an aid to subsequent development. Toward this end no attempt has been made to set forth any concrete analysis, since the subject is regarded as too complex for such a discussion at present However, as a preliminary step in this direction, an attempt is made to outline a manner of attack by which a proper conception of the phenomena may be realized. For the present a general discussion is made of the manner of working the metal with reference to its state and behavior in comparison with that found in drawing. A survey of the conditions observed in the forming of a stamping is also made along with a report of certain experimental investigations which were conducted by the author.

AN EXAMINATION of the behavior of metal in drawing finally resolves itself into a study of plastic behavior, which becomes specific when subjected to the conditions set up in a drawing operation. We are therefore interested, on the one hand, in the plastic state, and on the other, in the determination of the reactions existing within the draw under which the specific behavior is to be observed.

The Elastic and Plastic States

2 The behavior of a material not only is a function of the manner in which it is worked but also the state in which it exists during this work. The state, in turn, is governed by the physical characteristics of the material and the degree of work to which it

[1] Research Engineer, Laboratory of E. G. Budd Mfg. Co.

Contributed by the Machine Shop Practice Division and presented at the Annual Meeting, New York, N. Y., December 6 to 9, 1926, of The American Society of Mechanical Engineers.

is subjected. Hence, if expressions as to behavior are to be established, the physical properties and the nature of the imposed strains must first be determined.

3 The common metals may be regarded as existing in two possible cold states, the elastic and plastic. The elastic state does not necessarily imply that elasticity is complete but rather that it is predominant, and similarly with plasticity in the latter state. The division is engineering in character rather than physical. It

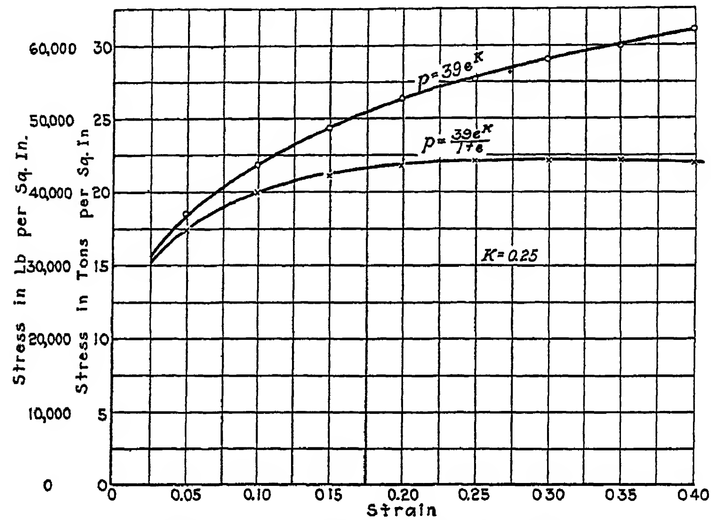


FIG. 1 CURVES SHOWING RELATION BETWEEN STRESS AND STRAIN

follows that in certain instances, especially with non-ferrous metals, such a division may not be successfully made. Either the predominance of the one phase over the other is small or else the change is gradual. In cast iron, a division is possible because, although it possesses neither an elastic limit nor a yield point, for practical purposes it may be regarded as elastic within a certain range. The upper limits of this range are defined by the region in which the transition becomes rapid.

4 In observing the behavior of a material from its stress-strain curve it must be remembered that the record is a manifestation of aggregate crystal action rather than of the individual behavior. The phenomenon of crystal slip is the basis of all plastic reactions. Beilby, Ewing, and Rosenhain have submitted an explanation through their well-known Amorphous Cement Theories. Since then a more tenable theory has been advanced by Jeffries from the point of view of slip interference. For a more fundamental explanation we may look to the physicist to describe the behavior in

terms of the space lattice. Because of the direct bearing upon the present discussion, the action of crystal slip is reviewed.

5 If a crystal be slightly strained so that no slip occurs, the system of molecular attraction arising from the distorted internal arrangement will cause the crystal to return to its original shape upon release of load. Further strain may cause slip to begin across the plane of minimum resistance. The two resulting crystalline elements behave elastically and are kept united by a definite attraction, which, with friction, offers increasing resistance to motion. (A manifestation of this attraction might possibly result when cold welding occurs between two clean moving surfaces of like material under pressure, the function of the motion being to insure complete metal-to-metal contact.) That an increasing resistance is present is evident from the increase in elastic strain. Moreover, constant relation between stress and elastic strain continues to hold. This procedure continues until a slip has taken place on a number of planes, depending upon the initial resistance offered. It is quite probable that slip on certain planes is arrested by interference from the juxtapositioning of neighboring grains.

6 If the applied load be released, the elastic subdivisions return to their original shapes, but reversal of slip does not similarly follow. The fact that upon repetition of loading (after a due time allowance) appreciable slip does not occur until a load equal to or greater than the previous one is reached, signifies that evidently conditions of slip resistance remain. It is to be observed that the resulting ultimate extension probably will be less than it would have been, had the load been steadily applied.

7 The increasing resistance to crystal slip appears to be a function of the material, possibly dependent upon conditions of friction and atomic structure. In addition there exists another resistance dependent upon the rate of loading, simulating the action of a viscous fluid under forced flow. Swain(1)[1] states that "a steady load considerably less than the usual ultimate will cause fracture if applied long enough." This effect is more common to the non-ferrous metals than to steel. Mathews has observed this condition in zinc. It is well known that a lower punch velocity is beneficial to a draw. It is felt, however, that other conditions, such as friction between the die and metal, are the ones mostly affected.

8 Regarding the behavior of the crystals in aggregate, the variation in crystal strength and state of initial stress causes slip to occur in sequence, remembering that a crystal in slip overloads its neighbors, thus promoting additional slip. Hence a smooth stress-strain curve for non-ferrous metals results. The increase in resistance to slip actually builds up much faster than the engineering stress-strain curve would indicate, due to the increase in stress from reduction in area.

[1] This and subsequent references refer to the bibliography in Appendix No. 2.

9 In steel a more uniform grain strength is present so that few crystals slip until a point is reached when the majority yield. The sequence effect above described similarly occurs in this instance but with greater rapidity. Above the yield point region slip occurs as with the non-ferrous metals. At the yield point, however, slip is attended by a decrease in stress, usually, which is reported by Kennedy(2) to be as much as 20 per cent of the yield point. This condition is characteristic of the individual crystals of pure iron, so that the fact that the steel crystal is composed of pearlite and ferrite can hardly explain this exceptional behavior.

10 The configuration of a plastically deformed crystal under no load results in a rearrangement of the original space lattice setting up internal stresses which endeavor against friction to restore the system. It is felt that around this phenomenon exists the physical explanation of exaggerated grain growth, which requires for its occurrence an initial critical strain and critical temperature, as shown by Kelley and Winlock(3). The application of heat lessens the effect of friction so that the return of state may be brought about quickly, in proportion to the degree of residual strain. Virtually a new crystal results which behaves as it did when originally unstrained. With lead, self-annealing occurs practically at room temperature, which accounts for the huge deformations possible. Such an action approaches that of viscous fluids in contrast to "the result of a limited number of separate slips," as described by Sauveur(4).

Elastic and Plastic Stresses

11 In elasticity the body under examination is regarded as being purely elastic. Elastic theories are built upon the assumption that the material is isotropic and that it possesses a constant modulus. Fortunately steel, which is most extensively used, possesses the properties desired to a reasonable extent. If the nature of the loading of a given member be known, the principal stresses may be arrived at and equated to the principal strains. From Poisson's ratio and the modulus of elasticity of material, the strain resulting from a given stress may be determined. If a cube be subjected to a tensile load the strains in all three directions may be definitely expressed. This holds regardless of the complexity of the system of loading. It is interesting to note that the cycle in such cases is reversible.

12 In elastic deformations, the determination of the shear and the tension stresses make it possible to keep the load well within the capacity of the metal. The ductile metals generally fail in shear, whereas the brittle ones are weakest in tension [as shown clearly by Upton(5) and Moreley(6)], so that the limiting stresses should be selected with regard to the material.

13 This requires that a test be employed which is indicative of the properties of the metal. Test specimens are regarded as

significant in establishing the elastic range of steel. Strictly speaking, specimens of widely different shapes may show small changes in elastic properties. The stress distribution in the specimens varies slightly with the shape instead of remaining constant as assumed in test calculations. Crystals of equal resistance are subjected to actually higher stresses in one case than in another. With imposed stresses considered nominally the same apparent variations in results occur. The fallacy, however, is of small magnitude and at present may be ignored. Apparent discrepancies between the behavior of the metal in the test specimen and in a given member are more correctly traced to the inability to determine the existing peak stresses, as in cases of high stress concentrations, so that entire specimens may be regarded to behave similarly throughout all sections, which condition again satisfies that of a truly elastic body. It may also be well to note that regardless of the manner of loading steel elastically no change results in the physical properties, for the present ignoring the action of fatigue.

14 It has been seen that in the plastic state the crystalline elements behave elastically. We have in effect elastic conditions superimposed upon plastic reactions. It is only logical to suppose that the elastic tendency present will influence the plastic behavior. For this reason an understanding of elastic conditions becomes essential to the more complex study of plasticity.

15 Unlike the analytical attack which has been so effectively employed in elasticity, most of the development pertaining to the plastic state has been made in an empirical fashion for the most part. The equations of viscous flow as given by Thompson and Tait furnish an excellent structure for the study of plastic deformation. This work may be regarded as an outstanding exception to the experimental work of other investigators in this field. Unfortunately metal in its plastic state does not behave exactly as a viscous fluid. Although crystal slip and viscous flow both proceed under shear, the elastic effect must be given due consideration. Recognizing the action of shear as common to both plastic deformation and hydraulic flow, Bingham(7) succeeded in applying the Law of Poiseulles with striking results. By taking the yield point, which represents the beginning of shear, as the origin of the flow curve, a close resemblance was obtained with curves of viscous fluids. Hence the laws for plastic deformation might fittingly be pictured as a compromise between those governing the action of elastic bodies and the flow of liquids. It is felt that a co-relation between equations of elasticity and flow, embodying the experimental data available, may exist, from which expressions for plastic behavior may be established.

16 In order to analyze plastic reactions in a manner resembling the elastic method of attack, the principal shear stresses (shear because of the ductile metal involved) and the characteristics of the material under these stresses should be determined. To ac-

complish this end a difficulty arises from the four following sources:

17 The slope of the stress-strain curve is continually varying with increasing stress, in contrast to the constant modulus existing in the elastic portion of the curve. Thus the relation between stress and strain is seen to vary with the stress (or strain). From

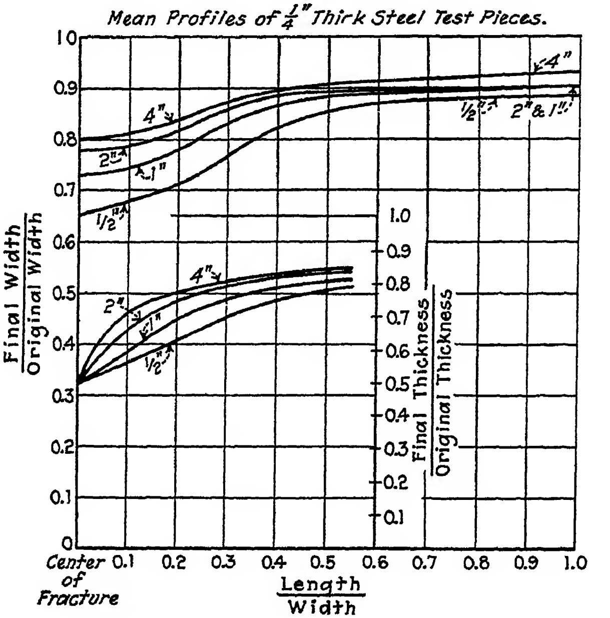


FIG. 2 CURVES SHOWING RELATION OF WIDTH OF SPECIMEN TO DEFORMATION

(From W. Gordon and G. H. Gulliver on the Strength and Ductility of Flat Steel Bars, in Trans. Royal Society of Edinburgh, vol. 48, part 1, no. 10.)

approximate calculations, this variation may be expressed in an equation so that a "typical curve" may be defined in terms of stress and strain. For sheet steel the following expression is suggested:

$$P = \frac{39e^k}{1+e}$$

where P is the stress, e the strain and k a constant ($= 0.25$).

18 The ratio between the strains of the three principal axes, resembling that of Poisson, is absent. True, a bar under tension for example will exhibit lateral strains somewhat in concordance with those defined by Poisson's ratio, but the variation may be great, depending considerably upon the initial dimensions of the

bar. It will be admitted that the lateral displacements become more individual to the physical dimensions in the region of contraction in which abnormal conditions of non-uniform extensions occur. On the other hand, it would be difficult to define the straining action occasioned in holding the width constant, the entire deformation resulting from a decrease in thickness. Gordon and Gulliver in an interesting experiment of pulling plates of constant thickness and of definite widths and lengths show that the per cent decrease in width becomes less as wider specimens are used. On the other hand, it will also be noticed that as the thickness has been held constant, a constant reduction results. These data appear encouraging toward establishing a plastic strain ratio analogous to that of Poisson.

19 There is a considerable question as to the manner of defining the characteristics of the metal. In the elastic curve it was seen that one section behaved as another practically independent of size and shape of test specimen. The engineering curve in which load intensity is plotted against elongation may be regarded as representative of average conditions. It indicates collective behavior and may be used with success for approximate calculations in which the general deformations are considered. The physical curve is obtained by plotting maximum strength intensity against minimum section. It denotes a particular behavior which occurs at the minimum section. In using the plastic curve as an agent representative of the behavior of the metal, it is seen that not only is a determination of the state of the strain in the stampings necessary, but also a definition of the conditions of stress in the test specimen.

20 It will be noticed that for very small gage lengths huge elongations result. If the metal were uniform in resistance, the piece as a whole would undergo the very great elongation which exists at the fracture point. Hence a high general or uniform elongation is desirable. On the other hand, in certain die formations in which the metal is so locked that the unconfined stretch is very small, a high local extension is required of necessity. Consequently, both types of extensions are desirable for drawing.

21 From Barba's law of similarity, the marked effect of shape of specimen upon recorded performance may be nullified. For instance, the effect of increasing the width with length and thickness fixed may be duplicated by decreasing the length with width and thickness fixed. The author applied this check to the actual results recorded by Gordon and Gulliver (8) and found that they consistently agreed with results as computed.

22 The law of similarity (9) permits of a comparison between dissimilar test bars, but the set of conditions that may be arbitrarily selected as standard are not necessarily most indicative of actual behavior. In the elongation equation $e = \frac{c\sqrt{A}}{L} + b$.

where e = the total per cent of elongation, c = constant, A = area of section, L = length of specimen and b = per cent of general extension, with $\frac{c\sqrt{A}}{L}$ = local extension. Unwin (10) gives a method of least squares for obtaining the most probable value of c by observing extensions in n gage lengths. Placing $a = \frac{\sqrt{A}}{L}$

$$c = \frac{n\Sigma ea - \Sigma e \Sigma a}{n\Sigma a^2 - (\Sigma a)^2}$$

$$b = \frac{\Sigma e \Sigma a^2 - \Sigma ea \Sigma a}{n\Sigma a^2 - (\Sigma a)^2}$$

If all elongation results are reduced to a standard set of conditions, represented by $L = K\sqrt{A}$, the value taken for K affects the significance of the answer as a measure of ductility. If K is large, more weight is given to the general extension in the answer, whereas if K is small, the local extension becomes more predominant. Since the general extension is usually of most consequence, K should be assigned a rather high value when one measurement for average conditions is to be used.

23 It has been found empirically that with sheet steel specimens $\frac{3}{4}$ in. in width, an extension measured in an 8-in. gage length is quite indicative of metal performance in the drawing operations. This follows, because with this shape of specimen the general extension is distinctly predominant.[1]

24 As a measurement for ductility the reduction of area may be regarded as a record of the very large extension at fracture. In fact, with sheet steel specimens in which the final area is difficult to determine, a suggested method is to measure the extension at fracture by initially ruling the specimen with fine lines closely and equally spaced. Referring to the behavior at rupture when the resistance to shear is less than in tension, as in the case of ductile materials, final failure usually starts in shear. Due to the rapid decrease in area and accordingly the rapid increase in stress, the tensile strength is exceeded; final failure occurring in tension. It is seen that a high reduction of area signifies that shear has proceeded relatively longer, due to a higher resistance to tension, with the result that the elongation of the smaller section is higher. Moreover, this behavior is independent of the original dimensions of the specimens. Hence the reduction of area provides information about the ductile properties of the metal. For this reason it is often taken by itself as indicative of ductility. But in such cases the history of extension is ignored unwarrantably. It is more common to regard reduction of area in conjunction with elongation in a given gage length, but difficulty arises in apportioning the proper weight to the two records. Since, in the final

[1] See also Appendix 1.

analysis, reduction of area defines the extremely local extension, the elongation equation may be made to correlate the two common measurements for ductility, if in determining the constants by Unwin's method smaller gage lengths be included.

25 In plastic reactions the manner of loading may influence the properties of strength, especially ductility. A new variable consequently enters for consideration as an alteration in the stress-strain curve. In order to find this effect specifically, it becomes necessary to determine the result of this work upon its behavior at the instant. One metal which may exhibit greater elongation than another when initially pulled may show inferior ductile properties when again compared after each has received the same amount of cold work. In short, the effective rate of work hardening may vary. Alterations in the stress-strain curve resulting from cold work vary considerably during the immediate time interval between reloadings. It may be pointed out that the intensity of stress upon an elementary section during drawing is not necessarily continually increasing. The intensity of stress may intermittently rise and fall as it increases. Such an instance is found as the section is pulled about a die contour and straightened. Again, the action of compression and tension may continually change at a given point in the metal during drawing. However, it should not be inferred that these conditions resemble those in which the load on a test specimen is entirely released and permitted to remain released for a given time and then re-applied. The test condition is only approached because at all times the section is probably under load. We are interested in the state of strain of a system only as this strain reveals the extent to which the metal is stressed in terms of its capacity. Consequently, any effect which alters this capacity is of direct importance since it changes the significance of the strain conditions.

Cold Working

26 Manifestations of plastic deformations are varied, being dependent upon the system of loading. Extrusion occurs under conditions of compression. Drawing results from the action of both tension and compression. (It is sometimes falsely regarded as pure stretch, whereas in reality stretch is only one phase of the alteration.) In rolling there is likewise produced a re-arrangement of the material, occurring under both tensile and compressive loadings. It differs from drawing in that compression is the predominant action in contrast to the elongation in drawing. These various forms of cold work may be identified as types of plastic deformation and it is not extraordinary that they should be closely allied. It is in this attitude that mention is made of the characteristics of some of these operations. The early investigations of Tresca(11) and the more recent ones of Bridgeman(12) show that steel may be made to flow in quite the same manner as a viscous

fluid. Under great hydrostatic pressure, jets of metal have been extruded through orifices. The salient feature of this type of deformation is the constraining element to which the surfaces of the metal are subjected. All faces are entirely enclosed with the exception of the small opening through which movement is directed. When pressure is exerted upon one of the faces displacement occurs. Assuming the walls of the vessel to be rigid, either the matter is compressed with a resulting increase in density, or an amount equivalent to that displaced is forced through the orifice. The resistance of steel above its yield point to change in density is greater than that by shear, with the result that the alteration finally proceeds by shear, giving rise to the phenomenon of flow.[1] Frequently the behavior of the material in forming operations is erroneously referred to as flow. Flow is more correctly characterized with the conditions of confinement as explained above.

27 In drawing, such a condition is seldom realized. The behavior is more appropriately conceived as a re-arrangement of the material. The underlying difference is one of directional effect attending the forces acting and the complimentary displacements. In extrusion a compressive force produces a displacement in directions entirely independent of the line of load application, whereas in drawing, change of shape results from a tensile load, alteration proceeding along the line of loading of the components thereof. The tendency toward metal continuity, even in extrusion, has been shown by Tresca. Strata of material are extruded in an order of sequence respective to the positions existing in the original system. In cold practice, extrusion is not frequently encountered, being confined chiefly to hot-working operations in which the metal is forced to flow in directions quite remote from the line of load, such as in die forging. However, in cold heading or upsetting the metal is made to flow, literally filling the cavities or pockets in the die. It is seen that extrusion is distinctive not in the mechanism of deformation but rather in the system of forces which produce the alteration. A special case of confined metal arises when all surfaces are subjected to a truly hydrostatic pressure. In such a case no shear can take place. The metal suffers no permanent deformation or change in physical properties, as observed by Jeffries.

28 Rolling is illustrative of another form of cold work and its mention is of special interest because of its priority to drawing in sequence of operations. In this way it affects the initial properties of the metal which subsequently is to be drawn. Modern stampings are of such difficult shape that optimum conditions of die design on the one hand and quality of metal on the other are

[1] It may be shown that in elastic reactions, according to Poisson's ratio, the resulting strains do not occur under a constant volume relation, so that increase in volume results. This increase, however, is extremely small and is generally ignored. Katuro Honda has measured this small density change magnetically.

required. However, the influence of rolling upon the metal will not be considered at this time. Tafel(13) has made some interesting observations regarding the behavior of the metal in rolling. In noting the constraining action from adjoining metal and friction upon the natural tendency of deformation, he succeeded in indicating a resultant flow which checked with experiments reasonably.

29 Rolling may be regarded as a composite of drawing and pressing. Puppe(14) and Fink(15) have shown that the work done in rolling resembles that in pressing, if slip in rolling be ignored. Distinctive from other operations, friction is necessary to rolling. It performs the drawing phase of the operation, pulling the metal in against the action of the horizontal component of compression. In such a system an elementary section is acted upon by components in compression exerted by the pressure of the rolls, and tension from the action of friction. It follows that the load in compression or in tension may each be lower than the elastic limit of the material. Moreover, the action of friction which resists the tendency to spread tends to prevent lateral strain of the section as it deforms. It is therefore seen that conditions of friction may limit the amount of draft. As the roll diameter is decreased, the effective horizontal component of friction likewise decreases. On the other hand, with decreased roll diameter decreased bearing pressures attend for a given draft, so that optimum conditions of these opposedly acting effects are desirable.

30 Tafel has shown that the action of spread is confined to the outer edges. It appears to be a function of the draft independent of the width of the sheet. Moreover, the spread increases proportionately as the edges are approached. Friction is probably not entirely accountable for this condition since the forward travel of the sections in the interior tend to maintain uniform width. Such an action of spread causes the outer edges to lag behind the more central region. The final sheet is therefore in a state of strain, the edges in tension and the middle portion in compression, which is generally manifest by a belly or buckle. This is one reason why it is regarded as almost impossible to produce a truly flat sheet by rolling. The tension to which the edges of the sheet are subjected may become serious for large drafts, so that fracture along the edges results. Hence the degree of draft is also limited by this condition.

31 As the metal is pulled into the rolls, slip between the metal and the rolls occurs. This follows logically when the surface travel of the metal is compared with that of the rolls. The surface speed of the metal in the rolling zone is a function of the speed at which the sheet enters the rolls, and the amount of draft. In one portion of the roll zone, the rate change in elongation is such as to cause the surface speed of both metal and rolls to be equal. As may be expected, this action of slip affects the conditions of friction.

Drawing

32 The distinction between deep drawing (or wire drawing) and the previously mentioned operations lies in the system of loading. The metal is called upon to produce its own deformation, so to speak. There is introduced more pronouncedly the element of strength or capacity which acts more rigorously in defining the elements of deformation. In extrusion, tenacity was found to be of relatively small consequence. In rolling, the elongation was considerably aided by the compressive action (although the tension may have been sufficient to cause failure).

33 Deformation in drawing is directed by the action of the tensile loads and the reactions set up within the external con-

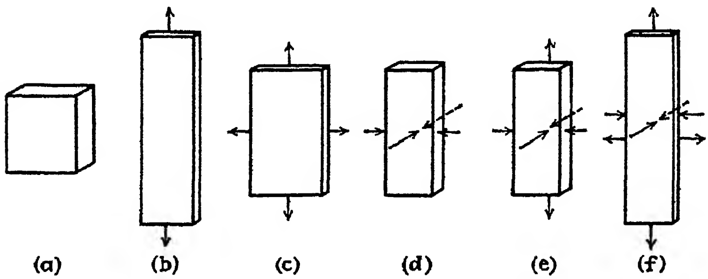


Fig. 3 Distortion Resulting from Different Forms of Loading

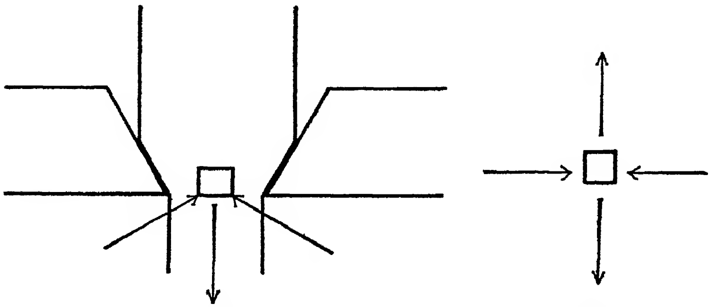
straints of the die. Hence, the alteration is a function of both the external loading and the reactions set up by the die.

34 Formations may be classified into various types with respect to the behavior in the alteration as apart from the physical shape resulting. A proper conception of this classification may be regarded as fundamental to the present discussion. Two identical contours may result from systems of loading quite unlike in character. It is therefore well to view the reactions in a bulk manner regardless of the particular dimension of interest. For example, in the elongation of a tensile test specimen cognizance should be given to the decrease of width and of thickness as well as to the increase in length. This fact may be demonstrated by examining the distortion of the body shown at (*a*) in Fig. 3 under different forms of loading.

35 We shall consider various methods for increasing the length of the block *a*. If the change in surface area observed in drawing be regarded as a multi-directional system of so many lineal extensions, an analogy may be drawn from the alterations effecting a change in the length of the block. In (*b*) of Fig. 3, the sheet is subjected to the pure stretch indicated, increase in length resulting

from a decrease in width and thickness. Such a condition seldom arises in drawing. If the original width is maintained constant (or increased) as in (*c*) of Fig. 3, the resulting elongation is obtained at the entire expense of reduction in thickness. Conceiving (*b*) as a pattern of so many minute sections, the transformation in (*c*) virtually becomes a re-arrangement of these figures, the surface area and thickness remaining constant. Similarly, drawing may be regarded as a re-arrangement of material into the positions producing an extension in the directions desired.

36 This form of deformation is observed in the Erichson cup test, in which the metal is subjected to tension in radial and circumferential directions. In this way, the cup test more nearly



(*a*) (*b*)

Fig. 4 Reactions Set Up by Die

resembles conditions as observed from the die, for the necking effect in width of the tension test specimen is absent, as in the case of actual conditions. Of course, reduction in thickness is not restricted, but this condition is common to both the stamping and the cup-test specimen. Operations of this type actually are relatively few. In automobile stampings, the formation of beads, rails, and sills may lie within this class. However, as a part of a more complex system this type of deformation is usually found in all forms of drawing. It is deserving of attention, because the final limits as to extension of draw are defined by the behavior of the metal under this manner of tensile loading.

37 So far, we have considered pure stretch in which both width and thickness were reduced and the modified form in which only a decrease in thickness results, the width remaining constant (or increasing). These alterations have proceeded in tension alone. We shall now go to a less common form of extension in which no change (or an increase) occurs in thickness, the entire reduction being confined to width. This type of deformation proceeds by compression, increase in thickness being restrained by external surfaces. Thus the elongation shown at (*d*) in Fig. 3 results. Such a condition is found in extrusion and may not be realized in draw-

ing, since the extension is produced under the application of compressive loads only.

38 If tensile forces of small magnitude be added, as shown at (*e*) in Fig. 3, the same alteration may be obtained if the compression loads are correspondingly decreased. The addition of the tensile component has consequently assisted in effecting the extension. Such a condition is common to drawing if the compressive forces be regarded as the reactions set up by the die when the specimen is loaded in tension. Wire drawing demonstrates this effect simply.

39 If the reactions set up by the die, Fig. 4 (*a*), are resolved into horizontal and vertical components in (*b*), the system is seen to resemble that of (*e*) in Fig. 3. As the total reduction is increased the character of deformation changes from one in which compression components are predominant to one in which the tension components are largest. It is usually desirable that the stress in the reduced section be kept within its elastic limit so that the total reduction is accordingly limited. Here, again, deformation occurs under the shear stresses which are set up by the compressive and tensile components. The individual components in either direction may be well below the elastic limit. Referring to Fig. 3, it is assumed that the tensile load in (*e*) only elastically strains the metal just outside of the altered region, so that occurrence of the entire deformation is restricted to the region indicated. This likewise holds in Fig. 4 (*a*). If the tensile load be increased so that the "outside" metal is plastically strained, the condition shown in (*f*) of Fig. 3 results. Assuming that no decrease in width over that shown in (*e*) is permitted, it follows that an additional decrease in thickness results. Thus an added extension occurs at the expense of the decreased factor of safety.

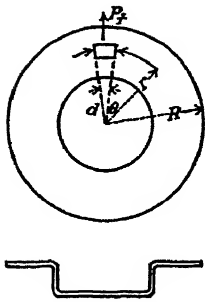


Fig. 5 Illustration of Action Accompanying Drawing of a Drum

40 Remembering that drawing is a re-shaping operation in which extension usually predominates, but as a component, it follows that the conditions observed in wire drawing are quite analogous. In drawing the drum shown in Fig. 5 the circumference of the blank is reduced to that of the drum, so that the metal confined between the surfaces of the dies is subjected to a compression and tension loading similar to that of the wedge shown in Fig. 4. Due to symmetry, no shear component exists. Resolving radially the compressive and tensile relations may be indicated by

$$(p_t+dp_t)(r+dr)d\theta-p_trd\theta+2p_cdr\sin\frac{d\theta}{2}=0$$

or

$$(p_t+p_c)dr+rdp_t=0$$

where p_t and p_c are the tensile and compressive components, respectively, and r is the distance from the center of the elementary section of width, $rd\theta$. The wall of the drum is evidently in tension between the action of the punch and the resistance to deformation of the blank, and, if the elastic limit is not exceeded, the system is similar to that of Fig. 3 (*e*). It is seen that the extension is equivalent to the amount of metal compressed. Under such conditions obviously failure cannot occur, but the urge for greater depths of draw requires that advantage be taken of the added elongation available in the wall metal. If the difference between the blank diameter and the drum diameter be increased so that the tension necessary in the wall causes a plastic strain, conditions shown at (*f*) in Fig. 3 result. This procedure if carried too far is likely to be dangerous. As the metal enters the wall it is in a plastically strained state so that its capacity for additional elongation is greatly reduced. Nominal variations in friction from differences of die pressures may cause variations in drag to bring about failure. Thus a limiting value for the ratio of blank diameter to drum diameter is imposed. If this ratio be excessively high, no blank movement will take place. The action will be one of blanking in which the bottom of the drum is blanked. A safe draw, therefore, is one in which the extension is produced for the most part in the blank, that in the wall being kept to a minimum.

41 The behavior of the metal under this system of deformation is regarded as the basis of the phenomena of drawing. A discussion of reactions set up will be given in more detail. The fact that the metal is worked in this manner rather than stretched may alter the physical properties required. In selecting a metal for a given job, the nature of its required performance is desirable, so that the test upon proper interpretation may express the characteristics required. Unquestionably the press operation requires that the metal possess high ductility. But ductility is difficult to define, especially when the exact manner in which the metal is worked is unknown. A test measurement is of value when the conditions of the test may be correlated with those existing in performance. Specifically, elongations in different gage lengths give most weight to the effect of local extension in one case, or general extension in another. Unless the type of extension existing in the performance be known, the gage length adopted may set up erroneous indications. In attempting to measure the ductility of a sheet for drawing operations it has been found that tensile test results vary as a specimen is taken in the direction of rolling or transverse to it. Moreover, a variation from one part of the sheet to another may frequently be observed. One advantage the cup test offers lies in the fact that the effect of the direction of rolling enters into the test in much the same manner as the stamping. However, in those stampings in which the major drawing occurs in a given direction the cup test may be less significant. For example, in the forming

of doors the blank if possible should be so placed that the elongation caused by the bead occurs in the line of rolling, due to the slightly better properties existing in this direction.

42 It therefore becomes necessary to analyze the reactions in the formation, if the test is to be effective. In the analysis, consideration should be given simultaneously to the state of the metal as previously mentioned. Dr. Langenberg(16), in his recent investigations on internal pressures of gun barrels, points out that this change in physical properties enters into considerations quite as much as a determination of the stress distribution.

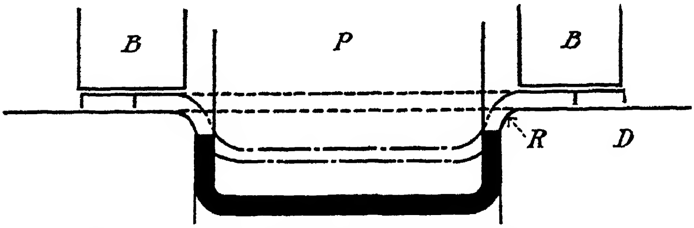


FIG. 6 MANNER OF HOLDING BLANK IN DRUM-DRAWING OPERATION

BEHAVIOR IN THE DIE

43 In discussing the conditions which are found in a drawing operation, the system as a whole will be first considered. In character these conditions are quite independent of each other and will be treated individually later. Isolated in this manner each may be resolved into an elementary state in which an analysis may be made. It then becomes necessary to draw from the physical determination a generalization in an effort to express the behavior under the more complex forms which enter into the draw. The reactions are closely related, each exerting an influence upon the other in their combined effect upon the result. It is with this aspect that we are at present interested. The various conditions as they prevail in the operation are defined from an analysis of the entire system. The physical behavior under such conditions may then be observed. The reactions set up in the forming of a drum may be quite appropriately taken as representative of the various types of drawing but for the element of symmetry. In cup and cylindrical formations, the system is symmetrical in contrast with the unsymmetrical shapes which are perhaps more commonly encountered, such as in the auto body stamping of door panels, deck lids, wheelhouse panels, etc. If allowance be given to the modifications arising from differences in symmetry, it will be seen that reactions are otherwise alike in character. Because of its symmetry, which lends itself more readily to analysis, the drum is selected for examination.

44 In drawing the drum shown in Fig. 6, the blank is held firmly between the blank holder B and the die D by springs or air, in the case of single action presses, or by toggles (sometimes supplemented with air) in the case of the double action presses. The punch P then descends, drawing the metal of the blank into the die and forming the successive steps indicated. Isolating the operation as it has partially proceeded, the following conditions are observed in the radial section shown in Fig. 7.

45 Fig. 7 may be divided into the sections indicated with respect to a classification of conditions which influence the behavior

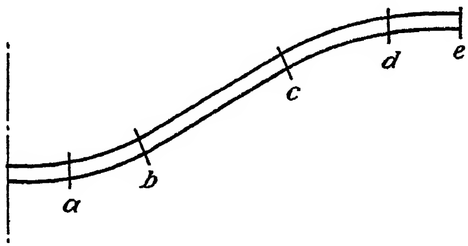


FIG. 7 ILLUSTRATION OF THE BEHAVIOR OF METAL IN DRAWING OPERATIONS

of the metal. The physical conditions involved and the individual effect that each produces upon the entire system will now be considered, commencing with *de*.

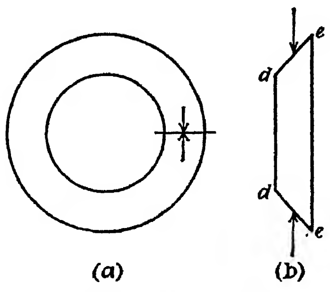


FIG. 8 ANALYSIS OF CIRCUMFERENTIAL STRIP

46 *Section de (Fig. 7)*.—As drawing proceeds, the circumferential strip *de* (Fig. 7) is subjected to a tangential compression as previously explained. A stress distribution may be indicated if a typical compressive stress-strain curve is assumed. Such a curve as obtained from tests is modified by the shape of the specimen. The constraining effect of face friction upon the true compressive curve is not quantitatively established. However, for approximate work, a typical curve may be assumed of the form

$$e = k + a \tan \alpha$$

where e is the strip, k the yield point, a the strain, and tan α the slope of the curve above the yield point (regarded for convenience as a straight line). It will be seen as the blank circumference is drawn in that the maximum stress intensity increases but the area of the radial section decreases so that the mean effective pressure also decreases. This condition may be expressed from the equation of the plastic curve. Consider a point in a blank circumference lying in an annulus of diameter δ and let this point move from the outside blank circumference to that of the drum. The plastic curve may now be expressed in terms of the blank and drum diameter as

$$e = k + \pi(D - \delta) \tan \alpha$$

where δ changes from D to d and where D is the blank diameter and d the drum or punch diameter. It should be remembered that the compressive action does not exist alone but with that of tension, so that a value for k should be modified accordingly.

47 Conceive the circumferential strip of width de (Fig. 7) straightened out as shown in Fig. 8, with the compressive forces still acting. Naturally the strip would buckle as the effect approaches that of Euler's long column. To resist this buckling tendency, we have either to increase the section (thickness) or maintain face parallelism by adequate support. Obviously, a thickness necessary for this condition would make drawing impossible, so the latter method is resorted to. As would be expected, the pressure required to prevent buckling decreases with increase in thickness. The friction between the blank holder and the die surfaces correspondingly drops, diminishing the tensile strength in bc (Fig. 7). That this factor is of consequence is evidenced by the fact that frequently increasing the metal thickness makes possible draws in which failure otherwise occurs. An attempt to show this convincingly by experiment has been unsuccessful, due to the entrance of external considerations. However, results of the experiment may be interpreted as indicating a tendency in this direction.

48 It is felt that from experiments data can be obtained for round blanks, expressing the necessary blank pressure in terms of blank diameter, punch diameter, and metal thickness. For unsymmetrical shapes, such an investigation appears futile. The conditions of pressure are individual to the contours, making it necessary to compile exhaustive data. More perhaps can be done by considering the condition in a physical light and applying this information to the particular case at hand.

49 It has been shown that the compressive reaction decreases as de (Fig. 7) narrows. This, together with the fact that the length also decreases, denotes that the blank pressure may become smaller. The manner in which the critical blank pressure varies under known conditions of width and length of strip may be examined for an elementary strip. Up to the buckling point, the

resistance to buckling increases with the load. Above this point certain assistance from the metal toward maintaining actual alignment still prevails. The question becomes: To what extent does this effect hold, or to what extent does the material behave hydrostatically? It is seen that if the amplitude of the wrinkle is kept a minimum, the effect of the resistance to bending offered by the metal is maximum, so that external face nominal forces may be small.

50 The toggle-linkage method meets this condition more favorably than either air or springs. Due to its relatively high rigidity small initial pressures may be used. The working pressure will vary with the tendency toward misalignment, and hence high pressures result only under such conditions. The resistance toward misalignment is increased by the action of the tensile component appearing simultaneously with the compression, due to the self-aligning action which is characteristic of tension.

51 The initial pressure when air or springs are used must be greater than the peak critical pressure, which means that throughout the greater part of the draw an unnecessary drag results. This condition is aggravated in the use of springs, since the load increases as the critical blank pressure decreases. Springs are used advantageously by making the free deflection great, and highly loading the spring initially, so that the increase in load under working deflections will be minimum. When a reducing blank pressure is required, if air is employed, a "leak-away" may be provided; with springs a toggle action may be employed.

52 An upsetting action accompanies the circumferential compression from the lateral strain which attends. Referring to (*d*) of Fig. 3, if the longitudinal edges alone are compressed elastically, the strains along the other two axes may be stated from Poisson's ratio. In the plastic region, as previously discussed, the extent to which this relation holds is questionable, becoming more so when the tensile reactions are included. In other words, the amount of pressure necessary to maintain uniform thickness under these conditions becomes a problem of physical determination. In attempting to arrive at a stress distribution, difficulty arises in definitely identifying the reaction (especially with unsymmetrical shapes), so that empirical data are of considerable value. Data which express the upsetting action in terms of blank diameter, punch diameter, and metal thickness would be directly applicable to cylindrical formations and, no doubt, would serve as a guide for analyzing the more difficult shapes. The increasing tendency of the metal to upset as the draw proceeds is resisted by an increased blank pressure intensity, since the area of the blank face decreases. Thus the blank holder not only serves to prevent wrinkling but also to maintain uniform wall thickness. However, the pressure

required to prevent, entirely, an increase in thickness would unduly stress the press and, more to the point, cause a higher drag to result. If the wall thickness is to be held uniform, the resulting upset may be reduced in subsequent "coining" operations under much better conditions.

53 In the above discussion the blank was considered to be drawn in by the punch. It frequently happens that the blank is locked, as in the formation of doors. In such cases, the desirability of a decreased or uniform blank pressure is absent; in fact, an increase in pressure may be quite acceptable.

54 *Section cd.* Let us now consider the portion *cd* of Fig. 7. If the tangential compression observed in *de*, and which continues

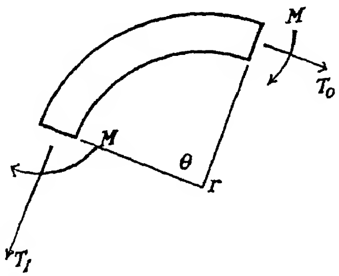


FIG. 9

throughout the arc of contact *cd* to a slight extent, be ignored, the proposition becomes one of confined bending and tension. Quantitatively, the effect in change of the radius of curvature of the arc is self-evident. With very small radii, shear at the die circle occurs, so that the drawing operation virtually becomes one of blanking. With radii slightly larger but still small, great resistance to motion results, so that fracture may occur in *bc*. In short, a generous radius is helpful in minimizing the tensile load in *bc*. As the radius becomes too large, however, the tangential squeezing effect becomes of more consequence, especially in the absence of an equivalent blank pressure, so that wrinkling may result. There are occasions in which small radii are employed to purposely resist the inward travel of the blank; but this restriction is more commonly produced by other methods which lend themselves to more sensitive regulation, such as the use of increased blank pressures, beads, shims, or inclined faces.

55 A study of the conditions surrounding *cd* illustrates the need of taking into consideration the behavior of the section as a component of the whole, as well as an individual unit. For example, increase in thickness acts unfavorably in *cd* and favorably in *de*, so that it is necessary to regard the resultant effect of the two functions.

56 In isolating *cd* for examination, consider that bending takes place about a straight axis rather than a curved, i.e., imagine the

die ring straightened, so that *cd* represents a section of a cylinder. Two conditions exist which influence the tension in *bc,* the plastic bending of the metal and the resistance to motion from the friction between the metal and the curved face of the die. The metal thickness enters into the bending consideration and the angle of contact, the locking tendency.

57 The reactions are indicated in Fig. 9. Let T_0 be the tensile resistance set up in *de* and let the arc of contact be defined as $r\theta$. We may assume for approximate calculations that the moment

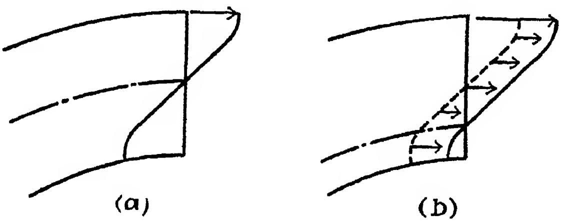


FIG. 10

necessary to bend the element to the radius of curvature may be expressed from the elastic formula.

$$M = k\frac{I}{C}$$

where $\frac{I}{C}$ is the section modulus and k represents the yield point.

We may also assume that the moment necessary to straighten the curved element is likewise M. Hence

$$T_1 = T_0 + \frac{2M}{r}$$

Strictly speaking, the elastic formula is not applicable, due to the presence of a tensile load. Observation of Fig. 10 shows that the neutral axis (*a*) becomes lower when a tensile load is added (*b*). The value for the section modulus is accordingly altered. However, the bending effect is small in comparison with that of locking, so that small errors may be overlooked in the bending calculations.

58 As the metal is caused to move, the frictional effect resembles that of the rope and windlass. For this condition we have:

$$T_2 = e^{\mu\theta}T_0$$

where μ is the coefficient of friction, and θ is the angle of contact. In this instance the sheet is considered to be infinitely small in thickness and offering no resistance to bending. The resultant tension in *bc* therefore becomes

$$T = T_0 + \frac{2M}{r} + T_0(e^{\mu\theta} - 1)$$
$$= (e^{\mu\theta})T_0 + \frac{2M}{r}$$

59 Since T_2 increases logarithmally with θ, it is seen that as the punch descends (Fig. 6) an increase results in T, due to the increasing angle of contact. In the following experiment an attempt is made to note the individual effects of thickness, radii of curvature, and angle of contact upon T, for definite values of T_0.

60 To obtain this information the apparatus shown in Fig. 11 was devised. Known constant loads T_0 were applied at one end of the specimen by attaching the hanging weight W, suitably multiplied, with wire rope. The other end of the specimen was gripped in the jaws of the tensile machine as indicated. The roll R was removable, so that rolls of different radii could be substituted. Also, provision was made to permit free rotation of the roll (on ball bearings) or to hold the roll fixed. The frame

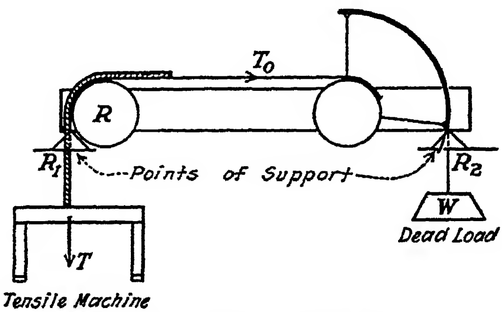


FIG. 11 TESTING APPARATUS

was supported at R_1 and R_2. R_1 was attached to the weight platform of the tensile machine in order to record the reaction of the load T. This was made possible by keeping R_1 and R_2 in the line of application of the external loads. The point R_2 was raised and lowered so that the effect in change in the angle of contact could be observed.

61 In order to estimate the effect of friction in the ball bearings, cloth strips were substituted for the specimen and wire rope, so that the results would not be affected by the bending of the metal. The friction proved to be negligible. A check on this value was obtained by reversing the motion so that the specimen was pulled by the weight.

62 It will be seen that conditions were comparable with those observed in the die. By changing the dead load the effect of drag in the blank was observed. The effect of change in die radius was demonstrated by the use of different rolls. An estimate of the locking effect was obtained by comparing results with the roll freely turning and held fixed. It was also possible to notice the effect of the angle of contact.

63 Table 1 indicates a reasonable agreement between the actual and computed total load necessary to pull the specimen under the

conditions noted with a dead load of 333 lb. (The weight of the apparatus supported by R_1 was subtracted.)

64 Table 2 shows that the actual load obtained by allowing the rolls to turn was in agreement with that computed for bending alone (allowance for the change in the neutral axis was made by modifying the section modulus slightly).

65 Examination of the curves of Fig. 12 reveals that the actual conditions are as expected. The effect of decreasing the radius is shown at (*a*). These curves also show that, for a given radius,

TABLE 1 ACTUAL TOTAL LOADS FOR COMBINED BENDING AND LOCKING

(Constant load = 333 lb.)

		1-in. radius		½-in. radius		¼-in. radius	
θ	Metal gage	Roll turning	Roll stationary	Roll turning	Roll stationary	Roll turning	Roll stationary
60°	.0395	365	425	395	460	450	515
	.050	380	435	440	485	495	535
	.061	390	465	475	530	500	545
	.072	425	480	535	570	565	595
	.088	470	530	555	570	...	..
	.125	495	665	640	665	...	...
90°	0395	345	445	395	485	450	555
	050	380	460	445	575	520	620
	.061	375	475	490	570	620	665
	072	435	520	565	635	650	840
	.088	475	575	645	685	...	...
	.125	645	735	915	935	...	...
120°	.0395	365	495	395	545	465	595
	.050	385	500	450	585	545	660
	.061	405	545	475	625	645	785
	.072	435	580	550	650	825	1000
	.088	485	585	665	730	...	..
	.125	625	740	965	1130	...	..

change in thickness varies with the section modulus. The curves of (*b*) show that the added effect from locking tends to straighten the curves of (*a*), as would be expected. Curve (*c*) shows that the angle of contact does not enter the conditions of bending (in accordance with the equation). The curves at (*d*) show that the effect of the angle of contact enters when locking occurs. It is shown at (*e*) that the magnitude of the dead load has no effect

TABLE 2 COMPARISON OF ACTUAL AND COMPUTED LOADS IN BENDING AND LOCKING

	Load from bending, roll turning						Load from locking, roll fixed θ = 90 deg., dead load, 333 lb					
	1-in. radius		½-in. radius		¼-in. radius		1-in. radius		½-in. radius		¼-in. radius	
Metal gage	Actual	Computed	Actual	Computed	Actual	Computed	Actual	Computed	Actual	Computed	Actual	Computed
.0395	17	24	41	48	95	93	445	485	485	520	555	565
.050	33	38	88	77	170	145	500	508	575	540	620	615
.061	52	57	116	112	267	213	510	527	575	580	605	685
.072	80	80	210	155	400	290	520	550	635	620	840	760
.088	128	120	287	228	...	...	575	600	700	800	..	...
.125	235	235	550	448	...	...	735	705	935	920	...	...

upon the conditions of bending. The curves at (*f*) show that the dead load alters conditions when locking occurs. It is seen that in every instance the trend of the curves is in agreement with the conditions expressed by the equations.

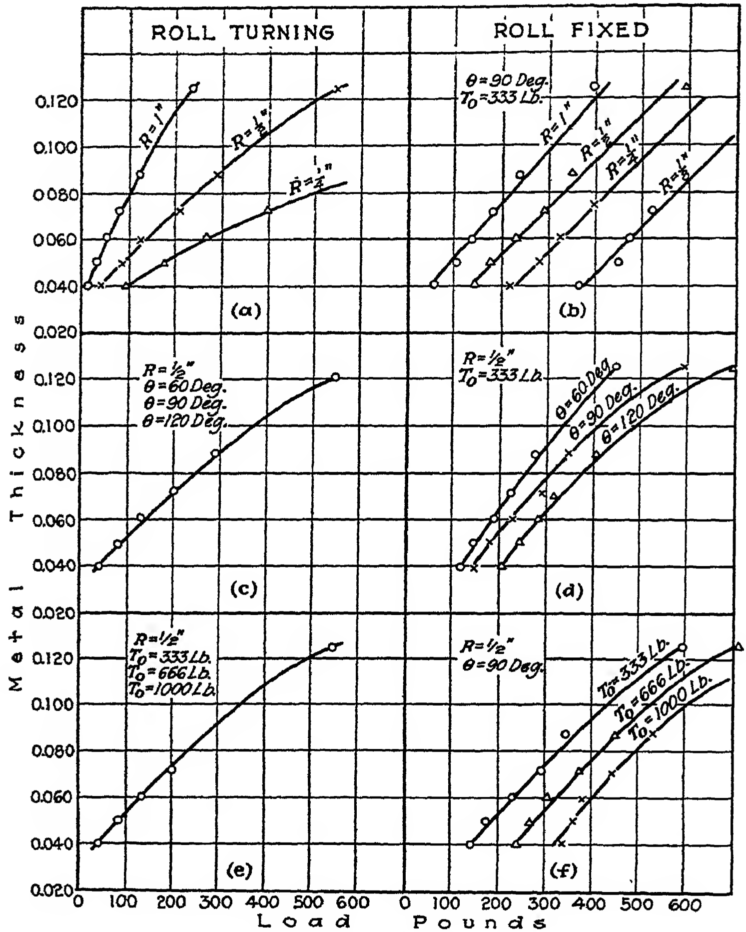


FIG. 12 CURVES SHOWING EFFECT OF VARIOUS CONDITIONS ON DRAWING

(a) Effect of decreasing radius
(b) Effect of locking on curves of (a)
(c) Relation of angle of contact to conditions of bending
(d) Relation of locking to effect of angle of contact
(e) Effect of magnitude of dead load on conditions of bending
(f) Effect of dead load on general conditions when locking occurs

66 *Sections ab and bc* (Fig. 7). Due to the close relation which exists between *ab* and *bc*, these sections will be treated simultaneously. In the present draw it will be expected that the elongation existing in *bc* has not entirely occurred in *de*, but rather an added extension in *bc* is present (Fig. 3*f*).

67 Regarding the system in plan, two opposing tangential actions are seen to occur. As the element (Fig. 13) moves from

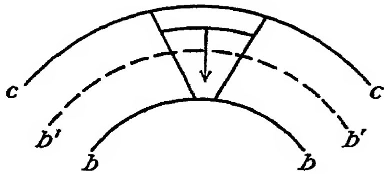


FIG. 13

cc to *bb* an action of circumferential compression results, but as it comes into contact with the punch it may be re-elongated, so that the deformation is not continuous.

68 It may be interesting to pause at this point and examine the extensions which result as the punch descends. This is more

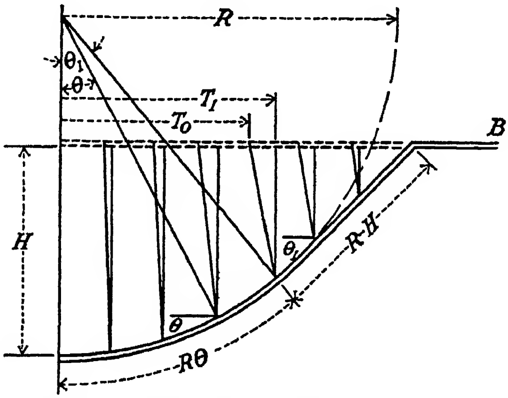


FIG. 14 ACTION OF HEMISPHERICAL PUNCH

simply described if for the time being we consider that no draw-in takes place in the blank, the entire deformation proceeding under tension as shown on Fig. 3 (*c*).

69 Let a hemispherical punch or radius R (Fig. 14) depress a blank held tightly in the blank holder into a die of the same radius to the depth H. Let θ_1 be the angle of contact and, for the present, regard this contact to be frictionless. From the geometry of the figure, the unconfined length is $R-H$, hence θ_1 may be expressed in terms of H

$$(R-H) = R(1-\sin\theta_1)(\sec\theta_1),$$
$$H = R(1-\sec\theta_1+\tan\theta_1)$$

70 If no change occurs in the circumferential length of equally spaced annuli, the corresponding ones in the formation would like-

wise be equally spaced, becoming the projections of the former. For a given value of H, the diametral extension in the contact zone would vary as $e = \sec \theta - 1$, reaching a peak when θ equals θ_1. The elongation in the confined section would be constant and equal to $e = \sec \theta_1 - 1$.

71 Actually, however, the radii of the annuli are extended from the action of the diametral increase, so that the radius of contact (r_1) is (r_0) extended. Hence, there results a circumfer-

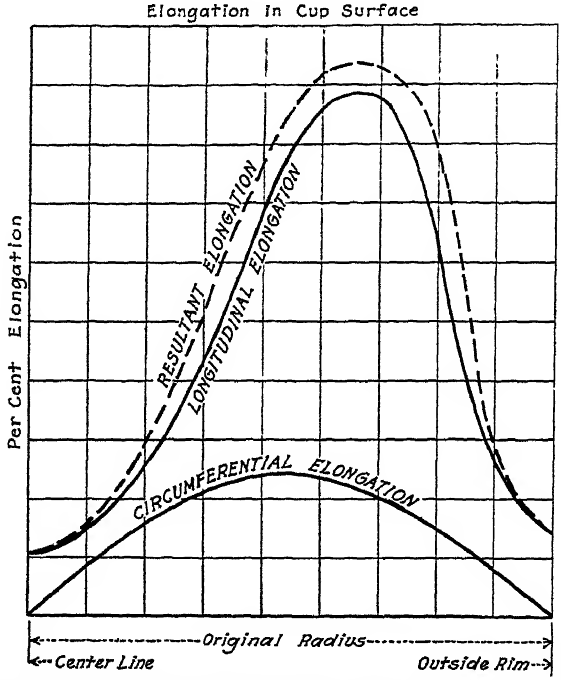


FIG. 15 CURVES SHOWING ELONGATION UNDER VARIOUS CONDITIONS

ential elongation and a corresponding increase in diametral extension. Then

$$e = \frac{(q+\Delta q) \sec \theta - q}{q}$$

Where q = an increment length. The total circumferential elonga tion is

$$c = \frac{r_1 - r_0}{r_0}$$

The resultant elongation at $\theta = \theta_1$ becomes $a = \sqrt{c^2 + e^2}$. It is difficult to determine the circumferential change unless conditions of locking be known. Experimental data (Fig. 17) indicate that the increase is relatively small. If it were not for the locking effect

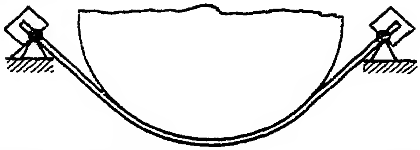


FIG. 16 MANNER OF TAKING HISTORY OF EXTENSIONS OF SMALL SECTIONS

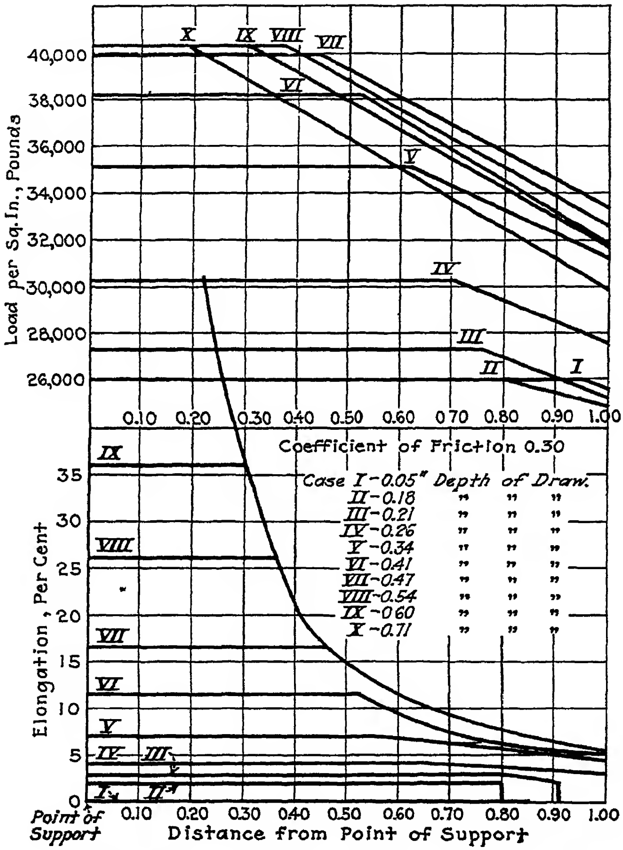


FIG. 17 VARIATIONS IN EXTENSION ALONG STRIP EXPRESSED FOR SUCCESSIVE STEPS OF DRAW

which is greatest at the center, evidently the elongation at this point would be greater, tending to nullify the peak which results at the point of tangency.

72 Since $R-r_0$ is reduced to $R-r_1$, the annuli are forced outward, each becoming somewhat longer circumferentially but closer spaced by an amount equal to r_1-r_0. This results in a decreased elongation in the open section, which is

$$e = \frac{(q-\Delta q)\sec\theta_1 - q}{q}$$

73 It is seen that this value may be quite small. With c and therefore A also small, a definite maximum elongation occurs in the region of tangency. This is borne out experimentally as shown in Fig. 15.

74 Returning now to the sections ABC of the drum, conditions of the cup are complicated by the circumferential compression already discussed. With the cup the complete deformation proceeded under change in area from reduction in thickness. With the drum a more nearly constant area, and likewise thickness, is maintained during the shaping of the blank. In ab locking takes place in much the same manner as it appeared in cd. Hence, any stretch which is caused to occur in abc must take place for the most part within bc, resulting in a greater elongation. This effect was fully demonstrated in an experiment of bead formation. Under the ordinary die conditions the greatest permissible depth for a given bead was increased 25 per cent by substituting rubber for steel in the punch. The effective worked length, instead of being confined to a small region, included a greater distance. This same condition is also observed when two sheets are formed simultaneously. The outer sheet, by virtue of this relative motion of the inner, elongates more uniformly.

75 The effect of locking enters into the consideration of all types of drawing and is of fundamental importance. It has been attempted experimentally to observe this condition by considering the behavior of a lineal strip under the action of a descending cylindrical punch. From this study it is anticipated that the conditions involving the effect of "area locking" as in contrast to "lineal locking" may be indicated.

76 Referring to Fig. 16, the history of extensions of small sections was made by taking readings at small advances of the punch. Similarly, using the locking equation and a typical stress-strain curve for obtaining relation of stress and strain, the computed behavior of these small sections at similar stages of punch travel was made. Comparison between the computed and actual results was found to be in reasonable agreement. The effect of bending of the specimen was ignored, due to the relatively large radius of the punch.

77 Referring to Fig. 17, the variation in extension along the strip is expressed (by computation) for successive steps of draw. It is to be observed that final conditions are effected cumulatively, so that a point-to-point method of calculation is required. Table 3 compares the actual results with the computed.

78 Considering now the tension in bc (Fig. 7): If D is the effect of drag from the blank pressure, c is the tensile component necessary to tangentially compress the blank circumference, R is the resistance set up to travel over the die curvature; then the tension in the wall becomes

$$T = D + C + R$$

Since the tension in the wall is equal to the punch pressure when the wall is vertical, a record of the variation in punch pressure should indicate the resultant effect from a change in any one of the above mentioned conditions.

79 Accordingly, a test was conducted in which the punch pressure was recorded in the forming of a brake drum when the following factors were allowed to vary:

a Blank diameter
b The thickness of the metal
c Blank pressure
d The radius of the die ring.

Readings were taken of all possible combinations for definite values of each factor. The individual effect of change in the factors upon resulting punch load was noted.[1] The following curves indicate the results obtained.

80 Fig. 18 shows the effect of change in metal thickness upon the final punch pressure for the cited values of blank diameter and radius of die ring. It is observed that the general characteristics of the curves are independent of die radius and blank diameter. Although the total punch pressure increases with the metal thickness, the pressure per metal thickness, which varies as the tensile stress in the wall, decreases. This decrease becomes less with increase in diameter and radius.

81 Fig. 19 shows the effect of blank diameter upon the punch pressure. The curve is representative of the combined results as

[1] The punch load was obtained by placing pressure pads between the ram and punch. Each pressure pad consisted of two plates separated by a number of balls which were located within a retainer plate. The thickness of the retainer plate was less than the diameter of the balls so that undisturbed indentation could occur. From the cited indentations, the load per ball was obtained by reference to a characteristic load-indentation curve obtained by recording indentations against known loads. Although a history of the loading is not given, the maximum load which is registered is generally the one of interest. The precision of this measurement is regarded as being sufficiently high for such determinations.

obtained for different values of die radius, blank pressure, and metal thickness.

82 Fig. 20 is similar to Fig. 18 except that the radius curves are obtained from the combined effect of blank diameter and pressure.

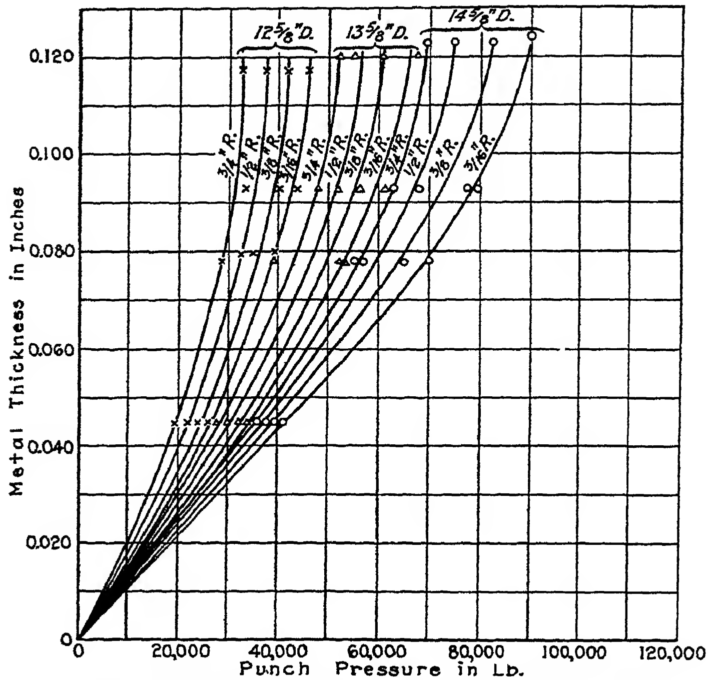


FIG. 18 EFFECT OF CHANGE IN METAL THICKNESS ON FINAL PUNCH PRESSURE

(Independent variables: Thickness of metal, T; diameter of blank, D; die radius, R
Dependent variables: Punch pressure, P.)

83 As in the case of Fig. 19, Fig. 21 is representative of the relation between blank pressure and punch pressure under the combined effect of the various conditions taken. For a given draw there exists a critical blank pressure which is defined as the

TABLE 3 COMPARISON OF ACTUAL AND COMPUTED PERCENTAGES OF ELONGATION OBTAINED IN THE CYLINDRICAL PUNCH TEST

Distance of punch from center	H = .25 in.		H = .375 in.		H = .500 in.		H = .563 in.		H = .625 in.	
	Actual	Computed	Actual	Computed	Actual	Computed	Actual	Computed	Actual	Computed
9	4	4	8	8	14	15	22	22	32	36
8	4	4	8	8	14	15	22	22	32	36
7	4	4	8	8	14	15	22	21.5	32	21.5
.6	4	4	8	8	14	14	21	14	15	14
.5	4	4	8	8	13	10.5	19	10.5	12	10.5
.4	4	4	7	7	12	8.5	14	8.5	10	8.5
.3	3	3	6	6	10	7.2	12	7.2	10	7.2
.2	3	3	6	5.5	9	6.5	9	6 5	9	6.5
.1	3	3	5	5	7	6	7	6	8	6

minimum pressure necessary to prevent the formation of wrinkles. Considerable difficulty was experienced in obtaining this critical value for a given set of conditions, due to the limited number of trials permissible.

Conclusion

84 In reiteration of the purpose of this paper an attempt has been made to bring out the major considerations important to the

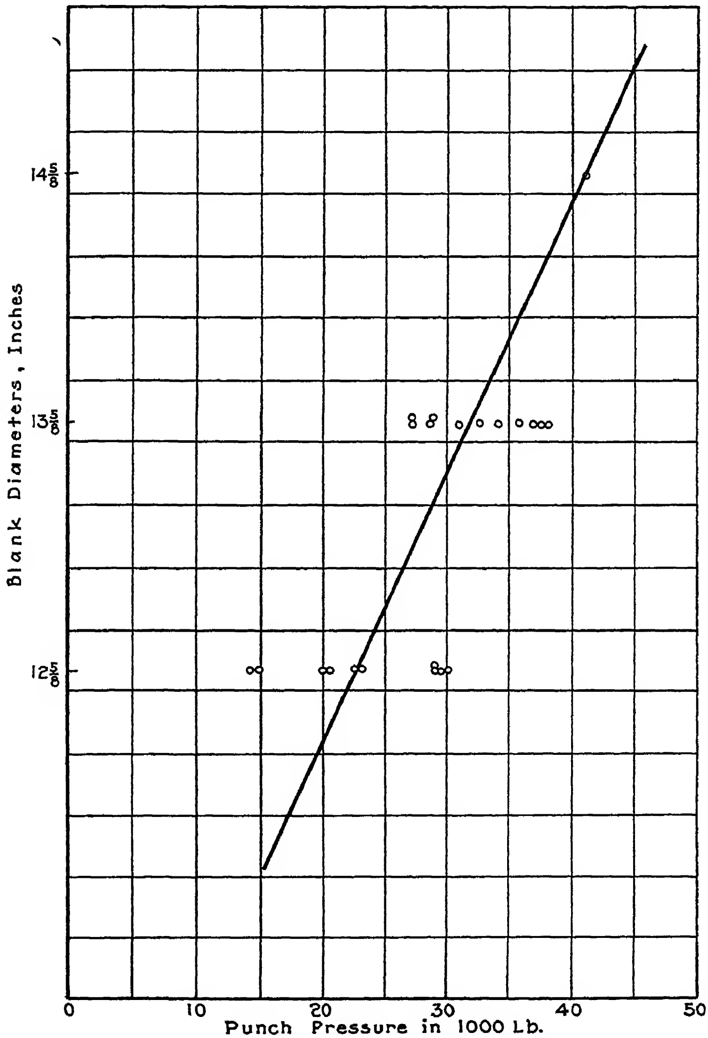


Fig. 19 Effect of Blank Diameter on Punch Pressure

(Transposed to common origin, which is that of $\frac{3}{16}$ in. Radius, $14\frac{5}{8}$ in. Diameter, 0.045 in. Thickness, 20,000 lb. blank pressure.)

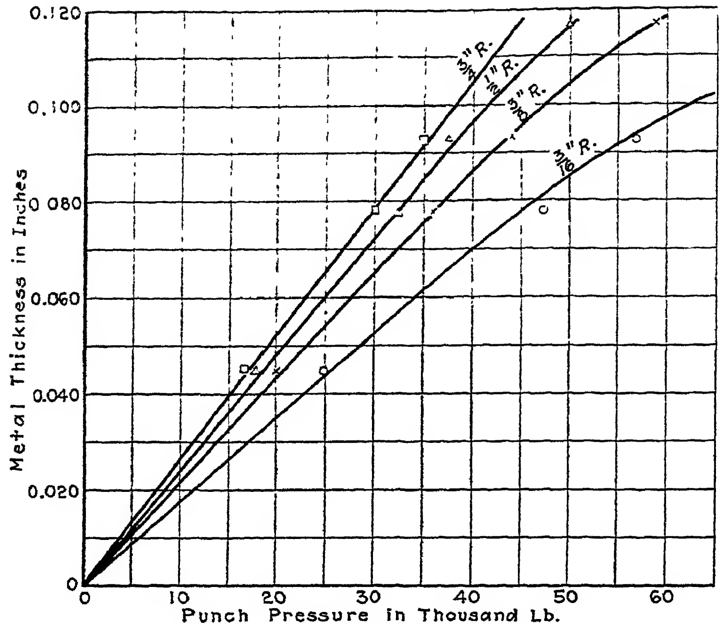


FIG. 20 EFFECT OF BLANK DIAMETER ON PUNCH PRESSURE

(Radius curves obtained from the combined effect of blank diameter and pressure. Constants: Blank pressure, 20,000 lb.; diameter, 12⅝ in. Independent variables: Thickness of metal; die radius.)

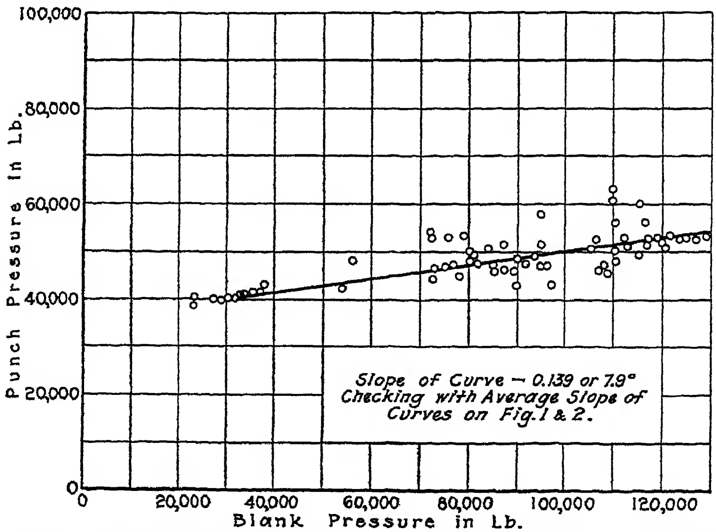


FIG. 21 RELATION BETWEEN BLANK PRESSURE AND PUNCH PRESSURE UNDER THE COMBINED EFFECT OF VARIOUS CONDITIONS

(Transposed to common origin, which is that of 14⅝ in. Diameter, 0.045 in. Thickness, $\frac{3}{16}$ in. Radius, 40,000 lb. blank pressure.)

study of plastic behavior of metal in drawing, so that a more comprehensive attack may be employed in the specific instances. The few specific examinations which have been given are to be regarded as illustrative of analysis rather than as concrete determinations of the particular conditions at hand. The experimental data have been introduced to support the contentions which were expressed.

Acknowledgments

85 The author is indebted to Dr. G. L. Kelley for his instructive direction in this work. He also wishes to express his appreciation to Messrs. Joseph Winlock and R. Eksergian for their helpful discussions and to thank Mr. W. S. Montgomery, Jr., for his assistance in the performance of the tests and the compilation of the data.

APPENDIX NO. 1

86 In line with Templin's investigation of aluminum test bars, the effect of shape of section under constant area was observed from the data of Gordon and Gulliver. Since these investigators worked with plates of constant thickness but varying widths, both shape and area

TABLE 4 EFFECT OF CHANGE IN SHAPE OF SECTION UPON EXTENSION WHEN GAGE LENGTH IS PROPORTIONAL TO SQUARE ROOT OF AREA

W/T	$\sqrt{A}$	b	c	e_1	e_2
3.81	0 488	19 9	88.5	25 8	26.5
5.64	0.593	20.2	99.2	26.8	27.0
7.57	0.688	20.9	92.4	27.1	26 5
9.57	0.773	20.4	89.7	26.4	25.5
11.14	0.834	20.0	85.5	25.7	25 0
13.32	0.912	19.8	86.0	25.5	25 0
15.39	0.978	20.9	87.3	26.7	26.0

W, Width of specimen
T, Thickness of specimen
A, Area of section
b, Constant in elongation equation (representing the general extension)
c, Constant in elongation equation (appearing in the local extension)
e_1, Computed extension
e_2, Interpolated extension

Both e_1 and e_2 represent extensions on gage lengths which are proportional to the square root of the indicated areas, i.e., on lengths where $L = 15\sqrt{A}$.

Values for e_1 were computed for each indicated value of W/T by the elongation equation. Hence $e_1 = c/15 + b$.

Values for e_2 were obtained from the data of Gordon and Gulliver. For each condition of W/T the extension on the proportional gage length ($L = 15\sqrt{A}$) was interpolated from readings as measured.

were affected by the ratio W/T. Referring to the curves in which gage length was plotted against measured elongation for given values of W/T, if extensions be interpolated for gage lengths proportional to the square root of the given areas, then by the law of similitude the resulting elongations should be the same. In each case the shape (ratio of W/T) was different, so that if any marked variation in the exten-

sions was observed it would be attributable exclusively to the effect of shape — as apart from area. These observations were made and the results tabulated in Table 4. It is seen that no appreciable variation exists in e_2, indicating that the effect of shape under constant area is negligible. Examination of the elongation equation shows that no account is taken of the effect of shape of section, and the above observation justifies this omission.

87 To further check the elongation equation, the extension e_1 was computed for equivalent gage lengths. The constants b and c were found by taking the elongations recorded on different gage lengths for each value of W/T and applying the method of least squares as previously described.

88 It will be noticed that the values in e_1 approximately agree with each other and with those in e_2. Two significant observations may be made. First, the elongation as computed agrees with that as measured, thereby establishing the validity of the equation. Second, the agreement of the two results indicates that the effect of shape alone is not a governing factor. In the computation the element of shape was absent, in contrast to its presence in the physical measurements, yet the results were alike.

APPENDIX NO. 2

1 Swain: "Structural Engineering — Strength of Materials," p. 75.
2 Kennedy: "Inst. Mech. Eng. Proc.," 1886.
3 Kelley & Winlock: On "Restraint of Exaggerated Grain Growth," Franklin Inst., Proc., 1924.
4 Sauveur: Metallography & Heat Treatment of Iron & Steel, p. 14.
5 Upton: Materials of Construction.
6 Morley: Strength of Materials.
7 Bingham: Fluidity and Plasticity.
8 Gordon and Gulliver: "Roy. Soc. of Edin. Trans.," vol. xlviii.
9 Barba: See Unwin "Tensile Tests of Mild Steel," Inst. Civ. Eng., Proc., vol. clv.
10 Unwin: (see 9).
11 Tresca: Franklin Inst., Jour., vol. 106, 1878.
12 Bridgeman: *Mech. Eng.*, vol. 47, No. 3, see also discussion by S. Timoshenko.
13 Tafel: *Iron Trade Review*, 1925-6.
14 Puppe: (see 13).
15 Fink (see 13).
16 Langenberg: "Effect of Cold Working on Strength of Hollow Cylinders," Trans. A.S.S.T., vol. viii, No. 4.
17 Templin: "Effect of Size and Shape of Test Specimen on Tensile Properties of Sheet Metal," Trans. A.S.T.M., 1926.

DISCUSSION

C. F. Nagel, Jr.[1] To a large extent, the metal-drawing industry is today still dependent upon cut-and-try methods. The writer believes that the technical phenomena functioning in the metal-drawing industry can be reduced to mathematical expressions which may be directly employed in the selection of metals, design

[1] Assistant Technical Director, Aluminum Co. of America, New Kensington, Pa.

of tools, and determination of operating conditions. The author suggests that elongation and reduction of area might be used to indicate the drawing qualities of the metal. To these might be added the ratio of yield point to tensile strength. In the case of aluminum and its alloys, workability of any particular alloy decreases as the yield point approaches the tensile strength. In a way, this is merely stating in different terms that workability, in general, varies with elongation and reduction of area, because in any given metal, as the yield point approaches the tensile strength, the elongation simultaneously decreases.

In Par. 7 the author quotes Swain as saying that "A steady load considerably less than the usual ultimate will cause fracture if applied long enough." The author further says that this effect is more common to the non-ferrous metals than to steel. Though this question may not be of extreme importance to the subject at hand, it is very important when considering the use of materials in engineering structures. The Aluminum Company of America has been conducting static tests on one of its high-strength alloys of the duralumin type, in which loads considerably above the yield point and up to 90 per cent of the ultimate have been imposed for almost a year, with as yet no measurable increase in extension above that noted immediately after putting the specimens under load. This initial extension, furthermore, is in accordance with the stress-strain curve. This type of aluminum alloy, and possibly others, apparently does not behave like zinc, and has as yet shown no evidence of creep.

The author recognizes the fact that as plastic deformation proceeds, the mechanical properties of the metal change. Mention is also made that cognizance must be taken of the rate of such alteration in properties. In the case of aluminum and its alloys, the tensile strength and yield point, of course, both increase, and the elongation decreases as cold work is put into the metal, but these various properties do not change at the same rate. The yield point and elongation are rapidly affected by the initial amount of stress, while the tensile strength is less rapidly affected. In the case of the tensile strength, if we should measure the amount of cold work put into the metal by the reduction in thickness produced in rolling, and should plot tensile strength against amount of cold work, we would find the curve to be a straight line until the reduction in thickness reaches approximately 80 per cent.

The author states, "One metal which may exhibit greater elongation than another when initially pulled may show inferior ductile properties when again compared after each has received the same amount of cold work. In short, the effective rate of work hardening may vary." The writer does not believe that Mr. Eksergian intends this to be construed as meaning that a given metal or alloy does not follow a definite rate-hardening law. Again, referring to aluminum and its alloys, if we have given a set of me-

chanical properties applying to the annealed state, we can predict accurately what those properties will be after a certain amount of cold work has been imposed upon that piece of metal.

The suggestion is made in the paper that the cup method, which the writer assumes to be the Erichsen test, might be used to select metal. This happens to be a subject to which we have given considerable thought and investigation, covering a great variety of methods including both the tensile and Erichsen test. Our investigations have included thousands of determinations, covering several months' production, comparing results obtainable whether the test was conducted by the usual plant inspection department or by a research laboratory. We arrived at the conclusion that the Erichsen test was not as accurate or reliable as the tensile test, especially for plant-control purposes. The end point in the Erichsen test is, to a large extent, a matter of personal judgment. The total spread in values between obviously good and obviously bad metal is fairly small. The Erichsen value, furthermore, is greatly influenced by the gage of the metal, with the result that the normal variations in thickness existing between different pieces of metal, together with the effect of thickness upon depth of draw, makes it quite difficult to set a definite Erichsen value to be used as a standard. Most of these deficiencies are largely overcome in a tensile test. Of course, as stated by the author, the Erichsen test to a degree causes the metal to behave in a manner similar to what it does in drawing, while if the tensile test were employed, one would have to translate mechanical properties to drawing qualities. The direction taken by the author's work, however, leads to just that situation where, as mathematical expressions are evolved, they will be based on mechanical properties of the metal.

In introducing aluminum into a metal-fabricating plant which has been accustomed to drawing steel, recognition must be given to the difference in properties between aluminum and steel. In the case of pure aluminum, we must recognize that the strength is considerably less than that of mild steel, and that the friction between the aluminum and the dies is greater than between the steel and the dies. Pure aluminum is also softer than steel. These differences must be compensated for by reducing the resistance offered against the movements of the metal. More consideration, therefore, must be given to the smoothness of the blank holder and die surfaces, the exact adjustment of blank holder pressure, more liberal die radius, and ample lubrication. With due attention to these details, the most intricate shapes can be successfully drawn in aluminum.

THE AUTHOR. The points which Mr. Nagle has raised, especially concerning the performance of aluminum, are regarded as a contribution to the subject. The ratio of yield point to tensile strength has been given some consideration — and perhaps should

be given more. So far evidence has not been sufficiently tangible to justify the drawing of conclusions.

Referring to the matter of work hardening, two metals identical in every respect when worked in the same manner will harden at the same rate, to be sure. But the question rests on the ability to establish identity with our present methods of test. May not a difference exist which cannot be detected in test but will be manifest in the rate of hardening? We have only to look at the elongation test to appreciate certain limitations. The fact that the extension of two metals in 8 in. is the same, is not indicative that their behavior in stretch is alike — as evidenced by the possible disagreement in the 2-in. elongations. It is noteworthy that the test has failed to reveal the very property for which it has been applied.

It was not intended to create an impression that the author advocates the Erichson cup test. Mention of this test was made to bring out the inherent advantages which it possessed. So far, no results have been obtained with this test which appear promising, even with the addition of a gage to accurately indicate the end point. It is felt that the failure of this test lies in the characteristics of the elongation curve, as shown on page 634. It is seen that only a small length is highly stretched. This condition is comparable to the elongation in a small gage length, with its attending shortcomings. It is to be observed that the employment of a large ball would not measurably aid this condition.

The author is in sympathy with Mr. Nagle in relying primarily upon simple elongation as a measure of ductility, providing that it be properly interpreted. To this end it is considered essential that readings be taken on short and long lengths so that the results may be given in terms of the general and local extensions. Elongations on one gage length do not sufficiently indicate behavior. When the two extensions are recorded, the ability of all sections to obey uniformly and the capacity of the weakest section in stretch, are respectively designated by the general and local elongations.

No. 2020

TESTS AND THEORY OF CURVED BEAMS

By A. M. Winslow,[1] Seattle, Wash.
Member of the Society

and

R. H. G. Edmonds,[2] Seattle, Wash.
Associate-Member of the Society

Despite the fact that curved beams comprise such important machine elements as crane hooks and punch frames, previous laboratory tests have been, in general, of meager scope and uncertain interpretation. The laws of stress distribution, therefore, are subject to diverse opinions among prominent engineers. The present tests consist of strain-gage observations at several points throughout the cross-sections of large steel specimens, determining complete strain curves. Radial strains, apparently not previously investigated, are definitely measured and a simple working theory of radial stress is developed. Stress in a radial direction is shown to be a vital factor in the design of curved beams of certain proportions. Finally, different theories of circumferential stress are investigated, both by comparison with tests and by analysis; prominent fallacies are pointed out and a more logical fundamental hypothesis advanced in support of the theory adopted.

TESTS of curved beams hitherto published have consisted mainly of attempts to determine maximum stress by observing the apparent proportional elastic limit of curved members in bending and comparing this result with the elastic limit of straight-bar tension specimens of the same material.[3] This method is indirect, the theoretical validity of the comparison has been questioned, and the observed proportional limit has been in many cases poorly defined. Furthermore, comparison of test results with theory has

[1] Associate Professor of Mechanical Engineering, University of Washington.
[2] Assistant Professor of Mechanical Engineering, University of Washington.
[3] Walter Rautenstrauch, Trans., A.S.M.E., vol. 31, p. 559.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

been hampered in some cases by the necessity for graphical calculation in the numerical application of theory. The graphical solution happens to involve terms so related that any lack of

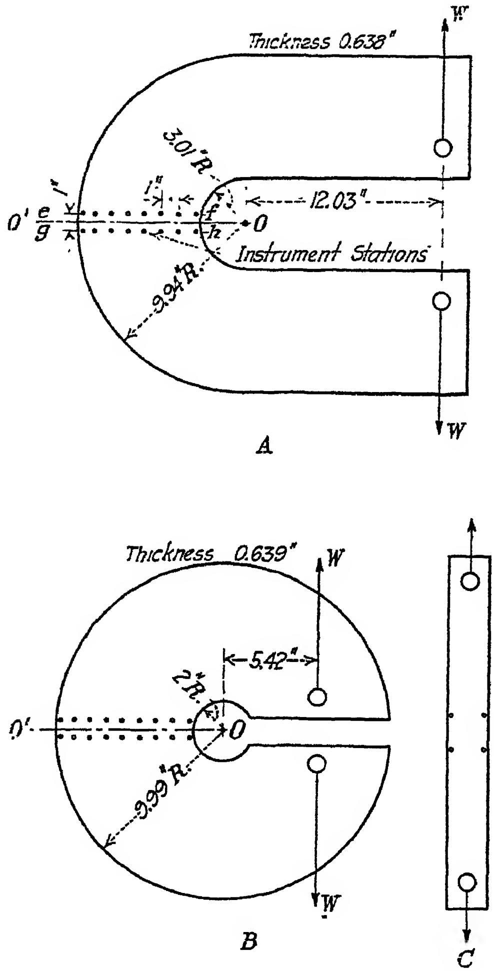


FIG. 1 DETAILS OF SPECIMENS

precision in graphics produces a greatly magnified error in the final result.[1] The purpose of the present tests was to investigate more definitely and completely the stresses in curved beams of assumed isotropic material. Specimens were made simple in shape to permit precise numerical computations without graphics,

[1] Arthur Morley, *Engineering*, vol. 98, p. 321.

and also sufficiently large for strain-gage observations at several points in the cross-section. One valuable result of this latter condition was the immediate check on the instrument work; with observations sufficient in number to determine a strain curve, a departure from the average curve was at once discovered and the point re-examined.

Description of Specimens and Testing Apparatus

2 The curved specimens were mild-steel plates, of rectangular cross-section, machined all over to finished dimensions noted in

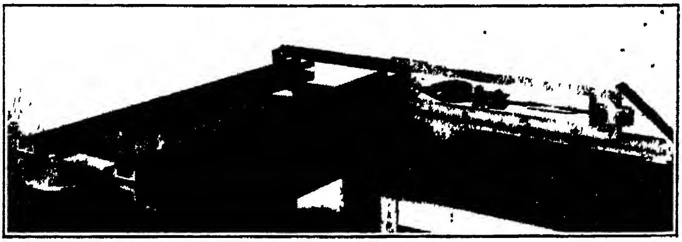


Fig. 2 Loading Frame

Fig. 3 Strain Gage

Fig. 1, *A* and *B*. Lack of a horizontal testing machine, together with the desirability of using the strain gage on a horizontal surface, led to the construction of the loading frame shown in Fig. 2. This frame consisted of a pair of steel channel levers, at the outer end of which force was applied by a turnbuckle through a

calibrated tension dynamometer. The levers were supported near the fulcrum points. The curved beam was supported on rollers. At times of applying load the levers were repeatedly raised slightly above their supports in order to eliminate the effects of friction as far as possible.

3 The strain-gage readings were taken on 1-in. spans at the stations indicated in Fig. 1, *A* and *B*. The strain observations were made both circumferentially and radially, the latter readings in specimen *A* being made along lines *ef* and *gh*. In specimen *B*, after the circumferential readings were completed, additional station points were made by adding center-punched holes along the radial line *OO′*. The reliability of strain-gage observations on a span as short as 1 in. has been questioned so often that some detailed descriptions of the instrument and methods of operation are warranted, together with a quite complete schedule of the test results. Much preliminary experimentation was carried on with various kinds of auxiliary clamping devices before the adoption of the arrangement of apparatus finally used. The Olsen strain-gage extensometer, shown in Fig. 3, has contact points adjustable for either a 1-in. or a 2-in. span. The gage is intended to be used either in center-punched holes or in holes drilled $\frac{1}{16}$ in. in diameter; the latter were used in all places except along the radial lines of specimen *B* already noted. The edges of the holes were smoothed with a plain conical pointed reamer and grinding compound, the apex angle of the reamer being greater than that of the instrument points. The reliability of the strain observations was greatly affected by the care used in the preparation of the holes. The reaming was observed under a microscope and was continued until the gage could be applied repeatedly with uniform dial readings. Fig. 3 also shows a lateral arm $\frac{1}{4}$ in. square by 6 in. long which was added in order to support the instrument normal to the surface. In taking observations the instrument points were pressed downward on the specimen and at the same time a slight lateral vibration was applied at the outer end of the arm in order to permit the instrument points to seat in complete contact. Finally, at the end of the arm a light transverse pressure was applied, always in the same direction, in order to take up any lost motion in a uniform manner. At each station the instrument was applied at least three times in succession until uniform consecutive readings were obtained. A single schedule of observations, for example with specimen *B*, consisted of sets of strain readings at four successive loads, of 100 lb. and 1200 lb. alternately, followed by comparisons of each two sets of readings at equal loads. Any station showing a pair of readings not in close agreement was investigated further. Next the strain increments for this schedule of readings were plotted and any point departing from a smooth curve was re-examined. In such cases of discrepancy, almost invariably inves-

tigation of the station holes in question disclosed the presence of dust or some foreign matter, the removal of which permitted uniform corrected observations. After completing the above procedure, before beginning another similar series of loadings, a resetting was made in the adjustable parts of the strain gage so as to change the dial readings and avoid the possible continued effect of any local irregularities in the construction of the instrument. In observations the position of the pointer on the dial was estimated to the nearest one-fourth of the smallest graduated interval, the one-fourth interval corresponding to a strain difference of 0.000015 in. per in.

Strain-Gage Results

4 The strain-gage readings are shown in Tables 1 to 4 inclusive and the average values are plotted in Figs. 5 and 7, with a smooth average curve drawn for each set of plotted points. The tables also show additional derived results which are explained later. Each radial reading noted in Table 2 is the average of readings along lines *ef* and *gh*, Fig. 1*A*. After ten sets of readings had been taken on specimen *A*, the uniformity of the results indicated that for specimen *B* six sets of readings would be sufficient to insure a reliable average. With reference to uniformity of strain-gage results, it may be worth while to note that Tables 1 to 4 inclusive represent in each case consecutive sets of observations and not a few best sets selected from a larger series.

5 The mathematical analysis of radial stresses developed later in this paper involves the necessity for determining Poisson's ratio of lateral strain. This value, together with the modulus of elasticity, was obtained by testing straight-bar tension specimens cut from the original plates *A* and *B*. These tension bars, $2\frac{1}{2}$ in. wide and approximately 21 in. long, had gage setting holes on 2-in. spans arranged as shown in Fig. 1*C*, permitting strain observations both longitudinally and transversely on both sides of the specimen. To save space in this paper, the twelve sets of observations for each tension bar are not tabulated; the average results were as follows:

	A	*B*
Load, lb per sq. in.	6,420.	11,310.
Longitudinal strain, in. per in.	0.0002208	0.000390
Transverse strain, in. per in.	0.0000546	0.000102
Modulus of elasticity	29,100,000.	29,000,000.
Poisson's ratio	0.247	0.262

For the two specimens the average value of Poisson's ratio is 0.255. For convenience the value 0.25 was used in all numerical computations; this is the theoretical value for isotropic materials and furthermore a small change in the value of Poisson's ratio produces only a very slight change in the final numerical results for curved beams.

6 Before interpreting further the results of these tests, it is necessary to consider certain theoretical relations. Up to the present time, three leading theories have been advanced to express the distribution of stresses in curved beams subjected to bending. The three mathematical formulas, already fully stated in several previous publications, need not be repeated in detail.[1] It is essential here simply to note the general principles and hypotheses underlying each theory, as follows:

TABLE 1 TESTS OF CIRCUMFERENTIAL STRAINS; SPECIMEN *A*. LOAD INCREMENT, 800 LB. AT 72 5 IN. LEVERAGE

Observed strain-gage increments

	Radius *R*, in.							
Test No.	3.09	4.08	5 08	6.07	7.07	8.07	9.07	9.83
				Side No. 1				
1	+2.10	+0.80	+0.30	—0.20	—0.35	—0.60	—0.85	—0.85
2	2.15	0.85	0.35	0.15	0.45	0.70	0.80	0.85
3	2.10	0.95	0.25	0.10	0.40	0.60	0.75	0.90
4	2.05	0.95	0.30	0.15	0.40	0 60	0 80	0.85
5	2.05	0.90	0.30	0.10	0.40	0.65	0.80	0.90
6	2.10	0.90	0.25	0.10	0.40	0.60	0.75	0.90
7	2.15	0.85	0.25	0.10	0.45	0.65	0.80	0.90
8	2.15	0.85	0.20	0.15	0.45	0.60	0.80	0.95
9	2.10	0.85	0.30	0.15	0.45	0.60	0 80	0.90
10	2.10	0.85	0.20	0.15	0.45	0.65	0.80	0.95
				Side No. 2				
1	1.95	0.85	0.25	0.15	0.40	0.60	0.80	0.90
2	1.90	0.85	0.20	0.15	0.40	0.65	0.80	0.95
3	1.90	0.85	0.20	0 15	0.40	0.60	0.80	0.90
4	1.95	0.80	0.25	0.10	0.45	0.65	0.75	0.90
5	1.95	0.80	0.25	0.15	0 40	0.70	0.80	0.90
6	1.90	0.85	0.25	0.15	0 35	0.65	0.80	0.90
7	1.90	0.85	0.25	0.15	0 40	0.70	0.80	0.90
8	1.95	0.85	0.20	0.15	0.40	0.70	0.85	0.95
9	1.95	0.85	0.20	0.10	0.35	0.65	0.75	0.90
10	+1.90	+0.85	+0.20	—0.15	—0.35	—0.65	—0.80	—0.90
(*a*) Average gage increment	+2.02	+0.86	+0.25	—0.14	—0.40	—0.64	—0.80	—0.90
(*b*) Unit strain, ($\times 10^4$)	+6.05	+2.57	+0.74	—0.41	—1.21	—1.92	—2.39	—2.70
(*c*) Unit strain modified by Poisson's ratio, Eq. [1], ($\times 10^4$)	+6.11	+2.84	+1.06	—0.12	—0.98	—1.75	—2.30	—2.70
(*d*) Observed circumferential stress; result (*c*) $\times$ mod. of elasticity, lb. per sq. in. ($\times 10^{-4}$)	+1.78	+0.83	+0.31	—0.08	—0.29	—0.51	—0.67	—0.79
(*e*) Theoretical circumferential stress, Eq. [8] lb. per sq in., ($\times 10^{-4}$)	+1.72	+0.84	+0.29	—0.07	—0.33	—0.53	—0.68	—0.78

1 The ordinary beam theory assumes that in the cross-section of a curved beam the stresses due to bending are distributed according to the same law as in the case of a straight beam

2 Winkler's theory takes account of curvature and is based on the hypothesis that plane transverse sections remain plane after loading

[1] Arthur Morley, *Engineering*, vol. 98, p. 321.

3 The Andrews-Pearson theory, proposed as a refinement of the Winkler theory, takes account of the additional consideration that the radial dimensions of the cross-section after loading are changed by the Poisson-ratio effect of transverse strains resulting from normal stresses.

It should also be noted that both the Winkler and the Andrews-Pearson theory, to reduce complexity, confine consideration to plane of maximum moment OO', Fig. 1, normal to direction of loading. On this plane the resultant external shear is zero and it is assumed that internal shearing stresses on this plane may be neglected without appreciable error.

Radial Stresses

7 The first investigations by the authors were with specimen *B*, on which strains were observed in a circumferential direction only.

TABLE 2 TESTS OF RADIAL STRAINS; SPECIMEN *A*. LOAD INCREMENT, 800 LB. AT 72.5 IN. LEVERAGE

Observed strain-gage increments

Test No	Average Radius *R*, in. 3.58	4.58	5.58	6.57	7.57	8.57
			Side No. 1			
1	−0.10	+0.25	+0.40	+0.40	+0.40	+0.35
2	0.05	0.25	0.45	0.40	0.40	0.40
3	0.10	0.20	0.40	0.45	0.40	0.40
4	0.05	0.25	0.45	0.45	0.40	0.40
5	0.10	0.25	0.40	0.40	0.40	0.35
6	0.10	0.20	0.40	0.40	0.40	0.40
7	0.05	0.25	0.40	0.35	0.40	0.35
8	0.10	0.20	0.45	0.35	0.40	0.35
9	0.05	0.25	0.40	0.40	0.40	0.35
10	0.10	0.25	0.45	0.40	0.40	0.35
			Side No. 2			
1	0.10	0.25	0.35	0.40	0.35	0.30
2	0.05	0.25	0.40	0.40	0.40	0.35
3	0.10	0.20	0.35	0.40	0.35	0.30
4	0.10	0.25	0.40	0.40	0.35	0.30
5	0.10	0.25	0.35	0.40	0.35	0.35
6	0.10	0.20	0.35	0.35	0.35	0.30
7	0.05	0.25	0.35	0.40	0.35	0.35
8	0.10	0.25	0.35	0.40	0.40	0.35
9	0.10	0.20	0.35	0.35	0.35	0.30
10	−0.10	+0.25	+0.40	+0.40	+0.35	+0.30
(*a*) Average gage increment	−0.08	+0.24	+0.39	+0.40	+0.38	+0.34
(*b*) Unit strain, ($\times 10^4$)	−0.23	+0.71	+1.18	+1.19	+1.14	+1.03
(*c*) Unit strain modified by Poisson's ratio, Eq. [1], ($\times 10^4$)	+0.79	+1.17	+1.29	+1.03	+0.78	+0.52
(*d*) Observed radial stress; result (*c*) × mod. of elasticity, lb. per sq. in., ($\times 10^{-4}$)	+0.23	+0.34	+0.38	+0.30	+0.23	+0.15
(*e*) Theoretical radial stress, Eq. [2] and [8], lb. per sq. in., ($\times 10^{-4}$)	+0.24	+0.37	+0.35	+0.29	+0.20	+0.12

These results, compared directly with the Winkler and Andrews-Pearson theories, continued to show pronounced unbalanced discrepancies, despite much experimental revision of testing procedure. Consideration finally indicated the desirability of measuring

TABLE 3 TESTS OF CIRCUMFERENTIAL STRAINS; SPECIMEN B. LOAD INCREMENT, 1100 LB.; EQUIVALENT TO 6300 LB. AT 11.42 IN. LEVERAGE

Observed strain-gage increments

Test No.	Radius R, in.								
	2.11	3.01	4.01	5.01	6.00	7.00	8.00	8.90	9.89
					Side No. 1				
1	+2.25	+1.00	+0.35	+0.10	−0.20	−0.35	−0.45	−0.55	−0.65
2	2.25	1.00	0.35	0.05	0.15	0.35	0.45	0.55	0.70
3	2.30	1.00	0.35	0.10	0.15	0.30	0.45	0.60	0.70
4	2.30	1.00	0.40	0.00	0.20	0.35	0.45	0.60	0.70
5	2.40	1.00	0.35	0.05	0.20	0.40	0.50	0.55	0.65
6	2.40	0.95	0.40	0.05	0.15	0.35	0.50	0.60	0.70
					Side No. 2				
1	2.25	1.05	0.35	0.05	0.15	0.35	0.50	0.60	0.60
2	2.25	1.00	0.35	0.05	0.15	0.35	0.55	0.60	0.60
3	2.25	1.00	0.35	0.05	0.15	0.35	0.50	0.60	0.65
4	2.20	1.00	0.35	0.05	0.20	0.35	0.45	0.60	0.60
5	2.30	1.05	0.35	0.00	0.15	0.35	0.45	0.60	0.65
6	+2.30	+1.00	+0.35	+0.05	−0.20	−0.35	−0.45	−0.60	−0.60
(a) Average gage increment	+2.30	+1.00	+0.30	+0.03	−0.17	−0.35	−0.47	−0.59	−0.65
(b) Unit strain, (× 10⁴)	+6.91	+3.01	+1.07	+0.10	−0.51	−1.05	−1.42	−1.76	−1.95
(c) Unit strain modified by Poisson's ratio, Eq. [1], (× 10⁴)	+6.98	+3.39	+1.49	+0.44	−0.23	−0.84	−1.29	−1.70	−1.94
(d) Observed circumferential stress; result (c) × mod. of elasticity, lb. per sq. in., (× 10⁻⁴)	+2.02	+0.98	+0.43	+0.13	−0.07	−0.24	−0.37	−0.49	−0.56
(e) Theoretical circumferential stress, Eq. [8], lb. per sq. in., (× 10⁻⁴)	+2.04	+1.05	+0.46	+0.12	−0.12	−0.28	−0.41	−0.51	−0.58

TABLE 4 TESTS OF RADIAL STRAINS; SPECIMEN B. LOAD INCREMENT, 1100 LB.; EQUIVALENT TO 6300 LB. AT 11.42 IN. LEVERAGE

Observed strain-gage increments

Test No.	Average Radius R, in.						
	2.61	3.61	4.61	5.61	6.61	7.61	8.61
				Side No. 1			
1	−0.00	+0.35	+0.40	+0.35	+0.30	+0.25	+0.20
2	0.00	0.35	0.40	0.30	0.30	0.25	0.25
3	0.00	0.30	0.35	0.35	0.30	0.25	0.20
4	0.00	0.30	0.35	0.40	0.35	0.25	0.25
5	0.00	0.30	0.40	0.35	0.35	0.25	0.20
6	0.00	0.35	0.40	0.35	0.35	0.25	0.25
				Side No. 2			
1	0.00	0.40	0.40	0.35	0.30	0.25	0.25
2	0.00	0.40	0.40	0.40	0.35	0.25	0.20
3	0.00	0.35	0.35	0.40	0.35	0.25	0.20
4	0.05	0.40	0.35	0.40	0.35	0.20	0.20
5	0.05	0.40	0.40	0.35	0.40	0.25	0.20
6	−0.00	+0.40	+0.40	+0.35	+0.35	+0.25	+0.25
(a) Average gage increment	−0.01	+0.36	+0.38	+0.36	+0.34	+0.25	+0.22
(b) Unit strain, (× 10⁴)	−0.03	+1.19	+1.28	+1.21	+1.18	+0.82	+0.74
(c) Unit strain modified by Poisson's ratio, Eq. [1], (× 10⁴)	+1.10	+1.73	+1.48	+1.21	+0.98	+0.54	+0.36
(d) Observed radial stress; result (c) × mod. of elasticity, lb. per sq. in., (× 10⁻⁴)	+0.32	+0.50	+0.43	+0.35	+0.28	+0.16	+0.10
(e) Theoretical radial stress, Eq. [2] and [8], lb. per sq. in., (× 10⁻⁴)	+0.33	+0.52	+0.47	+0.38	+0.28	+0.19	+0.11

strains in both directions of principal stress, that is, both in a radial and a circumferential direction. With normal unit stresses, C circumferentially and T radially, the resulting unit strains, denoted by small letters c and t, produced by Poisson's ratio m are as follows, E denoting modulus of elasticity:

$$c = \frac{1}{E}(C - mT)$$

$$t = \frac{1}{E}(T - mC)$$

from which

$$\left.\begin{aligned} C &= \frac{E}{1-m^2}(c+mt) \\ T &= \frac{E}{1-m^2}(t+mc) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [1]$$

To investigate the above relations more readily, specimen A was made with instrument-station holes arranged for observation of

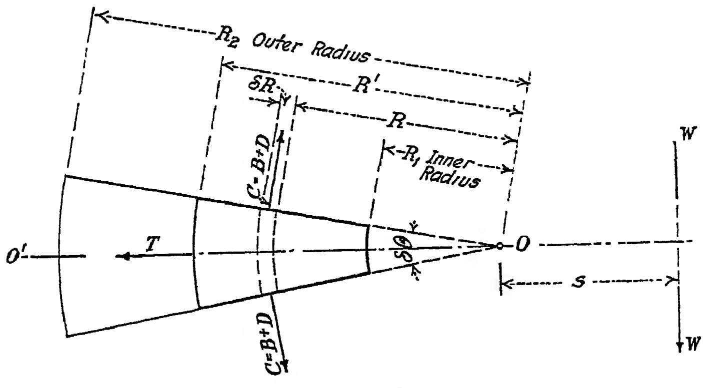


FIG. 4 ELEMENT OF CURVED BEAM

strains both radially and circumferentially. These test results are recorded in Tables 1 and 2, with the values of strains derived by Equations [1] shown in line c. As noted previously, specimen B was later arranged for testing radial strains; the results are shown in Tables 3 and 4.

8 The numerical values of radial stress should now be compared with theory. While radial strains have received comment by previous authors,[1] and a formula applicable only to rectangular cross-sections is given by Föppl,[2] theoretical radial stress analysis of general application to curved beams apparently is not found in current engineering literature, and the following solution is sub-

[1] A. Lewis Jenkins, Trans. A.S.M.E., vol. 32, p. 320.
[2] August Föppl, Technische Mechanik, 1907, vol. 5, p. 75.

mitted. Fig. 4 is a side elevation of a portion of a curved beam, such as Fig. 1, in which the plane of loading is a plane of symmetry. In accordance with general considerations by previous investigators, discussion will be limited to the immediate vicinity of transverse plane OO' and it will be assumed that shearing stresses may be disregarded. The normal unit stresses C on plane OO' consist of the sum of stresses B due to bending and D direct, the latter

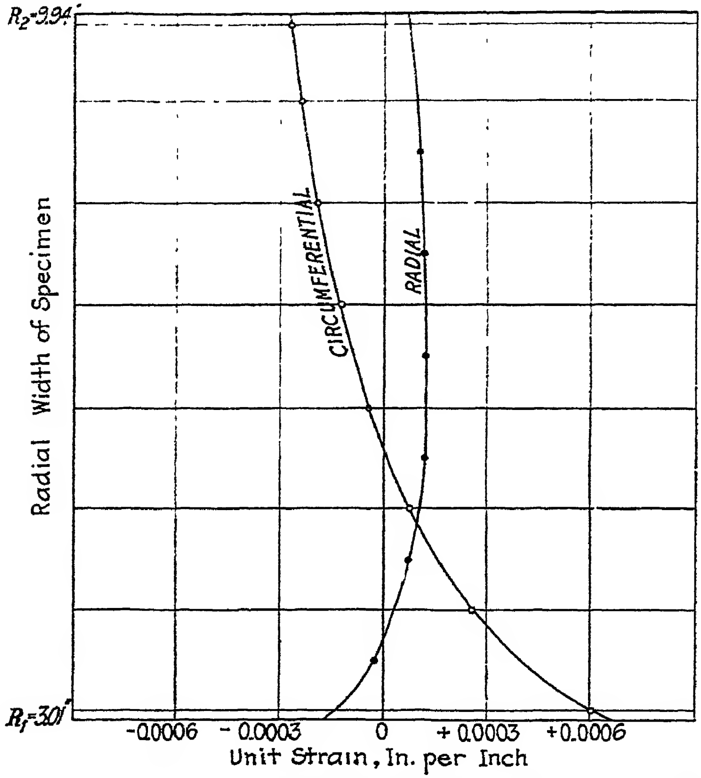


FIG. 5 CIRCUMFERENTIAL AND RADIAL STRAINS; SPECIMEN *A*. TEST VALUES WITH AVERAGE CURVES

over the total cross-section being equal to W, the external load. Stress D, in direction of W, has no radial component, and the relation between the unit stress B and the radial unit stress T is readily derived by referring to Fig. 4. Heavy lines indicate a portion of the curved beam included between inner radius R_1 and intermediate radius R' and bounded by transverse radial planes subtending an infinitesimal angle $\delta\theta$. Let w denote width of cross-section at radius R; w in general is not constant. Then consideration of equilibrium of component stresses in a radial direction leads to the relation

$$T = \frac{1}{w'R'}\int_{R_1}^{R'} Bw\delta R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]^{1}$$

It is seen that the radial stress is tension throughout the cross-section, becoming zero at the surfaces R_1 and R_2. The integral in Equation [2] denotes the sum of the stresses due to bending on the cross-sectional area included between radius R' and the

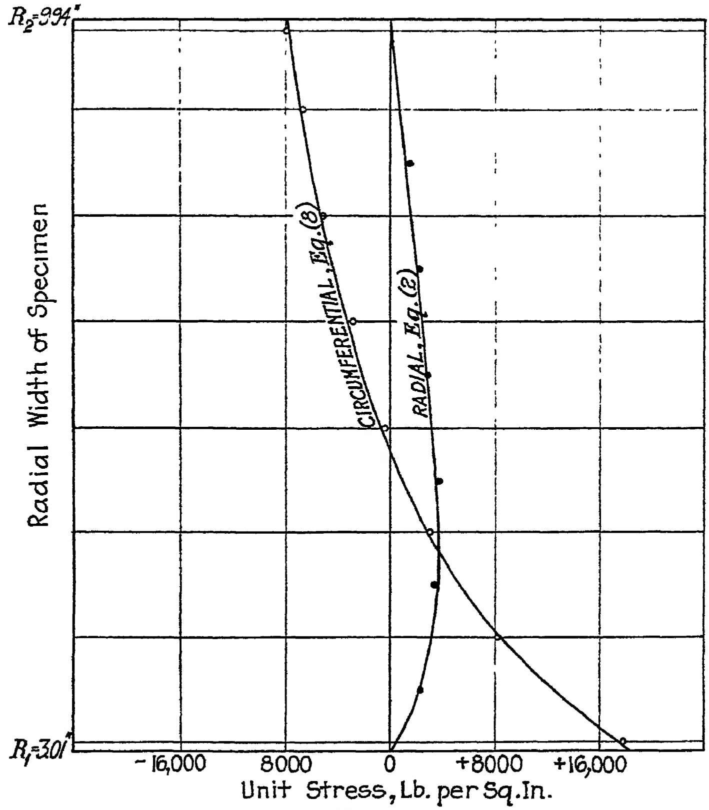


FIG. 6 CIRCUMFERENTIAL AND RADIAL STRESSES; SPECIMEN *A*. TEST RESULTS, INCLUDING EFFECT OF POISSON'S RATIO, COMPARED WITH THEORETICAL CURVES

inner radius R_1. Moreover, this relation for radial unit stress is perfectly general and applies with any theoretical value of B, the circumferential stress due to bending. For a flanged section with a thin web, where w is small, T is correspondingly large and the consideration of radial stress is important. The determination of the numerical value of T by Equation [2] involves no special

[1] It is shown later that the element of area may be taken equal to $w\delta R$.

difficulty; the summation represented by the integral may be obtained readily either graphically with a planimeter or in many cases with sufficient accuracy by approximate numerical computation.

THEORIES OF CIRCUMFERENTIAL STRESS

9 The relation [2] for radial stress shows that the Andrews-Pearson formula, as a refinement of the Winkler theory, is

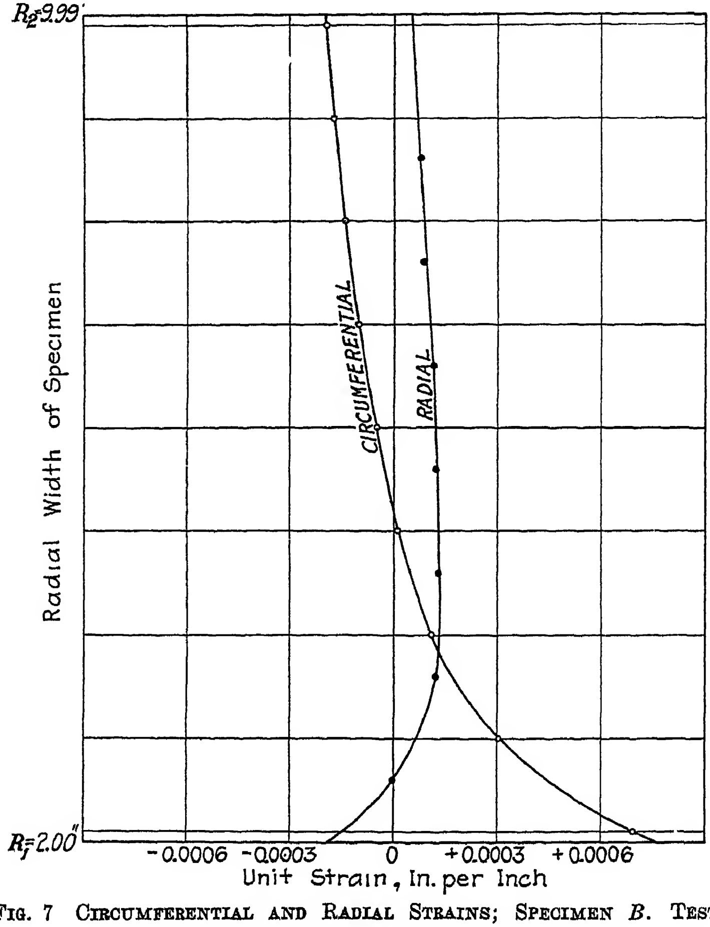


FIG. 7 CIRCUMFERENTIAL AND RADIAL STRAINS; SPECIMEN *B*. TEST VALUES WITH AVERAGE CURVES

erroneous and without value. The fundamental feature of the Andrews-Pearson formula is the additional consideration of changes in radial dimensions due to radial strain; it takes account, however, only of Poisson's lateral distortion due to circumferential

stress and omits entirely any consideration of strains due directly to radial stress. These latter strains in certain shapes of cross-sections may be very large, and in a considerable portion of any cross-section are in a direction opposite to the Poisson lateral strains.

10 Consideration of radial stresses not only overthrows the Andrews-Pearson formula, but also renders Winkler's solution inconsistent with the original hypothesis that plane transverse sections remain plane after loading. The plane-section relation, in agreement with Winkler's theoretical circumferential strains, is destroyed by the added strains due to the Poisson effect of radial stresses. The hypothesis regarding plane sections seems to be pure assumption without theoretical proof. The principle of least work, however, is gaining increasing recognition as a logical starting point in stress analysis, and in the following portion of this paper an attempt is made to apply this principle to the problem of circumferential stresses in curved beams. The inter-dependent relation of radial and circumferential stresses makes the problem complex and a direct analytical solution appears to be beyond the facilities of mathematics. However, a certain approximation leads to a definite solution showing important results when compared with Winkler's theory and with the present test observations. The application of the theory of least work to curved beams is simplified somewhat by the consideration, first, that in a straight bar of constant section subjected only to longitudinal stress the condition of least work is with the stress uniformly distributed over the cross-section. This is readily proved by simple applications of principles of equilibrium and of least work. From this result it follows that in a curved beam, Fig. 4, for least work the unit stress at any radius R is constant across the total width w of the cross-section. The considerations regarding neglect of shearing stresses are here the same as in the derivation of Equation [2]. Also, to reduce complexity for the time being, let the work due to radial stresses and strains be disregarded, so that the problem is limited to determining that distribution of circumferential stress which gives a minimum value for the work done by circumferential stress alone. In Fig. 4, referring to the elementary mass of radial dimension δR, the following three equations are obtained for vertical stress equilibrium, moment equilibrium, and least work, respectively:

$$\int_{R_1}^{R_2} Cw\delta R - W = 0 \quad . \quad . \quad . \quad [3]$$

$$\int_{R_1}^{R_2} C(R+s)w\delta R = 0 \quad . \quad . \quad . \quad [4]$$

$$\frac{\delta\theta}{2E}\int_{R_1}^{R_2} C^2Rw\delta R = \text{minimum} \quad . \quad . \quad . \quad [5]$$

The three conditions may be gathered into one relation, as follows:

$$\int_{R_1}^{R_2}(K_1C+K_2CR+C^2R)w\delta R = \text{minimum} \quad . \quad . \quad . \quad [6]$$

in which K_1 and K_2 are constants to be determined later. Making the derivative coefficient of Equation [6] with respect to C equal to zero,

$$K_1+K_2R+2CR = 0$$

Since the second derivative is positive, this is the condition for a minimum, or

$$C = -\frac{K_1}{2R} - \frac{K_2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [7]$$

Substituting this value of C in Equations [3] and [4], solving for K_1 and K_2, and substituting in Equation [7], the final result is

$$C = \frac{M\left(\dfrac{1}{R} - \dfrac{1}{A}\displaystyle\int_{R_1}^{R_2}\frac{w\delta R}{R}\right)}{R_g\displaystyle\int^{R_2}\frac{w\delta R}{R} - A} + \frac{W}{A} \quad . \quad . \quad . \quad . \quad . \quad [8]$$

in which R_g is the radius to the gravity center of the cross-section. A is the area of the cross-section, and $M = W(s+R_g)$. This result, Equation [8], is identical with Winkler's formula; in other words, neglecting radial stresses and considering circumferential stresses only, Winkler's formula may be derived from the hypothesis of least work without resorting to the assumption that plane transverse sections remain plane after loading.[1] The question now arises concerning the extent to which the circumferential stresses are theoretically modified in the least-work hypothesis by including the radial stresses. This point may be contemplated with some approximate speculations, dependent on the fact that, at the place in the cross-section where the circumferential stress is a maximum, the radial stress is zero. Such speculations, however, are doubtless out of place here and may better be left to the individual judgment of the reader. A derivation similar to that used in Equations [3] to [7] inclusive, but taking complete account of radial and circumferential stresses, gave much longer and more complex expressions which it seems unnecessary to publish in full. The resulting relation, for reference designated as Equation [8′], corresponding to Equation [8], contained on the right-hand side certain terms having integrals involving C, thus not giving a direct solution for C. To determine, in a special case, the variation between the approximate solution, Equation [8], and the precise

[1] Making R infinite reduces Equation [8] to the usual bending formula for straight beams, thus deriving the straight-beam formula from the hypothesis of least work.

solution, computations were made for specimen A by the method of successive approximations. Starting with Equation [8], this approximate value of C in terms of R was substituted where necessary in the integrals of Equation [8′], permitting solution for a revised value of C as a function of R. This revised value was again substituted, and so on. This numerical process, involving simple mathematical computations, is too long to warrant publication in detail and is a method too time consuming for purposes of practical engineering design. The final derived value of C, compared with the approximate solution, Equation [8], showed differences which were very slight and for practical purposes negligible. The above, of course, is only one special case, and too sweeping general conclusions should not be drawn from the result.

Comparisons of Tests with Theory

11 Adopting Winkler's Equation [8] as the most logical approximate solution at present available for circumferential stresses in curved beams, theoretical values of radial stresses are then obtained by Equation [2]. For specimen A, with test load, Equation [8] becomes

$$C = 19430\left(\frac{5.80}{R} - 1\right) + 181 \quad . \quad . \quad . \quad . \quad . \quad . \quad [8A]$$

The term $19430\left(\frac{5.80}{R} - 1\right)$ denotes the bending stress B. This value substituted in Equation [2] gives

$$T = \frac{19430}{R}\,(5.80 \log_e R - R - 3.38) \quad . \quad . \quad . \quad . \quad [2A]$$

For specimen B the corresponding results are

$$C = 14150\left(\frac{4.968}{R} - 1\right) + 1273 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [8B]$$

$$T = \frac{14150}{R}\,(4.968 \log_e R - R - 1.443) \quad . \quad . \quad . \quad . \quad [2B]$$

By the preceding equations, with the aid of Equations [1], the test observations comprised in this paper may be compared with the theoretical Equations [8] and [2]. This comparison is shown by Figs. 6 and 8, in which the curves are theoretical, while the plotted points represent test results. The corresponding numerical values are given in Tables 1 to 4 inclusive; in each table test results are shown in line [d] and theoretical values in line [e]. Few differences are found in excess of 0.04×10^4 lb. per sq. in., which is the stress corresponding to the smallest estimated interval used in strain-gage readings. These results, therefore, for the specimens tested appear to verify the theory of radial stress and to indicate the value of Winkler's formula as a sufficiently precise method of analysis for practical application.

DISCUSSION

Casper D. Meals.[1] Professor Andrews in his book, The Stresses in Hooks and Other Curved Beams, has written quite clearly on this subject and has shown by chart that there is very little difference in the results between the Winkler and Andrews-Pearson theories. In his Strength of Materials, Professor Andrews states that the additional accuracy over Winkler's theory is so small that its application to practical problems is not warranted, on account of the additional computation required and subsequent risk of error. Professor Morley in *Engineering*, September 25, 1914, arrived at the same conclusion.

Bach, in his Elasticitat und Festigkeit, elaborated upon Winkler's earlier works, and presents formulas that will give the same results. Bach's theory is given in Greene's Structural Mechanics of 1911, the fundamental equation for the stress intensity being

$$f = \frac{W}{A} + \frac{M}{RA} + \frac{M}{KRA} \cdot \frac{y}{R+y} \quad \ldots\ldots\ldots\ldots \quad [9]$$

R = radius of curvature of neutral axis of critical cross-section

y = distance from neutral axis to fiber of section under consideration

$M = Wl$

$$K = 1/A \int \frac{y}{R+y}\, \delta A = \frac{A'}{A} - 1$$

$$A' = \text{area of modified section} = R \int \frac{1}{R+y}\, \delta A.$$

In applying [9] the following must be observed:

W is positive when it produces tension and negative when it produces compression

M is positive when it increases the curvature, and negative when it decreases the curvature of the beam

Y is positive when measured toward the extrados and negative when measured toward the intrados or inside of the section.

For hooks, rings, links, etc., failure is due to tensile stresses, hence for a hook loaded as shown in Fig. 1 the maximum tensile stress intensity is

$$C = \frac{W}{A} - \frac{Wl}{RA} + \frac{Wld_t}{KARR_1} \quad \ldots\ldots \quad [10]$$

[1] Wire Rope Engineer, American Cable Co., New York, N. Y.

Where the line of application of load W coincides with origin O of R_1, there follows,

$$C = \frac{Wd_t}{KAR_1} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad [11]$$

d_t = distance from neutral axis to outermost fiber on inner or tensile side of section

$l = s+R$ = lever arm.

The fundamental equation for Winkler's theory as given in Andrew's The Stresses in Hooks and Other Curved Beams is

$$f = \frac{W}{A} + \frac{M}{RA} + \frac{M}{R^2A(K_1-1)} \cdot \frac{Ry}{R+y} \quad \ldots \quad [12]$$

which gives

$$C = \frac{W}{A} + \frac{Wl}{RA}\left\{\frac{d_t}{R_1(K_1-1)} - 1\right\} \quad \ldots \quad [13]$$

in which

$$K_1 = A'/A$$

Morley, in his Strength of Materials, gives a slightly different dress to Winkler's formula for C giving

$$C = \frac{W}{A} + \frac{Wl}{R(A'-A)}\left\{\frac{R}{R_1} - \frac{A'}{A}\right\} \quad \ldots \quad [14]$$

Resal, in his Resistance des Materiaux, gives it still another dress as follows:

$$C = \frac{WlRd_x}{I_1R_1} + \frac{W}{A} \quad \ldots \ldots \ldots \quad [15]$$

in which

$d_x = d_t - x_0$

x_0 = distance between neutral axis of the original and modified sections

$= \dfrac{R(A'-A)}{A'}$

I_1 = moment of inertia of modified section

$= \dfrac{R^2A(A'-A)}{A'}$

It can be proved algebraically that [9] and [12] are identical and that [13], [14] and [15] are identical with [10] or [11]; for example, by substitution of $(K_1-1)=K$, and $R=R_1+d_t$ in [13], [14] and [15], these formulas become [11].

Some values of K may be of interest; for the circle and ellipse,

$$K = \frac{d^2}{16R^2} + \frac{d^4}{128R^4} + \frac{5d^6}{4096R^6} + \ldots$$

for the square and rectangle,

$$K = \frac{d^2}{12R^2} + \frac{d^4}{80R^4} + \frac{d^6}{448R^6} + \ldots\ldots$$

For a square at 45 deg., as met with in chain grab hooks, the value of K is the same as given for the square section above, the decrease in strength of the hook being due to the increase in R.

For crane hook sections, A' may be determined with sufficient accuracy by

$$A' = R\sum\frac{\Delta A}{R+y}$$

Y is positive when measured toward the extrados and negative when measured toward the intrados of the section shown in Fig. 9. The depths Δd of the strips for ΔA may be taken at $d/20$ as a maximum. If the planimeter is handy, A' may be determined by transforming the critical section by the graphical method given in the various books on strength of materials.

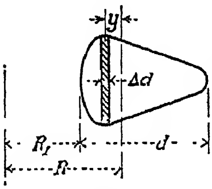


FIG. 9

A substitution of the first term $d^2/16R^2$ for K, $d/2$ for d_t, $\pi d^2/4$ for A, etc., in Equation [11] for hooks of circular stock leads to the very simple form of equation of $W = nd^2$ for the safe load capacities of the hooks.

Maximum values of C are as follows:

Wrought-iron hooks	20,000 to 25,000 lb. per sq. in.
Hammered steel hooks	
0.10 to 0.25 carbon steel..........	25,000 to 30,000 lb. per sq. in.
0.30 to 0.40 carbon steel..........	30,000 to 36,000 lb. per sq. in.
Drop-forged steel hooks	
0.30 to 0.40 carbon steel..........	36,000 to 42,000 lb. per sq. in.

Cast-steel hooks of 0 15 to 0 25 carbon steel have the same values as for wrought-iron hooks.

An analysis of commercial hooks, crane or hoist, will show these seemingly excessive tensile stresses C. These, however, are the extreme fiber stresses. The adjoining fibers, and those nearer the neutral axis, are subject to but little stress in comparison, and the conclusion drawn is that the extreme fibers are assisted in sustaining their maximum stresses by the large mass of adjoining fibers subject to lower stresses, and also that any slight bending of the curved beam results in some readjustment of the forces which tend to reduce the moments below the calculated values.

Tests of hooks made by the writer show that there is a very considerable reserve of strength in the hook after the load repre-

senting the elastic limit stresses of the extreme fibers as calculated by Equation [9] has been passed, and the maximum values of C given above are based on loads that give a factor of 2 on the elastic limit load capacities of hooks as determined by measuring the opening of the hook under loading.

A. K. Pedersen[1] in an article on the design of eye-bolts and lifting eyes, using Equation [9], noted the seemingly excessive values of C and accordingly proposed a "correction factor" of 2.20, so that the theory and practice could be reconciled. Compared to values of C above, this factor gives $30{,}000/2.2 = 13{,}650$ lb. per sq. in., a more reasonable fiber stress. It is, of course, a rather quieting artifice for those skeptical of higher values, but the results are identical.

An analysis of chain links by Equation [9] leads to the same high values of C even for loads of $W = 10{,}000d^2$.

Formulas [9], [10] and [11] have been used by the writer for a number of years in the designing of hooks, rings, eye-bolts, links and small open frames for plate lifting clamps, all of these being of steel.

As to their adaptability to large press and punch frames of cast iron, Equation [9] should be applicable, but lacking experience in the design of such members, some hesitancy is felt in recommending values of C for this purpose.

S. Timoshenko.[2] The purpose of this paper has apparently been to compare the stresses as obtained by experiment in curved bars with those computed on the basis of the usual theory of curved bars as developed by Winkler. It seems, however, that the authors of the paper are not aware that for the case of curved bars of rectangular cross-section an exact solution of the problem exists, which is based on the integration of differential equations of the theory of elasticity. This exact solution was obtained by Prof. H. Golovin as long ago as 1880.[3] It was published in Russian and remained unknown in other countries. At a later date, however, the same problem was solved by Prof. L. Prandtl of Göttingen, and this solution can now be found in the various books on the theory of elasticity.[4] In the case of pure bending this exact

[1] *American Machinist*, May 18, 1911.

[2] Research Engineer, Westinghouse Elec. & Manufacturing Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[3] See Bulletin of the Technological Inst. in St. Petersburg, 1880.

[4] See Encyklopädie der Math. Wissenschaften, vol. IV, 2, II, page 340. A. Föppl and L. Föppl, Drang und Zwang, vol. I (1924), page 303. S. Timoshenko, Theory of Elasticity, vol. I (1914), page 110.

solution yields the following expressions for the circumferential and radial stresses:

$$f_t = \frac{4M}{N}\left(R_2^2 - R_1^2 - R_1^2 \log \frac{R}{R_1} - R_2^2 \log \frac{R_2}{R} - \frac{R_1^2 R_2^2}{R^2} \log \frac{R_2}{R_1}\right) \quad . \; . \; . \; [16]$$

$$f_r = \frac{4M}{N}\left(-R_1^2 \log \frac{R}{R_1} - R_2^2 \log \frac{R_2}{R} + \frac{R_1^2 R_2^2}{R^2} \log \frac{R_2}{R_1}\right) \quad . \; [17]$$

in which

f_t, f_r = circumferential and radial stresses, respectively
M = bending moment per unit width of the cross-section
R_1, R_2 = inner and outer radii of the curved bar
R = an intermediate radius
$N = (R_2^2 - R_1^2)^2 - 4R_1^2 R_2^2 \left(\log \frac{R_2}{R_1}\right)^2.$

The comparison of expression [16] with Winkler's solution shows that although the error of the approximate formula increases with increase in the ratio R_2/R_1, it remains always very small. For instance, in the case $R_2/R_1 = 3$ the maximum stress as obtained from the exact Formula [16] will be about 6 per cent lower than the same stress as given by the usual Winkler's formula.

It should be also noted that the exact value of the ratio $f_{t\,min}/f_{t\,max}$ will be larger than that given by the approximate formula. For instance, for the specimen A used by the authors, this ratio will be equal to 0.466 instead of 0.450 as given by the approximate formula. Due to the discrepancy between the exact Formula [16] and Winkler's approximate formula for circumferential stresses there will be also a difference between the radial stresses. Correct values for these latter stresses will be somewhat less than as calculated in the paper. The exact solution of the problem shows also that in the case of pure bending, the cross-sections of the curved bar remain plane during bending, as Winkler assumed in developing his approximate theory. The error of this theory is due to the fact that the radial stresses were neglected. The exact theory mentioned above shows that the effect of these stresses is small and that Winkler's theory is accurate enough for practical application.

The authors claim that by using the principle of least work "a more logical fundamental hypothesis is advanced in support of the theory adopted," but in their development the radial stresses are neglected, which makes their result as approximate as that obtained by Winkler.

In order to get satisfactory results, both circumferential and radial stresses must be taken into consideration and this can be

done without difficulty. In order to satisfy the differential equation of equilibrium, these two stresses must be taken for pure bending in the following form.[1]

$$f_t = \frac{\partial^2 \phi}{\partial R^2}; \quad f_r = \frac{1}{R}\frac{\partial \phi}{\partial R} \quad . \quad . \quad . \quad . \quad . \quad . \quad [18]$$

in which ϕ denotes an arbitrary function of R. Then the expression for potential energy will be[2]

$$V = \frac{1}{2E}\int_{R_1}^{R_2} (f_t^2+f_r^2+2mf_rf_t)RdR$$

$$= \frac{1}{2E}\int_{R_1}^{R_2} \left[\left(\frac{\partial^2\phi}{\partial R^2}\right)^2 + \left(\frac{1}{R}\frac{\partial \phi}{\partial R}\right)^2 + 2m\frac{\partial^2\phi}{\partial R^2}\frac{1}{R}\frac{\partial\phi}{\partial R}\right]RdR$$

in which m denotes Poisson's ratio.

The condition that the first variation of expression V is equal to zero gives us the following equation:

$$\frac{\partial^4\phi}{\partial R^4} + \frac{2}{R}\frac{\partial^3\phi}{\partial R^3} - \frac{1}{R^2}\frac{\partial^2\phi}{\partial R^2} + \frac{1}{R^3}\frac{\partial\phi}{\partial R} = 0 \quad . \quad . \quad [19]$$

This is the equation from which the exact solution given above (Equations [16] and [17]) can easily be obtained.[3]

Referring to the experimental part of the work we do not agree with the authors that "previous laboratory tests have been, in general, of meager scope and uncertain interpretation." It is true that by observing the apparent proportional limit of curved specimens in bending, only a very rough comparison of test with theory can be obtained; but there have been tests in which very careful measurements of strain during bending were made with much more accurate measuring instruments[4] than the strain gage used by the authors.

The error of the strain gage used is, as stated in the paper, about 0.000015 in. per in., and with this instrument such small radial strains as 0.000025 to 0.000119 in bending tests, and such lateral contractions as 0.0000546 in tensile tests were measured. It is clearly seen that not only third but also second and sometimes the first figures in these readings can be affected by errors in measurements. For instance, Poisson's ratio, given as equal to 0.255, can be also equal to 0.29 or 0.30 as it is usually taken for mild steel and as it has been obtained by more careful experiments.[5]

[1] *Ibid.*

[2] The angle between the end cross-sections of the curved bar is taken equal to unity.

[3] These equations can be found in books mentioned in footnote 4, p. 667.

[4] See Preuss, *Zeit. des Vereines deutscher Ingenieure,* 1911, or Mitteilungen über Forschungsarbeiten, no. 126.

[5] See paper by Williams, *Phil. Mag.,* 1912.

The photoelastic method mentioned by the authors at the end of their paper is a very useful method in studying stress distribution and it has been applied many times in testing curved bars.[1] The results obtained, not only for maximum stress but also for stresses in intermediate points, have given very good agreement with the theory.

DIRK DEKKER.[2] The close agreement of results obtained analytically with those obtained experimentally proves the worth of the authors' work. We should be interested to see this borne out for various cross-sections of beam. In Fig. 4 the unit stress C on area wdr might be better presented as shown in Fig. 10, thereby indicating what the authors express in Par. 8, ninth line from bottom of page 656 and fitting in more logically with the last three lines of the same page.

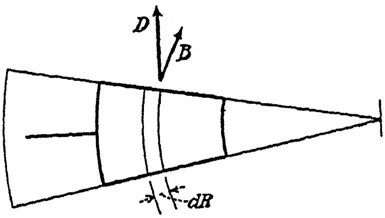


FIG 10

F. N. MENEFEE.[3] Any serious attempt to develop formulas for the stresses that come upon engineering structures today, without the use of the strain gage or some equivalent device, is in most cases open to the criticism of being one-sided. In the development of our elastic theory, from time to time, we have made certain assumptions which may have been accurate enough for the original case at hand, but which later have found their way into places where they were positively dangerous. Strain gages, photoelastic, or other methods for accurately determining strain or, at least, relative deformation, are being developed so rapidly now that one is hardly justified in attempting to develop stress formulas without the check of the gage, or even in preceding the development of formulas by determining the strain.

The investigation of the authors is a good illustration of this point, and should be followed by others on other types of bent beams and on planes differing from the one they chose, for it may

[1] See, for instance, O. Honigsberg, *Zeit. des Öst. Arch. u. Ing. Ver.*, vol. 58 (1906) p. 489. Prof. E. G. Coker, *Engineering*, 1911, p. 565. I. Aue, Zur Berechnung der Spannungen in gekrümmten Stäben unter Anwendung der optischen Methode, Dissertation, 1910, Dresden.

[2] Assistant Professor of Mechanical Engineering, Montana State College, Bozeman, Mont. Assoc-Mem. A.S.M.E.

[3] University of Michigan, Ann Arbor, Mich.

be that there is a compounding of stress on some diagonal plane, which is greater than on plane $O'O$, Fig. 1.

Little by little, we are reducing our assumptions by more exact information furnished by the strain gage. The assumption that the curved beams in question are isotropic is an incorrect one, but probably is well within the limits of our knowledge of E for the material.

Two very vital factors that engineers often overlook in the development of stress theory are: (*a*) Lack of knowledge of E for the material within five per cent, and (*b*) The impossibility of the assumed conditions to be realized in practice.

Concerning the first, in calibrating a strain gage to be used in determining the strains in an important bridge, a report, " accurate to within one-half of one per cent," caused great anxiety until a conference could be arranged and the client reminded of the use of the value of E, which he was assuming to be 30,000,000.

Of the second factor, the writer has recently conducted tests on columns and beams in a structure designed and built to support a generator, condenser, and steam turbine in a large steam plant. Measurements were made just after erection, after the concrete mat was laid, and again after the units were set. A complete study of the data leads to the conviction that not a beam nor a column is stressed in the way and amount that the designer intended. The writer feels that this condition in our structural field is fairly general, and probably arises from the fact that we make assumptions regarding the application of load which never can be realized. Too often we forget that the reaction at the end of a beam is a load, and that the end connections have as important a part in the stress functioning of the beam as the way in which the live load is placed.

Referring to the authors' procedure, it is not quite clear as to how the load W was obtained from the channels, unless they were fulcrumed against each other by a strut under or over the specimen. The care taken with the gage holes is essential to this kind of work, and sometimes temperature corrections should be made where the temperature varies between readings. On important work it has been found that unless great care is used, the gage is affected by the temperature of the hands and body of the experimenter.

There is little question in the writer's mind regarding the stress which the authors call " transverse radial." In a qualitative way it can be shown on an ordinary rubber eraser, by sticking a pin through each end near one edge and exerting a longitudinal pull on the eraser. That portion of the eraser on a line between the pins will elongate, and by Poisson's ratio for rubber, will diminish in cross-section. It is comparatively free to pull the rubber in from both flat sides and from the edge to which the line joining the pins is nearest. But such is not the case with the material between the line of pull and the far edge of the eraser. That material, if

pulled in, causes the eraser to bend and may even be in compression, as the O' portion of the authors' curved beams were. When in compression, that material in the edge of the eraser farthest from the line of pull is, by Poisson's ratio, being pushed farther away from the line of pull. Thus the tendency to bring all outer surfaces closer together, which in an axial pull meets with no resistance and sets up no conjugate stress, is counteracted, and a tension is set up. Of course, it is zero at each surface.

Another qualitative way of analyzing, which is probably more direct, is to draw the ordinary stress distribution diagram for a plane perpendicular to the authors' plane $O'O$. In Fig. 11 call it OP.

Picking a particle, as shown, and removing the same from the figure as at b gives us an opportunity to see what forces are acting to keep the particle in equilibrium. Taking off a small corner of the particle as shown at c, and inserting the force on the diagonal face that is necessary to put it in equilibrium, shows that on the

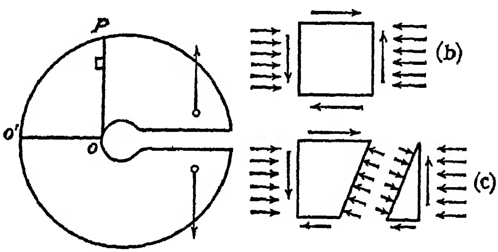


FIG. 11

diagonal we must have compression. The small triangular block is pushing to the left and upward. A similar force to the left and downward is set up below $O'O$. The result is some vertical and some radial or horizontal tension, just as was indicated by the gage.

Taking the authors' equations, it seems they are set up correctly, assuming, as they do, that there is no shear. Now of course there is shear. There is always shear in a beam and as the beam gets shorter, that theory regarding stresses which takes it into account will be the more accurate. It is due to shear that the results depart from a rule based on "planes before bending remain planes after bending." There is a vertical shear in the case and hence there must be horizontal shear, and with such short cantilever beams that shear is probably sufficient to produce the distortions which the authors have found and the stresses which accompany them.

In most cases the shear stresses would not be great enough to cause failure by that method, and hence it is thought that the development of formulas which give a more accurate measure of the tension and compression is important. The writer believes,

however, that a cast-iron curved beam of the same shape as used by the authors might fail by tension on a diagonal line running from O to the second or third quadrant of the specimens as shown in Fig. 1. If such were the case it would indicate that the plane of greatest tension was not $O'O$, but some other plane upon which shear increased the tension by compounding with some tension on a plane at right angles to its own direction.

This work of the authors is to be commended. It is not claimed by them that their equations will apply to all types of hooks, or curved beams, nor does it seem to the writer that such claims would be justified at this time. More tests and more measurements of strain on diagonals as well as planes of symmetry, and also similar tests on commercial hooks will be very valuable.

A. L. Kimball.[1] The matter of radial or transverse stresses brought out by the authors is very nicely shown in the photoelastic method of analysis. In fact, this problem is one which lends itself beautifully to that method of analysis. In such experience with that method as the writer has had, an interesting feature has been the presence of the transverse stresses mentioned by the authors where they would not always be expected. For accurate analysis of complex shapes or irregular shapes account should be taken of such stresses. The problem as given by the authors should be checked by the photoelastic method. It is not meant that the results as stated are not correct, but that the photoelastic method should be used merely as a matter of getting an additional check. We have had one or two cases in which accurate checks have been obtained in photoelastic analysis by the extensometer method, which would also give another means of insuring accuracy.

The Authors. Mr. Meals has contributed useful data on working stresses for crane hooks and has also stated several of the various forms in which Winkler's solution is given by different authors. As noted, by simple algebraic transformations each one of these solutions can be shown to be identical with Equation [8]. To assist any reader in deriving this result, it may be worth while to point out that the assumed plus direction of moments is opposite to that used in Par. 10. Also the integrals giving values of the constants K and A' are integrated in a direction to give positive numerical results.

On several phases of the work of this paper Dr. Timoshenko has submitted discussions, which require definite replies. Much emphasis is placed on Golovin's solution, which was not overlooked by the authors, as Footnote 2 in Par. 8 refers directly to the complete derivations of Equations [16] and [17]. At the beginning

[1] Research Engineer, General Electric Co., Schenectady, N. Y. Assoc. A.S.M.E.

of the present tests, the authors had seen a statement of Golovin's formula for circumferential stress, together with the information that the formula was limited to bars of rectangular cross-section, but no further features of importance were ascribed to it. Incidentally this explains why at first with specimen B no special provisions were made for measuring radial strains. In fact, it was not until after all the tests and computations comprised in this paper were finished that discovery was made, in Dr. Föppl's Technische Mechanik, of the complete derivations of Golovin's results, giving exact solutions for both radial and circumferential stress.[1] These derivations were subjected to careful scrutiny, leading to the conclusion that Golovin's formulas were so exceedingly limited in application that it was appropriate to cover them simply by a footnote reference without detracting from the continuity of the paper by extraneous discussion. The comments by Dr. Timoshenko, however, now make it necessary to point out the limitations of these formulas. This can be done by giving their derivation in brief outline, together with a more complete statement of the authors' solution by successive substitutions, which apparently was overlooked or misunderstood by Dr. Timoshenko.

Imposing the same limiting conditions as used by Golovin, Equations [16] and [17] can be derived readily from the authors' equations of least work. For bending alone without direct stress, using the notations of Par. 10, let M denote the bending moment per inch of width, make $w = 1$ in., and $C = B$. Equation [2] is unchanged; in Equation [3], $W = 0$; and Equation [4] becomes

$$\int_{R_1}^{R_2} CR\partial R - M = 0 \quad . \; . \; . \; . \; . \; . \; . \quad [4M]$$

For least work, taking complete account of radial stress,

$$\frac{\partial\theta}{2E}\int_{R_1}^{R_2}(C^2+T^2-2mCT)R\partial R = \text{minimum} \quad . \; . \; . \quad [5M]$$

It should be noted that the term containing m is negative, and not positive as shown by Dr. Timoshenko. This error did not affect Dr. Timoshenko's final result, as the integral of the term containing m may be shown equal to zero. Proceeding as in Par. 10, the equation corresponding to [6] is

$$\int_{R_1}^{R_2} (2K_3C+2K_4CR+C^2R+T^2R-2mCTR)\partial R$$
$$= \text{minimum} \quad . \; . \; . \; . \quad [6M]$$

Putting the derivative with respect to B equal to zero,

$$K_3+K_4R+CR+\int^{R_2} T\partial R - m\int_{R}^{R_2} C\partial R - mTR = 0 \quad . \quad [20]$$

[1] August Föppl, Technische Mechanik, 1907, vol. 5, p. 75.

For further reference the symbol C has been retained in these equations. In this solution, however, $C = B$. By differentiating Equation [20] with respect to R the integrals are removed, and the differentiation of Equation [2] with respect to R gives a value of B in terms of T. Substitution of this value of B results in the following differential equation in T alone:

$$\frac{\partial^2 T}{\partial R^2} + \frac{3\partial T}{R\partial R} + \frac{K_4}{R^2} = 0$$

Integration of this gives

$$T = -\frac{K_4}{2} \log_e R + \frac{K_5}{R^2} + K_6 \quad . \quad . \quad . \quad . \quad . \quad . \quad [21]$$

From Equation [2],

$$B = -\frac{K_4}{2} (\log_e R+1) - \frac{K_5}{R^2} + K_6 \quad . \quad . \quad . \quad . \quad . \quad [22]$$

By substitution of Equations [21] and [22] in [3], [4M] and the derivative of [20], solution for the constants K_4, K_5 and K_6 is obtained, giving values of B and T identical with Golovin's Equations [16] and [17].

Actual conditions, in which direct stress occurs together with bending, lead to no such complete and simple results. Making $w = 1$ in. in Equations [3], [4], [5M] and [6M], and putting the derivative of Equation [6M] with respect to B equal to zero gives an equation identical in form with Equation [20]. In this equation, however, $C = B+D$, complicating the solution. Immediate partial solution for C gives

$$C = -\frac{K_3}{R} - K_4 + \frac{1}{R} \psi(C) \quad . \quad . \quad . \quad . \quad . \quad . \quad [23]$$

in which $\psi(C)$ denotes terms involving C. By substituting Equation [23] in Equations [3] and [4], values of K_3 and K_4 may be obtained, which when inserted in Equation [23] give the equation referred to as [8′] in Par. 10. As there explained, by using successive values of C in $\psi(C)$ a numerical solution for C may be obtained. In deriving Equation [23] shearing stresses were disregarded. Plane OO', Fig. 1, is a plane of principal stress on which the shear is zero. On radial planes closely adjacent to OO' it seems that the shearing stresses will be small and that slight approximation will result from neglecting shear. With this exception, Equation [23] is an exact relation for circumferential stress. For specimen A, numerical application of Equation [23] was made at each instrument station. Results of the second approximation, in lb. per sq. in., at $R = 3.09$ in. and $R = 9.83$ in. are $+17{,}200$ and -8080, while Equation [8] gives $+17{,}220$ and -7790. The results show almost exact agreement for the maximum tensile stresses, while the precise Equation [23] gives slightly higher values for the com-

pressive stresses. In this manner numerical computations, taking complete account of radial stresses, were carried out to determine the percentage of approximation in Winkler's Equation [8].

Referring to the two preceding paragraphs, the limitations of Golovin's solution may now be pointed out. In order to obtain a complete mathematical solution, Golovin assumed the condition of pure bending. The word " never " may be challenged, but a curved beam subjected to pure bending without direct stress is at least very rarely encountered in practice. The question now arises whether this exact solution derived for the condition of pure bending applies also to the usual condition of bending combined with direct stress. This is a matter requiring investigation and proof, and it appears to depend primarily on the question whether, in compliance with the theory of least work, the direct stress D is constant throughout the cross-section. If D were constant, Equation [**23**] could be integrated, giving results identical with Equations [**16**] and [**17**], except that [**16**] would contain an additional term W/A representing direct stress. Mathematical investigations to determine whether for least work D is constant have resulted in expressions involving inconvenient complexity with the other variables, and success has not been achieved in proving that under the exact theory of elasticity D is uniformly distributed over the cross-section. Therefore, as the matter stands, Golovin's exact solution appears to be mainly of mathematical interest, practically without application to actual cases of curved beams. While it may be argued that the approximation obtained by Golovin's solution is small, the same may be said for Winkler's formula; for example, with pure bending, and $R_2/R_1 = 3$, the two formulas give a difference in maximum stress of only about 0.6 per cent, according to the authors' computations, instead of 6 per cent as noted by Dr. Timoshenko. As Golovin's solution is also limited completely to rectangular cross-sections, it seemed appropriate to refer to it only by footnote, and to devote more complete consideration to Winkler's formula, which has general application to various shapes of cross-sections. Furthermore, it should be noted that with Equation [**23**], giving the theoretical percentage of approximation of Winkler's formula, application is not limited to rectangular cross-sections. In deriving Equation [**23**], w may be considered variable, giving a similar general solution applicable to certain other shapes of cross-sections for which the required integration involved in $\psi(C)$ is possible.

As superior in accuracy to the present tests, Dr. Timoshenko has cited a single example, the work of E. Preuss. These tests, which had not previously come to the attention of the authors, consisted of measurements of circumferential strains at several radial positions on two crane hooks of 5-tons and 10-tons capacity. While the authors have no desire to disparage these interesting and important tests, a statement should be made regarding the pre-

cision attained. Four sets of measurements were taken; for the value of maximum tension one test showed a difference between test and theory of 5.5 per cent, the other three tests showed differences varying from 25.1 to 33 7 per cent. For the maximum compression the difference between test and theory varied from 14.2 to 22.2 per cent. Incidentally Preuss pointed out the uncertainty of his numerical results obtained by graphics, a difficulty already noted in Par. 1 and avoided in the specimens used by the authors. In view of the large variations from theoretical values obtained by Preuss, it seems unnecessary to discuss the accuracy of the strain gage, a mirror extensometer of modified Martens type used on an interval of 10 mm. In general, the experience of the authors has been that, with a strain gage of this arrangement, the pressure necessary to clamp the instrument in place is a very important source of difficulty in obtaining security against displacement and at the same time avoiding excessive friction. With the strain gage used by the authors several adjustable spring clamping devices were tried and abandoned.

In spite of comments at the end of Par. 5 regarding the unimportant effect of a small change in the value of Poisson's ratio, Dr. Timoshenko has criticised the magnitude of the measurements used for lateral strain. This criticism is the result of a misunderstanding due to the fact that the log of these tests was not published in full. For determining Poisson's ratio in bar *A*, the original log shows that the loads used varied slightly from a minimum of about 3000 lb. to a maximum of about 31,000 lb., giving a load increment of about 28,000 lb. To obtain comparable results, each reading was reduced to the equivalent value at a load of 10,000 lb., or 6420 lb. per sq. in. Without thought of creating false impressions, these reduced results were published, and the misapprehension caused thereby is regretted by the authors. The actual loads were nearly three times 6420 lb. per sq. in., also the strain interval in this case was 2 in., giving a precision practically six times as good as noted by Dr. Timoshenko. The one footnote reference to determinations of Poisson's ratio, by I. Williams, is mainly a description of mirror apparatus used to measure the change of angle between the two vertical sides of a small steel bar subjected to bending. Only a single specimen was used, giving an average value of 0.292 for Poisson's ratio. While no special study of Poisson's ratio had been made by the authors, their impression was that recent results were considerably lower than 0.3 and not far from the values obtained in the present tests. For example, in 1924 Professor T. M. Jasper at the University of Illinois published the results of tests on six specimens.[1] Values of Poisson's ratio were determined both by direct measurements of lateral

[1] T. M. Jasper, Proc. Amer. Soc. for Testing Materials, vol. 24, p. 1012.

strain and by comparisons of elastic moduli in torsion and tension, the results averaging close to 0.25.

Replying to Dr. Timoshenko's comments on the magnitude of the measurements of circumferential and radial strains, it should be noted that the minimum interpolated interval on the gage dial was only moderately small so that definite readings could be observed quickly without fatigue. In preliminary work a smaller interpolated interval was used at first, but later consideration led to the adoption of the larger interval, giving definite readings rapidly in numerical terms readily subtracted, compared and checked, and at the same time giving values small enough in proportion to the maximum strains to furnish sufficiently precise comparisons between test results and theory.

Regarding previous photoelastic investigations, there seems to be some discrepancy in the reference to the paper by Professor Coker in *Engineering,* 1911, as the discussion following page 565 relates to stress concentrations at holes and fillets, and does not pertain to curved bars. The other two references to photoelastic work are at present inaccessible to the authors, but it is hoped that their bearing on this paper requires no special discussion.

Fig. 10, submitted by Professor Dekker, gives a correct interpretation of Fig. 4. Apparently in Fig. 4 the conciseness of notation has caused some uncertainty. It is stated that the angle $\partial\theta$ is infinitesimal, in which case the lines of action of the forces B and D coincide.

The discussion by Professor Menefee considers stresses in portions of the curved beams not investigated by the authors. Information relative to these stresses could be obtained by the photoelastic method, and present plans of the authors contemplate these more comprehensive photoelastic investigations. This statement, together with the results noted in Par. 13, also constitutes a reply to the discussion by Mr. Kimball.

In conclusion, it seems desirable to mention one result of the present tests which has not yet been noted. Several previous authors, in discussing curved beams, have referred to the possible effect on stress distribution caused by the proximity of the applied load. No appreciable evidence of this effect is shown by the test results for specimen B, which is a curved beam of sharp curvature and short load leverage.

No. 2021

STRESSES AND DEFLECTIONS IN LARGE DYNAMO FRAMES DUE TO DEAD LOAD

By M. Stone,[1] East Pittsburgh, Pa.
Non-Member

Because of the continued tendency to enlarge the sizes of electrical rotating machinery, and necessarily the dynamo frames themselves, some thought has been given to the mechanics involved. The analysis given in this paper is the result of an investigation made by the Westinghouse Electric and Manufacturing Company. Two assumptions are made as to frame support: (1) *simply supported, and* (2) *built-in. The actual condition is probably between the two, and measurements are being made to determine the relative accuracy of the suppositions. However, since the first assumption imposes the more severe condition, size of air-gaps, and so forth, should be designed according to these results.*

The solution is obtained by two methods: Lord Rayleigh's theory, utilizing Fourier's series to give the final distribution of bending moments and deformations; and the theorem of Castigliano, which is simple and easy to use but which gives less general results. The compatibility of the two is established.

Tables and equations are given in the paper which will permit the accomplishment of satisfactory designs of machines much lighter than now in use. No analysis is made of the effect of unbalanced magnetic pull, all such effects being under the control of the designer in fixing his allowable air-gap variation.

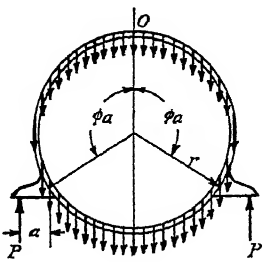
In the continued tendency to enlarge the sizes of rotating machinery, the question of mechanical design has often been the determining consideration. In the rotating parts themselves, the increased size always introduces higher centrifugal stresses, greater bearing pressures, etc., but with the constantly accruing knowledge of the behavior of materials under various conditions of load, temperature, continuance of load application, and so forth, we have been constantly raising our allowable limits in these instances.

[1] Research Laboratory, Westinghouse Electric and Manufacturing Company.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

2 In the case of the stationary parts of the machines, particularly the stators, comparatively little difficulty has been presented. As the machines became larger, in order to keep the stresses and deformations of a low order, it was only necessary to make the frames large and heavy. This type of construction involved the making of very heavy machines and necessitated, due to transportation limitations, their being made in two parts. These factors have increased the cost of the machines, involving in addition some loss of time.

3 With the idea of reducing the weight of these machines and enabling more of them to be transported in the assembled state,



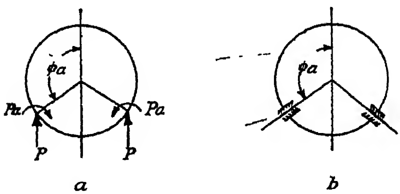


FIG. 1

while still retaining sufficient material to hold a uniform air-gap, some analysis of the mechanics involved for the designer's use is given in this paper.

4 The discussion is divided so that the main body contains tables of maximum deflections, maximum normal forces, and maximum bending moments. An example is also worked out for a proposed 75,000-kw. turbo-generator frame. The analysis itself appears in the form of two appendices.

Results

5 Skeletonizing the problem, as shown in Fig. 1, we attach the symbols as designated. The analysis has been made on two assumptions as to condition of support: (1) that the frame can be considered freely supported, which means that all cross-sections

are free to rotate during bending (see Fig. 1a), and (2) that the other extreme exists—the frame is built in at the points defined by angles ϕ_a, as in Fig. 1b.

6 Following the first assumption, Fig. 2 defines the positive directions of M_0 and S_0, where they represent the bending moment and normal reaction at the top of the frame, respectively. The shear force is always zero at this point, due to symmetry conditions. The moment arm of the supporting force P at the supporting section is a. Now, if K is the weight per inch of circumference on the center line of the frame, values of S_0 and M_0 can be written in the following forms:

$$S_0 = \alpha K r, \qquad M_0 = \beta K r^2$$

where α and β are different for various values of ϕ_a, the angle at the point of support. The actual expressions for α and β are given below:

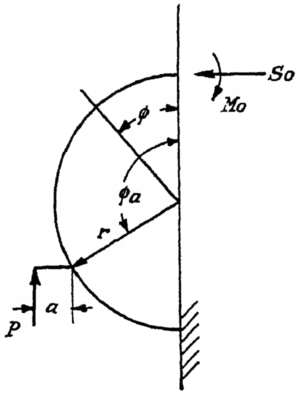


FIG. 2

$$\alpha = 2\frac{a}{r}\sin\phi_a + \sin^2\phi_a - \tfrac{1}{2} \quad \ldots \ldots \ldots \quad [1]$$

$$\beta = \pi\frac{a}{r} - \alpha - 1 + \pi\sin\phi_a - \frac{a}{r}\phi_a - \phi_a\sin\phi_a - \cos\phi_a \quad \ldots \quad [2]$$

Values are given in Table 1 for two extreme values of a, $a = 0$ and $a = r/3$; and for intermediate values of a, direct linear interpolation is sufficiently accurate.

TABLE 1

ϕ_a deg.	$\alpha\left(\frac{a}{r}=0\right)$	$\alpha\left(\frac{a}{r}=\frac{1}{3}\right)$	$\beta\left(\frac{a}{r}=0\right)$	$\beta\left(\frac{a}{r}=\frac{1}{3}\right)$
0	−0 500	−0.500	−1.500	−0.454
20	−0.383	−0.155	−0.602	+0.101
40	−0.087	+0.341	−0.109	+0.277
60	+0.250	+0.833	+0.065	+0.180
80	+0.470	+1.126	+0.076	−0.009
100	+0.470	+1.126	+0.079	−0.106
120	+0.250	+0.833	+0.160	−0.073
140	−0.087	+0.341	+0.310	+0.114
160	−0.383	−0.155	+0.443	+0.332
180	−0.500	−0.500	+0.500	+0.500

7 In order to evaluate the bending moment at any section (in Fig. 2) in the region $0<\phi<\phi_a$, we have the expression

$$M_1 = Kr^2[\phi\sin\phi - 1 + \cos\phi - \beta - \alpha(1-\cos\phi)] \quad \ldots \quad [3]$$

while for $\phi_a<\phi<\pi$ it becomes

$$M_2 = Kr^2\left[\phi \sin \phi - 1 + \cos \phi - \beta - \alpha(1-\cos\phi) + \pi\frac{a}{r} + \pi(\sin\phi_a - \sin\phi)\right] \quad [4]$$

8 Now we can give a table of changes of the vertical and horizontal diameters of the frame, δ and Δ, respectively. These, of course, are the variations in air gap that the machine will undergo, it being necessary to make $(\delta+\Delta)/2$ of such a value as not to exceed a predetermined design value. The general expressions are

$$\delta = \frac{Kr^4}{EI}\left[\frac{\pi}{2}(\phi_a + \sin\phi_a\cos\phi_a) + \pi\frac{a}{r} + \pi\sin\phi_a - 2(\alpha+\beta+1) - \frac{\pi^2}{4} + \pi\frac{a}{r}\cos\phi_a\right] \quad [5]$$

Now for Δ, we must write two expressions, dependent on the value of ϕ_a. For $0<\phi_a<\dfrac{\pi}{2}$, we have

$$\Delta = -\frac{2Kr^4}{EI}\left[\alpha\left(\frac{\pi+4}{4}\right) + \beta + 1 + \frac{3\pi}{8} - \pi\frac{a}{r} - \pi\sin\phi_a\right] \quad [6]$$

and for $\dfrac{\pi}{2}<\phi_a<\pi$

$$\Delta = -\frac{2Kr^4}{EI}\left[\alpha\left(\frac{\pi+4}{4}\right) + \beta + 1 - \frac{\pi}{8} - \frac{\pi a}{r}\sin\phi_a - \frac{\pi}{2}\sin^2\phi_a\right] \quad [7]$$

where I = moment of inertia of section of frame
E = modulus of elasticity.

For purposes of tabulation, let

γ = polynomial factor in Equation [5]
and λ = polynomial factor in Equations [6] and [7]

thus giving

$$\delta = \gamma\frac{Kr^4}{EI} \qquad \text{and} \qquad \Delta = \lambda\frac{Kr^4}{EI}$$

9 The tables and equations given heretofore have been based on an analysis using mainly the Castigliano theorem (see Appendix No. 1). A like analysis made according to the Rayleigh method of attack (see Appendix No. 2) yields the same tabulated data, but the general expressions appear in a different form. The latter method utilizes Fourier's series to express the distribution of moments and deformations, and yields more general expressions, so that we are able to obtain the radial deflections of the frame at any point instead of only along the vertical and horizontal diameters. If we have the case shown in Fig. 3 (where $a = 0$)

letting w = radial displacement at any angle ϕ, then Rayleigh's method leads to

$$w = \frac{2Kr^4}{EI} \sum_{2, 3, 4, \text{etc.}}^{\infty} \frac{(n \cos \phi_a \cos n\phi_a + \sin \phi_a \sin n\phi_a)(\cos n\phi)}{n(n^2-1)^2} \quad [8]$$

10 This value represents the displacement of any point relative to the center of the frame as considered fixed. But since we are only interested in relative displacements of the various parts of the frame, we proceed as follows: To get the total change of the vertical diameter, we must add the w's for $\phi = 0$ and $\phi = \pi$. Similarly one can get the change of the horizontal diameter by adding the values of w for $\phi = \pi/2$ and $\phi = 3\pi/2$. So

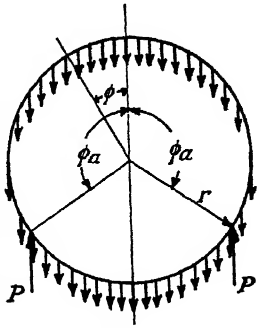


FIG. 3

$$\delta = \frac{2Kr^4}{EI} \sum_{2, 3, 4, \text{etc.}}^{\infty} \frac{(n \cos \phi_a \cos n\phi_a + \sin \phi_a \sin n\phi_a)(1 + \cos n\pi)}{n(n^2-1)^2}$$

and

$$\Delta = \frac{4Kr^4}{EI} \sum_{2, 4, 6, \text{etc.}}^{\infty} \frac{(n \cos \phi_a \cos n\phi_a + \sin \phi_a \sin n\phi_a)(\cos n\pi/2)}{n(n^2-1)^2}$$

11 These various series are rapidly convergent, two or three terms sufficing for computation purposes. Other cases are solved by use of this method and appear in Appendix No. 2.

The tabulation of γ and λ for various values of ϕ_a is found in Table 2.

TABLE 2

ϕ_a deg.	$\gamma\left(\frac{a}{r}=0\right)$	$\gamma\left(\frac{a}{r}=\frac{1}{3}\right)$	$\lambda\left(\frac{a}{r}=0\right)$	$\lambda\left(\frac{a}{r}=\frac{1}{3}\right)$
0	−0 468	−0.468	+0.430	+0.430
20	−0.373	−0.205	+0.362	+0.238
40	−0.196	+0 024	+0.210	+0.004
60	−0.053	+0.171	+0.060	−0.160
80	−0.005	+0.080	+0.002	−0.080
100	+0.002	−0.088	−0.002	+0.086
120	+0 046	−0.143	−0.070	+0.124
140	+0.165	−0.054	−0.226	−0.038
160	+0.366	+0.215	−0.366	−0.240
180	+0.468	+0.468	−0.430	−0.430

— elongation of diameter.
+ contraction of diameter.

12 The other case of assumed frame support — built-in at the supports — can be solved by both methods. If the set-up is given

by Fig. 4, we are able to tabulate, as in the first case, coefficients indicating the values of S_0 and M_0 for all angles ϕ_a. Letting

$$S_0 = \varepsilon K r$$
$$M_0 = \rho K r^2$$

and

$$\delta = \nu \frac{Kr^4}{EI}$$

where δ is the deflection of the top point relative to the supports, we have for these various coefficients the following equations:

$$\varepsilon = -\left[\frac{7\phi_a \sin\phi_a \cos\phi_a - 8\sin^2\phi_a + 2\phi_a^2 - \phi_a^2\cos 2\phi_a}{2\phi_a \sin\phi_a \cos\phi_a - 4\sin^2\phi_a + 2\phi_a^2}\right] \quad . \ [9]$$

$$\rho = \varepsilon\left(\frac{\sin\phi_a}{\phi_a} - 1\right) + 2\frac{\sin\phi_a}{\phi_a} - 1 - \cos\phi_a \quad . \ . \ . \ . \ . \ . \ . \ . \ [10]$$

$$\nu = (\varepsilon+\rho+1)(\cos\phi_a - 1) + \frac{(1+\varepsilon)}{2}(\sin^2\phi_a) + \tfrac{1}{4}(\sin^2\phi_a - 2\phi_a \sin\phi_a \cos\phi_a + \phi_a^2) \quad . \ [11]$$

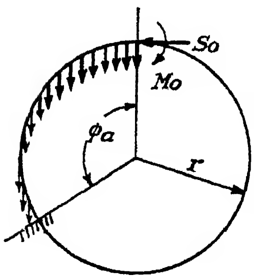


FIG. 4

These are given in Table 3.

13 The expressions given hold for both the upper and lower parts of the frame, where if the angle for the upper part is ϕ_a, it is $(\pi - \phi_a)$ for the lower. The actual change of vertical diameter will be the difference of the two values obtained, because both the top and bottom deflect downward.

14 From these two extreme cases of support assumption, we can get an idea of what is actually taking place in a dynamo frame, where the support conditions are indefinite. However, in order to check this analysis, measurements are being taken on many frames; but this work is not yet complete. Some earlier measurements indicate a leaning toward the built-in condition. In case of indecision, however, it is perhaps more conservative to

TABLE 3

ϕ_a	ϵ	ρ	ν
0°	......	......	......
20	+0.989	−0.000	+0.000
40	+0.979	−0.002	+0.001
60	+0.841	+0.009	+0.002
80	+0.745	+0.017	+0.003
100	+0 553	+0.062	+0.010
120	+0.349	+0.122	+0.037
140	+0.104	+0.216	+0.095
160	−0.180	+0.343	+0.220
180	−0.500	+0.500	+0.468

Positive directions are shown in Fig. 4.

imagine the most unfavorable conditions, and design as if freely supported.

15 An illustration of usage of the above equations and tables is given below. With the drawing and data as given in Fig. 5,

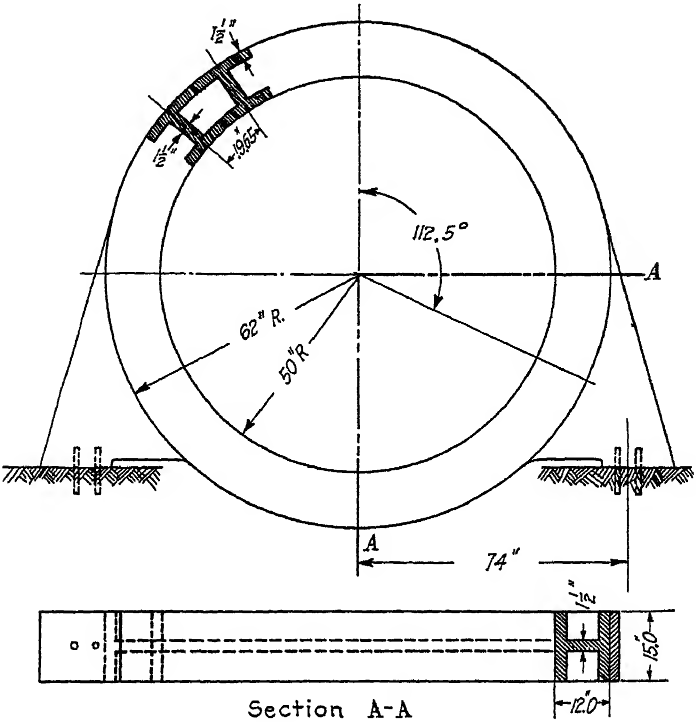


FIG. 5

Spacing of axial ribs = 19.65 in.
Spacing of circumferential ribs = 15.0 in.
Length = 210 in.

Weight of frame = 80,000 lb.
Weight of punchings = 176,000 lb
Weight of copper = 20,000 lb.

we have

$$
\begin{array}{ll}
K = 56 \text{ lb. per in. for each circular rib} & \\
\phi_a = 112.5 \text{ deg.} & \alpha = +0.943 \\
r = 56 \text{ in.} & \beta = -0.085 \\
a = r/3 & \gamma = -0.122 \\
I = 776 \text{ in.}^4 & \lambda = +0.096 \\
E = 25 \times 10^6 \text{ lb. per sq. in.} &
\end{array}
$$

Therefore

$$\delta = -0.122 \left(- \frac{(56)(56)^4}{(25 \times 10^6)(776)} \right) = 0.0035 \text{ in.}$$

$$\Delta = +0.096 \left(- \frac{(56)(56)^4}{(25 \times 10^6)(776)} \right) = 0.0027 \text{ in.}$$

$$M_0 = (-0.085)(56)(56^2) = 14{,}920 \text{ in-lb.}$$
$$S_0 = (0.943)(56)(56) = 2950 \text{ lb.}$$

In order to obtain the highest existent stress, however, we must have the distribution of bending moment. We can construct, therefore, the bending-moment diagram from Equations [3] and [4], giving us the distribution in Fig. 6 In this case, then, the maximum stress is $\frac{142{,}000 \times 6}{776} = 1100$ lb. per sq. in., while the maximum change in air-gap is $\frac{0.0035 + 0.0027}{2} = 0.0031$ in. Neither of these values is excessive.

Conclusions

16 Thus, we have two analytical methods of solving the problem of the deformation of frames by dead load. These methods

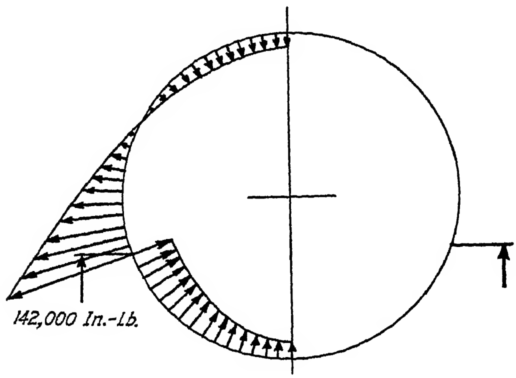


Fig. 6
(1 in. = 200,000 in-lb.)

may be generally applied to other types of problems, but they afford especial ease in solving those pertaining to circular rings, arcs and cylinders under various kinds of loading. In this particular application, it is evident that we are able, by the analysis, to aid materially in the designing of dynamo frames.

APPENDIX NO. 1

APPLICATION OF CASTIGLIANO THEOREM

17 Alberto Castigliano showed[1] that if the internal work of an elastic system is expressed as a homogeneous function of the second order of the external forces, the resulting expression is such that the differential coefficients give the relative displacements of their points of application. This means that if U represents internal work or potential energy of deformation due to forces P_1, P_2, etc., and δ_1, δ_2, etc., are the displacements of points of application of P_1, P_2, etc., then

$$\frac{\partial U}{\partial P_k} = \delta_k$$

The stored energy is usually composed of three parts: (1) energy of bending, (2) energy of direct stress, and (3) energy of shear. In the cases about to be discussed, the values of the last two are very small, of the order of 1 per cent or so of the total energy, and are neglected, only the energy of bending being considered.

18 It can be proved that if M is the bending moment existent in a beam, and can be expressed as a continuous function of the forces between certain limits a and b, then the stored energy in that part of the beam becomes

$$U_{a\text{-}b} = \frac{1}{2EI}\int_a^b M^2 dS$$

or if applied similarly to a circularly curved beam

$$U_{a\text{-}\beta} = \frac{r}{2EI}\int_a^\beta M^2 d\phi$$

where I = moment of inertia of cross-section of beam
E = modulus of elasticity
r = radius of curvature of curved beam.

In applying these relations to our frame, we choose the elastic system as shown in Fig. 7, considering the semi-circular beam to be built-in at the section shown, and applying the unknown reactions at the other end. From symmetry considerations we conclude that no shear force exists at this section and proceed to evaluate the stored energy. In the region $0<\phi<\phi_a$,

$$\text{Bending moment} = M_1 = -M_0 - S_0 r\,(1-\cos\phi) + Kr^2(\phi\sin\phi - 1 + \cos\phi)$$

where K = dead load per inch of circumference. And where $\phi_a<\phi<\pi$

$$M_2 = -M_0 - S_0 r\,(1-\cos\phi) + Kr^2(\phi\sin\phi - 1 + \cos\phi) + K\pi ra + K\pi r^2(\sin\phi_a - \sin\phi)$$

Now if U = stored energy, it is considered composed of two parts, U_1 and U_2, where

$$U_1 = \frac{r}{2EI}\int_0^{\phi_a} M_1^2 d\phi$$

$$U_2 = \frac{r}{2EI}\int_{\phi_a}^{\pi} M_2^2 d\phi$$

[1] Théorème de L'Equilibre des Systèmes Élastiques et ses Applications, Turin, 1879.

19 From the condition that there is no rotation of the top of the frame during deformation, we may extend the statement of Castigliano to give

$$\frac{\partial U}{\partial M_0}=0$$

or

$$\frac{\partial U}{\partial M_0}=0=\frac{r}{2EI}\int_0^{\phi_a}\frac{\partial(M_1^2)}{\partial M_0}\,d\phi+\frac{r}{2EI}\int_{\phi_a}^{\pi}\frac{\partial(M_2^2)}{\partial M_0}\,d\phi$$

$$=\frac{2r}{2EI}\int_0^{\phi_a}M_1\frac{\partial M_1}{\partial M_0}\,d\phi+\frac{2r}{2EI}\int_{\phi_a}^{\pi}M_2\frac{\partial M_2}{\partial M_0}\,d\phi$$

But

$$\frac{\partial M_1}{\partial M_0}=\frac{\partial M_2}{\partial M_0}=-1$$

therefore

$$0=\int_0^{\phi_a}M_1d\phi+\int_{\phi_a}^{\pi}M_2d\phi$$

Substituting the values of M_1 and M_2 obtained earlier, we have

$$\begin{aligned}0=&\int_0^{\phi_a}[-M_0-S_0r(1-\cos\phi)+Kr^2(\phi\sin\phi-1+\cos\phi)]d\phi\\&+\int_{\phi_a}^{\pi}[-M_0-S_0r(1-\cos\phi)+Kr^2(\phi\sin\phi-1+\cos\phi)+K\pi ra\\&+K\pi r^2(\sin\phi_a-\sin\phi)]d\phi\end{aligned}$$

Integrating this equation, we get

$$\begin{aligned}&(-M_0-S_0r-Kr^2)\phi_a+(S_0r+2Kr^2)\sin\phi_a-Kr^2\phi_a\cos\phi_a\\&-(S_0r+Kr^2)\sin\phi_a+(K\pi ra-M_0-S_0r-Kr^2+K\pi r^2\sin\phi_a)(\pi-\phi_a)\\&-Kr^2(\sin\phi_a-\phi_a\cos\phi_a)-K\pi r^2\cos\phi_a=0\end{aligned}$$

Solving this for M_0 in terms of S_0, we obtain

$$M_0=K\pi ra-S_0r-Kr^2+K\pi r^2\sin\phi_a-(Kra+Kr^2\sin\phi_a)\phi_a-Kr^2\cos\phi_a$$

20 It also follows from the condition that the top point moves neither to the right nor left during deformation of the frame that

$$\frac{\partial U}{\partial S_0}=0$$

And so we get

$$\frac{\partial U}{\partial S_0}=\frac{2r}{2EI}\int_0^{\phi_a}M_1\frac{\partial M_1}{\partial S_0}\,d\phi+\frac{2r}{2EI}\int_{\phi_a}^{\pi}M_2\frac{\partial M_2}{\partial S_0}\,d\phi=0$$

where

$$\frac{\partial M_1}{\partial S_0}=\frac{\partial M_2}{\partial S_0}=r(1-\cos\phi)$$

Therefore

$$\begin{aligned}0=&\int_0^{\phi_a}[-M_0-S_0r(1-\cos\phi)+Kr^2(\phi\sin\phi-1+\cos\phi)][1-\cos\phi]d\phi\\&+\int_{\phi_a}^{\pi}[-M_0-S_0r(1-\cos\phi)+Kr^2(\phi\sin\phi-1+\cos\phi)+K\pi ra\\&+K\pi r^2(\sin\phi_a-\sin\phi)][1-\cos\phi]d\phi\end{aligned}$$

which integrates into

$$0=(-M_0-S_0r-Kr^2+K\pi ra+K\pi r^2\sin\phi_a)\pi-K\pi r^2+K\pi r^2$$

$$+\frac{\pi K r^2}{4}-\left(\frac{S_0 r+K r^2}{2}\right)\pi-K\pi r^2\cos\phi_a$$
$$-(-M-S_0 r-K r^2+K\pi r a+K\pi r^2\sin\phi_a)\phi_a$$
$$-(M_0+2S_0 r+2K r^2-K\pi r a-K\pi r^2\sin\phi_a)\sin\phi_a$$
$$-K r^2(\sin\phi_a-\phi_a\cos\phi_a)+\frac{K r^2}{8}(\sin 2\phi_a-2\phi_a\cos 2\phi_a)$$
$$-\frac{K\pi r^2}{2}\sin^2\phi_a+(-M_0-S_0 r-K r^2)\phi_a+\left(\frac{S_0 r+K r^2}{2}\right)(\sin\phi_a\cos\phi_a+\phi_a)$$
$$+(M_0+2S_0 r+2K r^2)\sin\phi_a+K r^2(\sin\phi_a-\phi_a\cos\phi_a)$$
$$-\left(\frac{S_0 r+K r^2}{2}\right)(\sin\phi_a\cos\phi_a+\phi_a)-\frac{K r^2}{8}(\sin 2\phi_a-2\phi_a\cos 2\phi_a)$$

from which, by solving for M_0, we get

$$M_0=K\pi r a+K\pi r^2\sin\phi_a-\frac{3S_0 r}{2}-\frac{5K r^2}{4}-K r a\phi_a$$
$$-K r^2\phi_a\sin\phi_a+K r a\sin\phi_a+K r^2\sin^2\phi_a-K r^2\cos\phi_a$$

Equating these two values of M_0, and solving for S_0, we obtain

$$S_0=\left(2\frac{a}{r}\sin\phi_a+\sin^2\phi_a-\tfrac{1}{2}\right)Kr=\alpha Kr\quad.\ .\ .\ .\ .\quad[12]$$

And if this value of S_0 is substituted back into the first value for M_0, we have

$$M_0=\left(\pi\frac{a}{r}-\alpha-1+\pi\sin\phi_a-\phi_a\frac{a}{r}-\phi_a\sin\phi_a-\cos\phi_a\right)Kr^2=\beta Kr^2\quad.\quad[13]$$

Tables of α and β appear in the general discussion.

21 In order to obtain the deflection of the top point of the frame relative to the bottom, that is, the shortening or lengthening of the vertical diameter, we imagine a force X_0 (of zero magnitude) applied vertically at the top point in Fig. 7, and say that

$$\frac{\partial U}{\partial X_0}=\delta$$

The contribution of X_0 to M_1 and M_2, previously considered, is $X_0 r\sin\phi$, making

$$\frac{\partial M_1}{\partial X_0}=\frac{\partial M_2}{\partial X_0}=r\sin\phi$$

And the equation for δ becomes

$$\delta=\frac{r^2}{EI}\left[\int_0^{\phi_a}M_1\sin\phi\,d\phi+\int_{\phi_a}^{\pi}M_2\sin\phi\,d\phi\right]$$

Carrying out this integration, which is also very long, and substituting S_0 and M_0 by αKr and βKr^2, we get

$$\delta=\frac{Kr^4}{EI}\left[\frac{\pi}{2}(\phi_a+\sin\phi_a\cos\phi_a)+\pi\frac{a}{r}+\pi\sin\phi_a-2(\alpha+\beta+1)\right.$$
$$\left.-\frac{\pi^2}{4}+\pi\frac{a}{r}\cos\phi_a\right]=\gamma\frac{Kr^4}{EI}\quad.\quad[14]$$

In order to obtain the deformation of the horizontal diameter, we apply a force X (of zero magnitude) in much the same manner as above. Fig. 8 shows the set-up. Thus $\frac{\partial U}{\partial X}$ is the movement of point

$$S_0 = -\left[\frac{7\phi_a \sin\phi_a \cos\phi_a - 8\sin^2\phi_a + 2\phi_a^2 - \phi_a^2\cos 2\phi_a}{2\phi_a \sin\phi_a \cos\phi_a - 4\sin^2\phi_a + 2\phi_a^2}\right] Kr \quad . \quad [17]$$

Substituting in the first value for M_0, $S_0 = \epsilon Kr$, we get

$$M_0 = \epsilon\left[\left(\frac{\sin\phi_a}{\phi_a} - 1\right) + 2\frac{\sin\phi_a}{\phi_a} - 1 - \cos\phi_a\right] Kr^2 \quad . \quad . \quad [18]$$

27 The procedure to get the deflection of the topmost point relative to the built-in supports is the same as before. A fictitious force X is applied at the top, and the deflection directly obtained from

$$\delta = \frac{\partial U}{\partial X}, \qquad \text{where} \qquad U = \int_0^{\phi_a} \frac{M^2 r d\phi}{2EI}$$

From Fig. 9, we have for the bending moment

$$M = -M_0 - S_0 r(1-\cos\phi) + Kr^2(\phi\sin\phi - 1 + \cos\phi) + Xr\sin\phi$$

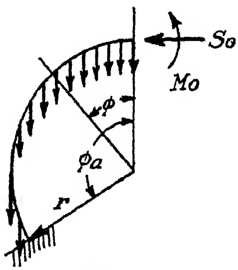


FIG. 9

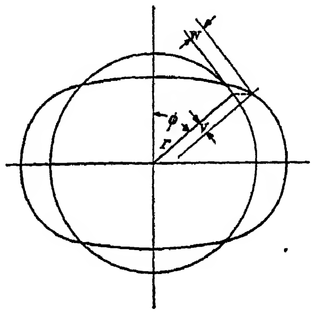


FIG. 10

Therefore

$$\delta = \frac{r^3}{EI}\int_0^{\phi_a} [-M_0 - S_0 r(1-\cos\phi) + Kr^2(\phi\sin\phi - 1 + \cos\phi)][\sin\phi]\, d\phi$$

Integrating this, and substituting S_0 and M_0 by ϵKr and ρKr^2, we obtain the final expression for δ:

$$\delta = \left[(\epsilon+\rho+1)(\cos\phi_a - 1) + \left(\frac{1+\epsilon}{2}\right)(\sin^2\phi_a) + \tfrac{1}{4}(\sin^2\phi_a - 2\phi_a\sin\phi_a\cos\phi_a + \phi_a^2)\right]\frac{Kr^4}{EI} = \nu\frac{Kr^4}{EI} \quad . \quad [19]$$

Tables of these values are given in the main discussion, and we see that the deflections in general are much smaller than for the case of the freely supported frame.

28 In the particular cases, when $\phi_a = 0$ or π, we obtain modified equations for [14], [15], and [16] which agree with solutions given by St. Venant[1] for these same cases.

[1] Barré de Saint Venant, Paris, 1843. Comptes Rendus, vol. 17.

APPENDIX NO. 2

APPLICATION OF RAYLEIGH METHOD

29 This method,[1] when applied to curved bars or plates, is essentially the expression of the radial and tangential displacements in terms of some generalized or normal system of coördinates In this case, Rayleigh utilized Fourier's series for this purpose and also introduced the simplification that the energy of deformation is only that of bending, which leads us to the statement that central lines of deformed sections do not change in length.

30 In Fig. 10, let

w = radial displacement of any given point
v = tangential displacement of same point
e = unit elongation.

From a general consideration of the deformation of a curved surface, we can obtain

$$e = \frac{w}{r} - \frac{\partial v}{r\partial\phi} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [20]$$

This method hypothesizes that $e = 0$, therefore $\frac{\partial v}{\partial \phi} = w$, and if we express w as a certain Fourier's series

$$w = -\sum nr(A_n \sin n\phi + B_n \cos n\phi) \quad . \; . \; . \; . \; . \; . \quad [21]$$

then it follows from the above relation that

$$v = \sum r(A_n \cos n\phi - B_n \sin n\phi) \quad . \; . \; . \; . \; . \; . \; . \quad [22]$$

31 Now we must study the change of curvature of the ring under bending. In general,

$$\frac{1}{\rho} = \frac{1}{\rho_0}\,\frac{1+\frac{2}{\rho_0^2}\left(\frac{d\rho_0}{d\phi}\right)^2 - \frac{1}{\rho_0}\left(\frac{d^2\rho_0}{d\phi^2}\right)}{\left[1+\frac{1}{\rho_0^2}\left(\frac{d\rho_0}{d\phi}\right)^2\right]^{\frac{3}{2}}}$$

where ρ_0 = radius of curvature at a point, and ρ = radius of curvature at any other adjacent point. By neglecting the squares of the quantity $d\rho_0/d\phi$, which is very small in the case of a deformed circle, we obtain the approximate equation

$$\frac{1}{\rho} = \frac{1}{\rho_0}\left[1 - \frac{1}{\rho_0}\left(\frac{d^2\rho_0}{d\phi^2}\right)\right]$$

And now, if $\rho_0 = r + w$, then we have

$$\begin{aligned}\frac{1}{\rho} &= \left(\frac{1}{r+w}\right)\left[1-\left(\frac{1}{r+w}\right)\left(\frac{d^2w}{d\phi^2}\right)\right] \\ &= \left(\frac{1}{r} - \frac{w}{r^2}\right)\left[1-\left(\frac{1}{r} - \frac{w}{r^2}\right)\left(\frac{d^2w}{d\phi^2}\right)\right] \\ &= \frac{1}{r} - \frac{w}{r^2} - \left(\frac{1}{r^2} - \frac{2w}{r^3} + \frac{w^2}{r^4}\right)\frac{d^2w}{d\phi^2}\end{aligned}$$

Neglecting terms of higher order, and dividing out $1/r$, we have

[1] Theory of Sound, Lord Rayleigh, vol. I, p. 412.

$$\frac{1}{\rho} = \frac{1}{r}\left[1 - \frac{w}{r} - \frac{1}{r}\frac{d^2w}{d\phi^2}\right]$$

or

$$\frac{1}{\rho} - \frac{1}{r} = -\left[\frac{w}{r^2} + \frac{1}{r^2}\frac{d^2w}{d\phi^2}\right]$$

Now, if k is the change of curvature,

$$k = \frac{1}{r} - \frac{1}{\rho}$$

therefore

$$k = \frac{1}{r^2}\left[w + \frac{d^2w}{d\phi^2}\right]$$

but since we are expressing w as a function of ϕ, A, and B, we must write

$$k = \frac{1}{r^2}\left[w + \frac{\partial^2 w}{\partial\phi^2}\right]$$

32 Now the energy of deformation per unit length is $\frac{1}{2}Mk$, where M is the bending moment, and can be expressed as $M = EIk$. Thus the total energy of deformation is obtained by integrating the above value over the entire length of the curved bar. In the case of a ring, this becomes

$$\text{Total energy} = V = \tfrac{1}{2}EI\int_0^{2\pi} k^2 r\,d\phi$$

where k must now be expressed as a function of ϕ. Since

$$w = -\Sigma nr\,(A_n \sin n\phi + B_n \cos n\phi)$$

then

$$\frac{\partial^2 w}{\partial\phi^2} = \sum n^3 r\,(A_n \sin n\phi + B_n \cos n\phi)$$

and

$$k = \frac{1}{r}\sum n(n^2-1)\,(A_n \sin n\phi + B_n \cos n\phi)$$

This gives us for k^2

$$k^2 = \frac{1}{r^2}\sum_{n=2}^{n=\infty}\ \sum_{k=2}^{k=\infty} nk(n^2-1)\,(k^2-1)\,(A_nA_k \sin n\phi \sin k\phi$$
$$+2A_nB_k \sin n\phi \cos k\phi + B_nB_k \cos n\phi \cos k\phi)$$

and so, knowing that the integrals of the terms where $k \neq n$ go to zero in the limits,

$$V = \frac{EI}{2r}\sum_2^{\infty} n^2(n^2-1)^2 \int_0^{2\pi} [(A_n^2 \sin^2 n\phi + 2A_nB_n \sin n\phi \cos n\phi$$
$$+ B_n^2 \cos^2 n\phi)\,d\phi]$$

which gives

$$V = \frac{EI\pi}{2r}\sum n^2(n^2-1)^2(A_n^2 + B_n^2)$$

33 For all symmetrical cases about the vertical axis, we have the condition (see Fig. 10) that when $\phi = 0$, $v = 0$, giving us $A_n = 0$, so

$$V = \frac{EI\pi}{2r}\sum n^2(n^2-1)^2 B_n^2 \quad . \ . \ . \ . \ . \ . \ . \ . \quad [23]$$

34 This last equation is a general expression for the stored energy of a deformed ring under any type of loading, when the deformations

are expressed as in Equations [21] and [22]. The only unknown in these two equations is B_n, and it is the method of evaluating this that concerns us now. The general idea is to imagine the ring to change its shape by a slight change in B_n, let us say. This changes that value of V, as given by Equation [23]. We may also use the idea that when the ring undergoes a slight change, the external forces on the ring move through corresponding small distances, doing work on the ring. These two energy changes must be equal.

35 Now in the case of the frame, we have the elastic system shown in Fig. 11, where K is the weight per inch, and the frame is simply supported $(a = 0)$. Expressing the change in V, with a change in B_n, we may write

$$\frac{\partial V}{\partial B_n}\,\delta B_n = \frac{EI\pi}{r}\,n^2(n^2-1)^2B_n\delta B_n$$

The change in external work done in additionally deforming the system is given by

$$\frac{\partial V}{\partial B_n}\,\delta B_n = -2P\cos\phi_a\,\frac{\partial w}{\partial B_n}\,\delta B_n + 2P\sin\phi_a\,\frac{\partial v}{\partial B_n}\,\delta B_n$$
$$-2\int_0^{\pi} Kr\cos\phi\,\frac{\partial w}{\partial B_n}\,\delta B_n d\phi + 2\int_0^{\pi} Kr\sin\phi\,\frac{\partial v}{\partial B_n}\,\delta B_n d\phi$$

Differentiating Equations [21] and [22] with regard to B_n, we have

$$\left.\frac{\partial w}{\partial B_n}\right|_{\phi=\phi_a} = -nr\cos n\phi_a$$
$$\left.\frac{\partial v}{\partial B_n}\right|_{\phi=\phi_a} = -r\sin n\phi_a$$

Substituting these values in the above equation, and equating the two expressions for change in energy, we have

$$\frac{EI\pi}{r}\,n^2(n^2-1)^2B_n = 2P[nr\cos\phi_a\cos n\phi_a + r\sin\phi_a\sin n\phi_a]$$
$$-2Kr^2\left[n\int_0^{\pi}\cos\phi\cos n\phi d\phi + \int_0^{\pi}\sin\phi\sin n\phi d\phi\right]. \quad [24]$$

36 For $n = 1$, we understand this term of the Fourier series to indicate a translation of the ring as a whole, and hence it does not enter our expression for energy of deformation; and for all other values of n, the last two integrals are zero. Therefore, this simplification allows us to write

$$\frac{EI\pi}{r}\,n^2(n^2-1)^2B_n = 2K\pi r^2(n\cos\phi_a\cos n\phi_a + \sin\phi_a\sin n\phi_a)$$

and solving for B_n, we obtain

$$B_n = \frac{2Kr^3}{EI}\left[\frac{n\cos\phi_a\cos n\phi_a + \sin\phi_a\sin n\phi_a}{n^2(n^2-1)^2}\right]$$

Substituting this value in Equations [21] and [22], we obtain the final equation of deformation:

$$w = -\frac{2Kr^4}{EI}\sum_{2,\,3,\,4,\,\text{etc.}}^{\infty}\frac{(n\cos\phi_a\cos n\phi_a + \sin\phi_a\sin n\phi_a)(\cos n\phi)}{n^2(n^2-1)^2}. \quad [25]$$

$$v = -\frac{2Kr^4}{EI}\sum_{2,\,3,\,4,\,\text{etc.}}^{\infty}\frac{(n\cos\phi_a\cos n\phi_a + \sin\phi_a\sin n\phi_a)(\sin n\phi)}{n^2(n^2-1)^2}. \quad [26]$$

37 Previously it has been stated that the bending moment at a section is EIk, and since we have proved that

$$k = \frac{1}{r}\sum n(n^2-1)\,(B_n \cos n\phi)$$

$$= \frac{2Kr^2}{EI}\sum \frac{n(n^2-1)\,(\cos n\phi)\,(n\cos\phi_a \cos n\phi_a + \sin\phi_a \sin n\phi_a)}{n^2(n^2-1)^2}$$

it follows that

$$M = 2Kr^2 \sum_{2,\,3,\,4,\,\text{etc.}}^{\infty} \frac{(n\cos\phi_a \cos n\phi_a + \sin\phi_a \sin n\phi_a)\,(\cos n\phi)}{n(n^2-1)} \quad . \; [27]$$

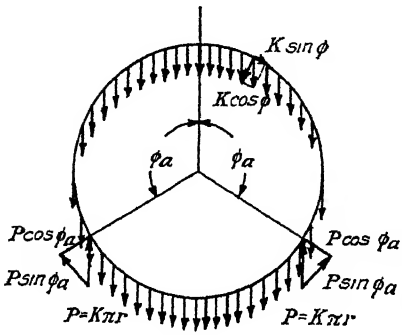


FIG. 11

38 Values of M calculated from the above equation check those made from Equations [2], [3], and [4] to within less than one per cent by using only three terms of the infinite series. Similar checks exist between Equations [25] and [5] and [6] for $a/r = 0$.

39 The set-up shown in Fig. 11 represents only the condition when the supporting force is applied at the center line of the frame. When the force is applied at a distance a away (Fig. 1), an additional couple Pa is introduced; and so, to solve the complete case, we must study the effect of such a couple.

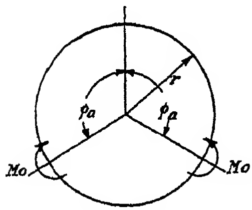


FIG. 12

40 If we proceed in our general attack as in the previous case, we get for the deflection at any point

$$w = \frac{2M_0 r^2}{\pi EI} \sum_{2,\,3,\,4,\,\text{etc.}}^{\infty} \frac{\sin n\phi_a \cos n\phi}{n(n^2-1)} \quad . \; [28]$$

41 It is impossible in this case to obtain the value of M at every section, by Rayleigh's method, as was done in the previous case. The expression

$$M = \frac{EI}{r^2}\left(w + \frac{\partial^2 w}{\partial\phi^2}\right)$$

is only true if the resultant Fourier series, expressing M, is uniformly convergent for every value of ϕ. This is obviously not true for $\phi = \phi_a$ where there exists a discontinuity in M, due to the concentrated couple However, the values of M obtained by the Castigliano theorem give here for $0 < \phi < \phi_a$

$$M = M_0\left(1 - \frac{\phi_a}{\pi}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [29]$$

and for $\phi_a < \phi < \pi$

$$M = M_0\left(-\frac{\phi}{\pi}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [30]$$

42 Thus in order to solve the general problem of a simply supported frame, the solution of the set-ups in Figs. 11 and 12 can be added directly, this being allowed by the principle of superposition.

43 The solution of the built-in frame presents the elastic system shown in Fig. 13. Here the reactions P, $K\pi r$, and M_0 must be so related that no rotation or lateral movement of the frame at the points of support take place. So, to complete our tools for the solution of this case, we solve the case shown in Fig. 14. Here again the procedure of starting with

$$V = \frac{EI\pi}{2r} \sum n^2(n^2-1)^2 B_n^2$$

is followed through. In this case, however, the work of the components of $P(P \sin \phi_a)$ is negative work, and we get

$$\frac{\partial V}{\partial B_n} \delta B_n = 2P \left[\cos \phi_a \frac{\partial v}{\partial B_n} \delta B_n - \sin \phi_a \frac{\partial w}{\partial B_n} \delta B_n \right]$$

or

$$\frac{EI\pi}{r} n^2(n^2-1)^2 B_n = -2Pr[n \sin \phi_a \cos n\phi_a - \cos \phi_a \sin n\phi_a]$$

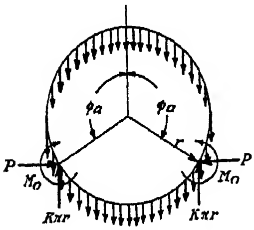


FIG. 13

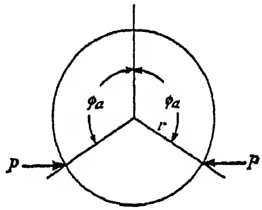


FIG. 14

thus

$$B_n = -\frac{2Pr^2}{\pi EI} \left[\frac{n \sin \phi_a \cos n\phi_a - \cos \phi_a \sin n\phi_a}{n^2(n^2-1)^2} \right]$$

and so

$$w = \frac{2Pr^3}{\pi EI} \sum_{2,\,3,\,4,\,\text{etc.}}^{\infty} \frac{(n \sin \phi_a \cos n\phi_a - \cos \phi_a \sin n\phi_a)(\cos n\phi)}{n(n^2-1)^2} . \quad [30]$$

and

$$M = \frac{2Pr}{\pi} \sum_{2,\,3,\,4,\,\text{etc.}}^{\infty} \frac{(n \sin \phi_a \cos n\phi_a - \cos \phi_a \sin n\phi_a)(\cos n\phi)}{n(n^2-1)} . \quad [31]$$

44 A comparison of the two methods of solution as applied to this problem reveals particular advantages to each. Although Castigliano's solution involves very long integrations, it entails comparatively little mathematical thinking. Rayleigh's attack, however, gives much more general results — in two equations giving the complete distribution of deformation and bending moment for any value of ϕ_a. Both are, however, powerful methods of solution of so-called "indeterminate elastic problems."

DISCUSSION

R. Eksergian.[1] The armature or field frame of an electric machine is subjected to two types of loading, gravity and loadings due to unbalanced magnetic pull, and the torque reaction, which may be neglected for the body sections of the frame. Due to deflection of the shaft, frame, etc., a uniform air gap is impossible, and often a very serious unbalanced magnetic pull may be set up which would overshadow the weight loading. Therefore, in the final proportioning of the field frame, careful consideration should be given this type of loading.

In the analysis of elastic structures we may proceed from three methods of attack: (1) the slope-deflection method, (2) by Castigliano's principle or the energy method, and (3) the variational method or the Hamiltonian principle. The Castigliano method is extensively used, particularly by European engineers in the solving of statically indeterminate problems, while English and American engineers incline more to the deflection method. The variation method offers a field of mathematical attack which has not been used extensively by engineers. It practically amounts to equating the work of the external forces on a small arbitrary additional deformation, or variation of a given deformation of the body, to the corresponding change of the internal work of deformation. The deformation may be an assumed curve constructed from a series, the latter of which may be expanded as a function of arbitrary coördinates. If we use a Fourier series, we have, in a sense, harmonic parameters or coördinates, the amplitude of each harmonic curve being a coördinate. We are concerned with the variation of this deformation curve and, in particular, with the partial variation or the variation of any single coördinate.

The author has chosen for a symmetrical solution horizontal and vertical axes, with the origin at the center. By this choice, however, the reactions of the supports have vertical displacements consistent with the variation, therefore do work. The work of deformation is correctly given in Par. 35. In Par. 43, however, the same deformation has been used. Therefore, the variation of the deformation causes both linear and angular displacements, and the bending moment and support reaction do work, which has been neglected in the analysis. The writer cannot agree with the analysis of Par. 43, or the statement that "the reactions P, $K\pi r$ and M_0 must be so related that no rotation or lateral movement of the frame at the points of the support take place" if the coördinate system of the deformation curves are maintained as in the problems of Par. 35.

[1] Engineer, Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

WILLIAM HOVGAARD.[1] The problem discussed by the author is entirely statical and consists in determining the deflections due to bending, under the action of gravity, of a circular structure of uniform section supported at two points. The author has obtained a check on his results by solving the problem, first by the method of Castigliano and second by that of Lord Rayleigh. The former involves the formation of an expression for the elastic energy and its differentiation, besides the introduction of fictitious forces of zero magnitude. The latter, as applied by the author, comprises the expression of infinite Fourier's series, a calculation of the elastic energy of deformation, and application of the principle of virtual velocities. Both methods are found to lead to the same result as of course they should.

There is, however, a third method of attack, which may be more straightforward and simple. This method is explained fully, elsewhere.[2] It consists in calculating, as for any curved bar, the deflections Δx and Δy along two coördinate axes, as well as the angular deflection. By making the deflections satisfy boundary or symmetry conditions, the unknown reactions are determined as by the other methods, but it is not required to calculate the energy or to introduce any fictitious quantities or artificial conceptions. The origin is placed at one of the points of symmetry where there is no angular deflection.

To illustrate the method, it will be applied to the case where the frame rests freely on the supports at two points on the circumference as in Fig. 3. Place the origin at the bottom of the frame, x positive to the left, y positive upward. We have then for any point P on the circumference: Angular deflections,

$$\Delta\phi_P = \int_0^P \frac{M}{EI}\, r\,d\phi$$

and the linear deflections,

$$\Delta x_P = -\int_0^P (y_P - y)\, \frac{M}{EI}\, r\,d\phi$$

$$\Delta y_P = \int_0^P (x_P - x)\, \frac{M}{EI}\, r\,d\phi$$

where

$$x = r \sin \phi$$
$$y = r(1+\cos \phi)$$

For the top point $\phi = 0$ we have

$$y = 2r \qquad x_P = 0 \qquad \Delta\phi_P = 0 \qquad \Delta x_P = 0 \qquad \Delta y_P = \delta$$

[1] Professor of Naval Construction, Mass. Institute of Technology, Cambridge, Mass.

[2] W. Hovgaard: Structural Design of Warships, E. and F. N. Spon, Ltd., London, 1915; p. 55 et seq.

For $\phi = \frac{\pi}{2}$, we have $\Delta x_P = \frac{1}{2}\Delta$, whence we can write at once

$$0 = \int_0^{\phi_a} M_1 d\phi + \int_{\phi_a}^{\pi} M_2 d\phi$$

$$0 = \int_0^{\phi_a} (1-\cos\phi) M_1 d\phi + \int_{\phi_a}^{\pi} (1-\cos\phi) M_2 d\phi$$

$$\delta = -\frac{r^2}{EI}\left[\int_0^{\phi_a} M_1 \sin\phi d\phi + \int_{\phi_a}^{\pi} M_2 \sin\phi d\phi\right]$$

$$\Delta = \frac{2r^2}{EI}\int_0^{\frac{\pi}{2}} \cos\phi M_1 d\phi = -\frac{2Kr^4}{EI}\left[\frac{\alpha(4-\pi)}{4} + \beta - \tfrac{3}{8}\pi + 1\right]$$

All of these expressions will be found to be identical with those deduced by the more laborious Castigliano process given in the paper. Notably the development of Δ, the elongation of the horizontal diameter, is much shorter and leads to a simpler expression. The deformation at any point can be readily determined.

This method has been employed with satisfactory results in many different problems, notably in calculating the strength of the closed frame rings of submarines, of the frames of light cruisers, including support by stanchions, and of the lattice keel girder of a semi-rigid airship. All of these cases were much more complicated than that considered in the paper, on account of variations in the transverse section of the girders, irregularities in the form of contour, and the more complex mode of loading and support, but the method here described proved simpler than the methods of least work and virtual velocities. The integrations were of course in all cases performed graphically.

When Lord Rayleigh's method is applied to a simple case as that here under consideration, it seems preferable to express the displacements by Ritz's method. The complete Fourier's series comprising both sines and cosines, covers all possibilities, but since the boundary or symmetry conditions are here fairly obvious *a priori,* we may at once express the displacements by a limited number of serial terms chosen so as to fulfil those conditions.

Since the tangential displacements must be zero at the top and bottom we cannot have any sine terms in v and may at once write

$$v = -B_2 \sin 2\phi - B_3 \sin 3\phi - \ldots$$

which, for inextensibility, gives

$$w = \frac{dv}{d\phi} = -2B_2 \cos 2\phi - 3B_3 \cos 3\phi - \ldots$$

It is clear that such an expression can be made to fulfil the requirements to radial displacements.

Including only the first two terms of the above series, we find

$$k = \frac{1}{r^2}\left(w + \frac{d^2w}{d\phi^2}\right) = -\frac{6}{r^2}\left[B_2 \cos 2\phi + 4B_3 \cos 3\phi\right]$$

$$V = \tfrac{1}{2}EIr\int_0^{2\pi} k^2 d\phi = \frac{18EI}{r}\int_0^{2\pi}(B_2 \cos 2\phi + 4B_3 \cos 3\phi)^2 d\phi$$

$$\therefore V = \frac{18EI\pi}{r}(B_2^2 + 16B_3^2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [32]$$

which checks with Equation [23].

In Par. 8 it is stated that the average air-gap, $\frac{\delta+\Delta}{2}$ is to be kept within a certain design value. This is because such value determines or influences the total magnetic flux. It would seem that it must be of interest also to investigate the distribution of the individual or local gap around the circumference, since this must determine the distribution of the magnetic flux. Such investigation can readily be made by the formulas here proposed. For instance, the displacement of the top of the frame relative to the support is the same as that relative to the rotor, and hence equal to the change of gap at this point. It is found very simply from

$$\Delta y_a = \frac{r^2}{EI}\int_0^{\phi_a} M_1 \sin \phi d\phi$$

C. A. Norman.[1] As a teacher of engineering, the writer is extremely interested in the type of mathematics used by the author. His problem is not practically a very complicated one — just the stress and strain conditions in an ordinary dynamo frame due to dead load. Any graduate engineer should be able to solve such a common problem, but it is doubtful whether many engineering schools in the country give to its undergraduate engineering students the mathematical and mechanical tools for solving it. Yet a great deal of material, and possibly also machine-shop labor, may be saved if the problem is solved with a fair degree of accuracy.

Similar situations occur continually in mechanical design. Should not our engineering books be so written that students can find help in them for the solution of these problems after graduation, even if we do not study them in the regular four-year courses? This has not been generally done in this country. In consequence, the average engineering textbook is of little, if any, value after graduation. Abroad, engineering teachers do not hesitate to put into their books very complete and up-to-date knowledge, and in consequence these books form valuable companions for the young engineers after graduation, and the engineering teachers

[1] Professor of Machine Design, Ohio State University, Columbus, Ohio. Mem. A.S.M.E.

No. 2022

WORK-HARDENING PROPERTIES OF METALS

THEIR RELATION TO METAL-CUTTING OPERATIONS AND CUTTING TEMPERATURES

By Edward G. Herbert,[1] Manchester, England

Non-Member

It is the principal object of the present work to bring into correlation with the operation of cutting tools certain groups of well established and generally recognized facts, the chief among them being: (1) the fact that metals are hardened by any process which deforms them so as to cause a permanent change of shape while they are at low or moderate temperatures, a process which is generally referred to as "cold work"; (2) the fact that metals are deformed and are therefore hardened by cutting tools; (3) the fact that heat is generated by deformation of metals and in a preëminent degree by metal-cutting operations; and (4) the fact which has now been known for some time, that the degree of hardness induced by working metals, with cutting tools, or otherwise, is greatly influenced by the temperature at which the deformation takes place.

An attempt will be made to show the bearing of these correlated facts on the resistance offered by metals to the cutting tool, and on the rate of wear of the cutting tool.

A further and most important branch of the same subject, the relation between the temperature generated in cutting and the heat resisting properties or "hot hardness" of the cutting tool, will be barely touched upon.

The limiting factor in the rapid removal of metal and therefore in the productivity of the metal-working industries is, ultimately, the temperature generated in cutting in relation to the capacity of the cutting tool to withstand high temperatures. The present work deals with the heat-producing properties of the work rather than with the heat-resisting properties of the tool.

[1] Edward G. Herbert, Ltd.

Contributed by the Machine Shop Practice Division and the Research Sub-Committee on Cutting and Forming of Metals and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

SECTION 1

The Hardness Induced by Cutting Tools

IN ORDER to investigate the hardness induced by the tool in the process of cutting metal, a uniform procedure was adopted. A bar of the metal under investigation was chucked in the lathe

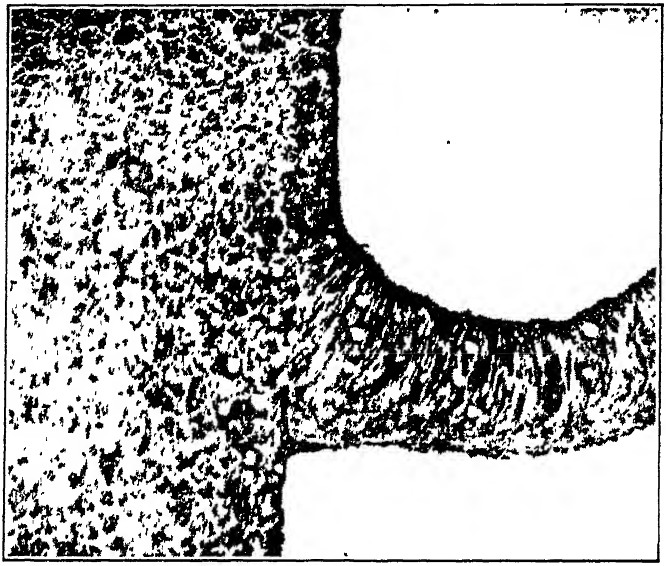


Fig. 1 Chip in Mild Steel A (×10)

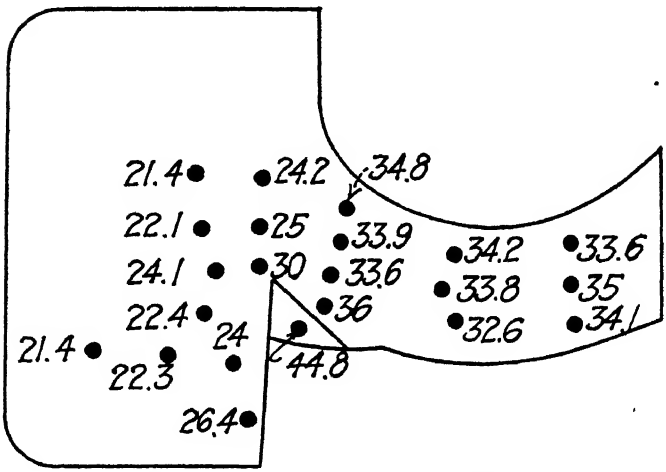


Fig. 1a Hardness Induced by Tool in Mild Steel A

and a cut $\frac{1}{4}$ in. deep was taken with a straight-edged side tool, the feed being $\frac{1}{27}$ in. and the speed 65 ft. per min. The lathe was quickly stopped while the cut was in progress and the tool with-

FIG. 2 CHIP IN STAINLESS IRON B (×10)

drawn, leaving a chip attached to the bar. A section was cut parallel to the axis of the bar through the chip and the adjacent metal, and the section was mounted in fusible metal, polished, etched, and tested for hardness at a number of points around the

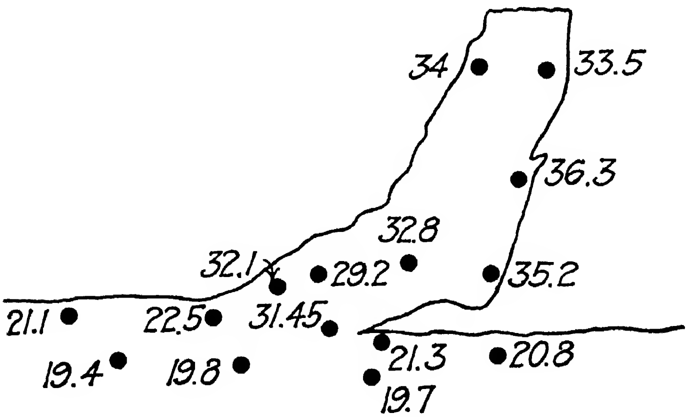


FIG. 2a HARDNESS INDUCED BY TOOL IN STAINLESS IRON B

zone of separation. The test used was the time test, made with the Herbert pendulum hardness tester. For the convenience of those not familiar with the pendulum time hardness numbers, conversion of these has been made into approximate Brinell num-

bers which are placed in brackets after the time numbers. The conversion was effected by the following formulas.

For brass $B = 0.29T^2$

For soft steels $B = 0.36T^2$

FIG. 3 CHIP IN STAINLESS STEEL C ($\times 10$)

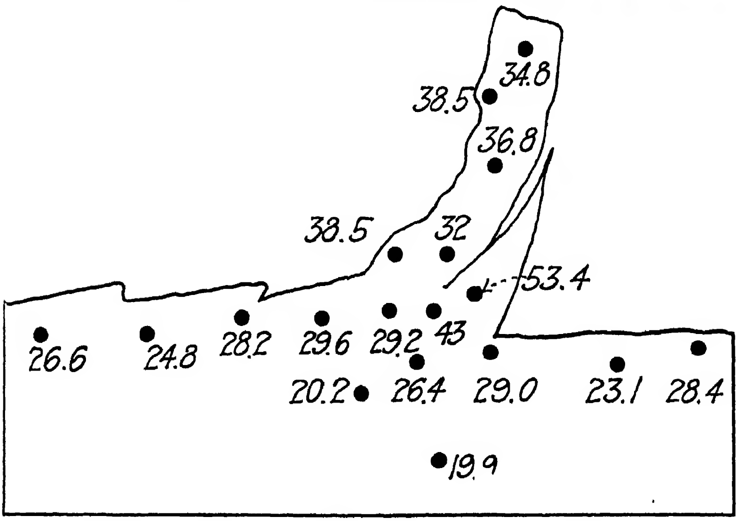


FIG. 3a HARDNESS INDUCED BY TOOL IN STAINLESS STEEL C

For steels harder than 280 Brinell $B = 10T$

or $B = 13.5dT$

where B is the Brinell hardness number, T the time hardness

number using the 4-kg. pendulum and 1-mm. steel ball, and *dT* the time hardness number with 4-kg. pendulum and 1-mm. diamond. The diamond, owing to its greater hardness and rigidity,

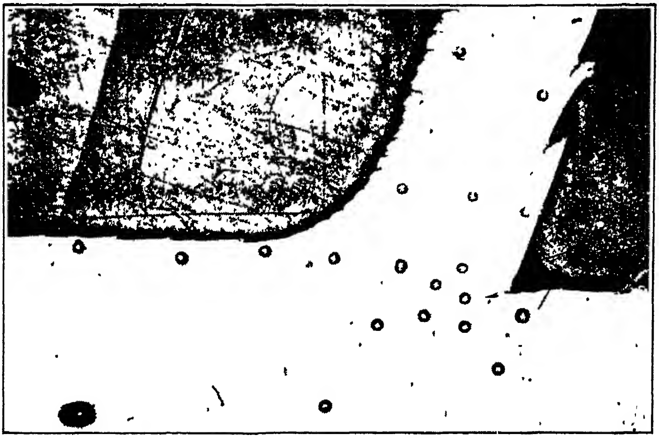


Fig. 4 Chip in Stainless Steel D (×10)

gives lower readings than the steel ball, hence the different conversion factors.

2 The metals selected for experiment were:

A — a mild steel of 35 tons per sq. in. tensile strength; time hardness 21.4 (163*B*)

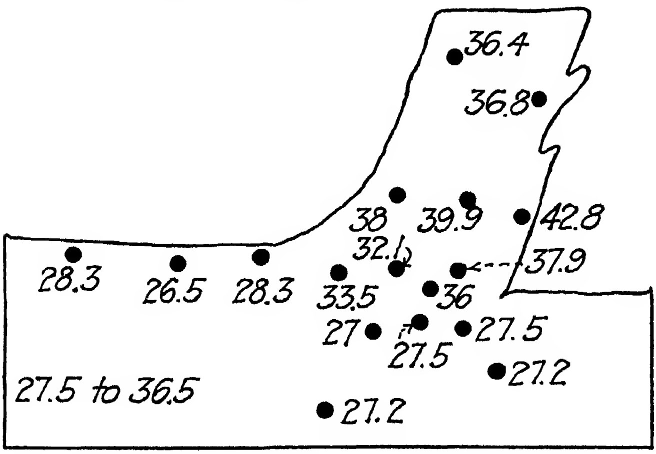


Fig. 4*a* Hardness Induced by Tool in Stainless Steel D

B — stainless iron (0.1C, 13.5Cr); time hardness 19.7 (140*B*)

C — stainless steel (0.1C, 17.2Cr, 7.9Ni); time hardness 19.9 (142*B*)

D — stainless steel (0.26C, 13.2Cr); time hardness 27.2 (265*B*)
E — stainless steel (0.11C, 15.7Cr, 10.5Ni); time hardness 18.0 (116*B*)

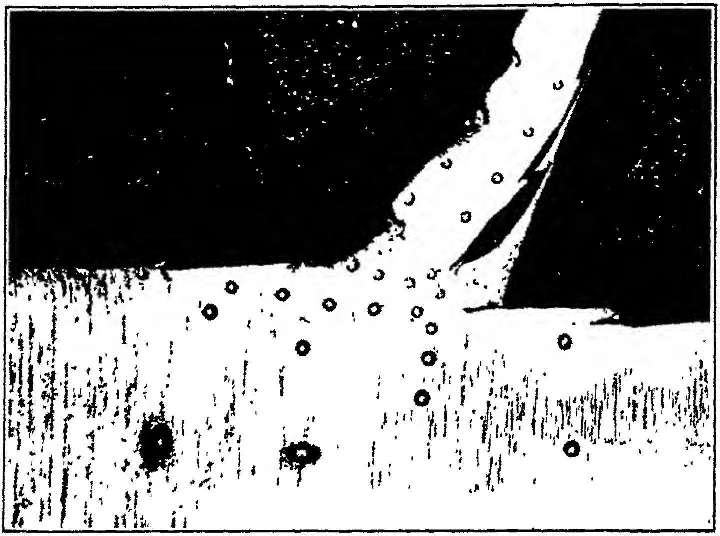


FIG. 5 CHIP IN STAINLESS STEEL E (×10)

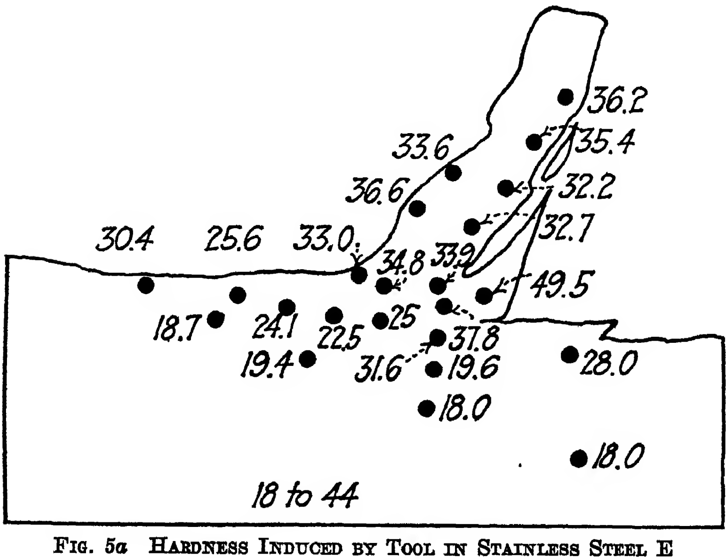


FIG. 5a HARDNESS INDUCED BY TOOL IN STAINLESS STEEL E

F — rod brass; time hardness 19.2 (106*B*)
G — cast iron; time hardness 29.3.

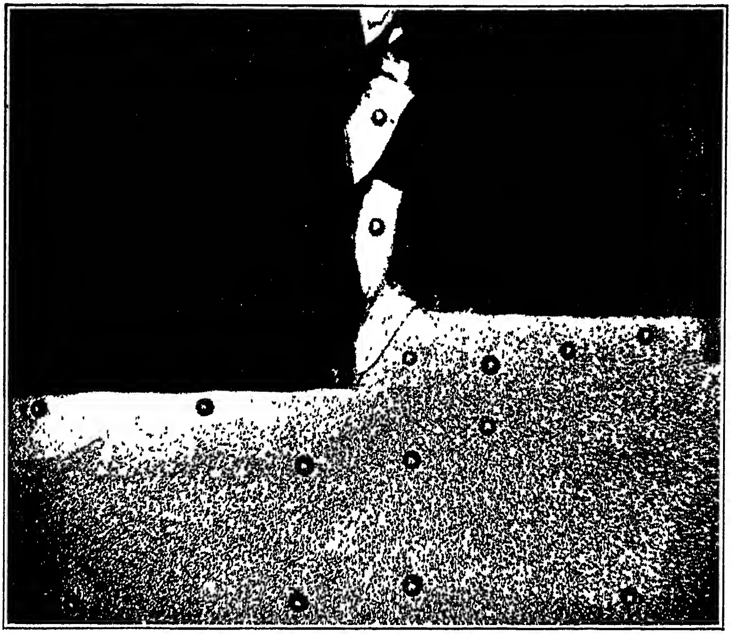


FIG. 6 CHIP IN BRASS F (×10)

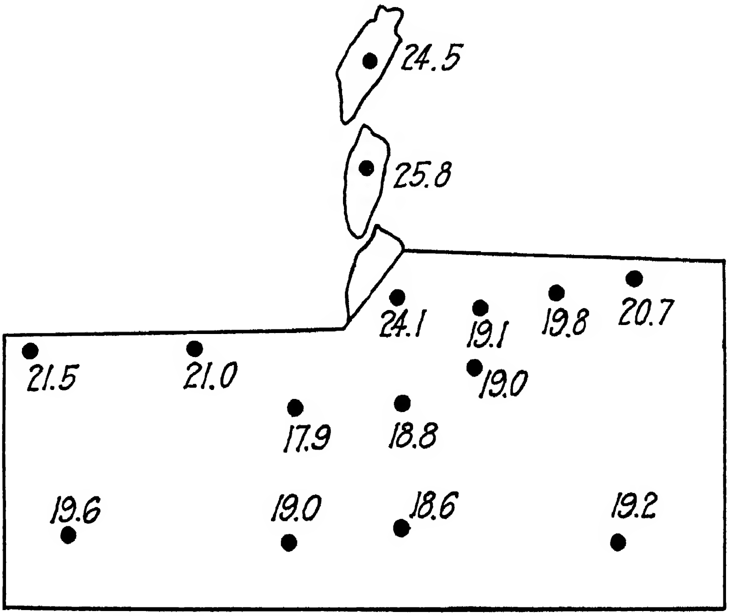


FIG. 6a HARDNESS INDUCED BY TOOL IN BRASS F

3 Photomicrographs of the sections are given in Figs. 1 to 6, and accompanying each photograph is a chart giving the pendulum time hardness of each spot tested. The brass chips in Fig. 6 appear separate, but were actually attached at one end to the bar. It was not possible to obtain an attached chip of cast iron, but a chip was sectioned and tested for hardness, the microstructure of the chip and of the bar being shown in Figs. 7 and 8.

4 From these figures it appears:

a That in the brittle metal, cast iron, the tool caused no distortion of the metal and no work hardening

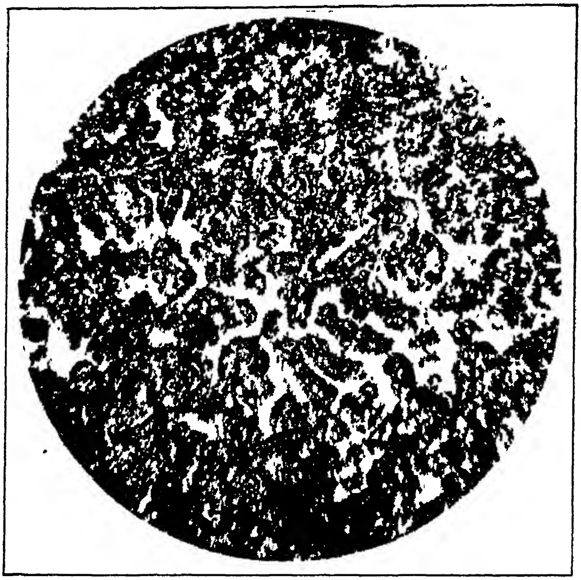


FIG. 7 UNDISTORTED STRUCTURE OF CAST-IRON CHIP. TIME HARDNESS 28.5 (×100)

b That in brass there was a slight distortion of the structure and a small amount of work hardening. The manner in which the brass chip is detached by shear with slight distortion of structure is shown in Fig. 9

c That in the steels the distortion of the structure was very severe, and the hardening of the metal in all cases considerable, but very different in degree. The work hardness figures are discussed in Section 2, but attention is here drawn to the hardness of the metal immediately ahead of the tool where separation of the chip was about to take place. It is evident that in every case except that of cast iron, the tool was actually cutting metal whose hardness was very different from that of the original bar. This is shown in Table 1 in which the metals are placed in the order of

their original time hardness. The hardness in the path of the tool and the percentage increase of time hardness at this point are given.

5 Five of the specimens were originally quite soft, and the specimen which had the lowest original hardness, stainless steel E ($T18$, $116B$), was hardened by the tool to $T37.8$ ($378B$), being exceeded in this respect only by stainless steel C, whose hardness was increased by the tool from $T19.9$ to $T43$ ($430B$). The steel which had the highest original hardness, stainless steel D, was only

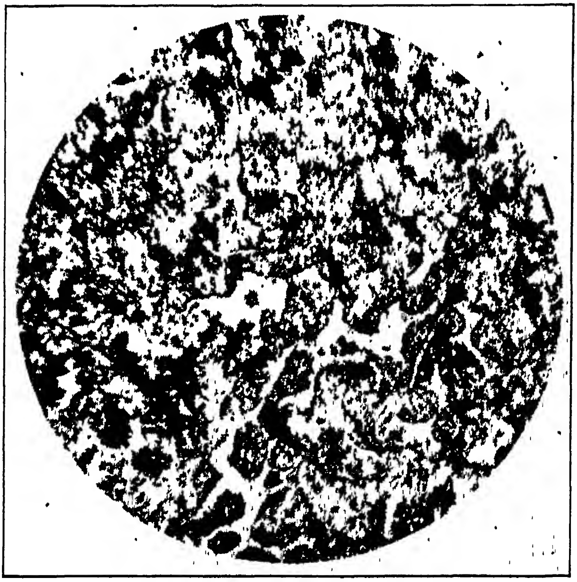


FIG. 8 STRUCTURE OF CAST-IRON BAR G. TIME HARDNESS 29.3 (×100)

slightly hardened, from 27.2 to 36. The mild steel A was originally harder than stainless iron B, but under the action of the tool the

TABLE 1 METALS ARRANGED IN ORDER OF ORIGINAL TIME HARDNESS

Specimen	Original hardness	Hardness in path of the tool	Time hardness increase, per cent
Stainless steel E.	18.0 (116B)	37.8 (378B)	110
Brass F	19.2 (106B)	24.1 (169B)	12.5
Stainless iron B........	19.7 (140B)	31.45 (315B)	60
Stainless steel C........	19.9 (142B)	43 (430B)	115
Mild steel A...........	21.4 (163B)	30 (300B)	40
Stainless steel D........	27.2 (265B)	36 (360B)	32
Cast iron G	29.3	29.3	0

order of hardness was reversed, while the hardest of all the specimens, cast iron G, was not hardened by the tool.

6 As to the general disposition of hardness, it is seen in Figs. 1a to 6a that there exists in front of the tool a hardness gradient

rising from the undistorted body of the bar toward the point of separation of the chip, and that this gradient is continued past the point of separation into the chip, where the greatest hardness is generally found. (The hardness of the "built-up edge" is discussed later.) Cutting takes place at a point on this hardness gradient which is always higher than the original hardness of the metal, but the hardness where cutting takes place differs from the original hardness according to the characteristics of the metal in respect to (*a*) its ductility or brittleness, and (*b*) its capacity for work hardening.

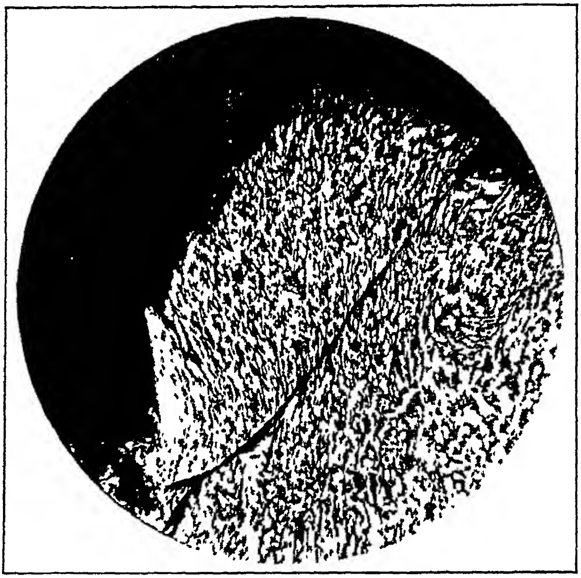


Fig. 9 Slight Distortion of Structure in Brass Chip (×50)

7 In attempting to estimate the resistance which a given metal will offer to a cutting tool it seems obvious that if we except metals such as cast iron and brass, which are broken into fragments rather than cut, the predominating factor must be the hardness of the metal which the tool is actually cutting, since this determines the force which the tool must exert in order to crush and finally rupture the metal. Likewise estimating the machinability of a ductile metal by its destructive action on the tool, three principal factors come into play, the force exerted on the tool face tending to cause wear, which depends on the hardness in the path of the tool; the heat generated, which depends on the hardness in the path of the tool, and the ratio between the hardness of the tool and the hardness of the metal in contact with it. Thus the resistance to cutting and the blunting of the tool are governed by the

hardness induced by the tool. The original hardness is of little account, since this hardness no longer exists where cutting takes place.

SECTION 2

Maximum Induced Hardness

8 In Section 1 it has been shown that the cutting tool hardens a ductile metal and that the actual separation of the chip takes place in a zone that has been so hardened. The capacity for being work-hardened differs greatly in different metals and it is convenient to have a means of measuring work-hardening capacity

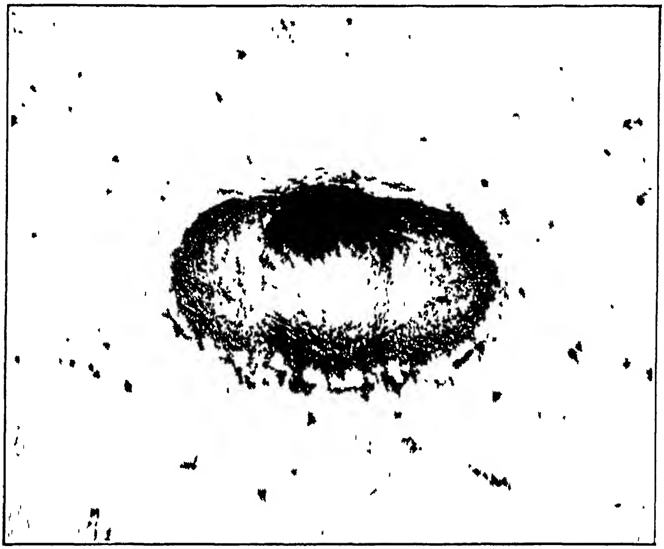


Fig. 10 Time-Work-Hardening Test Impression in Mild Steel. Original Hardness, 22; Maximum Induced Hardness, 29 (×100)

without resorting to a cutting test and the laborious method of sectioning and testing for hardness.

9 The measurement of the hardness capable of being induced by cold work is effected by a recently developed method of using the pendulum hardness tester. This instrument consists of an arched weight of 4 kilograms supported on a ball 1 mm. in diameter, the center of gravity of the pendulum being 0.1 mm. below the center of the ball. An illustration of the pendulum appears in Fig. 20. When the pendulum is placed gently on a specimen the ball makes an impression exactly like a Brinell impression (see Figs. 1 to 6), and when the pendulum is caused to oscillate through a small angle the ball rolls in this impression without slip and the time period of oscillation measures the hard-

ness of the specimen. The time-hardness number is the number of seconds occupied by 10 single swings, a hard specimen and a small impression being indicated by a slow rate of oscillation and a high time hardness number.

10 In order to measure the capacity of the specimen for being hardened by cold work, the pendulum is tilted, while still resting in the original impression, first to the right, then to the left, then back to its original vertical position. The effect of these motions is to elongate the impression into the form shown in Fig. 10, to roll the metal by two passes of the ball, and to harden it. The pendulum is now caused to oscillate as before through a small

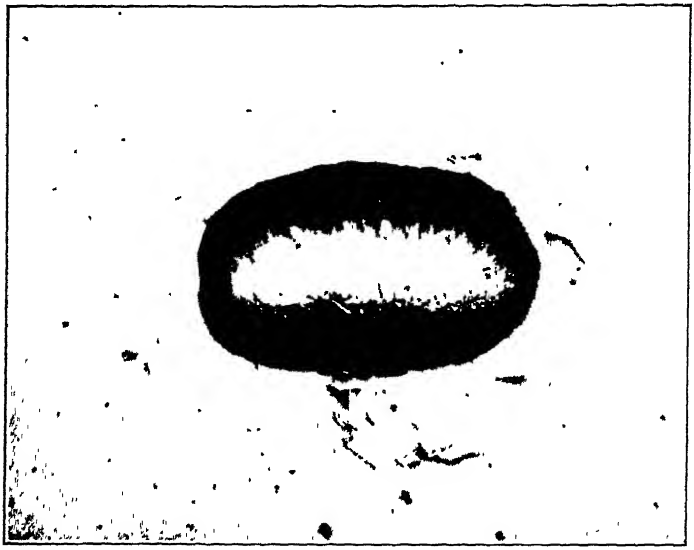


FIG. 11 TIME-WORK-HARDENING TEST IMPRESSION IN MANGANESE STEEL. ORIGINAL HARDNESS, 21; INDUCED HARDNESS (2 PASSES), 45.5 (×100)

angle while resting in the center of this rolled and hardened surface, and a second time test is made which gives the induced hardness due to two passes of the ball. The effect of this second time test is clearly seen in the bright band in the center of the impression, Fig. 10. Just as the size of the original circular impression was measured by the time of swing of the pendulum, so the induced hardness of the rolled surface is measured by the width of this band and by the number of seconds for 10 single swings of the pendulum while resting on it. If the metal has a very high work-hardening capacity, the central band is indistinguishable, just as the impression made by the time test on very hard steel is invisible. Fig. 11 shows the effect of two passes in manganese

steel, the original hardness being 21 (158*B*) and the hardness induced by twice rolling 45.5 (455*B*). The induced hardness due to two passes of the ball having been measured, the rolling process is repeated by tilting the pendulum again to the right, to the left, and back to the central position: a third time test is now made and this gives the induced hardness due to 4 passes of the ball. The process is repeated for 6, 8, 10, or more passes if necessary, but it is invariably found that after a certain number of passes, usually 6 to 10, the hardness reaches a maximum and then declines. Once the maximum has been passed no further increase of hardness can be induced by further rolling. The "maximum induced hardness" is then a measure of the capacity of the metal to be hardened by cold work.

11 Table 2 gives the actual readings in "time work-hardening tests" made on the various metals used in the cutting tests described in Section 1, and on certain other typical metals.

TABLE 2 TIME WORK-HARDENING TESTS

Passes of Ball	0	2	4	6	8	10	12	14	16	18	20	
Mild steel A.......	*21.2*	30.4	30.4	30 7	*31.5*	30 4						
Stainless iron B ..	*19.6*	*32.5*	31.6	30.4								
Stainless steel C...	*19.7*	41.2	45 4	47.3	48 2	49 8	50.3	51 5	52 3	52.7	*52.9*	49.6
Stainless steel D...	*27 5*	35.6	35.8	36.3	*36.5*	36.2						
Stainless steel E .	*18.0*	38.2	41 9	43.1	*44.0*	43.8						
Rod brass F........	*19.4*	*32.0*	32 0	31 8								
Cast iron G........	*30.0*	36.4	37.5	38 2	40.6	*40 8*	40 2					
Manganese steel . .	*21.0*	45.5	52 4	54.0	56.2	*57.2*	44 6					
Hard carbon steel..	*71.0*	*89.0*	88 0	86 0								
Aluminium bronze.	*22.4*	36 0	40.3	40 6	41 8	*42.3*	40 4					
Aluminium [1]	*5.5*	9.8	10.1	10 3	*10.7*	10.6						
Copper, cast [1]	*6.8*	17.5	19 8	20 8	21 5	23 5	23.5	23 5	23.7	*24.4*	24 4	24 2
Tungsten	*37.9*	56.6	57.9	*58.8*	58.5							

[1] Aluminium and copper were tested with a 3 mm. etched-steel ball to prevent slip.

12 It will be noticed that metals differ greatly not only in respect to the maximum hardness they are capable of attaining, but also in the rapidity with which they attain it. Thus the stainless iron B, hard carbon steel, and brass, attained their maximum hardness after two passes of the ball, whereas copper required 18 and stainless steel C 20 passes to harden them fully.

TABLE 3 TIME HARDNESS

	Induced by pendulum		Induced by tool		
	Maximum	2 passes	Path of tool	Chip	Original
Mild steel A..........	31.5	30.4	30.0	36.0	21.4
Stainless iron B	32.5	32.5	32.45	34	19.7
Stainless steel D.	36.5	35 6	36.0	39.0	27 2
Stainless steel E.... .	44.0	38.2	37.8	36 6	18.0
Stainless steel C......	52.9	41.2	43.0	38.5	19.9

13 In Table 3 the steel specimens used in the cutting tests, Section 1, are placed in the order of their maximum induced hardness measured with the pendulum. The hardness induced by the first two passes of the ball is also given, from Table 2, and the hardness in the path of the tool and the maximum hardness of the chip from Figs. 1*a* to 6*a*.

14 It is seen that the hardness in the path of the tool is always less than the maximum hardness induced by the pendulum work-hardening test, but approximates to the hardness induced by the first two passes of that test. The order of hardness induced by the pendulum and by the tool is precisely the same, and is totally different from the order of original hardness.

15 The chip hardness is variable and presents some anomalies, for example the mild-steel chip was abnormally hard. In this and two other cases the hardness of the chip was greater than the maximum hardness that could be induced by rolling with the

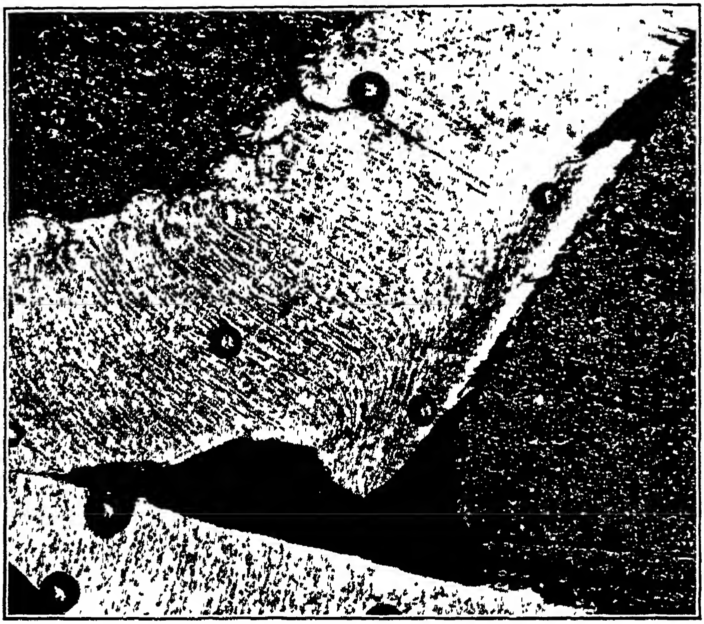


Fig. 12 Built-Up Edge in Stainless Iron B (×25)

pendulum, while in the cases of stainless steels E and C the chip was less hard than the metal immediately in front of the tool. Some of these anomalies, especially the last, may be due to the effect of heat. Cutting was done dry, and the chip attained a temperature which was in all cases high, and perhaps sufficient to cause annealing. Since the wear and eventual failure of the tool is normally caused by abrasion of the chip, it would appear that the ratio between tool hardness and chip hardness must be a principal determining factor in tool durability. It is not possible to determine the effective hardness of a chip which may have been raised to annealing temperature, but for the purpose of estimating the hardness which the tool must possess in order to resist the

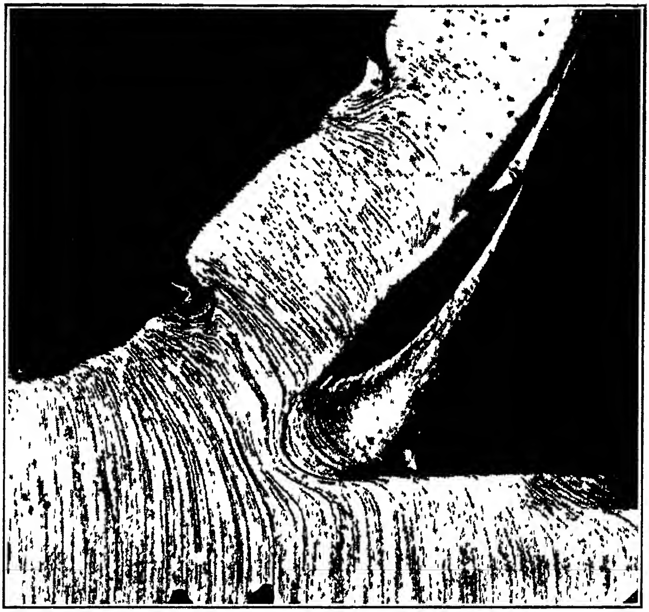


FIG. 13 BUILT-UP EDGE IN STAINLESS STEEL (×20)

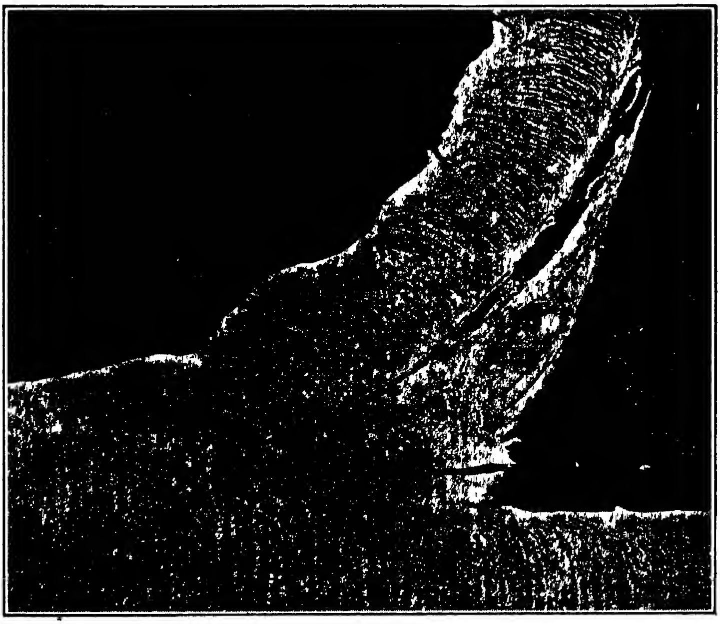


FIG. 14 BUILT-UP EDGE IN STAINLESS STEEL (×20)

abrasion of a chip, it may be safe to assume that the hardness of the latter is not less than the maximum induced hardness measured by the pendulum work-hardening test.

16 Referring to Table 2 it is seen that brass F and cast iron G both possessed a considerable capacity for work hardening. This can have had little adverse effect on their machinability, since they were not hardened by the tool, but under some cutting conditions the work-hardening capacity of such metals exercises a decisive influence, as all will be aware who have tried to file brass with a blunt file or to cut cast iron with a hacksaw which has lost its

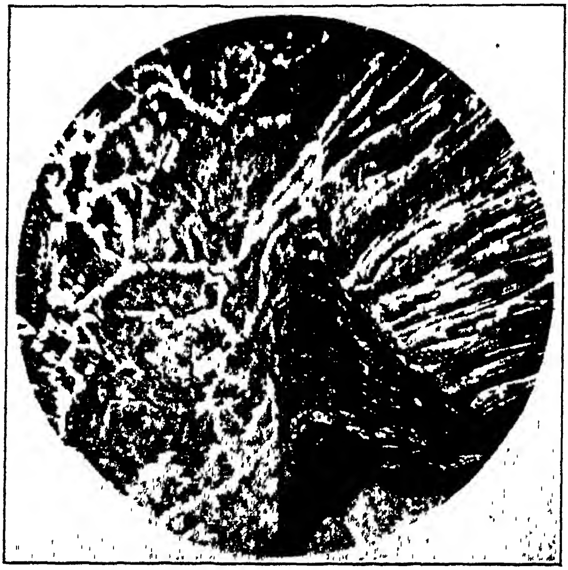


Fig. 15 Built-Up Edge in Mild Steel (×50)

sharpness. A blunt tool may harden brass, cast iron, and some bronzes to such an extent that they are almost impossible to cut.

SECTION 3

Chip Formation. The Built-Up Edge

17 In order to make clear what follows it is necessary to refer to the structure known as the "built-up edge" on cutting tools. This has often been supposed to consist of particles of metal scraped from the bar by the sharp edge of the tool, and accumulating on the top surface of the tool between the actual edge and the point where the chip impinges. The accompanying photographs, Figs. 12 to 17, show the successive stages of its formation and should suffice to dispose of the popular conception.

18 Fig. 12 is a chip of stainless iron a few seconds after cutting commenced. A large fissure runs ahead of the tool, and the surface of the chip where it impinges on the tool is dragged back by friction and consolidated into a hard layer whose structure is continuous with that of the chip, except at the upper part where separation has commenced. This confirms the statements of Rosenhain and Sturney (1)[1] whose experiments were conducted on chips in early stages of formation and who remarked on the continuity of structure of the " deformed area " and the chip.

19 A somewhat later stage is shown in Fig. 13 where the first layer has been almost worn away and has been overlaid by five

FIG. 16 BUILT-UP EDGE IN MILD STEEL (×75)

successive layers, the last of which is partly continuous with the chip, the separated portion having been broken in withdrawing the tool.

20 The successive layers are even more clearly shown in Fig. 14, where the edge is beginning to assume its characteristic triangular form and is seen to be collecting material not only from the chip but from the lower surface.

21 Figs. 15 and 16 show a later stage where the built-up edge actually surrounds the nose of the tool, while in Fig. 17 the completed edge, built up of innumerable layers and of almost razor-like sharpness, is cleaving a path through the solid steel. Professor

[1] This and similar numbers refer to the bibliography at the end of the paper.

Coker has shown (2) that the zone of separation is in a state of tension, and the edge may be regarded as severing metal which is already stretched almost to the breaking point.

22 The built-up edge is the actual cutting implement, the tool merely serving to support it. It is much harder than the metal it is cutting. Thus in mild steel A the original hardness was 21.4, the hardness in the zone of separation 30, the maximum hardness of the chip 36, and the hardness of the built-up edge 44 (440*B*). In stainless steel C the hardness of the built-up edge was 53.4 (534*B*).

Fig. 17 Built-Up Edge in Mild Steel (×100)

23 As to the manner in which the successive layers are united, Prof. Gerald Stoney has pointed out that welding can take place at quite moderate temperatures between clean metal surfaces under heavy pressure, and it is probable that the edge is a welded structure.

24 Its cutting angle is much more acute than that of the tool which supports it, and must be assumed to be the angle best suited for cutting the particular metal under conditions as to speed, feed, and so forth, in which it is formed. The tool angle might be inferred to be unimportant since the tool does no cutting, but actually the top rake angle of the tool determines the angle of incidence of the chip, and thus influences the pressure and the rate

at which the chip wears away the tool surface behind the built-up edge, this being the ultimate cause of failure of the tool. Dr. Klopstock has proposed (3) to lessen the angle of incidence without unduly weakening the tool by forming a hollow in the upper surface where the chip impinges.

25 This investigation has not been extended to acute-angled knife tools.

SECTION 4

Temperatures Generated in Cutting Metals

26 In a paper (4) on The Measurement of Cutting Temperatures the author has shown that the temperatures generated by tools cutting steel under normal conditions are very high, even when the operation is instantaneous as in the blow of a chisel,

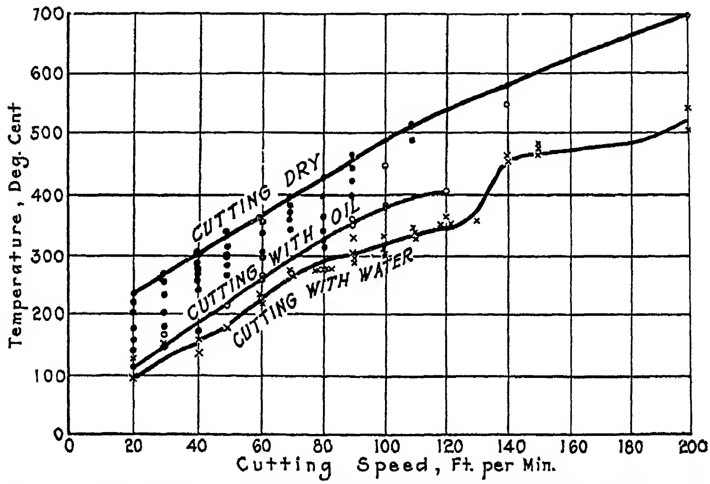


Fig. 18 Temperatures Generated on Tool-Steel Testing Machine

or when the tool and the work are flooded with oil or water. The method adopted for measuring cutting temperatures was that of the "tool-work thermocouple." The tool and the work being of different metals, as stellite and mild steel, they were treated as the two elements of a thermocouple, the tool being insulated with mica, and the thermoelectric voltage generated by the cutting temperature being measured with a millivoltmeter whose terminals were connected to the tool and to a convenient part of the machine.

27 Two diagrams are reproduced from this paper, Fig. 18 showing the temperatures generated on the tool-steel testing machine in which a cut $\frac{1}{16}$ in. wide by 0.0012 in. deep is taken on the end of a steel test tube, and Fig. 19 giving the temperatures generated in cutting a mild-steel bar on the bar lathe, the cut being $\frac{3}{8}$ in. deep and the feed 0.023 in. It will be seen from

Fig. 18 that a temperature of 500 deg. cent. (932 deg. fahr.) was generated by the tool taking a cut a thousandth of an inch deep at a speed of 200 ft. per min. when the tool was flooded with water, and a temperature of 700 deg. cent. (1292 deg. fahr.) when cutting dry under the same conditions. On the bar lathe temperatures of over 500 deg. cent. were measured when cutting with oil or water under normal workshop conditions. The instantaneous temperature generated at the edge of a file-cutting chisel was found to be 490 deg. cent. (914 deg. fahr.), the voltage in this case being measured with a string galvanometer.

28 The occurrence of these high temperatures under normal working conditions having been demonstrated, it is necessary to refer to the probable distribution of temperature.

29 The temperature measured by the tool-work thermocouple is necessarily that at a point or area of contact between the tool and the work material. Such contact always takes place where the chip impinges on the top face of the tool, a little distance behind the edge, and it does not necessarily take place elsewhere. It is reasonable to suppose, then, that the temperature measured by the tool-work thermocouple is the temperature between the tool and the chip, and that this is the maximum cutting temperature. It is the tool temperature — the temperature the tool has to withstand without becoming soft — and is therefore of supreme significance in relation to the hot hardness of the tool, which, however, is not now under consideration. The temperature in the work material results from the molecular friction due to deformation, and naturally increases from a minimum in the undeformed body of the bar, which is nearly cold, through a higher temperature where deformation commences considerably ahead of the tool, and so gradually rising, as the degree of deformation increases, to the zone where separation of the chip commences, rising further as the separated chip is crushed and bent back, and reaching a maximum where the already heated chip passes over the tool under heavy pressure. It is evident then that there is, properly speaking, no work temperature, but a temperature gradient in the work. The separation of the chip takes place somewhere on this gradient, but the actual temperature where separation is taking place cannot at present be measured. Reasons will be given for believing that it can be inferred with considerable accuracy, but for the present it must suffice to say that the separation of the chip takes place at some temperature intermediate between that of the cold bar and that of the very hot area of contact of chip and tool.

30 The existence of the temperature gradient may be inferred from a study of the progressive deformation and the gradual increase of hardness illustrated in Figs. 1 to 6. Its existence may be visually demonstrated by observing the coloration of a chip attached to a bar when the lathe has been suddenly stopped. The coloration will be seen to commence in the region ahead of the

tool where some deformation has already taken place, and to deepen progressively to a maximum in a portion of the chip which has traveled some distance from the tool.

SECTION 5

Work Hardening as Affected by Temperature

31 In 1923 work-hardening tests were first made with the pendulum hardness tester on specimens of steel at a series of rising temperatures, and the remarkable phenomenon was observed that the work-hardening capacity of the steel declined and sometimes almost disappeared at temperatures between 100 and 150 deg. cent. (212 and 302 deg. fahr.). This phenomenon has been under

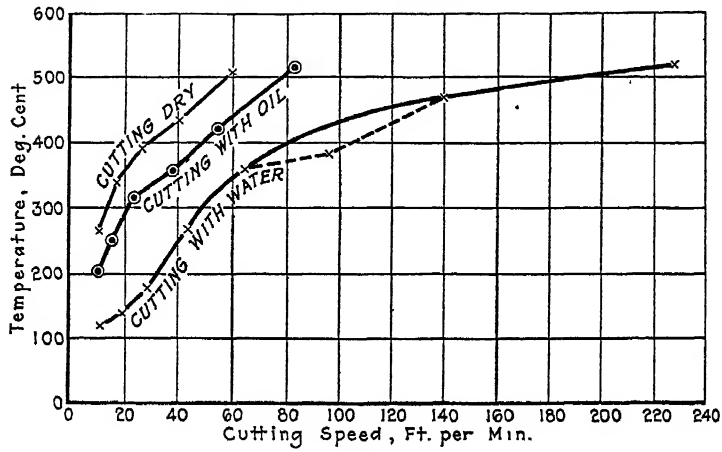


Fig. 19 Cutting Temperatures on Bar Lathe
(Cut $\frac{3}{8}$ in. deep. Traverse 43 per inch. Variable speed.)

continuous investigation since it was discovered. Its occurrence has been found to be almost universal in steels, and it has been found to occur also in many pure metals and non-ferrous alloys, usually at temperatures below 100 deg. cent.

32 The test used in these determinations is not the time work-hardening test described in Section 2 but the "scale work-hardening test," carried out with the same instrument in a different manner. The pendulum is placed on the specimen in a tilted position with the bubble at 0 on the scale and is released. It swings through a certain angle and stops when its energy has been absorbed in rolling out the original circular impression into an elongated form. The position of the bubble on the scale at the end of this first swing is a measure of the original scale hardness of the specimen, a harder specimen giving a longer swing and a higher reading. The pendulum is then tilted by hand to the left

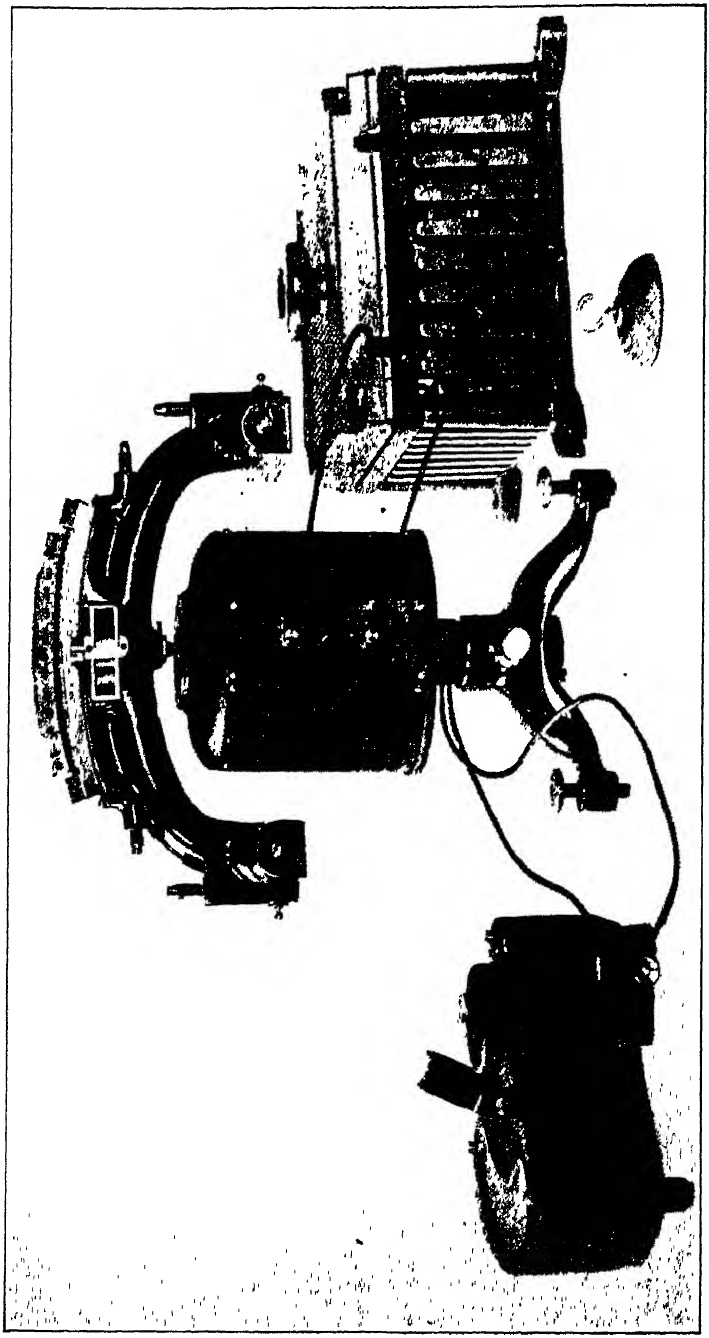


FIG. 20 PENDULUM HARDNESS TESTER AND ELECTRIC FURNACE

until the bubble is at 100 on the scale and is again released. In this case the ball rolls back along the impression which has been elongated, rolled, and work hardened, and the second reading

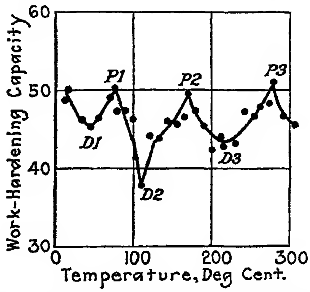


FIG. 21 TEMPERATURE-WORK-HARDENING CURVE OF VICKERS TEST-BAR STEEL

measures the scale hardness of the surface so hardened. The process is repeated until five readings have been obtained, the first on the original surface and four on the rolled and progressively work-hardened surface. Subtracting the first reading from the

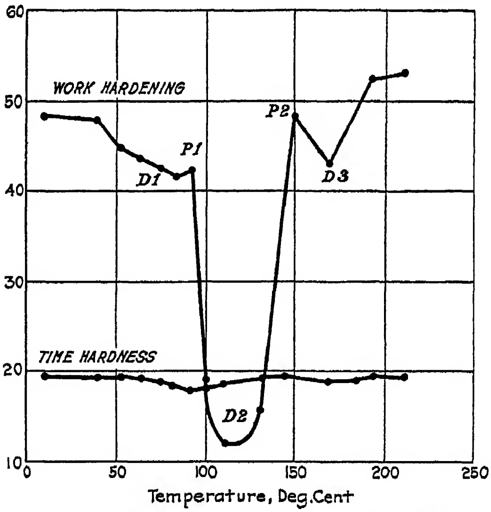


FIG. 22 TEMPERATURE-WORK-HARDENING CURVE OF MILD STEEL USED IN DEMPSTER SMITH'S LATHE TESTS

average of the next four a number is obtained which is a measure of the average increase of hardness due to rolling the specimen four times, and is called the "work-hardening capacity" of the specimen. Temperature-work-hardening tests are made with the equipment shown in Fig. 20. The specimen is placed in the electric furnace and remains there during the whole series of tests, which

are made at intervals of 5 or 10 deg. Sixty or more tests, each consisting of five hardness measurements, can be made on a surface $1\frac{1}{4}$ in. square.

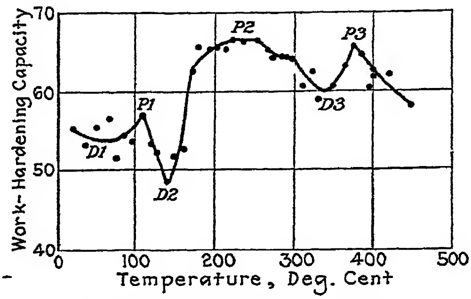


FIG. 23 TEMPERATURE-WORK-HARDENING CURVE OF SHEET STEEL

33 A typical temperature-work-hardening curve is given in Fig. 21. It is from a steel known as Vickers test bar steel, containing 0.65 carbon, 0.80 manganese, and 0.3 silicon, and is interesting because this steel has been used for the test tubes on which tool

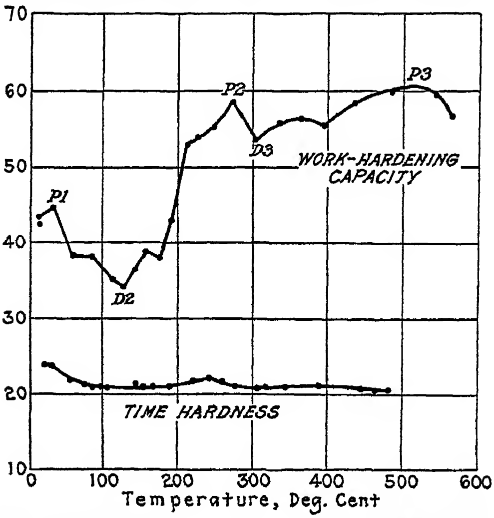


FIG. 24 FREE CUTTING STEEL. EFFECT OF TEMPERATURE ON HARDNESS AND WORK-HARDENING CAPACITY

steels have been tested during the past 17 years on the Herbert tool-steel testing machine. Further reference will be made to these tests in a later section. Fig. 22 is from a mild steel used by Dempster Smith for durability tests on the lathe; Fig. 23 from a very low carbon sheet steel; Fig. 24 from a free cutting steel; Fig. 25 from aluminum bronze; Fig. 26 from rod brass; Fig. 27, two curves from the same specimen of sheet brass, before and

after annealing; Fig. 28 from an aluminum-zinc alloy; Fig. 29 from tin (the dotted curve is a repeat test on the same specimen after annealing).

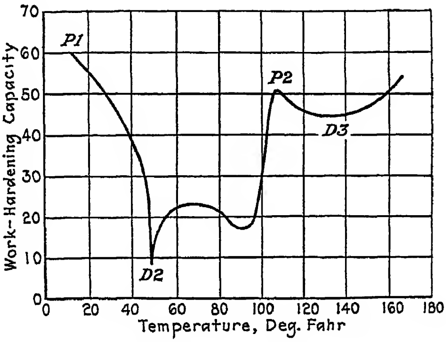


FIG. 25 TEMPERATURE-WORK-HARDENING CURVE OF ALUMINUM BRONZE

34 In the typical curve Fig. 21 there are six prominent features, three peaks, which are designated P1, P2, P3, and three depressions, D1, D2, D3. The principal depression D2 occurs in steels almost

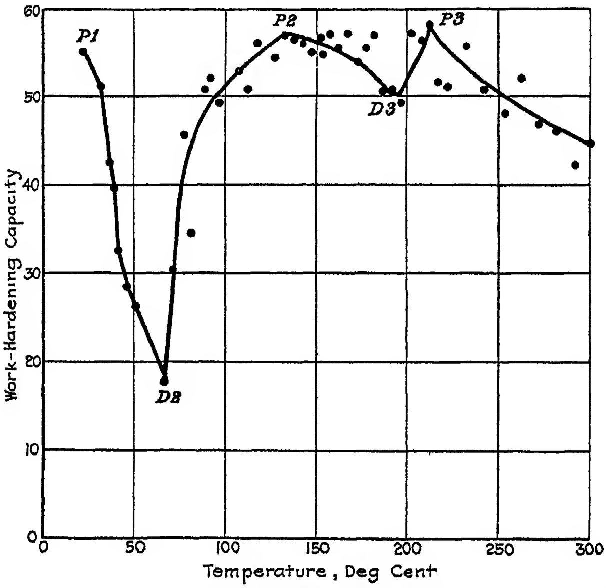


FIG. 26 TEMPERATURE-WORK-HARDENING CURVE OF ROD BRASS

always at 120 to 140 deg. cent. (248 to 284 deg. fahr.), though it has been found in high-speed steel at 150 deg. cent. (302 deg. fahr.). The curves of the non-ferrous metals are so similar that there is generally no difficulty in identifying the peaks and depres-

sions. D1 however is frequently absent, since D2 commences at atmospheric temperature. It is possible that D1 actually occurs

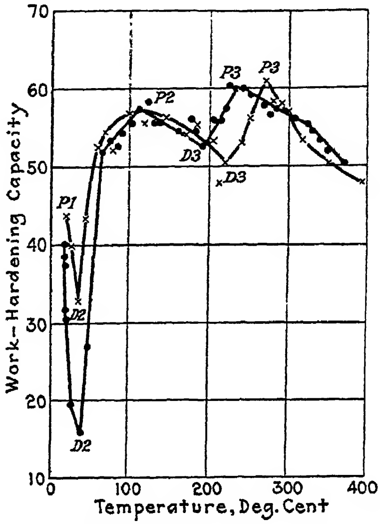


FIG. 27 TEMPERATURE-WORK-HARDENING CURVE OF SHEET BRASS

below atmospheric temperature. Some probability is lent to this view by Fig. 29 in which the curve for tin was moved bodily to

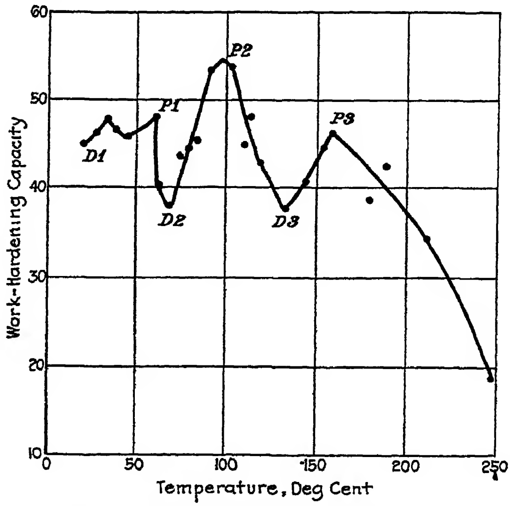


FIG. 28 TEMPERATURE-WORK-HARDENING CURVE OF ALUMINUM-ZINC ALLOY

the right by annealing, and D1, which was previously absent, emerged.

35 That phenomena so striking, and common to so many metals should have escaped the observation of metallurgists, will appear less remarkable when it is remembered that previous to 1923 the

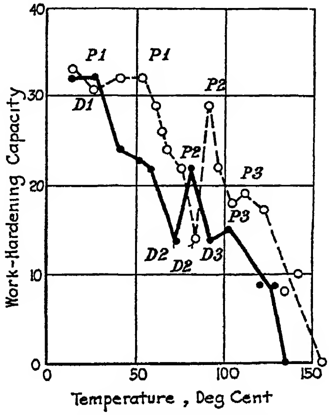


FIG. 29 TEMPERATURE-WORK-HARDENING CURVE OF TIN

work-hardening properties of metals had not been and could not be measured at a continuous series of temperatures a few degrees apart: no appliance capable of making such measurements was in existence. The observed changes in work-hardening capacity,

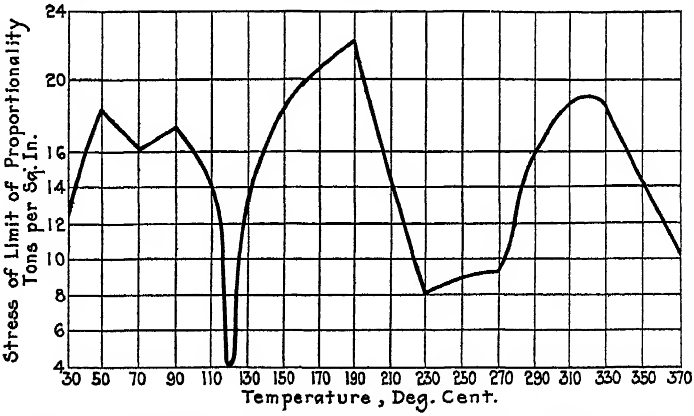


FIG. 30 LOW-TEMPERATURE CHANGES IN STEEL. GOFFEY AND THOMPSON'S TORSION TESTS

however, must be regarded as symptoms of some fundamental changes occurring in the structure of the metals, and it would be strange if these fundamental changes did not manifest themselves by influencing the results of physical tests other than the one described. Happily there exists independent evidence of a striking character.

36 In a paper on the Changes in Iron and Steel below 400 deg. cent., (5) Goffey and Thompson collected from many sources evidence of changes in steel at low temperatures, particularly at 120 deg. cent. (248 deg. fahr.), and described experiments of their own in which steel wires were subjected to torsion at a close succession of temperatures, and the limit of proportionality was ascertained at each temperature. One of Goffey and Thompson's diagrams is given in Fig. 30, and it is at once apparent that all the six features of the temperature-work-hardening curve, three peaks and three depressions, can be identified, the principal depression corresponding to D2 occurring at 120 deg. Such a similarity of results obtained independently by different investigators using totally different methods of investigation can scarcely be a coincidence. (Compare Figs. 28 and 30.)

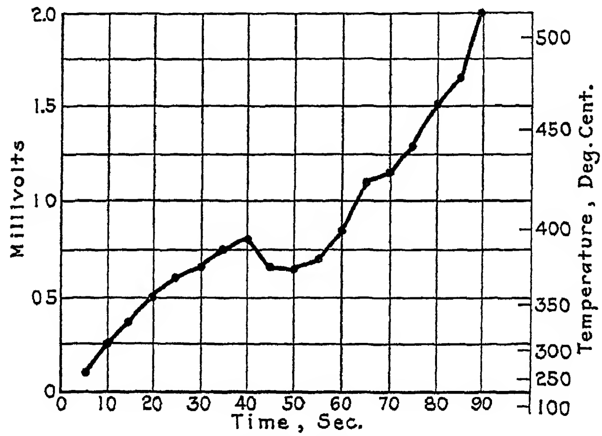


FIG. 31 VOLTAGES AND TEMPERATURE GENERATED IN SURFACING MILD-STEEL BAR

(Surface cut with water; 4-in. mild steel bar; feed, 170; cut, 0.03; speed, 223 r.p.m.)

37 The question as to which of the fundamental properties of metals undergo changes giving rise to these manifestations is one that has been the subject of much investigation. Obviously they must be properties common to the structure of the metals in which they occur, atomic, molecular, or crystalline. The evidence so far available points to some changes occurring at the crystal boundaries. It has not been possible to obtain temperature-work-hardening curves from metals in a coarsely crystalline state, chiefly because the orientation of individual crystals with respect to the direction of rolling affects their work-hardening capacity and causes erratic readings.

38 The changes, whatever their nature, are such as to affect the flow of metals strained beyond the elastic limit. They are not known to affect the structural use of metals, but are believed to

exercise a most important influence on the resistance of metals to severe deformation by cutting tools and machining operations generally.

SECTION 6

Work-Hardening Changes and Resistance to Cutting

39 It was shown in Section 1 that a metal-cutting tool work-hardens the metal before cutting it, and it was inferred that the

Fig. 32 Whitaker Ring on Mild Steel-Bar

work-hardening property of metals must be an important factor in their resistance to cutting and in their blunting effect on cutting tools. If these inferences were correct it follows that any change in the work-hardening property must be accompanied by a change

in the resistance and in the blunting effect. It has been shown in Section 5 that important changes occur in the work-hardening properties of metals as a result of changes in temperature, and in Section 4 that the range of temperatures generated in normal metal cutting comprises the range of temperatures within which the work-hardening changes occur. It remains to examine whether the observed changes in the resistance of metals to cutting, and in the durability of cutting tools, are such as would naturally result from the work-hardening changes.

40 The resistance which a metal offers to a cutting tool can be most directly measured by the vertical force on the tool while

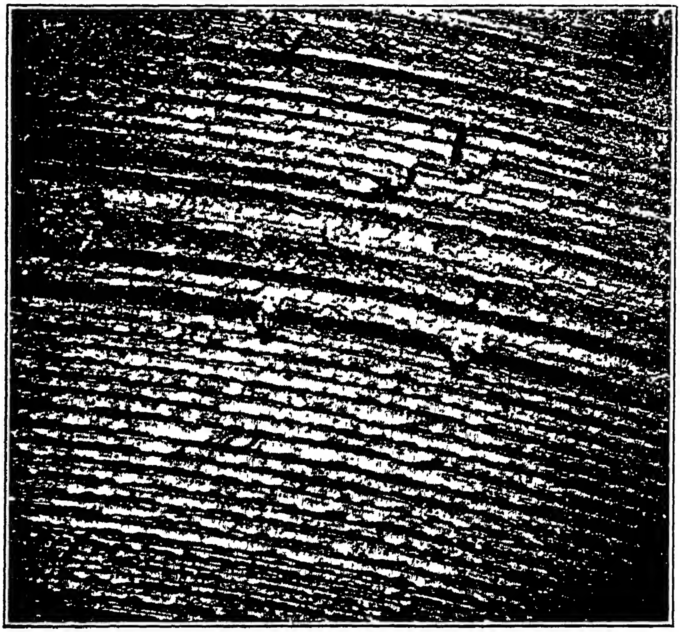


Fig. 33 Outer Edge of Whitaker Ring (×15)

cutting. Stanton and Hyde have shown (6) that when the cutting speed is progressively increased the vertical force on the tool declines to a minimum at a certain range of speeds, and rises at higher speeds. The decline in the vertical force was attributed to some effect of the cutting temperature on the steel operated upon. The cutting temperatures were not measured and the particular change in the work material was not identified.

41 Dempster Smith and Leigh (7) made similar tests and arrived at similar results, a fall in the vertical force to a minimum at a certain range of cutting speeds. It is a fair inference from these two independent investigations that the various steels used

in the experiments offered a lessened resistance to the cutting tools when they were within a certain range of cutting temperatures.

42 In order to test this matter further a method of investigation was suggested by Paul Whitaker. A surfacing cut was made on the end of a mild-steel bar, the cut starting at the center and ending at the periphery of the bar. Thus the cutting speed rose continuously from zero to any desired value, depending on the speed of the lathe. The vertical force on the tool was not measured,

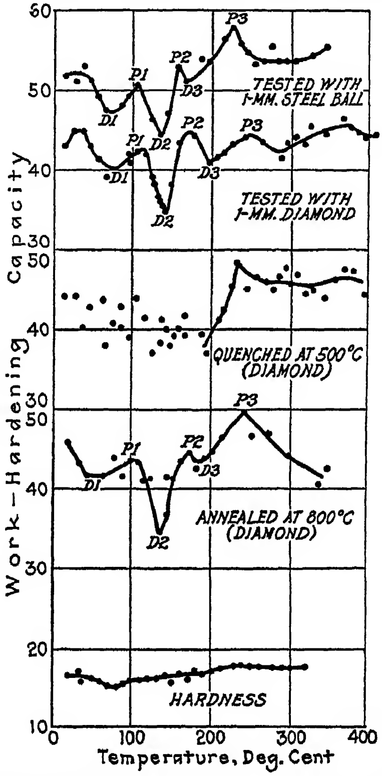


FIG. 34 TEMPERATURE-WORK-HARDENING CURVES OF MILD-STEEL BAR ILLUSTRATED IN FIG. 32

but the cutting temperature was measured by the thermoelectric method. The tool was insulated and a voltmeter was placed in circuit between the tool and the lathe. Voltage readings were taken at intervals of 5 seconds during the cut, and one of the resulting voltage curves is given in Fig. 31. The temperatures were ascertained by calibrating a couple made of the tool and a strip of the mild-steel bar, and are given on the right of the figure. Many such tests were made, and in each case it was found that the temperature at first rose steadily with the increasing speed, then

fell to a minimum, and rose to about 500 deg. cent. (932 deg. fahr.) at the periphery of the bar.

43 We have here clear evidence that within a certain range of speeds the generation of heat was less than at higher or lower speeds, and it is only possible to explain this on the supposition that the steel offered a lessened resistance to the tool when it was being cut within a certain range of temperatures, thus confirming the results of Stanton and Smith.

44 Fig. 32 is a photograph of the bar on which the surfacing cut was made, and the bright "Whitaker ring" coincides with the dip in the voltage curve. This ring is a phenomenon familiar to turners, and is certainly a temperature effect. By increasing or decreasing the lathe speed the ring can be produced at a smaller or larger diameter at will.

45 Fig. 33 is a photomicrograph of the outer portion of the Whitaker ring and of the adjacent surface. It shows that the ring is a smoothly finished surface contrasting with the plucked and torn surface outside it. Fig. 34 is a series of temperature-work-hardened curves made with the pendulum on a specimen of steel cut from the 4-in. bar, Fig. 32. It shows the usual deep D2 depression at 135 deg. cent. (275 deg. fahr.) where the work-hardening capacity of the steel was at a minimum. That the Whitaker ring coincides with the drop in cutting temperature was confirmed by observation and measurement. It is believed that the ring, the fall in cutting temperature, and the lessened resistance of the metal to the tool coincide with and are caused by the fall in the work-hardening capacity of the steel at the free cutting or D2 range of temperatures.

46 Unfortunately it is not possible to prove this by a comparison of temperatures. The significant range of cutting temperatures, Fig. 31, is always higher than the D2 range of temperatures, Fig. 34, but this was to be expected. It was pointed out in Section 4 that the cutting temperature measured with the tool-work thermocouple is the tool temperature, and that this temperature is the highest point of a temperature gradient, rising from the temperature of the cold bar. This being so it is certain that the D2 temperature of 135 deg. cent. (275 deg. fahr.) occurs at some point in the zone where cutting is taking place, and where the metal is being work-hardened by compression. Any decline in the work-hardening capacity of the metal occurring in this zone must be expected to lessen the resistance to cutting.

47 A possible clue to a more exact location of the D2 temperature in the cut is given by microscopic examination of the cut surface in and around the Whitaker ring. Observation of these surfaces, especially in metals other than steel, has strongly suggested that the formation of the ring is accompanied by some change in the formation or in the function of the built-up edge. It has been

shown that this structure normally performs a rôle secondary only to that of the tool. Rather it may be said that the tool merely acts as a support to the built-up edge, which is the actual cutting implement, and this is often apparent in the character of the machined surface. If it can be shown that when cutting is taking place at the D2 temperature the built-up edge ceases to function or functions abnormally, it may be possible to say with some certainty that the D2 temperature occurs at that point on the temperature gradient which coincides with the apex of the built-up edge, that is, with the precise zone where the chip becomes separated from the bar. This proof is not yet forthcoming, and there are formidable difficulties to be overcome before it will be possible to make a section through a chip in the act of being separated from a Whitaker ring. The nature of these difficulties will be sufficiently indicated by pointing out that the ring is associated with a definite temperature and that the lathe must be stopped so suddenly as to insure that the chip left attached to the bar has been produced at that temperaure.

SECTION 7

Work-Hardening Changes and the Durability of Cutting Tools

48 In a paper (8) presented to the Iron and Steel Institute in 1910 a description was given of a tool-steel testing machine which is essentially a vertical lathe with devices for measuring the blunting of the tool as the cut proceeds, and for recording the amount of metal cut from the work piece—a standard steel test tube. The durability of a cutting tool is measured at successive speeds, and a set of speed-durability curves is reproduced in Fig. 35. The characteristic features of these curves are the low durability of the tool at low speeds and its increase at higher speeds, and the occurrence of two peaks or maxima. The first of these characteristics is common to the speed-durability curve of every tool that has been tested on these machines in the course of seventeen years, and the second characteristic, the double peak, is of general, though not universal, occurrence.

49 Though these curves undoubtedly show the durability of tools cutting under certain standard conditions, it has been felt that some explanation of their principal characteristics was necessary before they could be accepted as representative of the behavior of tools in general, and there seemed to be only three directions in which this explanation might be sought: (1) In some obscure changes taking place in the tool as its cutting speed (and temperature) increased; (2) in some inherent difference between the testing machine and the lathe; or (3) in some changes taking place in the work piece (the test tube) as the speed and temperature increased.

(1) The changes in durability were for many years attributed to some unexplained changes taking place in the tool under the influence of cutting temperature, but a fresh difficulty arose when it was found that the same general form of curve was given by tools of carbon steel, high-speed steel, and stellite. Significant fluctuations were known to occur in the hardness and other properties of tool steel under the influence of temperature (7, 10), but these fluctuations were not in themselves capable of accounting for the form of the speed curve, and it seemed wholly improbable

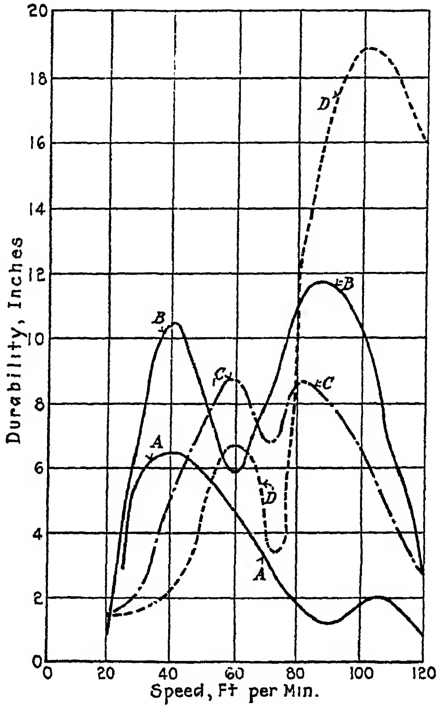


FIG. 35 SPEED-DURABILITY CURVES ON TOOL-STEEL TESTING MACHINE

that similar changes should take place at approximately the same cutting temperatures in metals so dissimilar in composition as tool steel and stellite.

(2) No essential difference was apparent between the vertical lathe known as the tool-steel testing machine and the lathes used by engineers such as would account for the peculiar form of the speed curve, and when in 1914 Denis published (9) a large number of speed-durability curves produced on an ordinary lathe and having all the essential characteristics of the testing-machine curves, including low durability at low speeds and the double peak, it became apparent that this form of curve could not be

attributed to anything in the design or construction of the testing machine. P. Denis has recently published an elaborate work (11) giving detailed instructions as to speeds, feeds, and cuts for all kinds of engineering operations, and all based on his typical double-peaked durability curves with low durability at low speeds. More recently Dempster Smith has conducted an elaborate investigation on the lathe (7, 12) and has produced a series of speed-durability curves similar in form to those originally produced on the testing machine. (See Fig. 36.)

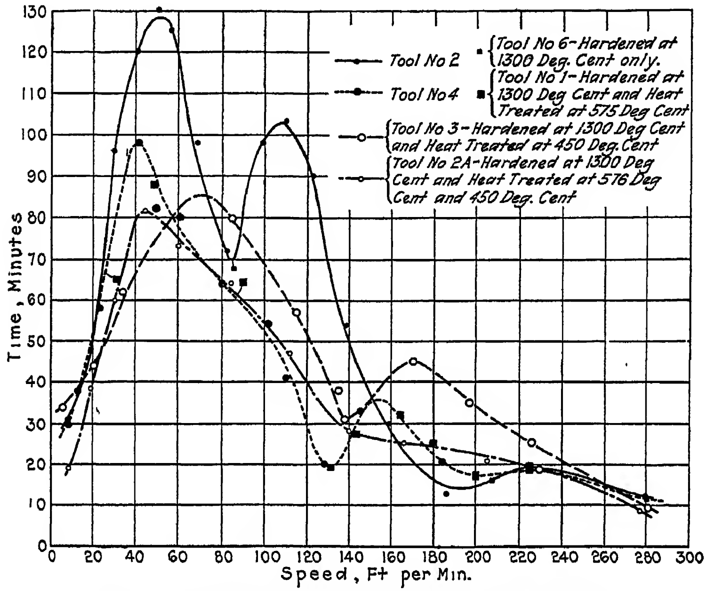


Fig. 36 Speed-Durability Curves on Bar Lathe

(3) That the peculiar form of the speed curve must be attributed to effects of temperature not only on the tool but also on the work material was early recognized, but a review of the known changes in the physical properties of unhardened steels and other metals occurring at various temperatures failed to produce any evidence of changes at low temperatures which could possibly account for the form of the speed curve. Highly significant changes, particularly in the elongation and reduction of area in tensile tests, were known to occur at temperatures as low as 300 deg. cent. (572 deg. fahr.) but there appeared to be no important changes at lower temperatures.

50 It was only when temperature-work-hardening tests were made, first on the steel used by Dempster Smith in his lathe tests, Figs. 22 and 36, and later on the Vickers test bar steel used for

the test tubes, and later still when Goffey and Thompson's results were brought into review, that light was thrown on the subject. The temperature-work-hardening curve of the test-tube steel is given in Fig. 21; the speed-durability curves produced in cutting this steel, in Fig. 35. Now if we assume, as we have grounds for assuming (Fig. 18), that a tool cutting with water at 20 ft. per min. on the testing machine generates a temperature rather below 100 deg. cent. (212 deg. fahr.), it will be seen (Fig. 21) that the test tube is then in the condition P1 in which its work-hardening properties are at a maximum. If we now suppose the speed and cutting temperature to be raised, and assume, as we must, that a decline in work-hardening of the tube will be accompanied by increased durability of the tool, it is apparent that the speed-durability curve produced on the Vickers steel will be one in which the durability increases from a low value at low speeds to a maximum durability corresponding to the minimum work-hardening at the D2 temperature, and that there will be a second maximum durability corresponding to the minimum work-hardening at the D3 temperature. The speed-durability curves of tools cutting this steel must assume the general form which experience has shown them to assume.

51 Similarly with Dempster Smith's lathe curves, Fig. 36, produced on bars of steel whose work-hardening characteristics are shown in Fig. 22, it is evident that the durability of tools cutting this steel should increase from a low value at the P1 temperature to a maximum at the D2 temperature and decline to a minimum corresponding to the P2 temperature. The general form of the curve can be thus accounted for.

52 It is not suggested that the durability of the tool and the changes taking place in the tool are of slight importance. The quality of the tool is indeed the predominating influence in the durability curve, whether produced on the testing machine or the lathe, but the features of the curve which are rightly attributed to the quality of the tool must be regarded as superimposed on a curve whose general form is determined by changes in the work-hardening properties of the material operated upon.

53 While it is true that this consideration introduces an element of complexity into the interpretation of the speed curves, it does not destroy their validity in estimating the quality of cutting tools, since the characteristic changes shown in the typical curve of Vickers steel, Fig. 21, have been found to occur in almost every variety of steel that has been tested, and the principal change, the D2 depression, always occurs in steel at approximately the same temperature, 120 to 140 deg. cent. (248 to 284 deg. fahr.). The behavior of the tool cutting Vickers steel test tube is therefore the behavior that may be anticipated in cutting any other steel.

SECTION 8

Conclusions

54 A tool cutting a ductile metal work-hardens the metal before cutting it. In so far as the machinability of a metal depends on hardness, it must therefore depend on the hardness induced by the tool and not on the original hardness.

55 Metals differ greatly in their capacity to work-harden. This capacity is capable of measurement by a test comprising a succession of alternate rolling operations and hardness measurements on the same spot. The hardness so induced and measured increases to a maximum, and declines with further rolling. The "maximum induced hardness" is a measure of the principal factor in the machinability of ductile metal.

56 Cast iron, though capable of being hardened by cold work, is not appreciably hardened by a sharp cutting tool. Its work-hardening capacity is not generally a factor in its machinability, though it may be under special conditions. Other metals, such as brass, are intermediate between cast iron and steel in respect to the degree of plastic deformation and of work-hardening caused by the tool, and in respect to the influence of their work-hardening capacity on their machinability.

57 The built-up edge is the actual cutting implement when obtuse-angled tools are used on ductile metal, and is formed by the welding together of successive layers of metal derived from the zone of separation. It is much harder than the metal from which it is built up, and assumes a cutting angle appropriate to the cutting conditions under which it is formed.

58 Tools of high-speed steel when cutting at maximum output generate temperatures approximating to 500 deg. cent. (932 deg. fahr.), even when artificially cooled. This temperature occurs where the chip impinges on the tool. It is the maximum cutting temperature and the temperature which the tool must be capable of withstanding without softening. A temperature gradient exists in the metal ahead of the tool, and the separation of the chip takes place at a point on this gradient which is not exactly determined, but is much below the maximum cutting temperature.

59 Steels, non-ferrous alloys, and pure metals undergo a series of remarkable changes at temperatures generally below 300 deg. cent. (572 deg. fahr.). These changes are manifested by fluctuations in their capacity for work-hardening. Six definite change points, three maxima and three minima, generally occur in steels, and of these at least five are usually capable of identification in non-ferrous metals and alloys. The principal change is a low work-hardening capacity occurring in steels at 120 to 140 deg. cent. (248 to 284 deg. fahr.), and in brasses and bronzes and some pure

metals at 40 to 70 deg. cent. (104 to 176 deg. fahr.). From the fact that analogous changes occur in metals dissimilar in physical characteristics, it is to be inferred that the changes are connected with that which is common to all of them, namely their crystalline structure. Changes analogous in character, in sequence, and in the temperatures at which they occur have been found in the limit of proportionality in torsion in the case of steel.

60 The changes in work-hardening capacity are not known to affect the structural uses of metals, but have a marked effect on their resistance to severe deformation such as occurs under the action of cutting tools. In particular the decline in work-hardening capacity of steel at the D2 or "free cutting range" of temperatures, about 130 deg. cent. (266 deg. fahr.), is accompanied by a lessened vertical force on the tool, a lessened generation of heat, a different character of chip, a smoother finish on the cut surface, and a great increase in the durability of the tool when cutting within that range of temperatures.

SECTION 9

Practical Applications

61 It is not generally practicable or desirable to determine the cutting conditions so that cutting takes place in the free cutting range of temperatures, though this may be done in the case of metals which are extremely difficult to cut under any other conditions. There is reason to believe, however, that the form of the work-hardening curve can be modified and in particular that the D2 depression corresponding to the free cutting range can be made deeper and wider by previous heat treatment of the metal, and perhaps by modifying its composition. A metal so treated offers a slight resistance to cutting over a wide range of cutting temperatures and can therefore be cut freely under a wide range of cutting conditions, speed, feed, and depth of cut. This is a subject ripe for investigation by metallurgists.

62 Although in metal-cutting operations it is not generally practicable to determine the cutting conditions with reference to the D2 range of temperatures, there may be other metal-forming operations, notably press work, in which, by a suitable regulation of working temperature, advantage may be taken of the D2 decline in work-hardening capacity. This has been found practicable in working certain refractory sheet metals, and the subject is now under investigation.

63 Since the limiting factor in the productivity of metal-cutting operations is the inability of the tool to withstand without injury the temperature generated in cutting, an advance in productivity may be sought (*a*) by providing metals which can be cut at higher

speeds without generating excessive temperatures, and (*b*) by providing tools capable of withstanding higher temperatures without being softened. These lines of advance can only be followed by studying (*a*) the effect of temperature on the work-hardening properties of metals, and (*b*) the effect of temperature on the hardness of tools.

64 In order that a tool may cut a given metal, the tool must be harder than the metal. In order that the tool may cut the metal without undue wear, the hardness of the tool must exceed the hardness of the metal in a certain minimum ratio. Since cutting heats the tool and hardens the metal, the significant hardness ratio is that between the hot hardness of the tool and the work-hardened or induced hardness of the metal. The cold hardness of the tool and the original hardness of the metal have little

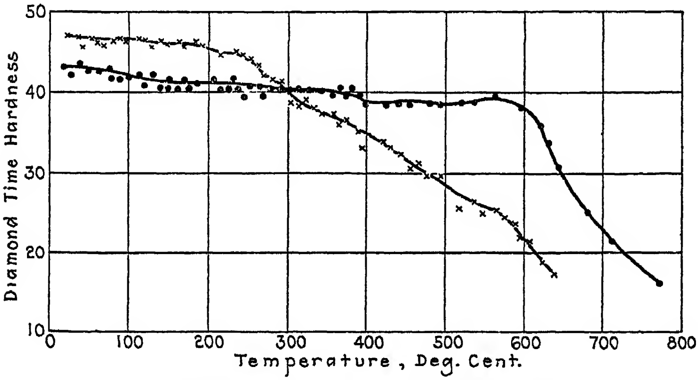


Fig. 37 Hot-Hardness Curves. Tool A, Crosses; Tool B, Dots

significance in relation to the ability of the one to cut the other under conditions of high productivity. This can be illustrated by an example taken from practice.

65 Two tools A and B were required to cut a certain steel X. Their respective cold hardnesses were,

Tool A, *dT* 47 = 635 Brinell. Tool A was hard.
Tool B, *dT* 43 = 580 Brinell. Tool B was moderately hard.
Steel X, *T* 18 = 116 Brinell. Steel X was dead soft.

66 The hot hardness of the two tools is shown in Fig. 37. They were placed in the electric furnace and tested with the pendulum and 1-mm. diamond at temperatures rising from cold to the softening temperatures.

67 Steel X was tested with the pendulum time work-hardening test and was found to have a maximum induced hardness 44 equal to 440 Brinell.

68 At 300 deg. cent. (572 deg. fahr.) (cutting temperature) tools A and B had equal hardness (see Fig. 37) *dT* 40 = 540 Brinell

Both were harder than the maximum induced hardness of X (440 Brinell) and would cut it.

69 At 400 deg. cent. (752 deg. fahr.) tool A had hardness dT 34 = 460 Brinell, which is little more than the maximum induced hardness of steel X. It could not cut it. Tool B had hardness dT 39.5 = 535 Brinell. It was harder than steel X and would cut it.

70 At 560 deg. cent. (1040 deg. fahr.) tool A still retained hardness 39.5 (535 Brinell). It would still cut steel X. Tool B was quite soft.

71 The inference is that the hard tool A would be incapable of cutting the dead soft steel X except at slow speeds. The softer tool B should be capable of cutting steel X at quite high speeds, but would wear rapidly owing to the rather low tool work-hardness ratio. A tool having higher hot hardness would be desirable.

Tool A was a hacksaw blade containing 2 per cent tungsten
Tool B was a hacksaw blade containing 18 per cent tungsten
Steel X was stainless steel.

72 Both blades were set to cut stainless steel dry at 146 strokes per minute. Blade A failed in a few seconds. Blade B made several cuts, during which it became red hot, but blunting was somewhat rapid.

73 It is self-evident that anyone concerned with the efficiency of high-speed tool steel should be familiar with the hot-hardness curve of the particular steel and hardening process employed. Such a curve is easily made, but takes time and softens the steel. It is neither practicable nor necessary to make a complete curve in order to check the hot-hardness of tools in quantity. It is practicable, however, to fix a limit below which the hot hardness must not fall, for example, diamond time hardness 40 at 550 deg. cent. (1022 deg. fahr.). This involves a single test, occupying a few seconds, on the hot tool. It can be carried out immediately after the secondary heat treatment. If, by improvement in the steel or in the hardening process, the limit can be raised, the efficiency and output of the plant can be correspondingly increased. The cold-hardness test gives no important information and can be omitted. The efficiency of high-speed steel depends on hot hardness and can only be ascertained by testing it hot.

74 The author expresses his grateful acknowledgment of valuable assistance rendered by Messrs. Brown Bayley's Steel Works, Ltd., Sheffield, in preparing and supplying many specimens of stainless steel, to Messrs. J. J. Saville, Ltd., Sheffield, for stainless iron, and to Messrs. Hadfields, Ltd., Sheffield, for manganese steel used in these experiments.

LIST OF REFERENCES

1 Flow and Rupture of Metals during Cutting, Rosenhain and Sturney. Proc., Instn. Mech. Eng., Jan. 1925, p. 141.

2 Report on the Action of Cutting Tools, E. G. Coker. Proc., Instn. Mech. Engrs., March 1925, p. 357.

3 Recent Investigations in Turning and Planing and a New Form of Cutting Tool, Hans Klopstock. Trans., A.S.M.E., vol. 47 (1925), p. 345.

4 Measurement of Cutting Temperatures, E. G. Herbert. Proc., Instn. Mech. Engrs., Feb. 1926, p. 289.

5 The Changes in Iron and Steel below 400 deg. cent., A. Goffey and F. C. Thompson. Iron & Steel Inst., May 1923, no. 1, vol. cvii, p. 465.

6 An Experimental Study of the Forces Exerted on the Surface of a Cutting Tool, Stanton and Hyde. Proc., Instn. Mech. Engrs., Jan. 1925, p. 175.

7 Experiments with Lathe Tools on Fine Cuts, Dempster Smith and A. Leigh. Proc., Instn. Mech. Engrs., March 1925, p. 383.

8 The Cutting Properties of Tool Steel, E. G. Herbert. Iron & Steel Inst., no. 1, 1910, p. 206.

9 Aciers à Outils, M. Denis. Revue de Métallurgie, xi, no. 1, Jan. 1914, p. 4.

10 The Influence of Heat on Hardened Tool Steels, E. G. Herbert. Iron & Steel Inst., no. 1, 1912, p. 358.

11 Organisation Scientifique de l'Usinage, P. Denis. Pub. H. Morin, Paris.

12 Some Hardness Observations of a Tool Steel, Dempster Smith and I. Hey. Proc., Manchester Assn. Engrs., March 1924.

DISCUSSION

R. S. Kerns.[1] The paper is incomplete in that it does not tell the method of cutting the specimens described in Par. 1. This cutting of the specimen, together with the polishing operation, seemingly would have a tendency to work harden the surface of the specimen. The author has attempted to show that a machine operation on metal will work harden the surface of the object as well as the metal in the path of the tool. If the machine operation has work hardened the specimens, then the values shown in this paper on increased hardness are not true values due to the tool alone, but result from a second work-hardening process. Hence the values over the area investigated are not all based on the same conditions. According to Table 3, the hardness of the specimen in the path of the tool is approximately the same as that produced by two passes of the pendulum. The double cold-working process produced by the tool and the subsequent cutting of the metal as described in Par. 1, give time hardness numbers almost identical with those produced by two passes of the pendulum, which is also a double cold-working process.

There is no doubt that the metal is hardened by the tool in making a cut. However, it is doubtful that the values shown by

[1] Pennsylvania State College, State College, Pa.

the author are the true hardness numbers of the metal caused by the tool alone. If these values are high, then how can it be shown what the true hardness is, when subject to the working tool and to no subsequent work hardening?

The author has reported on only one shape of tool. Would a different shape of tool nose work harden the metal in the same manner as the one described? Or, would a tool with an increased rake have a tendency to work harden the surface of the specimen or the metal in the path of the nose of the tool as did the tool described? It would seem that the author has opened up new fields in tool design, or methods of setting the tool for efficient cutting.

R. Poliakoff.[1] The author has shown numerically the changes in hardness that take place in the metal lying in the path of the tool that is cutting it. He shows that this induced hardness affects the process of cutting, and the life of the tool. If now, instead of considering the durability of the tool, as does the author, we consider only a predetermined or predescribed uniform condition of wear or bluntness of the tool, and take longitudinal cuts on a bar in a lathe at increasing speeds, the result must be that the lengths of the surfaces cut will follow the author's curves of durability and temperature-work-hardening curves. This conclusion naturally follows, if the author's deductions are correct, that the induced hardness is influenced by the temperatures at which deformation occurs, and changes with the cutting speeds.

Tests by the writer confirm this conclusion. In making finishing cuts in a lathe on a steel bar, both with carbon and high-speed steel tools, it was found that the lengths of surfaces machined with increasing speed follow a curve similar to those of the author. If the quality of the finish at different speeds could be represented on a diagram, the latter would have a form similar to the curve giving the relation between cutting speeds and surfaces machined. The phenomenon would be similar to the Whitaker ring described in Par. 42, the ring being translated, as it were, from a plane, as in the paper, into space. It is quite probable that on a bar of a large diameter, a second Whitaker ring probably would be observed, although of a different degree of brightness.

In Par. 24 the author mentions a hollow in the upper surface of the tool as proposed by Dr. Klopstock. As a matter of fact, this was proposed by the writer at the Cincinnati meeting of the Society, eight years before Dr. Klopstock proposed it.[2] The theory then advanced for this proposal was based on the formation of elementary, i.e., very thin, chips, of which the actual chip can be imagined as consisting. These thin chips must cause internal friction, displacement, and also heat formation.

[1] Mechanical Engineer, New York, N. Y. Mem. A.S.M.E.
[2] *Jour.* A.S.M.E., 1917, p. 700.

Jos. K. Wood.[1] The writer has a letter from the Brown-Bailey Steel Works, Ltd., of Sheffield, England, to the author, in which they make the following statement in reference to properties of steel D, whose time-hardness is given in Table 3: "This steel machines fairly easily and may be said to be closely comparable in machining properties with heat-treated nickel-chrome steel of 55 to 60 tons tensile strength."

In regard to steel E, they make the statement: "It is more difficult to machine E than steel D, and requires a higher speed and slower feed. It can be turned more easily than it can be drilled. Care has to be taken not to allow the tool to rub on the work surface and increase machining difficulties."

In regard to steel C, the corresponding statement is made: "Similar remarks apply as to steel E. It is a little more difficult all around to machine than E. From experiments other than cutting tests, we have determined that steels E and C do work harden much more rapidly than steel D, and that steel C work hardens more rapidly than E, and it is for this reason that we manufacture steel E in preference to steel D."

The Author. Mr. Kerns is concerned about a possible work-hardening effect due to the process of cutting the sections. In all metallographic work it is so much a matter of routine to adopt extreme precautions to avoid any disturbance of the metal under observation, such as might be caused by the process of section cutting, that it was not considered necessary to describe the process in detail. The sections were cut in the first place with a fine-toothed hacksaw. A considerable thickness of metal was then removed from the cut surface by careful hand polishing on coarse emery paper. All the metal affected by the coarse emery was then removed by hand polishing on emery paper of a finer grade, and this was repeated with emery papers of increasing fineness, followed by a final polishing with rouge and immersion in weak acid to dissolve the polished surface.

The ordinary process of preparing specimens for hardness tests is essentially one of section cutting, and no special precautions, such as those detailed above, are taken against work-hardening effects. It may therefore be doubted whether any hardness figures have ever been published which were less likely to have been affected in the manner suggested by Mr. Kerns, than those to which he refers.

Mr. Kerns is correct in stating that only one shape of tool was used in the experiments, though an exception was made in the case of the tool used for cutting brass. This tool had zero top rake in accordance with the usual practice in cutting that metal. Preparations are being made for further experiments in chip

---

[1] Consulting Engineer, New York, N. Y. Assoc-Mem. A.S.M.E.

analysis in which it is proposed to use acute-angled tools such as are commonly empolyed on the bar lathe, but no actual results have yet been obtained.

Mr. Poliakoff has given valuable confirmation of the peaked character of the durability curve of a tool cutting at increasing speeds, and especially interesting is his evidence that the peak of the durability curve coincides with a maximum in the quality of the finished surface produced by the tool. This not only adds an important link to the chain of evidence connecting the D2 depression in the temperature-work-hardening curve with the peak in the durability curve and with the smooth finish of the Whitaker ring, but also shows that the ring phenomenon may be turned to practical advantage by selecting the critical cutting speed which gives the maximum durability to a finishing tool, and the smoothest finish to the work.

His surmise that a second ring may be observed is correct. It not infrequently happens that three distinct rings occur, and it is believed that these are produced at cutting temperatures corresponding with the D1, D2, and D3 depressions in the temperature-work-hardening curve.

No. 2023

A RESEARCH IN THE ELEMENTS OF METAL CUTTING

CONFINED TO TOOL SHARPNESS, TOOL FORM, CHIP DIMENSIONS, AND THE FORCE INVOLVED

By Orlan W. Boston,[1] Ann Arbor, Mich.
Member of the Society

This paper gives an account of an investigation in the fundamental elements of metal cutting conducted in the Machine Tool Laboratory at the University of Michigan. The object of the investigation was to determine a relation between the force on the tool in the direction of cut for a constant cutting speed of 20 feet per minute, and the degrees of tool sharpness, the various tool angles, the width and depth of cut, and the physical properties of the materials cut. Nine representative types of material were cut including three carbon steels, three alloy steels, brass, and annealed and unannealed cast iron. The cutting was confined to straight-line motion on a planer and the tools used were of the end-cutting type. No cutting fluids were used and but one element was varied at a time.

The results show that the clearance angle has no influence on the force on the tool so long as the tool does not drag on the work, that the force on the tool remains constant for a wide variation of keenness of cutting edge, and for thick chips, particularly, the tool edge may be rounded to $\frac{1}{32}$-in. diameter without appreciable increase in the cutting force. It is also shown that the cutting force on the tool is reduced in direct proportion to the increase in front-rake angle, all other factors remaining constant. It is shown that thick chips are removed more efficiently than thin chips, and that narrow chips are removed more efficiently than wide chips. The results also indicate that there is an apparent relation between some of the physical properties of the metals and their machinability or the cutting force on the tool for the carbon steels in one group, the alloy steels in a second group, and cast iron in a third group.

A bibliography on metal cutting has been made a part of the paper to enable those interested in the subject readily to locate those articles of interest or value to them.

[1] Associate-Professor of Shop Practice and Director of Engineering Shops, University of Michigan.

Contributed by the Machine Shop Practice Division and the Research Sub-Committee on Cutting and Forming of Metals and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

DURING the last three years an investigation has been conducted as a project of the Department of Engineering Research in the Machine Tool Laboratory of the University of Michigan on elementary principles involved in the cutting of metals.

2 At the beginning, it was decided that the experiments should be confined to an investigation of a basic, scientific, rather than a practical, applied, nature. It was felt that the influence of curvature of the surface cut should be eliminated by confining the cuts to straight lines; also that each cut should be of sufficient length to permit the conditions of cutting to become uniform at the instant the reading was taken. In order to secure a straight-line cut having a constant cutting speed throughout the stroke, a planer was selected as the machine tool on which the work would be done.

3 The problems undertaken were as follows:

a To determine the influence of the degree of sharpness of the cutting edge of the tool, as the only variable, on the force on the tool or the energy required to remove a given volume of metal

b To determine the influence of the clearance back of the cutting edge of the tool, as the variable, on the force on the tool or the energy required to remove a given volume of metal

c To determine the influence of the front rake of the tool, as the variable, on the force on the tool or the energy required to remove a given volume of metal

d To determine the influence of side rake (skew), as the variable, on the force on the tool or the energy required to remove a given volume of metal

e To determine the influence of the depth of cut or width of cut, as the variable, on the force on the tool or the energy required to remove a given volume of metal

f To find a relation between the force on a tool of a given shape required to remove a specific chip of a given material and its physical or chemical properties.

The Tools

4 All tools used in these tests were of the cut-off or end-cutting type. They were prepared for each problem in groups identical as to geometric form, heat-treatment, grinding, etc., except for the single variable under consideration. In some instances the group of tools for one problem was reforged or machined into tools of another form for another problem. They were all machined or forged from carbon or high-speed steel bars $2\frac{1}{8}$ by $1\frac{1}{4}$ in. cross-section and about 12 in. long. Fig. 1 is presented to illustrate the terms used for the parts and angles of the tools

and the size of the chip. At *A* is shown a front and side view of a front-rake tool. The front-rake angle is the angle between the tool face and the vertical plane *A-A*. F. W. Taylor called this angle the back slope. The clearance is the angle between the body of the tool back of the cutting edge, or flank, and the work. The cutting angle is the sum of the lip and clearance angles. At *B* is shown a tool which has both front and side rake. The cutting edge is in a horizontal plane. The side-rake angle is best shown in the plan view as that angle between the cutting edge *BO* and the vertical line *BB*. It may also be measured in any plane *X-X* perpendicular to the paper.

5 A typical record sheet for each piece of steel from which a tool was made is given in Appendix No. 1-A. The first tool was designated as A-1, and its chemical composition, whether machined

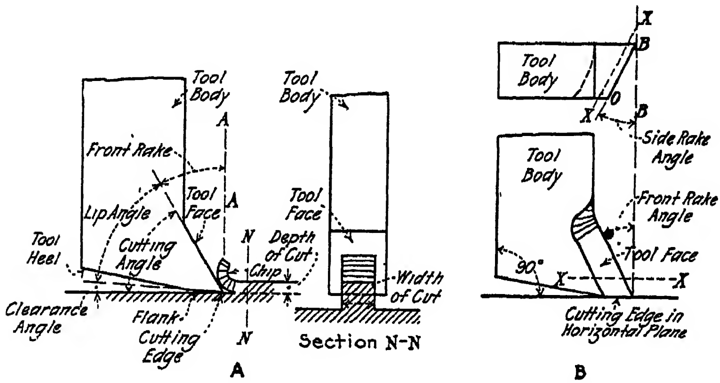


FIG. 1 TOOL PARTS AND ANGLES

or forged, heat-treatment, angles, hardness, etc., were given. When this tool was modified in any way, it was subsequently designated as another tool, A-2, etc. Every experiment sheet shows the tool used in that test. A table listing all tools and indicating the problems for which each was used is given in Appendix No. 1-B.

6 The tools were hardened and drawn in accordance with the specifications received from the manufacturer of the steel. Endurance was not an essential feature of the test. It was, however, necessary that a tool retain its geometric form with very little variation throughout a test. Usually the test was not of sufficient length to cause failure of the tool. Some of the tools were hardened in a forge fire and others in gas-fired furnaces, the temperatures of which were definitely determined by thermocouples. Our results show that the furnace-heat-treated tools give more consistent results than those treated in the forge fire. This is confirmed by tests conducted at the Bureau of Standards as described in their technical news bulletin No. 105 of January, 1925. It was

found that comparable performance was obtained when tools were raised to approximately equal temperatures for equal times in hardening. The Bureau test failed to show the superiority of the forge-fire hardening over gas, oil-fired, or electric furnace hardening.

Method of Measuring Forces

7 Fig. 2 shows the Liberty 30-in. by 36-in. by 8-ft. planer on which the tests were conducted. The power was furnished from

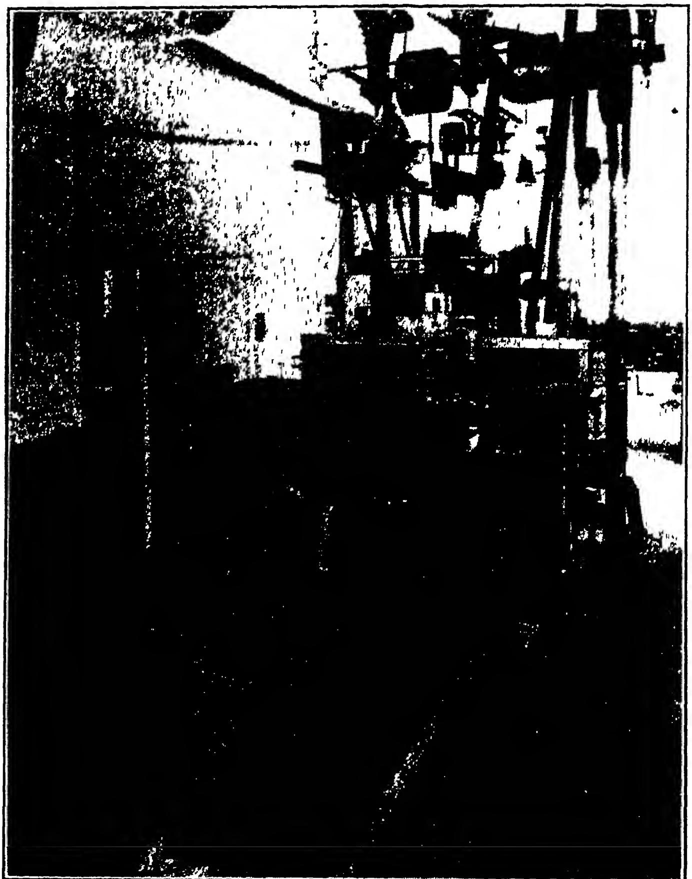


Fig. 2 Planer and Dynamometer Used in Experiments

a main drive shaft, a satisfactory arrangement in that the horse-power output of the machine was measured rather than the input. The cutting tool is shown mounted in the standard head on the cross-rail. The specially designed dynamometer is shown mounted

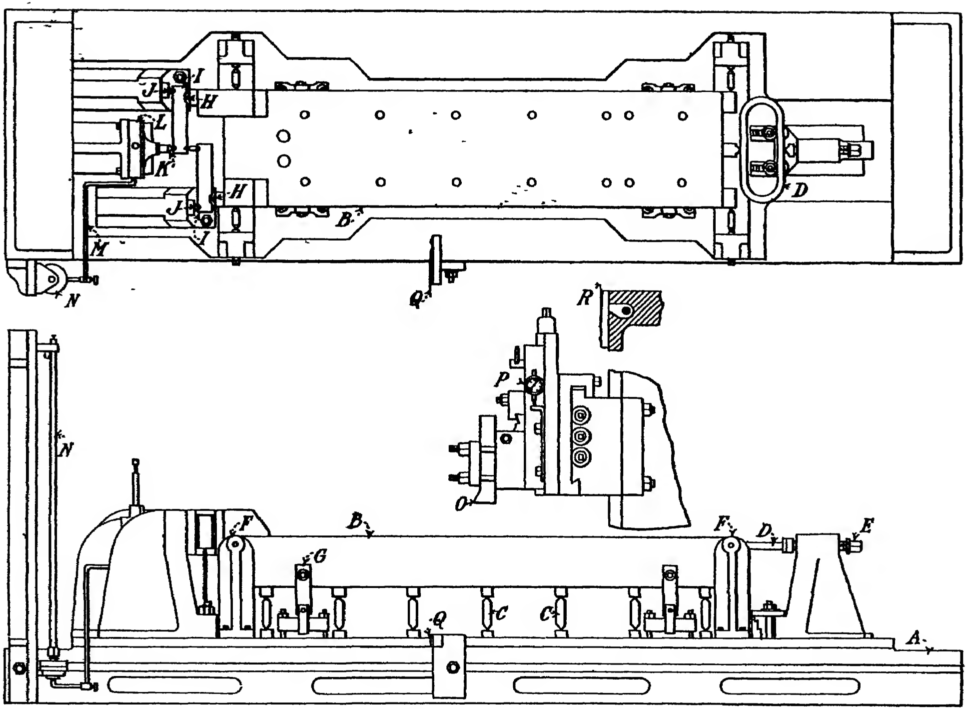


FIG. 3 LINE DIAGRAM OF DYNAMOMETER

on the planer table. The material being cut is clamped on the bed of the dynamometer.

8 Fig. 3 shows a line diagram of the dynamometer which consists essentially of a cast-iron bed B weighing about 1300 lb., mounted on seven horizontal knife edges C. The table is prevented from moving sideways by the four horizontal floating pins shown in the plan view which are supported by the brackets F. The bed is prevented from being lifted by four knife edges shown at G. An initial load of 2000 lb. tending to force the bed to the left was put on the bed by compressing the loop spring D. This loop spring was made of $\frac{3}{4}$-in. square spring steel having major and minor axes to center of bar of $11\frac{1}{2}$ and $4\frac{3}{4}$ in., respectively, and had previously been calibrated on a Riehlé testing machine. It

was found to give more satisfactory results than helical springs, as identical load-strain curves were obtained for increasing or reducing increments of load. Fig. 4 shows a close view of the

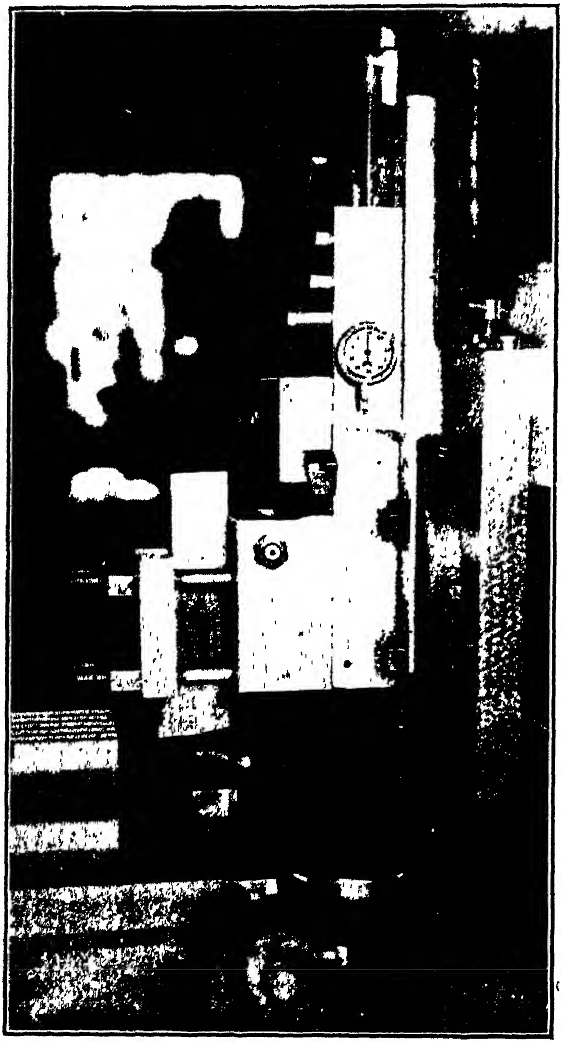


Fig. 4 Planer Tool Head and Loop Spring
(Also shows indicator for measuring depth of cut.)

loop spring in its position between the tail casting and the dynamometer bed. Fig. 5 shows the calibration curve up to 7000 lb. and gives consistent values of 0.0314 in. deflection per 1000

lb. of load. The deflections were read on a dial gage mounted across the minor axis. The 2000-lb. initial load was transmitted by the vertical knife edges *H* through the levers *I*, about the fulcrum *J*, to the pin *K*, which was mounted on a piston supported so as to bear against a rubber diaphragm contained in the cylinder *L*. This pressure was then transmitted hydraulically through the tube *M* to the mercury-column gage *N*.

9 Fig. 6 shows a separate view of the differential mercury-column gage. The point of zero reading on the mercury column corresponded with the initial spring load of 2000 lb. on the dynamometer bed. The mercury column was next calibrated to read in pounds the actual horizontal load in the direction of the

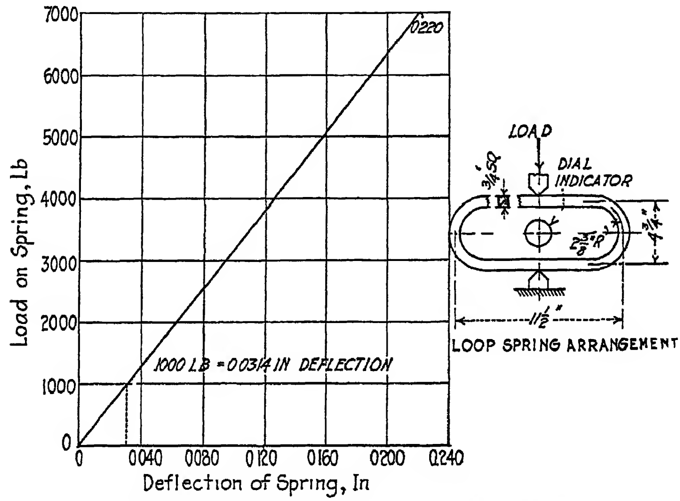


Fig. 5 Loop-Spring Arrangement and Calibration Curve

tool travel, by means of the loop spring *D*. Originally 100- and 200-lb. test gages were used but they gave unsatisfactory results.

10 The scale on the mercury column was made up by marking the mercury level for each successive load on the dynamometer bed as obtained by compressing the loop spring a definite amount with the adjustable screw *E*, Fig. 3. A 1000-lb. load of the spring produced a 2.539-in. rise of mercury in the column. While the dynamometer was designed for a load of 14,000 lb., it was calibrated only to 7000 lb.

11 In order that all readings might be taken in the same position of the bed, a spring contact was prepared, half of which was mounted on the vertical column as shown at *R* in the plan view of Fig. 3, the other half mounted on the bed of the planer shown at *Q*. As these springs passed, there was a snap, at which time the reading of the mercury column was taken. Originally all cuts

were made 4 ft. in length. It was found, however, that consistent readings could be obtained with a length of cut of 30 to 36 in.

12 The calibration was made with only the dead weights of the dynamometer parts plus a 300-lb. bar of material acting as vertical force. However, it is assumed that due to the ample support applied by the seven knife edges under the dynamometer table and

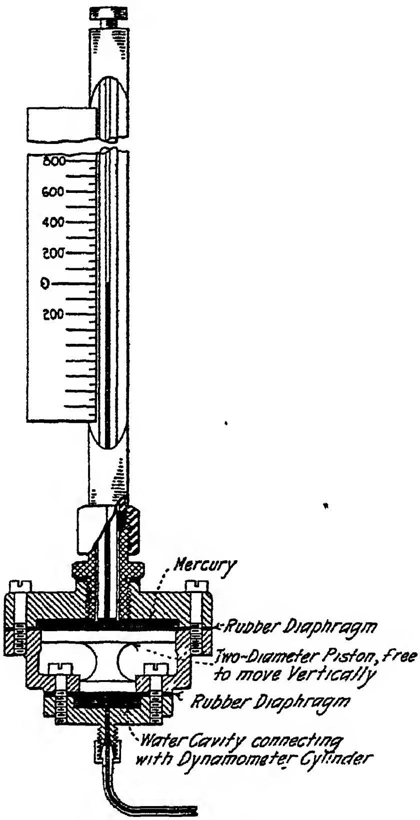


Fig. 6 Differential Column Gage

their very slight motion, any additional vertical loads caused by tool action would not cause sufficient friction in these knife edges to influence the force reading in a horizontal direction. This calibration demonstrated that a given increase in load applied to the dynamometer table caused a given rise in the mercury column regardless of what the total load acting might be, which proved that whatever friction existed did not influence the action of the dynamometer.

13 The dynamometer is sensitive to a force of ten pounds which may be read on the mercury-column gage. The variation of the actual chip size from the desired accounts for some variations in readings, as well as the variation caused by the material itself. Each test, however, was an average of at least six consecutive readings, so that the low force of a light chip would be compensated by the higher force of the heavier chip which would next be removed.

The Material

14 The materials selected to be cut in the experiments were confined to those in most common use which would give a wide range of physical characteristics. They included straight carbon steels (low, medium, and high carbon), $3\frac{1}{2}$ per cent nickel steels of high and low carbon, nickel-chromium steel, cast iron, and brass.

15 The steel was furnished in bars 4 by 6 in. and 48 in. long, prepared for the experiments by milling parallel grooves $\frac{3}{8}$ to $\frac{1}{2}$ in. wide and 1 in. deep in the upper surface, lengthwise of the bar. The material left between these grooves formed lands, the widths of which were equal to the width of chip to be cut. Fig. 2 shows one of these bars clamped on the bed of the dynamometer ready for testing. The tool was set over one land as shown in Figs. 2 and 4, and each successive cut was taken by feeding the tool vertically downward. The feed of the tool was accurately measured on the dial gage mounted on the tool holder as shown in Fig. 4.

16 The cast-iron bars were all cast from one ladle in the foundry laboratory in order to get bars as nearly alike as possible. Six bars were cast in a mold, each bar being $1\frac{3}{4}$ in. wide by 4 in. deep and 48 in. long. The lands were prepared on the top of the bar by milling.

17 Brass was furnished in rolled sheets, half-hard, of various thicknesses so that when cut on the edge, the thickness of the sheet represented the width of the chip. The 1-in. thick brass sheet was milled on its upper edge so as to present lands of desired widths. This also removed the cold-rolled skin from the side of the land.

18 A material-record sheet for each bar of material was kept, which gave all information as to manufacturer, chemical analysis, heat-treatment, hardness, and other physical properties. Appendix No. 2-A is a copy of the sheet for bar No. 1 of machine steel. Appendix No. 2-B is a material-record-sheet summary showing those problems for which each bar was used. Appendix No. 3 gives a list of test bars used for obtaining the physical properties of the materials cut.

19 Table 1 shows a list of the material, giving its physical and chemical properties. Table 2 is a summary sheet showing the cutting force for various tools for each material. These tables are referred to later in connection with individual problems.

TABLE 1 PHYSICAL AND CHEMICAL PROPERTIES OF MATERIALS CUT

Physical strengths given in lb per sq. in

		Tension					Compression			Shear	Hardness		
Material	Bar No.	Elastic limit	Ultimate strength[1]	Reduction of area, per cent	Elongation, per cent 2 in. g l.	Type of failure	Elastic limit	Ultimate strength[1]	Type of failure	Ultimate strength[1]	Brinell	Sclero-scope	Rock-well
S.A.E. 3120	31	47,500	77,000	70.2	30.3	cup and cone	41,700	116,750	...	54.150	155	21	48
S.A.E. 2345	29	60,000	99,670	42 7	25	cup and cone and plane ⊥ to axis	51,550	140,500	...	61,650	191	26	59
S.A.E. 2320	30	55,250	79,500	55 6	30	cup and cone and 45° shear	42,750	119,000	...	54.100	162	24	55
S A.E. 1035	28	19,900	54,000	60	35	...	16,000	93,000	...	37,700	99	19	48
0.15% C	1	25,300	52,400	67	41	cup and cone	22,000	80,500	...	39,400	106	21	54
0.15% C	2	24,500	52,000	66	41	cup and cone	...	...	...	89,400	102	22	56
0.15% C	3	28,000	51,300	67	40	cup and cone	...	...	...	39,400	101	21	58
0.15% C	4	20,900	52,000	65	40	cup and cone	...	...	...	39.400	102	21	45
0.15% C	5	22,400	50,800	68	42	cup and cone	19,075	85,350	...	39,410	101	26	56
0.15% C	6	22,500	52,750	63	40	cup and cone	...	...	...	39,400	99	21	56
1 03% C	32	39,170	80,000	56.7	32	cup and cone and plane ⊥ to axis	42,000	112,000	...	52,670	155	25	52
C. I.	1	none	16,160	none	none	plane ⊥ to axis	none	62,600	pl. 40° to axis	26,960	137	28	70
C. I.	2	none	18,200	none	none	plane ⊥ to axis	none	69,300	pl 33° to axis	26,000	156	28	83
C. I.	3	none	17,200	none	none	plane ⊥ to axis	none	70,000	pl 34° to axis	24,000	146	30	70
C I.	3A	none	14,000	none	none	...	none	57,200	pl. 34° to axis	24,700	118	25	65
C. I.	4	none	16,800	none	none	plane ⊥ to axis	none	72,000	pl 25° to axis	23,800	148	30	76
C. I.	4A	none	14,600	none	none	...	none	59,100	pl. 25° to axis	20,500	143	22	64
C. I.	5	none	17,300	none	none	plane ⊥ to axis	none	64,900	pl 32° to axis	27,000	153	31	78
C. I.	6	none	16,700	none	none	plane ⊥ to axis	none	65,700	pl. 32° to axis	28,400	151	32	...
C. I.	7A	none	14,520	none	none	plane ⊥ to axis	none	50,780	pl. 37° to axis	21,200	118	22	65
C. I.	8	none	17,200	none	none	plane ⊥ to axis	none	73,000	pl 33° to axis	22,700	149	31	85
C. I.	8A	none	16,900	none	none	plane ⊥ to axis	none	68,600	pl. 34° to axis	19,100	109	22	57
C. I.	9	none	21,300	none	none	plane ⊥ to axis	none	80,300	pl 27° to axis	28,300	170	32	88
C I	9A	none	13,400	none	none	plane ⊥ to axis	none	60,200	pl. 35° to axis	24,400	121	24	62
C I.	10	none	.	none	none	...	none	...	...	...	...	...	...
C. I.	11	none	18,900	none	none	plane ⊥ to axis	none	77,000	pl. 38° to axis	25,000	121	26	75
Brass	24	...	58,000	48	28	plane 45°	...	...	...	35,600	124	24	70
Brass	25	...	60,700	48	34	plane 45°	...	...	...	34,600	116	20	65
Brass	26	35,000	58,000	32	31	plane 45°	...	...	...	35,600	124	24	70
Brass	27	49,700	58,548	51	..	plane 45°	...	...	...	35,908	121	21	72
Brass	33	25,500	44,877	36		plane 45°	...	...	...	28,360	107	17	50
Brass	34	57,300	59,831	29	..	plane 45°	...	...	...	36,060	131	21	71

[1] Ultimate strength = maximum load/original area. Brinell Nos from 3000 kg load on 10-mm ball. Rockwell nos from $\frac{1}{16}$-in diam ball

CHEMICAL ANALYSES

S.A.E 3120 steel, bar No 31. C, 0 17; Mn, 0.60; P, 0 014; S, 0 019, Cr, 0.67; Si, 0.31; Ni, 1.28

S.A.E. 2345 steel, bar No. 29: C, 0 48; Mn, 0 59, P, 0 010; S, 0.024; Ni, 3.47.

S.A.E. 2320 steel, bar No. 30: C, 0 20, Mn, 0 65, P, 0.014, S, 0.018; Ni, 3 37, Si, 0.18.

S.A.E. 1035 steel, bar No. 28: C, 0 21; Mn, 0.27; P, 0 022, S, 0.030, Si, none

0.15 per cent carbon steel, bars Nos. 1 to 6 incl. C, 0 15, Mn, 0 24, Si, 0 13; S, 0.029; P, 0.014.

1 03 per cent carbon steel, bar No 32: C, 1 03; Mn, 0 28; P, 0.017; S, 0.018, Si, 0.18

Cast iron, all bars: C, 3 52; Si, 2 18; S, 0 108; Mn, 0 69; P, 0.489.

Brass, bars Nos 24 and 26: Cu, 61 34, Pb, 1 44; Fe, 0 034; Zn, remainder.

Brass, bar No 25: Cu, 61 48; Pb, 1.615; Fe, 0 069, Zn, remainder.

Brass, bar No. 27: Cu, 60 68; Pb, 1 635; Fe, 0 132, Zn, remainder.

Brass, bar No. 33: Cu, 61.98, Pb, 1 49, Fe, 0 050; Zn, 36 48.

Brass, bar No. 34: Cu, 64.10; Pb, 1.52; Fe, 0 012; Zn, 34.34

TABLE 2 CUTTING FORCES

Pounds per 0.001 sq. in. of cross-sectional area of cut

Material	Bar No.	Clearance in degrees, 30° front rake, 0° side rake, 1/2" width of cut, 0.024" depth of cut					Front-rake angle in degrees, 4° clearance, 0° side rake, 1/2" width, 0.012" depth of cut					Side-rake angle in degrees, 30° front rake, 4° clear., 1/2" width, 0.012" depth of cut				Width of cut, 30°-0°-4° tool, 0.012" depth of cut				Depth of cut in inches, 30°-0°-4° tool, 1/2" width					
		2	4	6	8	10	0	10	20	30	40	0	10	20	30	1/8"	1/4"	1/2"	3/4"	.003	.006	.012	.024	.032	.048
S.A.E. 3120	31	....	....	....	....	....	none	294	255	218	180	218	216	222	226	197	209	218	....	331	265	218	192	184*	172
S.A.E. 2345	29	....	....	....	....	....	none	314	293	275	247	255	265	—	—	225	246	275	....	385	307	275	214	203	....
S.A.E. 2320	30	....	....	....	....	....	none	295	260	229	195	229	241	244	254	195	210	229	....	348	276	229	191	184	..
S.A.E. 1035	28	....	....	....	....	....	none	296	245	214	172	214	—	220	216	210	....	214	217	—	253	214	180	....	....
0.15% C	1	....	....	....	207	210		....	....	....	....	246*	....	..	....	231*	244	246*	....	....	288*	246*	212*	199	....
0.15% C	2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	220		....	....	....	none		....	....
0.15% C	3	....	....	206	....	....	....	332	275	233	191	233	....	....	....	....	222*	233	....	....	283	233	208	....	....
0.15% C	4	209	207	....	....	....	....	307	—	234*	....	234*	....	....	....	206	—	234*	....	349	279	234	207	....	....
0.15% C	5	....	....	....	....	....	....	....	....	237*	....	237*	....	....	....	217	226	237*	252	....	292	237*	207	....	....
0.15% C	6	....	....	....	....	....	....	....	....	246	....	246	....	....	240	....	230	246	....	....	307	246	212	....	....
1.03% C	32	....	....	....	....	....	....	414	364	305	262	313	305	314	297	271	285	305	....	444	368	313	256	234	....
C. I.	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....		....	....	125	98	....	....	....
C. I.	2	....	....	....	....	....	....	....	....	122	....	122	131	128	120	....	....	122	....	....	....	122	....	....	....
C. I.	3	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	none	83	....	....
C. I.	3A	....	....	....		....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	none	72	....	....
C. I.	4	86	84	91	89	90	....	....	....	....	....	....	....	....	....	....	....	....	....	....	135	none	84	....	....
C. I.	4A	....	....		....	....	....	....	....	91*	....	91*	....	....	....	....	....	91*	....	....	115	91*	73	....	59
C. I.	5	....	....	....	....	....	....	164	120*	111	99.6	111	....	....	....	....	....	111	....	....	141	111		....	....
C. I.	6	....	....	....	....	....	....	....	....	....	....	....	....		....	....	....	....	....	....	125	none	82	....	....
C. I.	7A	....	....	....	....	....	129	....	....	86	....	86	....	....	....	....	91	86	94	....	....	80	....	....	....
		10° front rake tool																							
C. I.	8	125	125	125	130	127	....	....	....	103*	....	103*	....	....	....	....	....	103*	....	....	127	103*	82	....	65
C. I.	8A	....	....	....	....	....	....	....	....	90*	....	90*	....	....	....	....	....	90*	....	....	108	90*	74	....	60
C. I.	9	....	....	....	....	....	152	134	112	95	86	95	....	....	....	....	....	95	....	....	125	95	....	....	....
C. I.	9A	....	....	....	....	....	127	116	101	88	75	88	....	....	....	....	....	88	....	....	106	88	....	....	....
												0° front rake													
C. I.	10	....	....	....	....	....	143	....	....	....	....	143	145	142	147	....	....	....	....	....	....	....	....	....	....
C. I.	11	....		....	....	....	171	....	....	102	....	102	....	....	....	....	97.5	102	103	....	....	102	....	....	....
		0° front rake			0.012 in. deep																				
Brass	24	127	126	126	....	126	126	109	92	79	....	79	....	....	....	....	....	79	....	....	86	79	74	....	....
Brass	25	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	81	76	74	....	....
Brass	26	....	....	....	....	....	125	....	....	74	....	74	74	82	82.5		....	74	....	....	....	74	....	....	....
Brass	27	....	.	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.	....	....	....	....
Brass	33	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
Brass	34	....	....		....	..	....	....	..	....	....	....	....	....	....	....	....	....	....	....	.	....	....	....	....

* Value taken from curve Available data not reliable All cutting speeds, 20 ft. per min.

The Tests and Results

20 For clarity each problem is treated below individually.

Sharpness of Tool — Problem (a)

21 Problem (*a*) is "the determination of the influence of the degree of sharpness of the cutting edge of the tool, as the only variable, on the force on the tool or the energy required to remove a given volume of metal." This problem may also be to determine a standard practical degree of tool sharpness. It is divided into two parts. It was necessary to make sure that the tools as ordinarily heat-treated and ground would present a cutting edge which would give uniform results throughout any given test. For this purpose, tools having 0-, 10-, 20-, 30-, and 40-deg. front-rake angles with a clearance angle of 6 deg. were prepared, hardened, and drawn in accordance with the specifications of the manufacturer of the tool steel, some in a forge fire and others in a furnace. Depths of cuts of 0.005, 0.020, and 0.040 in. all having a width of 0.500 in. were made in open-hearth 0.15 per cent carbon steel fully annealed. The cutting speed was kept constant at 20 ft. per min.

22 The procedure was to grind the tool and cut with it under uniform conditions until the edge became dull and to observe the length of life of the cutting-tool edge which would make no increase in the reading of the force on the tool. It was found that in order to secure a cutting edge which would remain practically constant when cutting between 400 to 1600 linear feet that the degree of hardness was critical, that is, the temperatures of hardening and drawing must be very carefully controlled. It was also found that a properly hardened tool of either carbon or high-speed steel will present a cutting edge which will take all the cuts necessary for any of the following tests without dulling sufficiently to affect the gage reading.

23 Visual examination of the cutting edge was resorted to during most of these tests as the wear or change of condition of the edge was so small and affected the force on the tool so little that microscopic examinations were not considered necessary. It appeared that the original keenness of the ground edge, as is felt by drawing the finger along it, disappeared after the first two or three feet of cutting. The tool then seemed to remain in this new condition while cutting several hundred feet. It was found that when the cutting edge again changed, it wore rapidly and the failure of the tool soon followed. As we were interested in the life of the tool while sharp rather than the actual time and nature of failure, detailed accounts of the failure are not recorded.

24 In several instances, tools were prepared with definite degrees of tool sharpness, that is, some were carefully honed and

others ground on a tool grinder equipped with fine and coarse wheels. Photomicrographs were made of the cutting edges, looking down on the face of the tool over which the chip would slide. Cuts of $\frac{1}{4}$ or $\frac{1}{2}$ in. width and 1 ft., 3 ft., 10 ft., etc., of various depths were taken, after which photomicrographs were again taken. A comparison of these graphs showed no indications of wear on the tool for these short cuts, nor could it be detected just what part of the cutting edge, which was longer than the chip was wide, did the cutting.

25 The second part of the test was to determine the influence of a definite condition of the cutting edge on the force as registered by the dynamometer. The tools used for this test are shown in Fig. 7. Each one has a clearance angle of 4 deg. but front

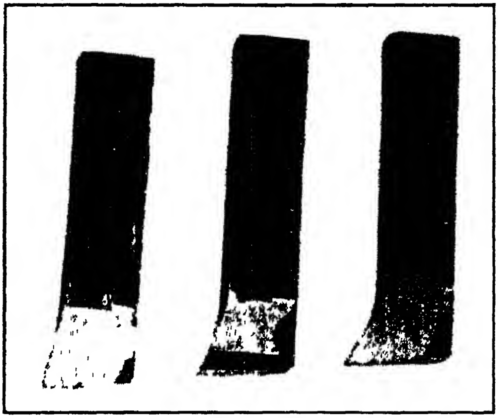


Fig. 7 Three Tools Used in Sharpness Test, Problem (a)

10-, 20-, and 30-deg. front rake angles, 4-deg. clearance.

rake angles of 10, 20, and 30 deg. respectively. They were hardened at 1500 deg. fahr. in water and tempered at 440 deg. fahr. in oil. Brinell and Rockwell hardness numbers taken about the cutting edges were consistent at 652 and 60-62, respectively, for all tools. The cutting speed was 20 ft. per min. in all cases. The tools were first carefully ground on a machine and then the cutting edges were honed so that under a microscope they appeared to be as even as the average razor blade. With this cutting edge, a series of cuts were taken in open-hearth 0.15 per cent carbon steel, annealed, and cast iron, with chips $\frac{1}{2}$ in. in width and 0.006 or 0.031 in. in depth. The force on the tool as indicated by the dynamometer was recorded. This force for comparative purposes was reduced to pounds per 0.001 sq. in. of cross-sectional area of the chip and plotted as ordinates over the condition of the cutting edge of the tool as abscissas. The tools were next ground in a tool

chip slippage for the 0 031-in. depth of cut in cast iron, but the surface is so smooth that the number of slips per inch cannot readily be seen nor can they be accurately counted. There seem to be approximately 20 slips per inch for honed tools and 28 slips per inch for the $\frac{1}{64}$-in. rounded-edge tools. As with steel, it appeared in every case that the initial degree of tool sharpness was changed by the first cut of 3 ft. but that this new condition was then maintained for the remainder of the test.

30 It was noticed that the beginning of each steel shaving had a smooth, bright surface on the underside for about $\frac{1}{8}$ to $\frac{1}{2}$ in.,

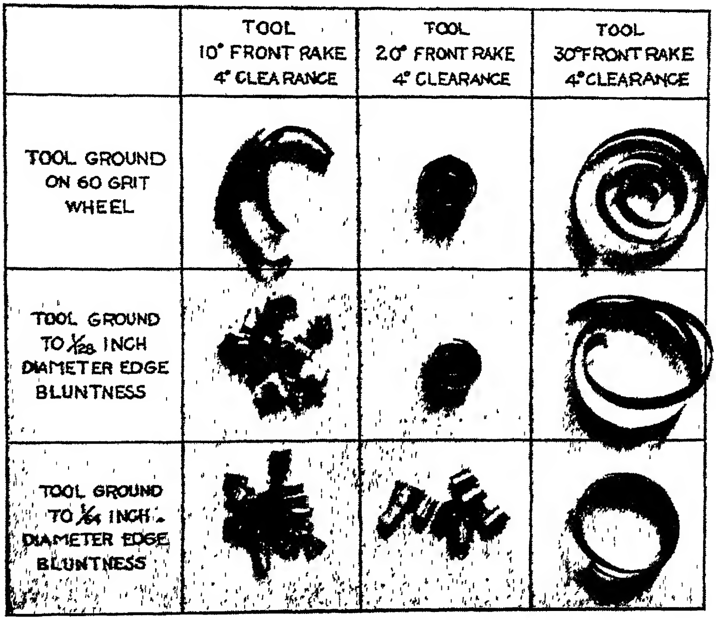


FIG. 10 TOOL SHARPNESS—FRONT RAKE, STEEL CHIPS

Material: 0.15-per cent carbon steel, annealed.
All chips 0.502 in. wide and 0.006 in. deep.

being longer for the greater thicknesses of chip, after which the rough slippage lines appeared and continued to the end of the cut. This seems to confirm the belief that after cutting has started the function of the cutting edge, particularly for the thicker chips, is to support a wedge of the material being cut. When the chip was broken off at the end of the cut, the small amount of material wedged on the cutting edge of the tool usually clung to the chips and was distinctly noticeable. The action of the cutting tool is not apparent to the eye; however an examination of the surface of the tool shows that there is a rubbing action a slight distance in from the edge, the distance varying with the thickness of the chip.

31 Fig. 10 shows some steel chips removed by the 10-, 20-, and 30-deg. front-rake tools, each having successively the three cutting edges, ground on 60-grit wheel, honed to $\frac{1}{128}$-in. diameter edge, and honed to $\frac{1}{64}$-in. diameter edge. For the 10-deg. front-rake tool, the chips appear to become more broken with the increase in bluntness. This is not true with the 30-deg. front-rake tool. It is also observed from this figure that the increased front rake for each degree of bluntness gives a more continuous type of chip. Fig. 11 shows a similar display of cast-iron chips. This, too, shows

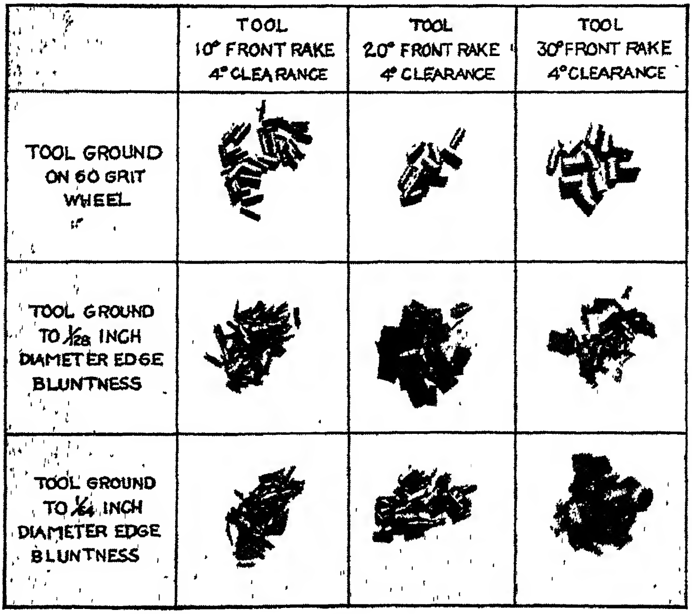


Fig. 11 Tool Sharpness — Front Rake, Cast-Iron Chips

Material: Cast iron. All chips 0.502 in. wide and 0.006 in. deep.

the chips to be more broken up as the cutting edge is increased in bluntness. For the well-ground tool (60-grit wheel) the chips appear to be more continuous with the increased front rake. The chips under the 10-deg. front-rake tool vary from $\frac{1}{8}$ to $\frac{3}{4}$ in. in length while those for the 30-deg. front-rake tool are for the full length of cut, 30 in.

Tool Clearance — Problem (b)

32 Problem (*b*) is "to determine the influence of the clearance back of the cutting edge of the tool, as the variable, on the force on the tool or the energy required to remove a given volume of metal." A number of tools were prepared having 0-, 10-, 20-, 30-

and 40-deg. front-rake angles, respectively, each in turn ground with clearance angles of 0, 2, 4, 6, 8, and 10 deg. The material cut for this problem was confined at first to the low-carbon steel fully annealed so that the results obtained would be based on a material uniform throughout. The width of the cut in all cases was 0.500 in., the depth of cut was successively 0.003, 0.006, 0.012, and 0.024 in. The cutting speed in all cases was constant at 20

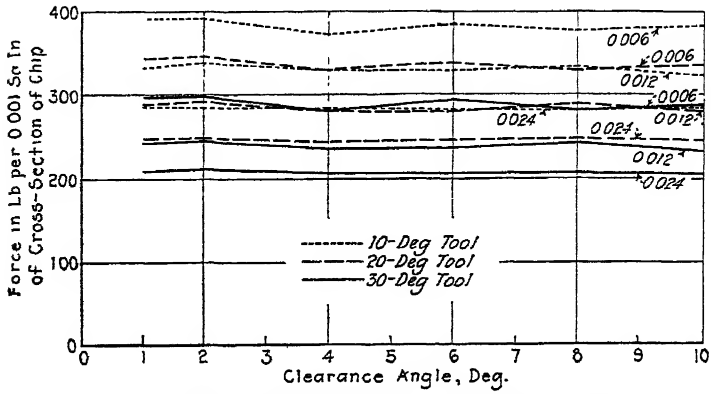


FIG. 12 UNIT CUTTING FORCE — CLEARANCE ANGLE

Material: Open-hearth 0.15 per cent carbon steel, annealed.
Tools: 10-, 20-, and 30-deg. front rake.
Speed: 20 ft. per min.

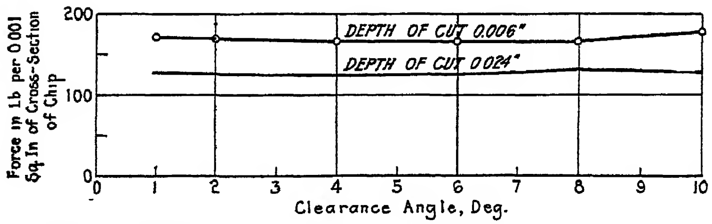


FIG. 13 CUTTING FORCE — CLEARANCE ANGLE, EXPERIMENT 19

Material: Cast iron, bar No. 8. Tool: 10-deg. rake.
Width of cut: 0.50 in. Speed: 20 ft. per min.

ft. per min. For comparative purposes, the force registered by the dynamometer was reduced to pounds per 0.001 sq. in. of cross-sectional area of the chip.

33 Representative values for the 10-, 20-, and 30-deg. front-rake tools cutting 0.006, 0.012, and 0.024 in. depth of chip for the low-carbon steel are shown in Fig. 12. While these curves are drawn through the actual values obtained, it appears obvious that their general tendency is to be straight horizontal lines. Results for the 0-deg. clearance for the three tools on low-carbon steel are not indicated because of their doubtful value, readings being alter-

nately high and low with an indication of considerable friction between the work and the clearance face of the tool. Also, values for a chip of 0 003 in. depth of cut are not reproduced, as the values for such thin chips appear to be too erratic to be of reliable assistance.

34 Fig. 13 shows unit-force values when cutting cast iron with a tool having a 10-deg. front rake angle for chips 0.006 and 0.024 in. deep and 0.500 in. in width. Fig. 14 shows the unit-force values for similar chips of cast iron for a tool having 30-deg. front rake.

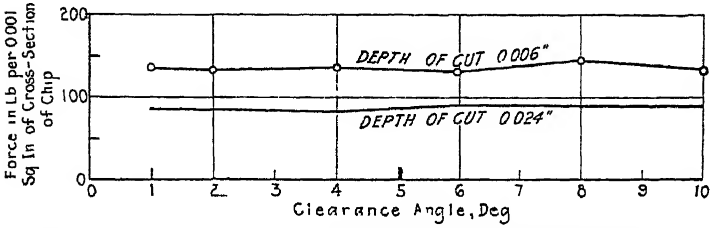


FIG. 14 CUTTING FORCE — CLEARANCE ANGLE, EXPERIMENT 19

Material: Cast iron, bar No 4. Tool: 30-deg. rake.
Width of cut: 0.50 in. Speed: 20 ft per min.

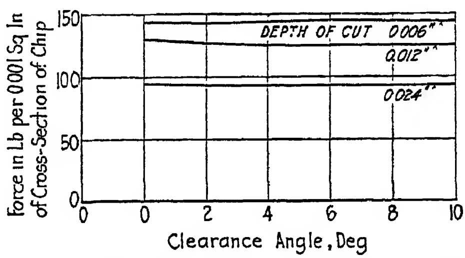


FIG. 15 CUTTING FORCE — CLEARANCE ANGLE

Material: Brass, bar No. 24 Tools: 0-deg rake.
Width of cut: 0.506 in. Speed: 20 ft. per min.

The unit forces for the 0.006- and 0.024-in. depths of cuts are 165 and 125 lb., respectively, for the 10-deg. front-rake tool, and 132 and 85 lb., respectively, for the 30-deg. tool.

35 Fig. 15 shows representative values for brass, the tool used having 0-deg. front rake and depths of cut of 0.006, 0.012, and 0.024 in. Here again are found straight horizontal lines. Unit forces of 144, 126, and 99.3 are representative values for each depth of cut. The 0.024-in. depth of cut caused chatter of the tool for all clearance angles. The chip slippage marks on the land after the 0.006- and 0.012-in. depths of cut chips had been removed were not countable as the surface was so smooth. After the 0.024-in. chip was removed, slippage or chatter lines were evident

and between 22 and 28 per linear inch. The clearance angle had little effect on the length of chip removed. A full-length (3-ft.) chip, in form of a coil, was removed with 0.006-in. depth of cut, about two inches in length for the 0.012-in., and $\frac{1}{32}$ in. for the 0.024-in. Fig. 16 is a summary sheet showing the unit forces plotted against clearance angles for steel, cast iron, and brass with tools having 10-, 10-, and 0-deg. front rake respectively. The depth of cut in each case is 0.024 in. The values given in Table 2, unless noted to the contrary, are for a 30-deg. front-rake tool. The curves give more values than the table because of the many combinations of rake and depths of cut.

36 Conclusions may be drawn from the above representative data to the effect that a variation of the clearance angle has no

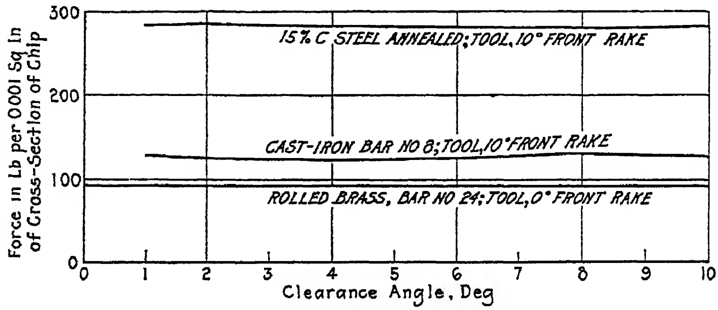


FIG. 16 SUMMARY UNIT CUTTING FORCE — CLEARANCE ANGLE CURVES, PROBLEM (b)

Materials: 0.15 per cent carbon steel, cast iron, and brass.
Depth of cuts: 0.024 in. Width of cuts: 0.5 in.
Speed: 20 ft. per min.

influence on the force required to remove the chip. Abrasion on the flank of the tool would probably prove excessive for low clearance angles, yet the less the clearance angle, the more the cutting edge is supported and the more metal there is to carry away the heat generated. It seems desirable, therefore, to select a clearance angle which will positively prevent the rubbing of the tool over the work and yet be as small as possible. In commercial practice, the accuracy with which these angles can be ground is one limiting feature, the feed of the tool is another. F. W. Taylor[1] recommended between 4- and 12-deg. clearance depending on the feed and the doubtful accuracy of grinding. As the feed, such as that given a side-cutting tool, does not enter into the work of these tests and as all tools are machine ground, a clearance angle of 4 deg. was approved as practicable for the tools used in the remainder of the work.

[1] **Ref. D-8, Par. 335. See Bibliography, Appendix No. 4.**

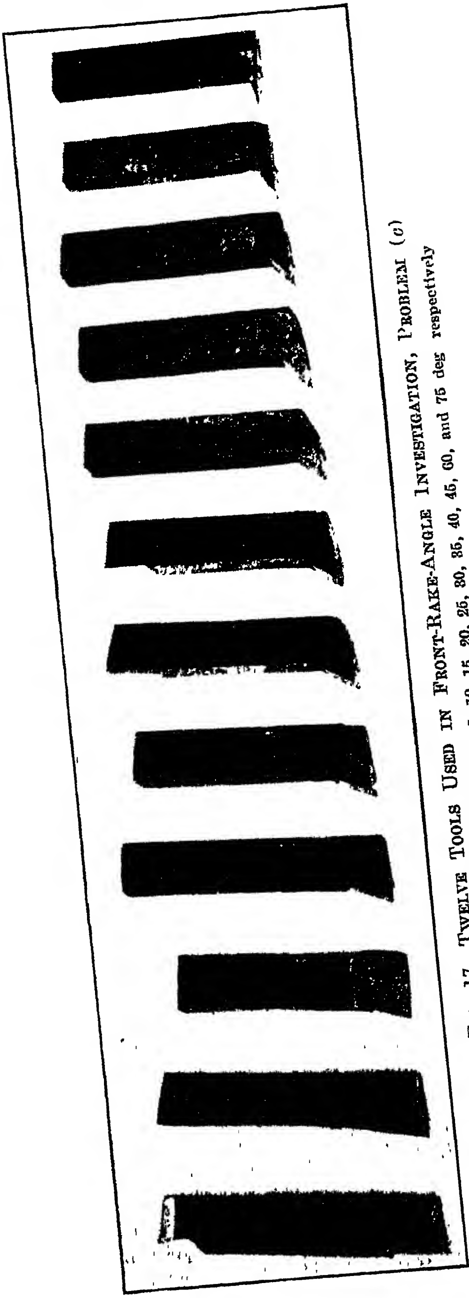


FIG. 17 TWELVE TOOLS USED IN FRONT-RAKE-ANGLE INVESTIGATION, PROBLEM (c)

From left, front-rake angles are 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 60, and 75 deg respectively

Front-Rake Angle — Problem (c)

37 Problem (c) is "to determine the influence of the front rake of the tool, as the variable, on the force on the tool or the energy required to remove a given volume of metal." Fig. 17 shows the 12 tools prepared for the front-rake investigation. These tools all have clearance angles of 4 deg. and a front-rake angle, starting from the left, of 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 60, and 75 deg. respectively. The data from these experiments have, for comparative purposes, also been reduced to the force in pounds per 0.001 sq. in. of cross-sectional area of the chip. The cutting speed in all cases has been confined to 20 ft. per minute, the width of cut to 0.50 in., the tool clearance to 4 deg., and the chip thicknesses to 0.006, 0.012, and 0.024 in., respectively. Representative data are shown for various metals cut. Fig. 18 shows nine of these

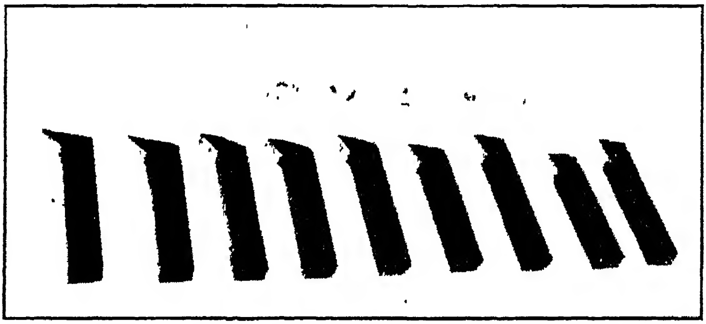


FIG. 18 NINE TOOLS HAVING FRONT RAKE ANGLES OF 45 TO 5 DEG., SHOWING STEEL CHIPS 0.500 IN. WIDE BY 0.012 IN. DEEP CUT BY EACH

tools with front-rake angles from 5 to 45 deg. and the steel chips, 0.50 in. wide by 0.012 in. deep cut by each.

38 Fig. 19 shows the unit force on the tool when cutting low-carbon steel, plotted as ordinates against the front rake angle of the tool in degrees, for three chip thicknesses. In this case the front rake was carried only to 45 deg., while the results obtained with the 0-deg. rake tool were so erratic and unreliable that they were omitted. Straight lines have been drawn through the points for each chip thickness and seem to indicate that the unit force for given conditions is reduced directly as the front-rake angle is increased. The straight line is believed to be the proper curve to represent the relation between the unit force and the front-rake angle. Were these curves extended as straight lines, they would intersect the abscissas axis within the 90-deg. front-rake-angle value. It was felt, however, that if these curves actually were continued, based on experimental results, they might extend to the right and downward but gradually become horizontal. Later

experiments confirm this. Fig. 20 gives a similar set of curves for S.A.E. 3120 steel with chip thicknesses of 0.006 and 0.012 in. These points also seem to fall reasonably well on inclined straight lines, as indicated. The curve for the 0.006-in. depth of cut would, if extended, intersect the abscissas axis to the right of the 90-deg. value. Fig. 21 shows the relation between the unit force and front-

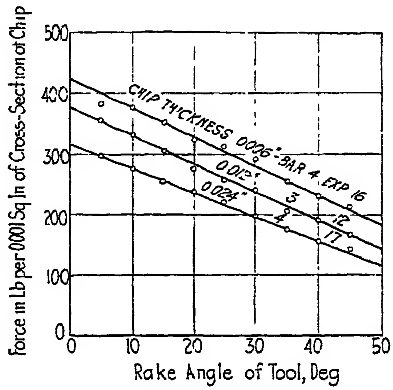


FIG. 19 FRONT-RAKE ANGLE, UNIT-FORCE CURVES FOR 0.15 PER CENT CARBON STEEL, EXPERIMENTS 16, 17, and 12

Material: 0.15 per cent carbon steel annealed.
Tools: 4 deg. clearance. Width of cut: 0.50 in.
Speed: 20 ft. per min.

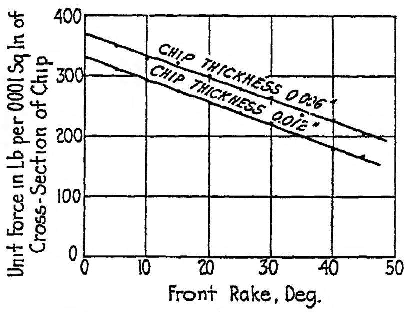


FIG. 20 UNIT FORCE—FRONT RAKE ANGLE CURVES FOR S.A.E. 3120 STEEL, EXPERIMENT 34

Material: S.A E. 3120 steel, bar No. 31.
Tools: 4-deg. clearance. Width of cut: 0.5 in.
Speed: 20 ft. per min.

rake angle for S.A.E. 2345 steel for 0.006- and 0.012-in. thicknesses of chip. These values seem to follow the straight-line rule, but appear to decrease in value less rapidly with the increased rake angle than is true with low-carbon or S.A.E. 3120 steel. The curves for both depths of cut would, if extended as straight lines, cross the abscissas axis at some distance to the right of the 90-deg. front-rake value.

39 Fig. 22 shows several unit-force, front-rake curves for brass. Chatter occurred with heavy chips (0.018, 0.024, and 0.030 in. depth of cut) and low front-rake angles. It is interesting to note that the force was reduced for the lower rake angles as a result of the chatter. It would appear to be more economical to cut brass under conditions which cause chatter. No attempt is made

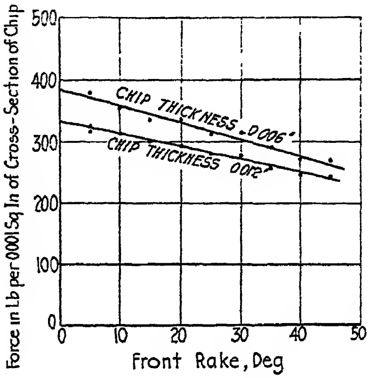


FIG. 21 UNIT FORCE—FRONT RAKE ANGLE CURVES FOR S.A.E. 2345 STEEL, EXPERIMENT 30

Material: S.A.E. 2345 steel, bar No. 29.
Tools: 4-deg. clearance. Width of cut: 0.5 in.
Speed: 20 ft. per min.

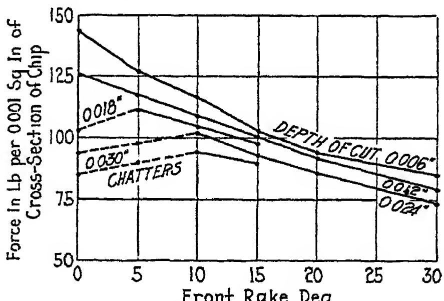


FIG. 22 UNIT FORCE—FRONT RAKE ANGLE CURVES FOR BRASS

Material: Brass, bar No. 24.
Tools: 4-deg. clearance. Width of cut: 0.506 in.
Speed: 20 ft. per min.

here to explain this, however. The points recording normal conditions for a given chip thickness seem to lie consistently on straight lines for rake angles below 20 deg. The 0.006-in. depth of cut, however, seems to have a tendency to become more horizontal between the 20- and 30-deg. front-rake angle, although with this thin chip the values obtained are less reliable than the others; the tool had a tendency to dig in on one cut and cut shallow on the next.

40 Fig. 23 shows the results of the influence of the front rake on the unit force when cutting annealed cast iron. These results seem to follow the straight-line law at least up to the 35-deg. angle and in Figs. 24 and 25 for unannealed cast iron seem to have a tendency to become more horizontal beyond the 25-deg. front rake angle. For the 0.024-in. depth of cut in Fig. 23, it appears that the curve has a tendency to become horizontal after the 35-deg. angle is reached. This is given considerable weight, as the material is uniform (annealed) and for heavy cuts the chance for error is less. In Fig. 25, some results are shown with the 60-deg.

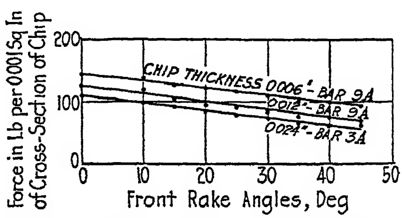


FIG. 23 UNIT FORCE—FRONT RAKE ANGLE CURVES FOR CAST IRON, EXPERIMENT 22

Material: Cast iron, annealed.
Tools: 4-deg. clearance. Width of cut: 0.5 in.
Speed: 20 ft. per min.

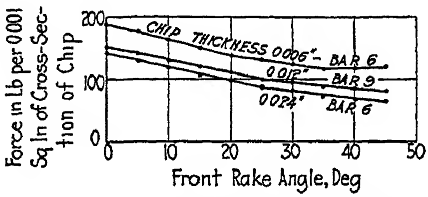


FIG. 24 UNIT FORCE—FRONT RAKE ANGLE CURVES FOR CAST IRON, EXPERIMENT 22

Material: Cast iron, unannealed.
Tools: 4-deg. clearance. Width of cut: 0.5 in.
Speed: 20 ft. per min.

front-rake tool. It is to be noted, however, that the results for the 60-deg. front-rake angle are slightly higher than those for the 45-deg. angle. This 60-deg. tool, while cutting with some difficulty in the cast iron, cut so poorly in the case of steel that no consistent results could be obtained. The 75-deg. front-rake tool which had a lip angle of 13 deg. (equivalent to that of a razor) could not be made to cut. It jumped up and slid over the top of the metal, or dulled instantly. It was too soft even when made of high-speed steel to yield reliable results. The results shown in Fig. 25, however, add weight to the evidence that beyond a certain front-rake angle there is no further reduction in the force on the tool.

41 Fig. 26 is a summary sheet showing the influence of the front-rake angle on the unit force for all materials cut with a depth of cut of 0.012 in. It is quite obvious from these results that the relation follows the straight-line law and the force on the tool is reduced as the front-rake angle is increased, till a certain limiting angle is reached. Beyond this angle, which seems

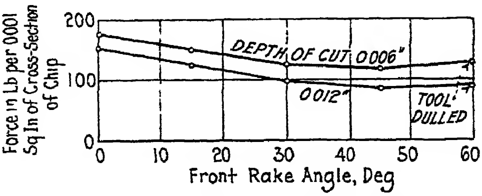


FIG. 25 UNIT FORCE — FRONT RAKE ANGLE CURVES FOR CAST IRON

Material: Cast iron, unannealed, bar No. 1
Tools: 4-deg. clearance. Width of cut: 0.5 in.
Speed: 20-ft. per min.

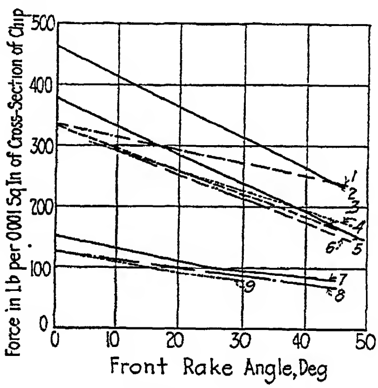


FIG. 26 SUMMARY UNIT FORCE — FRONT RAKE ANGLE CURVES

Tools: 4-deg. clearance. Depth of cut: 0.012 in. Width of cut: 0.5 in.
Speed: 20 ft. per min.

Curve 1: S.A.E. 2345 steel, bar No. 29.
Curve 2: 1.03 per cent carbon steel, annealed, bar No. 32.
Curve 3: S.A.E 3120 steel, bar No 31.
Curve 4: S.A.E. 2320 steel, bar No. 30.
Curve 5: 0.15 per cent carbon steel, annealed, bar No. 3.
Curve 6: S.A E. 1035 steel, bar No. 28
Curve 7: Cast iron, unannealed, bar No 9.
Curve 8: Cast iron, annealed, bar No. 9A.
Curve 9: Brass, bar No. 24.

to be between 20 and 30 deg. for brass, 25 to 30 deg. for cast iron, and above 45 deg. for steel, the influence of the front rake is less pronounced. Some metals are more readily influenced by the front rake than others.

Side-Rake Angle — Problem (d)

42 Problem *(d)* is "to determine the influence of side rake (skew), as the variable, on the force on the tool or the energy

required to remove a given volume of metal." Fig. 27 shows the first set of tools used in a side-rake problem. They were all of 30-deg. front rake and 4-deg. clearance, but had side-rake angles of 0, 10, 20, and 30 deg., respectively. The clearance was measured in the direction of travel of the tool.

43 A second set of side-rake tools, each having 0-deg. front rake and 0, 10, 20, 30, 45, 60, 75 deg. side rake, respectively, was later used. With these tools the clearance angle was ground at right angles to the cutting edge. The 30-deg. and 75-deg. side-rake tools,

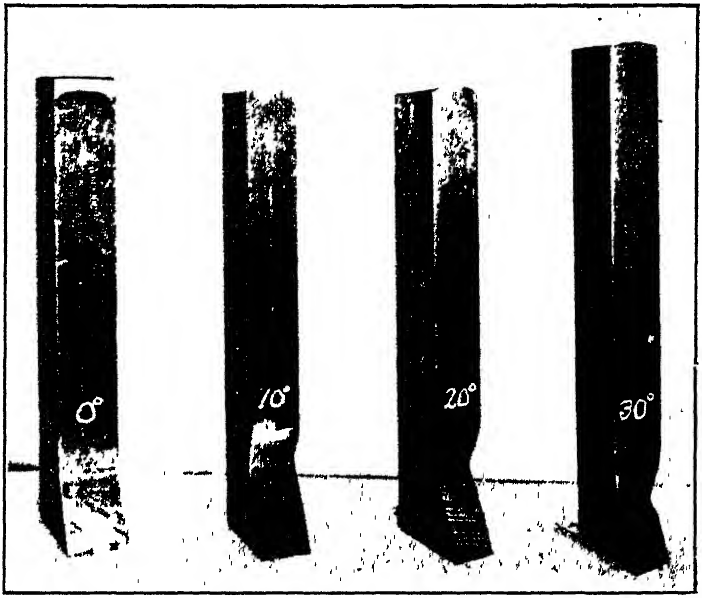


FIG. 27 FIRST SET OF TOOLS USED IN SIDE-RAKE INVESTIGATION, PROBLEM (*d*)

All tools have 30-deg. front rake and 0-, 10-, 20-, and 30-deg. side rake.

however, were later ground so that the clearance was measured from the cutting edge in the direction of tool travel for check purposes, as discussed later.

44 *First Set of Side-Rake-Angle Tools.* Fig. 28 shows a tool having 30-deg. front rake, 30-deg. side rake, and 4-deg. clearance in the process of removing a 0.15 per cent carbon-steel chip, which is 0.500 in. wide and 0.024 in. deep, from bar No. 6. Fig. 29 shows a number of chips of this material which have been cut under various conditions, the 30-deg. front-rake tools having side-rake angles of 0, 10, 20, and 30 deg., respectively, removing chips of 0.006, 0.012, and 0.024 in. deep and 0.500 in.

wide. All of these chips are the complete length of the bar, 36 in. It is noticeable that the diameter of the coil is increased slightly with the increase in depth of cut for a given side-rake angle. Also the diameter of the helix is reduced with the increase in side rake, but the length of the helix is increased as indicated by Table 3:

TABLE 3 MEASUREMENTS OF STEEL CHIPS PRODUCED BY SIDE-RAKE TOOLS

Side rake, deg. (30° front rake)	Diameter and length of chip coil, in. Depth of cut, in. 0.006	0.012	0.024
0	1 1/4 dia.	1 3/4 dia.	2 3/4 dia.
10	1 1/4 x 4	1 1/2 x 4 1/2	1 1/2 x 5
20	3/4 x 8	1 x 8	1 1/4 x 7
80	5/8 x 11	3/4 x 11	1 1/4 x 9 1/2

45 The slippage marks left on the land after the chip was removed were practically constant for each depth of cut for all of the tools, but decreased as the depth of cut was increased. The values as counted are shown in Table 4.

TABLE 4 SLIPPAGE ON STEEL OF SIDE-RAKE TOOLS

Depth of cut, in.	Approximate slippage
0.006	38–40 per in.
0.012	32–34 per in.
0.024	26–28 per in.

46 Fig. 30 shows a set of cast-iron chips for the various side-rake angles of the 30-deg. front-rake tools and for depths of cuts of 0.006, 0.012, and 0.024 in. The influence of the side-rake angle for each depth of cut is less marked than for the steel. Because of the brittleness of the material, the chips broke into small pieces rather than coming off in continuous lengths. For the thin chips, 0.006-in. depth of cut, the chips were longest, of the least diameter, and showed a tendency to become helical with the increase in side rake. The slippage of 0.006- and 0.012-in. depths of cuts was barely visible for any of the tools. On the 0.024-in. depth of cut, it varied from 30 slips per inch for the 0-deg. side-rake tool to 22 slips per inch for the 30-deg. side-rake tool.

47 With the first set of side-rake tools, each of which had a front-rake angle of 30 deg. as shown in Fig. 27 and side-rake angles of 0, 10, 20, and 30 deg., respectively, a series of cuts was made on brass bar No. 26. The width of cut was 0.512 in. and depths were of 0.006, 0.012, and 0.024 in., respectively. Table 5 shows the values obtained for the 0- and 30-deg. side-rake tools for all three depths of cut. The values for all four tools are given in Table 2.

48 The results show that the total force on the tool for the 0.006-in. depth of cut increased from 260 lb. for 0-deg. side rake to 300 lb. for 30-deg. side rake. This value divided by the area

of chip (0.512 multiplied by 0.006) all divided by 1000 gives the unit force or the force in pounds per 0.001 sq. in. of the cross-sectional area of the chip, which is 84.5 lb. for 0-deg. side-rake tool and 97.4 lb. for the 30-deg. side-rake tool. For all three depths

TABLE 5 TOTAL AND UNIT FORCES ON SIDE-RAKE TOOL FOR BRASS

Depth of cut, in.	Side rake (front rake 30 deg.) 0 deg. Total	Unit	30 deg. Total	Unit
0 006	260	84.5	300	97.4
0.012	452	74 0	506	82.5
0.024	890	72.3	965	78.5

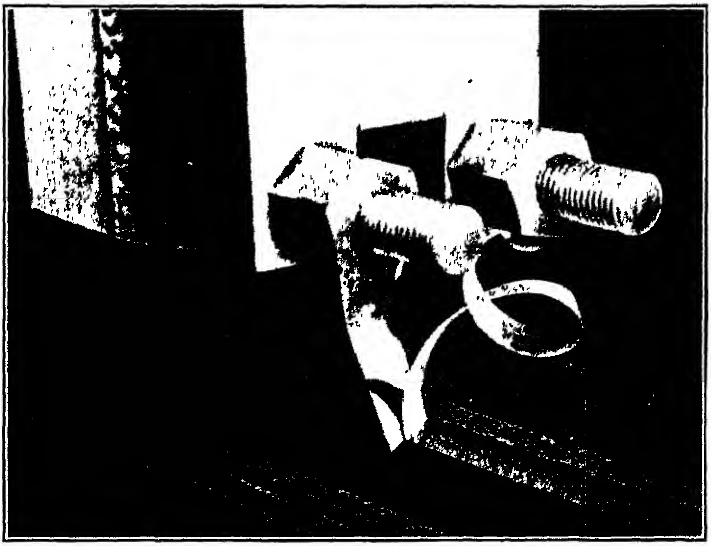


FIG. 28 A 30-DEG. SIDE RAKE, 30-DEG. FRONT RAKE, 4-DEG. CLEARANCE ANGLE TOOL REMOVING A 0.15 PER CENT CARBON-STEEL CHIP 0.5 IN. WIDE AND 0.024 IN. DEEP

of cut there is a very slight increase in the total and unit force on the tool. The 30-deg. front-rake tool, however, appeared to be excessive for brass as alternately uniformly thin and thick chips were cut.

49 For the 0.012-in. depth and 0.512-in. width of cut, the chips for the 0-, 10-, 20-, and 30-deg. side-rake tools were $1\frac{1}{8}$ in. diameter spiral, $\frac{1}{2}$ in. diameter by 6 in. long, $\frac{5}{16}$ in. diameter by $11\frac{1}{2}$ in. long, and $\frac{1}{4}$ in. diameter and $14\frac{3}{8}$ in. long. The original length of cut was 33 in. These values do not compare favorably with those for steel, Table 3.

50 Another series of cuts was made with the same set of tools on cast iron, bar No. 2. The width of cut was 0.500 in. and the depths of cut 0.006, 0.012, and 0.024 in. The cutting speed, as in

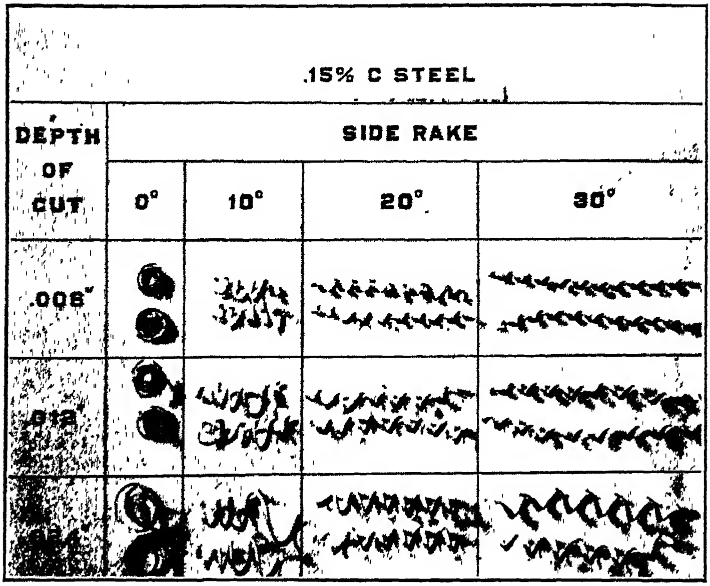


FIG. 29 0.15 PER CENT CARBON-STEEL CHIPS 0.5 IN. WIDE REMOVED BY TOOLS HAVING 30-DEG. FRONT-RAKE AND VARIOUS SIDE-RAKE ANGLES

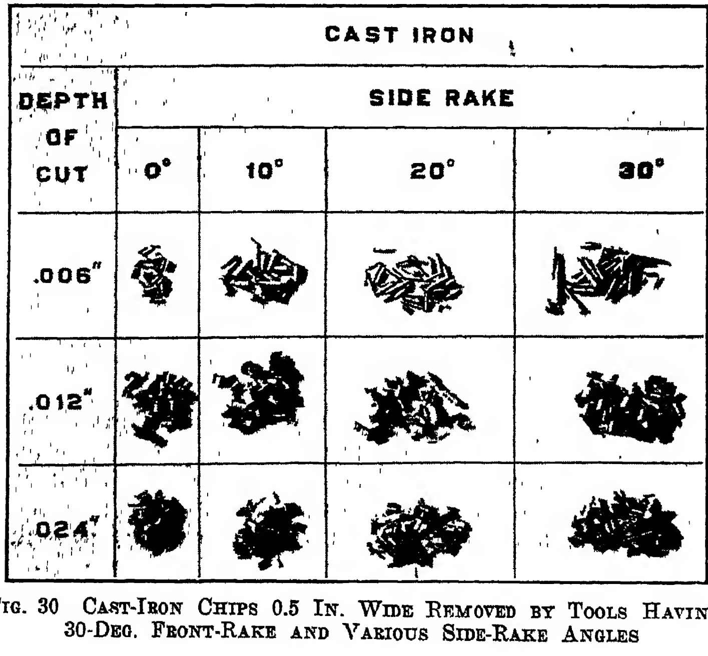


FIG. 30 CAST-IRON CHIPS 0.5 IN. WIDE REMOVED BY TOOLS HAVING 30-DEG. FRONT-RAKE AND VARIOUS SIDE-RAKE ANGLES

all other tests, was 20 ft. per min. The unit forces are shown plotted in Fig. 31. The total force for the 0.006-in. depth of cut is increased from 431 lb. for the 0-deg. side-rake tool to 441 lb. for the 30-deg. side-rake tool. The corresponding values of the unit forces as plotted are 146 and 150 lb. The increase in the unit force for the 10-deg. side-rake tool for 0.012- and 0.024-in. depths of cut is undoubtedly due to the fact that the tool was burned in grinding and was considerably blunted while cutting. The cuts were first made with a 0-deg. side-rake tool, then the 10-, 20-, and

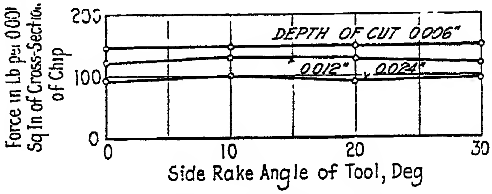


FIG. 31 UNIT FORCE — SIDE RAKE ANGLE CURVES FOR CAST IRON, EXPERIMENT 48

Material: Cast iron, bar. No 2.
Tools: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.5 in. Speed: 20 ft. per min.

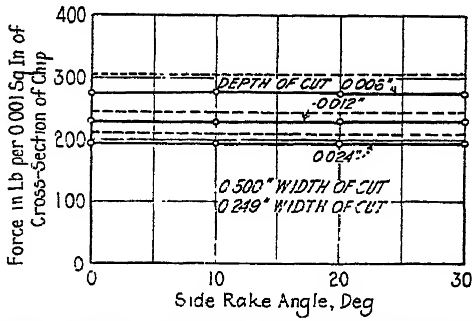


FIG. 32 UNIT FORCE — SIDE RAKE ANGLE CURVES FOR STEEL, EXPERIMENT 49

Material: 0.15 per cent carbon steel, annealed, bar No. 6.
Tools: 4 deg. clearance, 30-deg. front rake.
Widths of cuts: 0.249 and 0.5 in. Speed: 20 ft. per min.

30-deg. tools in order. After cutting with the 30-deg. side-rake tool, the force for the 0-deg. side-rake tool was confirmed. It was observed that the unit force for most check tests varied not more than 1 or 2 lb. For cast iron, the value of the check test was about 2 lb. less, due probably to the increased softness of the metal as the cut approached the center of the bar on account of its being annealed in the mold.

51 A similar series of cuts was made on 0.15 per cent carbon steel, annealed, bar No. 6. Two widths of cut, 0.249 and 0.500 in., respectively, were used for each depth of 0.006, 0.012, and 0.024

in. The results are shown plotted in Fig. 32. The unit forces for the 0.249-in. width are uniformly less than those for the 0.500-in. width. This suggests that the unit force for a side-rake-angle tool is reduced as the width of cut is reduced and is referred to again under the width-of-chip problem.

52 All cuts of the same depth were made on one land and run consecutively from 0- to the 30-deg. side-rake tools and then the 0-deg. tool was again checked. On the 0.500-in. land, check tests were made for each of the three depths of cut for the 0-deg. and 30-deg. side-rake tools with results shown in Table 6.

TABLE 6 UNIT FORCES ON SIDE-RAKE-ANGLE TOOLS FOR 0.15 PER CENT CARBON STEEL

Depth of cut, in.	Unit force (front rake 30 deg.) Side rake angle 0 deg.	30 deg.
0.006	307	308
0.012	246	246
0 024	212	210

These values together with the intermediate values are given in Fig. 32. With the 0.024-in. depth of cut, the tools had a tendency to hog in, every other cut, so that readings were alternately high and low. The values given are averages, however. These curves appear to be straight horizontal lines.

53 Another series of tests was made on what was purchased for S.A.E. 1035 steel, bar No. 28, but the analysis showed the carbon content to be only 0.22 per cent. The values of the unit forces obtained are shown in Fig. 33. The results for the 10-deg. side-rake tool for the 0.006- and 0.012-in. depths of cut are low. For the 0.024-in. depth of cut there appears to be a uniform increase in the unit force from 180 lb. for the 0-deg. side-rake tool to 194 lb. for the 30-deg. side-rake tool. This increase, however, is very small.

54 Fig. 34 is a summary sheet showing the values of the unit forces for the 30-deg. front-rake tools having 0-, 10-, 20-, and 30-deg. side rake for eight different materials. The width of cut in each case was 0.500 in., the depth of cut in all cases was 0.012 in., so that all values in the figure are directly comparable. The curve for brass has the same values as those shown in Table 6 for the 0.012-in. depth of cut. The curve for cast iron is a duplicate of the middle curve in Fig. 31. The last curve is for S.A.E. 1035 steel and is a duplication of the middle curve of Fig. 33. The curve for S.A.E. 3120 steel for the 0.012-in. depth of cut was the only one obtained for that material. The curve for the 0.15 per cent carbon steel is a duplication of the center dashed curve of Fig. 32. The curve for the S.A.E. 2320 steel, bar No. 30, is the only one obtained for this metal with this set of tools. The curve for the S.A.E. 2345 steel, bar No. 29, is one of two curves obtained.

This curve for the 0 012-in. depth of cut shows a high value for the 20-deg. side-rake tool. Uniform results, however, were obtained with all tools for a chip depth of 0.006 in., for which the unit force for all side-rake angles was 307 lb. It is, therefore, thought that the rise for the 0.012-in. depth of cut for the 20-deg. side-rake angle is due to experimental error or some other outside influence. The highest curve, that for the 1.03 per cent carbon steel, is the only one obtained for this material with this set of side-rake tools.

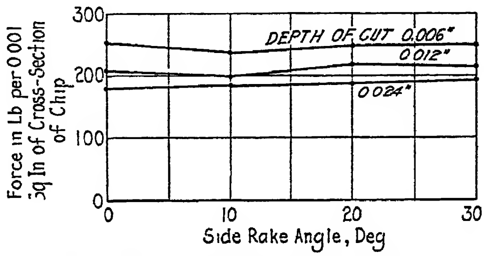


FIG. 33 UNIT FORCE — SIDE-RAKE-ANGLE CURVES FOR STEEL, EXPERIMENT 28

Material: S.A.E. 1035 steel, bar No. 28.
Tools: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.5 in. Speed: 20 ft. per min.

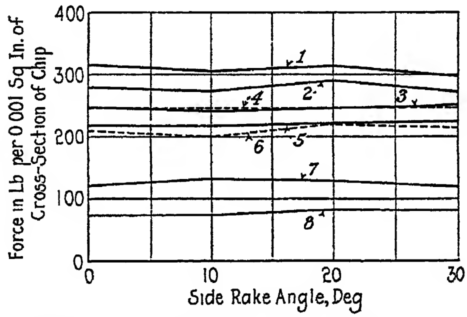


FIG. 34 SUMMARY UNIT FORCE — SIDE-RAKE-ANGLE CURVES PROBLEM *(d)*

Tools: 4-deg. clearance, 30-deg. front rake.
Depth of cut: 0.012 in. Width of cut: 0 5 in. Speed: 20 ft. per min
Curve 1: 1.03 per cent carbon steel, annealed, bar No. 32
Curve 2: S.A.E. 2345 steel, bar No. 29.
Curve 3: S.A.E. 2320 steel, bar No 30.
Curve 4: 0.15 per cent carbon steel, annealed, bar No. 6.
Curve 5: S.A.E. 3120 steel, bar No. 31.
Curve 6: S.A.E. 1035 steel, bar No. 28.
Curve 7: Cast iron, unannealed, bar No. 2
Curve 8: Brass, rolled, bar No. 26.

55 All of the results which are available for the first set of tools, namely, those having 30-deg. front rake and 0-, 10-, 20-, and 30-deg. side rake, indicate that the value of the side-rake angle has very little influence on the unit or total force on the tool in

removing a given chip. All of the curves shown are drawn to connect the points obtained experimentally. Smooth curves through these points, however, would eliminate many of the irregularities referred to, leaving nearly all of the curves as straight horizontal lines.

56 *Second Set of Side-Rake-Angle Tools.* Because of the small cutting angles of these tools and the tendency of such angles to fail, dig in, or otherwise cause trouble, the side-rake angle of the first set of tools was not carried beyond 30 deg. The values obtained, however, would indicate that all of these curves would continue to be straight horizontal lines for side-

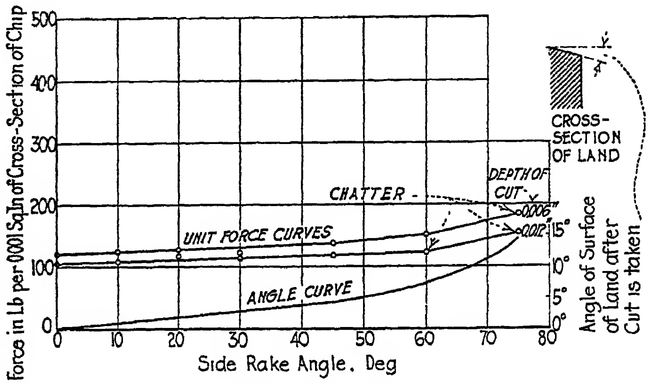


FIG. 35 UNIT FORCE—SIDE RAKE ANGLE CURVES FOR BRASS, EXPERIMENT 57

Material: Brass, bar No. 27.
Tools: 4-deg. clearance, 0-deg. front rake.
Width of cut: 0.312 in. **Speed:** 20 ft. per min.

rake angles greater than 30 deg. In order, however, to make sure of this point and to obtain comparable data with another front-rake-angle tool, the second series of side-rake tools having 0-deg. front rake was prepared.

57 Fig. 35 shows the results of the force in pounds per 0.001 sq. in. cross-sectional area of chip as a function of the side-rake angles for this set of tools when cutting brass, bar No. 27. The brass was rolled to a width of 0.312 in. Two depths of cut were taken, 0.006 and 0.012 in., respectively. The curves indicate a slight increase in the unit force for side-rake angles up to 45 deg., with a slightly accelerated increase for the 60- and 75-deg. angles. The increase up to the 45-deg. side-rake angle, however, is so small that it may be due to experimental error, as smooth curves drawn through the points up to the 30-deg. angle might be horizontal. The cutting action seemed quite free for most tools. There was much chatter on the 0.012-in. depth of cut with the 60- and

75-deg. tools and some chatter on the 0.006-in. depth of cut with a 75-deg. tool.

58 The tools having the 4-deg. clearance angle ground at right angles to the cutting edge left the top of the land not horizontal but slightly inclined to the side as shown in the section, Fig. 35. A curve is plotted at the bottom showing the value of these side angles for the various side-rake angles of the tool. It is less than $4\frac{1}{2}$ deg. when the side-rake angle is 45 deg., but increases rapidly to $14\frac{1}{2}$ deg. for a side-rake angle of 75 deg. In the worst case, that for a 75-deg. side-rake-angle tool, the actual thickness of the cut is reduced to 0.9636 (cosine $14\frac{1}{2}$ deg.) multiplied by the vertical depth of cut. The width of the chip would then equal the

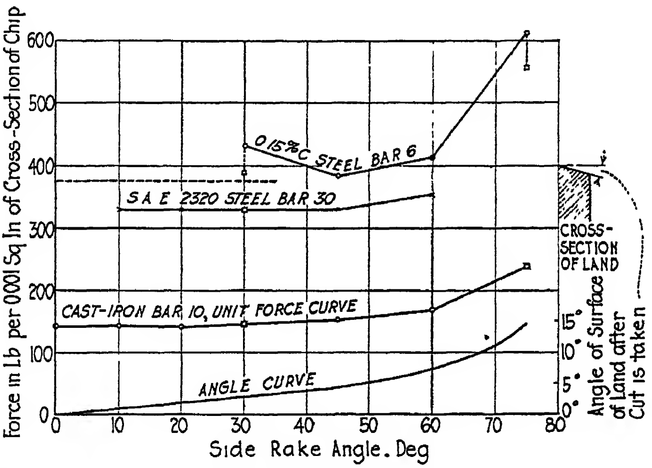


FIG. 36 UNIT FORCE — SIDE RAKE ANGLE CURVES, EXPERIMENT 58

Tools: 4-deg. clearance, 0-deg. front rake.
Width of cut: 0.5 in. Depth of cut: 0.012 in. Speed: 20 ft. per min.

horizontal width 0.312 divided by 0.9636 or 0.324 inches. This variation would appear to be of negligible influence on the force on the tool, but for check purposes the 30- and 75-deg. side-rake tools were reground so that the 4-deg. clearance angle was measured in the plane of tool travel and so that the land would be horizontal after a chip was removed. The values for brass in Fig. 35 for both depths of cut were obtained with these corrected tools and found to agree with the first within a few pounds, so it was not considered necessary to regrind the other tools for this correction.

59 Fig. 36 shows the unit-force curves as a function of the side-rake angle of the 0-deg. front-rake tools for cast iron, S.A.E. 2320 steel, and 0.15 per cent carbon steel. The values for the cast iron, bar No. 10, appear to be constant at 143 lb. up to the 30-deg. side-rake angle for which 147 lb. is registered. A slight

increase to 152.5 lb. is noted for the 45-deg. angle with still greater increases to 169 and 238 lb. for the 60- and 75-deg. angles, respectively. The points marked with a circle and square for the 30- and 75-deg. side-rake tools indicate that these values were obtained with the side-rake tools before and after correcting the tools for the slope of the top of the land.

60 The curve for the S.A.E. 2320 steel, bar No. 30, appears to be horizontal until the 45-deg. side-rake angle is passed, when a slight increase from 330 to 355 lb. for the 60-deg. tools is shown. The square on the curve indicates that the value obtained for the 30-deg. tool was with the tool corrected so as to leave the top of the land horizontal. This point falling on the horizontal line indicates the slight influence of the slope of the top of the land for the 30-deg. side-rake tool. The surface left on the S.A.E. 2320 steel was quite rough for the 10-deg. tool but satisfactory in all other cases. The 0-deg. front-rake and 0-deg. side-rake tool was not used because of erratic results usually obtained with this tool when cutting steel. No cuts were taken with the 75-deg. side-rake tool because of trouble experienced with the excessive side thrust on the chip which produced erratic results, and heated excessively.

61 The upper curve shows the unit forces obtained for the various side-rake angles when cutting 0.15 per cent carbon steel, bar No. 6. The value of the unit force for side-rake angles below 30 deg. is shown by a straight dashed line. Actual test values for the 0-deg. front- and side-rake tool were so erratic that they were thought unreliable. The value of 375 lb. as the unit force was taken from the front-rake investigation curves, Fig. 19. The values represented by the solid line were obtained with difficulty and are not considered wholly reliable. The points for the 30-deg. and 75-deg. side-rake-angle tools were checked with the tools corrected for clearance, the new values being lower as indicated by the square. It is interesting to note that in Fig. 36 the unit force for the 0.15 per cent carbon steel is 375 lb. and for S.A.E. 2320 steel only 330 lb. In Fig. 34, where the 30-deg. front-rake tools were used, the values for the unit forces of the two materials are almost equal at 245 lb., the same bars being used in each case.

62 The values presented for both sets of side-rake tools for the various metals cut seem to indicate decisively that for moderate values, the side-rake angle, all other factors remaining constant, has no influence on the total or unit force on the tool. When the side-rake angle reaches 60 deg., a definite increase is noted and for greater values of side rake the force is markedly increased.

Force — Chip Width, and Depth of Cut, Problem (e)

63 Problem (*e*) is " to determine the influence of the depth of cut or width of cut, as the variable, on the force on the tool or the energy required to remove a given volume of metal." In order

to have but one variable at a time, the problem has been divided into two parts so as to determine, first, the influence on the force on the tool of a change in width of chip, and, secondly, the influence on the force due to a change in the depth of cut.

64 *Force — Chip Width.* The 30-deg. front-rake tool having a 4-deg. clearance angle was found to give consistently reliable results in previous tests on most materials. It was therefore selected as the tool to be used in the chip-width series of tests.

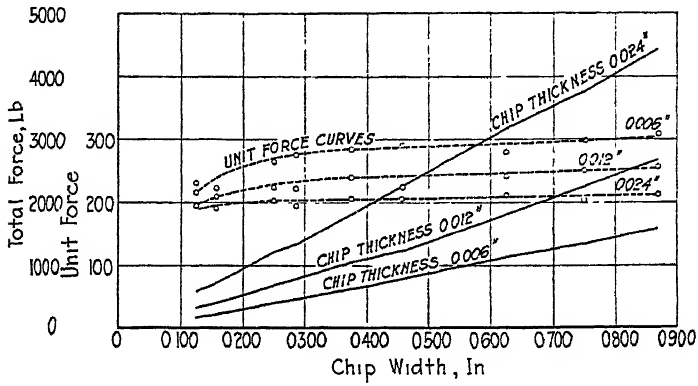


FIG. 37 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES FOR STEEL, EXPERIMENT 20

Material: 0.15 per cent carbon steel, annealed, bar No. 5.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

For brass, however, in order to avoid chatter, which occurs in taking heavy cuts with small-front-rake-angle tools, and to avoid hogging in, or taking of alternately thick and thin chips, which occurs with tools of excessive front rake, such as the 30-deg. front-rake tool, a tool having 15-deg. front rake and 4-deg. clearance was used. A 0-deg. front-rake tool was used with the 30-deg. tool on annealed cast iron and on brass as noted.

65 The usual cutting speed was 20 ft. per min. This is rather low for cutting brass. A number of check runs were made at 35 ft. per min. for brass as well as some of the steels, and identical results were obtained in every case for the two speeds.

66 Depths of cut were confined in most instances to 0.006, 0.012, and 0.024 in. The width of cut ranged between $\frac{1}{8}$ in. and 1 in. Fig. 37 shows the total- and unit-force curves for various chip widths for 0.15 per cent carbon steel, annealed, bar No. 5. The total-force curves are shown as solid lines, the lines having been drawn between the actual experimental points rather than as a smooth curve. The total-force curves appear to be straight

lines, although they are slightly concave upward. The slight variations are due probably to experimental error. If the curves were extended to a chip having zero width, they would pass through the origin. This point is discussed later. The unit-force curves shown dashed are drawn as smooth curves through the points as the points are well scattered. A slight variation of the total force from the smooth curve shows up considerably magnified in the unit-force curve which is plotted on a much larger scale. It appears that the unit-force curves to the right of 0.3-in. chip width are nearly horizontal, but to the left they have a tendency to drop. It was difficult to secure constant readings with a narrow chip width, as a small error in the reading of the total force represented a large percentage of the reading. However, the values for the

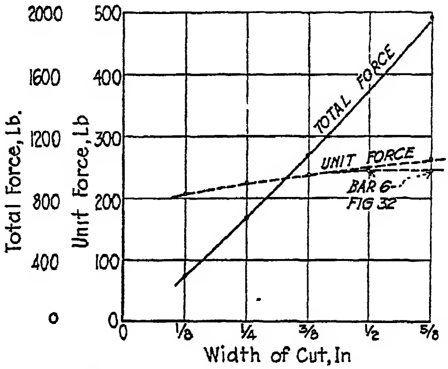


FIG. 38 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES

Material: 0.15 per cent carbon steel, bar No. 5.
Tool: 4-deg. clearance, 30-deg. front rake, 20-deg side rake.
Depth of chip: 0.012 in.

0.158-in. width are believed to be a little too low. The values for a chip width of $\frac{1}{8}$ in. are high for all three depths of cut.

67 In Fig. 32 it was noticed that the unit forces for side-rake tools on a land of 0.5 in. were higher than those for the 0.25-in. width. As a check, a tool having 30-deg. front-rake and 20-deg. side-rake angle was selected for a test on a 0.15 per cent carbon steel, bar No. 5, the depth of cut being 0.012 in. and the width $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{8}$ in. respectively, as these lands were already prepared. The results of this test are shown in Fig. 38, the solid line representing the total-force curve and the dashed line, the unit-force curve. The total-force curve appears to be slightly concave upward, causing the unit-force curve to be slightly concave downward for the $\frac{1}{8}$- and $\frac{1}{4}$-in. widths. The value of the total forces for the $\frac{5}{8}$-in. width of cut may be slightly high, as it was difficult to obtain consistent readings. Two points were taken from Fig. 32, bar No. 6, for the $\frac{1}{2}$- and $\frac{5}{8}$-in. widths, which also agree with the values in Fig. 37, and the unit-force curve drawn through them.

The unit-force curve is unquestionably lower with the narrow widths of cut and has a tendency to become horizontal with the greater widths of cut. The 235 lb. for the ¼-in. width and the 245 lb. for the ½-in. width check very well with the unit forces for the 0.012-in. depth of cut on Fig. 32. The finish left on the lands after a chip had been removed seemed to be finer for the narrower widths of cut. Slippage lines were counted as follows: 40 to 60 for the ⅛-in. width; 33 to 40 for the ¼-in. width; 28 to 32 for the ⅜-in. width; 24 to 30 for the ⅝-in. width; and about 12 to 20 for the ¾-in. width, the force values for which were too erratic to plot. The first number of these slippage values corresponds to the continuous lines across the land at an angle of about 20 deg. The second and larger figure corresponds to the number of lines countable, some of which were not continuous.

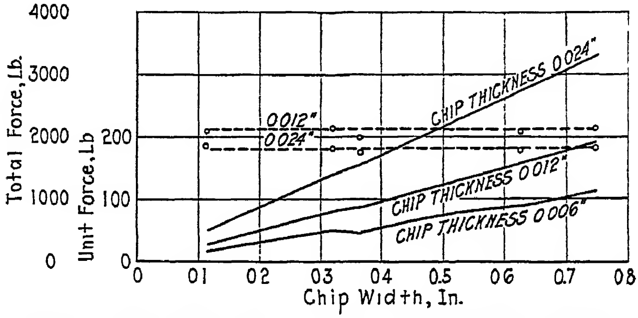


Fig. 39 Unit and Total Force — Chip Width Curves, Experiment 29

Material: S.A.E. 1035 steel, bar No. 28.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft per min.

The chip coils in each case were ¾ in. outside diameter by 2½ in. long for the ⅛-in. land, 1¾ by 3½ in. for the ¼-in. land, 2 by 3¾ in. for the ⅜-in. land, 3 by 4½ in. for ⅝-in. land (chip was conical) and 3 by 5 in. for the ¾-in. land (chip was conical).

68 Fig. 39 shows the total and unit forces for various chip widths for S.A.E. 1035 steel, bar No. 28. The total-force curve for the 0.006-in. chip is somewhat irregular, probably due to experimental error, associated with light cuts, therefore, unit-force curves have been drawn for only the 0.012- and 0.024-in. depths of cut. Both unit-force curves appear to be horizontal straight lines with no tendency to dip for the narrow chip widths. Fig. 40 shows the total and unit forces obtained for the various chip widths for 1.03 per cent carbon steel, annealed, bar No. 32. The total-force curve is obviously a straight line increasing directly with the width of cut. It is noted here that the curve when extended to the axis of abscissas as a straight line, passes not through the origin, but crosses the axis a short distance to the

right. This accounts for the tendency of the unit-force curve, shown in dotted lines, to dip with the low values of chip width. Several of the following figures for steel show the total-force curve to be similar to this one. All steel chips were noticed to have on the side which slides over the tool face a narrow band at either edge which appeared somewhat smoother than the surface in between. It was thought the intensity of stress, due to the removal of the chip by the tool, was less on these two narrow bands than

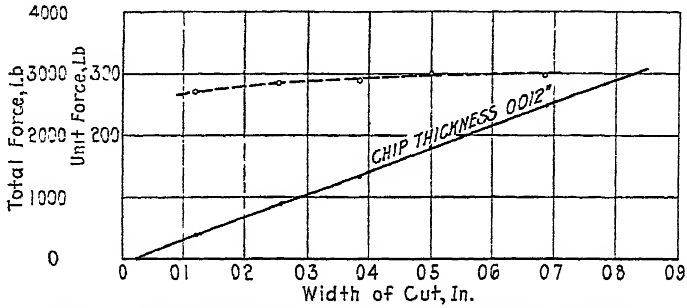


FIG. 40 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES, EXPERIMENT 45

Material: 1.03 per cent carbon steel, annealed, bar No. 32
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

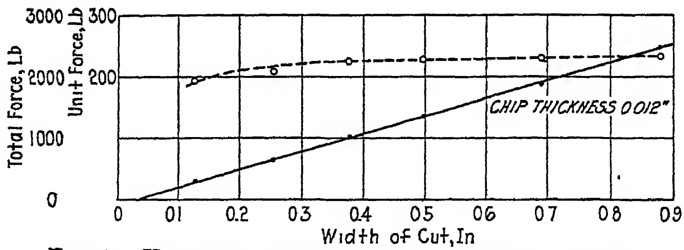


FIG. 41 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES, EXPERIMENT 41

Material: S.A.E. 2320 steel, bar No. 30
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

on the portion of the chip between them. As the width of cut approaches zero the total force would approach zero so that all of these total-force curves should pass through the origin by turning slightly concave upwards for the very thin chips. The reason for these edges on the steel chips cannot be accounted for unless it is due to lubrication. In milling the steel test bars in a milling machine, they were lubricated with lard oil and although wiped dry and sometimes cleaned with gasoline, there may have been a trace of oil left. This trace, so small as not to be seen, would cause some smoke during a heavy cut. It may be that the small amount of oil actually lubricated the chip at either side.

69 Fig. 41 shows a similar total- and unit-force curve for S.A.E. 2320 steel, bar No. 30. The unit force appears to be practically horizontal until a width of cut of less than ¼ in. is reached Fig. 42 shows a total- and unit-force curve for S.A.E. 2345, bar No. 29. The 30-deg. front-rake tool is used in all cases. It is only for a chip width of ⅛ in. that the unit force drops perceptibly, although it is probably gradually reduced from the ¼-in. width. Fig. 43 shows similar curves for the S.A.E. 3120 steel, bar No. 31. The unit-force curve here slightly slopes downward to the left.

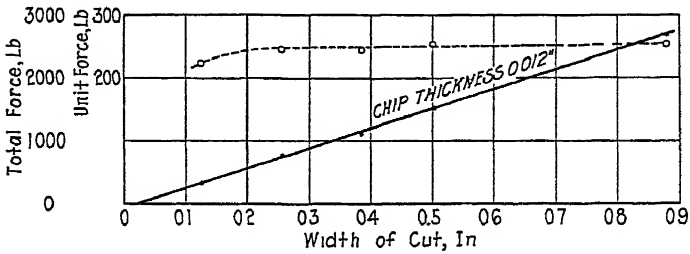


FIG. 42 UNIT AND TOTAL FORCE—CHIP WIDTH CURVES, EXPERIMENT 33

Material: S.A.E. 2345 steel, bar No. 29.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

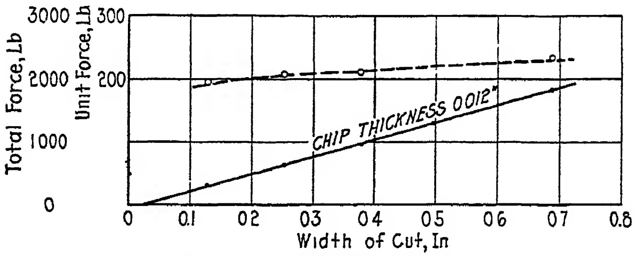


FIG. 43 UNIT AND TOTAL FORCE—CHIP WIDTH CURVES, EXPERIMENT 37

Material: S.A.E. 3120 steel, bar No. 31.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

The value for the ⅛-in. width, however, is not far below the values for greater widths.

70 Values for three depths of cut for cast iron, annealed, bar No. 7A, are shown in Fig. 44. A 0-deg. front-rake tool was used. The narrowest chip cut was ¼ in. in width. The values of the unit curves appear to be horizontal except for the points at the extreme right for a width of cut of 0.775 in. where a distinct increase is noted. This is attributed to some experimental error, as the increase occurs for all three depths of cut for that width only. Fig. 45 shows a similar set of curves for the same annealed cast iron, bar No. 7A, for a 30-deg. front-rake tool. While the unit

values are somewhat lower than those for similar depths of cut in Fig. 44, they have the same characteristics except for the increase for the 0.775-in. width of cut. The values for the $\frac{1}{4}$-in width of cut in Fig. 45 for the 0.012- and 0.024-in. depths of cut appear slightly above the normal unit-force curve. This is probably due to experimental error or variation in material, which for cast iron is a common source of annoyance.

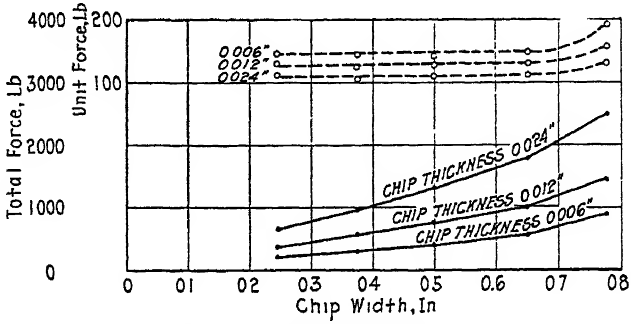


FIG. 44 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES, EXPERIMENT 25

Material: Cast iron, annealed, bar No. 7A.
Tool: 4-deg. clearance, 0-deg. front rake. Speed: 20 ft. per min.

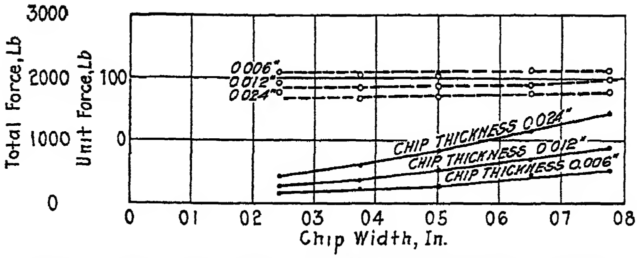


FIG. 45 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES, EXPERIMENT 25

Material: Cast iron, annealed bar No. 7A.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft per min.

71 Fig. 46 shows three total- and unit-force curves for cast iron, unannealed, bar No. 11. The tool used in this case had 0-deg. front rake. The results given in Fig. 47 are for the same material, but with a tool having 30-deg. front rake. The values are greater for the 0-deg. front-rake tool as expected. The characteristics of the unit-force curves, however, still indicate no influence due to the change in width of chip when the depth remains constant. The unit-force curves for the 0.024-in. depth of cut in both figures would turn downward to the left as the total-force curve must be concave upwards in order to pass through the origin.

72 Figs. 48 and 49 show total and unit forces for brass, bars Nos. 33 and 34, respectively, the tool in each case having 15-deg. front rake. Values for the unit-force curves for the 0.006- and 0.024-in. depth of cut only are shown in both cases, as the unit-force values for intermediate depths of cut would lie in between and cause congestion. The total-force curves in both figures appear to be straight lines passing through the origin and, therefore, the unit-force curves would continue horizontally to the left.

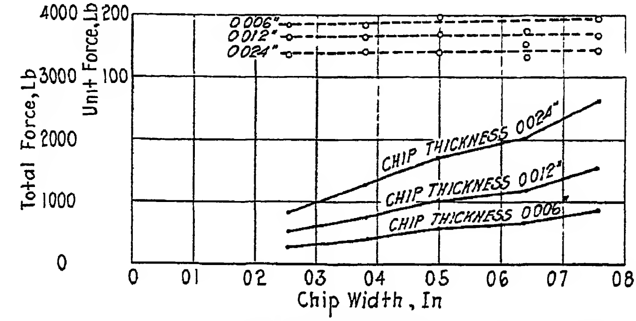


FIG. 46 UNIT AND TOTAL FORCE—CHIP WIDTH CURVES, EXPERIMENT 25

Material: Cast iron, unannealed, bar No. 11.
Tool: 4-deg. clearance, 0-deg. front rake. Speed: 20 ft. per min.

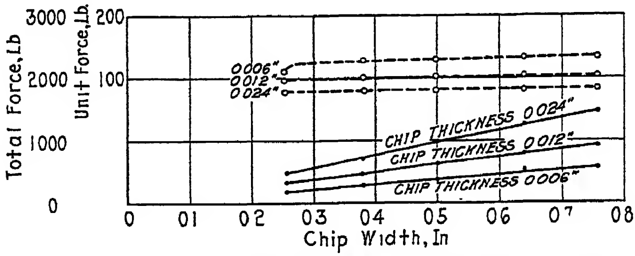


FIG. 47 UNIT AND TOTAL FORCE—CHIP WIDTH CURVES, EXPERIMENT 25

Material: Cast iron, unannealed, bar No. 11.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

73 Fig. 50 is a summary sheet of total forces for a depth of cut of 0.012 in. for various widths of cut for all of the materials used. A 30-deg. front-rake tool was used in all cases except for curve 7 for brass, for which a 15-deg. front-rake tool was used. The total-force curves, which are concave upward, cause the unit-force curves to be concave downward and to drop for small widths of cut, also, those total-force curves which cross the abscissa axis to the right of the origin, cause the unit-force curve to drop for small widths of cut. Fig. 51 is another summary sheet showing the unit forces for the curves of Fig. 50, plotted over the widths

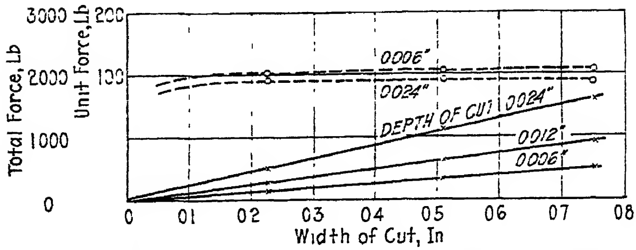


FIG. 48 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES, EXPERIMENT 56

Material, Brass, bar No. 33.
Tool: 4-deg clearance, 15-deg. front rake. Speed: 20 ft. per min.

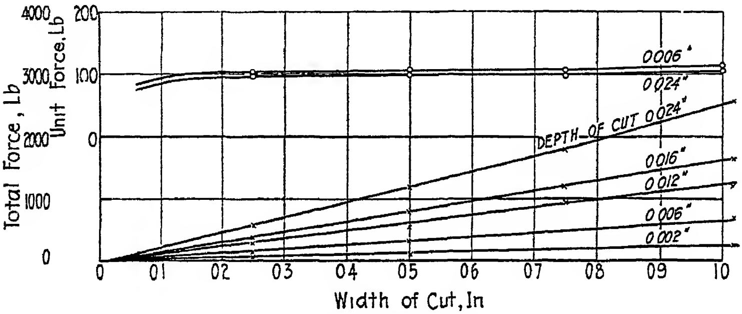


FIG. 49 UNIT AND TOTAL FORCE — CHIP WIDTH CURVES, EXPERIMENT 56

Material: Brass, bar No. 34.
Tool: 4-deg. clearance, 15-deg. front rake. Speed: 20 ft. per min.

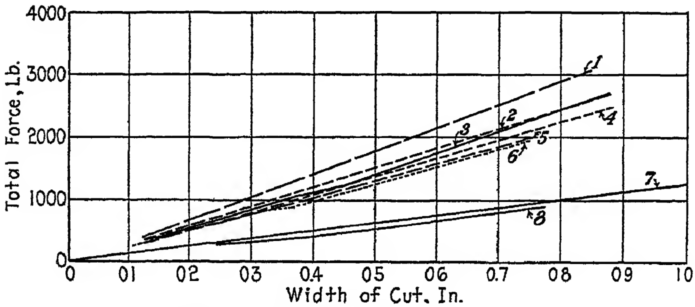


FIG. 50 SUMMARY OF TOTAL FORCE — CHIP WIDTH CURVES, PROBLEM *(e)*

Tools: 4-deg. clearance. Depth of cut: 0.012 in. Speed: 20 ft. per min.
Curve 1: 1.03 per cent carbon steel, bar No. 32, front-rake angle 30 deg.
Curve 2: S.A.E 2345 steel, bar No. 29, front-rake angle 30 deg.
Curve 3: 0.15 per cent carbon steel, bar No. 5, front-rake angle 30 deg.
Curve 4: S.A.E. 2320 steel, bar No. 30, front-rake angle 30 deg.
Curve 5: S.A.E. 3120 steel, bar No. 31, front-rake angle 30 deg.
Curve 6: S.A.E. 1035 steel, bar No. 28, front-rake angle 30 deg.
Curve 7: Brass, bar No. 34, front-rake angle 15 deg.
Curve 8: Cast iron, annealed, bar No. 7A, front-rake angle 30 deg.

of cut as abscissas. All curves except 6 for S.A.E. 1035 steel and 9 for annealed cast iron are shown concave downward for the small widths of cut. Both curves would, however, turn downward if continued to the left as the total-force curves in Fig. 39 and 45 would have to turn concave upward in order to pass through the origin. From this evidence it is concluded that the total and unit forces are lower for the narrower width of cut. The unit force increases rapidly with an increase in width of cut for most metals until a width of 0.24 in. is reached, after which the increase is less and in some instances there is no increase.

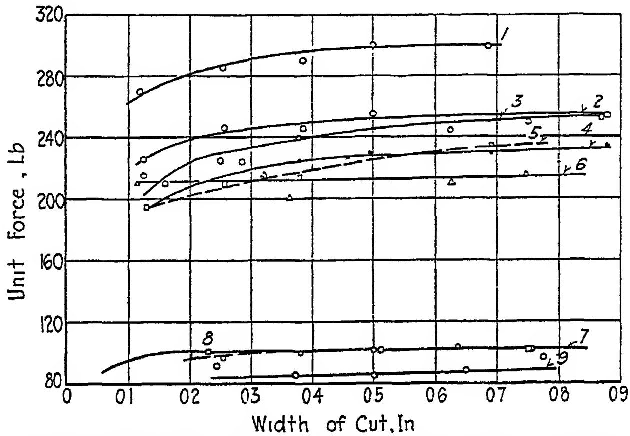


FIG 51 SUMMARY OF UNIT FORCE—CHIP WIDTH CURVES, PROBLEM *(e)*

Tools: 4-deg. clearance. Front-rake angle 30 deg. (except brass, 15 deg.). Depth of cut: 0.012 in. Speed: 20 ft. per min.
Curve 1: 1.03 per cent carbon steel, bar No. 32.
Curve 2: S.A.E. 2345 steel, bar No. 29.
Curve 3: 0.15 per cent carbon steel, bar No. 5.
Curve 4: S.A.E. 2320 steel, bar No. 30.
Curve 5: S.A.E. 3120 steel, bar No. 31.
Curve 6: S.A.E. 1035 steel, bar No. 28.
Curve 7: Brass, rolled, bar No. 33.
Curve 8: Cast iron, unannealed, bar No. 11.
Curve 9: Cast iron, annealed, bar No. 7A.

74 *Force—Depth of Cut.* The results of the second part of Problem (*e*), i.e., to show the influence on the force on the tool due to a change in the depth of cut or chip thickness for a constant width of cut, are given below. Again the 30-deg. front-rake tool having 4-deg. clearance was used for all metals except brass, when the 0-deg. and 15-deg. front-rake tools were used. A 0-deg. tool was used with the 30-deg. front-rake tool on cast iron as is noted.

75 Fig. 52 shows the total- and unit-force curves for 0.15 per cent carbon steel, bar No. 4. The width of cut is $\frac{1}{8}$ in. and the depth of cut varies between 0 and 0.100 in. The total-force curve has been drawn between the actual experimental points and ap-

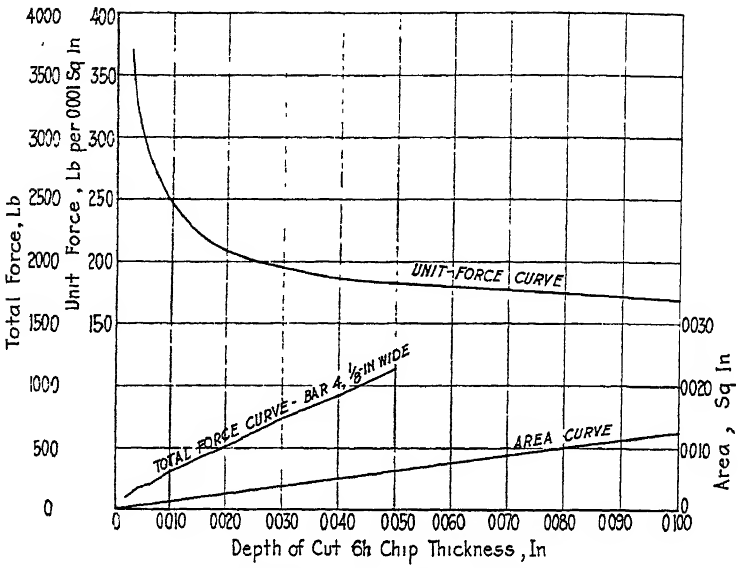


Fig 52 Chip-Thickness Curves, Experiment 14

Material. 0 15 per cent carbon steel, bar No 4.
Tool: 4-deg. clearance, 30-deg. front rake.
Chip width: ⅛ in. Speed: 20 ft per min.

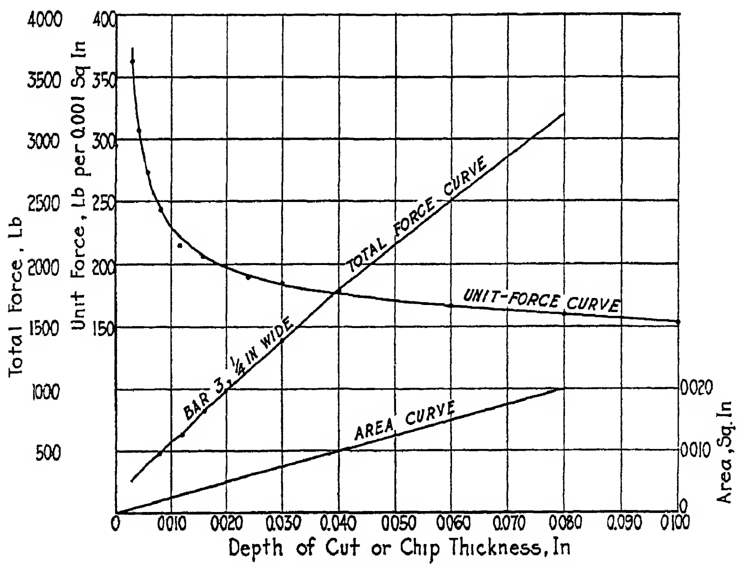


Fig. 53 Chip-Thickness Curves. Experiment 18

Material: 0.15 per cent carbon steel, bar No. 3.
Tool: 4-deg. clearance, 30-deg. front rake.
Chip width: ¼ in. Speed: 20 ft. per min.

pears to be somewhat irregular. If continued to the left and downward, it would pass through the coördinate axis, but would have to dip down considerably. The curve is obviously concave downward. Values for extremely small depths of cut were obtained with difficulty and in many cases proved unreliable. An error of a small fraction of a thousandth of an inch in depth of cut for such small depths of cut proved to be excessive. A slight error in reading the gage would also cause a wide variation in the total and unit force. The unit-force curve is shown plotted in the upper part of Fig. 52 and appears to be nearly horizontal for great

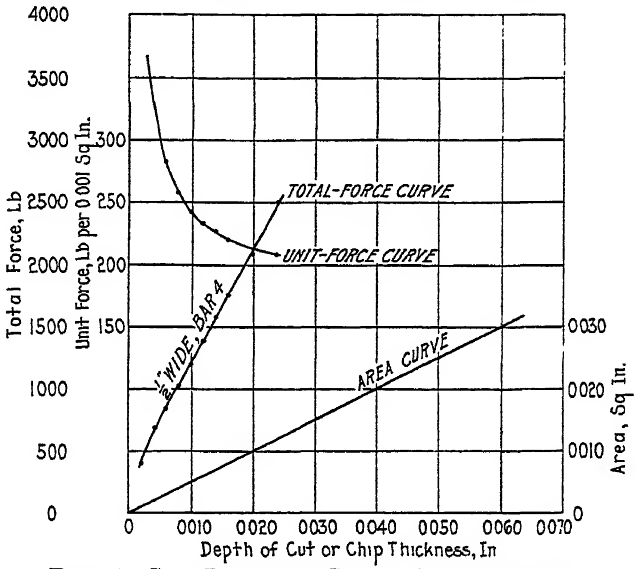


FIG. 54 CHIP-THICKNESS CURVES, EXPERIMENT 13

Material: 0.15 per cent carbon steel, bar No. 4.
Tool: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.500 in. Speed: 20 ft. per min.

depths of cut, but turns upward as the depth of cut is reduced below 0.030 in. Fig. 53 shows a similar total- and unit-force curve for 0.15 per cent carbon steel, bar No. 3, the width of land being $\frac{1}{4}$ in. The corresponding values for a $\frac{1}{2}$-in. width of land on bar No. 3 are shown in Fig. 54, while the values in Fig. 55 are produced from a 1-in. width of land on bar No. 4.

76 For the sake of comparison, the total-force curves, on the 0.15 per cent low-carbon steel for all four chip widths are shown in Fig. 56. The curves for the $\frac{1}{8}$-, $\frac{1}{2}$-, and 1-in. widths being of bar No. 4, are comparable. The total force for a 0.012-in. depth of cut for each of the three widths is 350, 1400, and 2725 lb., respectively. The value for the $\frac{1}{4}$-in. width of bar No. 3 is 655 lb. and is slightly lower than a smooth curve through the three values

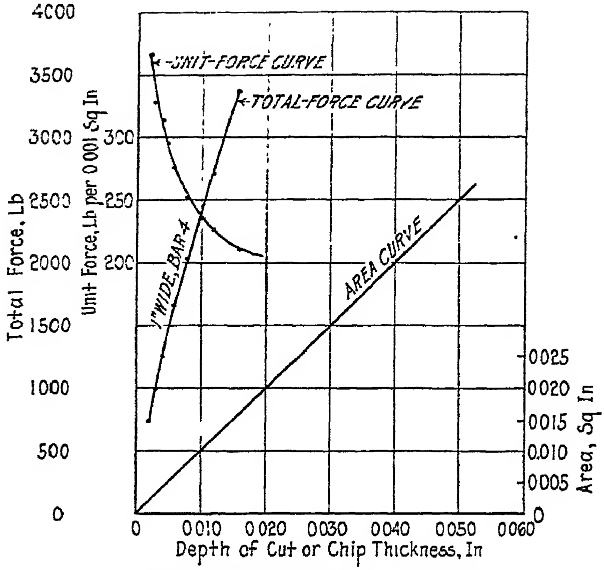


Fig. 55 Chip-Thickness Curves, Experiment 13

Material: 0.15 per cent carbon steel, bar No. 4.
Tool: 4-deg. clearance, 30-deg. front rake.
Width of cut: 1 in. Speed: 20 ft per min.

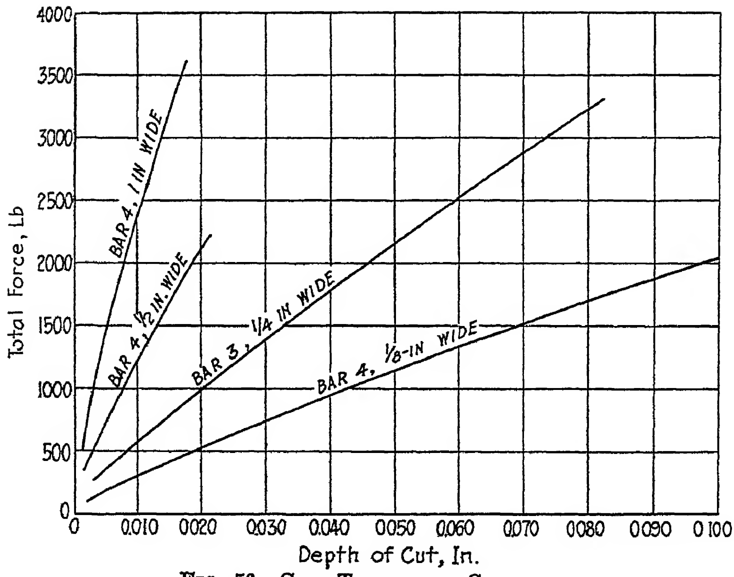


Fig. 56 Chip-Thickness Curves

Material: 0.15 per cent carbon steel.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

for bar No. 4 plotted over their chip widths. Fig 57 shows the unit-force curves for the total-force curves of Fig 56 for the four widths of cut of the 0.15 per cent carbon steel. The depth of cut was 0.012 in. and the 30-deg. front-rake tool was used. These unit curves indicate the tendency of the unit forces to increase as the depth of cut is reduced for all widths of cut. The ordinate scale is large and the curves, particularly for bar No. 4, are close together for $\frac{1}{8}$-, $\frac{1}{2}$-, and 1-in. widths.

77 Fig. 58 shows a total- and unit-force curve as a function of the depth of cut for S.A.E. 1035 steel, bar No. 28. The value for

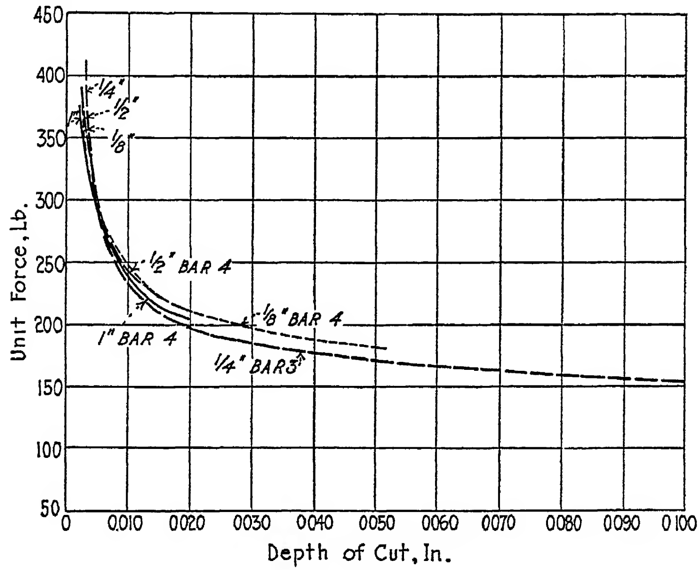


Fig. 57 Chip-Thickness Curves

Material: 0.15 per cent carbon steel, bars Nos. 3 and 4.
Tool: 4-deg. clearance, 30-deg. front rake. Speed: 20 ft. per min.

both the total and unit forces for the 0.003 in. depth of cut is obviously in error. Otherwise, the curves are similar in character to those for 0.15 per cent carbon steel. Fig. 59 shows a similar set of curves for the 1.03 per cent carbon steel, bar No. 32; Fig. 60 for S.A.E. 2320 steel; bar No. 30; Fig. 61 for S.A.E. 2345 steel, bar No. 29; and Fig. 62 for S.A.E. 3120 steel, bar No. 31. In all of the curves for the steel, there is a distinct tendency for the unit force to curve upward for depths of cut below 0.024 in. For depths greater than this, the curves become more and more horizontal, but seem to slope to the right for all depths.

78 Values for the total and unit forces when cutting cast iron annealed, bar No. 8A with both the 30- and 0-deg. front-rake tools, are shown in Fig. 63. The total-force curve for the 0-deg. front-

rake tool is considerably higher than that for the 30-deg. front-rake tool. The curves, however, appear to be slightly concave downward, becoming more so with the reduced depth of cut. The unit forces for the 0-deg. front-rake tool are also higher than

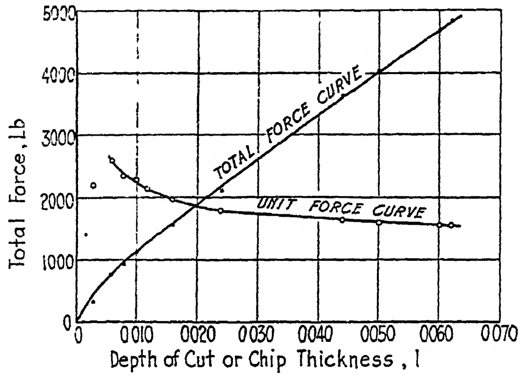


FIG. 58 CHIP-THICKNESS CURVES, EXPERIMENT 27

Material: S.A.E. 1035 steel, bar No. 28.
Tool: 4-deg. clearance, 30-deg. front rake. Width of cut: 0.5 in.
Speed: 20 ft. per min.

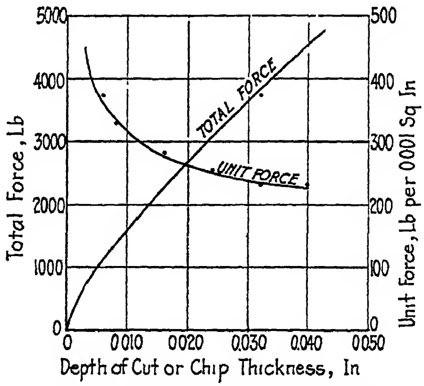


FIG. 59 CHIP-THICKNESS CURVES, EXPERIMENT 44

Material: 1.03 per cent carbon steel, bar No. 32.
Tool: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.5 in. Speed: 20 ft. per min.

those for the 30-deg. front-rake tool. However, both turn upward decidedly for depths of cut less than 0.024 in. Similar values are shown for unannealed cast iron, bar No. 8, in Fig. 64. These values appear more uniform than those for the annealed bar. The total-force curves for both the 0- and 30-deg. front-rake-angle tools have a slightly greater curvature concave downward than for

the annealed bar. The characteristics are quite similar. The unit-force curves, as for the annealed cast iron, are lowest for the greatest depth of cut, are concave upward at all points, and are accelerated upward with a reduction in depth of cut. Below 0.025-in. depth of cut, there is a marked change in the unit force for a given change in depth of cut, while for values above 0.025 in., the change is less marked.

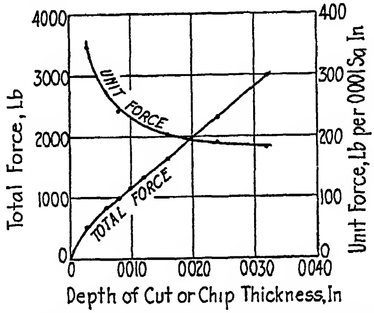


FIG. 60 CHIP-THICKNESS CURVES, EXPERIMENT 40

Material: S.A.E. 2320 steel, bar No. 30.
Tool: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.5 in. Speed: 20 ft. per min.

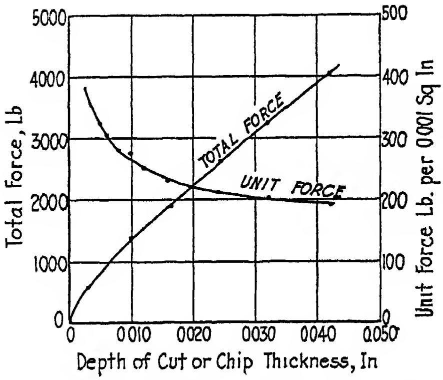


FIG. 61 CHIP-THICKNESS CURVES, EXPERIMENT 32

Material: S A E 2345 steel, bar No. 29.
Tool: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.5 in. Speed: 20 ft per min.

79 Fig. 65 shows the total forces on 15-deg. front-rake tool when cutting brass, bar No. 34, for widths of cut of $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 in., respectively. All curves are drawn as straight lines and seem to fit the conditions quite well. The curves if extended as straight lines would cross the ordinate axis above the zero point. They would, however, as true curves, turn downward so as to pass through the origin when the depth of cut is zero. The values for the total force for the 0.024-in. depth of cut are 570, 1175, 1800,

and 2570 lb., respectively. When plotted over the widths of chip as abscissas, a curve slightly concave upward is obtained as shown by the total-force curve in Fig. 49. It is for this reason that the unit-force curves in Figs. 48 and 49 turn down for small widths of cut. Two unit-force curves are drawn in Fig. 65 for the 1.015-in. and 0.250-in. widths. No intermediate values are shown, as they lie in between these two curves which in themselves are close together. The unit-force curves show the usual tendency to turn upward for small values of depth of cut.

80 Fig. 66 shows the total-force curves for 0.006-, 0.012-, and 0.024-in. depths of cut, for four widths of cut. Each width, however, is from a different bar, so that the results are not directly comparable. The unit-force values for the 1.015-in. width, bar

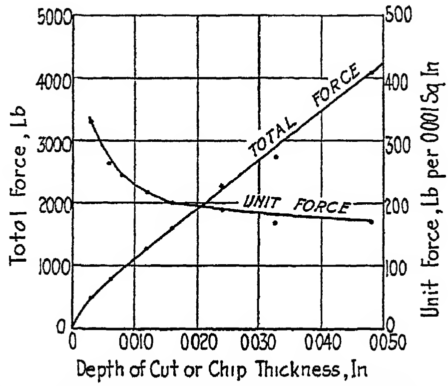


FIG. 62 CHIP-THICKNESS CURVES, EXPERIMENT 36

Material: S.A.E. 3120 steel, bar No. 31.
Tool: 4-deg. clearance, 30-deg. front rake.
Width of cut: 0.5 in. Speed: 20 ft. per min.

No. 34, are 134 lb. for the 0.006-in. chip and 123 lb. for the 0.012-in. depth of cut. The unit-force values for the other curves also show those for the 0.006-in. depth of cut to be greater than those for the 0.012-in. depth of cut, indicating that the unit-force curve would turn upward for the thinner chips. The curves shown are drawn through the actual experimental points. Notation is made for those points at which chatter occurred. When chatter occurs the force on the tool is reduced.

81 Fig. 67 shows the total-force curves for a 0.308-in. width of cut, bar No. 27, and a 0.512-in. width of cut, bar No. 26, plotted on depth of cut in inches. The lower portions of the curves have been drawn as straight lines and would cross the ordinate axis, if continued as straight lines, slightly above the origin. The dashed lines indicate the total-force curves if continued upward as straight lines without chatter. The full lines are drawn through the actual experimental points. The points at which chatter occurred are

indicated. Unit-force curves are shown for both widths of cut for the straight-line, total-force curves. That for the 0.512-in. width is shown above that for the 0.308-in. width. While it is expected that the unit-force curve for the wider chip will be higher, the marked difference in values is accounted for in this case by the difference in materials.

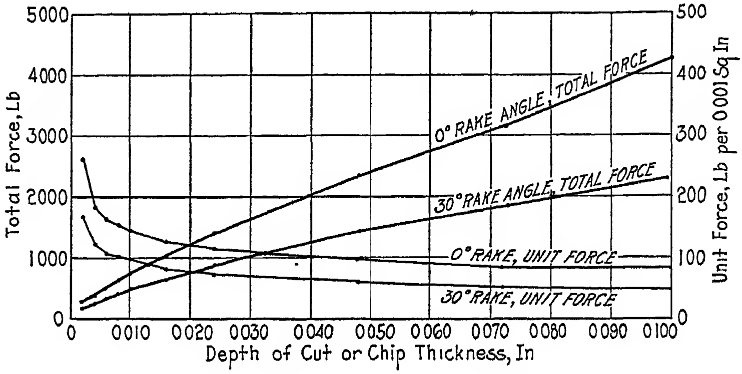


FIG. 63 CHIP-THICKNESS CURVES, EXPERIMENT 23

Material: Cast iron, annealed, bar No 8A.
Tools: 4-deg. clearance. Width of cut: 0.5 in. Speed: 20 ft. per min.

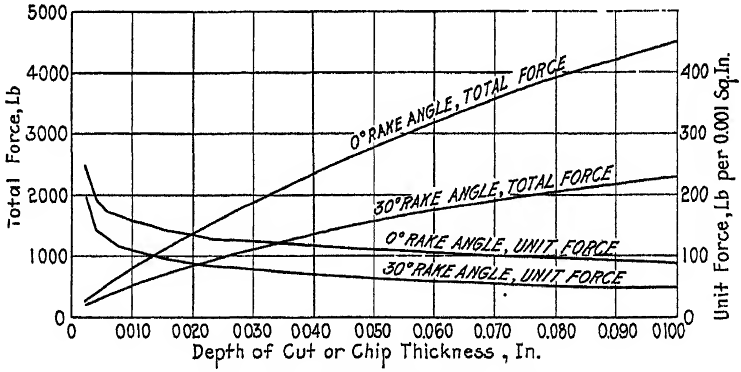


FIG. 64 CHIP-THICKNESS CURVES, EXPERIMENT 23

Material: Cast iron, unannealed, bar No. 8.
Tools: 4-deg. clearance. Width of cut: 0.5 in. Speed: 20 ft. per min.

82 Fig. 68 shows all total-force curves for the 0.5-in. width of cut for the nine different materials cut, plotted on the depths of cut as abscissas. All of the curves are decidedly concave downward with increased curvature for the thinner chips. Curve 7 for brass is drawn as a straight line, although it is actually very slightly concave downward. A 30-deg. front-rake tool was used in all cases except for brass, for which a 15-deg. front-rake tool

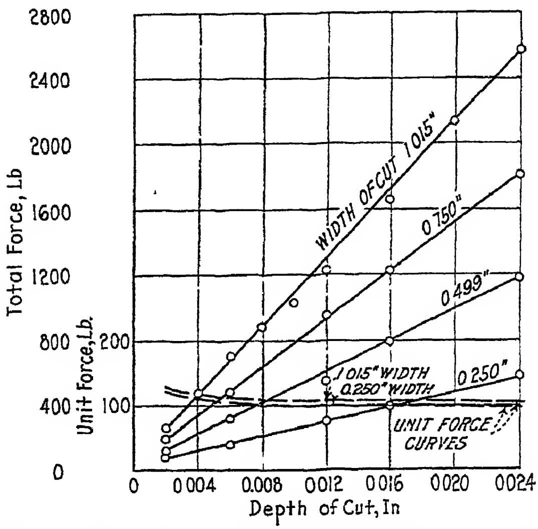


FIG. 65 CHIP-THICKNESS CURVES, EXPERIMENT 56

Material: Brass, bar No. 34.
Tool: 4-deg. clearance, 15-deg. front rake. Speed: 20 ft. per min.

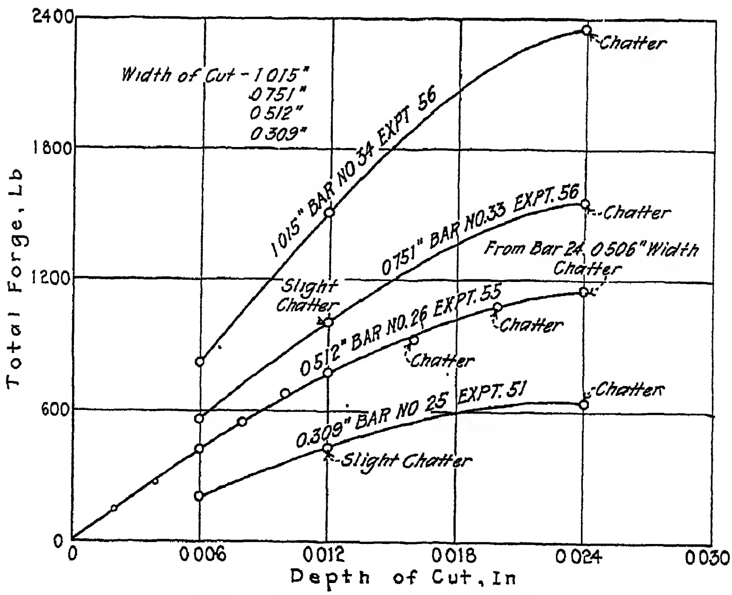


FIG. 66 CHIP-THICKNESS CURVES

Material: Brass, bars Nos. 25, 26, 33, and 34.
Tool: 4-deg. clearance, 0-deg. front rake. Speed: 20 ft. per min.

was used. Corresponding values for the unit forces for the various metals cut, plotted on the depths of cut in inches as abscissas, are shown in Fig. 69. These curves show that the unit forces for the very thin chips (0.003 in. and less) are relatively high, but are reduced rapidly with an increase in depth of cut. This reduction in unit force becomes more retarded as the depth of cut is increased so that for depths of cut of 0.030 in. or greater, there appears to be little reduction in unit force for an increase in depth of cut.

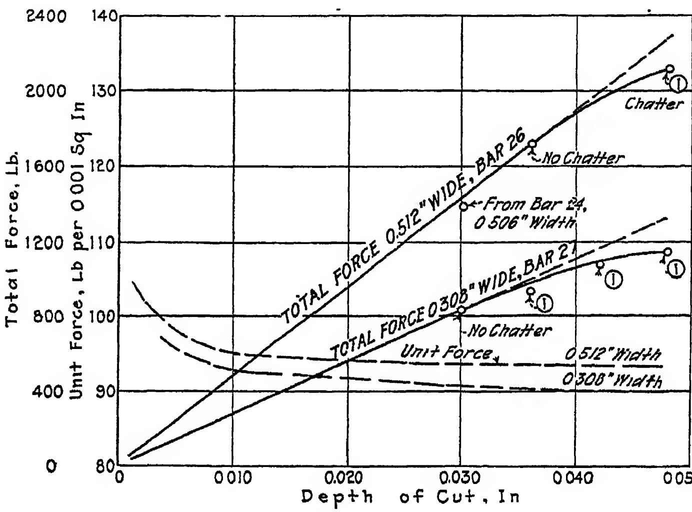


FIG. 67 CHIP-THICKNESS CURVES, EXPERIMENT 52

Material: Brass, bars Nos 26 and 27.
Tool: 4-deg. clearance, 15-deg. front rake. Speed: 20 ft. per min.

Physical Properties of Materials Cut—Problem (f)

83 Problem (*f*) is "to find the relation between the force on the tool of a given shape required to remove a specific chip of a given material and the physical or chemical properties of the material." For this purpose, the unit force, that is, the force per 0.001 sq. in. of cross-sectional area of chip for a 30-deg. front-rake tool, has been selected for each material to compare with the other physical properties of the material. The unit force for the 30-deg. tool was selected, as that tool gave satisfactory results for all metals except brass. Even that for brass with the 0.012-in. depth of cut is considered reliable and representative of the properties of the material. All unit forces were determined from a cut 0.5 in. wide and 0.012 in. deep. These unit forces are shown in Fig. 26.

84 In Fig. 70 several curves are plotted to show any relation which might exist between the unit forces and the other physical properties of the materials. The abscissas 1, 2, 3, etc., represent

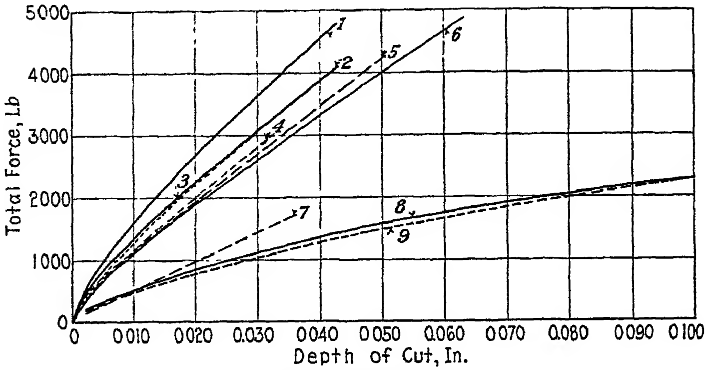


FIG. 68 SUMMARY OF TOTAL FORCE — CHIP THICKNESS CURVES PROBLEM *(e)*

Tools: 4-deg. clearance, 30-deg. front rake (except brass, 15-deg.)
Width of cut: 0.5 in. Speed: 20 ft. per min.
Curve 1: 1.03 per cent carbon steel, bar No. 32.
Curve 2: S.A.E. 2345 steel, bar No. 29.
Curve 3: 0.15 per cent carbon steel, bar No. 4.
Curve 4: S.A.E. 2320 steel, bar No. 30.
Curve 5: S.A.E. 3120 steel, bar No. 31.
Curve 6: S.A.E. 1035 steel, bar No. 28.
Curve 7: Brass, rolled, bar No. 26.
Curve 8: Cast iron, unannealed, bar No. 8.
Curve 9: Cast iron, annealed, bar No. 8A.

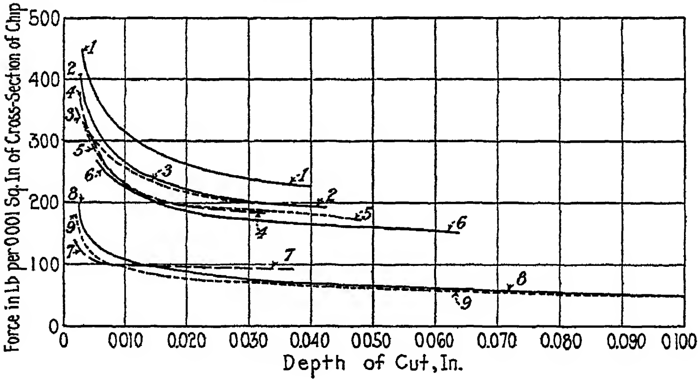


FIG. 69 SUMMARY OF UNIT FORCE — CHIP THICKNESS CURVES

Tools: 4-deg. clearance, 30-deg. front rake (except brass, 15-deg.)
Width of cut: 0.5 in. Speed: 20 ft. per min.
Material: Curve numbers same as in Fig. 68.

the various materials referred to in the table in the figure. These materials are representative of all of those cut and are arranged in order of intensity of unit forces. Curve *A* joins the unit-force values for each material. The values decrease with each material

from left to right. Curves *B*, *C*, and *D* indicate the Brinell, Rockwell, and scleroscope hardness numbers, respectively, for each material. The Brinell numbers for the steels 1, 3, and 6 are low on

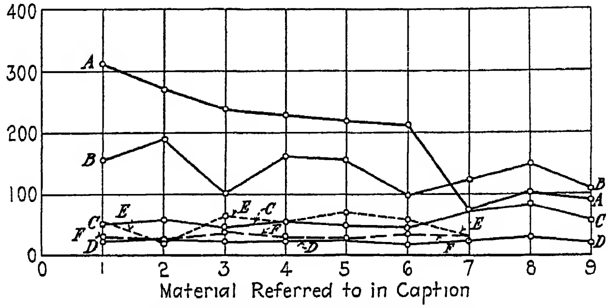


FIG. 70 PHYSICAL-PROPERTY CURVES

CURVES

A Unit force for 0.5 by 0.012 in chip 30-deg. front rake tool.
B Brinell hardness number.
C Rockwell hardness number.
D Scleroscope hardness number.
E Percentage reduction of area (tension).
F Percentage elongation in 2 inches.

MATERIALS

1 1.03 per cent carbon steel, bar No. 32
2 S.A.E. 2345 steel, bar No. 29.
3 0.15 per cent carbon steel, bar No. 4
4 S.A.E. 2320 steel, bar No. 30.
5 S.A.E. 3120 steel, bar No. 31
6 S.A.E 1035 steel, bar No. 28.
7 Brass, rolled, bar No. 26.
8 Cast iron, unannealed, bar No. 8.
9 Cast iron, annealed, bar No. 8A.

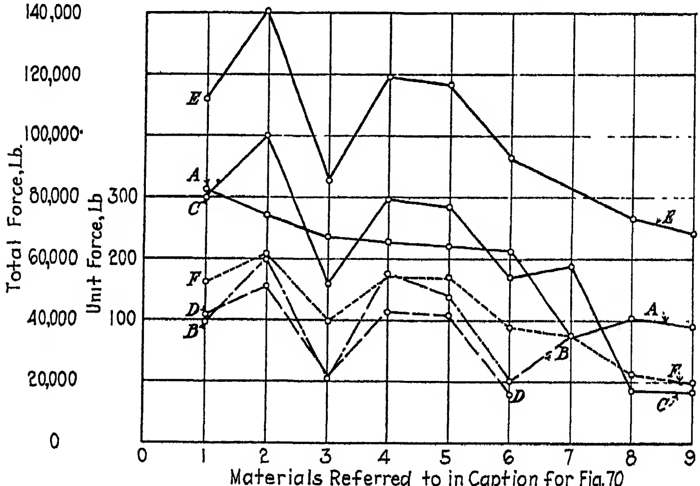


FIG. 71 PHYSICAL-PROPERTY CURVES

(See Par. 85. Material number same as in Fig. 70.)

the curve, while the corresponding unit forces remain relatively even with the others. The unit forces for brass, 7, unannealed cast iron, 8, and annealed cast iron, 9, are less than half of those for steel and are even greater than those for steels 3 and 6. It is appreciated that for other heat treatments of the materials the

unit force and hardness numbers would probably be different. There appears to be no helpful relation between the unit-force and the hardness-number curves. The highest point on the Rockwell and scleroscope hardness curves is for cast iron, 8. Curves *E* and *F* shown in dashed lines represent, in per cent, the reduction of area and elongation in 2 in., respectively, for each material. The reduction of area appears to be highest for the steels which have the lowest unit forces except the S.A.E. 2345 steel, 2, but drops with the unit force for brass, 7, as the unit force also does. No relation between the unit force and these factors seems to be suggested.

85 Fig. 71 shows the unit force and strength plotted over the materials referred to in the table in Fig. 70 as abscissas. The

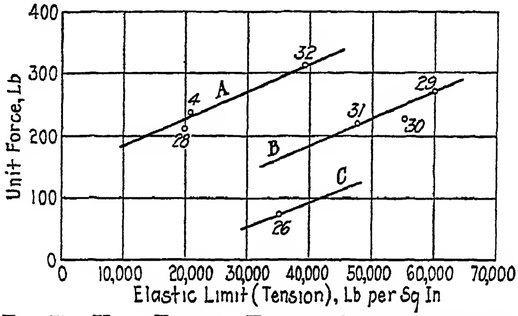


FIG. 72 UNIT FORCE — ELASTIC LIMIT (TENSION)

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min

Bar No. 32: 1.03 per cent carbon steel. Bar No. 30: S.A.E. 2320 steel.
Bar No. 29: S.A.E. 2345 steel. Bar No. 31: S.A.E. 3120 steel.
Bar No. 4: 0.15 per cent carbon steel. Bar No. 28: S.A.E. 1035 steel.
Bar No. 8: Cast iron, unannealed. Bar No. 8A. Cast iron, annealed.
Bar No. 26: Brass, rolled.

unit-force curve *A* is identical with that for Fig. 70. Curve *B* represents the elastic limits in tension for each material. The elastic limit for the 1.03 per cent carbon steel, 1, is 39,170 lb., which is lower than that for steels 2, 4, and 5, while its corresponding unit-force value is highest of all steels. The elastic limits of bars Nos. 3 and 6, both low-carbon steels, are relatively very low while the respective unit forces are even with those of the other steels. The elastic limit for brass, 7, is not altogether reliable, although it was determined very carefully. Curve *D* represents the elastic limits in compression for the materials and follows closely curve *B*, the elastic limits in tension. Curves *C*, *E*, and *F* represent the ultimate strength (maximum load divided by original area) in tension, compression, and shear, respectively (for test-bar shapes, see Appendix No. 3). These curves show high values for the alloy steels 2, 4, and 5, and relatively low values for the carbon steels, 1, 3, and 6, and indicate no consistent

relation to the cutting unit-force curve. The ultimate-strength curves E and C show a reduction for annealed and unannealed cast iron, which is not proportional to the reduced cutting-unit forces of curve A. The elastic limit and ultimate strength in tension, curves B and C are relatively high for brass, 7, however, which has the lowest cutting unit force. Figs. 70 and 71 indicate no consistent relation between the physical properties of the various metals and the unit forces. It does suggest, however, that it may be of advantage to divide the materials into four groups, namely, alloy steels, carbon steels, brasses, and cast irons for further study.

86 Figs. 72 to 80, inclusive, show the location on the coördinate axes of each material. The ordinates equal the unit force for the material for the 30-deg. tool and the abscissas equal the physical property in question. Fig. 72 shows the points plotted over the elastic limits in tension as abscissas. This shows bars Nos. 28, 4,

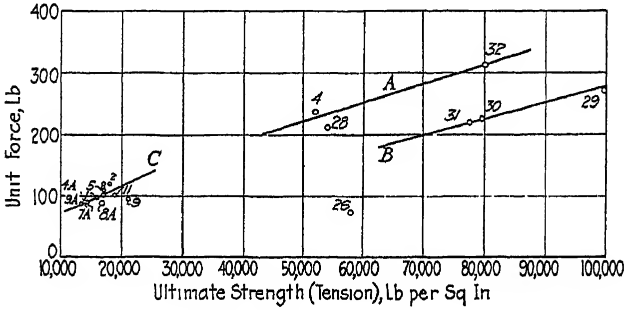


FIG. 73 UNIT FORCE — ULTIMATE STRENGTH (TENSION)

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

and 32, which are all straight carbon steels, lying close to the straight line A, while bars Nos. 31, 30, and 29, all alloy steels, indicate line B, and the brass, bar No. 26, being isolated, might indicate the third line C. In Fig. 73, the points are plotted over the ultimate strengths in tension. Again the straight carbon steels, bars Nos. 4, 28, and 32, indicate the possibility of the curve A, while the alloy steels, bars Nos. 29, 30, and 31, indicate the possibility of curve B. The cast irons, bars Nos. 8 and 8A, and the brass, bar No. 26, do not seem to conform in any way with the values of the steels. Points for other cast-iron bars Nos. 1, 2, 4A, 5, 7A, and 9A all fall within half an inch to the left or above the point 8A, while bar No. 9 falls half an inch to the right of point 8A. This suggests a third line C which gives a relation between the unit cutting force for a 30-deg. front-rake tool for cast iron and its ultimate strength in tension. Fig. 74 shows these points again plotted over the elastic limit in compression and Fig. 75 the same for the ultimate strength in compression. In each case

a curve A through the carbon steels is indicated, and also a curve B through the alloy steels. Again, points for all available data for cast iron are plotted. Except for bar No. 9, the points indicate that a line C expresses a relation between the unit force on the 30-deg. tool and the ultimate strength in compression of the cast iron tested. Curve C conforms well with curve B extended. Fig. 76 shows the points plotted over the ultimate strength in shear, and

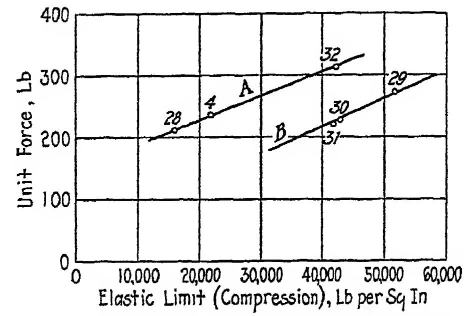


FIG. 74 UNIT FORCE—ELASTIC LIMIT (COMPRESSION)

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

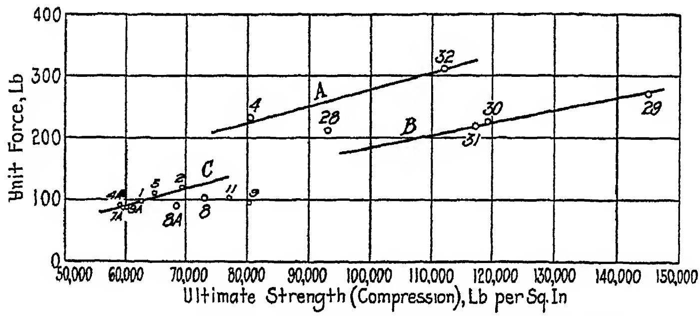


FIG. 75 UNIT FORCE—ULTIMATE STRENGTH (COMPRESSION)

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

again there is an indication of a curve A for carbon steels and another curve B for the alloy steels. Points for all available data on cast iron are again plotted, and, while they fall close together, a curve C is indicated. Curve C might be shown to become a part of curve B extended, were more data available. The single point for brass, bar No. 26, seems to be quite independent.

87 Values of the unit forces for the various materials are plotted over the Brinell, the scleroscope, and the Rockwell hardness numbers in Figs. 77, 78, and 79, respectively. The Brinell

figures in Fig. 77 also seem to indicate a line *A* through the points 28, 4, and 32 for the straight carbon steels and another line *B* through the alloy steels 31, 30, and 29. The points for cast iron, bars Nos. 8 and 8A, and for brass, bar No. 26, seem to have the same unit-force values over a range of Brinell hardness numbers from 110 to 150, but the unit forces increase as the Brinell number increases above 150. The suggested curve *C* is of little value

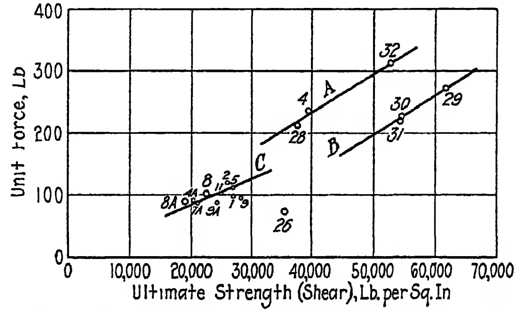


FIG. 76 UNIT FORCE — ULTIMATE STRENGTH (SHEAR)

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

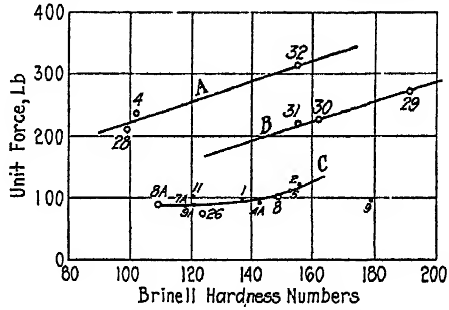


FIG. 77 UNIT FORCE — BRINELL HARDNESS NUMBERS

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

because of the slight change in unit force for a wide range of Brinell reading. The scleroscope hardness numbers shown in Fig. 78 seem to group the steels together in one group and the brass and cast irons together in a second. There appears to be no relation which would predict the unit-force value for a given scleroscope hardness for the steels, but curve *C* through the cast-iron points is indicated. There appears to be no useful relation between the unit-force points plotted over Rockwell hardness numbers in Fig. 79, as indicated curve *C* is horizontal until a Rockwell number of 70 is reached, after which there is a small

increase in the unit force for an increase in hardness. The points for the steels are grouped above the lower values of Rockwell hardness numbers, but in the higher unit-force values. The points for cast iron and brass extend over a wide range of Rockwell numbers from 57 to 88, but over a very limited range of unit force values which are of low magnitude.

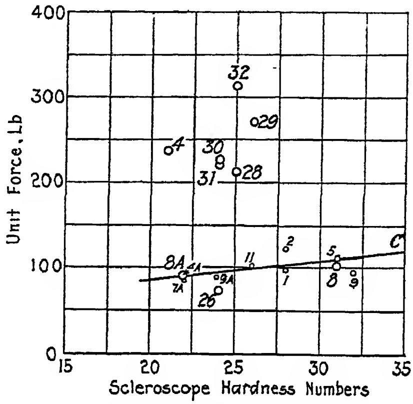


FIG. 78 UNIT FORCE — SCLEROSCOPE HARDNESS NUMBERS

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft per min.
(Material bar numbers same as in Fig. 72.)

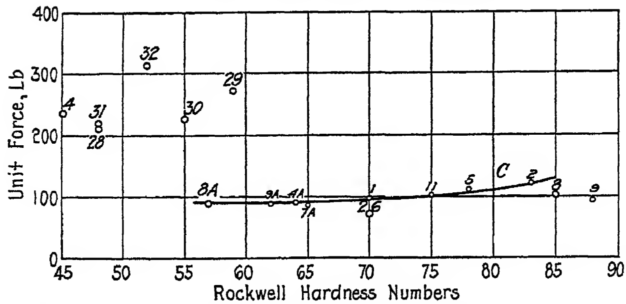


FIG. 79 UNIT FORCE — ROCKWELL HARDNESS NUMBERS

Tools: 4-deg. clearance, 30-deg. front rake.
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

88 In Fig. 80 are shown plotted for each material the unit forces as ordinates over the reduction of area in per cent and elongation in two inches in per cent as abscissas. The reduction-of-area points cover the whole field from 32 per cent for brass, bar No. 26, to 70 per cent for S.A.E. 3120 steel, bar No. 31. The range in unit forces likewise covers the whole field and there appears to be no indication of the value of the unit force for a material having a known reduction of area. The percentage-of-elongation points

are well grouped together to the left of the figure. Line *A* has been drawn through the points for the alloy steels, bars Nos. 29, 30, and 31, and may represent a relation between the elongation and the unit cutting force. A similar relation is not apparent, however, for the carbon steels, as bar No. 28 falls too far below a line *B* drawn through points 32 and 4.

89 These deductions have been made in most cases for only three points for each line. Unquestionably, additional data for both carbon and alloy steels would permit more definite conclusions to be drawn. The unit forces are those for a 30-deg. front-rake tool. This in itself has an influence on the lines *A* and *B* of the various figures. If a 15-deg. front-rake tool had been selected instead of the 30-deg. tool, the unit-force curve would be higher throughout but the values for some materials would be relatively

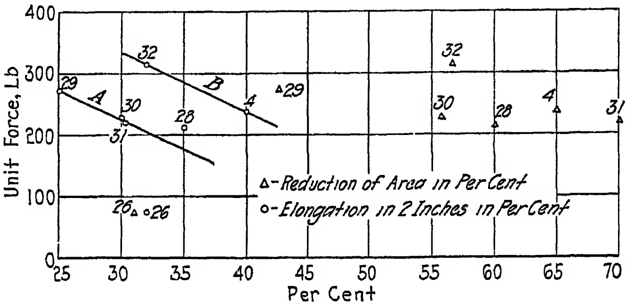


FIG. 80 UNIT FORCE — ELONGATION AND PERCENTAGE OF REDUCTION OF AREA

Tools: 4-deg. clearance, 30-deg. front rake
Chip: 0.012 in. deep by 0.5 in. wide. Speed: 20 ft. per min.
(Material bar numbers same as in Fig. 72.)

higher than those for others, as it was seen in Fig. 26 that the slope of the unit-force curve for front-rake angles is greater for some materials than others. For another heat-treatment of each of these bars, the unit force would be greater or less. This, too, might destroy the apparent relation between the physical properties and the cutting forces. The fact that the straight carbon steels seem to separate themselves from the alloy steels as far as the cutting properties are concerned is entirely new to the author, and appears to be a problem of atomic structure of the materials.

SUMMARY

90 To enable one to compare the results obtained above, three sets of curves have been prepared as a function of cubic inches of metal removed per minute per horsepower. Fig. 81 shows these values for various front-rake angles in degrees as abscissas. For all metals, there is a distinct increase in the amount of metal removed per minute per horsepower as the front-rake angle is

increased, i.e., the cutting angle is reduced. For all of these curves the depth of cut is 0.012 in., the width of cut, 0.5 in., and the cutting speed, 20 ft. per min. It is observed from the figure that the 1.03 per cent carbon steel has the least amount of metal removed per horsepower-minute, increasing from 0.94 cu. in. for the 10-deg. front-rake tool to 1.52 cu. in. per min. for a 40-deg. front-rake tool. The unit forces are obtained from Fig. 26 and the cubic inches per minute per horsepower equals 396 divided by the unit force. These curves also show that the influence of the front-rake angle is less for S.A.E. 2345 steel than it is for the 0.15 per cent carbon steel, the latter showing an increase from

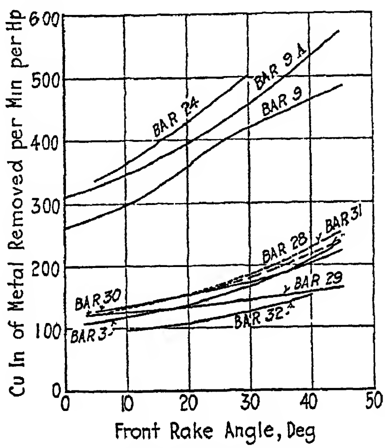


FIG. 81 CUBIC INCHES OF METAL REMOVED PER MINUTE PER HORSEPOWER — CHIP WIDTH

Tools: 4-deg. clearance. Width of cut: 0.5 in. Depth of cut: 0.012 in. Speed: 20 ft. per min.
(Bar No. 9, cast iron, unannealed; bar No. 9A, cast iron, annealed. Other material bar numbers same as in Fig. 72.)

1.12 cu. in. per min. per horsepower for a 5-deg. front-rake tool to 2.37 cu. in. for a 45-deg. front-rake tool, while the corresponding figures for S.A.E. 2345 are 1.23 and 1.65 cu. in. The brass and cast iron are shown much higher on the scale than the steels.

91 Fig. 82 shows the cubic inches of metal removed per minute per horsepower plotted over the width of cut in inches for the various metals for the 30-deg. front-rake tool; for brass a 15-deg. tool was used. These curves fall off slightly for widths of cut between 0.1 and 0.3 in. The values for 0.15 per cent carbon steel (refer to Fig. 51 for unit forces) for a 0.2-in. width of cut is 1.77 cu. in. per min. per horsepower, and for a 0.7-in. width is 1.57 cu. in.; again the 1.03 per cent carbon steel is the lowest with about 1.5 cu. in. of metal removed per min. per horsepower for a 0.1-in. width, and 1.32 cu. in. for a 0.7-in. width of cut.

92 Fig. 83 shows a similar set of curves plotted over the depth of cut in inches for the 30-deg. front-rake tool; for brass a 15-deg. tool was used. In all cases, the amount of metal removed in cubic inches increases rapidly until a depth of cut of 0.010 in. is reached and then less rapidly until a depth of cut of about 0.025 in. is reached, after which the increase for the steels is less marked. Again, the lowest curve is for the 1.03 per cent carbon steel, which has a value of 1.06 cu. in. removed per min. per horsepower for 0.005-in. depth of cut and increases to 1.25 cu. in. for 0.010-in. depth of cut and to 1.73 cu. in. for 0.040-in. depth of cut. The 0.15 per cent carbon steel increases from 1.15 in. for a 0.0025-in. depth of cut to 1.86 cu. in. for a 0.025-in. depth of cut.

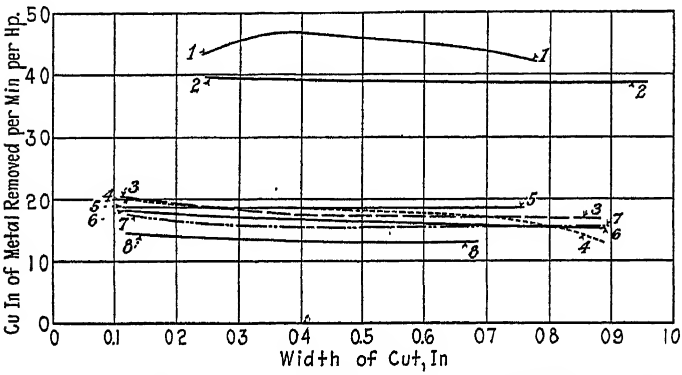


FIG. 82 CUBIC INCHES OF METAL REMOVED PER MINUTE PER HORSEPOWER — CHIP WIDTH

Tools: 4-deg. clearance, 30-deg. front rake (except brass, 15-deg.).
Depth of cut: 0.012 in. Speed: 20 ft. per min.
Curve 1: Cast iron, annealed, bar No. 7A.
Curve 2: Brass, bar No. 34.
Curve 3: S.A.E. 2320 steel, bar No. 30.
Curve 4: S.A.E. 3120 steel, bar No. 31.
Curve 5: S.A.E. 1035 steel, bar No. 28.
Curve 6: 0.15 per cent carbon steel, bar No. 5.
Curve 7: S.A.E. 2345 steel, bar No. 29.
Curve 8: 1.03 per cent carbon steel, bar No 32.

93 These curves indicate clearly the influence of the variables involved on the cutting efficiency of the tool. For these curves the cubic inches removed per horsepower-minute has been used as the ordinate, but some authors have used the inversed ratio, that is, the horsepower per cubic inch of metal removed per minute. These values are readily interchangeable, one being the reciprocal of the other.

CONCLUSIONS

94 The following conclusions are based only on the data collected by the author as presented above, and are not influenced by the work of other investigators. They are, however, compared

later with results of others. In reading these conclusions it should be kept in mind that this is an investigation of forces on the tool as a function of some one variable, rather than a study of tool endurance. A conclusion quite favorable from the standpoint of force on the tool may be decidedly unfavorable to the life of the tool. The conclusions follow:

1 The burred keenness of a newly ground tool edge disappears after the first two or three feet of cutting. The tool edge main-

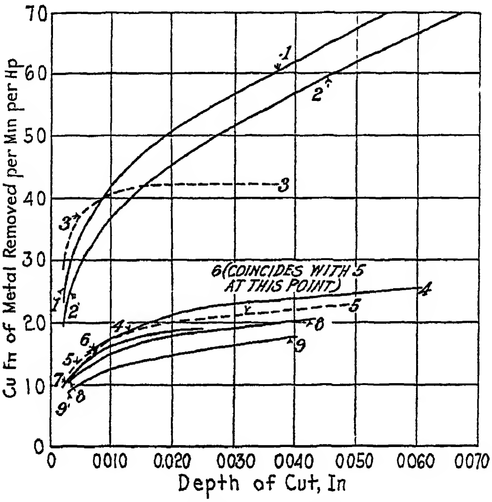


Fig. 83 Cubic Inches of Metal Removed per Minute per Horsepower — Depth of Cut

Tools: 4-deg. clearance, 30-deg. front rake (except brass, 15-deg.)
Width of cut: 0.5 in. Speed: 20 ft. per min.
Curve 1: Cast iron, annealed, bar No. 8A.
Curve 2: Cast iron, unannealed, bar No. 8.
Curve 3: Brass, rolled, bar No. 26.
Curve 4: S.A.E. 1035 steel, bar No. 28.
Curve 5: S.A.E. 3120 steel, bar No. 31.
Curve 6: S.A.E. 2320 steel, bar No. 30.
Curve 7: 0.15 per cent carbon steel, bar No. 3.
Curve 8: S.A.E. 2345 steel, bar No. 29
Curve 9: 1.03 per cent carbon steel, bar No. 32

tains this new condition while cutting several hundred feet. When a second change occurs, the failure of the tool soon follows.

2 The force on the tool for a given material and size of cut is the same for a tool carefully heat-treated and ground on any standard commercial abrasive wheel. A tool with a carefully honed cutting edge gives the same force as a tool ground on a coarse-grain wheel; also a slight rounding of the edge with a honing stone causes no increase in the force on the tool, other conditions being constant. The thicker the chip, the blunter the cutting edge may be without producing a noticeable increase in the force on the tool (see Fig. 8).

3 A variation of the clearance angle between 1 and 10 deg. has no influence on the force on the tool in the direction of the cut, all other conditions being constant (see Fig. 16).

4 The force on an end cutting tool for all materials is reduced in direct proportion to the increase of the front rake angle, until a certain limiting angle is reached. Beyond this limiting angle, which appears to be between 20 and 30 deg. for brass, 25 to 30 deg. for cast iron, and above 45 deg. for steel, the influence of the front rake angle is less pronounced, i.e., for the same increase in front-rake angle, the unit force is reduced a smaller amount. For front-rake angles above 60 deg., that is, for cutting angles less than 30 deg., the total pressure on the tool in the direction of cut is increased (see Fig. 25). For some materials, however, such as S.A.E. 2345, the force on the tool is influenced less for a given increase in front-rake angle than others (see Fig. 26).

5 The introduction of side rake on an end-cutting tool has no influence on the unit or total force on a tool in removing a given chip. For a tool having front and side rake, cleaner cutting was noted than for the 0-deg. front-rake tool with various angles of side rake (see Fig. 34). For values of side rake of 60 deg. or greater on the 0-deg. front-rake tools, an increase in the force on the tool in the direction of cut occurs (see Figs. 35 and 36).

6 For the same material, tool, and depth of cut, the total force on the tool increases with an increase in width of cut, but at a greater rate. The total force on the tool in the direction of the cut plotted over the width of cut gives a curve slightly concave upward, passing through the origin; the curvature increases as the width approaches zero (see Fig. 50). In other words, all other conditions remaining constant, a chip less than 0.25 in. in width is removed more efficiently than a chip wider than 0.25 in. This applies for rectangular chips removed with an end cutting tool (see Fig. 51).

7 For the same material, tool, and width of cut, all other conditions being constant, the total force on the tool increases as the depth of cut increases, but at a slower rate. The total force on the tool in the direction of cut plotted over the depth of cut gives a curve concave downward and passing through the origin; the curvature increases as the depth approaches zero (see Fig. 68). The force on the tool per 0.001 sq. in. of cross-sectional area of chip is much higher for a thin chip than for a thick chip; this difference is greatest, however, for increments in the small depths of cut. When the depth of cut is in excess of 0.030 in., the unit-force curve becomes almost horizontal. The unit force increases rapidly as the depth of the cut is reduced below 0.030 in. This applies for rectangular chips removed with an end cutting tool (see Fig. 69).

8 When the unit cutting forces are plotted against the various physical properties, no solution is indicated when all metals are

taken as a group. For certain physical properties the materials divide themselves into three groups, namely, straight carbon steels, alloy steels, and cast iron, for each of which a relation is indicated. For the straight carbon steels it appears that the elastic limit in tension and compression, the ultimate strength in tension and shear, and the Brinell hardness are functions of the unit force on the tool. For the alloy steels the elastic limit in tension and compression, the ultimate strength in tension, compression, and shear, the Brinell hardness, and the percentage of elongation in two inches, each indicates a relation to the unit force. For cast iron a similar relation is suggested for the ultimate strength in tension, compression, and shear, and the scleroscope hardness number (see Figs. 70 to 80 incl.).

Comparison and Discussion of Results

95 Investigations in metal cutting have been made under so many different conditions, by so many individuals, and over such a long period of time that it is with considerable circumspection that the author attempts any summarization and comparison of results. References to a few papers, however, which deal directly with the problems of this paper are made. The author believes that a thorough digest and summarization of existing data is one of the most important steps for the benefit of the future of the metal-cutting art. The theories should be substantiated by results from practice. Such a summarization would doubtlessly indicate that more data are needed on some subjects to confirm, explain, or augment those already available, and certainly would bring to light much that has never received sufficient publicity, and that which has been forgotten or never published in this country. If possible, a theory of metal cutting which underlies all types of cutting should be developed, as certainly there is something in common in the removal of a chip by milling, drilling, turning, etc. All this is a tremendous problem and should be undertaken in a big way.

96 The terms applicable to metal cutting should be standardized, and the size and shape of test specimens for deriving physical properties of the metals cut should be simplified and standardized.

97 Cutting fluids should receive a due portion of study, as they are a necessary part of cutting metals. We know that cutting fluids have a great influence, but the best cutting fluid (lubricant and coolant) for one job is hopelessly inadequate for the next; why?

Comparison: Sharpness of tool—Problem (a)

98 Just what happens when a tool cuts has been very clearly shown by E. G. Coker[1] by passing polarized light through nitro-

[1] Ref. D-35, p. 575.

cellulose when being cut by a steel or glass tool of the end-cutting type when milling, turning, and planing. The resultant color effect caused by the strained material indicated definitely the strains set up in the tool and the material. He observed that a tool ground in the ordinary way (later referred to as badly ground) with an angle of 45 to 60 deg. produced an action quite different from one with a similar edge but finished on a stone as perfectly as possible. In the former case the cutting action was irregular and imperfect, material being broken away from the piece by a wedge action in which apparently the shaving was bent away from the disk being cut by the upper face of the tool. The finished tool gave a continuous, smooth-flowing, clean-cut chip. Coker reports further, "the edge of the disk is left in a ragged condition and somewhat wavy in outline, so much so, in fact, that in some cases the main shaving broken off in this way is accompanied by a very thin shaving quite separate from the main one, and pared off by a true cutting process. That it is possible to obtain a somewhat similar effect in the turning of cast iron is well known, only in this case the cast-iron chippings are accompanied by a fine powder of cast iron falling from the cutting edge of the tool." Whether there is a difference in the forces acting on the tool in the two cases is not stated by Coker. Fig. 8 of the present paper would indicate that the force should be the same unless nitrocellulose has cutting properties different from cast iron and steel, also unless the cutting tool is considerably duller than the one referred to as being "ground in the ordinary way." Fig. 9 shows that the finish for the honed tool is best of all, which agrees with Coker's results, but does not indicate that there should be such a difference in chip distortion due to the increased dullness. With tools definitely dulled, such excessive distortion was not observed as is shown in Fig. 9. The wedge action referred to by Coker was observed by the author, and for a given degree of tool sharpness occurs for deep cuts but not for fine cuts. A heavy chip shows the slippage lines due, evidently, to the shearing of the metal by the wedge. A very light or finishing cut is perfectly smooth on the bottom of the chip, which appears clean cut and burnished rather than torn. The inner strip of the chip referred to by Coker is undoubtedly that part of the material removed by actual cutting action of the point, whereas the major portion of the chip is removed by a combination of tear and shear. Coker also observed that the total work done was a linear function of the speed. The author noticed for many different conditions that the force on the tool was the same for speeds of 20 and 35 ft. per min., the two available cutting speeds of the planer.

99 Crompton,[1] reported that with heavier cuts the keen-honed edge lasted when cutting steel for only a few feet of cutting, after

[1] Ref. D-35, pp. 597-598.

which the edge became slightly rounded, in which condition it remained for some time. Owing to the scouring action of the chip on the upper face of the tool, concavities were formed which, as they deepened, advanced toward the cutting edge and eventually met it in such a manner that the included angle between the curved top surface and the front face of the tool was lessened, the cutting efficiency being thereby much increased. This condition continued for a long time and eventually the scouring action of the chip, together with the wear on the leading face, so reduced the lip angle of the tool that it broke away. He also reported that with a slightly blunted tool, the chip, in all cases of steel, consisted of a very thin inner unbroken layer which was continuous, on the top of which the rest of the chip was piled in a series of slabs. Dean Ripper[1] found that the wear of the cutting edge was continuous while cutting, even though a false lip or wedge of cut material protected it from most of the work of cutting. Many curves are given showing that the relation between the units of wear of the edge and time of cutting for a constant cutting speed is a curve passing through the origin, slightly concave downward. The curve is steepest for the highest speeds.

100 Fred A. Parsons[2] reported that "the condition of the cutter has a pronounced effect on the amount of power required for removing metal. Tests showing the variations for sharp and dull milling cutters cannot be expected to show uniform results, as there is no standard for dullness. However, a number of tests show that the power may be expected to increase as much as 40 per cent or more before the appearance of the cut warns the operator that it is time to resharpen in the case of milling cutters." Parsons further states that other tools such as drills and lathe tools also cause an increase in power as they become dull. The author believes that the increase in power consumption for continuous-cutting tools as they become dulled is not so marked as in the case of milling cutters. Milling is peculiar in that each tooth removes only a small chip, which starts with zero thickness and increases to a maximum; before the tool starts to cut the tooth exerts a rubbing action over the work, which is certainly a function of the keenness of the edge. In continuous turning, a false cutting edge is built up on the tool from the material being cut, which partially protects the cutting edge and also relieves the cutting edge of most of the work. In milling, most of the chip would be removed before this false edge is built up to assist the cutting and protect the cutting edge. Coker's and Crompton's results agree with the observations of the author. Ripper's statement that tool wear was uniformly continuous does not agree with Crompton or the author. Parson's report on milling cutters

[1] Ref. D-16, p. 1074.
[2] Ref. C-3, par. 50.

does not agree. The variance seems plausible, however, because of the rubbing action of each tooth of the milling cutter before it starts to cut.

Comparison: Clearance Angle of Tool—Problem (b)

101 F. W. Taylor[1] stated: "in seeking for the proper clearance angles for tools, we have as yet been unable to devise any type of experiment which would demonstrate in a clear-cut manner which clearance angle is the best." Taylor reported further that "on the one hand, it is evident that the larger the clearance angle, the greater will be the ease with which the tool can be fed into its work, the first action of the tool when brought into contact with the forging being that of forcing the line of cutting edge into the material to be cut. On the other hand, every increase in the clearance angle takes off an equal amount from the lip angle and therefore subjects the tool to a greater tendency to crumble or spall away at the cutting edge. It must be remembered also that the tool travels in a spiral path around the work which it is cutting in the lathe, and that the angle of this path with a perpendicular line in the case of coarse feeds taken upon small diameters of work, becomes of distinctly appreciable size. In all cases, therefore, the clearance angle adopted for standard shop tools must be sufficiently large to avoid all possibility from this source of rubbing the flank of the tool against the spiral flank of the forging. The clearance angles for roughing tools in common use vary between 4 and 12 deg. We have had experience on a large scale in different shops with tools carefully ground with clearance angles of 5, 6, and 8 deg. In the case of one large machine shop which had used clearance angles ground to 8 deg. through a term of years, they finally adopted the 6-deg. clearance angle with satisfaction. For many years past our experiments have all been made with the 6-deg. clearance angle, and this has been demonstrated to be amply large for our various experiments. On the other hand, a 5-deg. clearance angle in practical use in a large shop has appeared to us through long-continued observations to grind away the flank of the tool just below the cutting edge rather more rapidly than the 6-deg. angle. We have therefore adopted the 6-deg. clearance angle as our standard." Taylor points out further that "a clearance angle of from 9 to 12 deg. should be used in shops in which each machinist grinds his own tools." In straight-line cutting, such as planer and shaper work, the 4-deg. clearance angle, as used by the author, is considered ample. For broaching, another form of straight-line cutting, even smaller angles are used. H. F. Donaldson[2] states that the clear-

[1] Ref. D-8, par. 336 and 337.
[2] Ref. D-6, p. 7.

ance angle should under no circumstances be less than 3 deg., but should be greater as the diameter of the work in the lathe increases. The influence of the diameter of the work was discussed by Orcutt, on page 29, who believed the statement to be an error. Ashton, on page 38, agreed with Donaldson in that "for a small radius of curvature the surface of the work was less in contact with the tool than with the large radius, that is to say, that it could stand a flatter and squarer angle." In the Manchester experiments of 1922, reported on by Dempster Smith,[1] a 6-deg. clearance angle was used. Airey and Oxford[2] reported that "excessive clearance does not reduce the cutting force, does not appreciably affect the life of the tool in normal wear, but does tend to chatter in action; also it weakens the cross-section near the cutting edge and increases liability to snip. Clearance should therefore be kept to a minimum. Three degrees was found to be ample, but 5 deg. was usually used. In many tool cribs it might be desirable to standardize at 8 deg. due to the possibility of an error of a few degrees." The results of the experimenters mentioned above are in accordance with the author's. Taylor's 6-deg. clearance angle for turning was based on wear or tool life with no reference to the force on the tool involved.

Comparison: Front-Rake Angle — Problem (c)

102 Robert H. Smith,[3] before 1880, found that in turning with a round-nose tool, a side rake angle — with practically no front rake (for turning this is equivalent to the author's front rake angle) — of 24 deg. for cutting cast iron and 46½ deg. for steel and wrought iron gave very satisfactory results. The tools lasted admirably. He determined that the smaller the cutting angle of the tool edge, the smaller was the horsepower required for cutting; but the angle actually required depended upon the necessity of avoiding, first, too frequent breakage, and second, overheating and consequent loss of temper at the edges. He further reported: "it was well known that with a large amount of top rake there was risk of 'digging-in.' If the depth of cut were small, the work would push the tool outwards from the center, a push which the tool clamp must resist. If the cut were deep, it would draw the tool in toward the center, to prevent which the tool clamp must exert an outward pull. For each top rake and for each quantity of material cut — perhaps varying slightly with the cutting speed — there was one definite depth of cut, to maintain which steadily the tool clamp needed to exert no radial force whatever. Similarly with the transverse feed."

[1] Ref. D-33, p. 9.
[2] Ref. C-1, par. 75.
[3] Ref. D-6, p. 52.

103 Airey and Oxford[1] report for milling that "from a power-consumption standpoint, rake is increasingly beneficial as it becomes greater. The advantage of rake, though, does not increase at so fast a rate after about 15 deg. is passed. Further, this is influenced by the kind of material being cut. On the other hand, the life of the cutter might be influenced disadvantageously by excessive rake." They show in Fig. 4 that for milling, the metal removed in cubic inches per horsepower-minute is increased due to increased front rake, but not as a direct function of the rake angle. Calling the production of the zero-degree-rake tool over a certain region of chip weight as 100, then the productions for the 10-, 20-, and 30-deg. tools are, for

Cast iron	133,	138.4,	and 142.1
Bronze	113.2,	123,	and 118
Machine steel	118.7,	157.2,	and 172
Carbon tool steel	106,	112.2,	and 112

104 The Manchester experiments[2] show that the vertical force on the tool when turning increases almost directly as the cutting angle in the plane of shaving increases. This true cutting angle is actually a function of the front and side rake of the tool and corresponds quite favorably with the front rake of the author's experiments, although the chip formation itself is somewhat different, inasmuch as a round-nosed tool was used in the Manchester experiments, while a straight-edged, end-cutting tool was used by the author. Smith[3] shows that the cutting speed for a definite tool life for a number of steels is maximum when the cutting angle in the plane of shaving is between 70 and 75 deg. and that the intensity of pressure on the top surface of the tool increases with the cutting angle (reduced rake) resulting in a higher rate of wear, but at the same time the amount of metal to be worn away before the cutting edge is reached increases.

105 Klopstock[4] shows that the power consumption is considerably reduced for small contained angles, that is, for large front-rake angles, but that there is little difference in power consumption for large contained angles. This work refers to Taylor's statement,[5] which says: "The writer believes that it would be profitable to experiment with more acute lip angles than 61 deg. in cutting dead-soft steel." It may be that with this extremely soft steel, still higher cutting speeds could be obtained with more acute angles, in which case it would be advisable, of course, to make special tools for cutting this quality of metal, in shops where

[1] Ref. C-1, p. 28.
[2] Ref. D-33, p. 75.
[3] Ref. D-33, p. 51.
[4] Ref. D-72, p. 14.
[5] Ref. D-8, p. 91.

large amounts of it are used. Klopstock's interpretation of Taylor's statement as quoted by DeLeeuw does not agree with the results shown in this paper. The author believes, however, that Taylor's statement is correct.

106 The results of tests conducted by Parsons[1] seem to show that for milling cutters, "the power required varies directly as the ratio of the coversed sine of the rake angle, or the cubic inches of metal removed per tool horsepower varies inversely in the same way." Parson's tests carried the front-rake angle only to 12 deg., however.

107 Rosenhain and Sturney report[2] that the average depth of cut is very much greater than that intended (0.010 or 0.015 in. as a constant) when small top-rake angles are used, but decreases rapidly as the top-rake (front-rake) angle increases until, with an angle of about 15 deg., when cutting mild steel, the average approximates closely to the intended depth of cut. The conclusions of the experimenters referred to above, with the exception of Klopstock, are in agreement with those of the author. Their conclusions, however, are based on the more limited practical cutting range, while the author's are based on the whole range that the tool could be made to cut.

Comparison: Side Rake — Problem (d)

108 Airey and Oxford[3] state that "it is difficult to understand the growth of the belief that spiral cutters are more efficient, because abstract analysis leads to the conclusion that spiral cutters must be inferior to straight cutters. By 'inferior' the meaning intended is in reference to power efficiency only. The use of a spiral angle is heartily recommended, for it results in continuity of action, tends to avoid chatter, and keeps the driving power more smoothly constant. The smallest angle consistent with smoothness of action should be used."

109 For milling, the side-rake angle is comparable with the side-rake angle of the tools used by the author. In turning, however, the side-rake angle is really the front-rake angle as used by the writer, so that it is the side-rake angle in turning that is of most importance as far as power efficiency is concerned. The front-rake angle in turning is of value, however, as it affects the surfacing force on the tool and the direction of flow of chip.

110 Parsons[4] states that "the effect of adding a spiral angle to a milling cutter, as far as power efficiency is concerned, seems to be confined to reducing the bumping action of the cut and

[1] Ref. C-3, par. 42.
[2] Ref. D-67, p. 27.
[3] Ref. C-1, p. 19.
[4] Ref. C-3, par. 47.

thereby somewhat reducing the maximum power required. From the standpoint of power required, this effect is not important." "The spiral angle is of considerable importance, however, since a large spiral angle enables fewer teeth to be used in a spiral or slabbing cutter." The only available results on the influence of side rake alone, those for milling, as given above, agree with the author's in that side rake alone has little influence on the power consumption but does permit some desirable practical features.

Comparison: Chip Width and Depth of Cut — Problem (e)

111 F. W. Taylor[1] points out that for depths of cut less than $\frac{3}{16}$ in., an error is likely to be made in maintaining a uniform depth of cut, which becomes so large a percentage of the total depth as materially to affect the accuracy of the experiment. He states further that "this is true to such an extent that as a result of our experience we should consider practically worthless for determining laws all cuts that are as shallow as $\frac{1}{16}$ in. and we should much prefer a $\frac{3}{16}$-in. depth to $\frac{1}{8}$-in. depth." A cut $\frac{3}{16}$ in. in depth by $\frac{1}{16}$ in. feed is recommended by him. The author has referred, in connection with his data, to the irregularities or fluctuations when depths of cut less than 0.006 in. were taken. Taylor further states that "a feed which is finer than $\frac{1}{16}$ in. is also undesirable for experiments, because any trifling imperfection or flaw at or near the cutting edge of the tool will more seriously affect the results with a smaller feed than this and also small hard spots or other blemishes in the metal that is being tested have a much worse effect on the tools with a fine than with a coarse feed." It was for this reason that the author felt it necessary to carry out the test on problem (*a*), involving the effect of the force on a tool as a function of its sharpness for various depths of cut, in which it was shown that a dulled tool increases the cutting force on thin chips but not on thick ones (see Fig. 8). Taylor further reports (par. 291) that "it is the thickness of shaving, then, which must be first considered, as this element has more effect upon the design of our standard tools, and in fact upon the whole problem of cutting metals, than any other single item which is completely under the control of those who are managing a shop." Taylor concluded that the pressure in pounds per square inch of chip area increases as the chip becomes thinner. This agrees with the early experiments conducted in Germany but disagrees with those of Nicolson[2] who concluded that the pressure on the tool per square inch of cross-sectional area of chip was a direct function of the area of the chip and was the same whether light or heavy cuts were taken, and that it did not depend upon

[1] Ref. D-8, p. 66.
[2] Ref. D-7.

either the thickness of the feed or the depth of cut. As to the chip thickness affecting the cutting speed, Taylor wrote: "to make it more apparent that the element affecting the cutting speed the most is the thickness of the shaving, the writer would call attention to the fact that dividing the thickness of the shaving by 3 increases the cutting speed in the ratio of 1 to 1.8, while dividing the depth of cut by 3 only increases the cutting speed in the ratio of 1 to 1.27."

112 Smith[1] reported from the Manchester experiments, for the vertical force on a tool when turning in a lathe, that "the results obtained in these trials are plotted on a base of depth of cut in Fig. 44, and it will be observed that for each traverse the vertical force increases with the depth of cut according to a linear law. It will also be seen that for a given depth of cut this force does not vary directly as the traverse, but is proportionately greater for finer traverses." Smith showed in his Fig. 30 that for a constant depth of cut the cutting speed for a definite tool life was maximum for fine traverses, but was reduced very rapidly for small increases in traverse and gradually turned from a steep curve for small traverses to nearly horizontal for heavy traverses. Also, for a constant traverse with variable depth of cut, the cutting speed for a definite tool life was also shown to be maximum for the smallest depth of cut, reducing rapidly for the first increase in depth and less rapidly with additional increases until the curve becomes almost horizontal for depths between $\frac{3}{16}$ and $\frac{3}{8}$ in. Airey and Oxford[2] state, "it has been demonstrated earlier that the force required to remove metal (by milling) does not increase in proportion to the chip thickness. It follows directly from this that as feed is increased, the force will not increase in proportion."

113 Klopstock[3] states, "It will be seen that a chip of 1 mm. depth of cut and 10 mm. feed will require only about one-half the actual power (delivered to machine) as compared with a chip of 10 mm. cut and 1 mm. feed." For these same cuts it is seen from his Fig. 13 that the actual power absorbed by the first chip is only one-third or one-fourth of that absorbed by the second chip. In Klopstock's Fig. 18 is shown the total and unit vertical pressures on the tool for wrought iron and cast iron, the curves of which compare favorably with those of the author (Figs. 60 to 64). In his Figs. 19 and 20 are shown the total force plotted against the cross-sectional area of the chip as abscissas for various experimenters such as Taylor, Nicolson, Ripper, and Klopstock. These curves, while plotted on the cross-sectional area of chip as a basis, fall within the limits of the results of the author. It is observed, however, that if the chip area is made up with a constant

[1] Ref. D-33, p. 80.
[2] Ref. C-1, par. 51.
[3] Ref. D-72, p. 12.

width, the area being increased by an increase in depth, a curve slightly above those shown is obtained, which is concave downward. As the area of chip is increased, by increasing the width, that is for constant depth, a curve below those shown, which is concave upward, is obtained. (See author's Figs. 43 and 62 for nickel-chrome steel for example.)

114 Ripper[1] concluded (p. 1092, No. 6) that for carbon-steel tools, with which cuts of small area only are possible, the influence of the depth of cut upon the cutting speed for certain tool life is exactly the same as that of the feed, also (p. 1118, No. 5), "if the area of cut is kept constant, a higher cutting speed for given tool life is obtainable when the cut is deep and the feed fine, than when the cut is shallow and the feed coarse." This effect is undoubtedly due to greater length of cutting edge to dissipate the heat than because of smaller forces involved.

115 Parsons[2] states that "a test run first with several narrow cutters set up with all the teeth on a line, and again run with the teeth staggered (to give the effect of a spiral cut) showed that the power reduced in the ratio of 1.42 to 1.27." It seems very probable to the author that this reduction in power is due not to the effect of a spiral, but to the effect of narrower widths of chip as is pointed out above.

116 Rosenhain and Sturney[3] show that for a given front rake angle, the average depth of cut for an end-cutting tool is greater than the intended depth of cut as a result of the tearing of the chip ahead of the cutting edge but slightly deeper into the metal being cut. This condition is increased as the depth of cut is increased from 0.01 to 0.035 in. Most of the published results on the relation of cutting force to chip depth agree in general with those of the author. The author's method, however, permits a careful study of the single variable and shows the limiting values of both width of cut and depth of cut under which the unit force is increased as the depth is reduced or decreased as the width is reduced.

Comparison: Physical Properties—Problem (f)

117 Many experimenters have compared physical properties of materials cut with the force on the cutting tool. The difficulty in comparing such results, however, is that the conditions have not been properly standardized.

118 Machinability is a term which during the last few years has gained considerable prominence, but, like the hardness of metals, is not clearly defined. The author for the sake of

[1] Ref. D-16, p. 1092 and 1118.
[2] Ref. C-3, par. 48.
[3] Ref. D-67, p. 27.

comparison selected the force on a 30-deg. front-rake tool for a chip 0.012 in. deep and 0.5 in. wide as representing the machinability of the material. A machinability tester, described in Airey and Oxford's paper on The Art of Milling, has a cutter functioning as a single tooth of a milling cutter, which takes a width of cut of $\frac{1}{8}$ in., a radius of cut of $1\frac{3}{4}$ in., with a feed per chip of 0.004, 0.008, 0.012, and 0.016 in. for a constant depth of cut. This machinability tester measured the energy in foot-pounds required to remove a chip of a given metal.

119 Other types of machinability testers of the drill type which have been used by Bauer,[1] Keep,[2] Leyde,[3] Reininger,[4] Kurrin, and Kessner[5] use some factor of the penetration of the drill as the indication of machinability of a metal. Grossmann[6] states that "the drill test can give only comparative, not absolute, values for machinability, since the angle a $\left(\tan a = \frac{\text{revolutions}}{\text{depth}}\right)$, Fig. 255, will depend not only on the material under test, but also on the material of the tool, the cutting angles, the pressure P, the rate of rotation and the progressive dulling of the tool."

120 M. A. Grossmann in his Physical Metallography,[6] p. 326, gives a very complete discussion of the relation of machinability to hardness and workability. He states that "by workability is understood the capacity of a material for undergoing plastic deformation without rupture. The elongation in a tension test, the extent of bending before fracture in the notch-impact and various cold-bend tests, and the extent of deformation in the other mechanical tests all represent this quality of workability." After explaining Thieme's conception of chip removal, which is a pressure process, he adds (p. 328), "There comes into question not only the resistance to penetration, but also the resistance to the pressing aside of the chip element," and concludes with, "we arrive thus at the law that machinability depends on both hardness (e.g., ball hardness) and workability, the difficulty of machining increasing with an increase in either property." It is explained that the test may be carried out on a planer, lathe, or drill press, all factors constant so that the travel of the tool for a given force on it in a specific time, is the measure of machinability. A case is given in Fig. 256 to show the machinability as determined by means of the Kessner drill apparatus and the ball hardness of a copper (2)-zinc (1) alloy with various amounts of lead from zero to

[1] Ref. D-2.
[2] *Iron Age,* 1899, p. 9, and 1900, p. 16.
[3] *Z. d. Ing.,* 1904, p. 169.
[4] Giessereizeitung 1904, pp. 217 and 627.
[5] Ref. D-60.
[6] Ref. D-86, pp. 326 and 330.

11 per cent. The machinability (drill penetration) curve is a smooth curve concave downward, increasing in height rapidly as lead is first added, but gradually becoming nearly horizontal when the lead content is 11 per cent. The ball-hardness curve, on the other hand, is practically the same for the 0 and 11 per cent contents, but bends upward to a maximum for a lead content of 2 per cent.

121 Klopstock[1] shows, in his Fig. 23, Brinell-hardness curves and chip-pressure curves for seven materials, including Kurrin's drilling test. He states that "it appears that the intersecting points of the hardness curves and chip-pressure curves will be found near two approximately parallel straight lines; one located considerably below the other. The upper line connects the intersecting point of curves relating to materials forming continuous chips such as steel, wrought iron, copper, etc., while the lower line connects the intersecting points of curves relating to materials such as cast iron and brass." "These observations permit the determination of chip pressure and, therefore, power requirements in the turning and planing of materials of which the Brinell hardness characteristics are known."

122 The Erichsen test supplemented with a photomicrograph was selected as the best indication of drawability (workability) of the ten samples of sheet metal recently submitted by the Boston Sheet Metal Company to the Pennsylvania State College[2] for tests.

123 The following is taken from a letter to the author from E. G. Herbert of Manchester, England: "I think it is quite well established that there is no direct relationship between the hardness of metals and their resistance to cutting, and it has been my endeavor in 'The Pendulum' and elsewhere to show why such a relationship cannot exist. The principal reasons are, according to my experience, two:

"(1) When a metal is cut it is work-hardened in the process of cutting, and its resistance to cutting depends far more on its work-hardening properties, that is on the hardness induced in it by the tool, than on its original hardness. As a familiar example, manganese steel is shown by the Brinell test to be soft. This is confirmed by the pendulum time test which gives its hardness 24, the hardness of ordinary mild steel being 20. But manganese steel cannot be cut, and the reason is immediately made apparent by the pendulum 'work-hardening test' which gives its original scale hardness 14 and its scale hardness after being rolled with the pendulum ball 80 to 90, i.e., equal to the hardness of hardened tool steel. All other metals are similarly hardened by cutting tools, but in very different degrees, and their resistance to cutting must therefore depend on their 'work-hardening capacity.'

[1] Ref. D-72.

[2] *Mechanical Engineering*, Feb. 1926.

"(2) Metals are heated in cutting, and their resistance to cutting must therefore depend on their properties when in a heated state — not cold. The hardness of steels and other metals does not generally change much within the ordinary range of cutting temperatures, but their work-hardening properties do change in a remarkable manner, almost disappearing in many cases at temperatures (in mild steel) between 100 and 150 deg. cent. Any study of machinability which fails to take these facts into account must, in my opinion, lead to negative results."

124 Herbert also writes,[1] "The depression in the work-hardening curve coincides with the free-cutting range of temperatures Metal cut within this range is only slightly hardened by the tool with the following results:

1 The metal cuts freely, leaving a smooth finish
2 Less heat is generated
3 A flowing helical chip is produced
4 A bright Whitaker ring is formed.

These results confirm the observations of Stanton and Dempster Smith (Bulletin of the Institution of Mechanical Engineers, No. 2, 1925) that the vertical force on the tool falls to a minimum at certain cutting speeds and also at higher speeds. Stanton found that this effect disappeared when the steel was normalized." It is not possible to compare the relations between physical properties of materials and their machinability as outlined above with those of the author because of the difference in methods or standards used. It appears obvious, however, that there is no satisfactory method extant which is wholly reliable for all metals.

Acknowledgments

125 The author wishes to take this opportunity to express his appreciation to Prof. A. E. White, Director of the Department of Engineering Research, University of Michigan, for his encouragement and assistance, as it was through his department that this work was sponsored by the Manufacturing Methods Committee, Howard L. McGregor, Chairman, of the Michigan Manufacturers' Association. Thanks are accorded to the following companies, which contributed financially to the project:

Detroit Steel Products Company, Detroit
Detroit Twist Drill Company, Detroit
Ford Motor Company, Detroit
General Motors Company, Detroit
Hupp Motor Company, Detroit
Michigan Screw Company, Lansing
National Twist Drill & Tool Company, Detroit

[1] Ref. D-88 and D-90, p. 364.

Packard Motor Car Company, Detroit
Reo Motor Car Company, Lansing
Russell Wheel & Foundry Company, Detroit
Timken Detroit Axle Company, Detroit
Detroit Copper & Brass Rolling Mills, Detroit
General Manufacturing Company, Detroit
Wilton Tool & Manufacturing Company, Detroit.

126 Acknowledgment is also made of the valuable assistance rendered in the preparation of equipment, execution of the experiments, and the compilation of data by Messrs. Carleton H. Currie, Donald L. Perkins, and Clarence J. Swigert.

127 Lastly, acknowledgment is made of the assistance and coöperation of the members of the staff of the Department of Engineering Shops, University of Michigan.

APPENDIX NO. 1-A

TOOL LIST

Manufacturer of steel, ...

Size of tool, $2\frac{1}{8} \times 1\frac{1}{4} \times 12$ in.

Grade of tool, Special No. 14, p. 27, Cat. No 28

Chemical composition, C, 1.05%; Mn, 0.22%; Si, 0.10%; S, 0.018%; P, 0.017%

Remarks	Tool No.	Angles of tool form, deg. Clearance	Front rake	Side rake	Forged or machined	Annealing	Forging	Hardening	Tempering	Hardness, Rockwell
First end	A-1	6	0	0	M	1380–1440	....	1400–1450	450	61
Rehardened and reground.....	A 1[a]	8	0	0		....	...	....	..	
	A-2	2	0	0	M	. .	.	1430	450	60
Reground	A-3	$3\frac{1}{2}$	0	0	M	.. ..		....	.	..
Reground	A-3[a]	10	0	0		.	.	....	...	..
Reground	A-4	6	0	0	M			.	.	.
Hardened and reground.......	A-5	4	5	0	F	.. . .	.	1500	440	60
Other end	AA[1]-6	4	0	45	M	... .	.	1475	425	61

[1] Double letter indicates other end of bar.

APPENDIX NO. 1-B

TOOL SUMMARY SHEET SHOWING PROBLEMS USING THE TOOL

Tool No.	Angles of tool form: Clearance	Angles of tool form: Front rake	Angles of tool form: Side rake	Problem tool was used on
A-1	6	0	0	*b*
1*a*	8	0	0	*b*
2	2	0	0	*b*
3	$3\frac{1}{2}$	0	0	*b, c*
3*a*	10	0	0	*b*
4	6	0	0	*b*
5	4	5	0	*b, c*
AA-6[1]	4[2]	0	45	*d*
B-1	6	10	0	*b*
1*a*	8	10	0	*b*
2	2	10	0	*b*
2*a*	10	10	0	*b*
3	4	10	0	*a*
4	4	10	0	*a, b*
5	4	10	0	*a*
6	4	10	0	*a*
7	4	10	0	*a*
8	4	10	0	*a*
9	4	10	0	*a*
C-1	6	20	0	*b*
1*a*	8	20	0	*b*
2	2	20	0	*b*
3	$3\frac{2}{3}$	20	0	*b*
3*a*	10	20	0	*b*
4	6	20	.	*b*
5	4	20	0	*c*
D-1	6	30	0	*b*
1*a*	8	30	0	*b*
2	2	30	0	*b*
2*a*	10	30	0	*b*
3	4	30	0	*b*
DD-4[1]	4	0	30	*d*
5[1]	4[3] at rt. angle to edge	0	30	*d*
E-1	6	40	0	*b*
1*a*	8	40	0	*b*
2	2	40	0	*b*
3	$3\frac{2}{3}$	40	0	*b*
3*a*	10	40	0	*b*
4	6	40	0	*b*
5	4	20	0	*c*
EE-6	4	0	0	*d*
F-1	..	20	0	..
2	4	20	0	*a*
3	4	20	0	*a*
4	4	20	0	*a*
5	4	20	0	*a*
6	4	20	0	*a*
7	4	20	0	*a*
8	4	20	0	*a*
9	4	20	0	*a, c*
G-1	..	50	0	..
2	4	30	0	*a, c, d*
3	4	30	0	*a*
4	4	30	0	*a*
5	4	30	0	*a*
6	4	30	0	*a*
7	4	30	0	*a*
8	4	30	0	*a*
9	4	30	0	*a, c, d*

[1] Opposite end of bar.
[2] Angle measured in plane of motion.
[3] Angle measured in plane normal to cutting edge.

TOOL SUMMARY SHEET—CONTINUED

Tool No.	Angles of tool form: Clearance	Front rake	Side rake	Problem tool was used on
H-1	6	0	0	*b*
2	6	10	0	*b*
3	4	0	0	*c*
4	4	0	20	*d*
HH-5	4	0	60	*d*
I-1	6	40	0	..
2	6	30	0	.
3	6	30	0	.
II-4	4	0	75	*d*
5	4 at rt. angles to edge	0	75	*d*
J-1	4	15	0	*c*
K-1	4	25	0	*c*
KK-2	4	0	10	*d*
L-1	4	35	0	*c*
M-1	4	40	0	*c*
N-1	4	45	0	*c*
O-1	4	30	30	*d*
P-1	4	30	20	*d*
R-1	4	30	10	*d*
S-1	4	60	0	*c*
T-1	2	75	0	*c*

APPENDIX NO. 2-A

MATERIAL RECORD SHEET FOR EACH BAR OF MATERIAL CUT

Bar Number; machine steel 1
Material, O. H. machinery steel (one of a six-bar order)
Manufacturer, ... Steel Company
Chemical composition; C, 0.15; Mn, 0.24; Si, 0 13; S, 0.029; P, 0.014
Condition at factory, fully annealed, Brinell 101

Bar No.	Heat treatment	Hardness: Brinell	Rockwell	Scleroscope	Location
1	Fully annealed	101	..	21-22	At factory
		107 end			on surface
		105 center	54	21-22	Half-way
		107 end			through

$\frac{3}{4}$-in cube removed from end of bar for photomicrograph specimen

Physical properties:

Elastic limit, lb. per sq. in.	25,300
Yield point, lb. per sq. in.	25,300
Ultimate strength in tension, lb per sq. in.	52,400
Percentage of reduction in area	67
Percentage of elongation in 2 in.	41
Ultimate shear stress from torsion	
Ultimate shear stress by die method, lb. per sq. in.	39,400
Elastic limit in compression, lb. per sq in.	22,000
Ultimate compressive strength, lb. per sq. in.	85,350

APPENDIX NO. 2-B

MATERIAL-RECORD-SHEET SUMMARY SHOWING THOSE PROBLEMS FOR WHICH EACH BAR WAS USED

Material identification: Name	Bar No.	Problems for which material was used
S A E 3120 steel	31	c, d, e, f
S A E. 2345 steel	29	c, d, e, f
S A.E. 2320 steel	30	c, d, e, f
S A.E. 1035 steel	28	c, d, e, f
0.15% carbon steel	1	b, f
0.15% carbon steel	2	b, c. f
0 15% carbon steel	3	a, b, e, f
0.15% carbon steel	4	b, c, f
0.15% carbon steel	5	a, e
0 15% carbon steel	6	d
1 03% carbon steel	32	c, d, e, f
Cast iron	1	..
Cast iron	2	d
Cast iron	3	c
Cast iron	3A	c
Cast iron	4	b
Cast iron	4A	f
Cast iron	5	e
Cast iron	6	e
Cast iron	7	a
Cast iron	7A	e
Cast iron	8	b, f
Cast iron	8A	f
Cast iron	9	e
Cast iron	9A	e
Cast iron	10	a, d
Cast iron	11	e
Brass	24	b, e, f
Brass	25	e also cutting speeds
Brass	26	d, f
Brass	27	c, d, f
Brass	33	c, f
Brass	34	e, f

APPENDIX NO. 3

TEST BARS USED FOR OBTAINING PHYSICAL PROPERTIES OF MATERIALS CUT

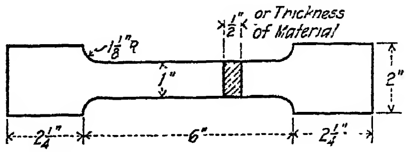
1 Steel tensile-test specimen.
(Standard A.S.T.M. bar, 0.505 in. diameter.)

2 Cast-iron tensile-test specimen.
(Standard A.S.T.M. bar, 0.800 in. diameter.)

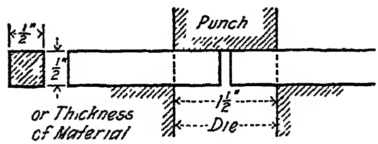
3 Cast-iron compression-test specimen.
(Cylinder 1.128 in. diameter and 2.5 in. high.)

4 Steel compression-test specimen.
($\frac{3}{4}$ in. square section and 3 in. high.)

5 Brass tensile-test specimen.



6 Shear test specimens for all metals.
($\frac{1}{2}$ in. square and 3 to 4 in. long.)



APPENDIX NO. 4

BIBLIOGRAPHY OF RESEARCH IN CUTTING TOOLS AND METAL CUTTING

General

Bibliography of the Manufacture, Heat Treatment, Uses and Tests of High Speed Steel, American Society for Steel Treating, October 1922, pp. 47-75.

A — Cutting Fluids

A-1 Lubrication of Cutting Tools, E. K. Hammond.
Machinery, Jan. 1917.

A-2 Cutting Oils, Their Properties, Composition, Examination, and Industrial Application, C. W. Copeland, Remington Arms Co.
Chemical & Metallurgical Engineering, July 1, 1917, p. 25.

A-3 The Action of Lubricants on Metal Cutting Tools, Albert Kingsbury and others.
Discussion of Mr. DeLeeuw's paper, A Foundation for Machine Tool Design, *Journal,* A.S.M.E., July, 1917, p. 582.

A-4 Memorandum on Cutting Fluids, Bulletin No. 2, Dept. of Scientific and Industrial Research, London, 1918.

A-5 Klean Kut Facts, and
Up to the Minute Scientific Comments on Oil Products Used for Metal Working and Heat Treating.
D. A. Stuart & Co., Inc., 2727 S. Troy St., Chicago.

A-6 Oil Analysis, A. H. Gill.
J. B. Lippincott Company, 1919.

A-7 Industrial Oil Engineering, J. R. Battle.
J. B. Lippincott Company, 1920, pp. 635-638.

A-8 Cutting Fluids, E. C. Bingham, Chemist.
Bureau of Standards Technologic Paper No. 204, 12-20-21.

A-9 Infection from Cutting Oils, A. L. DeLeeuw.
American Machinist, Dec. 14, 1922, p. 915.

A-10 Cutting Lubricants and Their Application; Principles of Coolants; Suitability to Material Being Cut; Factors Such as Power Consumption and Removal of Chips, A. A. Dowd and F. W. Curtis.
American Machinist, Oct. 25, 1923, p. 613.

A-11 Use and Importance of Cutting Lubricants; Proper Application to Tool and Work; Value of Lubricants in Thread Cutting, A. A. Dowd and F. W. Curtis.
American Machinist, Jan. 24, 1924, p. 135.
A-12 Lubrication and Cooling the Cutting Tool, Allan F. Brewer, Texas Oil Company.
Industrial Management, April 1924, p. 197.
A-13 Storing and Clarifying Oil in Shops.
A-14 Modern Cutting and Grinding.
Oakley Chemical Co., Thames St., New York City.
A-15 Industrial and Automotive Lubrication, Waverley Oil Works Co., Pittsburgh, Penna. (Second Edition 1925.)
A-16 Compounds for Cutting, Grinding, and Rust Prevention, J. A. Maguire.
Western Machinery World, Feb. 1926, p. 65.
A-17 Machinery's Handbook, p. 489, and p. 878.
A-18 Mark's Handbook, p. 645.
A-19 Patents on Cutting Oil, Maitland, Sun Oil Company.

B — Drilling, Tapping, and Reaming

B-1 A Twist Drill Dynamometer, W. W. Bird and H. P. Fairfield.
Trans. A.S.M.E., vol. 26, 1905, p. 355.
B-1*a* Experiments on the Forces Acting on Twist Drills — When Operating on Cast Iron and Steel, Dempster Smith and R. Poliakoff.
Proc., Inst. of Mech. Engrs., March 1909, p. 315.
B-2 Radial Drilling Machines Break Drills and Records, F. E. Bocorselski.
American Machinist, vol. 33 (1910), p. 379.
B-3 High Speed Steel Drill Tests, F. R. Norris.
American Machinist, vol. 34 (1911), p. 719.
B-4 The Development of the Drill Test as a Means of Ascertaining the Machining Properties of Iron and Other Metals and for the Investigation of Tool Steels, A. Kessner.
Iron and Steel Inst., Carnegie Schol. Mem., vol. 5 (1913), p. 10.
B-5 An Efficiency Testing Machine for Testing Drills, Taps, and Dies, T. Y. Olsen.
Proc. A.S.T.M., vol. 14, pt. 11 (1914), p. 541.
B-6 Experiments with Drilling Dynamometer. (New drilling data from 6-ft. radials — results of Dempster Smith of England. Drills ¾-in., 1-in., to 3-in. diam. forged-twisted. H. M. Norris.
American Machinist, Aug. 13, 1914, p. 265.
Kent's Handbook, 9th ed. pp. 1286-1288.
B-7 An Investigation of Twist Drills, Bruce W. Benedict and W. Penn Lucas.
Bull. No. 103, Engrg. Exp. Sta., Univ. of Illinois, 1917.
B-8 Twist Drill Dynamometer, R. Poliakoff.
American Machinist, June 12, 1919, p. 1132.
B-9 Effect of Design on Drilling Machine Efficiency; Discusses speeds and feeds and effect of high-speed steel on design of machine and spindle of drive, F. E. Johnson.
Machinery, Aug. 1922, p. 964.
B-10 Significance of Tool Temperatures (Thermocouple in point of drill), H. H. Schwartz and W. W. Flagle.
A.S.T.M. *Journal*, June 1923.
B-11 Foreign Progress in Cutting Metals, C. A. Beckett.
Apparatus developed by Drs. Heym and Kessner to determine machinability factors affected by (a) drill hardness, (b) durability of cutting edge, and (c) variable cutting angle of drill.
Mechanical Engineering, Oct. 1924, p. 624.

C — MILLING

C-1 On the Art of Milling, John Airey and C. J. Oxford.
Mechanical Engineering, vol. 45 (1921), p. 549.
C-1a Milling Cutters and Their Efficiency, A. L. Deleeuw.
Trans., A.S.M.E. vol. 33 (1911), p. 245.
C-2 Effect of Variation in Design of Milling Cutters on Power Requirements and Capacity, B. P. Graves and J. A. Hall.
Mechanical Engineering, vol. 45 (1923), p. 155.
C-3 Power Required for Cutting Metals by Milling, Fred A. Parsons, Kempsmith Milling Machine Company.
Mechanical Engineering, January 1923, p 35.
C-4 Milling with Stellite Cutters.
Machinery, April 1923, May 1923, p. 717.
C-5 Chart for Calculating S. & F. of Solid and Inserted-blade Milling Cutters, Frank W. Curtis, Haynes-Stellite Co.
American Machinist, Sept. 27, 1923, p. 472.
C-6 Chip Formation by Milling Cutters (with photographs of successive stages rake 0 to 20 deg.), C. F. Roby (thesis at Cincinnati Univ.).
Mechanical Engineering, Oct. 1923, p. 586.
C-7 A Treatise on Milling and Milling Machines, Cincinnati Milling Machine Co., 1922.

D — TURNING, STRAIGHT LINE, AND GENERAL

D-1 Cutting Tools, Robert H. Smith.
Cassell & Co., 1881. (See Proc., Inst. of Mech. Engrs. 1903, p. 48.)
D-2 A Novel Method of Testing C. I. for Hardness, Chas. A. Bauer.
American Machinist, 1-4, 1897, S-245.
D-3 Schnelldrehstahl — Bericht des vom Berliner Bezirksverin deutcher Ingenieure Gebildeten Ausschusses.
Zeitschrift des Vereines deutscher Ingenieure, vol. 45 (1901), p. 1377.
D-4 Experiences Sur Le Travail Des Machines — Outils, Pour Les Metaux, C. Codron.
Vol. I (1901-1902) and vol. II (1903-1905). Extrait du *Bulletin de la Societe d' Encouragement pour l' industrie Nationale.* Vol. III (1906-1910), Extrait de la *Revue de Mecanique.* Reprints: vol. I (1902) considers drilling, vol. II (1906) considers reaming, vol. III (1910) considers general machine tools, shearing, and punching. G. E. Stechert & Co., 33 E. 10th St., New York, N. Y.
D-5 Notes on High Speed Steels (as used in Union Pacific Railroad Shops at Omaha, Nebr.), Henry H. Suplee.
Proc. of Inst. of Mech. Engrs., July 1903, p. 457.
D-6 Cutting Angles of Tools for Metalwork, as Affecting Speed and Feed (lathe dynamometer used), H. F. Donaldson.
Proc. of Inst. of Mech. Engrs., Jan. 3, 1903, p. 5.
D-7 Experiments with a Lathe Tool Dynamometer, J. T. Nicolson.
Trans. A.S.M.E., vol. 25 (1903), p. 637.
D-8 On the Art of Cutting Metals, F. W. Taylor.
Trans., A.S.M.E., vol. 28 (1907), p. 31.
D-8a Discussions by H. R. Towne, J. M. Dodge, C. W. Rice, J. S. Bancroft, H. K. Hathaway, J. T. Hawkins, L. P. Breckenridge.
Proc. A.S.M.E., Jan. 8, 1907, p. 929.
D-8b Discussions by J. E. Stead, Oberlin Smith, W. H. Blawelt, D. Adamson, and W. S. Huson.
Proc., A.S.M.E., Mar. 1907, p. 1159.

D-8c Discussions by H. LeChatelier, C. Codron, W. Lewis, R. A. Skeggs J. T. Nicolson, F. W. Taylor.
Proc., A.S.M.E, April 1907, p. 1339.

D-9 The Cutting Properties of Tool Steels, Edward G. Herbert.
Jour. Iron & Steel Inst., 1910, pt. 1, p. 206.

D-10 Power Required to Remove Metal, Chas. Robbins.
Trans., A.S.M.E., vol. 32 (1910), p. 202.

D-11 High Speed Steel, O. M. Becker.
McGraw-Hill, 1910.

D-12 Boring Tools, Dempster Smith.
Trans. Manchester Assn. of Engrs., Oct. 1911.

D-13 Cutting Force, Power, and Drive in Lathe Work, G. Lindner.
Zeits. f. Werkzeugmaschinen u. Werkzeuge, Feb. 5, 1912.

D-14 The Influence of Heat on Hardened Tool Steels, Edward G. Herbert.
Jour. Iron & Steel Inst., vol. 85 (1912 pt. 1), p. 358.

D-15 Tool Steel for the U. S. Navy, Lewis H. Kenney.
Society of Naval Arch. & Marine Eng., 1912.

D-16 Cutting Power of Lathe Turning Tools, William Ripper and C. W. Burley.
Proc., Inst. of Mech. Engrs., 1913, p. 1067. For further discussion, see *Prac. Engr.*, Dec. 11, 1913 and later (serial).

D-17 Die Fortschritte deutscher Stahlwerke. (The Progress of German Steel Industry). *Stahl und Eisen*, 1913.

D-18 Power Requirements of Lathes, A. Bodi.
Industria, Sept. 14, 1913.

D-19 A Basis of Measuring Lathe Capacity, L. R. Pomeroy.
American Engineer, March 1913 (discusses Nicolson's results).

D-20 Past Experimental Work on Cutting Tools, Dempster Smith.
Proc., Manchester Assn. Engrs., March 1915.

D-21 Notes on the Relations Between the Cutting Efficiencies of Tool Steels and Their Brinell and Scleroscope Hardness, J. O. Arnold.
Jour., Iron & Steel Inst., vol. 93 (1916 pt. 1), p. 102.

D-22 A Foundation for Machine-Tool Design and Construction, A. L. DeLeeuw.
Trans., A.S.M.E., vol. 39 (1917), p. 185.

D-23 Molecular Constituents of High Speed Steels and Their Correlation with Lathe Efficiencies, J. O. Arnold and Fred Ibbotson.
Jour. Iron & Steel Inst., vol. 99 (1919, pt. 1), p. 407.

D-24 Die Werkzeugmaschinen, 3 Aufl., Hulle.
(Machine Tools—3 Editions) Springer, Berlin, 1919.

D-25 Der Schnellbetrieb (High Speed Work), L. Walther.
R. Oldenbourg, 1919, Munchen.

D-26 Die Dreherei und ihre Werkzeuge (Turning and its Tools), 3 ed., Willie Hippler.
Springer, Berlin, 1919.

D-27 Cutting Edges, R. E. Crompton.
Engineer, vol. 130 (1920), p. 207.

D-28 Uber Dreharbeit und Werkzeugstahle (About Turning and Cutting Tools), Taylor-Wallichs.
Julius Springer, 1920.

D-29 Comparative Tests upon High Speed Steels, A. J. Langhammer.
Chemical & Metallurgical Engineering, vol. 22 (1920), p. 889.

D-30 Cutting Pressure and Form of Chip, H. Sack.
Betrieb, vol. 3, Sept. 15, 1921.

D-31 Hardening High Speed Steel, A. H. d'Arcambal.
Chemical & Metallurgical Engineering, vol. 25 (1921), p. 1168.

D-32 Metal Cutting Tools, Their Principles, Action, and Construction, A. L. DeLeeuw.
McGraw-Hill, 1922.
D-33 Report of Lathe Tools Research Committee, Dempster Smith.
Bul. Manchester Assn. of Engrs. 1922.
D-34 Lathe Breakdown Tests of Some Modern High Speed Steels, H. J. French and Jerome Strauss.
Trans., A.S.S.T., vol. 2 (1922), p. 1125.
D-35 An Account of Some Experiments on the Action of Cutting Tools, E. G. Coker and K. C. Chakko.
Excerpt Minutes of Proceedings of Meetings, Bul. pub. by Inst. of Mech. Engrs., April 1922.
D-36 Are Metal Workers Hidebound?
Editorial, *Amer. Mach.*, 1922, vol. 57, p. 154.
Discussion by Alfred Herbert, Ltd., *American Machinist*, 1922, vol. 57, p. 768.
Discussion by F. W. Curtis, *American Machinist*, vol. 58, 1923, p. 162.
D-37 Considerations Generales sur Les Machines-outile. *Outillage*, vol. 6, nos. 37, 38, 39, 43, 44, 45, 47, Sept. to Nov. 1922. (Power required, theory of design, etc.)
D-38 Causes of Failure of High Speed Tools, J. F. Kayser.
Chemical & Metallurgical Engineering, vol. 25 (1922), p. 847.
D-39 Performance of High Speed Cutting Tools, Jerome Strauss.
Iron Age, vol. III (1923), p. 1103.
D-40 Power Required for Cutting Metal, Fred A. Parsons, Kempsmith Milling Machine Co.
Mechanical Engineering, vol. 45 (1923), p. 35.
D-41 Significance of Tool Temperatures as a Function of the Cutting Resistance of Metals, H. A. Schwartz and W. W. Flagle.
Proc. A.S.T.M., vol. 23, pt. 11 (1923), p. 27.
D-42 Effect of Heat Treatment on Lathe Tool Performance and Some Other Properties of High Speed Steels, H. J. French, Jerome Strauss, and T. C. Digges.
Trans. A.S.S.T., vol. 4 (1923), p. 353.
D-43 Engineering Problems Solved by Photo-Elastic Methods, E. G. Coker.
Jour. Franklin Inst., 1923, vol. 196, p. 433.
D-44 Metal Cutting Tools (discussion of tools shapes, angles, and speeds), W. E. Splain.
American Machinist, p. 663, vol. 58, May 3, 1923.
Discussion by F. W. Elstub, *American Machinist*, p. 64, vol. 59, July 12, 1923.
Discussion by R. Poliakoff, *American Machinist*, p. 783, vol. 59, Nov. 15, 1923.
D-45 Practical Notes on Speeds and Feeds, Giving Tables of Speeds for Different Processes and Metals.
American Machinist, Sept. 27, 1923, p. 467.
D-46 Metallurgical Aspect of Cutting Metals.
Iron Age, Sept. 27, 1923, p. 805.
D-47 Lathe Tool Angles and Speeds for Cutting Monel Metal and Nickel.
Machinery, Dec. 1923, p. 287.
Also see International Nickel Co.'s Working of Monel.
D-48 Carbon Tool Steel Specifications, A.S.T.M.
American Machinist, Dec. 20, 1923, p. 914.
D-49 Results of Extended Work on Tool Form, Force, Chip Formation, Speed, Etc., Hans Klopstock.
Werkstättstechnik, Dec. 15, 1923, Part 1. Berlin, Part 2.

D-50 Progress Report on the Present Status and Future Problems on the Art of Cutting Metals, C. A. Becket.
A.S.M.E. Special Research Committee, A.S.M.E. Dec. 3, 1923.
D-51 Investigation of Turning, Hans Klopstock.
Werkstättstechnik, vol. 19, Dec. 1 and 15, 1923.
D-52 Tacho-Scope, Direct Reading R.P.M. Counter and F.P.M.
American Machinist, vol. 59, p. 155.
D-53 Schnellstahl und Schneidmetall (High Speed Steel and Tool Steel), F. Rapatz.
Maschinenbau, 1924.
D-54 Roto-stat, A Means of Slowing up Motion of Cutting so as to Make it Readable.
American Machinist, vol. 59, p. 349, and Jan. 17, 1924, p. 103.
D-55 Speeds and Feeds for Materials, Effect of Lubrication, Etc., Dowd and Curtis.
American Machinist, Feb. 1924.
D-56 Zur Normung der Schneidstahle Ruckblick und Ausblick (Standardization of Cutting Tools, Terms, etc.).
N.D.I., Feb. 28, 1924, p. N-63-65.
D-57 Influence of Temperature on Work-Hardening of Metals, E. G. Herbert.
Engineer, April 4, 1924, p. 356.
Also see p. 366 Smith & Hey's article.
D-58 Hardness and Cutting Trials of Tool Steel, Dempster Smith and I. Hey.
(Manchester Assn. of Engrs, March 28, 1924.)
Engineer, April 4, 1924, p. 363.
D-59 The Thermal Action of Cutting Tools, Dempster Smith and I. Hey.
Editorial, April 4, 1924, *Engineer*, p. 363.
D-60 Testing Machinability by Drilling, Kessner.
Testing, April, 1924.
D-61 Chip Formation, as Tested at National Physical Laboratory, England.
Engineering, May 16, 1924.
D-62 Definition of Terms Used in Heat Treatment.
Tentatively approved by Recom. Prac. Comm. of A.S.S.T.
American Machinist, Mar. 20, 1924, p. 420.
D-63 Foreign Progress in Cutting Metals, C. A. Beckett.
Mechanical Engineering, Oct. 1924, Progress report of Special Research Committee, covering work of Smith and Hey, and Herbert hardness tester.
D-64 Schneidversuche mit Schnellarbeitsstahlen, R. Hohage and A. Grutzner.
Stahl und Eisen, vol. 45, 1925, p. 1126.
D-65 Report on the Action of Cutting Tools, E. G. Coker.
(Use of polarized light on lathe, planer, and milling tools.)
Engineering, vol. 119 (1925), p. 365.
I.M.E. paper.
D-66 Experimental Study of Forces Exerted upon Surfaces of a Cutting Tool, T. E. Stanton and Mr. J. H. Hyde of National Physical Laboratory, London.
Engineer, Jan. 30, 1925, p. 126.
Engineering, Jan. 30, 1925, p. 139, 148, and 151.
D-67 Flow and Rupture of Metals during Cutting, W. Rosenhain and Mr. A. C. Sturney of National Physical Laboratory, London.
Engineering, Jan. 30, 1925, p. 151.
Engineer, Jan. 30, 1925, p. 126.
D-68 Thermoelectric Measurement of Cutting Tool Temperatures, Henry Shore.
Jour., Washington Academy of Sciences, March 4, 1925, p. 85.

D-69 Factors Influencing the Machinability of Cast Iron, John W. Bolton, Metallurgist, Niles Tool Wks.
Machinery, March 1925, p. 533.
D-70 Der heutige Stand der Zerspanungs wissenschaft (The Present State of Metal Cutting), W. Hippler.
Zeitschrift fur Maschinenbau, 1925.
D-71 Experiments with Lathe Tools on Fine Cuts and Some Physical Properties of the Tool Steels and the Metal Operated Upon, Dempster Smith and Arthur Leigh.
Engineering, March 20, 1925, p. 364.
Engineer, March 30, 1925, p. 319.
Paper of Institution of Mech. Engr.
D-72 Recent Investigations in Turning and Planing and a New Form of Cutting Tool, Hans Klopstock.
Mechanical Engineering, June 1925.
Zeits des Vereines deutcher Ingenieure, Feb. 21, 1925.
D-73 A Lathe Dynamometer Described, A. Sahnazaroff.
Die Werkzeugmaschine, May 15, 1925, p. 254.
D-74 Tools, Speeds, Feeds, and Depth of Cut for Finishing Commutators (Copper and Micaplate).
Machinery, June 1925, p. 817.
D-75 Wissenschaftliche Gestaltung der Werkzeuge (Scientific forming of Cutting Tools).
Zeitschrift des Vereines deutcher Ingenieure, 1925.
D-76 Spanalhebende Werkzeuge fur die Metallbearbeitung und ihre Hilfsernrichtungen (Chip taking tools for the Working of Metals and their Accessories), J. Reindl.
Springer, Berlin, 1925.
D-77 Cutting Speeds of Crank Shapers.
Machinery, Aug. 1925, p. 957.
D-78 A New Method of Measuring Temperatures Generated by Metal Cutting Tools, Edward G. Herbert.
(Thermoelectric, stellite tool cutting mild steel.) Description, *The Pendulum*, June 1925.
Some data, *The Pendulum*, Sept. 1925.
D-79 Experiments with Nickel, Tantalum, Cobalt, and Molybdenum in High Speed Steel, H. J. French and T. C. Digges.
Annual Meeting A.S.S.T., Sept. 1925.
D-80 Die Prufung der Gewindebohres (The Investigation of Taps, at Charlottenburg Laboratory for Machine Tools "Torque"), Von M. Kurrein.
Werkstattstechnik, Sept. 1, 1925.
D-81 Hardness Governs Machinability, James Ward.
Foundry, Sept. 1, 1925, p. 685.
D-82 Why Do We Know so Little About Metal Cutting?
American Machinist, Sept. 17, 1925, p. 476.
D-83 Machinability of Brass Rod in Screw Machines, A. C. Lusher, Scoville Mfg. Co., Providence, R. I.
Paper before Providence Section, A.S.M.E., Oct. 1925.
D-84 Hardness Numbers and Their Relation to Machinability.
Mechanical Engineering, Nov. 1925, p. 919.
Discussion at Cleveland Meeting of A.S.S.T. in Sept. 1925.
D-85 Research in Metal Cutting, O. W. Boston.
American Machinist, Nov. 19, 1925, p. 805.
D-86 Physical Metallography, E. Heyn. Translated into English by M. A. Grossmann.
(Excellent bibliographical index.)
(Relation of Machinability to Hardness and Workability), p. 326.
John Wiley & Sons, 1925 — United Alloy Steel Corp.

D-87 Die Messung der Schneidentemperature beim Abdrehen von Flusseisen (Measuring the cutting temperatures in the cutting of low carbon steel), Gottwein.
Maschinenbau, Nov. 19, 1925, p. 1129-1135
D-88 The Measurement of Cutting Temperatures, E. G. Herbert.
Paper, Institution of Mech. Eng., Feb. 19, 1926.
D-89 Cutting Tests of Tool Steels, U. S. Naval Gun Factory, Jerome Strauss.
Trans., A.S.S.T., April 1926.
D-90 Supplementary paper on The Measurement of Cutting Temperatures, E. G. Herbert.
American Machinist, March 4, 1926, p 363.
D-91 Cutting, Principles of Cutting Edges, W. R. Ward.
Trans, A.S.S.T., March 1926, p. 482.

DISCUSSION

EDWARD G. HERBERT.[1] The author is to be congratulated on his courage in using the planer rather than the lathe for his cutting-tool experiments. He is also to be congratulated on the ingenuity with which he has solved the problem of measuring the cutting force exerted by a reciprocating machine.

Interest attaches to the fact established in Par. 39 that in cutting brass the force on the tool is least when the tool has little or no rake. It is common workshop practice to cut brass with a tool with zero top rake. Such a tool removes the metal in a shower of small detached particles. Its general adoption, doubtless, is based on experience of the greater durability of the tool, which naturally results from the lessened vertical force. The writer has attempted to analyze the process of chip formation in brass and in his experiments used a tool without rake.[2] It was shown that the chip is detached by shear, with but little distortion, and hence with but slight work-hardening of the metal. It may be that in this fact lies the explanation of the reduced force on the tool. The right-angled tool removes metal whose hardness is very little increased in the process of cutting, whereas a tool with rake distorts and hardens the metal before cutting it.

The relatively low force exerted in cutting cast iron, as compared with the force required to cut steel of lesser hardness, may be similarly explained. The cast-iron chip is separated by shear without distortion and without hardening. The steel chip consists of metal harder, and in some steels very much harder, than the body from which it was separated.

The writer endorses the author's suggestion that "if possible a theory of metal cutting which underlies all types of cutting should be developed." Such a theory should take account of

[1] Edward G. Herbert, Ltd., Manchester, England.
[2] See paper on Work-Hardening Properties of Metals, page 705, this volume.

the very different behavior of different metals under the action of cutting tools, and would necessarily base itself on a study of chip formation.

A. L. DE LEEUW.[1] The writer does not believe that tests made with the dynamometer used by the author of the paper will give dependable results. When the work that is to be cut is supported on a dynamometer, the same conditions do not exist in the experiment as obtain in actual cutting conditions in the shop.

In the first place it has been found by the author of the paper that when the width of the cut is increased, the amount of power required, or the pressure required, is not in proportion to the width of the cut. The writer doubts the accuracy of the observations and the results; in fact, he knows that they are not true. Since these results may honestly be doubted the writer is inclined also to doubt the other results. He is inclined to believe that the results obtained are due to the use of the dynamometer. If that is the case all other results obtained with the dynamometer are also open to question.

Another point to which the writer wishes to call attention is the shear angle of the tool, which the author denotes as side rake. The shear angle on the planer tool, so far as the writer knows, has never failed to give results comparable with those obtained by the author. There is, however, one other result that has not been mentioned. If the shear angle is very much increased, say to 70 or 80 degrees, a decided effect is produced on the finish of the metal being worked. Materials which do not permit of a nice finish with a square tool will produce a beautiful polished finish with a shear tool. The same results as obtained with a shear tool were obtained in some of the experiments made by the writer a number of years ago with a rotary tool in which the shear angle was very large. That tool also produced a beautiful finish on the surface of the work. The helical mill is another example of a shear tool, and it also produces a much better finish than the ordinary plain or spiral cutter.

The author has failed to mention the effect of polishing the top of the tool. In a paper read before the Society in 1917[2] the writer described a result obtained by polishing the tool and also by cutting out the tool according to the shape which the chip ordinarily would produce. While no tests were made at the time to determine the results as to quality, it was found that these procedures had a strong effect upon the life of the tool. The polished and cut tool lasted from three to four times as long as the ordinary tool.

[1] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.
[2] Trans. A.S.M.E., vol. 39 (1917), p. 185.

The author has stated that less power is required per unit of section with a heavy cut than with a light one. As far back as 1898 the writer made experiments to determine the power required for different machine tools. During these experiments it was found consistently that the minimum power is required when the section cut approaches a perfect square. For instance, a cut $\frac{1}{8}$ in. deep and $\frac{1}{8}$ in. wide will require less power than a cut $\frac{1}{4}$ in. wide and $\frac{1}{16}$ in. deep, or $\frac{1}{4}$ in. deep and $\frac{1}{16}$ in. wide. The further we recede from the perfect square the greater is the power required for equal sections of metal cut.

The fact that chatter was developed when cutting brass is of interest. As a result of a few experiments the writer inclines to the idea that chatter is caused by synchronism between the variations in pressure when separating the chip and the natural period of oscillation of some machine parts. It may be that this synchronism existing between some parts of the mechanism used by the author and the period of separation of the individual chips diminished the amount of power required.

The term "machinability" has been used in two different ways. This is one of the many cases where the engineering profession takes an existing term out of practice and gives it a new meaning. This procedure is highly objectionable. If there is such a thing as machinability it should be given a new name. In the shop machinability means whether or not the workman is able to machine the piece. You may have a piece of hard steel, which machines beautifully, while a piece of wrought iron will not machine at all. The wrought iron may be perfectly machinable when milled and not at all machinable when it is tapped. The use of the same word in two different meanings is objectionable. Let us get another term for it and then define its meaning. The sense in which the word machinability is used in the paper is not the sense in which the shop man uses it.

A. L. Davis.[1] The writer has had an experience similar to the author's in regard to the amount of power required not being in proportion to the work done. This experience was encountered in passing metal through rolls, where the thrust was taken on an Olsen machine. A metal strip twice the width of another strip does not always require just twice the pressure. Of course, there are reasons why this is so, one of which is that there is room for the metal to escape sidewise, which is more effective on the narrow bar. Clearly this same action occured in the author's experiments. Mr. De Leeuw admits this when he states that there is an optimum cross-section, the square which is removed with less pressure, or less work, than the oblong in either direction.

[1] Research Engineer, Scovill Mfg. Co., Waterbury, Conn. Mem. A.S.M.E.

JOSEPH G. BERCSI.[1] The writer desires to submit the following comments on Professor Boston's paper, inasmuch as metal-cutting tests that he later refers to were made by him some time ago at the Mechanical Technological Laboratory of the Hungarian

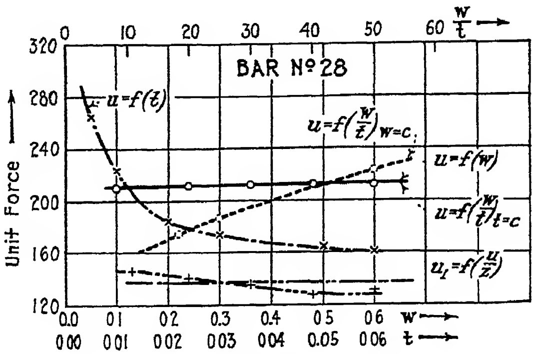


FIG. 84

Royal Polytechnic University, Budapest, under the direction of Prof. Alexander Rejtö.

As the author's tests show, the total force necessary for metal cutting depends not solely upon the material being cut, upon the tool used (assuming that all the conditions are the same), and upon the cross-section of the chip, because the computed unit

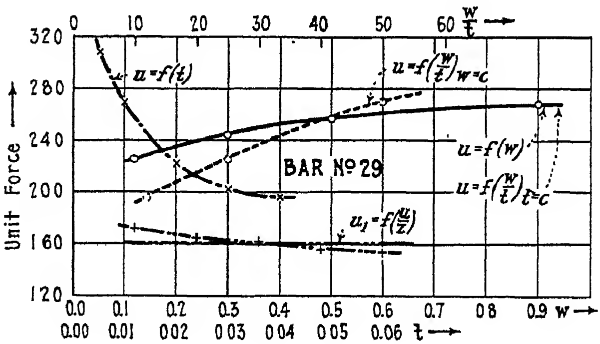


FIG. 85

forces do not have the same values in different cases, but vary between wide limits. All similar tests made furnish proof of this observation.

A similar phenomenon is involved in the compression of a piece of metal of, say, cylindrical shape whose height h is small

[1] Westinghouse Elec. & Mfg. Co., Sharon, Pa. Jun. A.S.M.E.

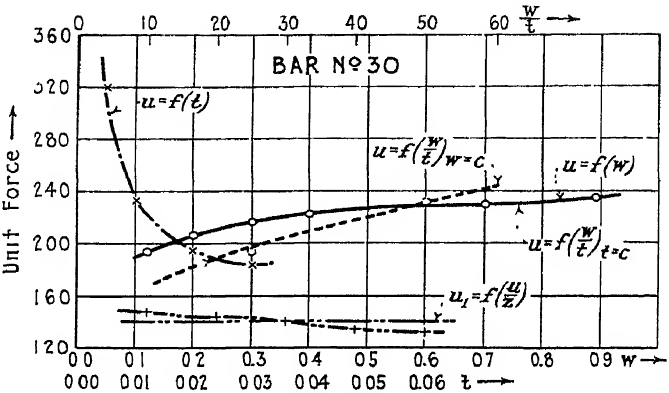


FIG. 86

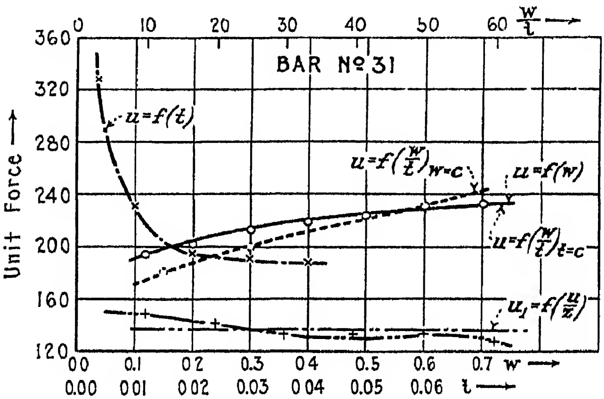


FIG. 87

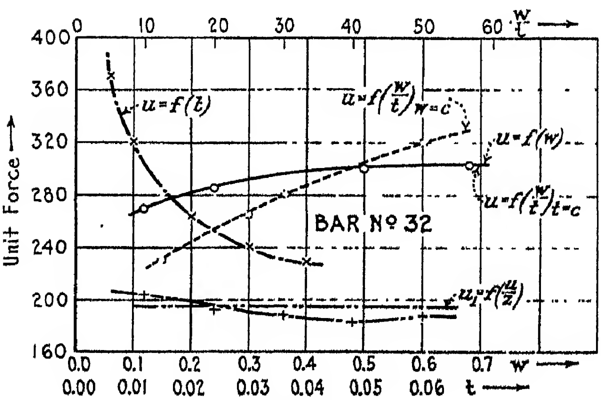


FIG 88

compared with its diameter d. If $d>2h \cot \beta$, the test piece[1] will not decrease the same percentage in height corresponding to a compression stress as it does when $d<2h \cot \beta$.

The same is the case when cutting a piece of metal. The larger the ratio between the width w and the thickness t of the chip, the larger is u the unit force in the cross-section. This means that the unit force depends upon the ratio w/t. Employing this assumption, the writer constructed five diagrams (Figs. 84-88) for bars Nos. 28 to 32, using the author's data. In these diagrams are two curves denoted by $u = f(w)$ and $u = f(t)$ [i.e., unit force vs. chip width and chip thickness], from which are computed two other curves, denoted by $u = f\left(\frac{w}{t}\right)_{w=c}$ and $u = f\left(\frac{w}{t}\right)_{t=c}$ [i.e., unit force as function of the ratio w/t, figured from the curve $u = f(t)$, when w is constant, and unit force as function of the same ratio, figured from the curve $u = f(w)$, when t is constant]. For the first of these curves the curve $u = f(w)$ was used.

As can be seen, these last two curves lie quite close together in the case of bars 29, 30, and 31 at the point where $w/t = 50$. When $w/t > 50$ the thickness is so small that these parts of the curves are uncertain. The correspondence is not so good in the other two cases, but the difference between the two unit forces is always less than 15 per cent.

It is obvious that an approximate relation exists between the unit force and the ratio w/t, and for the same material, if the tool and all the other circumstances are identical, the unit force of cutting is about alike if the ratio w/t is the same.

The next step would be to look for a relation between the mechanical properties of the metal and the unit force necessary for cutting. According to tests made in the above-mentioned laboratory, this can be expressed as follows:

$$u_1 = \frac{u}{z}$$

where

$$u_1 = S(1+f_1 \tan^2 \beta) f(m, T)$$

Here S is the maximum compression stress corresponding to the changed cross-section (this is the work-hardened value of the stress), f_1 the value of the friction between the metal and the tool, β the slip angle, and z a factor depending upon the geometrical shape of the chip and having the following value:

$$z = \sqrt[4]{\frac{(n+1)(2n+1)}{6n}}$$

[1] The angle β is the slip angle, determined by the direction of traction and the direction of slipping.

where $n = w/2t$. Applying this relation to the author's data, if

	$n = w/2t = 5$	10	15	20	25	30
then	$z = 1.21$	1.4	1.52	1.64	1.73	1.8

and we get the curves denoted by $u_1 = f\ (u/z)$, which in all five cases can be replaced by a straight line, parallel with the base line. This last curve was calculated from the average values of the two $u = f\ (w/t)$ curves. In this way it is possible to compute the unit and the total force of metal cutting from the compression stress. The total force can be expressed as follows:

$$F = AS(1+f_1 \tan^2 \beta)zf\ (m,\ T)$$

where m represents the properties of the metal, which are not taken into consideration, and T those of the tool. According to the tests, these are the elongation of the metal corresponding to the maximum compression stress, the cutting angle of the tool, and the temperature. This last factor was not taken into consideration in the Budapest tests.

THE AUTHOR. So far as the use of the dynamometer is concerned the author agrees that it not only is a nuisance, but an expensive luxury. However, any one who has had anything to do with an investigation of metal cutting would be glad to receive information as to methods of carrying on such an investigation without a dynamometer. The author does not agree with Mr. De Leeuw, however, as to the unreliability of the dynamometer. He believes that good results have been obtained, which are reliable. Certainly, for an individual dynamometer, the results are comparable.

The 75-deg. side-rake tool that Mr. De Leeuw spoke of is being commonly used to give a good finish in planer work. Its use is confined almost entirely to finishing, which means that a depth of cut of probably 0.001 in. is taken. It has been found that the 75-deg. side-rake tool with a zero-degree front rake did not give the finish that was expected. Perhaps some front rake would have changed the conditions.

Mr. De Leeuw has referred to a polished tool. In the author's experiments this type of tool was used. It is referred to in the paper as a honed tool. As stated in the paper, only the force on the tool was determined and not the life of the tool.

In regard to machinability, the author believes that he defined what was in mind when the word was used. He does not believe that any apologies to the shop man are necessary in using this word for the reason that, if the shop man has defined it as to whether a metal can or cannot be machined he has ascribed a very indefinite meaning to it. Very often one mechanic can machine a piece that another mechanic has said could not be machined. The author has used the word machinability to indicate the rela-

tion between various metals as shown by the force on the tool. See Par. 83. The term also has been used by Mr. French in his paper[1] to show a relation between various metals as indicated by the cutting-tool life He carefully defined the term as the speed for a given life of a definitely formed tool while cutting a chip of specific depth and traverse. The term has been used also in connection with the finish produced on various metals. Other methods, including that of drilling, have been referred to in the paper, Pars. 117 to 124, inclusive In the author's opinion, each method has been devised to indicate certain machining properties of metals, and its use is justified until some better method is devised or a standard definition is accepted.

In the paper, the literature on the results of other experimenters, as they have touched on the problems therein considered, has been presented. The results have been stated as concisely as possible, and the results obtained by the author have been compared. This was not done to confirm the results presented in the paper, but simply for the purpose of correlating all work that already had been done along that particular line.

[1] Rough Turning with Particular Reference to the Steel Cut, p 533, this volume.

No. 2024

THE INFLUENCE OF RADIATION IN COAL-FIRED FURNACES ON BOILER-SURFACE REQUIREMENTS, AND A SIMPLIFIED METHOD FOR ITS CALCULATION

By W. J Wohlenberg,[1] New Haven, Conn
Member of the Society

AND

E. L. Lindseth,[2] Cleveland, Ohio
Junior Member of the Society

This paper includes a simplified method of dealing with the energy problem of the boiler furnace. Its application yields information concerning that division of total surface between cold furnace walls and convection zone which results in the greatest overall energy absorption. The influences of fuel type, air preheat, and certain other factors on furnace conditions and surface requirements are also discussed.

IN RECENT years several important contributions have been made to the literature on radiation, and a few papers and books dealing directly with radiation[3] in the boiler furnace have also been published. Unfortunately, and no doubt because of insufficient available data at the time of publication, most of the latter deal with the subject in a fragmentary manner. Thus a number of authors[4] show excellent methods for calculating the radiation discharge from the fuel bed in stoker-fired furnaces but leave out of consideration radiation from the gaseous constituents.

[1] Associate Professor of Mechanical Engineering, Sheffield Scientific School, Yale University.

[2] Test Department, Cleveland Electric Illuminating Company.

[3] M. Gerbel, Die Grundgesetzte der Würmestrahlung, Verlag von Julius Springer, Berlin, W9. See also Broido, Radiation in Boiler Furnaces, Trans. A.S.M.E., 1925.

Rosak and Veron. *Revue de Métallurgie*, Aug., Sept., and Oct., 1924. See also Broido, Radiation in Boiler Furnaces, Trans. A.S.M.E., 1925.

[4] Munzinger, Die Leistungssteigerung von Grossdampfkesseln, Verlag von Julius Springer, Berlin W9.

Contributed by the Power Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

Of the various methods of dealing with the radiation from the gaseous constituents the most comprehensive is that developed by Schack,[1] a translation of which is included in an appendix to the paper by Broido[2] entitled Radiation in Boiler Furnaces.

2 Most of the treatments referred to above originated with foreign engineers and scientists. In America, Christie, dealing directly with the boiler furnace, has discussed the problem from the qualitative rather than the quantitative point of view. Of the attempts made in this country to obtain a simple solution of the energy quantities involved, the methods of Orrok[3] and of Broido may be mentioned. Both of these are based on a large amount of data collected from experiment. Thus the paper by Broido, referred to above, contains a considerable amount of such data. Orrok, in several discussions, presents a large quantity of data taken from tests on both stoker-fired and pulverized-fuel furnaces. Both authors have attempted to set up empirical methods for dealing with the energy problem of the boiler furnace, and no doubt for a limited range such methods are applicable. Broido proposes a mean curve from which the energy absorption in the furnace may be approximated, and Orrok has modified Hudson's formula in an attempt to obtain a solution. Empiricism must always follow in the wake of experimental results, but it is, so to speak, hedged within a certain explored region, and as a tool it may not be applied with confidence outside of this field.

3 This paper is an extension of the general method outlined in the preceding paper[4] by Wohlenberg and Morrow entitled Radiation in the Pulverized-Fuel Furnace. It includes the effects on energy emission of fuel type, temperature of air admitted for combustion, and manner of burning the coal, that is, whether it is burned on a stoker or in the pulverized form. Although reference is made to the former paper, the present paper is a unit in itself for those primarily interested in the application and results of a method of solving the energy problems of the furnace. Those interested also in the more complete theoretical basis are referred to the preceding paper. The nomenclature is chosen so that, wherever possible, a given symbol represents the same thing in both papers.

4 In the light of better available data certain changes have been made to the method outlined by Wohlenberg and Morrow. Noteworthy among these is the substitution of Schack's method of dealing with radiation from the furnace gases for the Stefan-Boltzman law with a reduced coefficient of radiation. The reasons

[1] Schack, Mitteilung, Nr. 55 der Wärmestelle Düsseldorf des Vereines deutscher Eisenhüttenleute.

[2] Broido. Trans., A.S.M.E., 1925.

[3] Orrok. Trans., A.S.M.E., 1925.

[4] Wohlenberg and Morrow, Trans., A.S.M.E., 1925.

for this are made apparent later in the paper. A second modification is the result of information furnished by the application of the general method in evaluating a certain type of curve. It concerns the manner of solving for the energy emission in furnaces having partially cold envelopes. This solution is now effected by first evaluating the emission which would occur in a furnace totally enclosed by cold walls and then applying a reference factor whose value is found for the conditions involved by reference to the curves above mentioned.

PART I DISCUSSION OF RESULTS[1]

5 The consideration in the present work of the influence of several additional conditions not treated in the preceding paper has made necessary the introduction of certain additional coefficients. Two of these, designated as μ_5 and μ_6, are now defined.

The Coefficients μ_5 and μ_6

6 The coefficient of emission μ_5 is the ratio of the net energy emission within the furnace, exclusive of that absorbed by convection at the cold face forming the aperture for the escaping gases, to the liberated energy. The coefficient μ_6 is similar but contains in the denominator the sum of the liberated energy and that brought in by the air above atmospheric temperature. If the air

[1] Attention to the order followed by the authors in presenting their material will facilitate the reader's comprehension of it. Part I presents the results of the solutions of hypothetical problems in the design of furnaces by the method of the authors. These results, in graphical form, are analyzed, and general conclusions deduced from them. Part II develops the general method used by the authors in the solution of the hypothetical problems. An analysis is first made of the energy quantities involved for a type *E* furnace (all surfaces cold), and a general energy equation satisfying the condition of equilibrium is established. It is then shown how solutions for other types of furnaces in which the fraction cold Ψ is less than unity (that is, furnaces in which the surfaces are partly made up of refractory material) may be found by introducing into the solution for the type *E* furnace a reference factor γ. In Appendix No. 1 the hypothetical problems for the type *E* furnace are solved, substituting the numerical quantities involved in the equations, and in Appendix No. 2 curves are developed by means of which the solutions for other types of furnaces may be obtained. Appendix No. 3 shows how the most effective fraction cold Ψ may be obtained. Certain coefficients necessary to the solutions are derived in Appendix No. 4, and Schack's method for determining the radiation from the furnace gases is discussed in Appendix No. 5. Appendix No. 6 considers the influence of air preheat on surface requirements, and Appendix No. 7 shows how the equivalent fractions cold may be determined for furnaces other than those cubical in shape for which the general equations apply. — Editor.

is not preheated, μ_5 and μ_6 have the same value, but otherwise the value of μ_6 is obviously the smaller of the two.

7 In the paper by Wohlenberg and Morrow the symbol μ_4 was used to denote the *furnace absorption efficiency* which is defined as the relation of the energy actually absorbed by the cold furnace walls, exclusive of the absorption by convection at the cold face forming the gas aperture, to the liberated energy. The coefficient μ_5 therefore differs from μ_4 because there is a certain loss from the outside furnace walls to the atmosphere. This loss is usually very small in a well-designed wall, and hence μ_5 is a very close approximation to the furnace absorption efficiency, particularly in the furnace with all interior walls cold. Furthermore the emission coefficients μ_5 and μ_6 are the correct ones to use for a determination of the mean temperature of the gases escaping from the furnace. For these reasons they are used by the author in this paper rather than the corresponding furnace absorption efficiencies μ_4.

THE "FRACTION COLD" Ψ

8 The mean air, flame, and refractory temperatures are denoted as before by the symbols[1] T_A, T_U, and T_R. The furnaces are designated as *A*, *B*, *C*, *D*, and *E*. In the first one, type *A*, one of the six faces is cold and in the last, type *E*, all of the exposed faces are cold. A distinction is necessary as between the type *E* stoker-fired furnace and the type *E* pulverized-coal fired furnace. The former has five cold faces because the bottom face is considered as occupied by the fuel bed, while the pulverized-coal fired furnace has six cold faces because all of the fuel is in suspension. Therefore if we let

$$\Psi = \frac{S_C}{S_C+S_R} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [1]$$

in which S_C represents the sum of interior cold surfaces projected on the faces at which they are placed and S_R the sum of interior refractory surfaces projected on the faces at which they are placed, the "fraction cold" Ψ assumes the following values:

[1] It should be emphasized that the symbols T_A, T_U, and T_R represent the mean values of the temperatures in question. Thus T_A is the mean value of the air entering the furnace. T_U and T_R are radiation mean values. The radiation mean of the flame temperature T_U is also the mean temperature of the gases escaping from the furnace. Actual temperatures in the various zones of the furnace will be both above and below this value. Likewise, certain refractory areas may be above and others below the value as computed for T_R. For a more complete discussion of these temperature coördinates, see paper by Wohlenberg and Morrow, Trans. A.S.M.E., vol. 47 (1925), p. 127.

VALUES OF Ψ FOR CUBICAL FURNACES

	Ψ for stoker furnace	Ψ for pulverized-coal furnace
Type *A*, one face cold	$\frac{1}{5}$	$\frac{1}{6}$
Type *B*, two faces cold	$\frac{2}{5}$	$\frac{2}{6}$
Type *C*, three faces cold	$\frac{3}{5}$	$\frac{3}{6}$
Type *D*, four faces cold	$\frac{4}{5}$	$\frac{4}{6}$
Type *E*, five faces cold for stoker; six faces cold for pulverized coal	$\frac{5}{5}$	$\frac{6}{6}$

THE FURNACES AND THE FUELS

9 Solutions are made in the present paper for three furnaces of different sizes; and in each furnace it has been assumed that three different coals have been burned at various rates of energy liberation, various air temperatures, and two excess-air fractions, 20 and 40 per cent. The air quantity in relation to the theoretical requirement is usually denoted simply as 120 and 140 per cent respectively for 20 and 40 per cent excess.

10 The furnaces are designated as follows:

Furnace No. 1, 10 ft. x 10 ft. x 10 ft. = 1,000 cu. ft.
Furnace No. 2, 20 ft. x 20 ft. x 20 ft. = 8,000 cu. ft.
Furnace No. 3, 30 ft. x 30 ft. x 30 ft. = 27,000 cu. ft.

TABLE 1 COALS

Name	Coal No. 1, Ill. Bit., Saline Co	Coal No 2, McDowell, West Va.	Coal No. 3, Pure Carbon
B t.u. per lb.	12,800	14,500	14,520
Moisture, per cent	6.0	3 00	...
Volatile, per cent	32.4	13 00	...
Fixed carbon, per cent	54 3	78 80	100
Ash, per cent	7.3	5.23	...
Ultimate Analyses			
Carbon, per cent	71 6	82 84	100
Hydrogen, per cent	5.3	4 46	...
Oxygen, per cent	12.8	5.94	...
Sulphur, per cent	1.7	0.48	...
Nitrogen, per cent	1 3	1.05	...
Ash, per cent	7.3	5.23	...
Pounds per cu. ft. Raw	84	90	100
Pounds per cu. ft. Coked	75	75	100
Coking diam ratio	0.884	1.003	1.00

11 Table 1 gives the characteristics of the coals used. Coal No. 1 is that on which the results of the paper by Wohlenberg and Morrow are based. This coal contains a high percentage of volatile matter. Coal No. 2 is of the high-grade low-volatile eastern bituminous variety. Coal No. 3, although not actually available for use in boiler furnaces, was chosen because it represents a limiting condition as to volatile and the resulting radiating gaseous constituents in that the products show a maximum CO_2 content, and a minimum of H_2O, the other radiating constituent of furnace gases. Table 2 gives the air requirements and burned-gas mixtures of the three coals.

Influence of Coal Type in Pulverized-Coal Furnaces

12 The influence of fuel type on energy emission and flame temperature is illustrated by curves on Figs. 1, 2, and 3. These are based on the type *E* pulverized-coal furnace, and the same initial mean diameter d_0 of the ground particles is used in all cases. This diameter has been assumed to be 0.002 in. for all coals, in accordance with the analysis in Appendix No. 6 of Wohlenberg and Morrow for coal pulverized so that 75 per cent passes through a 200-mesh sieve.

13 On this basis it is noted that the coefficients of emission μ_5 and μ_6 are not materially affected by the type of fuel used

TABLE 2 PRODUCTS OF COMBUSTION OF COALS OF TABLE 1

		Coal No. 1	Coal No. 2	Coal No. 3
Air per pound of coal, lb...	120% air	11.58	13 10	13 90
	140% air	13.51	15.28	16 22
CO_2 in flue gas, per cent....	120% air	14.23	14.72	17 3%
	140% air	12.27	12.67	14.82%
H_2O in flue gas, per cent . .	120% air	6 32	4.74	....
	140% air	5.45	4.09	...
O_2 in flue gas, per cent......	120% air	3.31	3.34	3 46
	140% air	5.70	5.76	5 92

when other conditions are comparable. The flame temperature T_U, however, is appreciably higher for coal No. 3 than it is for coals No. 1 and No. 2. It is to be noted also that the small variation in emission coefficients exists in spite of the fact that, due to the larger H_2O constituent, the combustion products of coals No. 1 and No. 2 radiate more at a given temperature than do the combustion products of pure carbon. This, in the case of the latter fuel, is partially compensated for by the greater mean diameter of the solid particles as they pass through the furnace, due now to the reduced early expulsion of volatile portions; but the larger compensation is probably caused by the higher temperature which the combustion products of carbon may have for the same energy content, because of the absence of latent heat of water vapor. This results in a somewhat higher flame temperature for the same emission coefficient which in itself is perhaps the chief factor in raising the radiation discharge of pure carbon to that of the other coals.

14 The apportionment of emitted energy to convection and to radiation from gases and solid-carbon particles varies as shown by Figs. 4*a*, 4*b*, 4*c*, 5*a*, 5*b*, and 5*c*. Figs. 4*a*, 4*b*, 4*c*, 5*a* and 5*b* are all based on coal No. 1, and should represent with fair accuracy what may be expected in this respect, because under the same conditions the coal should be ground to the same mean initial diameter in all cases. Fig. 5*c*, however, is based on the same mean initial diameter d_0 for all coals whereas the coal itself may have an influence on what the actual mean diameter will be even though

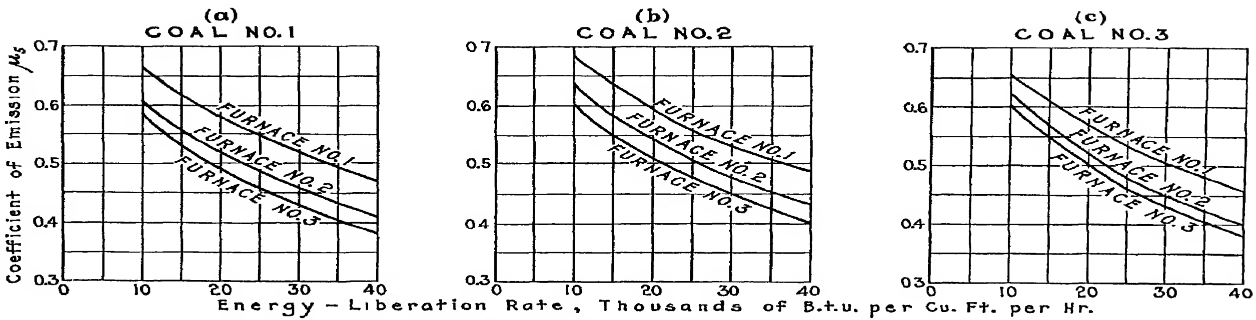


FIG. 1 INFLUENCE OF ENERGY-LIBERATION RATE ON COEFFICIENT OF EMISSION μ_5 FOR SEVERAL FUELS
(Air temperature, 500 deg. fahr ; air, 120 per cent.)

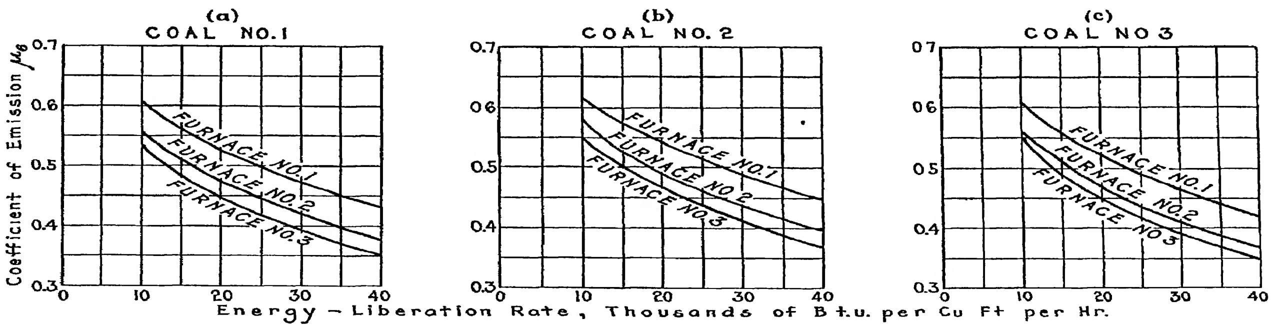


FIG. 2 INFLUENCE OF ENERGY-LIBERATION RATE ON COEFFICIENT OF EMISSION μ_6 FOR SEVERAL FUELS
(Air temperature, 500 deg fahr.; air, 120 per cent)

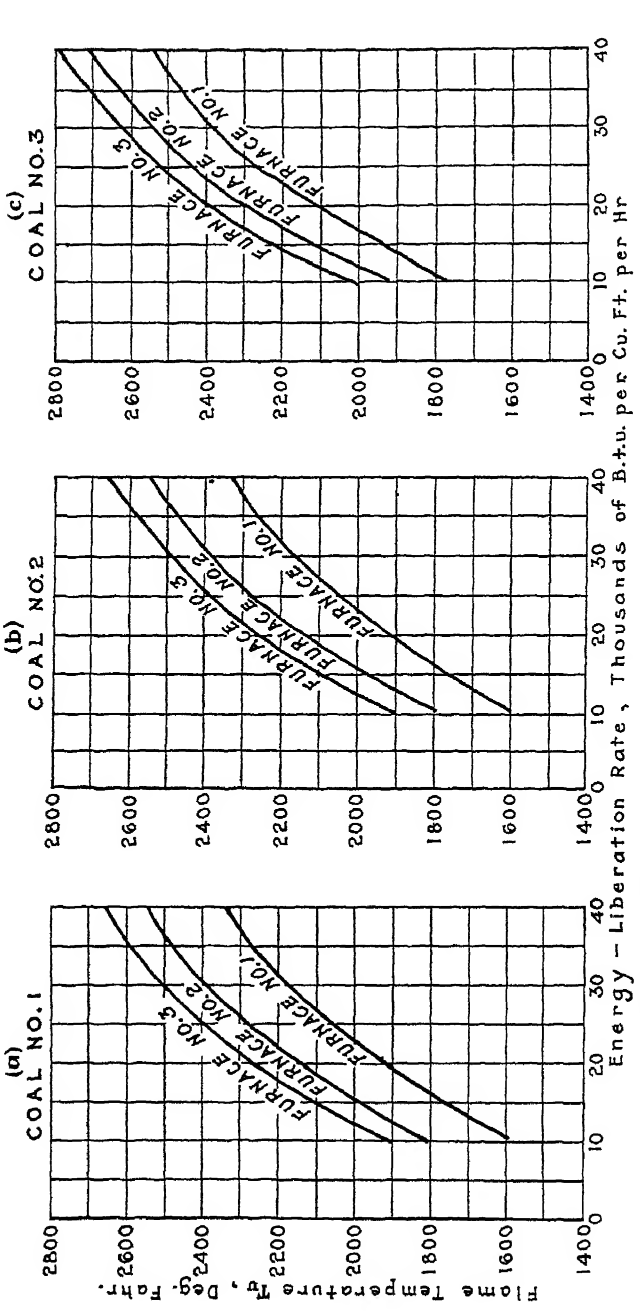


FIG. 3 INFLUENCE OF ENERGY-LIBERATION RATE ON FLAME TEMPERATURE FOR SEVERAL FUELS

(Air temperature, 500 deg fahr ; air, 120 per cent)

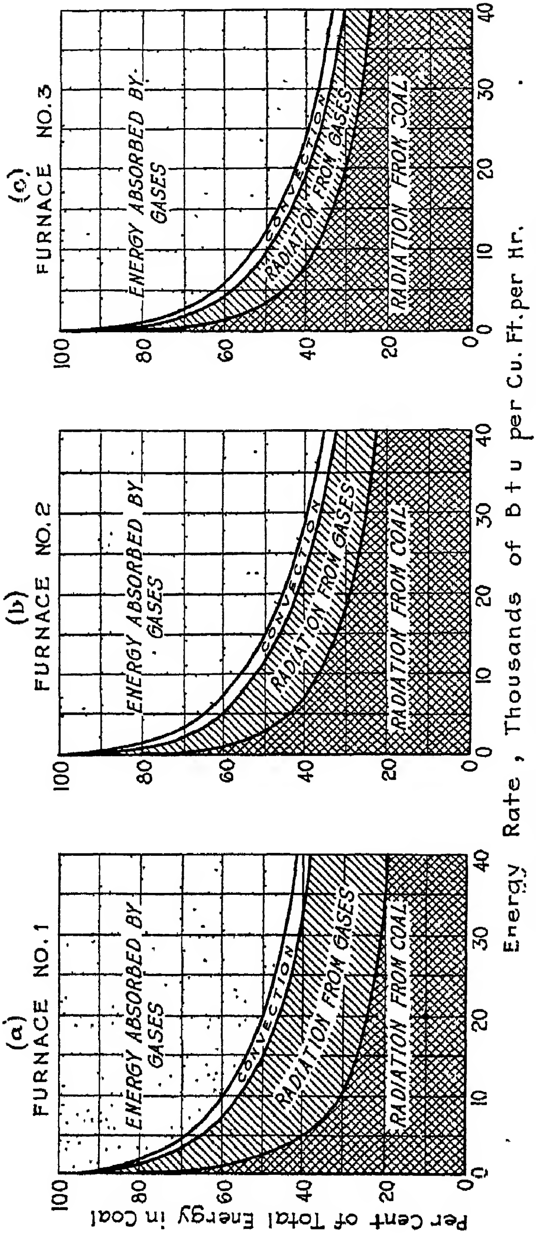


FIG. 4 INFLUENCE OF THE ENERGY-LIBERATION RATE ON THE APPORTIONMENT OF ENERGY WITHIN THE TYPE *E* PULVERIZED-COAL FURNACE

(Mean initial diameter of particles = 0.002 in. for 75 per cent through 200-mesh screen Coal No 1; air, 120 per cent at 70 deg. fahr.)

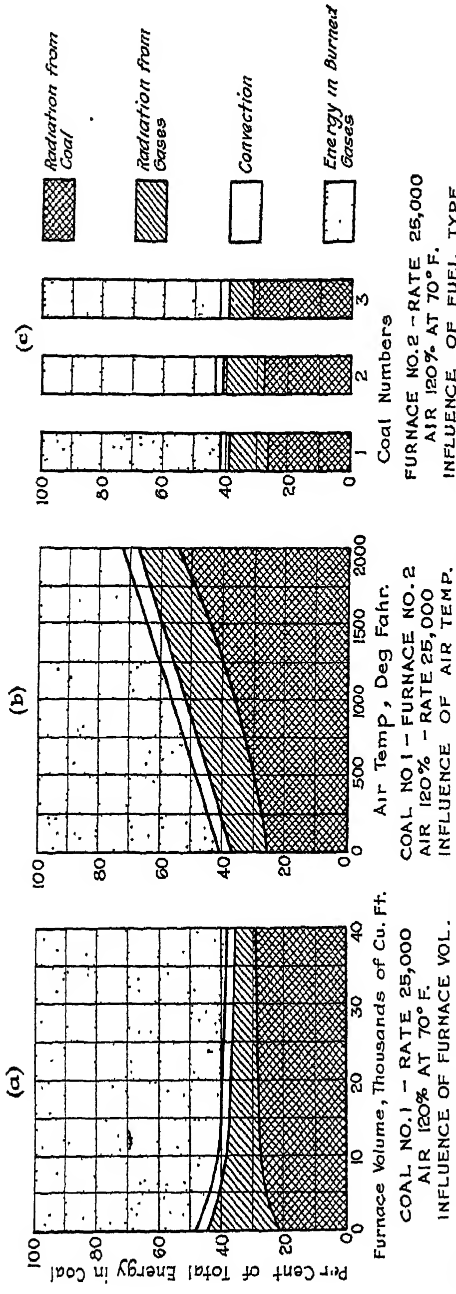


FIG. 5 INFLUENCE OF VARIOUS FACTORS ON THE APPORTIONMENT OF ENERGY WITHIN THE TYPE *E* PULVERIZED-COAL FURNACE

in all cases the same percentage passes through a given mesh screen. A detailed discussion including the influence of coal type on mean diameter d_0 and the resulting influence on energy emission, although referred to in connection with the relative radiance[1] μ_1, of the flame surface by Wohlenberg and Morrow, must be left for a later date.

Influence of Coal Type in Stoker-Fired Furnaces, and Some Comparisons with Pulverized-Coal Furnaces

15 In stoker-fired furnaces the radiating fuel-bed area may remain constant regardless of fuel type. Furthermore, in large furnaces the area of a fuel bed, considered as the bottom face of the furnace, is less than would be the net unshielded area of pulverized-fuel particles in suspension if coal should be burned in the pulverized form in the same furnace volume. Therefore, in large stoker-fired furnaces, other conditions being the same in both cases, the total emission must be less, whence the fraction of radiation from the gaseous products may be considerably more than is the case for pulverized coal. A few results are shown in Tables 3 and 3A.

TABLE 3 APPORTIONMENT OF ENERGY EMITTED WITHIN STOKER-FIRED TYPE *E* FURNACES

(Coal No. 1; air, 140 per cent; air temperature, 70 deg. fahr., rate, 40,000 B.t.u. per cu. ft. per hr.)

Percentage of total emission in furnace as	Furnace No. 1	Furnace No. 2	Furnace No. 3
Convection, per cent	8 2	6 7	8.6
Radiation from gases, per cent	40.7	41 1	38 4
Radiation from solids, per cent	51.1	52 2	53 0
Total, per cent	100.0	100.0	100.0

TABLE 3A APPORTIONMENT OF ENERGY WITHIN THE STOKER-FIRED TYPE *E* FURNACE

(Coal No. 1; air, 140 per cent; air temperature, 70 deg fahr., rate, 40,000 B.t.u. per cu. ft per hr.)

Percentage of total energy in fuel as	Furnace No. 1	Furnace No. 2	Furnace No. 3
Convection, per cent	3 0	1 7	1 7
Radiation from gases, per cent	14 9	10 6	7 6
Radiation from solids, per cent	18 8	13 5	10.5
Heat in burned gases, per cent	63.3	74 2	80 2
Total, per cent	100 0	100 0	100.0

16 The emission coefficients μ_5 and flame temperatures T_U are compared in Table 4 with those of the pulverized-fuel furnace for the above conditions. The latter are identical in each case except for the air requirement which for the pulverized coal is 120 per cent and for the stoker 140 per cent.

[1] Defined as the radiation intensity of the flame surface compared to that of a black body. Trans., A.S.M.E., vol. 47 (1925), p. 128.

Since T_U for the stoker in furnaces No. 2 and No. 3 is higher than it is for pulverized coal it is apparent that the smaller value of μ_5 for the stoker is due to the smaller area of the radiating fuel bed rather than to the larger excess-air fraction.

Influence of Energy Rate, Furnace Volume, Air Preheat, and Other Conditions in the Pulverized-Fuel Furnace

17 In order to illustrate the trend of values showing the influence of energy rate, furnace volume, and air preheat, a few of the results are now included in the form of curves shown on Figs. 6 to 14. All of these apply to the type E furnaces. The trends of values shown for these furnaces are about the same as they are in those having partially cold envelopes, and since results for the

TABLE 4

	Furnace No. 1	Furnace No 2	Furnace No 3
Stoker (air = 140 per cent)			
Value of μ_5	0.37	0.26	0.20
T_U, deg. fahr.	2080	2440	2630
Pulverized coal (air = 120 per cent)			
Value of μ_5	0.41	0 35	0.33
T_U, deg. fahr	2220	2420	2530

type E furnace are used in this paper as a basis for computing the partially black furnaces, it is necessary to include results for the former covering a wide range of conditions. The larger part of such data is included in tabular form in Tables 10 to 14 of Part II.

18 The variations shown by the curves on Figs. 6 to 15 for the most part need no further explanation, as inspection generally reveals the important facts. A few points however are worthy of discussion. Referring to Fig. 12, it is noted that the coefficient μ_5 increases very considerably with an increase in the air temperature, while Fig. 14 shows corresponding small rise in flame temperature. The coefficient μ_6 as shown in Fig. 13 remains nearly constant, indicating that most of the extra input by the preheated air is absorbed because of a considerable increase in radiation with only a small increase in the flame temperature. It is also to be noted that on Fig. 14 the curves for 120 and 140 per cent air cross, indicating that within a limited range of increase of excess air, for air temperatures above that shown by the intersection, the flame temperature will increase rather than decrease.

19 The fact that the rise in flame temperature lags behind the rise in air temperature which causes it, indicates that the curves may cross. In other words, if the air is preheated a sufficient amount, the radiation rate within the furnace will be sufficiently high so that practically all of the energy liberated by the fuel is absorbed within the furnace itself. The curves of Fig. 15 illustrate the conditions in this respect. It is noticed that the intersections

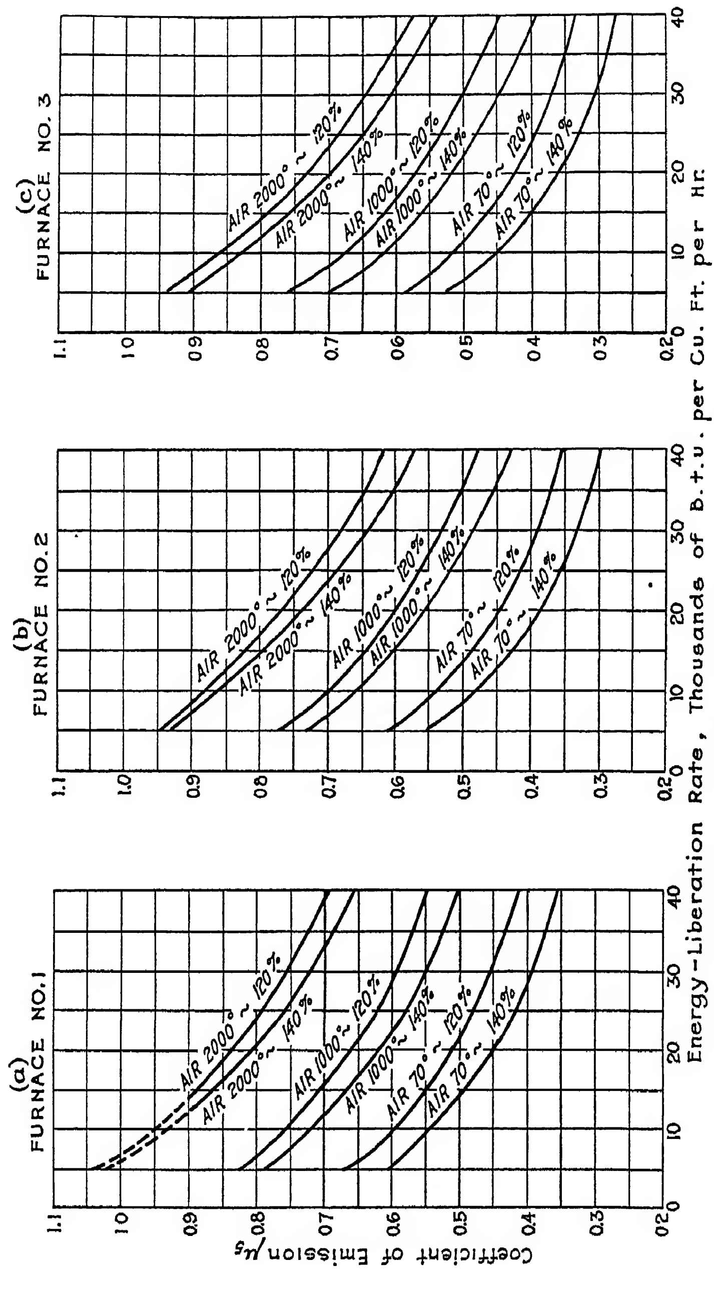


FIG. 6 INFLUENCE OF ENERGY-LIBERATION RATE ON COEFFICIENT OF EMISSION μ_5 FOR COAL NO. 1 PULVERIZED

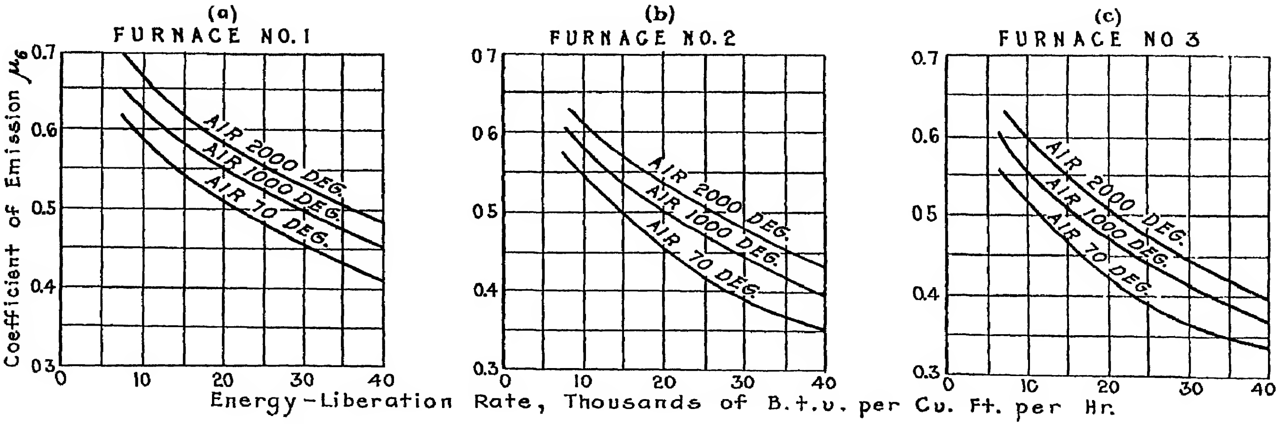


FIG. 7 INFLUENCE OF ENERGY-LIBERATION RATE ON COEFFICIENT OF EMISSION μ_6 FOR COAL NO. 1 PULVERIZED; AIR, 120 PER CENT

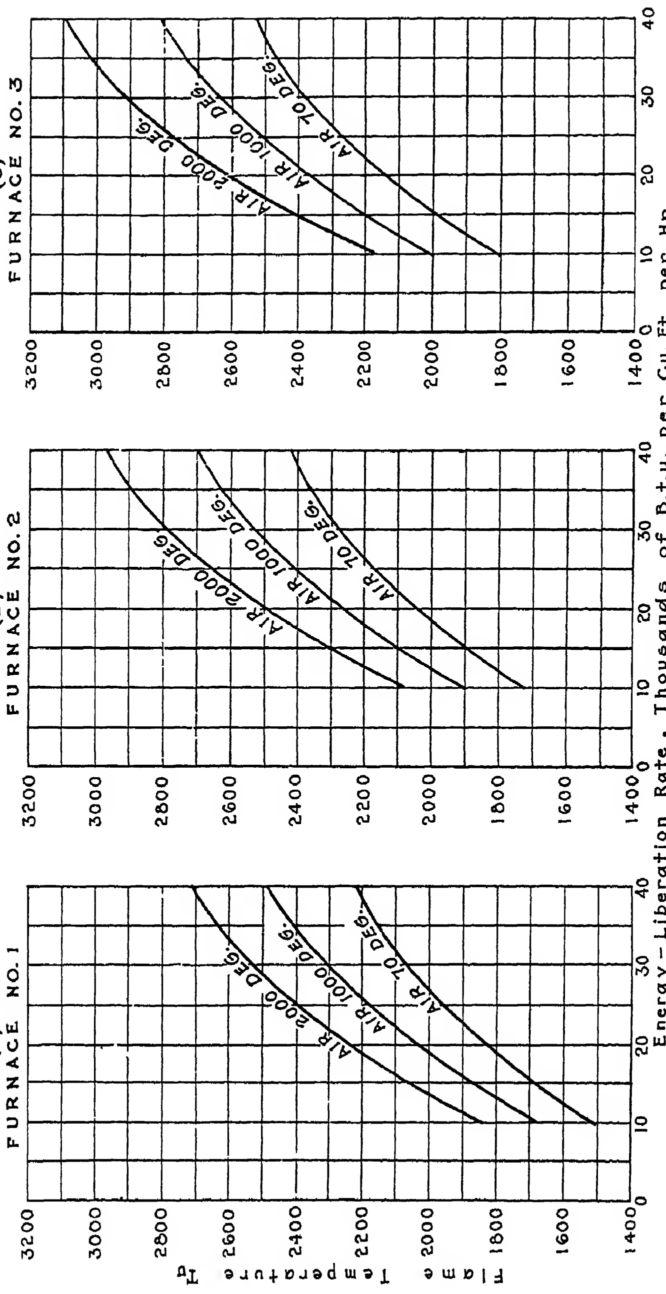


FIG. 8 INFLUENCE OF ENERGY-LIBERATION RATE ON FLAME TEMPERATURE FOR COAL NO 1 PULVERIZED; AIR, 120 PER CENT

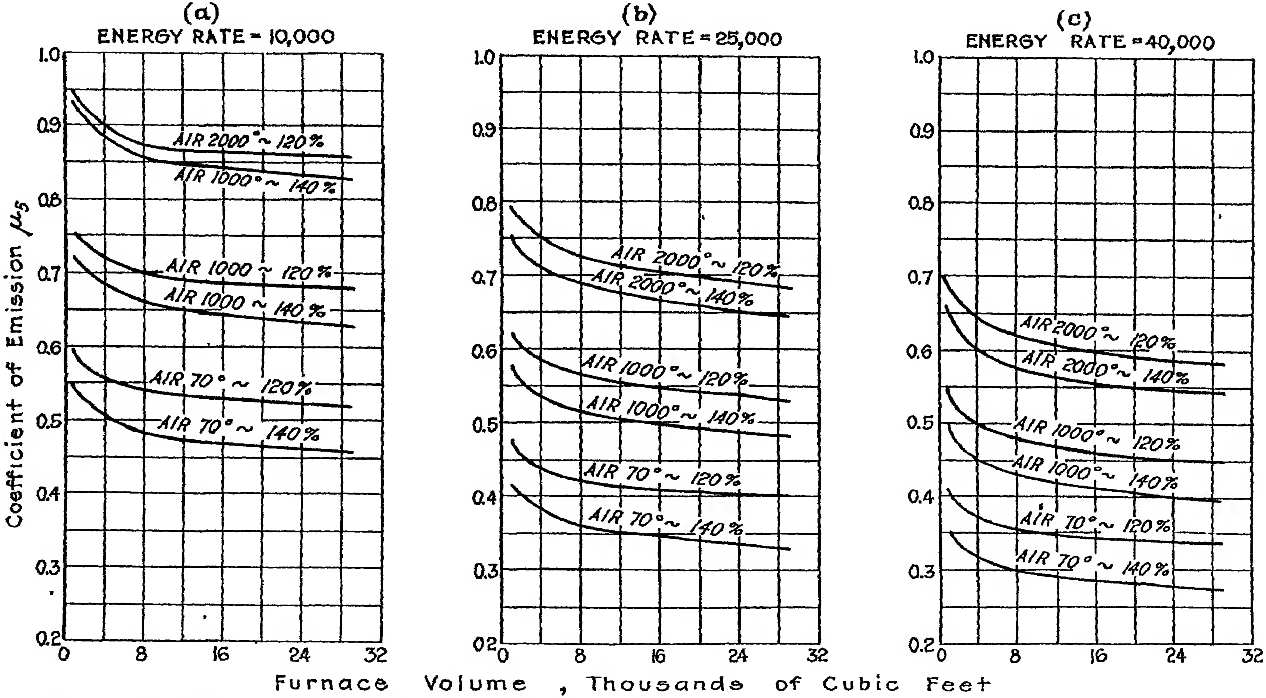


FIG. 9 INFLUENCE OF FURNACE VOLUME ON COEFFICIENT OF EMISSION μ_5 FOR COAL NO. 1 PULVERIZED

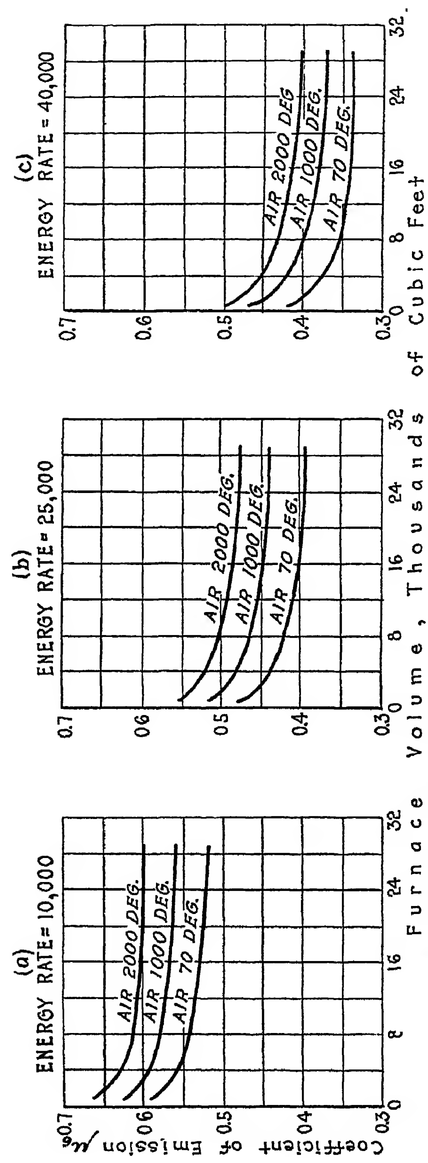


FIG. 10 INFLUENCE OF FURNACE VOLUME ON COEFFICIENT OF EMISSION μ_6 FOR COAL NO. 1 PULVERIZED; AIR, 120 PER CENT

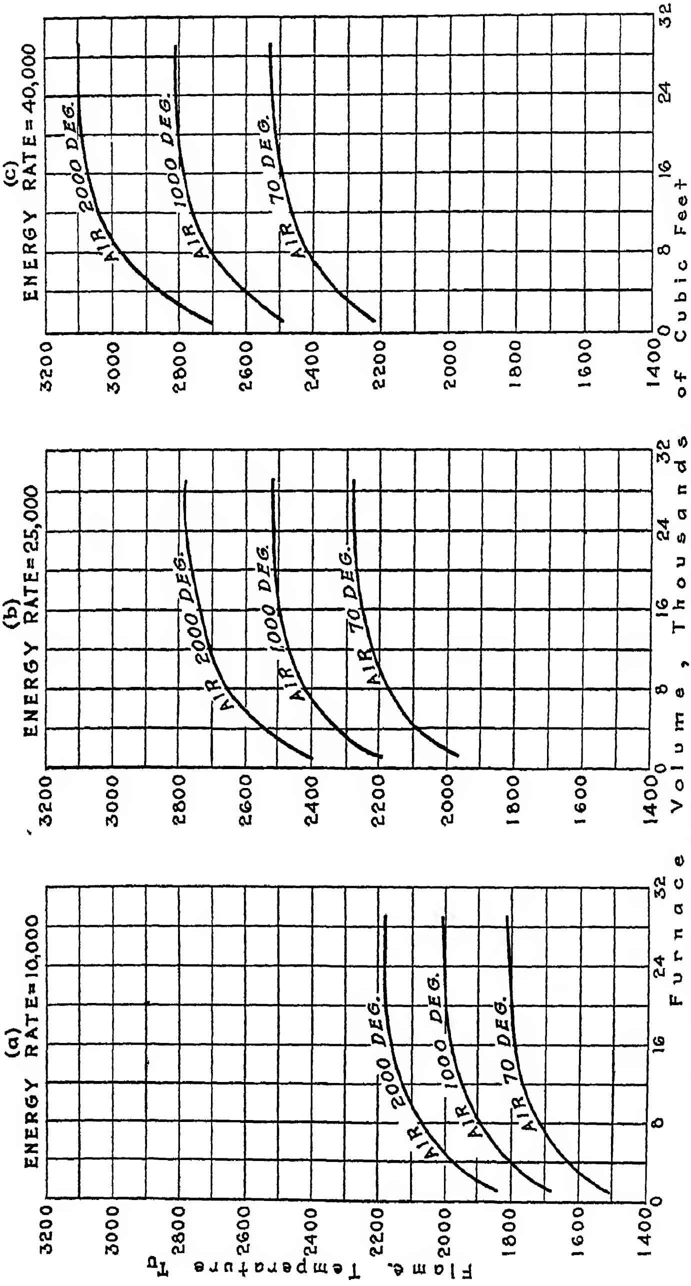


Fig. 11 Influence of Furnace Volume on Flame Temperature T_U for Coal No. 1 Pulverized; Air, 120 Per Cent

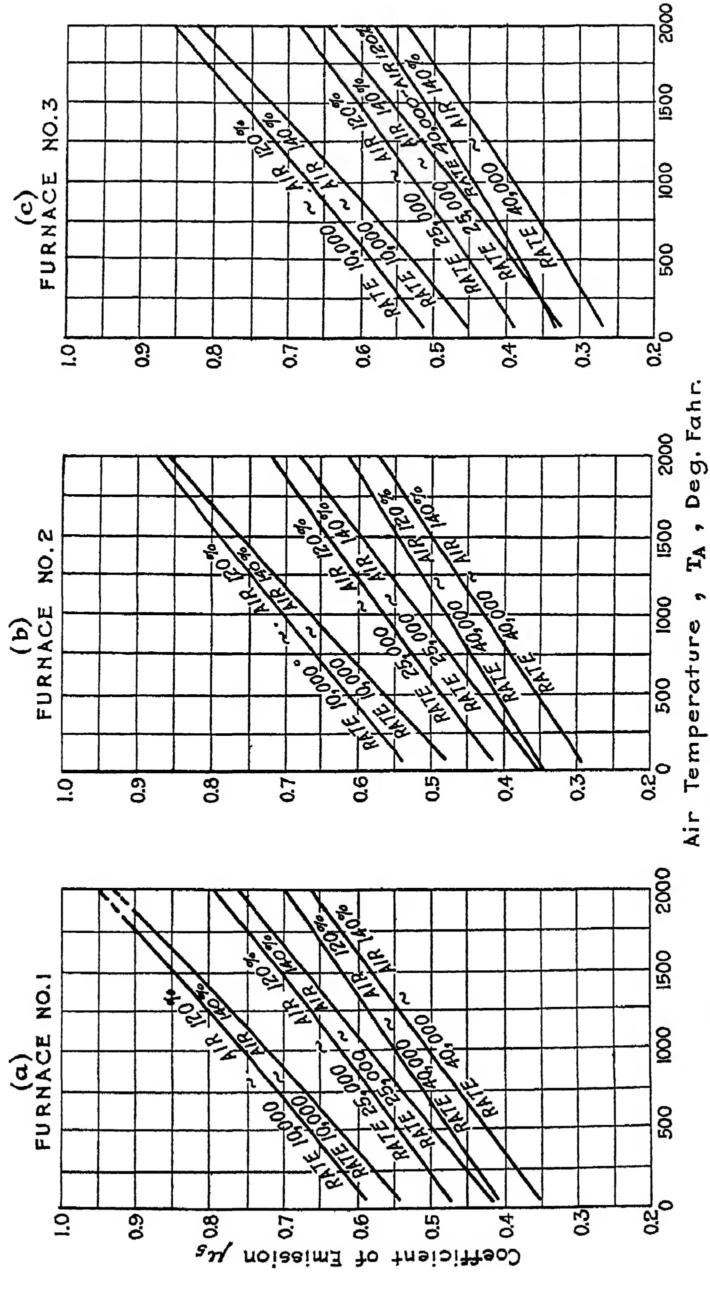


FIG. 12 INFLUENCE OF AIR TEMPERATURE T_A ON COEFFICIENT OF EMISSION μ_5 FOR COAL NO. 1 PULVERIZED

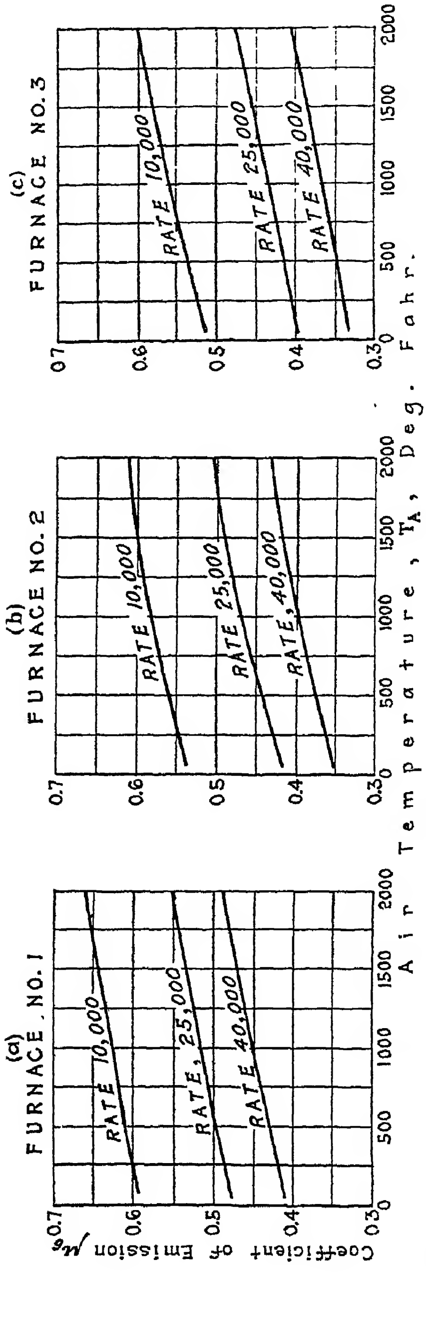


FIG 13 INFLUENCE OF AIR TEMPERATURE T_A ON COEFFICIENT OF EMISSION μ_0 FOR COAL NO. 1 PULVERIZED; AIR, 120 PER CENT

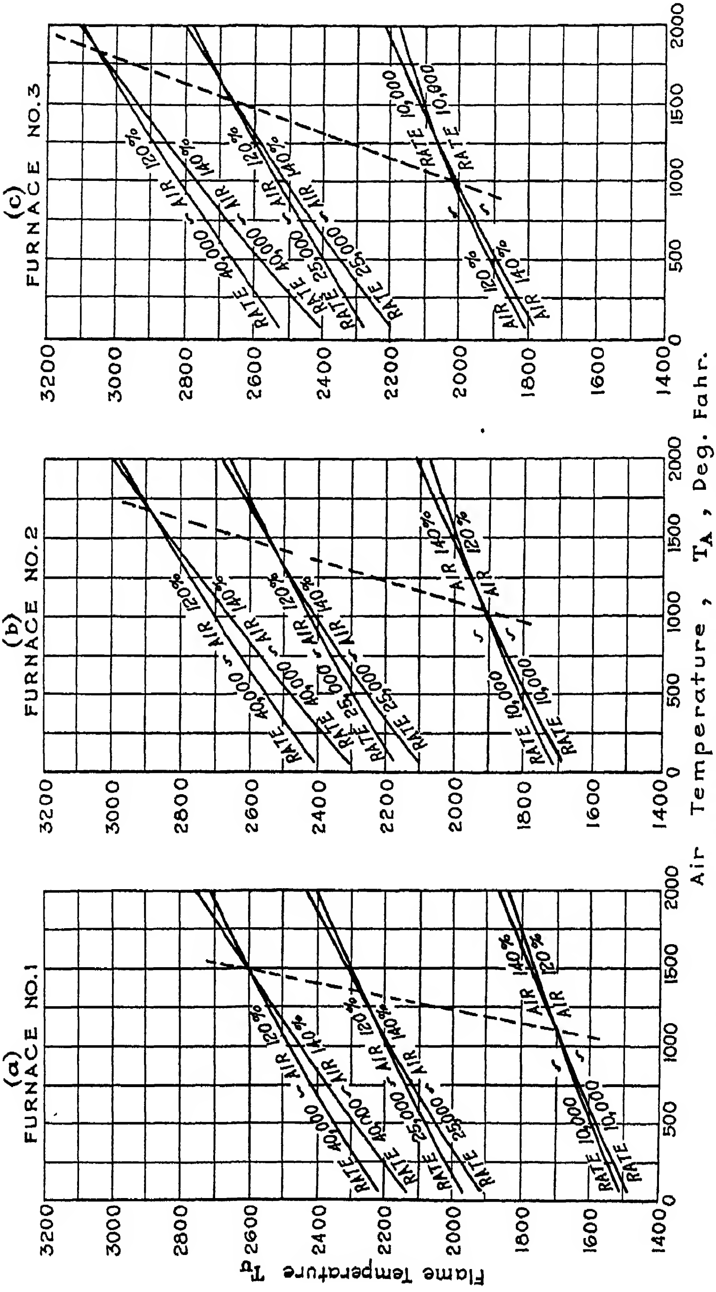


FIG. 14 INFLUENCE OF AIR TEMPERATURE T_A ON FLAME TEMPERATURE T_U FOR COAL NO. 1 PULVERIZED

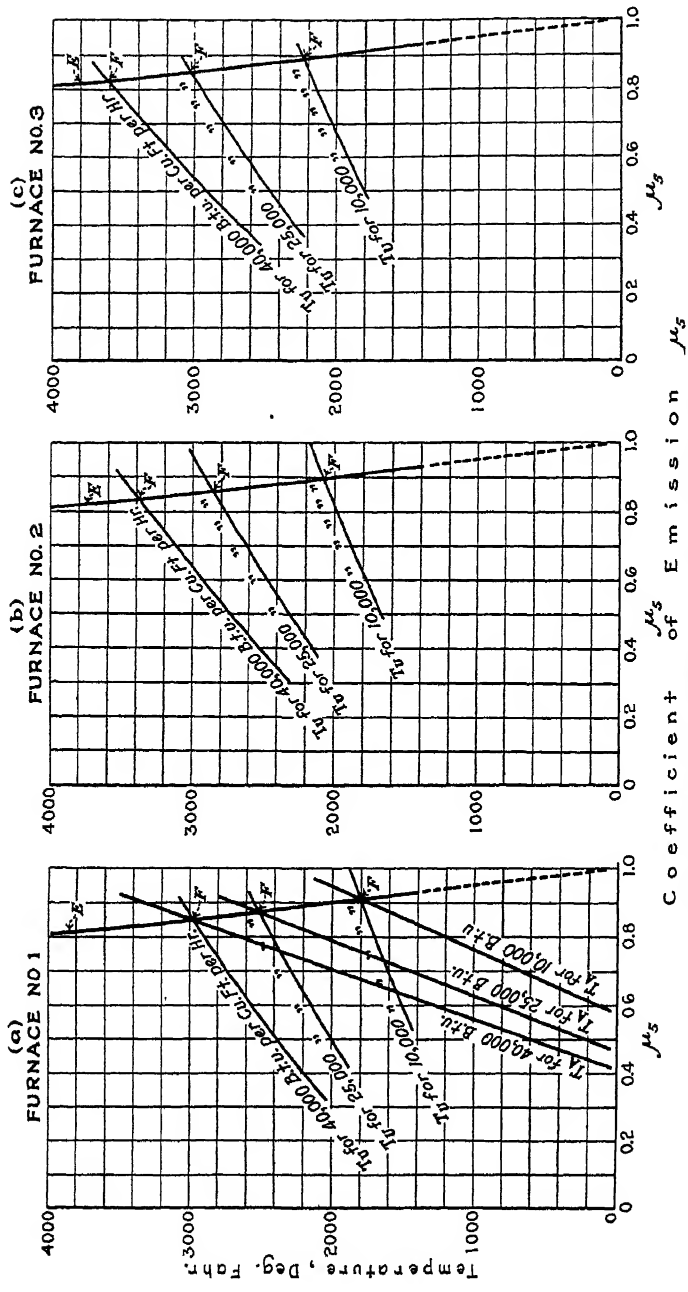


FIG. 15 COEFFICIENTS OF EMISSION μ_s WHEN $T_A = T_U$

(T_U = flame temperature; T_A = air temperature. Coal No. 1; air, 120 per cent; coal temperature at entrance = 70 deg. fahr.; Type E furnace for pulverized coal.)

at points F of the T_U and T_A curves occur at values of μ_5 below unity. A value of μ_5 at the point F is the maximum which can occur in a system in which all of the energy for preheating the air is taken from the products of combustion. If we designate it as μ_{5F}, the difference $(1-\mu_{5F})$ may be called the "retention coefficient" and represents the fraction of liberated energy of combustion which is used to raise the temperature of the coal constituents as products of combustion from entrance temperature to the temperature at point F. It may be computed as follows:

$$(1-\mu_{5F}) = \frac{(\text{B.t.u. in products per lb. coal at } T_U = T_A) - (\text{B.t.u. in air per lb. coal at } T_A = T_U)}{\text{B.t.u. per lb. coal}} \quad . \quad [2]$$

20 The curve E which passes through the points F thus represents the maximum values of the emission coefficients μ_5 for $T_A = T_U$, when coal No. 1 enters the preheated air at 70 deg. fahr. Its trend is therefore independent of the furnace type or size, and

TABLE 5 INFLUENCE OF VARIOUS FACTORS AND CONDITIONS IN THE FURNACE ON FLAME TEMPERATURE T_U AND COEFFICIENTS OF EMISSION μ_5 AND μ_6

	Influence on		
Influence of	Flame temperature T_U	Coefficient, μ_5	Coefficient, μ_6
Increase in furnace volume	Increase, Fig. 11	Decrease, Fig. 6	Decrease, Fig. 10
Increase in excess air	Decrease up to certain air temperature, then increase, Fig. 14	Decrease, Fig. 6	Decrease
Increase in energy rate	Increase, Fig. 3	Decrease, Fig. 1	Decrease, Fig. 2
Increase in air temperature	Increase	Increase, Fig. 12	Increase, Fig. 13
Increase in fraction cold	Decrease	Increase	Increase
Fuel type	Bituminous: Practically none Anthracite: Increase	Practically none, Fig. 5	Practically none

it has been plotted as the result of data computed by means of Equation [2] for all furnaces burning coal No. 1 with 120 per cent air when the coal is introduced at 70 deg. fahr.

21 It is noticed that on the right side of curve E the flame temperatures fall below the air temperatures T_A, indicating that if the air temperature from some outside source should be raised above the temperature indicated at points F, the flame temperature in the furnace would actually be lower than the temperature of the incoming air. This is true because radiation is approximately proportional to the fourth power of the absolute temperature, whence above the temperatures at points F the radiation discharge will be so great that the temperature T_U will fall below the temperature T_A. A summary of the influence of the foregoing conditions may be found in Table 5.

Influence on the Total Surface Required of the Fraction of the Furnace Envelope Cold

22 In order to discuss this problem it is necessary to introduce the " coefficient of emission reference factor " γ. Concisely stated

$$\gamma_A = \frac{\mu_{5A}}{\mu_{5E}} = \frac{\mu_{6A}}{\mu_{6E}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

in which μ_{5A} and μ_{6A} represent the emission coefficients μ_5 and μ_6 for the type A furnace and μ_{5E} and μ_{6E} the emission coefficients

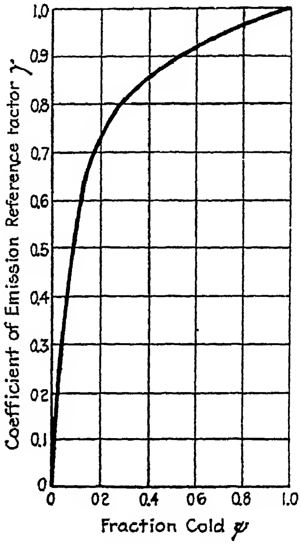


Fig. 16 Influence of the Fraction Cold Ψ on the Coefficient of Emission Reference Factor γ

(Coal No. 1 pulverized; air, 120 per cent at 70 deg. fahr.; furnace No. 2; energy rate = 25,000 B.t.u.)

μ_5 and μ_6 of the type E furnace, the energy rate, fuel, air quantity, air temperature, furnace volume, and furnace shape being the same in both cases. Similarly

$$\gamma_B = \frac{\mu_{5B}}{\mu_{5E}}; \qquad \gamma_D = \frac{\mu_{5D}}{\mu_{5E}}; \qquad \text{etc.} \; . \; . \; . \; . \; . \; . \quad [4]$$

The factor γ thus represents the relation between the energy emitted in a given furnace cavity for given combustion conditions for a particular fraction cold ($\Psi <$ unity) of envelope and that emitted in the same furnace cavity when Ψ is unity and other conditions are the same.

23 If γ is plotted against the fraction cold Ψ the result is a curve of which that shown in Fig. 16 is typical. Inspection of

this curve shows that the value of γ does not decrease in proportion to the decrease in Ψ. It follows that, depending upon the particular conditions, there may be a certain fraction cold Ψ which will yield for a given final gas temperature the least amount of total surface. The value of Ψ with which this result is realized may be called the most effective fraction cold.

24 A convenient basis on which to compare results in this respect is furnished if in all cases we compute the surface necessary to reduce the gas temperature to that temperature T_E of the gases leaving the type E furnace ($\Psi = 1.00$) when they have

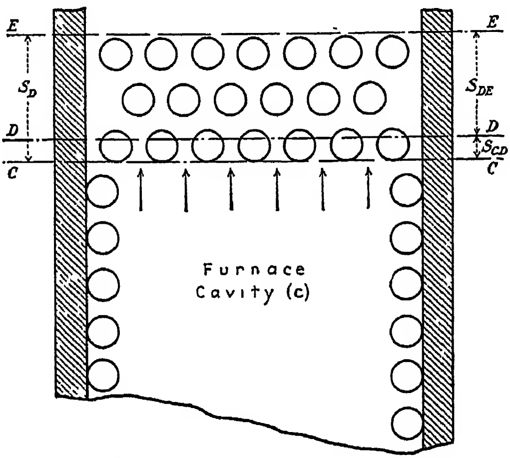


FIG. 17 DATUM PLANES IN PATH OF GAS TRAVEL

Tube surface between planes E-E and C-C = S_D, the total convection surface required to cool gases to temperature T_E. Plane E-E coincides with plane D-D when $\psi = 1$. When $\Psi < 1$, E-E is beyond D-D.

Tube surface between planes E-E and D-D = S_{DE}, the extra convection surface required to reduce gas temperature to T_E when Ψ is less than unity.

Tube surface between planes D-D and C-C = S_{CD}.

Furnace No. 1, S_{CD} = 100 sq. ft.; furnace No. 2, S_{CD} = 400 sq. ft.; furnace No. 3, S_{CD} = 900 sq. ft.

traversed an amount of cold surface S_{CD} (Fig. 17) in the aperture equal to the area of a plane section covering the cold face at the aperture. For a reduced fraction cold Ψ, other conditions being the same, the gases will not be cooled to the temperature T_E until they have traversed the extra convection surface S_{DE}. If now the cold surface in the furnace is considered as flat plates projected on the faces at which they are placed, and S_C represents the combined interior plane areas of surfaces projected in this manner, then the total effective surface required to reduce the temperature of the gases to T_E is equal to

$$S_{CE} = S_C + S_{DE} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

There may be some criticism of the unit adopted for measuring the surface S_C in the furnace proper. For computing radiation quantities, whatever surface is present must, however, be reduced to these terms, and it seems desirable, therefore, to state the surfaces in terms of such projected areas. If now a given type of construction makes it necessary to include two or three square feet of surface in the furnace for each square foot of projected area this will, of course, have to be charged into the cost, but it must be considered in the energy problem of the furnace as one square foot.

25 The surface S_D as shown in Fig. 17 represents the total required convection surface, beginning at the aperture, to cool the gases from the flame temperature to the temperature T_E. The surface S_{DE} then is equal to the difference $(S_D - S_{CD})$. It is to be noted that the surface S_{CD} within the aperture absorbs a large amount of heat both by radiation and by convection as the gases escape. The following additional terms are introduced also in discussing this phase of the problem. The symbol η_{CE} represents the net energy emission in furnace and in convection zone in cooling the gases to the temperature T_E when expressed as a fraction of the liberated energy. The symbol θ_E represents the energy remaining in the gases, expressed likewise as a fraction of the liberated energy, and Φ is the relation between the energy above atmospheric temperature brought into the furnace by the preheated air and the energy liberated by the fuel. For zero preheat, Φ is zero. In general:

$$\Phi = \frac{Q_A}{Q_U} = \frac{\text{B.t.u. brought in by air}}{\text{Liberated energy}} \quad . \; . \; . \; . \; . \quad [6]$$

$$(1+\Phi) = \frac{Q_{UA}}{Q_U} = \frac{\text{Total input to furnace}}{\text{Liberated energy}} \quad . \; . \; . \; . \; . \quad [7]$$

and

$$(1+\Phi-\theta_E) = \eta_{CE} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [8]$$

26 In Figs. 18 to 23 the total effective surface S_{CE} required to bring the gases to the temperature T_E is plotted against the fraction cold Ψ as the abscissa. On the same charts are shown curves of the surface S_C and the extra convection surface S_{DE} plotted against the same abscissa. For a given value of Ψ the S_{CE} ordinate is thus the sum of the S_C and S_{DE} ordinates and it is to be noted that when a minimum exists for a value of Ψ less than unity, it always falls directly above the intersection of the S_C and S_{DE} curves. At this point S_C equals S_{DE}. When a minimum condition exists, therefore, for values of Ψ less than unity, that is for other than an all-cold furnace envelope, *the extra convection surface, as above defined, which is necessary to cool the gas temperature — from the flame temperature T_U to the temperature*

T_E—*is exactly equal to the projected cold surface in the furnace as above defined.* It should be noted that all the curves on these Figs. 18 to 23 are based on the (γ, Ψ) relation as developed for the cubical furnace, a factor which of course must be considered in the use to which the information is put. Special cases are discussed in Appendix No. 7.

27 The dotted curves shown on these charts, as distinguished from the solid-line curves, represent conditions for a convection heat-transfer coefficient $2z$ just twice the corresponding coefficient z used in computing the latter. The values used for these coefficients are stated in the lower tables shown above the curves. The flame temperatures for *A*, *B*, *D* and *E* furnaces are also shown. These are the furnaces which were computed in arriving at the results. The upper table includes in each case, as shown, the θ_E, n_{CE}, and T_E values for the solid-line curves (coefficient $= z$) and dotted-line curves (coefficient $= 2z$). The values for the coefficient z (Fig. 28) are no doubt low, as later explained. The values $2z$, however, are high, whence inspection of the figures shows the whole probable range through which the most effective fraction cold Ψ may vary.

28 Since the same surface is required to cool the gases from temperature T_E to any lower temperature T_N, other conditions being the same, ordinate values taken from curves S_{CE} will at once indicate, by differences, what influence the fraction cold Ψ has on the total surface requirements. Thus in Fig. 18*b* it is shown that the least total surface is required with Ψ equal to 0.39 for the solid-curve conditions, and to 0.25 for the dotted curves. If for the solid-curve conditions Ψ is reduced to 0.1, it will require $(665-470) = 195$ sq. ft. additional surface. The variation in flame temperature with Ψ for this condition may be ascertained by reference to the T_U column of the lower table.

29 Even for large furnaces, if high rates of heat transfer exist in the convection zone, there is, as shown on Figs. 19 and 20, a minimum surface requirement for Ψ less than unity. It should be noted, however, for moderate rates of heat transfer in the convection zone, that the increase in total surface requirements, because of decreasing the fraction cold, assumes considerable proportions as Ψ is decreased below 0.2.

30 Fig. 21 shows that an increase of the air temperature does not greatly alter the conditions discussed above, except that about the same total surface is required (solid curves) for values of Ψ between 0.5 and 1.0. As before, values of Ψ below 0.2 result in a considerable addition to the surface requirements.

31 When comparing stokers and pulverized coal, Figs. 22 and 23, it is seen that, other conditions being the same, the stoker requires about the same fraction cold, but, for the higher rates of energy liberation which usually exist, the required surface

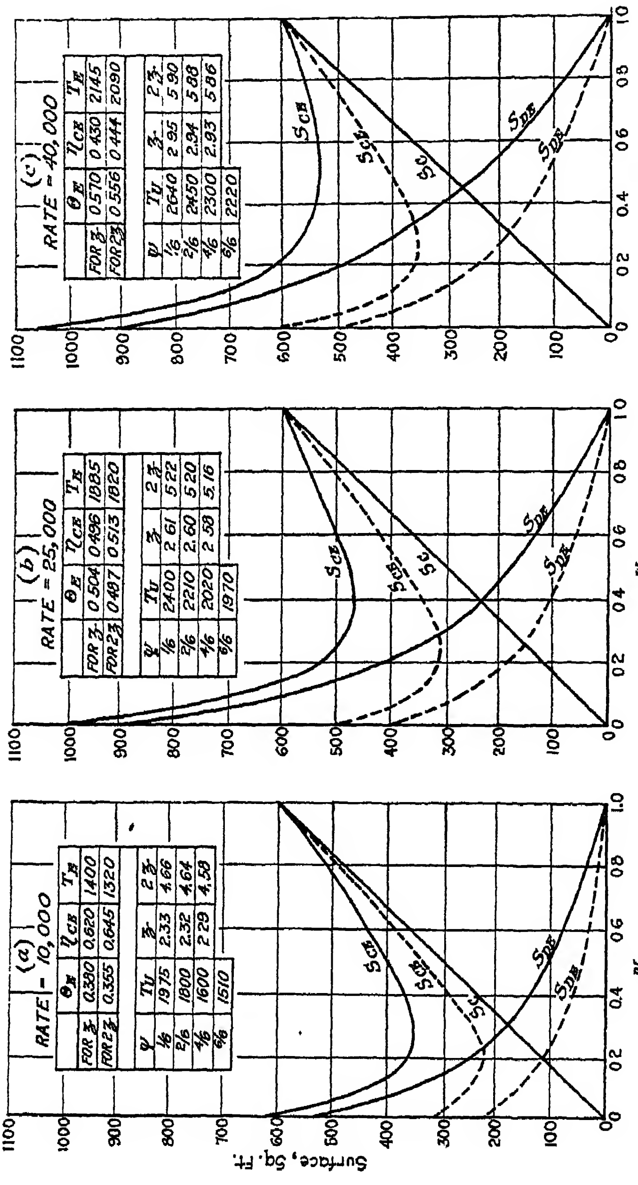


FIG. 18 INFLUENCE OF ENERGY-LIBERATION RATE ON TOTAL SURFACE REQUIRED TO REDUCE GASES TO TEMPERATURE T_E

(Pulverized coal; furnace No. 1; coal No. 1; air, 120 per cent; air temperature = 70 deg. fahr.)

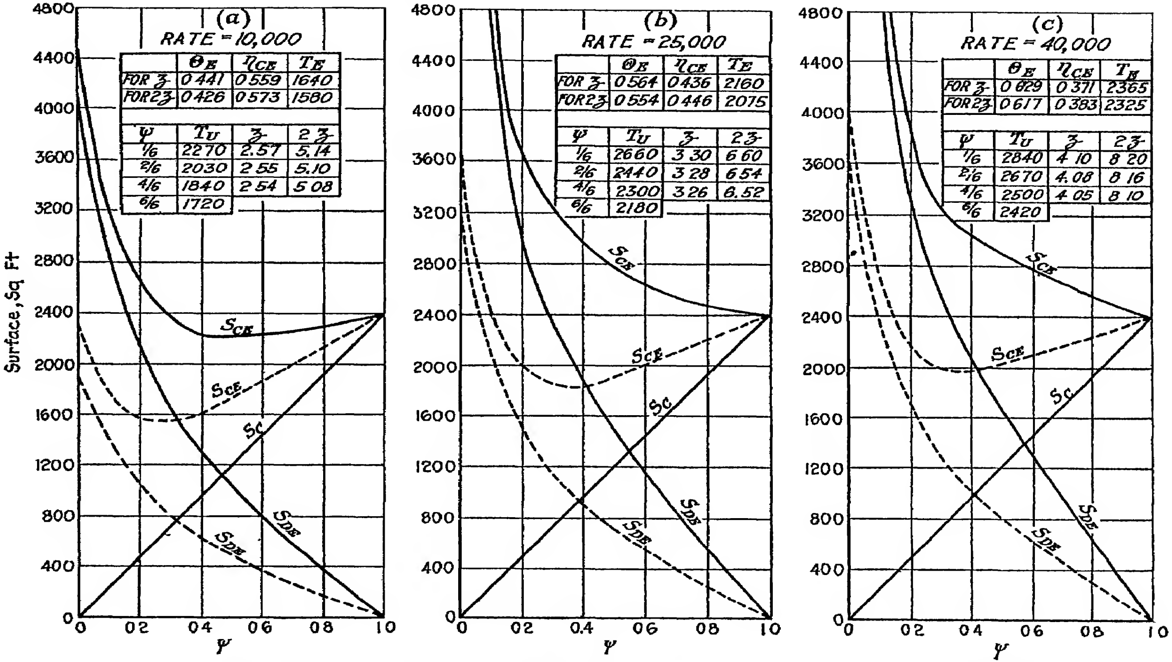


Fig. 19 Influence of Energy-Liberation Rate on Total Surface Required to Reduce Gases to Temperature T_E

(Pulverized coal; furnace No. 2; Coal No. 1; air, 120 per cent; air temperature = 70 deg. fahr)

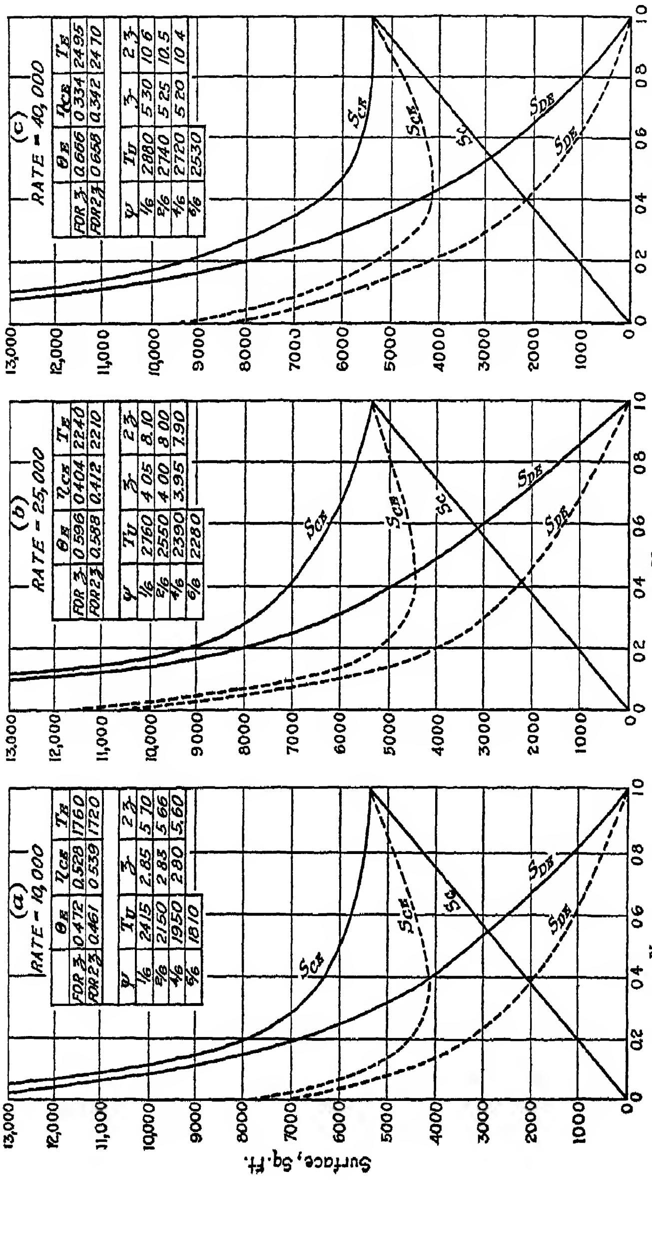


FIG. 20 INFLUENCE OF ENERGY-LIBERATION RATE ON TOTAL SURFACE REQUIRED TO REDUCE GASES TO TEMPERATURE T_E

(Pulverized coal; furnace No 3; coal No. 1; air, 120 per cent; air temperature = 70 deg. fahr)

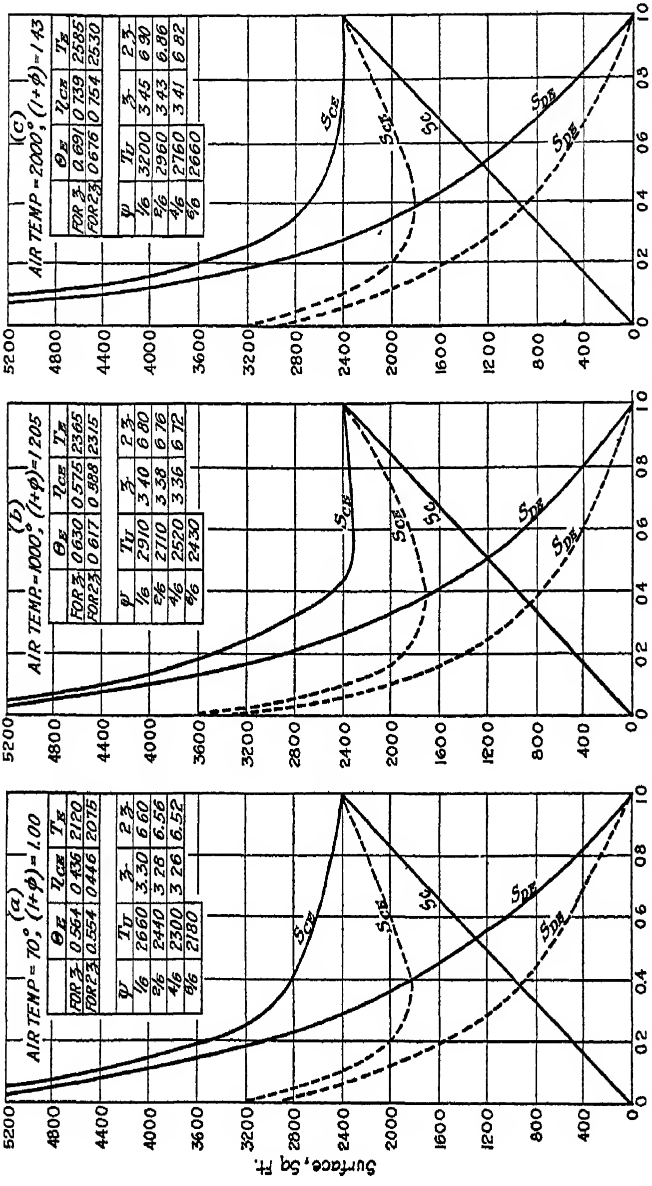


FIG. 21 INFLUENCE OF AIR TEMPERATURE ON TOTAL SURFACE REQUIRED TO REDUCE GASES TO TEMPERATURE T_E

(Pulverized coal; furnace No 2; coal No. 1; air, 120 per cent; rate = 25,000 B.t.u.)

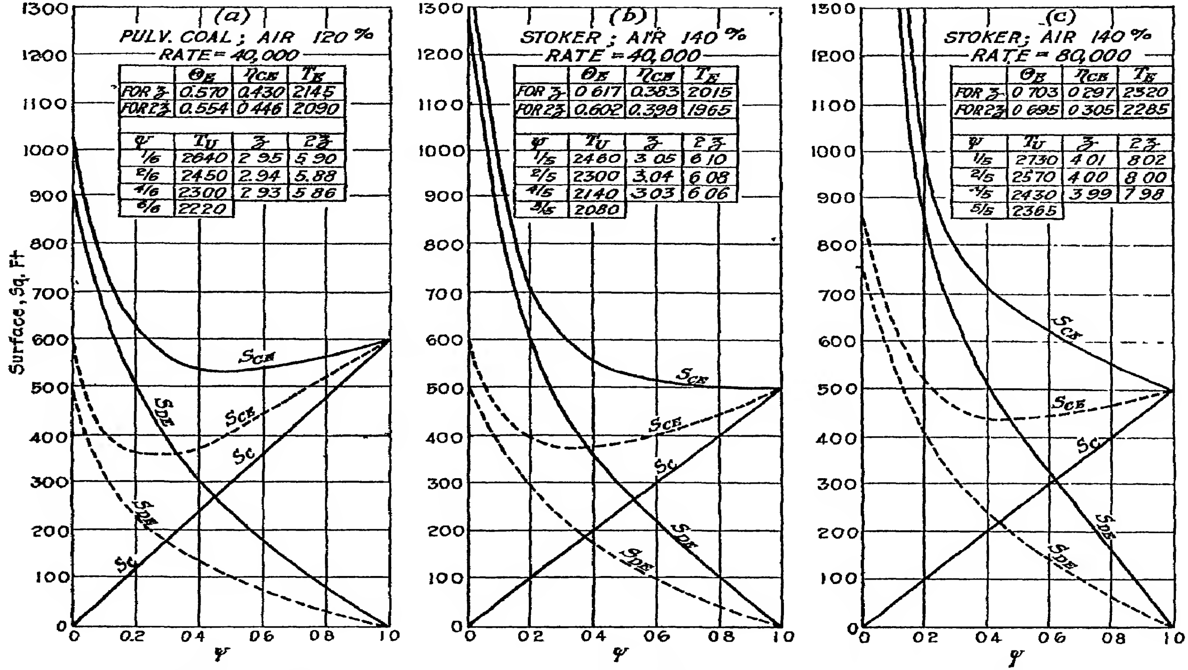


Fig. 22 Comparison of Total Surface Required with Stokers and with Pulverized Coal to Reduce Gases to Temperature T_E

(Furnace No. 1; coal No. 1; air temperature = 70 deg. fahr.)

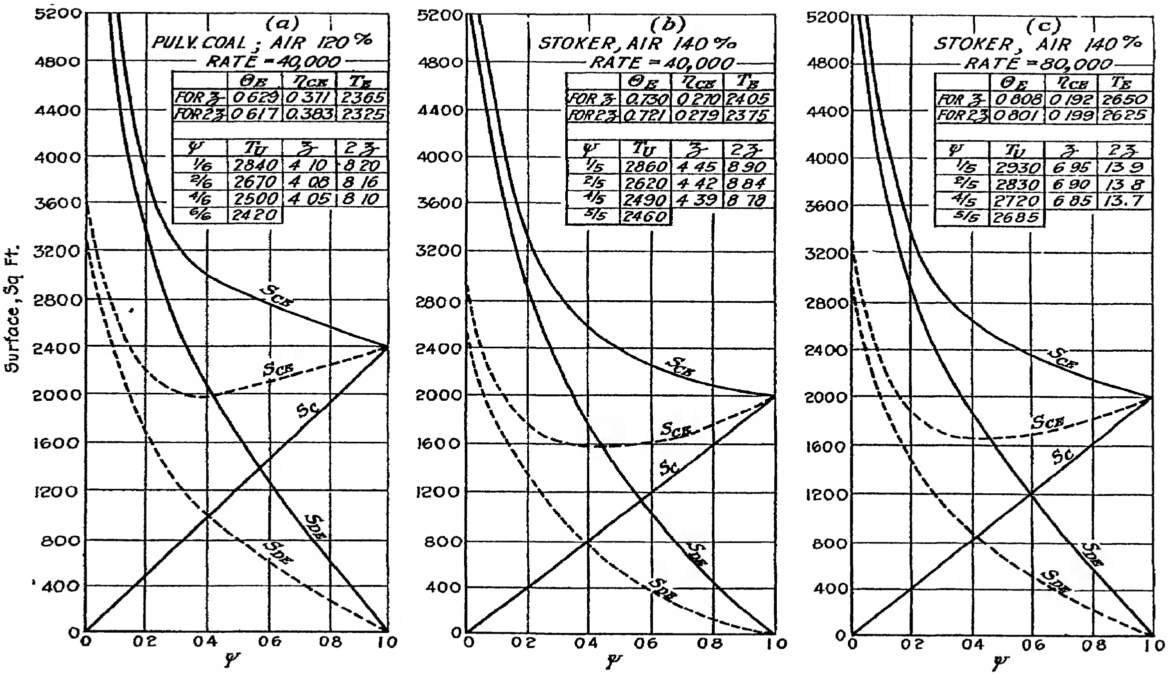


FIG. 23 COMPARISON OF TOTAL SURFACE REQUIRED WITH STOKERS AND WITH PULVERIZED COAL TO REDUCE GASES TO TEMPERATURE T_E

(Furnace No. 2; coal No. 1; air temperature = 70 deg. fahr.)

eventually may be materially reduced by increasing the fraction cold.

Influence of Air Preheat on Surface Requirements and Distribution

32 For a complete solution the problem of influence of air preheat must be treated in each instance as a special case, inasmuch as surface arrangements, flow areas, and a number of other factors enter into it. If, however, the discussion is limited to an investigation of the relative magnitudes of the fractional parts of the total energy which must be absorbed in the different sections of the steam generator, including the air preheater, the results become more or less generally significant. If, in addition, the necessary mean temperature differences between gases and surfaces are evaluated, a very fair basis is established for comparison under different conditions.

33 Such an analysis has been included in Appendix No. 6. The results show that as the air temperature is increased, a larger and larger fraction of the total energy input must be absorbed in the air preheater at a smaller difference in temperature between the gases and the surfaces at which the energy is absorbed. At the same time the fraction of the liberated energy which is absorbed in the furnace rises very materially, whence the necessary boiler surface interposed between the furnace and air preheater is very greatly reduced. Eventually, for high air temperatures, this part of the surface may be reduced to nothing more than that required for superheating the steam when superheated steam is required.

34 Preheating the air therefore reduces the requirements in pressure-vessel surface, but the total surface requirements are increased, and very considerably so, for high air temperatures. It is apparent, therefore, that the cost of a steam-generating system for a given amount of steam will first decrease as the air temperature is increased, because of the predominance of reduction of cost in pressure-vessel surface and relatively smaller influence of increasing low-cost air-preheating surface. For higher air temperatures the necessary increase in air-preheater surface becomes so large as to more than offset the lower cost of pressure vessels. It appears, however, considering the beneficial effect on combustion, and providing the ash fusion temperature is high enough to permit it, that it may prove economical to operate with air temperatures up to and perhaps even exceeding 1000 deg. fahr.

PART II GENERAL METHOD

35 Since by means of the modified method the net energy emission in any furnace may be referred to that in a type E furnace (see Par. 22), the solution for any particular case may

always begin by solving for the flame temperature T_U which establishes equilibrium in the furnace considered as surrounded by cold walls.

The General Energy Equation

36 The energy diagram for such a furnace is represented by Fig. 24. In this diagram Q_U represents energy liberated by the fuel; Q_A, energy brought in by the preheated air; $Q_{UA} = (Q_U + Q_A)$, the total input to the furnace; $\mu_5 Q_U = \mu_6 Q_{UA}$, energy emitted in the furnace; and $Q_B = (Q_{UA} - \mu_6 Q_{UA})$ the energy remaining in the gases as they escape from the furnace.

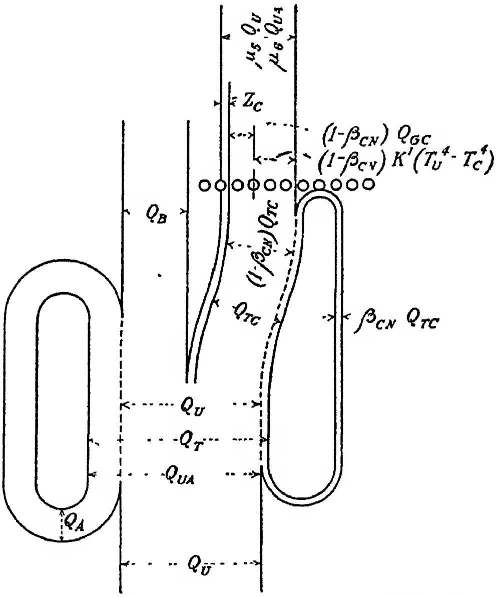


Fig. 24 Energy Diagram for Type *E* Furnace

37 The total radiation from the fuel and flame mass is designated as Q_{TC}. Of this the part $\beta_{CN} Q_{TC}$ is recirculated by reflection at the cold surface which thereby serves to increase the flame temperature T_U. Thus β_{CN} denotes the coefficient of net reflection at the cold surface whence the total absorption from radiation may be expressed as $(1-\beta_{CN})Q_{TC}$. This quantity is divided into two parts such that

$$(1-\beta_{CN})Q_{TC} = (1-\beta_{CN})Q_{GC} + (1-\beta_{CN})K'(T_U^4 - T_C^4) \quad . \quad [9]$$

The terms Q_{GC} and $K'(T_U^4 - T_C^4)$ represent the total emission by radiation from the gaseous and solid radiating fuel masses respectively. If to these is added the energy Z_C, transferred by

convection at the cold furnace walls, exclusive of that forming the aperture, the sum

$$(1-\beta_{CN})Q_{GC}+(1-\beta_{CN})K'(T_U^4-T_C^4)+Z_C = \mu_5 \cdot Q_U = \frac{dQ_F}{dt} \quad . \quad [10]$$

represents the net emission in the type E furnace.

38 The general energy equation applicable to the furnace when preheated air is used may be stated in the form:

$$\frac{dQ_U}{dt}+\frac{dQ_A}{dt}=\frac{dQ_B}{dt}+\frac{dQ_F}{dt} \quad . \; . \; . \; . \; . \; . \; . \quad [11]$$

in which dQ_U/dt, dQ_R/dt, dQ_B/dt, and dQ_F/dt represent the time rates of the energy quantities involved. Thus

$$\left.\begin{array}{l} dQ_U/dt = G_U \cdot U = \text{lb. of fuel per hr. } (G_U) \times \text{B.t.u. per lb. of fuel } (U) \\ dQ_A/dt = G_A \cdot \Delta h_A = \text{B.t.u. per hr. input above atmospheric temperature by } G_A \text{ lb. air per hr entering at temperature } T_A \\ dQ_B/dt = \Sigma G_B \Delta h_B = \text{B.t.u. per hr. absorbed in raising } \Sigma G_B \text{ lb. of gas per hr. from atmospheric temperature to mean flame temperature } T_U, \text{ the symbol } \Sigma \text{ denoting the several constituents} \end{array}\right\} \quad [12]$$

and dQ_F/dt = as defined by Equation [10].
By combining [10], [11], and [12] the energy equation appears as

$$G_U \cdot U+G_A \cdot \Delta h_A = \Sigma G_B \cdot \Delta h_B+(1-\beta_{CN})Q_{GC} + (1-\beta_{CN})K'(T_U^4-T_C^4)+Z_C \quad . \quad [13]$$

for which the quantities Q_{GC}, K', and Z_C require further resolution in order to effect a solution. Before taking up the application of the general equations to particular systems, however, it is desirable to discuss in somewhat more detail the methods of evaluating the radiation from gases and the energy transferred by convection.

Radiation Intensity from Gaseous Masses

39 In the paper by Wohlenberg and Morrow the radiation from gases was assumed to obey the well-known Stefan-Boltzman fourth-power law. Experimental evidence shows that energy radiated from gases resulting in the constant-pressure combustion process, does not vary as the fourth-power law, which means that for reliable results, the use of this law necessitates an adjustment of the radiation coefficient for any considerable range in conditions.

40 It has long been known that the radiation emitted in furnace gases is chiefly due to carbon dioxide and water vapor. The amounts of radiation from each gas are found, by examination of their spectra, to lie in three bands of wave lengths Dr. Schack, employing this information, determined the intensity of radiation in each wave length for the given temperature of the gas and integrated these over the width of the band, thus determining the gross emission from the gas. This emission is found to be a function of the percentage by volume content of the gas, thickness of the gas column, and its temperature. In following the development of this method by application in a considerable number of cases, Dr. Schack has found it to yield exceedingly good results. It appears, therefore, that this method of dealing with radiations from the gases will yield better information than does the method employed in the former paper, and for this reason it has been adopted here. A discussion of Schack's method is found in Appendix No. 5.

41 By Schack's method, the *radiation intensity* q_{GC}, in B.t.u. per square foot of radiating gaseous surface per hour, can conveniently be plotted against the temperature T_U. Such curves, based, however, on the receiving surface, are later included in Fig. 31.

Heat Transfer by Convection Z_C at the Cold Furnace Walls

42 The principal difficulty in evaluating the heat transfer by convection Z_C at the cold furnace walls term lies in a proper determination of the mean temperature of the gases which sweep over the furnace walls. However, by assuming the furnace to be large enough so that the flames do not actually come in contact with cold walls, it is possible, with what experimental data are available on explorations of the furnace cavity for the temperature variations, to arrive at a curve showing approximately what the expectations are in this respect. Such a curve is shown later in Fig. 32.

43 With a knowledge of the quantity of gas passing through the furnace and the temperature differences in the vicinity of the wall, between the gases and the wall, it now becomes possible to fix the value of the coefficient of heat transfer z. The area of the cold walls is then the only remaining factor which, with the others, will yield the heat transfer by convection Z_C.

Application of General Formulas for Type E Furnace to Particular Cases

44 *Pulverized Coal.* For convenience in calculations the equilibrium Equation [**13**], as applicable to type E furnaces, is now stated in the final form

$$
\begin{aligned}
&(G_U \cdot U + G_A \cdot \Delta h_A) \\
&\quad = \Sigma G_B \cdot \Delta h_B + \{I_G(1-I_S)S'_U\}\{\alpha'_U(1-\beta_{CN})[T_U{}^4 - T_C{}^4]\} \\
&\qquad + \{(1-I''_S) \cdot S''_{UC}\}\{(1-\beta_{CN})q_{GC}\} \\
&\qquad + R_Z(T_U - T_C) \cdot z(S_C - S_{CD}) \quad \ldots\ldots\ldots\ldots \quad [14]
\end{aligned}
$$

It is to be noted that the index prime as distinguished from the index double prime is used to designate quantities concerned with radiation from solids, whereas the latter is used to designate those concerned with radiation from gaseous masses. The several coefficients are defined in the following:

I_G = radiation penetration coefficient through the gaseous media

I_S = mean shielding coefficient of solid particles for each other

S'_U = total surface area of particles in suspension at any instant

$[I_G(1-I_S)S'_U]$ = the equivalent effective radiating area of solids

α'_U = radiating coefficient at the fuel surface when radiating through an unclouded medium

β_{CN} = net reflection coefficient at cold surface

$[\alpha'_U(1-\beta_{CN})(T_U{}^4-T_C{}^4)]$ = absorption intensity at receiving surface with radiating fuel bed immediately in front of it. For coal radiating surfaces α'_U should be taken equal to $\alpha_0 = 1.62 \times 10^{-9}$ = coefficient for black body

I''_S = mean shielding coefficient by the suspended particles for the gaseous radiation

S''_{UC} = square feet of flame surface which can "see" the cold surface

$[(1-I''_S)S''_{UC}]$ = equivalent effective radiating surface of the gaseous masses

q_{GC} = gaseous radiation intensity through a perfectly clear flame to a black body absorber, B.t.u. per sq. ft. per hr.

$[(1-\beta_{CN})q_{GC}]$ = radiation absorption intensity at cold surface by radiations from gases through unclouded medium, B.t.u. per sq. ft. per hr.

$$R_Z = \left(\frac{\Delta T_{GC}}{T_U - T_C}\right) = \frac{\text{mean temperature difference at cold walls between gases and walls}}{\text{difference between mean flame temperature and cold-wall temperature}}$$

z = coefficient of heat transfer, B.t.u. per sq. ft. per hr. per deg. fahr.

$(S_C - S_{CD})$ = cold surface at walls in furnace exclusive of that at the aperture.

45 For a given temperature T_C, the magnitudes of heat capacities Δh_B, coefficients I_G, I_S, and I''_S, surfaces S'_U and S''_{UC}, and radiation intensity q_{GC} all depend on the temperature T_U. The solution for T_U is therefore the result of a series of trials. In solving the equation a preliminary value of T_U is assumed

TABLE 6 VALUES OF G_UU AND $G_A\Delta h_A$ FOR ENERGY-LIBERATION RATE = 10,000 B.T.U. PER CU. FT. PER HR

Coal Furnace Air excess	G_U	G_UU, thousands	Lb air per lb. coal	G_A	$G_A\Delta h_A$, thousands B t.u. for air temperature shown 500° F.	1000° F.	1500° F.	2000° F.
1-1-20	569	7,290	11 58	6,590	687	1,496	2,327	3,143
1-1-40	......	......	13.51	7,684	799	1,745	2,714	3,666
1-2-20	5,356	68,590	11.58	62,000	6,446	14,080	21,890	29,580
1-2-40	......	.. ...	13 51	72,360	7,522	16,430	25,570	34,570
1-3-20	19,010	243,389	11.58	220,800	22,960	51,400	77,960	105,400
1-3-40	......	......	13.51	257,400	26,780	60,800	90,880	122,800
2-1-20	502	7,290	13 10	6,578	684	1,493	2,322	3,138
2-1-40	......	......	15.28	7,676	798	1,743	2,710	3,662
2-2-20	4,730	68,590	13.10	61,960	6,442	14,070	21,870	29,550
2-2-40	......	......	15.28	72,300	7,520	16,410	25,520	34,500
2-3-20	16,830	243,389	13 10	220,500	22,930	51,200	77,800	105,200
2-3-40	......	......	15.28	257,300	26,770	58,400	90,840	122,700
3-1-20	501	7,290	13.90	6,960	724	1,580	2,458	3,320
3-1-40	......	......	16.22	8,122	845	1,845	2,868	3,875
3-2-20	4,720	68,590	13.90	65,600	6,820	14,900	23,170	31,600
3-2-40	......	......	16.22	76,560	7,960	17,380	27,030	36,500
3-3-20	16,800	243,389	13.90	233,600	24,290	53,000	82,440	111,400
3-3-40	.. ...	......	16 22	272,600	28,340	61,860	96,200	130,000
	(B.t.u. per lb. air above 70°) — Δh_{70} =				104	227	353	477

TABLE 7 VALUES OF G_UU AND $G_A\Delta h_A$ FOR ENERGY-LIBERATION RATE = 25,000 B.T.U. PER CU. FT. PER HR.

Coal Furnace Air excess	G_U	G_UU, thousands	Lb. air per lb. coal	G_A	$G_A\Delta h_A$, thousands B.t.u. for air temperature shown 500° F.	1000° F.	1500° F.	2000° F.
1-1-20	1,423	18,230	11.58	16,500	1,716	3,746	5,824	7,872
1-1-40	......	......	13.51	19,230	2,000	4,367	6,790	9,176
1-2-20	13,390	171,500	11.58	155,200	16,140	35,230	54,800	74,000
1-2-40	......	......	13.51	180,900	18,810	41,090	63,840	86,240
1-3-20	47,630	608,500	11.58	552,000	57,400	125,300	194,800	263,300
1-3-40	......	......	13.51	643,500	66,920	146,000	227,200	307,000
2-1-20	1,257	18,230	13.10	16,470	1,713	3,740	5,816	7,860
2-1-40	......	......	15.28	19,200	1,996	4,360	6,780	9,160
2-2-20	11,820	171,500	13.10	154,800	16,100	35,140	54,660	73,840
2-2-40	......	......	15.28	180,600	18,780	41,000	63,800	86,160
2-3-20	42,050	608,500	13.10	550,800	57,300	125,000	194,500	262,800
2-3-40	......	......	15.28	642,400	66,820	145,900	226,900	306,600
3-1-20	1,254	18,230	13 90	17,430	1,813	3,959	6,156	8,320
3-1-40	......	......	16.22	20,340	2,116	4,620	7,180	9,700
3-2-20	11,800	171,500	13 90	164,000	17,060	37,220	57,920	78,200
3-2-40	......	......	16.22	191,400	19,900	43,480	67,600	91,260
3-3-20	41,980	608,500	13.90	583,400	60,660	132,400	205,900	278,200
3-3-40	......	......	16.22	681,000	70,800	154,600	240,400	325,000
	(B.t.u. per lb. air above 70°) Δh_A =				104	227	353	477

for which all of the depending quantities are solved. Such values are substituted in Equation [14] which, when the input (left-hand side) is equal to the output (right-hand side), checks the assumed trial value of T_U. The solution for T_U is then complete and correct in all its details for the assumed furnace conditions

46 To reduce the labor required in computation, Tables 6 to 9 and the curves of Figs. 25 to 33 are included. From these, values for the several coefficients, intensities, and other quantities may be directly obtained when the conditions of the problem have been stated. The heat-capacity curves, Figs. 25 to 28, are based on the data included in Appendix No. 1 of Wohlenberg and Morrow and need no further comment. The coefficients I_G and $(1-I''_S)$, Fig. 30, have practically equal values for coals No. 1 and No. 2, but for coal No. 3, as shown, separate curves must be plotted. Of course a variation in the particle size from that assumed will influence such magnitudes. The derivation of these curves is discussed in Appendix No. 4. The curves of radiation

TABLE 8 VALUES OF $G_U U$ AND $G_A \Delta h_A$ FOR ENERGY-LIBERATION RATE = 40,000 B.T.U. PER CU. FT PER HR.

Coal Furnace Air excess	G_U	$G_U U$, thousands	Lb air per lb. coal	G_A	$G_A \Delta h_A$, thousands B.t.u. for air temperature shown: 500° F.	1000° F.	1500° F.	2000° F.
1-1-20	2,277	29,160	11.58	26,360	2,742	5,982	9,300	12,580
1-1-40	......	......	13.51	30,740	3,198	6,980	10,860	14,660
1-2-20	21,420	274,300	11.58	248,000	25,800	56,300	87,560	118,300
1-2-40	......	......	13.51	289,400	30,100	65,700	102,300	138,100
1-3-20	76,040	973,600	11.58	883,000	91,820	200,500	311,700	421,100
1-3-40	......	......	13.51	1,030,000	107,200	227,000	363,700	491,400
2-1-20	2,010	29,160	13.10	26,300	2,736	5,970	9,280	12,550
2-1-40	......	......	15.28	30,700	3,192	6,970	10,830	14,650
2-2-20	18,910	274,300	13.10	247,800	25,770	56,240	87,450	118,200
2-2-40	......	......	15.28	289,200	30,090	65,660	102,200	138,000
2-3-20	67,300	973,800	13.10	882,000	91,760	200,300	311,300	420,800
2-3-40	......	......	15.28	1,028,000	107,000	233,400	363,000	490,400
3-1-20	2,005	29,160	13.90	27,830	2,896	6,320	9,826	13,280
3-1-40	......	......	16.22	32,500	3,380	7,380	11,480	15,500
3-2-20	18,880	274,300	13.90	262,300	27,300	59,580	92,600	125,200
3-2-40	......	......	16.22	306,200	31,850	69,500	108,200	146,200
3-3-20	67,200	973,800	13.90	934,000	97,160	212,000	329,700	445,800
3-3-40	......	......	16.22	1,091,000	113,400	247,700	385,000	520,400
		(B.t.u. per lb. air above 70°) Δh_A =			104	227	353	477

absorption intensity $(1-\beta_{ON})q_{GO}$ (Fig. 31) of coals No. 1 and No. 2 are applicable to any fuels whose products contain 10 per cent of CO_2 (by volume) or more and 5 per cent or more of water vapor when the thickness of the gas column is ten feet or more. The curve for coal No. 3 is applicable to any fuel whose products contain 10 per cent of CO_2 or more by volume in a column 5 feet thick or more when there is no water vapor present. The derivation of these curves is explained in Appendix No. 5.

47 The heat-transfer curves, Fig. 33, are based on the Babcock and Wilcox experiments.[1] To account approximately for turbulence the equivalent mass-flow rate at the walls has been taken as that which would result if all of the gases were made to flow through an area equal to one-half of the cross-sectional area of the flame. See Appendix No. 1, Par. 63. This part of the analysis

[1] Experiments on The Rate of Heat Transfer from a Hot Gas to a Cooler Metallic Surface. Babcock & Wilcox Company.

probably involves the greatest error, but fortunately the energy quantities involved are of relatively small importance.

48 By application of equilibrium Equation [14] together with the foregoing tables and curves, solutions for T_U may be

TABLE 9 VALUES OF I_s AND S'_U

Coal Furnace Air excess	λ	I_s	Γ^2_{3F}	S'_U	Coal per hour G_U: 10,000 B.t.u per cu. ft	25,000 B t.u. per cu. ft.	40,000 B.t.u. per cu. ft.
1-1-20	3.32	$\frac{289.1}{T_U} - \frac{70{,}700}{T_U^2}$	81	$\frac{255{,}300}{T_U}$	569	1,423	2,277
1-1-40	2.888	$\frac{251.6}{T_U} - \frac{53{,}600}{T_U^2}$	81	$\frac{222{,}200}{T_U}$	.....	......	....
1-2-20	7.02	$\frac{611.6}{T_U} - \frac{316{,}300}{T_U^2}$	361	$\frac{2{,}408{,}000}{T_U}$	5,356	13,390	21,420
1-2-40	6.10	$\frac{531.6}{T_U} - \frac{239{,}000}{T_U^2}$	361	$\frac{2{,}092{,}000}{T_U}$	.....	.....	......
1-3-20	10.70	$\frac{932.0}{T_U} - \frac{737{,}000}{T_U^2}$	841	$\frac{8{,}550{,}000}{T_U}$	19,060	47,630	76,220
1-3-40	9.31	$\frac{811.0}{T_U} - \frac{557{,}000}{T_U^2}$	841	$\frac{7{,}440{,}000}{T_U}$	......	......	......
2-1-20	3.567	$\frac{310.7}{T_U} - \frac{81{,}800}{T_U^2}$	81	$\frac{274{,}300}{T_U}$	502	1,257	2,010
2-1-40	3.088	$\frac{269.0}{T_U} - \frac{61{,}300}{T_U^2}$	81	$\frac{237{,}400}{T_U}$	......	......	......
2-2-20	7 53	$\frac{656.0}{T_U} - \frac{364{,}000}{T_U^2}$	361	$\frac{2{,}582{,}000}{T_U}$	4,730	11,820	18,910
2-2-40	6 52	$\frac{568.0}{T_U} - \frac{273{,}000}{T_U^2}$	361	$\frac{2{,}236{,}000}{T_U}$	......	.....	......
2-3-20	11.50	$\frac{1{,}002.0}{T_U} - \frac{849{,}000}{T_U^2}$	841	$\frac{9{,}190{,}000}{T_U}$	16,830	42,050	67,300
2-3-40	9 96	$\frac{868.0}{T_U} - \frac{637{,}000}{T_U^2}$	841	$\frac{7{,}960{,}000}{T_U}$	......	......	......
3-1-20	3 02	$\frac{263.0}{T_U} - \frac{58{,}600}{T_U^2}$	81	$\frac{232{,}000}{T_U}$	501	1,254	2,005
3-1-40	2.612	$\frac{227.7}{T_U} - \frac{43{,}700}{T_U^2}$	81	$\frac{201{,}000}{T_U}$	......	......	.....
3-2-20	6.378	$\frac{555.4}{T_U} - \frac{261{,}000}{T_U^2}$	361	$\frac{2{,}186{,}000}{T_U}$	4,720	11,800	18,880
3-2-40	5.516	$\frac{480.6}{T_U} - \frac{195{,}200}{T_U^2}$	361	$\frac{1{,}891{,}000}{T_U}$	......	......	.....
3-3-20	9.73	$\frac{848.0}{T_U} - \frac{607{,}000}{T_U^2}$	841	$\frac{7{,}770{,}000}{T_U}$	16,800	41,980	69,200
3-3-40	8.42	$\frac{733.6}{T_U} - \frac{455{,}200}{T_U^2}$	841	$\frac{6{,}730{,}000}{T_U}$	......	......	......

arrived at throughout a wide range of conditions. Having solved for T_U, solutions for the emission coefficients μ_5 and μ_6 follow by substitution in the fractions

$$\mu_5 = \frac{dQ_F/dt}{dQ_U/dt} \quad \ldots\ldots\ldots\ldots \quad [15]$$

and

$$\mu_6 = \frac{dQ_F/dt}{dQ_U/dt + dQ_A/dt} \quad \ldots\ldots\ldots \quad [16]$$

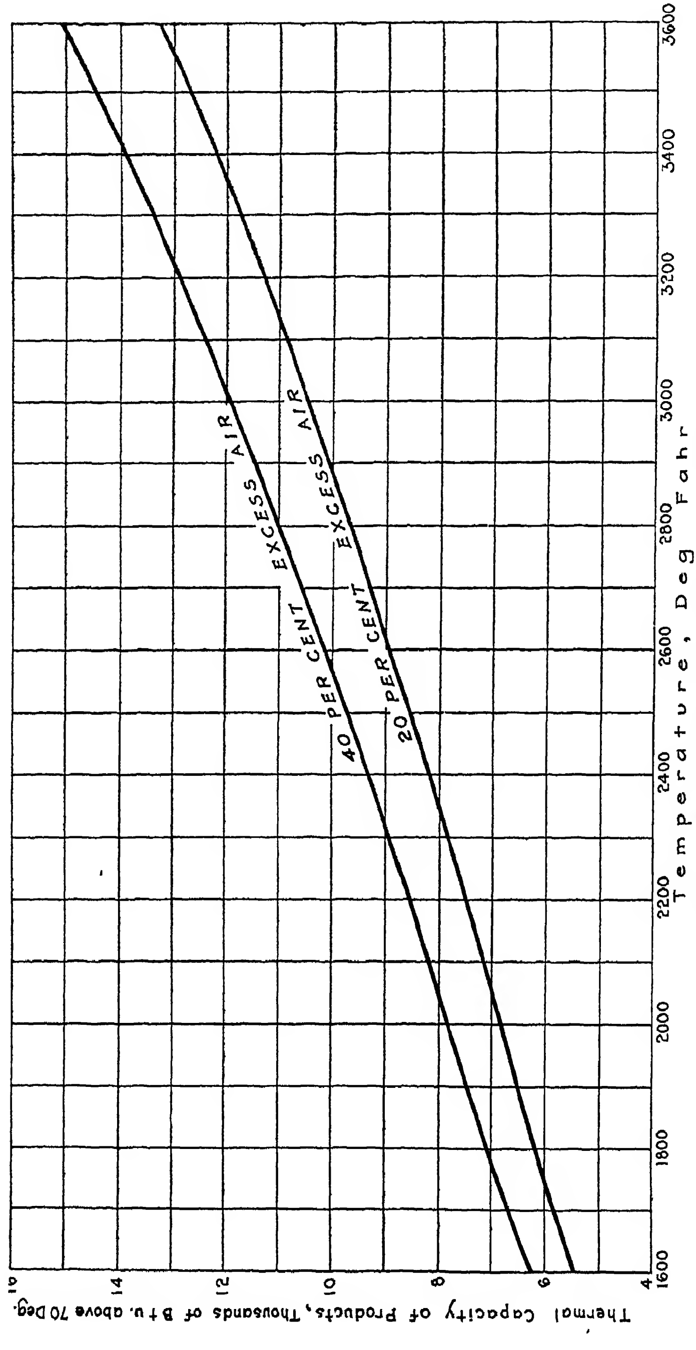


FIG. 25 THERMAL CAPACITY OF PRODUCTS OF COMBUSTION
(Saline County, Illinois, bituminous. Coal No. 1, 12,800 B.t.u.)

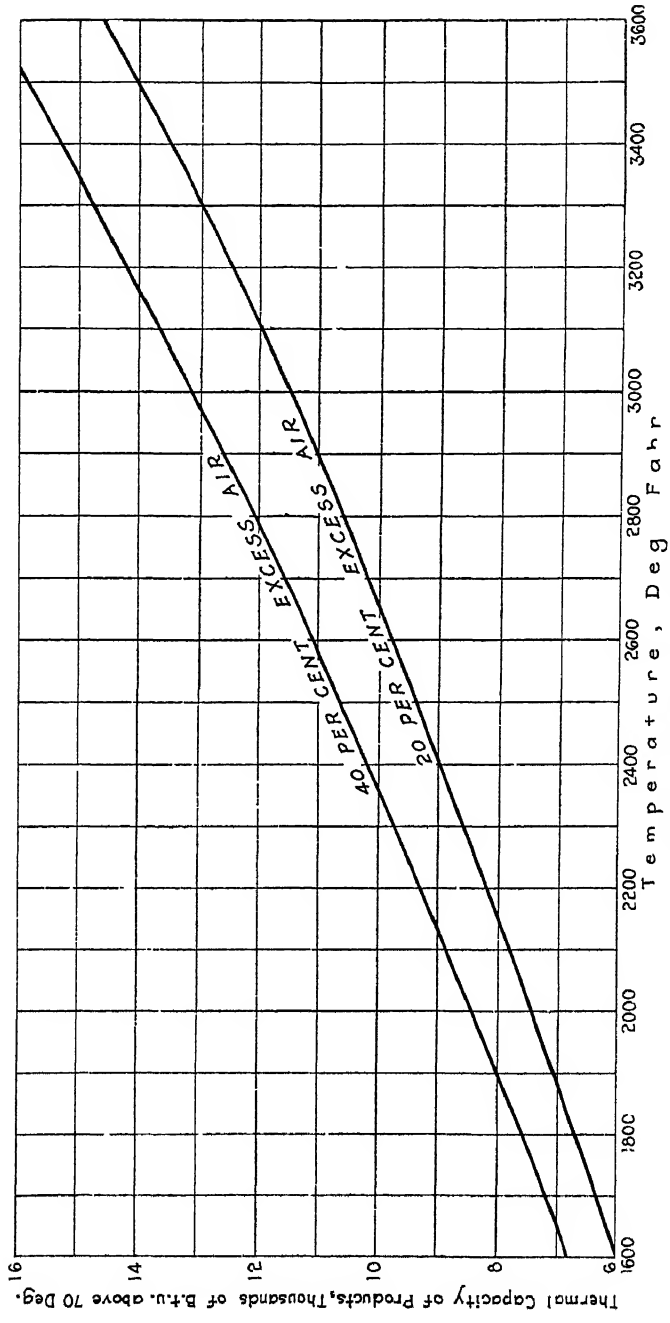


FIG. 26 THERMAL CAPACITY OF PRODUCTS OF COMBUSTION
(McDowell County, West Virginia, bituminous. Coal No. 2, 14,500 B.t.u.)

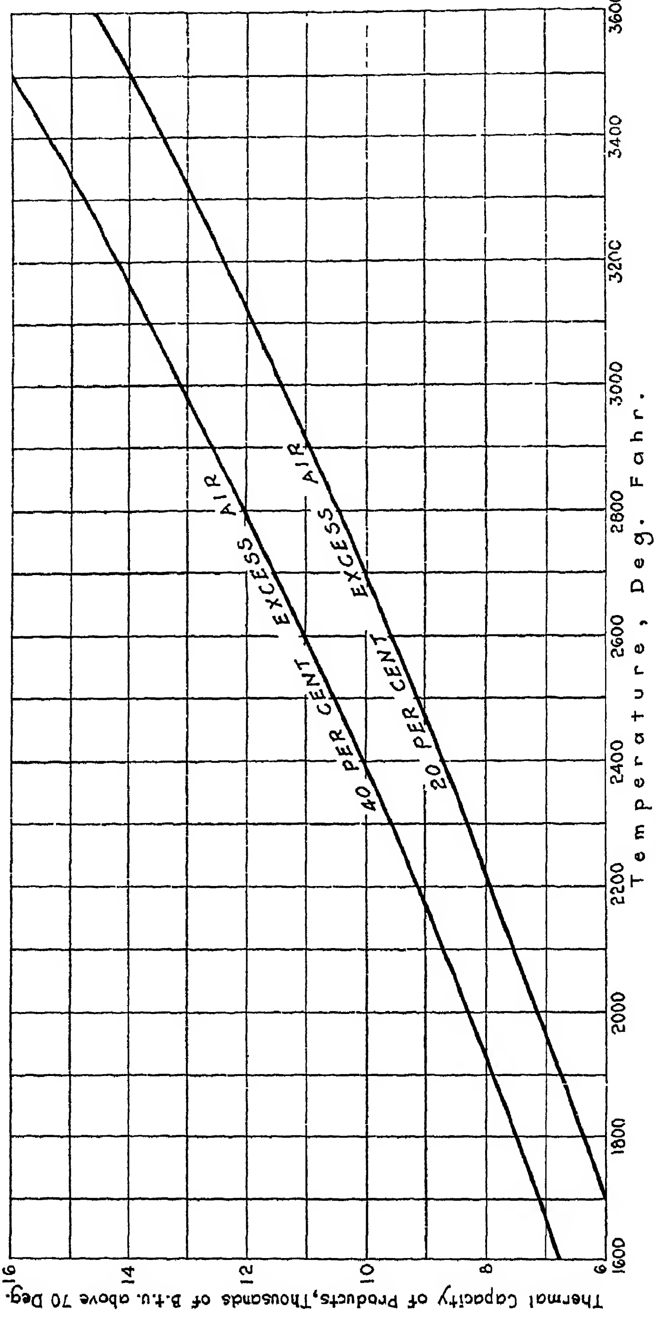


FIG. 27 THERMAL CAPACITY OF PRODUCTS OF COMBUSTION
(Amorphous carbon. Coal No. 8, 14,520 B.t.u.)

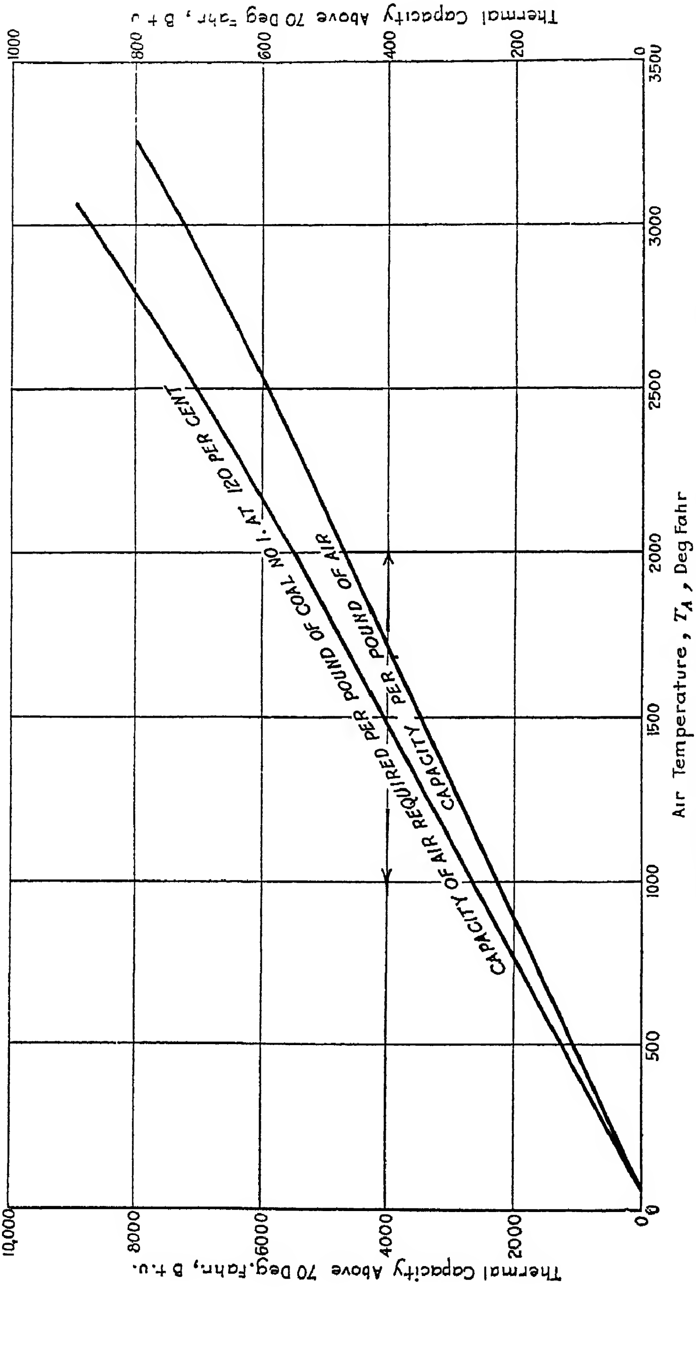


FIG. 28 THERMAL CAPACITY OF AIR IN B.T.U. ABOVE 70 DEG. FAHR.

where

$$dQ_F/dt = \{I_G(1-I_S)\cdot S'_U\}\{\alpha'_U(1-\beta_{CN})[T_U{}^4-T_C{}^4]\}$$
$$+\{(1-I''_S)S''_{UC}\}\{(1-\beta_{CN})q_{GC}\}$$
$$+R_Z[T_U-T_C]z(S_C-S_{CD}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [17]$$

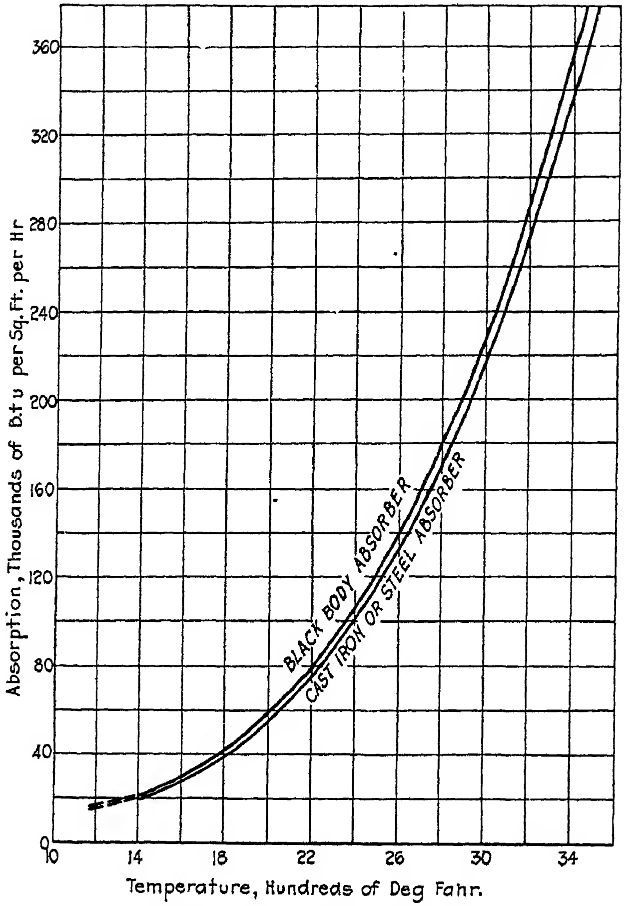


FIG. 29 RADIATION ABSORPTION AT 440 DEG. FAHR. COLD SURFACE (900 DEG. FAHR. ABS.)

From black-body radiator at fahrenheit temperature shown, $Q = a_0\ (T_U{}^4 - T_C{}^4)$.
From cast-iron or steel absorber, $Q = a_0\ (1 - \beta_{CN})(T_U{}^4 - T_C{}^4)$ $a = 1.62 \times 10^{-9}$ $(1 - \beta_{CN}) = 0.95$.
T_U = Absolute temperature of radiator.
T_C = Absolute temperature of absorber.

and in which all terms have been evaluated when the correct value of T_U has been found.

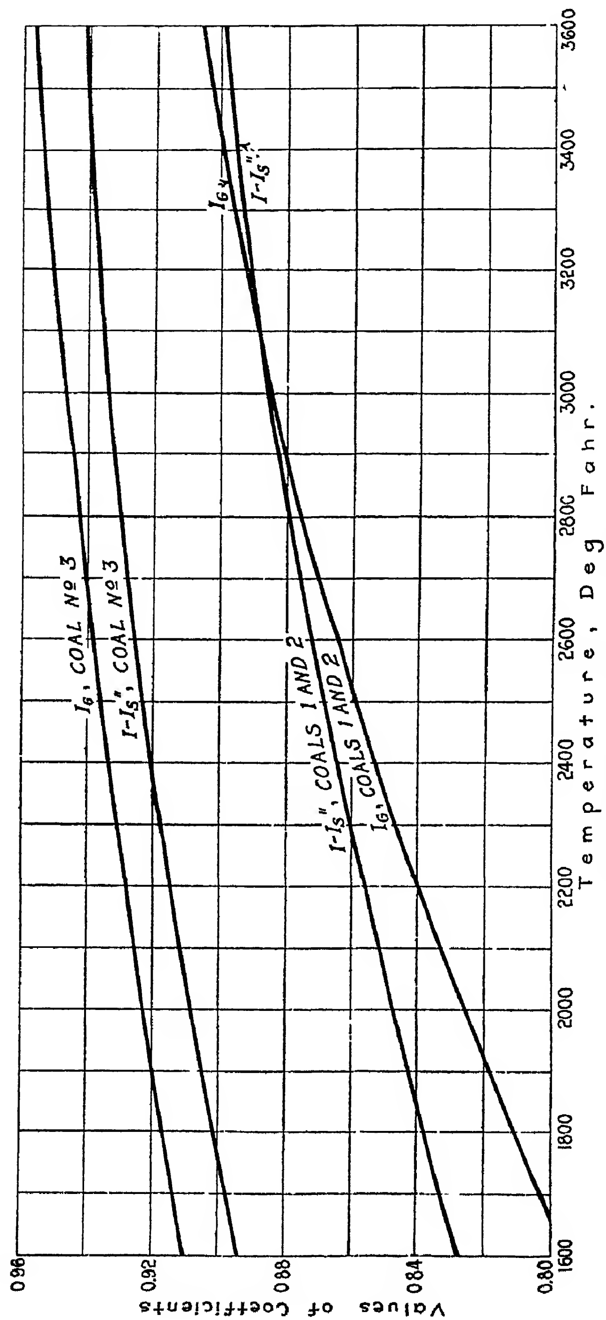


FIG. 30 COEFFICIENTS I_G AND $(1-I''_S)$ FOR COALS 1, 2, AND 3

49 *Stokers.* Since for stokers both I_S and I''_S vanish, Equation [14] assumes, for the type E furnace, the form

$$\begin{aligned}(G_U U + G_A \cdot \Delta h_A) \\ = \Sigma G_B \cdot \Delta h_B + \{I_G S'_U\}\{\alpha'_U (1-\beta_{CN})(T_U{}^4 - T_C{}^4\} \\ + S''_{UC}\{(1-\beta_{CN}) q_{GC}\} \\ + R_Z (T_U - T_C) z (S_C - S_{CD}) \quad \ldots \ldots \ldots \quad [18]\end{aligned}$$

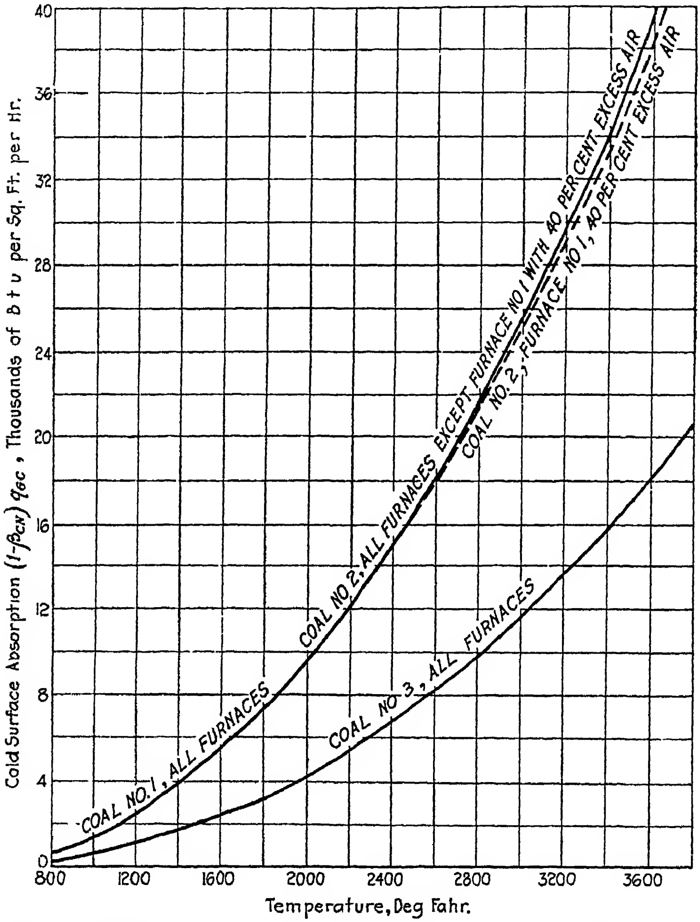


FIG. 31 RADIATION-ABSORPTION INTENSITY $(1-\beta_{CN})\, q_{GC}$ FROM FURNACE GASES THROUGH AN UNCLOUDED MEDIUM

and

$$\begin{aligned} dQ_F/dt = \{I_G S'_U\}\{\alpha'_U (1-\beta_{CN})(T_U{}^4 - T_C{}^4)\} \\ + S''_{UC}\{(1-\beta_{CN}) q_{GC}\} \\ + R_Z (T_U - T_C) z (S_C - S_{CD}) \quad \ldots \ldots \quad [19]\end{aligned}$$

The quantities shown are taken from the foregoing tables and curves, with the exception of S'_U, which now may be taken equal to that of a smooth envelope that just covers the fuel bed with no points of inflexion in its surface. In the furnaces considered this has been taken equal to the cross-sectional area at the bottom.

Solution for Furnace in Which the Fraction Cold Ψ Is Less than Unity

50 *The coefficient of emission reference factor* γ has been discussed in Par. 22. The first evaluation of this factor is accomplished by actually solving for the emission coefficients in

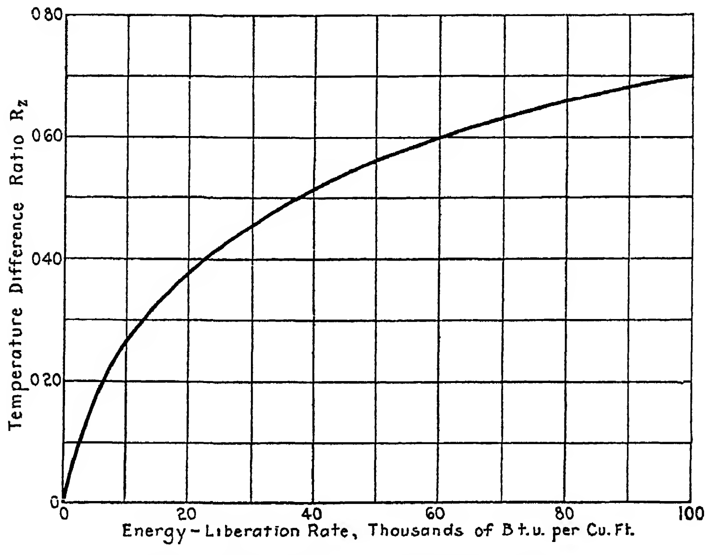


Fig. 32 Convection Temperature Difference, Gases to Cold Surface, as a Fraction of the Temperature Difference between Mean Flame and Cold Surface

furnaces having the fractions cold Ψ less than unity, and the method employed is outlined in Appendix No. 2. For the present let it be sufficient to illustrate in a general way how the factor may be used in solving furnace problems.

51 The three quantities μ_5, μ_6, and T_U are usually sought in such a case. The solution is first completed for the type E furnace for which the values μ_{5E}, μ_{6E}, and T_{UE} have been obtained. For the type A furnace with pulverized coal Ψ_A is $\frac{1}{6} = 0.167$. For the stoker $\Psi_A = \frac{1}{5} = 0.20$. For the given combustion conditions, air temperature, and furnace, the values of γ may be found directly from the curves shown in Figs. 34 to 39. For given conditions

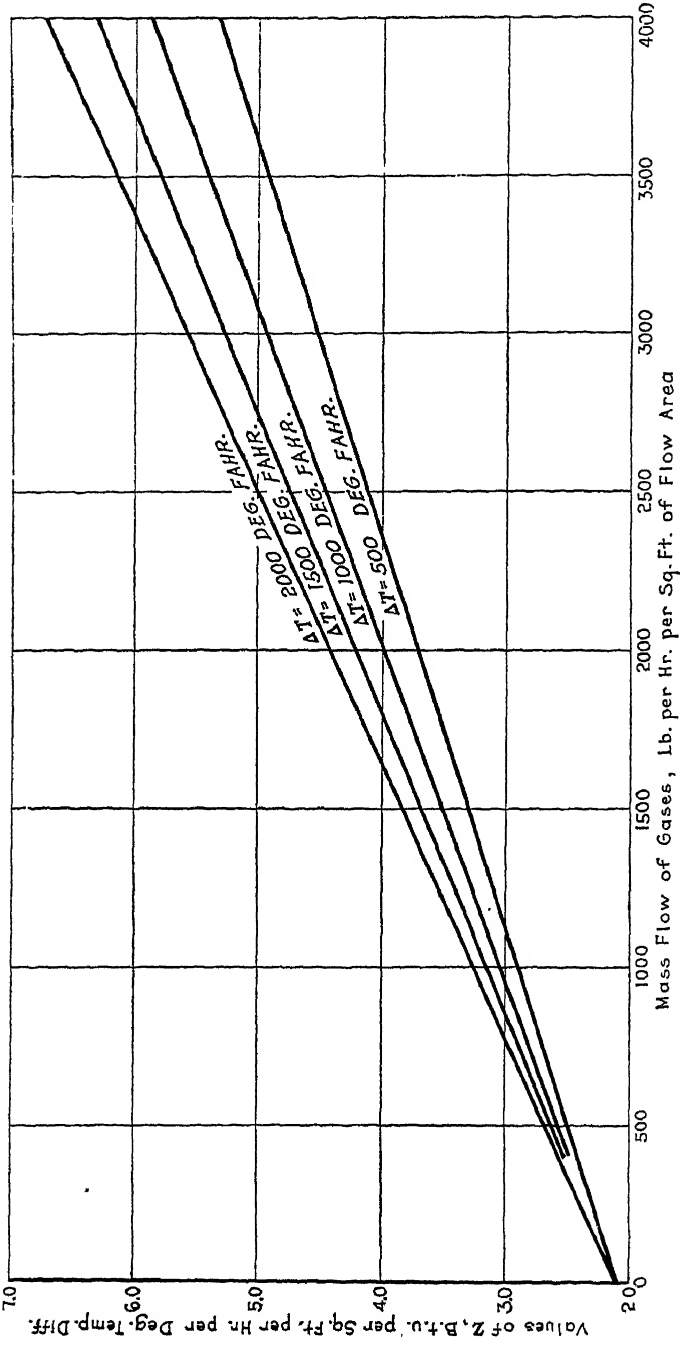


FIG. 33 CONVECTIONAL COEFFICIENT OF HEAT TRANSFER *z*
(Data from B. & W. heat-transfer experiments.)

this value may be designated as γ_A. The μ_5 and μ_6 values of the type A furnace are then obtained from the products

$$\mu_{5A} = \gamma_A \mu_{5E} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [20]$$

and

$$\mu_{6A} = \gamma_A \mu_{6E} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [21]$$

The fraction of the liberated energy which remains in the products of combustion may then be represented by the formula

$$\theta_A = (1+\Phi-\mu_{5A}) \quad . \; . \; . \; . \; . \; . \; . \; . \quad [22]$$

in which as before

$$(1+\Phi) = \frac{Q_{UA}}{Q_U} = \frac{G_U \cdot U + G_A \cdot \Delta h_A}{G_U \cdot U} \quad . \; . \; . \; . \quad [23]$$

and the thermal capacity of the gases per pound of coal equals

$$\theta_A U = H_U \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [24]$$

U as before representing the B.t.u. per lb. of coal. The product, Equation [24], is exactly the ordinate of such curves as are included in Figs. 25, 26, and 27, whence the temperature T_U is read on the abscissa for the ordinate value as above computed.

52 It is to be noted that the (γ, Ψ) curves here included have been plotted from results as computed for cubical furnaces in which coal No. 1 has been burned with 120 per cent air. For cubical furnaces, however, the curves are applicable with considerable accuracy for most bituminous coals and for air quantities between 100 and 160 per cent. For pulverized coal they may also be applied directly for any furnace cavity which does not contain partially closed-in ineffective pockets. Thus, as an example, for pulverized coal furnaces the above curves may be applied without introducing any considerable error, even though the furnace cavity should be, for instance, two or three times as high as it is wide or deep, providing the cavity is not divided into essentially distinct chambers by constrictions or necks. This, however, is not true for exceedingly high furnaces of small cross-sectional area if stoker fired, because in these the radiating solid-fuel area is spread out at one face of the furnace cavity rather than distributed throughout the whole furnace volume. For the latter cases either (γ, Ψ) curves, developed for the particular proportions of the furnace cavity under consideration, must be employed, or the value of an equivalent fraction cold or an equivalent γ must be computed. Special cases for both the pulverized-coal and stoker-fired furnace are discussed somewhat more in detail in Appendix No. 7.

Results in Tabular Form

53 Values of μ_5, μ_6, and T_U as worked out for the type E pulverized-coal furnace are included in Tables 10, 11, and 12. When applying them it should be remembered that all are based

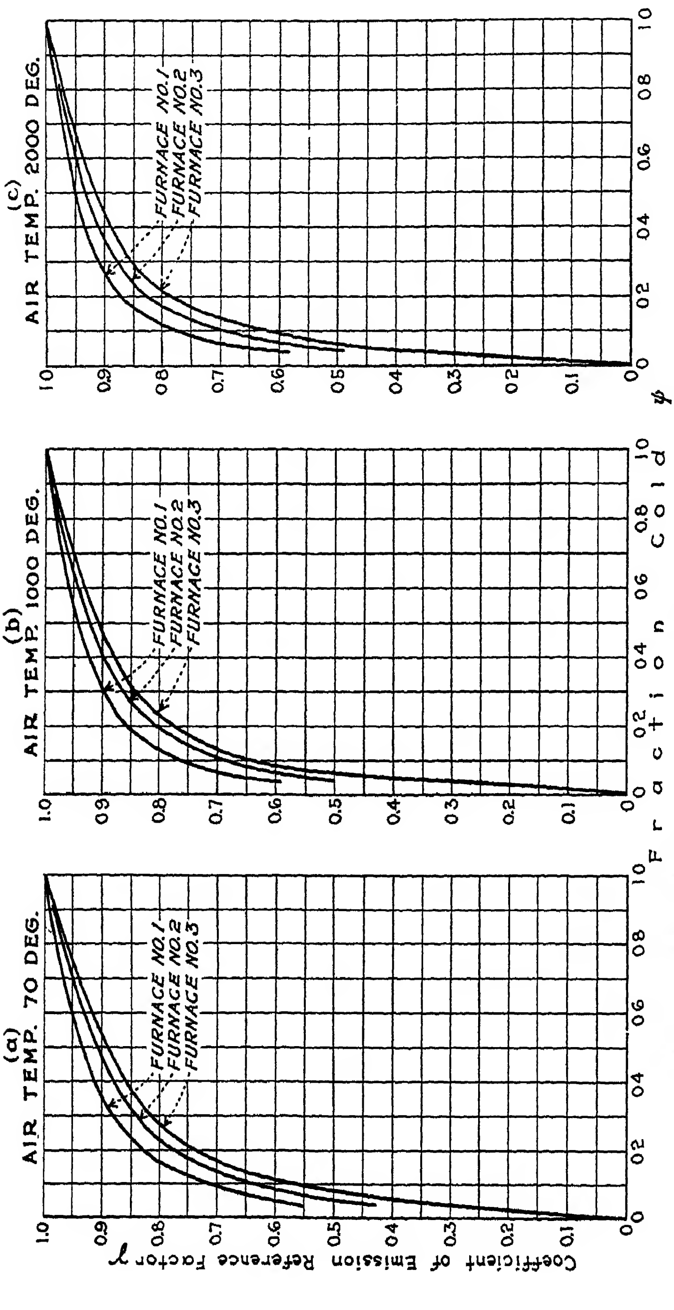


FIG. 34 INFLUENCE OF FRACTION COLD Ψ ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ

(Coal No. 1, pulverized; air, 120 per cent; energy rate = 10,000 B.t.u. Applicable in general to all coals with air up to 160 per cent.)

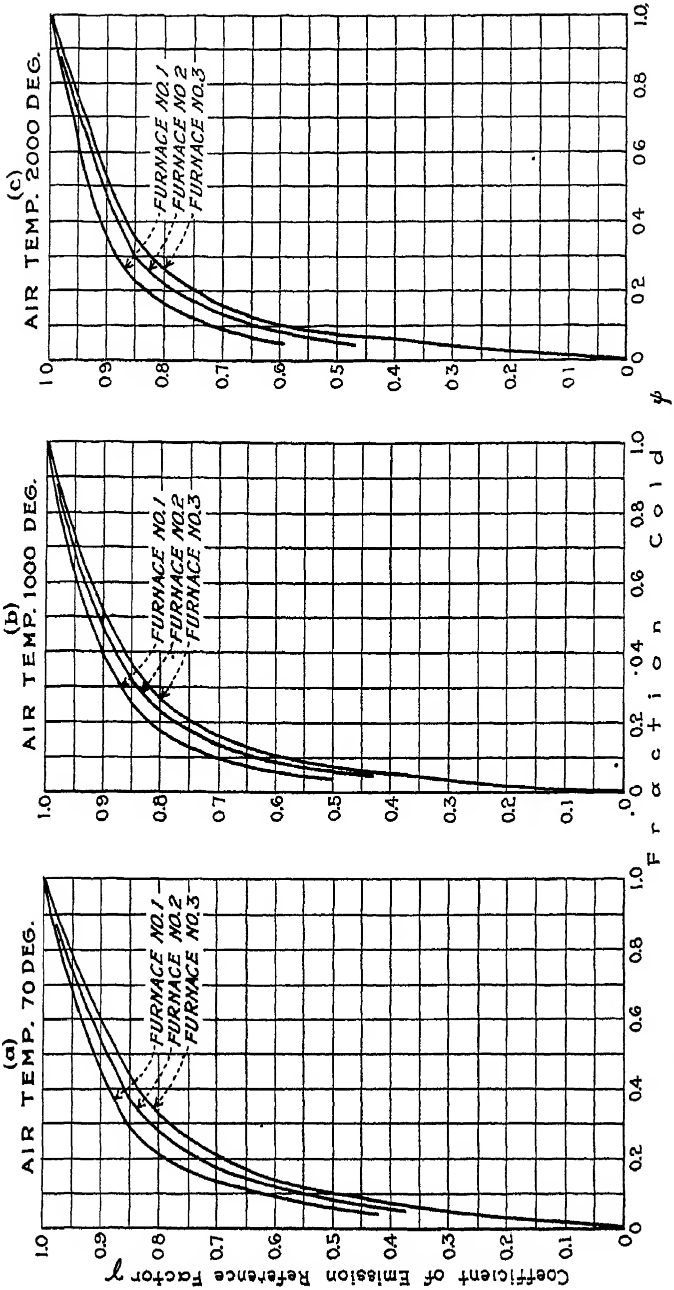


FIG. 35 INFLUENCE OF THE FRACTION COLD Ψ ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ

(Coal No. 1, pulverized; air, 120 per cent; energy rate = 25,000 B.t.u. Applicable in general to all coals with air up to 160 per cent.)

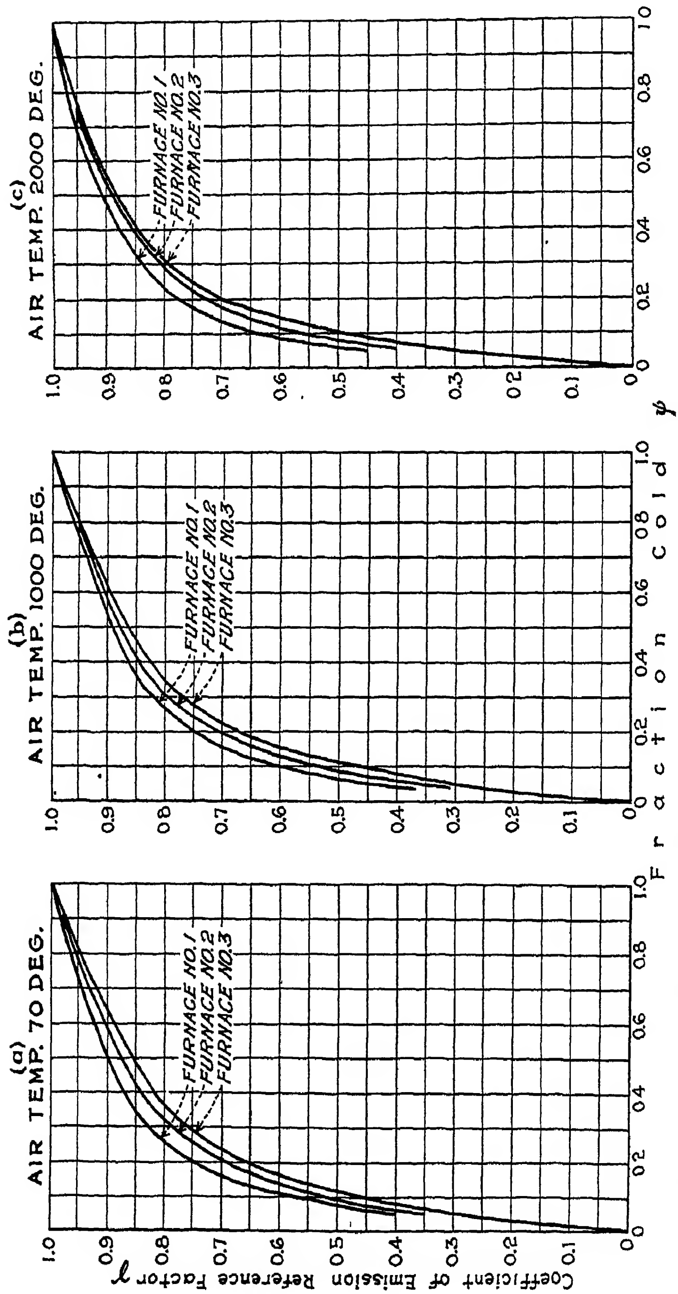


FIG. 36 INFLUENCE OF THE FRACTION COLD Ψ ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ

(Coal No. 1, pulverized; air, 120 per cent, energy rate = 40,000 B t u Applicable in general to all coals with air up to 160 per cent.)

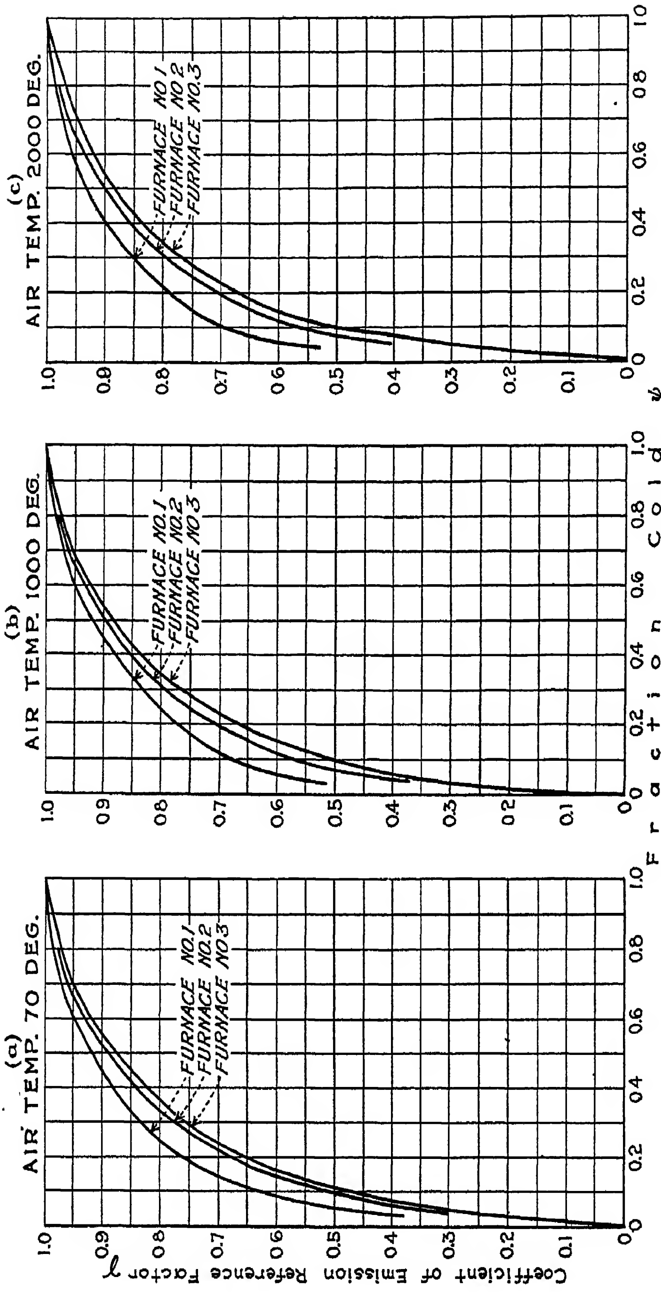


FIG. 37 INFLUENCE OF THE FRACTION COLD Ψ ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ

(Coal No. 1, stoker-fired; air, 140 per cent; energy rate = 25,000 B t.u. Applicable in general to all coals with air up to 160 per cent.)

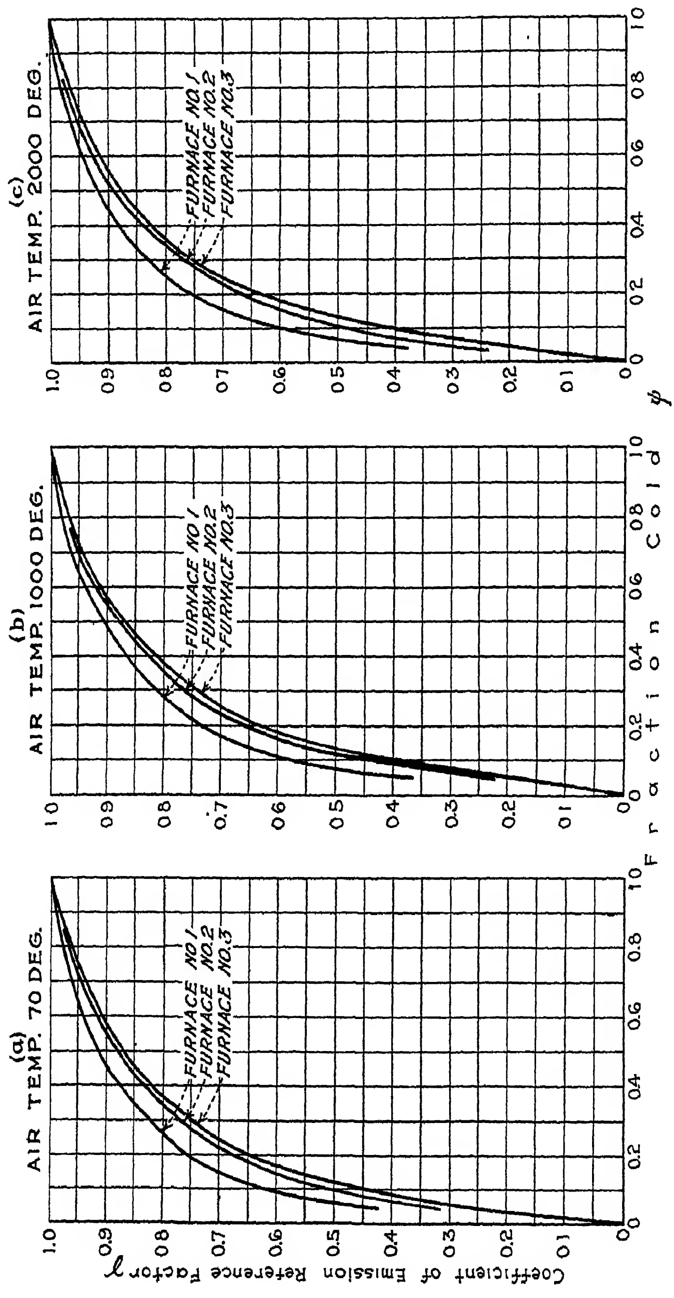


FIG. 38 INFLUENCE OF THE FRACTION COLD Ψ ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ

(Coal No. 1, stoker-fired; air, 140 per cent; energy rate = 40,000 B t.u. Applicable in general to all coals with air up to 160 per cent.)

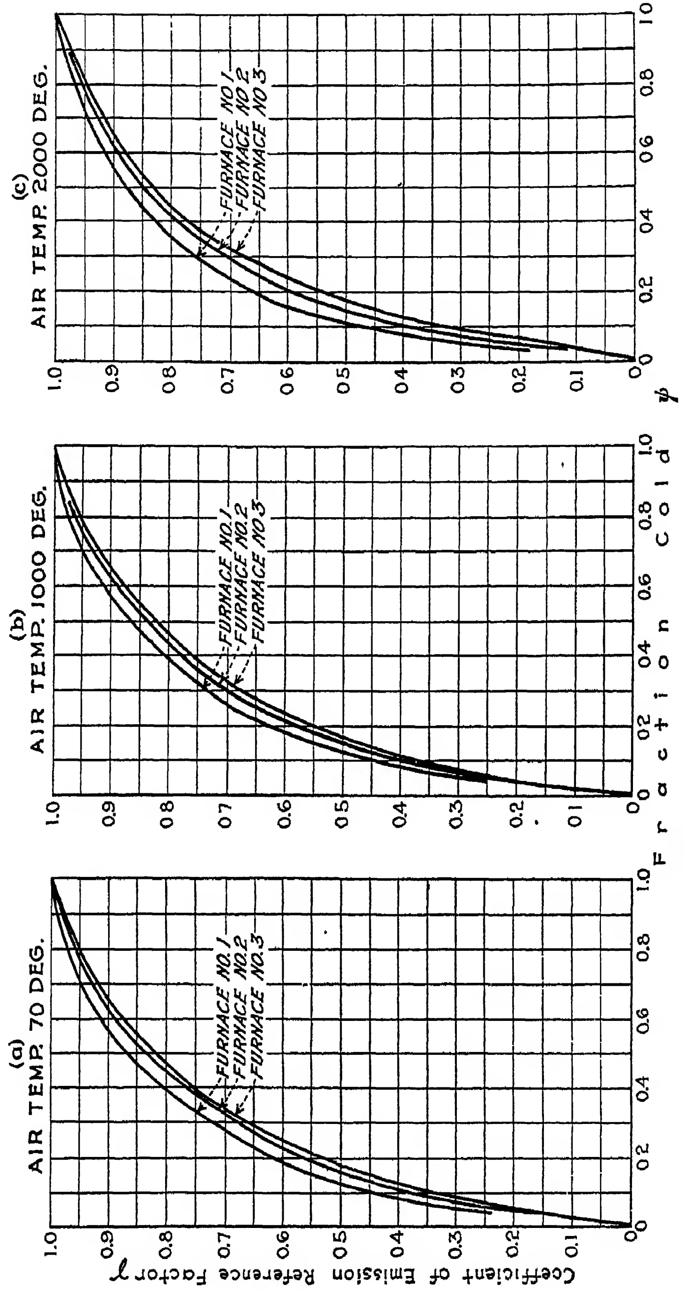


FIG. 39 INFLUENCE OF THE FRACTION COLD Ψ ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ

(Coal No. 1, stoker-fired; air, 140 per cent; energy rate = 80,000 B t.u. Applicable in general to all coals with air up to 160 per cent.)

TABLE 10 VALUES OF μ_5, μ_6, AND T_U FOR ENERGY-LIBERATION RATE = 10,000 B.T.U. PER CU. FT. PER HR. IN TYPE *E* FURNACE

Coal Furnace Air excess	70 deg. fahr.			500 deg. fahr.			1000 deg. fahr.			1500 deg. fahr.			2000 deg. fahr.		
	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6
1-1-20	1510	0.590	0.590	1590	0.665	0.608	1680	0 754	0 626	1760	0.852	0.646	1840	0.946	0.662
1-1-40	1490	0.541	0.541	1580	0.624	0.562	1680	0.720	0.581	1780	0.822	0.600	1870	0.987	0.624
2-1-20	1510	0.608	0.608	1590	0.688	0.618	1670	0.768	0.638	1760	0.867	0.658	1830	0.968	0.677
2-1-40	1490	0.558	0.558	1590	0.647	0.583	1690	0 745	0.601	1790	0.850	0 620	1870	0.961	0.639
3-1-20	1690	0.592	0.592	1770	0.658	0.608	1870	0.762	0.626	1960	0.858	0.642	2030	0 960	0.660
3-1-40	1660	0.525	0.525	1770	0.610	0.547	1880	0.718	0.570	1990	0 827	0.593	2080	0 933	0.609
1-2-20	1720	0.541	0.541	1800	0.608	0.556	1900	0.700	0.580	1990	0.789	0.597	2080	0.875	0.612
1-2-40	1690	0.482	0.482	1790	0.556	0.500	1900	0.661	0 532	2020	0.754	0.556	2110	0.804	0.575
2-2-20	1720	0.559	0.559	1800	0 637	0 579	1890	0.720	0.598	1980	0.811	0.614	2060	0 908	0.635
2-2-40	1710	0.506	0.506	1800	0.586	0.528	1910	0.684	0.552	2010	0 786	0.573	2100	0.891	0.592
3-2-20	1840	0.554	0.554	1930	0.626	0.569	2050	0.710	0.583	2140	0.817	0.610	2230	0.912	0.624
3-2-40	1810	0.481	0.481	1920	0.568	0.504	2050	0.670	0.535	2160	0.772	0.554	2270	0 879	0.574
1-3-20	1810	0.516	0.516	1900	0.583	0.538	2010	0.678	0.560	2100	0.762	0.577	2180	0.854	0.596
1-3-40	1780	0.452	0.452	1890	0.530	0.477	2010	0.628	0 508	2120	0.724	0.527	2220	0 826	0.549
2-3-20	1810	0.538	0.538	1900	0 604	0.552	2000	0.698	0.577	2090	0.786	0.596	2170	0.873	0.610
2-3-40	1790	0.477	0.477	1900	0.558	0.503	2010	0.652	0.526	2120	0.757	0.551	2220	0.856	0.570
3-3-20	1920	0.534	0.534	2010	0.604	0.550	2110	0.694	0.570	2220	0.789	0.590	2330	0.899	0.583
3-3-40	1890	0 463	0 463	2000	0.543	0.487	2130	0.644	0.514	2250	0.748	0 537	2370	0.854	0 557

TABLE 11 VALUES OF μ_5, μ_6, AND T_U FOR ENERGY-LIBERATION RATE = 25,000 B.T.U. PER CU. FT. PER HR IN TYPE *E* FURNACE

Coal Furnace Air excess	70 deg. fahr.			500 deg. fahr			1000 deg. fahr.			1500 deg. fahr.			2000 deg. fahr		
	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6
1-1-20	1970	0.477	0.477	2070	0.543	0.496	2190	0.624	0.518	2300	0.707	0.535	2400	0.790	0.552
1-1-40	1910	0.415	0.415	2040	0.490	0.441	2180	0.579	0.467	2300	0.672	0.480	2430	0.758	0.506
2-1-20	1980	0.495	0.495	2070	0.559	0.511	2190	0.646	0.536	2300	0.729	0.553	2390	0.816	0.570
2-1-40	1930	0.440	0.440	2050	0.517	0.464	2190	0.606	0.489	2310	0.701	0.511	2430	0.792	0.528
3-1-20	2160	0.461	0.461	2270	0.537	0.489	2400	0.618	0.508	2540	0.699	0.523	2650	0.784	0.538
3-1-40	2090	0.400	0.400	2240	0.475	0.426	2400	0.559	0.446	2540	0.656	0.472	2680	0.745	0.486
1-2-20	2180	0.418	0.418	2290	0.482	0.441	2430	0.560	0.464	2550	0.644	0.488	2660	0.722	0.504
1-2-40	2100	0.357	0.357	2250	0.427	0.385	2410	0.513	0.414	2550	0.598	0.436	2680	0.684	0.455
2-2-20	2180	0.439	0.439	2290	0.502	0.459	2430	0.583	0.484	2550	0.663	0.503	2660	0.742	0.519
2-2-40	2120	0.382	0.382	2260	0.449	0.404	2420	0.533	0.430	2550	0.620	0.452	2680	0.708	0.472
3-2-20	2310	0.423	0.423	2440	0.477	0.434	2590	0.566	0.465	2720	0.643	0.480	2840	0.723	0.497
3-2-40	2240	0.357	0.357	2400	0.424	0.380	2570	0.504	0.403	2720	0.593	0.425	2870	0.678	0.443
1-3-20	2280	0.394	0.394	2400	0.454	0.415	2520	0.528	0.438	2660	0.604	0.458	2780	0.682	0.476
1-3-40	2200	0.329	0.329	2350	0.394	0.355	2510	0.478	0.384	2660	0.558	0.407	2800	0.643	0.427
2-3-20	2280	0.411	0.411	2400	0.475	0.434	2520	0.547	0.454	2660	0.628	0.476	2780	0.710	0.496
2-3-40	2210	0.353	0.353	2370	0.417	0.376	2520	0.501	0.404	2670	0.585	0.427	2810	0.664	0.442
3-3-20	2390	0.398	0.398	2520	0.462	0.420	2670	0.534	0.439	2810	0.614	0.459	2930	0.690	0.474
3-3-40	2310	0.332	0.332	2470	0.399	0.358	2650	0.480	0.383	2810	0.559	0.400	2960	0.641	0.418

TABLE 12 VALUES OF μ_5, μ_6, AND T_U FOR ENERGY-LIBERATION RATE = 40,000 B.T.U. PER CU. FT. PER HR IN TYPE *E* FURNACE

Coal Furnace Air excess	70 deg. fahr.			500 deg. fahr.			1000 deg fahr.			1500 deg. fahr.			2000 deg. fahr.		
	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6	T_U	μ_5	μ_6
1–1–20	2220	0.412	0.412	2340	0.473	0.432	2430	0.549	0.455	2600	0.623	0.472	2710	0.694	0.485
1–1–40	2130	0.353	0.353	2290	0.422	0.380	2450	0.502	0.407	2600	0.583	0.425	2750	0.658	0.438
2–1–20	2220	0.429	0.429	2340	0.490	0.448	2430	0.506	0.469	2600	0.642	0.487	2720	0.717	0.501
2–1–40	2150	0.375	0.375	2300	0.441	0.398	2460	0.521	0.420	2600	0.602	0.439	2750	0.698	0.461
3–1–20	2400	0.393	0.393	2540	0.468	0.421	2700	0.528	0.434	2850	0.604	0.451	2980	0.676	0.461
3–1–40	2320	0.336	0.336	2490	0.398	0.356	2670	0.472	0.376	2840	0.552	0.396	2990	0.635	0.414
1–2–20	2420	0.353	0.353	2550	0.413	0.378	2700	0.479	0.398	2840	0.549	0.416	2980	0.617	0.431
1–2–40	2310	0.296	0.296	2480	0.358	0.318	2660	0.429	0.346	2830	0.500	0.364	3000	0.572	0.372
2–2–20	2420	0.375	0.375	2500	0.432	0.395	2700	0.499	0.414	2840	0.568	0.431	2980	0.644	0.450
2–2–40	2330	0.319	0.319	2490	0.375	0.338	2670	0.447	0.361	2830	0.525	0.384	3000	0.601	0.400
3–2–20	2570	0.336	0.336	2710	0.406	0.370	2870	0.474	0.389	3040	0.538	0.404	3170	0.613	0.421
3–2–40	2460	0.291	0.291	2640	0.348	0.311	2830	0.415	0.332	3010	0.489	0.351	3170	0.562	0.367
1–3–20	2530	0.335	0.335	2660	0.380	0.350	2810	0.446	0.370	2960	0.514	0.389	3100	0.580	0.405
1–3–40	2400	0.271	0.271	2570	0.327	0.294	2750	0.391	0.317	2930	0.464	0.338	3110	0.541	0.360
2–3–20	2520	0.348	0.348	2660	0.402	0.367	2810	0.470	0.390	2970	0.531	0.402	3090	0.602	0.421
2–3–40	2400	0.294	0.294	2580	0.341	0.307	2770	0.411	0.332	2940	0.484	0.352	3100	0.560	0.372
3–3–20	2630	0.330	0.330	2780	0.382	0.348	2950	0.449	0.365	3110	0.517	0.386	3250	0.586	0.402
3–3–40	2510	0.267	0.267	2690	0.321	0.287	2890	0.387	0.309	3080	0.457	0.327	3250	0.532	0.387

TABLE 13 VALUES OF μ AND γ FOR PULVERIZED-COAL FURNACES

Coal Furnace Air excess	70 deg. fahr.					500 deg. fahr.					1000 deg. fahr.					1500 deg. fahr					2000 deg. fahr.					Energy rate
	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	
1-1-20	0.590	0.590	0.800	0.885	0.960	0.655	0.608	0.815+	0.895	0.963	0.754	0.626	0.835	0.910—	0.965	0.852	0.646	0.849	0.915	0.970	0.946	0.662	0.855	0.920	0.970	10,000
1-2-20	0.541	0.541	0.734	0.850	0.940	0.608	0.556	0.760	0.865	0.950	0.700	0.580	0.783	0.880	0.950	0.789	0.597	0.796	0.885	0.960	0.875	0.612	0.805	0.890	0.960	
1-3-20	0.516	0.516	0.692	0.830	0.930	0.583	0.533	0.725	0.845—	0.985	0.678	0.560	0.750	0.860	0.945	0.762	0.577	0.762	0.870	0.950	0.854	0.596	0.765	0.870	0.950	
1-1-20	0.477	0.477	0.753	0.865	0.945	0.543	0.496	0.770	0.870	0.950	0.624	0.518	0.786	0.880	0.955	0.707	0.535	0.799	0.890	0.960	0.740	0.552	0.810	0.895	0.960	25,000
1-2-20	0.418	0.418	0.691	0.830	0.930	0.482	0.441	0.720	0.845	0.935	0.562	0.466	0.738	0.850	0.945	0.644	0.488	0.750	0.860	0.945	0.722	0.504	0.755	0.865	0.950	
1-3-20	0.394	0.394	0.651	0.805	0.918	0.454	0.415	0.680	0.820	0.925	0.528	0.438	0.706	0.835	0.935	0.604	0.458	0.717	0.840	0.935	0.682	0.476	0.720	0.845	0.940	
1-1-20	0.412	0.412	0.712	0.840	0.936	0.473	0.432	0.725	0.840	0.930	0.549	0.455	0.744	0.855	0.945	0.623	0.472	0.756	0.865	0.950	0.694	0.485	0.760	0.865	0.950	40,000
1-2-20	0.353	0.353	0.653	0.805	0.920	0.413	0.378	0.675	0.820	0.925	0.479	0.398	0.698	0.830	0.935	0.549	0.416	0.709	0.835	0.935	0.617	0.431	0.715	0.840	0.940	
1-3-20	0.335	0.335	0.616	0.780	0.905	0.880	0.350	0.640	[illegible]	0.915	0.446	0.370	0.663	0.815	0.925	0.514	0.389	0.679	0.820	0.925	0.580	0.405	0.690	0.830	0.980	

γ_A, γ_B, and γ_D respectively for A, B, and D furnaces μ_5 and μ_6 for E furnaces. (See Par. 8.)
(μ_6 for E) × γ_A = (μ_5 for A), etc (See Pars. 51 and 52)

TABLE 14 VALUES OF μ AND γ FOR STOKER-FIRED FURNACES

Coal Furnace Air excess	70 deg. fahr					500 deg. fahr.					1000 deg. fahr					1500 deg. fahr.					2000 deg. fahr.					Energy Rate
	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	μ_5 for E	μ_6 for E	γ_A	γ_B	γ_D	
1-1-40	0.421	0.421	0.730	0.850	0.965	0.480	0.434	0.750	0.845+	0.970	0.596	0.481	0.769	0.870	0.970	0.695	0.506	0.774	0.875	0.975	0.790	0.525	0.780	0.875	0.975	25,000
1-2-40	0.317	0.317	0.640	0.795	0.950	0.380	0.342	0.660	0.810	0.955	0.464	0.374	0.674	0.820	0.958	0.552	0.402	0.679	0.820	0.958	0.650	0.432	0.600	0.830	0.958	
1-3-40	0.261	0.261	0.610	0.775	0.940	0.320	0.288	0.630	0.790	0.945	0.386	0.311	0.642	0.795	0.950	0.462	0.336	0.647	0.800	0.950	0.540	0.359	0.655	0.805	0.950	
1-1-40	0.367	0.367	0.688	0.825	0.955	0.440	0.396	0.705	0.835	0.960	0.530	0.428	0.725	0.845	0.965	0.620	0.451	0.730	0.850	0.965	0.730	0.485	0.740	0.855	0.965	40,000
1-2-40	0.258	0.258	0.605	0.770	0.940	0.330	0.297	0.625	0.785	0.945	0.410	0.331	0.637	0.795	0.950	0.500	0.364	0.042	0.795	0.950	0.590	0.392	0.640	0.795	0.950	
1-3-40	0.198	0.198	0.576	0.750	0.930	0.250	0.225	0.595	0.765+	0.940	0.320	0.258	0.616	0.780	0.942	0.390	0.357	0.620	0.780	0.945	0.470	0.312	0.625	0.785	0.945	
1-1-40	0.282	0.282	0.582	0.755	0.935	0.360	0.324	0.590	0.765—	0.935	0.450	0.363	0.614	0.780	0.945	0.540	0.393	0.618	0.780	0.945	0.640	0.425	0.620	0.785	0.945	80,000
1-2-40	0.180	0.180	0.518	0.710	0.915	0.250	0.225	0.535	0.725	0.920	0.320	0.258	0.546	0.735	0.925	0.400	0.291	0.550	0.735	0.925	0.490	0.326	0.555	0.740	0.925	
1-3-40	0.132	0.132	0.495	0.695	0.905	0.190	0.171	0.510	0.705	0.910	0.260	0.210	0.522	0.715	0.915	0.330	0.240	0.526	0.725	0.920	0.400	0.266	0.530	0.720	0.920	

γ_A, γ_B, and γ_D respectively for A, B, and D furnaces. μ_5 and μ_6 for E furnace. (See Par. 8)
(μ_5 for E) $\times$ γ_A = (μ_5 for A), etc. (See Pars 51 and 52.)

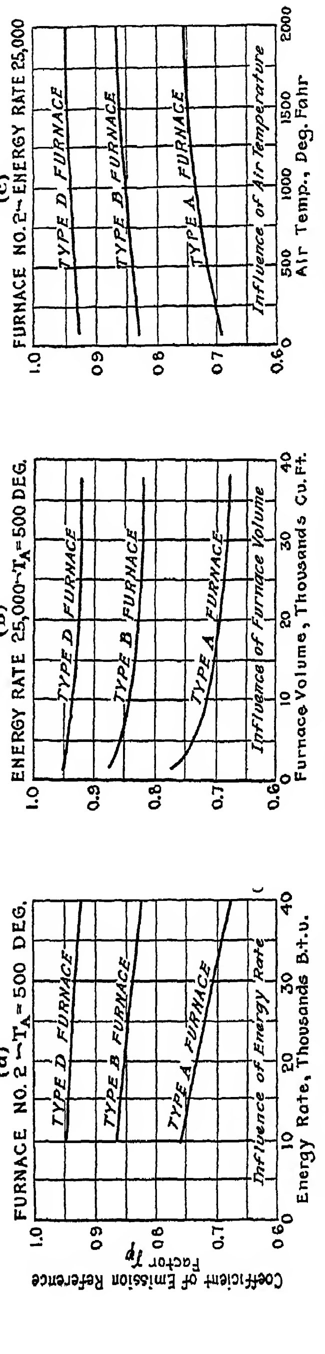


FIG 40 INFLUENCE OF VARIOUS FACTORS ON THE COEFFICIENT OF EMISSION REFERENCE FACTOR γ
(Coal No 1, pulverized; air, 120 per cent)

on the same mean initial diameter of the ground-fuel particles which has been taken as 0.002 inch[1] for 75 per cent through a 200-mesh screen. Tables 13 and 14 include the values of the reference factors γ, as computed for coal No. 1 with 120 and 140 per cent air, respectively for the pulverized-coal and stoker-fired furnaces. It may be interesting to note that with γ values for the *A*, *B*, and *D* furnaces there are five known points on each of the (γ, Ψ) curves, the other two in all cases being values of $\gamma = 1$ for $\Psi = 1.00$, and $\gamma = 0$ for $\Psi = 0$. These tables contain considerable data that have not been included elsewhere in curve form.

54 Finally it may be of interest to note that a coefficient of emission reference factor γ for a given cold fraction Ψ may be plotted to show its variations with other coördinates, such as energy rate, air temperature, furnace volume, etc. In each case all coördinates, except those between which the curve is plotted, are, of course, held constant. Representative variations are shown by curves in Fig. 40. It is seen that γ_Ψ plotted against energy rate results in practically straight-line variations and that the air temperature has only a very small influence on the value of γ_Ψ. The subscript is, of course, used to denote that the cold fraction Ψ is held constant for such variations.

[1] See Pars. 12 and 14.

APPENDIX NO. 1

SAMPLE CALCULATIONS FOR TYPE *E* FURNACE

55 Following are the data for this illustrative case: Furnace No. 2, type *E*, 20 ft. x 20 ft. x 20 ft.; flame, 19 ft. x 19 ft. x 19 ft.; air, 120 per cent; air temperature $T_A = 1000$ deg. fahr.; energy rate, 25,000 B.t.u. per cu. ft. per hr.; fuel, Coal No. 1, Saline County, Illinois, bituminous

Heating value, B.t.u. per lb.	12,800
Air, pounds per pound of fuel	11 58
Analysis of products (by volume)	
CO_2, per cent	14 23
H_2O, per cent	6 32

Pulverized, 75 per cent through 200-mesh screen, $d_0 = 0.002$ in.

56 For a cubical furnace 20 ft. x 20 ft. x 20 ft. the volume will be 8000 cu. ft. For the assumed conditions, the flame dimensions will represent a volume of 6859 cu. ft. The flame surfaces parallel to exposed surfaces will be 361 sq. ft. each, and the total flame surface will be 2166 sq. ft. The area of each furnace face will be 400 sq. ft., and the total convection surface exclusive of the area of the aperture, 2000 sq. ft.

57 For an energy-liberation rate of 25,000 B.t.u. per cu. ft. per hr.

$$G_U \cdot U = 25{,}000 \times 6859 = 171{,}500{,}000 \text{ B.t.u. per hr. (Table 7)}$$

and the fuel required per hour will be

$$G_U = \frac{171{,}500{,}000}{12{,}800} = 13{,}390 \text{ lb. (Table 7)}$$

The total air required for combustion will be

$$G_A = 13{,}390 \times 11.58 = 155{,}200 \text{ lb. (Table 7)}$$

and the heat per pound above 70 deg. fahr. in the preheated air[1] at temperature $T_A = 1000$ deg. will be

$$\Delta h_A = 227 \text{ B.t.u. (see Fig. 28)}$$

Thus the total heat,

$$G_A \Delta h_A = 155{,}200 \times 227 = 35{,}230{,}000 \text{ B.t.u. per hr. (Table 7)}$$

and total energy input per hour

$$G_U U + G_A \Delta h_A = 171{,}500{,}000 + 35{,}230{,}000 = 206{,}730{,}000 \text{ B.t.u. per hr.}$$

58 The coking diameter ratio is found from the relationship[2]

$$K_0 = \sqrt[3]{(1-p)\,\frac{\delta_0}{\delta_1}} \quad \text{(see Equation [51])}$$

For coal No. 1 containing 38.4 per cent moisture and volatile, $(1-p) = 0.616$, $\delta_0 = 84$ lb. per cu. ft. raw, and $\delta_1 = 75$ lb. per cu. ft. coked, hence

$$K_0 = 0.884$$

[1] See Appendix No. 1, Wohlenberg and Morrow, Trans., A.S.M.E., 1925.

[2] See Appendix No. 4, Par. 91.

For the given conditions

$$V_F^{\frac{1}{3}} = 19$$
$$K_0^2 = 0\ 7815$$
$$g_B = 12.58$$
$$\delta_0 = 84$$
$$d_0 = 0.002$$

and

$$\lambda \text{ (see Appendix No. 4, Par. 82)} = \frac{V_F^{\frac{1}{3}} K_0^2}{g_B \delta_0 \cdot d_0} = \frac{19 \times 0.7815}{12.58 \times 84 \times 0.002}$$
$$\lambda = 7.02 \text{ (Table 9)}$$

59 Assuming a flame temperature $T_U = 2430$ deg. fahr. (2890 deg fahr. abs.)

$$I_S = 87.1 \frac{\lambda}{T_U} - 6420 \left(\frac{\lambda}{T_U}\right)^2 \quad \text{(see Equation [47])}$$
$$= 87.1 \frac{7.02}{2890} - 6420 \left[\frac{7.02}{2890}\right]^2$$
$$= 0.212 - 0.038 = 0.174$$

and

$$1 - I_S = 0.826$$

The surface[1] of the particles in suspension

$$S'_U = 950 \frac{V_F^{\frac{2}{3}} \lambda}{T_U} = 950 \frac{19^2 \times 7.02}{2890} = 833 \text{ sq. ft. (see Equation [48])}$$

60 The coefficient of penetration[2] I_G is obtained from the relation

$$I_G = 1 - \sigma$$

where σ is the blackness fraction $\frac{q_{GC}}{q_{CB}}$. From Fig. 31, q_{GC} for coal No. 1 at 2430 deg. fahr. is $\frac{15{,}500}{(1-\beta_{CN})}$ B.t.u. and q_{CB} for the same absorber is found from Fig. 29 to be $\frac{106{,}500}{(1-\beta_{CN})}$ B.t.u. Hence

$$I_G = 1 - \frac{15{,}500}{106{,}500} = 1 - 0.146 = 0.854$$

Values of I_G may be read directly from curves in Fig. 30.

61 Thus the equivalent effective radiating area of the coal particles

$$I_G (1 - I_S) S'_U = 0.854 \times 0.826 \times 833 = 588 \text{ sq. ft.}$$

and the absorption intensity at the receiving surface

$$\alpha'_U (1 - \beta_{CN}) (T_U^4 - T_C^4)$$
$$= 1.62 \times 10^{-9} \times 0.95 [(697{,}600 - 6{,}561) \times 10^8]$$
$$= 106{,}500 \text{ B.t.u. per sq. ft. per hr.}$$

The total radiation absorption at the cold surface from the solid particles is then

$$588 \times 106{,}500 = 62{,}400{,}000$$

62 The coefficient[3] $(1 - I''_S)$ of the coal particles for the gases is found from Fig. 30 to be 0.866 and the equivalent effective radiating surface of the gaseous masses

[1] See Appendix No. 4, Par. 84.
[2] See Appendix No. 4, Pars. 88, 89.
[3] See Appendix No. 4, Par. 86.

$$(1-I''_S)S''_{UC} = 0.866\times2166 = 1876 \text{ sq. ft.}$$

The radiation-absorption intensity $(1-\beta_{CN})q_{GC}$ at the cold surface, due to radiation from the gases, is calculated by Schack's method, and the results are shown in Fig. 31 from which the value for 2430 deg. fahr. is found to be 15,500 B.t.u. per sq. ft. The total absorption due to radiation from the gases is thus

$$(1-I''_S)S''_{UC}(1-\beta_{CN})q_{GC} = 1{,}876\times15{,}500 = 28{,}600{,}000 \text{ B.t.u. per hr.}$$

63 The heat transferred to the walls by convection[2] is given by the relation

$$Z_C = R_Z(T_U-T_C)z(S_C-S_{CD})$$

in which R_Z, the temperature-difference ratio, is found from Fig. 32. to be 0.42 and the mean temperature difference at the wall between gases and wall

$$\Delta T_{GC} = (T_U-T_C) = 0.42(2890-900) = 836 \text{ deg. fahr.}$$

The mass flow[3] for the given furnace and energy rate is

$$\frac{G_B}{\frac{1}{2}V_F^{\frac{2}{3}}} = \frac{168{,}590}{180.5} = 934 \text{ lb. per sq. ft. per hr.}$$

and the corresponding convectional coefficient of heat transfer z for a temperature difference of 836 deg. fahr. is found from Fig. 33 to be 2.95 B.t.u. per sq. ft. per hour per degree difference in temperature. The convectional cold surface independent of the area at the aperture, $S_C-S_{CD} = 2400-400 = 2000$ sq. ft., and the heat absorbed at the cold surface by convection becomes

$$(S_C-S_{DC})\Delta T_{GC}\cdot z = 2000\times836\times2.95 = 4{,}930{,}000 \text{ B.t.u. per hr.}$$

64 The energy in the burned gases[4] $\Sigma G_B\Delta h_B$ resulting from the combustion of 13,390 lb. of fuel per hr. is found from Fig. 25 to be

$$13{,}390\times8{,}300 = 111{,}000{,}000 \text{ B.t.u. per hr.}$$

65 The equilibrium Equation [14] may now be set up:

$$\begin{aligned}(G_UU+G_A\cdot\Delta h_A = \Sigma G_B\Delta h_B & \\ &+\left\{I_G(1-I_S)S'_U\right\}\left\{\alpha'_U(1-\beta_{CN})[T_U^4-T_C^4]\right\}\\ &+\left\{(1-I''_S)S''_{UC}\right\}\left\{(1-\beta_{CN})q_{GC}\right\}\\ &+R_Z(T_U-T_C)z(S_C-S_{CD}) \quad \text{(see Equation [14])}\end{aligned}$$

$$\begin{aligned}171{,}500{,}000+35{,}230{,}000 &= 111{,}000{,}000\\ &\quad+62{,}400{,}000\\ &\quad+28{,}600{,}000\\ &\quad+4{,}930{,}000\\ 206{,}730{,}000 &= 206{,}930{,}000\end{aligned}$$

Thus the energy output for the assumed temperature of 2430 deg. fahr. checks the input within the limits of accuracy required, and the equilibrium equation may be said to be satisfied.

66 The net energy emission within the furnace (i.e., radiation from coal and gases plus convection) is then

$$62{,}400{,}000+28{,}600{,}000+4{,}930{,}000 = 95{,}930{,}000 \text{ B.t.u. per hr.}$$

[1] See Appendix No. 5.
[2] See Pars. 42, 43, 47.
[3] See Par. 47.
[4] See Par. 46.

and the coefficient of emission is

$$\mu_5 = \frac{95{,}930{,}000}{171{,}500{,}000} = 0.560$$

The corresponding coefficient μ_6 is

$$\mu_6 = \frac{95{,}930{,}000}{206{,}730{,}000} = 0.464$$

67 Following is the condensed form of calculation used by the authors:

Calculation number 113*a*	Date 6/28/26
Coal 1	Air preheat 1000 deg. fahr.
Furnace 2	Assumed temperature 2430 deg. fahr.
Excess air 20 per cent	Assumed temperature 2890 deg. fahr. abs.
Energy rate 25,000 B.t u.	

Energy liberated by fuel $G_U U$	171,500,000 B.t.u.
Energy in preheated air $G_A h_A$	35,230,000 B.t.u.
Total energy input	206,730,000 B.t.u.

$$I_S = \frac{611.6}{2890} = \frac{316{,}300}{2890} = 0.212-0.038 = 0.174 \quad (1-I_S) = 0.826$$

$$S'_U = \frac{2{,}408{,}000}{2890} = 833 \text{ sq. ft.}$$

$I_G = 0.854$

$M = (1-I_S)\times I_G \times S'_U = 588$

$J = (1-\beta_{CN})\,a_0(T_U^4 - T_C^4) = 106{,}500$

Radiation from solid particles $= M\times J =$ 62,400,000 B.t.u.

$(1-I''_S) = 0.866$

$S''_{UC} = 6\times 361 = 2166$

Gas radiation $= 0.866\times 2166\times 15{,}500 =$ 28,600,000 B.t.u.

$z = 2.95$

$(S_C - S_{CD}) = 2400-400 = 2000$

$\Delta T_{GC} = 0.42\times 1990$

Convection $= 2.95\times 2000\times 836 =$ 4,930,000 B.t.u.

Total absorption by cold surface Z_C	95,930,000 B.t.u.
Energy of burned gases 13,390×8,300	111,000,000 B.t.u.
Total energy output	206,930,000 B.t.u.
Total energy input	206,730,000 B.t.u.

$$\mu_5 = \frac{95{,}930{,}000}{171{,}500{,}000} = 0.560$$

$$\mu_6 = \frac{95{,}930{,}000}{206{,}730{,}000} = 0.464$$

APPENDIX NO. 2

EVALUATION OF *A*, *B*, AND *D* FURNACES FOR PURPOSE OF FINDING (γ, Ψ) CURVES

68 Since no reference factors γ are available at the outset, it is necessary to compute the *A*, *B*, and *D* furnaces by means of the general theory outlined in Wohlenberg and Morrow. For this purpose the equilibrium equation is stated in the form

$$(G_U U+G_A\Delta h_A) = \sum G_B\cdot\Delta h_B+(1-\beta_{CN})\left[\frac{K_1K_3}{(1-\beta_{RN})K_1+K_3}+K_2\right][T_U^4-T_C^4]+Z_C \quad [25]$$

In the pulverized-fuel furnace, the constants in this equation involve the following quantities:

$$K_1 = (K'_1+K''_1) \quad [26]$$

in which

$$K'_1 = a'_U I_G(1-I_S)\frac{S'_U}{2}\cdot\frac{\omega'_{UR}}{2\pi} \quad [27]$$

$$K''_1 = a''_U(1-I''_S)S''_{UR} \quad [28]$$

$$K_2 = K'_2+K''_2 \quad [29]$$

in which

$$K'_2 = a'_U I_G(1-I_S)\frac{S'_U}{2}\cdot\frac{\omega'_{UC}}{2\pi} \quad [30]$$

$$K''_2 = a''_U(1-I''_S)S''_{UC} \quad [31]$$

and

$$\left.\begin{aligned} K_3 &= a_{RF}I_G(1-I_S)S_C\frac{\omega_{CR}}{2\pi}\\ &\text{or}\\ K_3 &= a_{RF}I_G(1-I_S)S_R\frac{\omega_{RC}}{2\pi}\end{aligned}\right\} \quad [32]$$

In these formulas ω'_{UR}, ω'_{UC} and ω_{CR} represent, respectively, the mean solid angles subtended by the refractory from the fuel particles, by the cold surface from the fuel particles, and by the refractory from the cold surface.

69 The radiation coefficients at the fuel surface, from the gases, and from the refractory to the gases, are designated respectively as a'_U, a''_U and a_{RF}. The first is taken as $= a_0 = 1.62\times10^{-9}$ and $a_{RF} = 0.95a_0$. The radiation coefficient a''_U is really an equivalent radiation coefficient whose value itself depends on the flame temperature. It is that value which, when applying the fourth-power temperature law to radiation from the gases, will yield the correct radiation absorption intensity, and is evaluated for each trial value of T_U by first finding the actual radiation absorption intensity from such curves as are shown in Fig. 31, and then applying the equation

$$a''_U(1-\beta_{CN})(T_U^4-T_C^4) = (1-\beta_{CN})q_{GC} \quad [33]$$

The value of the right-hand side of this equation is found for a given temperature T_U by reference to Fig. 31. The value of a''_U is thus the result of a solution for a given temperature T_U.

70 All other quantities involved, with the exception of the net reflection at the refractory, have been defined in Par. 44 and are evaluated either in curve form or in tables. The solution for a given

case is thus, as before, the result of a series of trials. The net energy emission is

$$dQ_F/dt = (1-\beta_{CN})\left[\frac{K_1K_3}{(1-\beta_{RN})K_1+K_3}+K_2\right][T_U^4-T_C^4]+Z_C \quad . \quad [34]$$

and the emission coefficients as before

$$\mu_5 = \frac{dQ_F/dt}{dQ_U/dt} \qquad \text{and} \qquad \mu_6 = \frac{dQ_F/dt}{dQ_U/dt+dQ_A/dt}$$

71 The mean temperature of the refractory, as in Wohlenberg and Morrow, may be solved for from the relation

$$T_R^4 = \left[\frac{(1-\beta_{RN})K_1T_U^4+K_3T_C^4}{(1-\beta_{RN})K_1+K_3}\right] \quad . \; . \; . \; . \; . \; . \quad [35]$$

in which β_{RN}, the net reflection from the refractory, as explained in Wohlenberg and Morrow, is equal to $\frac{1}{2}\beta_R$, where β_R is the coefficient of reflection at a point on the refractory.

72 Since for stoker-fired furnaces I''_S and I_S vanish, the constants assume the form

$$K'_1 = a'_UI_GS'_U\,\frac{\omega'_{UR}}{2\pi} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [36]$$

$$K''_1 = a''_U \cdot S''_{UR} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [37]$$

$$K'_2 = a'_UI_G\;S'_U \cdot \frac{\omega'_{UC}}{2\pi} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [38]$$

$$K''_2 = a''_U \cdot S''_{UC} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [39]$$

$$\left.\begin{aligned} K_3 &= a_{RF}I_G \cdot S_C\,\frac{\omega_{CR}}{2\pi} \\ &\text{or} \\ K_3 &= a_{RF}I_GS_R \cdot \frac{\omega_{RC}}{2\pi} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [40]$$

73 It is to be noted that for $\frac{1}{2}S'_U$, the fuel surface S'_U has been substituted in this case. This is because in the pulverized-fuel furnace the particles radiate in all directions whence surfaces $\frac{1}{2}S'_U$ see through a total solid angle 4π, or surfaces S'_U see through a solid angle 2π. (See Par. 37, Wohlenberg and Morrow.) In the stoker now only one side of the hot coal bed sees into the furnace, whence the total seeing solid angle is 2π for the exposed net surface S'_U. The solid angles are not alike, however, in both cases; for instance, values of the solid angles assume the values for the *A*, *B*, *D*, and *E* furnaces shown in Table 15. The method of evaluating the solid angles is outlined in Appendix No. 2 of Wohlenberg and Morrow.

TABLE 15 SOLID ANGLES IN *A*, *B*, *D*, AND *E* FURNACES

	Pulverized coal				Stokers			
	A	*B*	*D*	*E*	*A*	*B*	*D*	*E*
For ω	Top cold	Top and bottom cold	Top, bottom, and two sides cold	All surfaces cold	Top cold	Top and back cold	Top, back, and two sides cold	All surfaces cold
ω_{UC}	$\frac{4}{8}\pi$	$\frac{8}{8}\pi$	$\frac{13}{8}\pi$	4π	$\frac{1}{4}\pi$	$\frac{11}{16}\pi$	$2\pi - \frac{11}{16}\pi$	2π
ω_{UR}	$4\pi - \frac{4}{8}\pi$	$4\pi - \frac{8}{8}\pi$	$4\pi - \frac{13}{8}\pi$	zero	$2\pi - \frac{1}{4}\pi$	$2\pi - \frac{11}{16}\pi$	$\frac{7}{16}\pi$	zero
ω_{CR}	2π	$2\pi - \frac{1}{4}\pi$	....	zero	$2\pi - \frac{1}{4}\pi$	$2\pi - \frac{25}{32}\pi$	......	zero
ω_{RC}	....	....	$2\pi - \frac{1}{4}\pi$	zero	......	......	$2\pi - \frac{7}{16}\pi$	zero

74 For furnaces in which the cold-surface distribution is such that it cannot be dealt with in terms of cold faces, evaluation of the solid angles is a very difficult problem. This is entirely obviated if the (γ, Ψ) curves are employed. Since the latter have been formulated on a scientific basis the errors involved in their use are no doubt less than would be those introduced by necessary approximations of solid-angle values.

75 Having found values of μ_5 for the E, A, B, etc. furnaces, the values of γ are, of course, directly the result of substitution in the formulas $\gamma_A = \frac{\mu_{5A}}{\mu_{5E}}$, $\gamma_B = \frac{\mu_{5B}}{\mu_{5E}}$, etc.

APPENDIX NO. 3

METHOD OF DETERMINING THE MOST EFFECTIVE FRACTION COLD

76 A determination of this value of Ψ involves the spacing of tubes in the convection zone; and properly to show the influence of variations of air preheat, energy rate, furnace volume, etc., it is necessary to use the same spacing and tube size in all cases. For this purpose 3¼-in. tubes, spaced 5 in. between centers, have been assumed. The approximate flow areas in the aperture for this particular arrangement are given in Table 16.

TABLE 16

Furnace No. 1	40 sq. ft.
Furnace No. 2	160 sq. ft.
Furnace No. 3	360 sq. ft.

77 In the solution of the problem for a given furnace volume, fuel, air quantity, and air temperature, it is first necessary to determine the temperature of the gases T_E at the plane D-D, Fig. 17, when they have traversed an amount of surface S_{CD} as they pass into the aperture. For furnace No. 1, $S_{CD} = 100$ sq. ft. The temperature drop of the gases between T_U and T_E is for no case very great, and the differences $\Delta T_{UC} = (T_U - T_C)$ and $\Delta T_{EC} = (T_E - T_C)$ are comparatively large. Therefore, for all practical cases, the mean temperature difference may be determined by the law of arithmetic temperature differences, whence the temperature T_E may be determined by trial from the relations

$$\left.\begin{aligned} \frac{\Delta T_{UC} + \Delta T_{EC}}{2} (z) \cdot S_{CD} &= G_U (H_U - H_E) \\ T_E &= (T_C + \Delta T_{EC}) \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad [41]$$

in which H_U and H_E represent, respectively, the thermal capacities of the products per pound fuel at temperatures T_U and T_E. Values of these are found from such curves as are shown in Figs. 25, 26, and 27 The value of the heat-transfer coefficient z may be obtained for a given mass-flow rate from such curves as are shown in Fig. 33, although, as previously mentioned, these probably represent the lower limits of values which should be used.

78 Having determined T_E, the above system is investigated for Ψ less than unity, other conditions being the same. It will now require an extra amount of convectional surface S_{DE}, Fig. 17, to reduce the gas temperature to the above value of T_E.

79 Considering furnace B, for instance, the mean flame temperature T_U in this furnace must first be determined. This is accomplished as follows:

$$\mu_{5B} = \gamma_B \mu_{5E}$$

and the remaining energy fraction θ_G at entrance to the aperture is

$$\theta_G = (1+\Phi-\mu_{5B}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [42]$$

in which, as before,

$$(1+\Phi) = \frac{Q_{Ut}}{Q_U} = \frac{\text{Input to furnace}}{\text{Liberated energy}} \text{ (see Par. 26)}$$

The energy remaining in the gases per pound fuel is then equal to

$$\theta_G \cdot U = H_{UB} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [43]$$

whence T_{UB} is found by reference to heat-capacity curves Figs. 25, 26, and 27 for the thermal capacity H_{UB}.

80 For the B furnace, the total convection surface S_D, beginning at the entrance to the aperture, required to cool the gases from the flame temperature T_{UB} to the temperature T_E is now found from the relation

$$\frac{\Delta T_{UBG}+\Delta T_{EG}}{2} (z) \cdot S_D = G_U \cdot (H_{UB}-H_E) \quad . \quad . \quad . \quad . \quad . \quad [44]$$

The extra convection surface is equal to

$$S_{DE} = (S_D - S_{CD}) \text{ (see Par. 25)}$$

and the total surface required to cool the gases to T_E is equal to

$$S_{CE} = (S_C + S_{DE}) \text{ (see Eq. 5)}$$

81 To obtain the most effective fraction cold Ψ the solution is carried through as above outlined for a number of values of Ψ between zero and unity, and a curve is plotted to determine the minimum point.

APPENDIX NO. 4

THE COEFFICIENTS I_S, I''_S, I_G, K_0, AND THE FUEL SURFACE S'_U

82 *The Mean Shielding Coefficient I_S.* The value of the mean shielding coefficient I_S of the coal particles for each other in the pulverized-fuel furnace is obtainable from the relation

$$I_S = 87.1 \frac{V_F \frac{1}{3} K_0^3}{g_B T_U \delta_0 d_0} - 6420 \left[\frac{V_F \frac{1}{3} K_0^2}{g_B T_U \delta_0 d_0} \right]^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad [45]$$

or, if

$$\lambda = \frac{V_F \frac{1}{3} K_0^2}{g_B \delta_0 d_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [46]$$

$$I_S = 87.1 \frac{\lambda}{T_U} - 6420 \left[\frac{\lambda}{T_U} \right]^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [47]$$

in which

V_F = flame volume in cubic feet
g_B = pounds of burned gas per pound of fuel
T_U = radiation mean of flame temperature, deg. fahr. absolute
d_0 = initial mean diameter of the fuel particles, in.
K_0 = coking diameter ratio (see Par. 91).
δ_0 = weight of fuel per cubic foot.

The development of this relation is included in Appendix No. 4 of the paper by Wohlenberg and Morrow.[1]

83 In the stoker-fired furnace the shielding I_S reduces to zero and the coefficient $(1-I_S)$ becomes unity.

84 *The Fuel Surface S'_U.* In the pulverized-coal furnace the area S'_U of particles in suspension is obtainable from the relation

$$S'_U = 950 V_F^{\frac{2}{3}} \left(\frac{\lambda}{T_U} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [48]$$

The development of this expression is likewise included in Appendix No. 4 of the paper by Wohlenberg and Morrow.

85 The corresponding fuel surface in a stoker-fired furnace is that of a smooth envelope that just covers the fuel bed with no points of inflexion in its surface.

86 *The Mean Shielding Coefficient I''_S.* To determine the mean shielding effect I''_S of the coal particles in suspension in the pulverized-fuel furnace upon the radiation from the gases, the same principle was employed as in the case of the shielding of the coal particles for each other. The mean critical radiation thickness[2] of the gaseous products of the combustion of coals 1 and 2 was found to be about 7½ feet for the amounts of excess air employed. Hence since 7½ feet was the maximum effective thickness of radiating gas column, the shielding effect of the suspended coal particles on this radiation was calculated for a point at the center of a furnace of semi-diameter equal to this critical thickness. That is, for coals 1 and 2 the average shielding of the coal particles for the gases was determined to be that of the coal particles for each other in a furnace 15 ft. x 15 ft. x 15 ft. For coal No. 3 where the gaseous radiation is from carbon dioxide alone, there being no water vapor present, the critical radiation thickness was found to be about 5 feet, hence the corresponding shielding effect was determined for a furnace 10 ft. x 10 ft. x 10 ft. These results, of course, are all mean values but are within the limits of accuracy of the necessary assumptions. Curves of $(1-I''_S)$ for all coals are to be found in Fig. 30.

87 In the stoker-fired furnace the shielding I''_S reduces to zero and the coefficient $(1-I''_S)$ becomes unity.

88 *The Coefficient of Penetration I_G.* The coefficient of penetration I_G for radiant energy directed through the gas mass, developed in Appendix No. 3 of the paper by Wohlenberg and Morrow, is modified in the light of the results obtained by the use of Schack's method for determining the radiation from the gases. The principle employed, namely that of surface intensities, is precisely that of the previous paper, the result differing only in the form which the equation assumes.

89 If q_{KG} and q_{WG} be the radiation emission intensities in B.t.u. per square foot per hour from the carbon dioxide and water vapor respectively in the furnace gases (as calculated by the method of Appendix No. 5), and q_{GB} be the black-body radiation emission intensity in B.t.u. per square foot per hour corresponding to the temperature of the gases, found from Fig. 29, the fraction of black-body radiation emitted at that temperature by the gases is obviously

$$\sigma = \frac{q_{KG}+q_{WG}}{q_{GB}} = \frac{q_{GG}}{q_{GB}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [49]$$

where σ is the degree of "blackness" or "blackness fraction" of the gases. Hence from the fundamental considerations, when the gases are

[1] See Trans., A.S.M.E., 1925.

[2] See Appendix 5, par. 90.

able to emit a fraction σ of black-body radiation at a given temperature, they are likewise able to absorb or shield to the same extent, and the coefficient of penetration for radiation through the gases is

$$I_G = 1-\sigma = \left(1-\frac{q_{GC}}{q_{CB}}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [50]$$

Values of the fraction $(1-\sigma)$ have been calculated for the coals used, and may be found from Fig. 30 for any temperature.

90 For the stoker-fired furnace the coefficient of penetration I_G assumes the same form as above.

91 *The Coking-Diameter Ratio.* The coking-diameter ratio K_0 is defined as the coefficient such that $K_0 d_0$ is equal to the mean particle diameter with moisture and volatile expelled. That is

$$K_0 d_0 = d_1$$

where d_0 = the diameter of a single particle raw

and d_1 = the diameter of a single particle coked

If now w_0 and w_1 be the weights of a single particle raw and coked, v_0 and v_1 be the volumes of a single particle raw and coked, and δ_0 and δ_1 be the densities raw and coked

$$w_0 = v_0\delta_0 = \frac{\pi d_0^3}{6}\delta_0$$

and

$$w_1 = v_1\delta_1 = \frac{\pi d_1^3}{6}\delta_1$$

But if the coal contains a fraction p of volatile and moisture by weight

$$\frac{w_1}{w_0} = \frac{1-p}{1} = \frac{v_1\delta_1}{v_0\delta_0} = \frac{d_1^3\delta_1}{d_0^3\delta_0}$$

and

$$\frac{d_1}{d_0} = K_0 = \sqrt[3]{\frac{w_1\delta_0}{w_0\delta_1}} = \sqrt[3]{(1-p)\frac{\delta_0}{\delta_1}} \quad . \quad . \quad [51]$$

APPENDIX NO. 5

RADIATION FROM FURNACE GASES

92 Schack[1] gives for the determination of the radiations from furnace gases two equations, one applicable to carbon dioxide and the second to water vapor. The equation for carbon dioxide[2] is of the form:

$$Q_K = \frac{R}{4.9}\left[\Phi_1(K_1-K'_1)\left(1-\frac{1-e^{-16c}}{16c}\right)+(K_2-K'_2)\right.$$
$$\left.+\Phi_3(K_3-K'_3)\left(1-\frac{1-e^{-80c}}{80c}\right)\right] \quad . \; . \quad [52]$$

in which Q_K is the absorption in kilogram calories per square meter per hour, R is the radiation absorption coefficient for the receiving

[1] Schack, Mitteilung Nr. 55 der Wärmestelle Düsseldorf des Vereines deutscher Eisenhüttenleute.

[2] See Broido: Trans. A.S.M.E., 1925.

surface, and c is a thickness-concentration product of the gas column equal to $\frac{p}{100} \times S$, where p is the per cent by volume concentration of the carbon dioxide and S is the thickness of the column in meters.

93 The exponential functions of the form $1-\frac{1-e^{-kc}}{kc}$ are the thickness-concentration functions which enter according to the law of variation of radiation absorption in gases with varying thicknesses of column. This law states that

$$E = E_0(1-e^{-kS}) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [53]$$

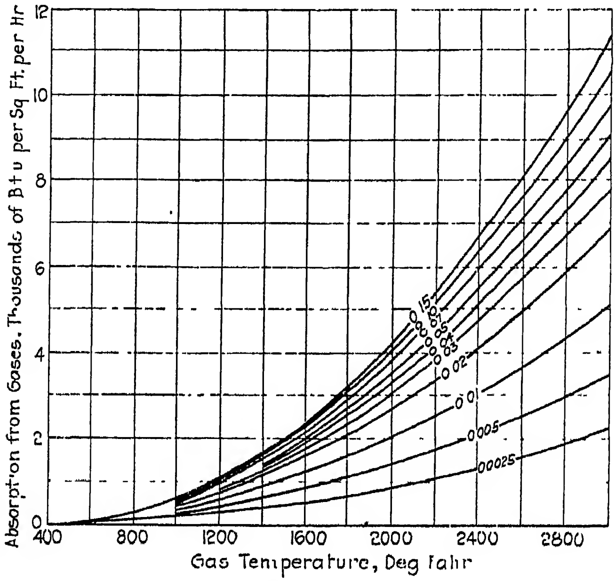


FIG. 41 COLD-SURFACE ABSORPTION FROM CARBON DIOXIDE, THOUSANDS OF B.T.U. PER SQ. FT. PER HR. FROM GASES WITH c AS SHOWN

$\left(c = \frac{p}{328} \times S\right.$; where p = per cent by volume of CO_2 in gases, and S = thickness of gas column in feet.$\left.\right)$

where E is the absorbed energy, E_0 the incident energy, k the absorption constant for the gas, and s the length of the radiation path in the gas, that is, the thickness. These exponentials have been evaluated by Schack and graphically expressed for both water vapor and carbon dioxide.[1]

94 The symbols K_1, K'_1, K'_2 are the temperature coefficients denoting the radiation in the various absorption bands if produced by infinitely thick layers of carbon dioxide, the values of which have also been evaluated by Schack from experimental data. The symbols Φ_1, Φ_2, etc. are coefficients depending upon the shape of the radiating gas body and the value of the function c.

[1] See Broido: Trans. A.S.M.E., 1925.

95 For water vapor precisely the same method is employed as in the case of carbon dioxide, but with different numerical values of the exponential and temperature functions.

96 Employing these equations, the radiation absorptions from furnace gases have been calculated for all values of the function c, the results of which are shown in Figs. 41 and 42. The maximum effective value of c is found in all cases to be about 0.15, that is, when the product $\left(\frac{p}{328} \times s\right)$ of the percentage of concentration and the thickness is 0.15 or greater, no further addition of either concentration or thickness produces any increase in the radiation intensity from the gases. For

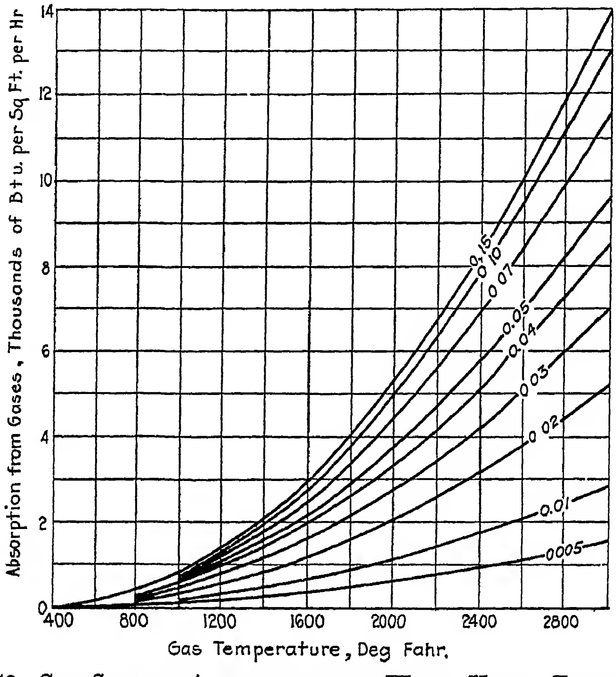


FIG. 42 COLD-SURFACE ABSORPTION FROM WATER VAPOR, THOUSANDS OF B.T.U. PER SQ. FT. PER HR. FROM GASES WITH c AS SHOWN

$\left(c = \frac{p}{328} \times S\right.$; where p = per cent by volume of H_2O in gases, and S = thickness of gas column in feet.$\left.\right)$

a gas of given carbon-dioxide or water-vapor content, that thickness which will bring the function c to a value 0.15 is known as the critical radiation thickness of the gas. Obviously then, thicknesses of gas columns greater than the critical thickness are not effective in producing any added radiation intensity.

97 The absorption curves are calculated for a cast-iron or steel absorber with a surface coefficient of reflection β_{CN} of 0.05. That is to say, 95 per cent of the rays emitted by the gases are absorbed at the receiving surface. Furthermore, the curves are based on a receiving-surface temperature of 900 deg. fahr. absolute (440 deg. fahr.), corresponding to a boiler pressure of 365 lb. per sq. in. gage. Any

application of these results to other conditions must of necessity involve a correction which in the case of the surface-reflection coefficient is one of direct proportion, but which grows to some complexity for variations in receiving-surface temperature. It is satisfactory to note, however, that the application of these curves anywhere in the range of boiler pressures from 85 lb. per sq. in. (315 deg. fahr.) to 600 lb. per sq. in. (485 deg. fahr.) for a furnace temperature of 2000 deg. fahr. or higher, results *per se* in an error of less than one per cent, an accuracy quite in keeping with that of the necessary assumptions.

98 For the purpose of the investigation in the body of this paper, Fig. 31 was calculated from the data of Figs. 41 and 42 to give the results for the particular coals and furnaces used. The critical thickness of gas column was exceeded in all cases, except in the smallest furnace, when the products were diluted with 40 per cent excess air. Hence the curves for coals 1 and 2 are practically identical, being plots of the sums of the intensities for carbon dioxide and water vapor when c exceeds 0.15. The curve for coal 3, which is pure carbon, involves, of course, radiation from carbon dioxide alone and is less throughout the range than the curves for coals 1 and 2.

APPENDIX NO. 6

INFLUENCE OF AIR PREHEAT ON SURFACE REQUIREMENTS AND ITS DISTRIBUTION

99 Let C, D, and E represent the several cross-sections in the gas stream described in Fig. 17, and F, G, ... N, additional stations, the last of which represents the plane in which the gases finally escape. Q_U, Q_F, and Q_B denote, as before, liberated energy, net energy lost to furnace walls and energy retained in the gases. The symbol η_{xy} denotes the energy transfer from gases between stations indicated by the subscripts, as before, in terms of the liberated energy taken as unity, and θ_C, θ_E, ... θ_N the energy remaining in the gases, expressed as a fraction of the liberated energy.

100 The transferred energy fractions η_{xy} may be expressed in terms of the remaining energy fraction θ and the coefficient Φ, thus

$$\eta_{CE} = (1+\Phi-\theta_E); \qquad \eta_{CF} = (1+\Phi-\theta_F); \; \ldots . \; \eta_{CN} = (1+\Phi-\theta_N) \quad . \; [54]$$

$$\eta_{EF} = (\theta_E-\theta_F); \qquad \eta_{FG} = (\theta_F-\theta_G); \; \ldots . \; \eta_{MN} = (\theta_M-\theta_N) \quad . \; . \; . \; . \; [55]$$

101 A systematic statement of the distribution of the transferred energy in terms of the transferred energy fractions may be formulated as follows:

$$\left.\begin{array}{lll} \text{Furnace plus surface} & S_{DE} = Q_F+Q_B\Big/^{T_U}_{T_E} & = \eta_{CE}Q_U \\ \text{Convection surface} & S_{EF} = \ldots . Q_B\Big/^{T_E}_{T_F} & = \eta_{EF}Q_U \\ \text{Convection surface} & S_{MN} = \ldots . Q_B\Big/^{T_M}_{T_N} & = \eta_{MN}Q_U \\ \text{Escaping with gases} \ldots\ldots\ldots\ldots & Q_B\Big/^{T_N}_{T_{70°}} & = \eta_{N70°}Q_U \end{array}\right\} \; . \; . \; [56]$$

$$\text{Total} \ldots . (1+\Phi)Q_U = \eta_{C.70°}\cdot Q_U = \ldots . \sum_{y=70°}^{x=c}\eta_{xy}\cdot Q_U \quad . \; . \; [57]$$

102 The purpose of this part of the investigation is to find: (1) How the fractions, η_{CE}, η_{EF}, ... η_{MN} vary with the temperature

of the air supplied for combustion and (2) how the corresponding requirements in heat-absorbing surfaces vary. This is best accomplished by dealing with a definite example.

103 Furnace No. 2, fired with coal No. 1, pulverized, burned at a rate of 25,000 B.t.u. per cu. ft. per hr. with 120 per cent air, will now be investigated and considered as part of a system in which the steam is generated at 365 lb. per sq. in. gage pressure, 650 deg. fahr., and which is supplied with feedwater from bleeder heaters at 300 deg. fahr. The superheater is assumed to be directly beyond the surface S_{DE}. The gases therefore have a temperature T_E at entrance to the superheater. In all cases the escaping-gas temperature is taken as 350 deg. fahr., as any further reduction in temperature will require the same additional surface in all cases to be considered for the same gas velocities and surface arrangements.

104 This system will now be considered as operating at the following four air temperatures.

Case (*a*) .... T_A = 70 deg. fahr.
Case (*b*) .... T_A = 500 deg. fahr.
Case (*c*) .... T_A = 1000 deg. fahr.
Case (*d*) .... T_A = 2000 deg. fahr.

For Case (*a*) an economizer must be used, as the temperature of the saturated steam is 440 deg. fahr. Sufficient economizer surface will be installed to raise the feedwater from 300 to 400 deg. fahr.

TABLE 17

	Case (*a*)	Case (*b*)	Case (*c*)	Case (*d*)
Entrance to superheater	T_E	T_E	T_E	T_E
Exit from superheater	T_F	T_F	T_F	T_F
Exit from boiler	T_G	T_G	T_G	T_G
Exit from economizer	T_N	....	...	..
Exit from air preheater	....	T_N	T_N	T_N

Results of the analysis are shown in Tables 18 to 21.

TABLE 18

Case (*a*) air temperature = 70 deg. fahr $(1 + \Phi) = 1.00$

Station	Temperature, deg. fahr.	B.t.u. in gas per lb. coal	θ_x	η_{xy}
E	2120	7230	$\theta_E = 0.565$	$\eta_{OE} = 0.435$
F	1725	5920	$\theta_F = 0.463$	$\eta_{EF} = 0.102$
G	683	2200	$\theta_G = 0.173$	$\eta_{FG} = 0.290$
N	350	1000	$\theta_N = 0.078$	$\eta_{GN} = 0.095$
70°	70	0000	....	$\eta_{N70} = 0.078$
				$(1 + \Phi) = 1.000$

TABLE 19

Case (*b*) air temperature = 500 deg. fahr. $(1 + \Phi) = 1.094$

Station	Temperature, deg. fahr.	B.t.u. in gas per lb. coal	θ_x	η_{xy}
E	2228	7600	$\theta_E = 0.594$	$\eta_{OE} = 0.500$
F	1835	6289	$\theta_F = 0.492$	$\eta_{EF} = 0.102$
G	746	2420	$\theta_G = 0.190$	$\eta_{FG} = 0.302$
N	350	1000	$\theta_N = 0.078$	$\eta_{GN} = 0.112$
70°	70	0000	....	$\eta_{N70} = 0.078$
				$(1 + \Phi) = 1.094$

105 Thus the gas temperatures at the stations have been fixed by the conditions of the problem and the proper notation is given in Table 17.

106 Analysis of the above columns of η_{xY} in conjunction with the gas-surface temperatures shows that as the air temperature is increased the requirements in boiler surface are considerably decreased, whereas the requirements in air-preheater surface are very greatly increased.

107 Thus reference to Fig. 21 indicates that the surface S_{CE} is very nearly the same for all cases if the furnace is at least 30 per cent

TABLE 20

Case (c) air temperature = 1000 deg. fahr. $(1 + \Phi) = 1.205$

Station	Temperature, deg. fahr.	B.t.u. in gas per lb. coal	θ_x	η_{xy}
E	2365	8050	$\theta_E = 0.630$	$\eta_{CE} = 0.575$
F	1975	6739	$\theta_F = 0.528$	$\eta_{EF} = 0.102$
G	1210	4050	$\theta_G = 0.316$	$\eta_{FG} = 0.212$
N	350	1000	$\theta_N = 0.078$	$\eta_{GN} = 0.238$
70°	70	0000	....	$\eta_{N70} = 0.078$
				$(1 + \Phi) = 1.205$

TABLE 21

Case (d) air temperature = 2000 deg. fahr. $(1 + \Phi) = 1.430$

Station	Temperature, deg. fahr.	B.t.u. in gas per lb. coal	θ_x	η_{xy}
E	2585	8850	$\theta_E = 0.691$	$\eta_{CE} = 0.730$
F	2210	7539	$\theta_F = 0.589$	$\eta_{EF} = 0.102$
G	2126	7250	$\theta_G = 0.567$	$\eta_{FG} = 0.022$
N	350	1000	$\theta_N = 0.078$	$\eta_{GN} = 0.480$
70°	70	0000	....	$\eta_{N70} = 0.078$
				$(1 + \Phi) = 1.430$

cold. For values of Ψ greater than 0.3, the conditions of Table 22 exist for the fraction η_{EG} that must be absorbed on the boiler and superheating surfaces.

TABLE 22

Air, temperature, deg. fahr.	η_{EG}	Gas, temperature, E to G, deg. fahr.	Surface temperatures, Boiler, deg. fahr.	Surface temperatures, Super-heater, deg. fahr.
500	$0.102 + 0.302 = 0.404$	2228 to 746		440 +
1000	$0.102 + 0.212 = 0.314$	2365 to 1210	440 +	to
2000	$0.102 + 0.022 = 0.124$	2585 to 2126		650 +

108 The result for equal gas velocities is as shown in Table 23 when expressed in terms of the surface S_{EG} of Case (b) taken as unity.

TABLE 23

Air, temperature, deg. fahr.	Surface S_{EG}
500	1.00
1000	0.60
2000	0.16

109 For the air preheater however the conditions are reversed as in Table 24.

TABLE 24

Air temperature, deg. fahr.	η_{GN}	Gas temperature G to N, deg. fahr.	Approximate surface temperature, N to G, deg. fahr.
500	0.112	746 to 350	220 to 630
1000	0.238	1210 to 350	220 to 1100
2000	0.480	2126 to 350	220 to 2070

110 Inspection shows that as the air temperature goes up the mean temperature difference in the preheater is decreased, which means that

as the air temperature is increased it becomes necessary to absorb a larger and larger quantity of energy at lower temperature differences. The results may again be expressed as in Table 25 as a ratio, taking requirements for Case (*b*) as unity.

TABLE 25

Air temperature, deg. fahr.	Surface in preheater (S_{GN})
500	1.00
1000	2.25
2000	5.90

111 Comparing next Cases (*a*) and (*b*) it is again found that S_{DE} is very nearly the same for the two cases. For the remainder refer to Table 26.

TABLE 26

Case	η_{EG}	Gas temperature, E to G deg. fahr.
Case (*a*), Economizer	0.392	2120 to 683
Case (*b*), Air preheater	0.404	2228 to 746

112 Hence the requirements in boiler surface S_{EG} should be slightly less when preheated air is used. Comparison of the economizer and air-preheater conditions bring out the figures of Table 27.

TABLE 27

Case	η_{GN}	Gas temperature, G to N deg fahr.	Approximate surface temperatures, deg. fahr.
Case (*a*) Economizer	0 095	683 to 350	400 to 300
Case (*b*) Air preheater	0.112	746 to 350	630 to 220

113 Inspection shows that the approximate mean temperature differences, gas to surface, are in the ratio 166/123, economizer to air preheater, and since less energy is absorbed here in the ratio 0.095/0.112 it will require approximately 1.6 times as much surface in the air preheater as it will in the economizer, providing the mean air and gas velocities are the same for all cases. However, to reduce draft losses the proportions in this respect vary widely and each case is therefore a special one.

114 To proceed further with the analysis it is necessary to specify definite conditions as to the heat-transfer rate. Beyond this point, therefore, every case is a special one and should be solved for the particular conditions involved. However, in view of the increased combustion efficiency thereby resulting in the decreased requirements in pressure-vessel surface, we may conclude that it will often prove economical to preheat the air for combustion up to and perhaps over 1000 deg. fahr., in spite of the increase in total surface which this involves. Of course the fusing point of the ash must be considered in this connection, and it will be one of the conditions to limit the practical degree of preheating the air.

APPENDIX NO. 7

EQUIVALENT FRACTIONS COLD FOR SPECIAL CASES

115 (γ, Ψ) *Relation for the Cubical Furnaces Included in Part II.* In applying the reference factor values as here computed it should be noted, as before mentioned, that they are based on cubical furnaces.

For pulverized coal the first one-sixth of the cold part of the envelope ($\Psi = \frac{1}{6}$) has been considered as placed at the top of the furnace and the second one-sixth ($\Psi = \frac{1}{3}$) at the bottom of the furnace. What furnace-cavity faces are taken as increments to the cold surface however is of little consequence in the pulverized-fuel furnaces. In the stoker furnaces the first one-fifth of the envelope cold ($\Psi = \frac{1}{5}$) has been considered as at the side opposite the fuel bed. This of course is at the top. The other increments are then necessarily taken on the back, front, and side walls. In the stoker furnace the influence of the first cold increment depends somewhat upon its position, since the solid angle subtended by the side walls from the fuel bed is not quite equal to that subtended by the top. In general, however, the first one-fifth of the cold surface would be taken at the top, whence the curves may be considered generally applicable for cubical furnaces. For certain odd-shaped furnace cavities and sometimes also for peculiar surface dis-

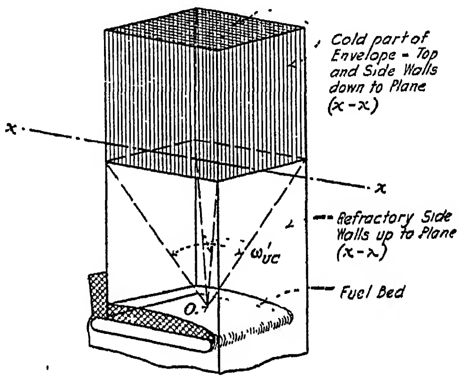


FIG. 43 FURNACE X — COLD CAVITY OVER HOT CAVITY

tributions the general method as outlined in Par. 51 is applied in a modified form.

116 *General Procedure when Furnace Cavity Is not Cubical.* In all cases this consists, as before, of evaluation of the emission coefficient μ_{5B} for the actual furnace cavity considered first, as totally enclosed by cold surface. If this cavity is only partially enclosed by cold surface, the next step involves employment of the (γ, Ψ) relation as developed for that rectangular parallelopiped which with certain faces or parts of faces cold subtends the same solid angles between the cold parts and the radiating solid-fuel masses. The procedure is based on the fact that

$$\frac{\mu_{5\varpi}}{\mu_{5B\varpi}} \text{ (is practically) } = \frac{\mu_{5P}}{\mu_{5BP}} = \gamma_P \quad . \quad . \quad . \quad . \quad . \quad . \quad [58]$$

In this expression $\mu_{5\varpi}$ and μ_{5P} denote the emission coefficients respectively for the actual cavity and equivalent cavity both partially enclosed by cold surface as above specified. The denominators, $\mu_{5B\varpi}$ and μ_{5BP}, denote the corresponding values for the cavities when totally enclosed by black surface. Now both $\mu_{5B\varpi}$ and γ_P are evaluated without difficulty, the former for the cavity under consideration and the latter for the equivalent envelope. The product

$$\mu_{5B\varpi} \cdot \gamma_P = \mu_{5\varpi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [59]$$

is a very close approximation to the true value of the emission coefficient in the odd-shaped cavity under consideration. It should perhaps be reiterated that even for stokers the error involved in using the (γ, Ψ) relations as shown in Figs. 37, 38, and 39 and in Table 14 is small for usual furnace proportions. If a cavity is exceptionally high in relation to its horizontal area, or if any dimension is large in comparison with the other two, then the (γ, Ψ) relation for an equivalent envelope other than a cube should be employed. The procedure is further discussed in connection with particular cases.

117 *Influence of Proportions of a Rectangular Parallelopiped on* (γ, Ψ) *Relation.* In Fig. 43 we shall consider first that only the top face of a tall rectangular parallelopiped cavity, having a fuel bed at the bottom, is cold. The inequality

$$\frac{\Psi_{xA}}{\Psi_A} \neq \frac{(\omega'_{UC})_{xA}}{(\omega'_{UC})_A} \qquad [60]$$

states that the fraction-cold value does not change in proportion to the change in value of solid radiating angles, subtended by the cold

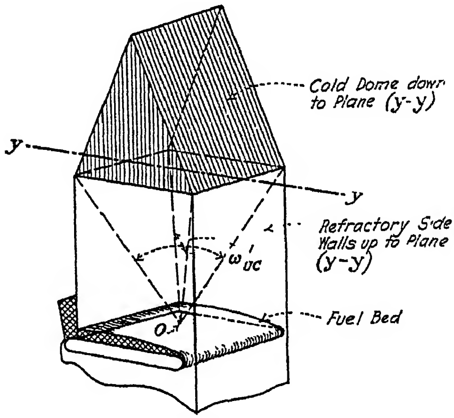


FIG. 44 FURNACE Y — COLD DOME OVER HOT CAVITY

surface from the fuel bed, as a given type A furnace, of a given volume, has its shape altered from that of a cube to that of a tall parallelopiped. It follows that for strict accuracy in stoker-fired furnaces, a special set of (γ, Ψ) curves should be evaluated for each particular set of furnace proportions. However, relatively few cases will serve to cover the whole field of rectangular parallelopipeds. Such relations may then be taken as basic for other furnace cavities, as it will presently be shown that most furnace cavities may be either broken up into, or compared with, rectangular parallelopipeds from which the equivalent fractions cold, or equivalent γ values, may be computed.

118 *Stoker-Fired Furnaces with Cold Pockets.* As a second example the cold surface in furnace X as shown in Fig. 43 is considered as extending over the top and down the side walls to the plane $(x\text{-}x)$. The furnace cavity as a whole is thus divided into the part above the plane $(x\text{-}x)$ and the part below this plane. The upper part of the cavity is really a partially closed-in cold pocket, and, for this reason, should be treated as a special case if a high degree of accuracy is essential.

A (γ, Ψ) curve plotted for this furnace between values $\Psi = 0.2$ and $\Psi = 0.7$, will fall perceptibly below another (γ, Ψ) curve for the same parallelopiped in which the increments to the cold surface, following the top face as cold, are taken, as covering in succession the interior facial areas of each side wall. For the usual furnace cavities, fired with pulverized coal, this difference, practically speaking, is hardly perceptible unless the flame does not strike appreciably into the so-called cold pocket. This condition is discussed in a later paragraph. The procedure for stokers is outlined in the following paragraph:

119 The method outlined for furnace X is, in general procedure, identical with that which is employed in dealing with furnace Y, Fig. 44. This cavity, as the former, contains a cold pocket at the top. A close approximation of the energy emission for both cases may be obtained by evaluating conditions for an equivalent envelope in which the cold surface is projected into the planes $(x\text{-}x)$ and $(y\text{-}y)$. The equivalent energy-liberation rate to be used in such calculations is equal to the total rate, B.t.u. per hour, divided by the flame volume contained within the equivalent envelope. The equivalent fraction cold is thus the plane area of the cold surface projected, as above specified, divided by the sum of this area plus the side walls up to planes $(x\text{-}x)$ or $(y\text{-}y)$. If the volume within the cavity below these planes approximates a cube, the reference factor γ may be taken directly from the data included in Part II of this paper. Otherwise a special (γ, Ψ) curve, applicable to the particular parallelopiped proportions of the equivalent envelope, should be employed. If now μ_{sBx} represents the emission coefficient for the actual total envelope considered as totally cold, other conditions the same, then the product $(\mu_{sBx} \cdot \gamma_P)$ very closely approximates the actual coefficient of emission.

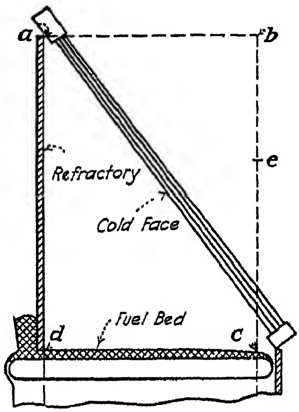


FIG. 45 FURNACE YB

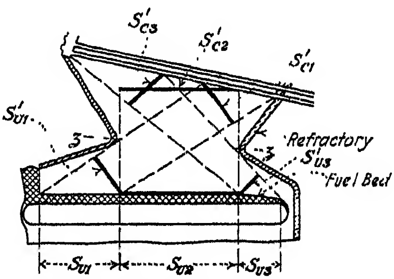


FIG. 46 FURNACE Z — CONSTRICTED SECTION IN PLANE z—z

120 *Pulverized-Coal Furnaces with Cold Pockets.* In the pulverized-coal furnace if the flame does not strike appreciably into a cold pocket or dome, the particles in suspension above a plane such as $(y\text{-}y)$, Fig. 44, have been reduced to minute ash cores, which, because of their small size (number of ash cores per cubic foot being taken as equal to number of coal particles per cubic foot), present a greatly reduced radiating surface. For the cold surface above the plane $(y\text{-}y)$ the treatment is then exactly as outlined in the preceding paragraph. The fine ash particles suspended within the cold dome will, of course, radiate some energy which has not been taken into account.

121 *Stoker-Fired Furnaces with Sloping Cold Faces.* In another case a stoker-fired furnace (*YB*) with a sloping face of cold surface, as shown in Fig. 45, is considered. For absorption of radiant energy from the fuel bed this furnace is obviously equivalent to one in which the faces (*a-b*) and (*b-c*), shown dotted, are cold. The procedure is obviously the same as that outlined in Par. 119 in which μ_{5E} is again evaluated for the actual total envelope considered as all cold. The reference factor γ may then be evaluated for the equivalent envelope (*a, b, c, d,*) in which the sides *ab* and *cd* are cold. Should the sloping cold face terminate at its lower end at the point *e* the cold part of the equivalent envelope should be taken as (*abe*). Obviously, for ordinary furnace proportions, cavities of this type may be evaluated with sufficient accuracy by applying the μ_{5E} and γ values directly from the data included in Part II.

122 *Stoker-Fired Furnaces with Constricted Section in Cavity.* One other furnace, designated in Fig. 46 as *Z*, is analyzed for the equivalent reference factor γ. It is characterized by the constricted section in a plane (*z-z*) between the fuel bed and cold surface. In such a furnace cavity the fuel bed may be divided into the parts S_{U1} under the front arch, S_{U2} under the constricted area in plain view of the cold surface, and S_{U3} under the back arch. The partial surface S_{U1} projected as shown at S'_{U1} sees projected cold surface at the end of an equivalently effective cavity as shown at S'_{C1}. Similarly the partial surface S_{U3} projected as shown at S'_{U3} sees projected cold surface at the end of an equivalently effective cavity as shown at S'_{C3}. The partial surface S_{U2} without projection sees projected cold surface as shown at S'_{C2}. The furnace cavity as a whole thus contains within it the three equivalently effective cavities having at their bottoms the surfaces S'_{U2}, S'_{U1} and S'_{U3}, and at their tops, respectively, the projected cold surfaces S'_{C2}, S'_{C1} and S'_{C3}. For any one of these equivalent cavities, taken by itself, the fraction cold Ψ may be evaluated. Letting Ψ_2, Ψ_1 and Ψ_3 represent the respective fractions cold, the corresponding reference factor values are γ_2, γ_1 and γ_3. These are of course taken from (γ, Ψ) relations applicable to the particular set of proportions involved in each of the equivalently-effective cavities. The approximate mean value of γ for the cavity as a whole may then be found from the relation

$$\gamma_P = \frac{\gamma_1 S_{U1} + \gamma_2 S_{U2} + \gamma_3 S_{U3}}{S_U} \quad \ldots\ldots\ldots \quad [61]$$

where

$$S_U = (S_{U1} + S_{U2} + S_{U3}) \quad \ldots\ldots\ldots\ldots \quad [62]$$

DISCUSSION

B. N. Broido.[1] In Table 3A for Furnace No. 2, stoker-fired, the heat left in the burnt gases before entering the boiler is given as 74.2 per cent, or the heat absorbed in the furnace is 26 per cent. This is also repeated in Table 4. It is also stated there that the temperature in the furnace is 2440 deg. fahr. Coal No. 2 with 14,500 B.t.u. requires 15.28 lb. of gases at 140 per cent excess air, as stated in Table 2. Adding to it one pound of gas coming from the fuel itself, there will be about 16 lb. of gas per pound of coal.

[1] Consulting Engineer, The Superheater Co., New York, N. Y. Mem. A.S.M.E. Deceased, February 10, 1927.

If there were no radiation in the furnace the temperature of this amount of gas would be increased 3630 deg. fahr., or with the addition of the 70 deg. of the entering air, the temperature of the gases would be 3700 deg. fahr. If the gases of Table 4 have a temperature of 2440 deg. fahr. entering the boiler, or 2370 deg. above the temperature of the entering air in the furnace, they contain only 65 per cent of the total heat liberated in the furnace Hence the heat absorbed in the furnace is 35 per cent, instead of 26 per cent as stated in Tables 3A and 4.

While the efficiency of the furnace is not 100 per cent, and some heat is lost by radiation or incomplete combustion, it certainly would not be as much as 10 per cent. This indicates that the total heat absorption in the furnace is higher than that calculated by the authors. As a matter of fact, anyone who has had experience in the operation of a boiler furnace, tests, and heat balance, knows that the actual heat absorbed in a cubical furnace, 20 ft. on all sides, and with all five sides covered. with water-cooled walls, will absorb more than 26 per cent.

The authors mention repeatedly the temperature of the gases in the furnace. They do not state, however, where these temperatures are taken. From experiments with stokers, as well as with pulverized fuel furnaces, particularly those having water-cooled walls, the temperature of the gases near the walls is considerably lower than in the center. This should be taken into consideration in calculating both the radiation and the convection heat absorbed in the furnace. The 240 deg. fahr., for instance, mentioned in Table 4 for furnace No. 2, applies probably to the center of the furnace. It probably will be less near the walls, so that the total heat actually contained in the gases will be still less than mentioned in the paper, which shows further that the actual heat absorption in the furnace is greater than that calculated by the authors.

The authors in their calculations did not use the actual surface of the water-cooled wall but the projected area. The writer suggested using this method of calculation in his paper presented last year,[1] and it seems to be the only correct method of calculation where radiation in furnaces is concerned.

As already mentioned, the method of calculating the radiation in pulverized fuel furnaces, taking into consideration the size of the particles of the coal, does not seem to be quite consistent, because these particles, while in the furnace, may change their size considerably. This is particularly true with some kinds of coal which become rather sticky while burning, so that many particles of the coal will probably combine.

The method of calculating the radiation from the furnace in the present paper should be considered a considerable improvement as

[1] Radiation in Boiler Furnaces. Trans. A.S.M.E. (1925), vol. 47, p. 1123.

compared with the paper presented by Wohlenberg and Morrow, as it takes into full consideration the radiation of the gases.

In their calculations for the radiation of gases, as presented in Fig. 31 the authors have figured with 2430 deg. fahr. As already stated, this temperature will probably exist at a given condition, in the center of the boiler. However, it will be less near the water-cooled walls, as shown in the writer's paper above mentioned. It is shown that the gases are not transparent to their own radiation above a thickness of 3 ft., and as the temperature at the walls is considerably lower, the figures on actual absorption by radiation from the gases must be corrected accordingly.

In calculating a new boiler or installation, the question often comes up as to the influence of the water-cooled walls on the exit gas temperatures and also on the efficiency of the boiler. The writer made a few calculations in this respect and some interesting results were obtained. As would be expected, the reduction in temperature of the exit gases is considerably less than the reduction in the temperature of the gases entering the boiler, due to the water-cooled walls.

There is a certain proportion between the boiler surfaces and the water-cooled surface that will give the best results with respect to the total efficiency of the boiler. If this proportion or relation is increased or decreased, the temperature of the gases is increasing or the efficiency decreasing. The examples which we calculated have shown that with water-cooled surface, amounting to from 4 to about 15 per cent of the boiler heating surface, the exit gas temperatures in the boiler decreased between 20 and 75 deg. fahr. For boilers provided with economizers the exit gas temperature of the economizer remains practically constant. That is, in many cases, up to 15 per cent of water-cooled surfaces will have very little effect upon the final temperature of the exit gases with an economizer. The actual efficiency of the boiler is slightly increased, however, due to the reduced radiation losses in the furnace because of the lower furnace temperatures.

One of the reasons for the comparatively small effect of the water-cooled furnace surfaces on the efficiency of the boiler is the fact that with increasing surfaces in the furnace, the heat absorption per unit or per square foot of heating surface is considerably reduced. Examples have shown that with an increase in the surface of from 4 to 15 per cent, the heat absorbed per square foot of surface was reduced from 60,000 to 22,000 B.t.u. The efficiency of a boiler, however, would again be increased with the increased radiation surface in the furnace, if the area of the pass for the gases in the boiler is correctly proportioned. That is, the larger the surface in a furnace, the cooler will be the gases leaving the furnace from a given heat liberation in the furnace, and, correspondingly, the smaller must be the area for the gases passing through the boiler. This point, up to the present, has not been given due consideration.

Par. 19 is of particular interest. It indicates that the present tendency toward the use of highly preheated air with a corresponding decrease in the boiler surface, is based on sound theoretical grounds, in spite of the fact that many consider it still debatable.

J. G. Coutant.[1] The authors intimate that they intend the calculations to cover the complete design of water-cooled surface. In this, one or two omissions have been made which it is believed are very important, namely, the furnace conditions, and whether or not the gases are transparent. We have been given a furnace volume of 19 x 19 x 19 ft. for a furnace flame, which if true would be nothing more than burning combustible hydrocarbons, producing smoke and ash, and which would not permit the ready transfer of heat by radiation. This is an important point, because it is responsible for the low rates of operation which are obtained on boiler furnaces today with powdered coal.

It also has caused considerable foreign comment. At a recent meeting of the French Society of Civil Engineers, Mr. Orengo discussed the subject of powdered coal, the unit system, storage system, and large combustion chambers in which the coal was burned by the long-flame method and also with short-flames. With the long-flame method, the combustion chambers were twice the size of those used with the short-flame. Professor Roszak, in commenting on Mr. Orengo's paper, said that these combustion chambers, as used in America, were a source of worry because of the cost; and that the fragile construction and the maintenance on such apparatus in the language of his department, Normandy, was a *bête à chagrin*. It seems that while we have large combustion chambers, there is no reason for their being uneconomical. It is only a question of the method of burning the coal for us to get a clear atmosphere in the furnace and utilize the water-cooled surfaces.

In the Comine power station in France the unit system of pulverized coal is used, and the combustion chambers are one-half the size of those used in the United States. They are driven at rates equally as high as ours without any water-cooled surface. While the writer recommends the water-cooled surface, he also recommends boilers operating at higher capacity. We should investigate the method of firing, together with the water-cooled surface, so that we get a clear atmosphere in the furnace.

The Authors. The values in tables 3A and 4 for heat absorbed in the furnace and in the gases, and to which Mr. Broido refers in the first paragraph of his discussion, are all based on coal No. 1. Mr. Broido bases his arguments on the erroneous assumption that

[1] Furnace Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

coal No. 2 has been or may be used in arriving at such values. Referring to Fig. 25, it is noted that for a gas temperature of 2440 deg. fahr. with 40 per cent excess air, the products contain 9500 B.t.u. per pound of coal. The heating value of coal No. 1 is 12,800 B.t.u., whence 74 per cent of the liberated energy is still in the gases as they escape from the furnace. The values reported in this table are thus correct for the assumed conditions.

With reference to Mr. Broido's statement in the latter part of the second paragraph of his discussion, it should be noted that the larger the furnace for a given energy liberation rate (B.t.u. per cu. ft. per hr.), the smaller will be the fraction of the liberated energy which is absorbed in the furnace itself. The furnace in question is a very large one for a stoker, which fact must not be lost sight of.

The gas temperatures to which Mr. Broido refers several times are not, as he assumes, those at the center of the furnace. In all cases the temperature T_U is the mean temperature of the gases escaping from the furnace. This is also the radiation mean of the furnace temperature. A more thorough discussion of this temperature coördinate is included in the paper on this subject by Wohlenberg and Morrow.[1]

In the latter part of his discussion Mr. Broido calls attention to the relatively small influence on escaping gas temperature of a reduction in furnace temperature when caused wholly by an increase in the extent of the cold surface in the furnace walls. Figures 18 to 23 show more specifically what this influence is. It is there stated in terms of a difference in total surface requirements when in each case the finally escaping gases are reduced to the same temperature. This difference in surface requirements is constant regardless of what the final gas temperature is. Thus the authors have assumed for their temperature datum on gases, that temperature which will result in the gases leaving an all-cold furnace, the type E furnace. In all cases where the furnace has less cold surface than a completely surrounded cold envelope, the gases will issue from the furnace aperture at a somewhat higher temperature than with an all-cold surface. The convection surface calculated in this case is that additional surface required to reduce the gases to the temperature of the datum condition; that is, the gases leaving the type E or all-cold furnace. From this point on, be it 2000 deg., or 1800 deg., or 200 deg., the additional convection surface required to reduce the gases to the same exit temperature is the same in all cases. Hence, the curves (Figs. 18-23) of the authors may be used, not to determine the total surface required for a given efficiency, but the difference in two different installations. The curves are curves of difference in total surface.

[1] Radiation in the Pulverized Fuel Furnace, Trans. A.S.M.E., vol. 47 (1925), p. 127.

Mr. Coutant has mentioned the fact that the transparency of the flame is of vital importance in calculating the radiation. The authors have recognized an elaborate method, the interference in radiation, or the shielding of one particle of coal by another in the furnace, and hence the interference with the transparency. In the matter of gases it long has been recognized that the CO_2 and water vapor are not perfectly transparent to radiation, and hence there have been calculated, also, the interferences due to the absorption by the carbon dioxide, and to the absorption by the water vapor, of the radiation emission from the coal particles. Further investigation of the material of the paper will reveal, in the appendices, perhaps, the method used by the authors in calculating this transparency.

No. 2025

ACCURACY OF THE V-NOTCH-WEIR METHOD OF MEASUREMENT

By D. Robert Yarnall,[1] Philadelphia, Pa.
Member of the Society

The paper presents the results of tests made on a V-notch weir tank having a capacity of one million pounds of water per hour in which the accuracy was guaranteed to be within ½ of 1 per cent. The tests reported were carried out at the University of Pennsylvania. The details of the weir tank, the notches and hook-gage used, and the arrangement of the calibrating apparatus are described. The results are plotted and compared with those obtained by James Barr, published in "Engineering" in 1910.

IN 1912, at the Annual Meeting of the Society, the author presented a paper[2] on the subject The V-Notch Weir Method of Measurement, and now after fourteen years of further experience with V-notch weirs, it seems appropriate to present a second paper, which will have to do more particularly with the degree of accuracy which can be counted upon when using this method of measuring fluids.

Object of Test

2 We recently had occasion to build for the new Richmond Station of the Philadelphia Electric Company a V-notch meter with a maximum capacity of a million pounds of water per hour. This meter was to be used as a standard piece of testing apparatus for calibrating orifice and other types of meters, which are used for measuring various rates of flow in this station. As we were obliged to guarantee accuracies of this meter of ½ of 1 per cent over its entire range, whether using full 90-deg. weir plates or fractions thereof, it seemed advisable to make a special calibration of it. The Civil Engineering Department of the University of Pennsylvania kindly placed at our disposal their hydraulic laboratory for making the tests.

[1] Yarnall-Waring Company.
[2] See Trans. A.S.M.E., Vol. 39 (1912), p. 1055.

Contributed by the Power Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

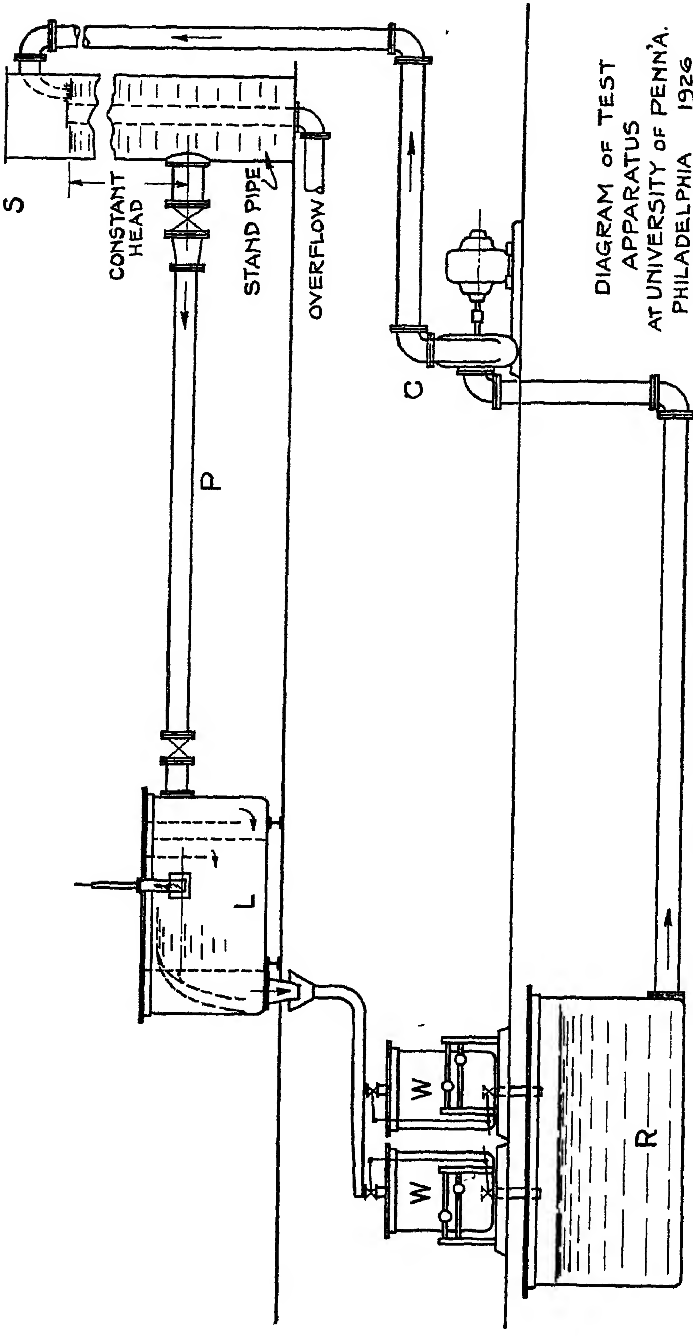


FIG. 1 DIAGRAM OF TEST APPARATUS

3 The paper deals with these tests, therefore, and it is hoped that the interesting results obtained may be of value to engineers who may have occasion to use this method of measurement.

Personnel

4 Acknowledgment is made to those who were instrumental in carrying through the investigation: Prof. W. S. Pardoe, of the

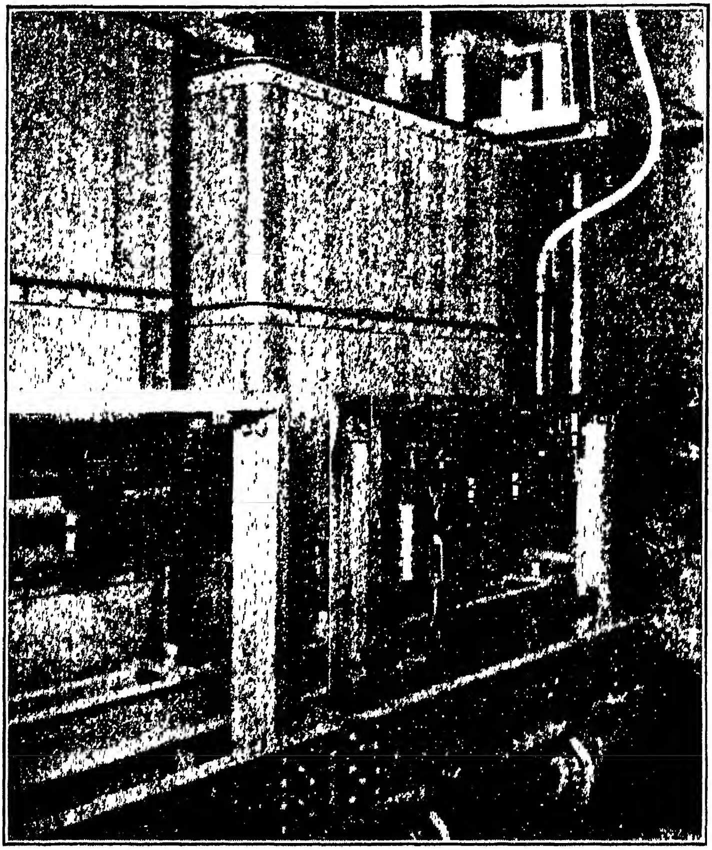


Fig. 2 Weighing Tanks

University of Pennsylvania, F. G. Ely, test engineer of the Philadelphia Electric Company, and A. L. Aicher, chief draftsman of the Yarnall-Waring Company; it is through the patience and painstaking work of these men that the highly satisfactory results shown in this paper were obtained.

Description

5 Fig. 1 shows diagrammatically the apparatus used in making the tests, and the arrangement. On account of the design of the stand-pipe *S*, which was located on the floor of the hydraulic laboratory, we were able to maintain constant heads at all times on the inlet of the meter.

6 On this same floor was placed a large V-notch meter tank *L*, as shown, which was connected by means of a horizontal pipe *P*

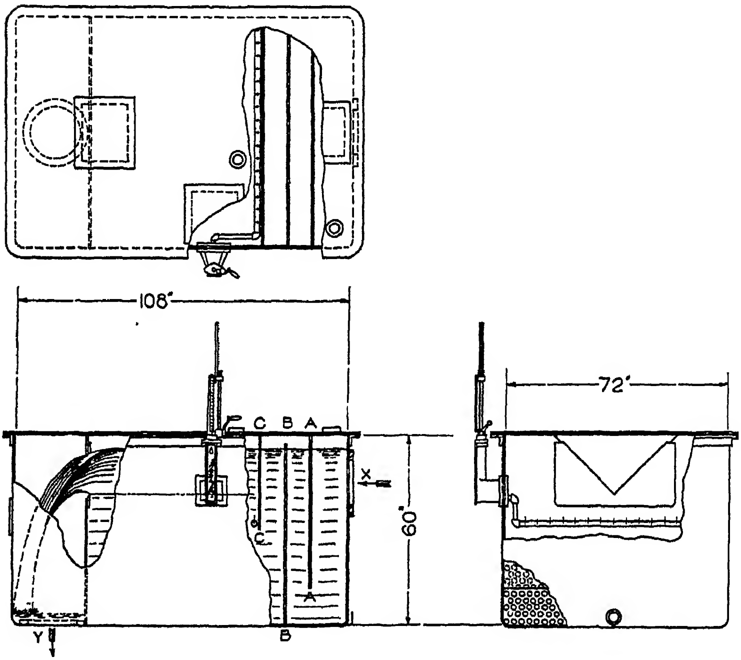


Fig. 3 The V-Notch Weir Tank

to the stand-pipe, this horizontal pipe having two gate valves for throttling the rate of flow. As a matter of fact, the gate valve near the V-notch tank was left open, except for regulating the lower rates of flow — 8-in. head and below.

7 The V-notch-meter tank discharged directly into a small hoppered flume, which in turn discharged into the weighing tanks *W*, shown on the floor below. These in turn discharged into the concrete reservoir *R* under the laboratory.

8 The water was circulated by means of a motor-driven centrifugal pump *C*.

Weighing Tanks

9 The weighing tanks in the laboratory were conveniently arranged, so that even at the maximum rate of flow of one million pounds per hour, no difficulty was found in accurately measuring the water in each tank in turn. Fig. 2 shows in detail the construction of the weighing tanks; and also shows the lever on the right of the weighing scale, which enabled the operator to discharge the tank; another lever controlled the inlet valve.

10 The accuracy of the weighing tanks with scales had been previously determined by means of a separate calibration.

Meter Tank

11 Fig. 3 gives in some detail the construction of the tank of the V-notch meter, which was the subject of this investigation.

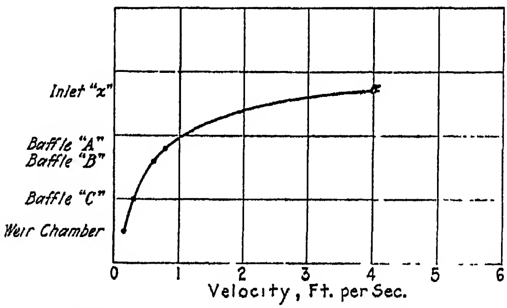


Fig. 4 Velocities in Approach Chamber of Weir Tank

The tank measures 108 in. long, 72 in. wide, and 60 in. deep, these dimensions being adequate for a 15-in., 90-deg. V-notch weir having a maximum capacity of one million pounds per hour.

Triple System of Baffling

12 Many engineers have the impression that in order to obtain reliable accuracy in weir measurements, a long flume or approach chamber must be provided, in order to smooth out the lines of flow and insure a uniform velocity of approach, as well as to obtain a dependable still water surface from which to measure the head. Eighteen years of experience with this method of measurement has shown that, if baffle plates are carefully designed, it is quite possible in a comparatively short tank to handle relatively large volumes of water without serious disturbance of the water surface in the approach chamber. In modern, rather condensed, power stations the advantages of this design are obvious. In the

meter here shown, it is believed that the baffling system is about as satisfactory as one could desire. It will be noticed that opposite the inlet X is provided a vertical baffle plate, which runs down to

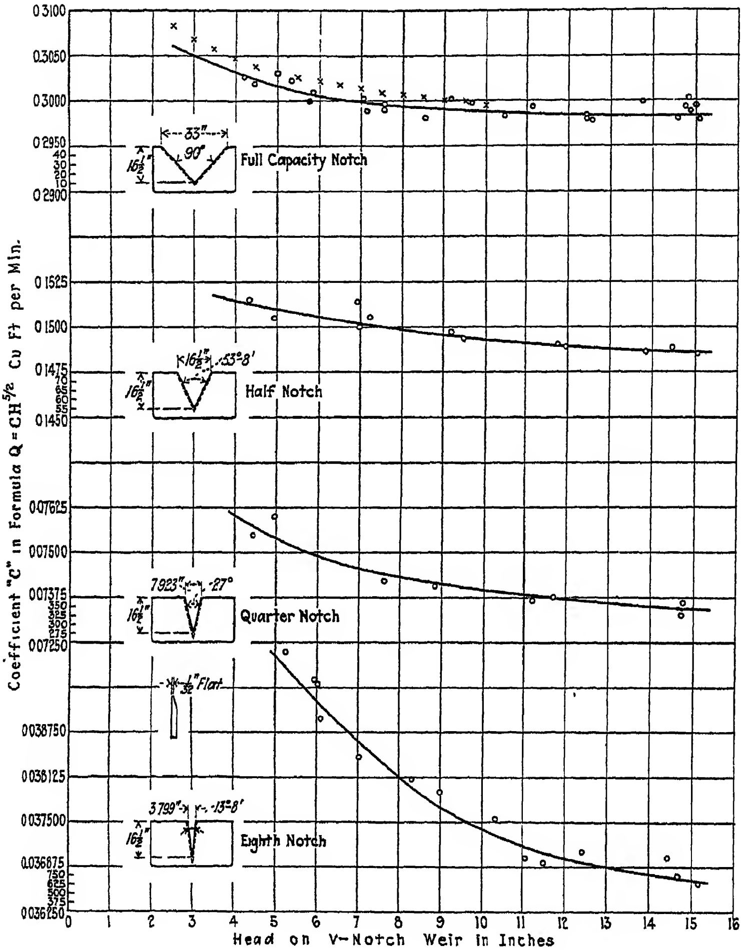


FIG. 5 RESULTS OF TESTS — COEFFICIENTS FOR THE VARIOUS NOTCHES
x indicates values obtained by James Barr. See *Engineering*, April 8 and 15, and Oct. 28, 1910.

within 12 in. of the bottom of the tank, the opening extending across the entire (72 in.) width of the tank. Eight inches in front of this first baffle *A-A* is a second vertical baffle *B-B*, which is perforated over its entire surface, and which extends almost to the cover of the tank. Then 8 in. in front of this second perforated

baffle is a third baffle *C-C*, which extends down from the cover plate about one-half the depth of the tank.

13 This " triple system " of baffles is so designed as to produce a gradually decreasing velocity of flow of the water from the

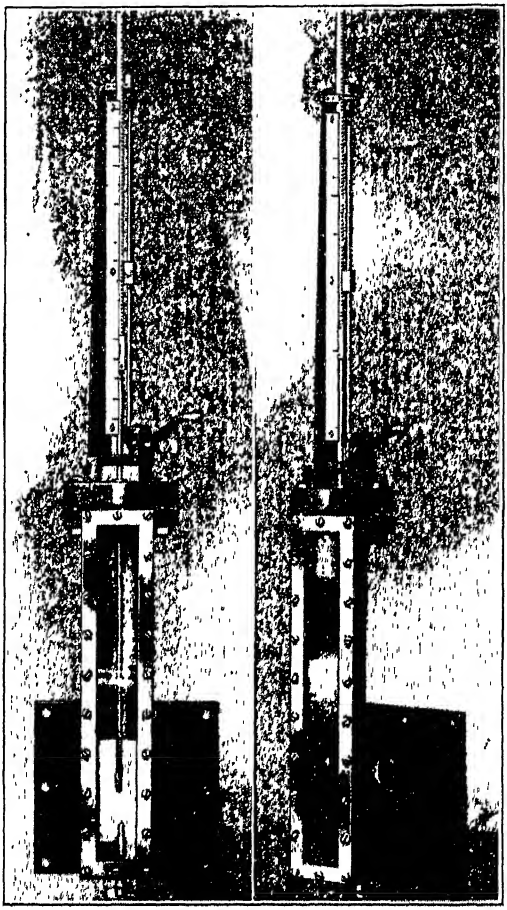


FIG. 6 THE HOOK-GAGE

inlet *X* to the approach chamber. These velocities are plotted on the diagram shown in Fig. 4. The results obtained with this system were so satisfactory that on the water surface of the approach chamber, even at the rate of flow of one million pounds per hour, there was practically no disturbance of the water surface, and most accurate hook-gage readings were possible.

V-Notches

14 Details of the four sharp-edged V-notch weir plates (actually measuring $\frac{1}{32}$-in. wide) used for these tests are shown in Fig. 5. The outlet of the tank at *Y*, as shown, discharges directly into the weighing system below.

Hook-Gage

15 As very great accuracy in the reading of the head was required for such a calibration, a special hook-gage, shown in Fig. 6, was designed and built. This hook-gage was mounted as shown on the side of the weir chamber just at the zero level of the weir. It is built in the form of a "bay window" and bolted to the side of the steel tank. The front of the projecting casting is of glass, so that the hook-gage position can be conveniently observed as it is moved up and down to meet the water level by means of the small hand-operated crank, shown in Fig. 6. A vernier is attached to the movable rack and passes in front of the vertical scale in such a position as to be very easily read by the operator standing in front of the hook-gage. The permanent zero adjustment indicator at the bottom of the bay window is clearly shown in Fig. 6.

16 In order to obtain a more perfect still-water surface in the hook-gage chamber, a flange was bolted across the inlet of the bay window, to which was attached a perforated $1\frac{1}{4}$-in. pipe, as shown in Fig. 3. This pipe extended across the tank and was attached to and just above the bottom of baffle plate *C*, about 6 in. below the zero level of the meter.

17 In making zero adjustments before each test a hook-gage, furnished by the University, was used as an auxiliary for setting the hook-gage attached to the weir tank.

18 The cover plate of the tank was conveniently provided with three manholes, one over the inlet, one adjacent to the hook-gage, and one over the V-notch weir plate, so that during the tests, by means of electric lights placed just under the cover inside the tank, it was quite easy to observe the conditions of flow. These manholes also afforded means for making zero adjustments.

Test Data

19 In the experiments with the V-notch meter under discussion there were actually seventy-nine complete tests made, covering about one month in actual time consumed. These tests were made to check the accuracy of the coefficients of discharge used in the regular V-notch-weir formula:

$$Q = \text{Coefficient} \times H^{5/2}$$

in which *Q* equals the quantity in cubic feet per minute; *H* equals head in inches.

20 Four complete series of tests were made, the first series on the full-capacity 90-deg. V-notch, the second series on the "$\frac{1}{2}$ notch," the third series on the "$\frac{1}{4}$ notch," and the fourth series on the "$\frac{1}{8}$ notch." The exact dimensions of the notches tested are shown in Fig. 5.

21 Tests Nos. 1 to 17 inclusive were used to perfect the apparatus and the method of conducting the tests. The results of these first tests, therefore, were not used in plotting the coefficient curves of Fig. 5.

22 Test No. 32 was an experiment to try out, at a high rate of flow, the method of controlling the head by means of the 12-in. gate valve in the inlet connection next to the meter tank. The result of this test also was not used in plotting the coefficient curve.

23 During the tests there was very little variation in temperature of the water used. Actual tests for the density per cubic foot of the water were made by means of the U. S. Bureau of Standards weighing device, but there was no appreciable variation from the figure 62.34 lb. per cu. ft., which was used in all of the calculations.

24 In view of the convenience of the weighing apparatus described and shown in this paper, which is a part of the hydraulic-laboratory equipment of the University, it was possible to use the flying-start and flying-stop method; the time interval was obtained by an accurate stop watch, which was in the hands of Professor Pardoe, who actually operated the weighing tank levers, and hence was in a position to control the interval of each test.

25 Head measurements were made with very great care, by means of the hook-gage previously described. Two operators actually made the observations, one operator adjusting the hook-gage at fifteen-second intervals, the other operator recording the head at the same intervals. In this way many head readings were obtained and the average for the test used. It should be stated, however, that before any tests were made very careful adjustment of the inlet valve had been made to insure the head desired, and during the test there was, therefore, very little variation of this head.

26 Table 1 gives the actual results obtained.

Test Results

27 Fig. 5 shows graphically the results of the various tests made on the four different V-notch weir plates.

28 On the upper curve it will be noticed that we have indicated by means of crosses, the coefficients which were published by James Barr in *Engineering,* April 8 and 15, and October 28, 1910. Barr's tests were largely made on the lower rates of flow, but were made with the same 90-deg. V-notch weir plate as used in our tests. For some years past we have felt that his coefficient values were

too high and it is interesting, therefore, to notice that our curve of coefficients is slightly lower than his curve. To be sure, the

TABLE 1 RESULTS OF V-NOTCH WEIR TESTS

Date	Test No.	V Notch	Water weighed, lb.	Time period min.	Time period sec.	Water, lb. per hr	Head, in.	Coefficient
3/17/26	18	Full	134510	8	30 6	948360	14.814	0.30017
3/17/26	19	Full	137355	8	28 2	973010	14.9855	0.29924
3/18/26	20	Full	145450	9	09.8	952380	14.8671	0 29878
3/18/26	21	Full	150005	9	08.7	984186	15.083	0.2978
3/18/26	22	Full	93415	12	06.2	463080	11.1329	0 29936
3/18/26	23	Full	57250	12	03.2	284986	9.1605	0.3002
3/18/26	24	Full	131215	10	06.3	779100	13.718	0 2998
3/18/26	25	Full	179150	10	58.8	978975	15.0522	0 29775
3/18/26	26	Full	15000	14	17.2	62996	4 988	0.303085
3/19/26	27	Full	257505	16	29.7	936676	14.76	0 29919
3/19/26	28	Full	83170	15	21.7	324848	9.6574	0.29965
3/19/26	29	Full	151855	14	59	608100	12.439	0.29795
3/19/26	30	Full	121305	11	58.3	607955	12.429	0 29845
3/19/26	31	Full	166120	11	00 5	905425	14.5833	0.29805
3/20/26	33	Full	155895	15	01.2	622750	12.562	0.29768
3/20/26	34	Full	150395	14	29.5	622680	12 5597	0.29778
3/20/26	35	Full	31530	10	48.7	174976	7.5446	0.2992
3/20/26	36	Full	15000	10	08	88816	5.7462	0 3000
3/20/26	37	Full	31650	10	46.4	176267	7 5628	0.2996
3/20/26	38	Full	106555	16	08	396280	10.475	0.29833
3/23/26	39	Full	30750	12	12.8	151065	7.117	0.298875
3/23/26	40	Full	30685	12	22	148880	7.063	0.30022
3/23/26	41	Full	62245	15	45 6	236970	8.529	0.2981
3/23/26	42	Full	15000	9	46.4	92088	5.822	0.30102
3/23/26	43	Full	13000	16	30 9	47229	4.452	0.30192
3/23/26	44	Full	15000	12	06	74378	5.3364	0.302275
3/23/26	45	Full	8500	12	26 4	40997	4.203	0.302645
3/24/26	46	1/2 90	5000	13	41.3	21917	4.316	0.15141
3/24/26	47	1/2 90	14000	11	47.4	71247	6.9177	0.15133
3/24/26	48	1/2 90	117800	16	01 8	440930	14.440	0.14877
3/24/26	49	1/2 90	83010	18	08 2	274613	11.9455	0 14887
3/24/26	50	1/2 90	28780	12	03 3	143242	9.1876	0.14967
3/25/26	51	1/2 90	106220	16	09 3	394500	13.8196	0.14858
3/25/26	52	1/2 90	53580	12	12 6	263290	11.7412	0.11901
3/25/26	53	1/2 90	124385	15	20.9	486250	15.0315	0.1484
3/25/26	54	1/2 90	30615	11	55.3	154080	9.469	0.1493
3/25/26	55	1/2 90	15000	12	29 3	72067	6.9741	0.150
3/25/26	56	1/2 90	15000	11	26 8	78625	7.2154	0.15031
3/25/26	57	1/2 90	6000	11	48.3	30495	4.9378	0.15048
3/26/26	58	1/4 90	15000	3	55.3	229500	14.737	0.07859
3/26/26	59	1/4 90	61840	16	21.8	226753	14.6947	0.07324
3/26/26	60	1/4 90	30420	14	09.7	128880	11.688	0.07376
3/26/26	61	1/4 90	30790	16	04.7	114900	11.171	0.07365
3/26/26	62	1/4 90	15000	13	54.3	64725	8.8606	0.074045
3/26/26	63	1/4 90	8000	10	51.8	44186	7.60	0.07419
3/26/26	64	1/4 90	4500	17	18.2	15604	4.964	0.075985
3/26/26	65	1/4 90	3000	15	20.2	11737	4.442	0.07546
3/27/26	66	1/8 90	15000	7	22 5	122033	15.131	0.036635
3/27/26	67	1/8 90	15000	8	16.3	108805	14.395	0.037
3/27/26	68	1/8 90	15000	8	00.2	112453	14.625	0.036755
3/27/26	69	1/8 90	11000	11	45 2	56154	11.033	0.03713
3/27/26	70	1/8 90	11000	10	42.2	61663	11.478	0.036935
3/27/26	71	1/8 90	3500	15	41.8	13379	6.1	0.03892
3/27/26	72	1/8 90	3500	11	10.9	18780	7.0237	0 038405
4/ 7/26	73	1/8 90	2000	12	36 1	9523	5.2738	0.03986
4/ 7/26	74	1/8 90	2500	11	26 5	13110	6.02	0.039416
4/ 7/26	75	1/8 90	2500	11	35 8	12935	5.9846	0.03947
4/ 7/26	76	1/8 90	6000	10	30.2	34275	8.9804	0.037915
4/ 7/26	77	1/8 90	6000	12	43	28310	8.3043	0.038085
4/ 7/26	78	1/8 90	7500	9	21.1	48119	10.327	0.03754
4/ 7/26	79	1/8 90	10000	7	59 6	75062	12.309	0.037072

difference is very small; nevertheless, we must seek to eliminate all factors which may introduce even slight errors in the measurement of rates of flow.

29 In the experiments which we have made we did not devote any time to determining the coefficients for the lower rates of flow, because there are already many data other than James Barr's covering this particular phase of the question.

30 Attention is called to the coefficient curves for the ½ notch, ¼ notch, and ⅛ notch, which were obtained in exactly the same way, and with the same apparatus as used for the tests on the full 90-deg. notch.

31 Fig. 7 shows the V-notch meter, which was used in these tests, as it is installed in the testing laboratory of the new Richmond Station of the Philadelphia Electric Company, where it is used in the regular routine of station testing of orifice and other meters.

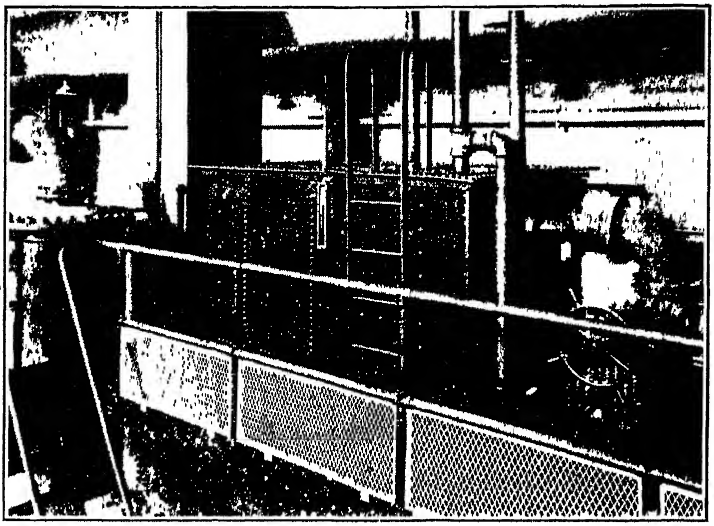


FIG. 7 THE V-NOTCH WEIR TANK IN ITS PERMANENT LOCATION

CONCLUSION

32 From the many tests which have been made with the apparatus described in this paper, we are convinced that accuracies within ½ of 1 per cent may be obtained by anyone who uses the coefficients corresponding to any given head, shown by the curves in Fig. 5, because all of the points from which the curves were plotted are within this range of accuracy.

33 At some future time we hope that it may be possible to carry these tests still further by establishing coefficients at rates of flow corresponding to 20-in. and even 24-in. head. We presume, however, that such coefficients will be only slightly different from the coefficients at 15-in. head, in view of the fact that all four of the

curves seem to approach the horizontal at this point, as will be noted by observing Fig. 5.

DISCUSSION

W. S. Pardoe.[1] The author has laid before us data that are usually considered private, with all the experimental points plotted. He has thus given the reader a chance to estimate or compute the probable error involved in the use of the V-notch weir.

The curves in Fig. 8 are a record of tests by various students on a 90-deg. V-notch weir in the Civil Engineering Laboratory of the University of Pennsylvania. The cross-section of the weir tank is as shown, the length being 34 ft. Hence, it is much longer and smaller in cross-section than the author's tank. The value of the coefficient C plotted was obtained from the expression

$$Q = C\frac{8}{15}\tan\frac{\theta}{2}\sqrt{2g}H^{\frac{5}{2}}$$

On the same sheet the author's curve, reduced to the same basis, is shown in a dotted line. The author's curves show a higher degree of accuracy than the University of Pennsylvania curve. Probably the experimenters were more careful and accurate with their work, but the V-notch weir is not a device of high accuracy for measuring water. We cannot have a high degree of accuracy and a great variation in discharge for a relatively small variation in head. This is evidenced by the spotty nature of the points on all the curves.

In experimental work the errors which creep in are due largely to the measurement of the head H. If the expression for the discharge of a V-notch weir be differentiated, thus

$$Q = CH^{\frac{5}{2}}$$

$$dQ = \tfrac{5}{2}CH^{\frac{3}{2}}dH = \tfrac{5}{2}CH^{\frac{5}{2}}\frac{dH}{H}$$

or

$$\frac{dQ}{Q} = \tfrac{5}{2}\,\frac{dH}{H}$$

That is, if the error in measuring H is 1 per cent, the error in Q is $2\frac{1}{2}$ per cent.

Again, if the expression for the right weir $Q = CH^{\frac{3}{2}}$ be differentiated similarly, the result will be

$$\frac{dQ}{Q} = \tfrac{3}{2}\frac{dH}{H}$$

or $1\frac{1}{2}$ per cent error in Q for a 1 per cent error in measuring H.

[1] University of Pennsylvania, Philadelphia, Pa.

Similarly for the orifice $Q=C\sqrt{H}$ and

$$\frac{dQ}{Q}=\frac{1}{2}\frac{dH}{H}$$

or but $\frac{1}{2}$ per cent error in Q for a 1 per cent error in measuring H.

Although it may not be possible to measure H with the same percentage of accuracy with each of these flow meters, nevertheless numerous plots of experiments indicate that such a relation exists.

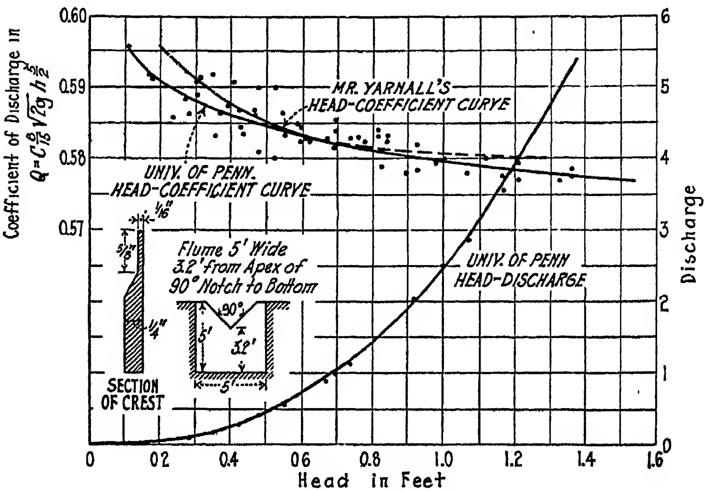


FIG. 8 HEAD-COEFFICIENT AND HEAD-DISCHARGE CURVES FOR 90-DEG. V-NOTCH WEIR

The V-notch weir finds its greatest usefulness in the measurement of a quantity which varies greatly and still gives moderate heads for the largest rates.

CARROLL F. MERRIAM.[1] The diagrams, Figs. 9 and 10, illustrate the relative merits of metering apparatus in which the flow varies with the $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ powers of the head, respectively. This would be typified by the orifice or rectangular weir and the V-notch weir. Fig. 9 shows the variation of head for percentages of capacity discharge.

In Fig. 10 an attempt has been made to show the effect of a constant error, which in each case is equal to $\frac{1}{10}$ of 1 per cent of the full capacity head. It will be readily seen that although at full capacity the orifice, assuming all other conditions equal,

[1] Pennsylvania Water & Power Company, Baltimore, Md. Mem. A.S.M.E.

has five times the accuracy of the V-notch, still at fractional loads this advantage is soon lost.

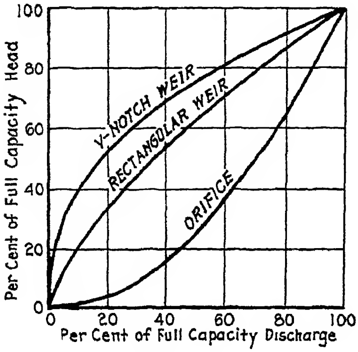


Fig. 9

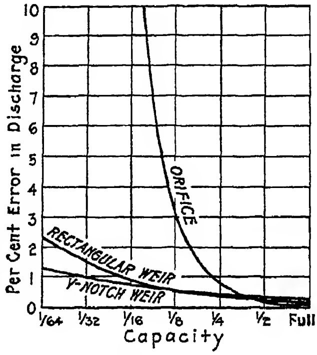


Fig. 10

Fig. 9 Variations of Head for Various Fractions of Capacity Discharge

Fig. 10 Effect of Constant Error in Discharge

For the purpose for which the weir described by the author is apparently intended, i.e., for a standard for checking meters of various ranges, the V-notch is admirably adapted.

A. G. Christie.[1] This paper contributes some valuable engineering data in the form of accurately determined constants for V-notch weirs of large capacity.

The author states that a hook gage was used as an auxiliary in making zero adjustments before each test. This zero adjustment is one of the most important adjustments in the whole test. Can the author tell us whether the hook gages were set by means of spirit levels or whether the water level at no flow over the weir was used in determining the zero reading? Also was there any allowance made for the meniscus of the water at the weir and at the hook gage if adjustment was made by water levels?

Some years ago the writer endeavored to measure a cold brine solution by means of a V-notch weir and hook gage. This solution changed in density and viscosity from time to time. Our tests indicated that there might be some definite relation between the coefficient of the weir and the density, and also the viscosity of the brine, but sufficient data were not available to draw definite conclusions. Has the author any information regarding the effect of viscosity on discharge coefficients?

The weir tank used by the author is so arranged that the discharge from the weir flowed freely into the discharge pipe and

[1] Professor of Mechanical Engineering, Johns Hopkins University, Baltimore, Md. Mem. A.S.M.E.

out into the weighing scales. When V-notch weirs are used in commercial work, the level on the discharge side of the weir rises and falls depending on operating conditions. It may, at times, approach quite closely to the bottom of the V-notch. Does this varying water height have any reaction effect on the flowing stream from the weir and in that way modify the coefficient of the weir?

J. M. Spitzglass.[1] In Par. 2 the author states that the meter was to be used as a standard piece of testing apparatus for calibrating *orifice and other types of meters.* In general, he does not explain how the other devices were to be tested by this meter. The coefficients plotted in Fig. 5 probably represent the true accuracy of the weir proper. The question then arises, if the tests to be followed in the plant will involve certain mechanical recording and integrating devices, will there be any more certainty in this assumed standard than in any one of the orifice meters, or any other meters that are to be tested by it? If, on the other hand, the comparative tests are to be made by reading the head on the weir with scientific precision, as outlined in the paper, there is no necessity for using the weir, as similar tests can be made directly on the orifice meter or any other differential device.

The paper describes a scientific test on what we call the primary device of the flow meter. It does not appear from the paper that a secondary device was in any way considered for this test. How then will this weir be used as a standard for testing other meters? Will it be used with or without a secondary device? If it is to be used with a secondary device of the recording and integrating type, it no doubt will be accurate enough for practical purposes, but it can no more be considered as a standard for testing other meters.

Fig. 4 shows the velocities of approach in the chamber. How were these velocities obtained? That is, what method was used in determining the actual velocities at the given sections of the chamber?

Fig. 5 shows the varying coefficients for the various notches. The coefficient is shown to vary, both with the head of water and with the shape of the notch. The second variation depends, of course, upon the relation between the area and the height. Has the author ever tried to incorporate the geometry of the figure into the equation, so that he could obtain the same coefficients for the various notches? It would be well to determine the relation, if such exists, so that a test on a given shape of weir notch would apply to all other shapes of similar structure. One thing is certain, that in all cases of weir measurement the integrating and recording devices must be so constructed as to take care of the variation

[1] Vice-President, Republic Flow Meters Co., Chicago, Ill. Mem. A.S.M.E.

of the coefficient with the head on the weir. The writer agrees with the conclusion that the coefficients obtained are accurate within ½ of 1 per cent, provided those coefficients are taken to correspond with the given head, and provided the readings are obtained with the precision outlined in the paper.

In comparing the weir with the orifice, Professor Pardoe brought out very clearly where the two will agree and where they will not, by stating that one varies with the 5/2 power of H, the other one with the one-half power of H. In the case of the weir the readings are very accurate at the bottom, whereas with the orifice they are very accurate at the top. There may be a point where one can be tested with the other, but in general the writer suggests that they be tested separately.

V. M. Frost.[1] Mr. Spitzglass, in his discussion, has touched one of the most important elements in regard to the accuracy of V-notch meters or, in fact, of almost any form of orifice meter.

In this particular case, the head was measured by a hook gage which involved only the personal equation of the operator in determining the zero setting of the gage and the moment at which the hook point pierced the water surface. The amount of flow was then determined by inserting the average observed head in a formula which contained a constant for the particular meter.

In the V-notch meter, as more commonly used, the head is determined by means of a float, which in turn operates a cam device which contains in its construction the formula and constant for the meter, so that as a result of any vertical movement of the float the quantity flowing is indicated directly on a scale or chart.

The accuracy of the result is thus affected by other things than the mere measurement of the head but includes, for instance, such things as the accuracy of the machine work on the cam motion and the lost motion and friction of the moving parts.

Hence the real accuracy of any such meter depends upon the accuracy with which the head causing the flow is measured and converted into the quantity flowing.

In a few cases, where we were unable to install weigh tanks of the proper capacity, we have checked the float indication by means of a hook gage installed temporarily, and have been satisfied if the two results agreed.

C. C. Trump.[2] Viscosity affects the coefficients of weirs and orifices with sharp edges. The higher viscosities give less contraction and higher coefficients; perhaps contrary to the usual expectation.

[1] Assistant to General Superintendent of Generation, Public Service Electric & Gas Co., Newark, N. J. Mem. A.S.M.E.

[2] Engineer of Tests, The Atlantic Refining Co., Philadelphia, Pa. Assoc-Mem. A.S.M.E.

With a constant viscosity, a coefficient may be applied to sharp-edged weirs and orifices. This is shown in a report to Goulds Pumps, Inc., made by Professor Daugherty on pumps and orifices used with various oils.

With varying viscosities, however, a rounded edge has been found to reduce the contraction for thin liquids. Hence, high and low viscosities have coefficients nearly enough the same for all practical purposes, and of the order of accuracy of plus or minus ½ of 1 per cent. A rounded-edge orifice is easier to make than a weir, and a series of orifices may be arranged to replace a weir with proportional flow.

M. C. STUART.[1] In order to determine the relation between notch angle and performance of the V-notch weir, the writer has computed Mr. Yarnall's data upon the basis of the fundamental

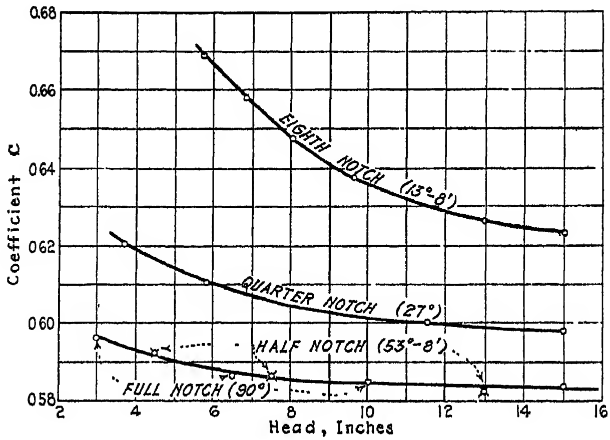


FIG. 11 CURVES SHOWING RELATION BETWEEN COEFFICIENT C AND HEAD ON WEIR FOR VARIOUS ANGLES OF NOTCH

(Formula used given in text of discussion.)

triangular-weir formula, which includes the notch angle. This formula, the derivation of which may be found in any standard treatise on hydraulics, is

$$Q = C\tfrac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}\,H^{\frac{5}{2}}$$

in which

Q = discharge, cu. ft. per sec.
θ = angle of notch
H = head on weir, ft.

[1] Professor of Experimental Engineering, Lehigh University, Bethlehem, Pa. Mem. A.S.M.E.

The coefficient C in the above formula is a general triangular-weir coefficient which is indicative of the amount of contraction and energy losses in the weir of any angle.

The results of the computation for C from Mr. Yarnall's data, as given on his smooth curves, are given in Fig. 11.

It is very interesting to note that the coefficient for the 90-deg. and the 53-deg. (half-notch) weirs are practically identical, which means that these weirs, and presumably weirs of any intermediate angle, have the same proportionate contractions and losses. This is obviously a very valuable feature of the V-notch. The quarter-notch and the eighth-notch weir coefficients are somewhat larger, but of the same order of magnitude. By cross plotting these coefficients against angle of notch, the coefficient for any angle may be determined and thus the discharge for weirs of any angle may be computed with a good degree of accuracy.

Morrough P. O'Brien.[1] When the values representing head in inches are plotted against the pounds discharge per hour (Table 1) on logarithmic paper, as in Fig. 12, the points come very close to lying on straight lines which seem to be parallel. This indicates that, within the range of the experiment, the discharge may be expressed in the form

$$\log q = \log m + n \log h$$

which is equal to

$$q = mh^n$$

where

q = the rate of discharge

h = the head

m and n = pure numbers to be determined from the experimental data.

Taking the logarithms of the head and discharge and applying the centroid method[2] for determining the best line joining the points, the following formulas are obtained:

Angle	Formula: h, ft., Q, cu. ft. per sec.	Formula: head, in., Q, lb. per hr.
90°	$Q = 2.48\, h^{2.49}$	$Q = 1144.3\, h^{2.49}$
53° 8′	$Q = 1.242\, h^{2.48}$	$Q = 570.0\, h^{2.48}$
27°	$Q = 0.613\, h^{2.475}$	$Q = 293.1\, h^{2.475}$
13° 8′	$Q = 0.3312\, h^{2.42}$	$Q = 168.3\, h^{2.42}$

Since the formulas for the discharge in pounds per hour are based on a water density of 62.34 lb. per cu. ft., in determining the discharge of water having another density, the result as given by the formula should be multiplied by the density divided by 62.34.

[1] Engineering Experiment Station, Purdue University, Lafayette, Ind.

[2] Measurement of Pipe Flow by the Coördinate Method. Jour. Amer. Water Works Assn., vol. 13, no. 3, March, 1925.

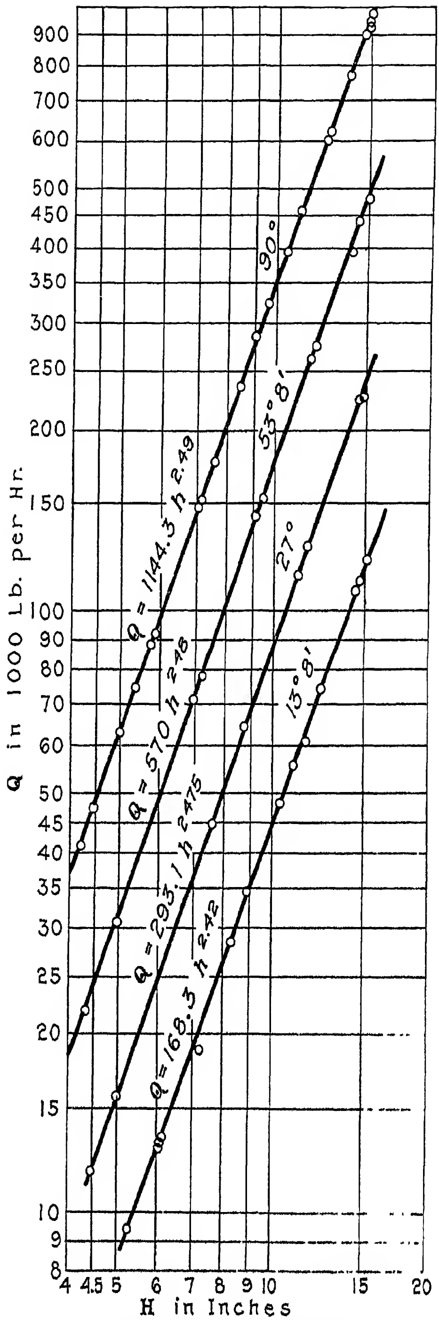


FIG. 12 HEAD-DISCHARGE CURVE

A general formula for the discharge has been developed by W. V. Cone[1] of the form

$$Q = (0.025+2.462S)H^{\left(2.5-\frac{0.0195}{S^{0.75}}\right)}$$

where

$$Q = \text{discharge in cu. ft. per sec.}$$
$$S = \text{side slope}$$
$$H = \text{head in feet}$$

Substituting the values of S for the angles used in Mr. Yarnall's experiments gives:

Angle	Formula
90°	$Q = 2.49\ H^{2.48}$
53° 8′	$Q = 1.256\ H^{2.47}$
27°	$Q = 0.5912\ H^{2.443}$
13° 8′	$Q = 0.283\ H^{2.40}$

The values of both m and n are smaller for the 27° and 13° 8′ notches than were obtained from Mr. Yarnall's data. This disagreement might have been expected, since these angles are outside the range for which the Cone formula was developed, namely, 28° 4′ to 120°.

Considering the two remaining angles (90° and 53° 8′), the average difference between the values of discharge for either angle by the Cone formula and the formula previously developed from Mr. Yarnall's data, may be obtained by determining the difference in the discharges where $H = 1$ and the value of H where the discharges are equal.

Taking the 90° notch,

$$2.49H^{2.48} = 2.48H^{2.49}+E$$
$$H = 1$$
$$E = \frac{0.01}{2.49} = 0.4\%$$
$$Q_c - Q_y = 0$$
$$2.49H^{2.48} = 2.48H^{2.49}$$
$$\frac{2.49}{2.48} = H^{0.01}$$
$$H = \left(\frac{2.49}{2.48}\right)^{100} = 1.496 \text{ ft.}$$

Since the logarithms of these two equations are straight lines intersecting at the point $H = 1.496$, the general equation for the percentage difference is

$$E = K \log H + C$$
$$H = 1, \quad \log H = 0, \quad E = 0.4\%, \quad C = 0.4$$
$$H = 1.496, \quad \log H = 0.175, \quad E = 0.0\%, \quad C = 0.4$$
$$K \log H = -0.4$$
$$K = 2.29$$
$$\text{For } H = 0.2, \quad E = 2\%$$

[1] *Journal of Agricultural Research*, vol 5, no. 23.

In the case of the 53° 8′ V-notch, when

$$H = 1.0' \qquad E = 1.14\%$$
$$H = 3.07' \qquad E = 0.0$$
$$H = 0.2' \qquad E = 2.7\%$$

The value of the use of weir formulas of the type $q = mh^n$ lies in the fact that through them we are able to eliminate a large part of the erratic errors in the experimental data, which is not possible when we use a convenient but arbitrary power of one of the measured quantities and compute a so-called "constant." When an equation is derived by the centroid method or more accurately by the method of least squares, we can compute the probable error of any particular reading by comparing it with the general formula. This has been done for the 27° weir, with the results shown in Table 2.

TABLE 2

27° V-notch, $Q = 0.613\,H^{2.47}$

Run	Lb. per hr.	Cu. ft. per sec.	Head, in.	Head, ft.	$H^{1.47}$	Q	Difference	Per cent
58	229,500	1.023	14.737	1 227	1.3508	1.016	+0.007	0.69
59	226,753	1.010	14 695	1.225	1.3476	1.012	—0.002	0.20
60	128,880	0.574	11.688	0.974	0.9620	0.574	0	0.0
61	114,900	0.5120	11.171	0.931	0.9002	0 514	—0.002	0.30
62	64,725	0.2884	8.861	0.738	0.6308	0.2895	—0.001	0.35
63	44,186	0.1969	7.60	0 633	0.5106	0.1980	—0.0011	0.56
64	15,604	0.06953	4.964	0.4135	0.2730	0.0692	+0.0003	0.43
65	11,737	0.05230	4.442	0.3700	0.2319	0.0521	+0.0002	0.38
							TOTAL =	3.00

$$\text{Average error} = \frac{3}{8} = 0.375$$

$$\text{Probable error} = \frac{0.375}{\sqrt{8}} = 0\ 13$$

The average error expresses the probable error of any future observation, with an even chance that, on a single observation of head, the discharge will be in error by more or less than 0.37 of one per cent.

The probable error of the equation is the average error of the individual readings divided by the square root of the number of observations;[1] therefore, the equation may be written

$$Q = 0.613H^{2.47} \pm 0.13\%$$

This probable error expresses the degree of accuracy of the series of experiments as a whole and should be very useful in weighing different experimenters' results when combining them into a single equation.

The practical difficulty of using a formula having a power of the head other than 2.5 is in raising the number representing the head to this odd power. However, if these formulas come into general use, tables can be made up giving the ordinary ranges

[1] An Elementary Treatise on Precision of Measurement. Wm. S. Franklin, Franklin & Charles, Publishers, Lancaster, Pa.

of heads raised to the most commonly found powers as has already been done for the 1.47 power in King's Handbook of Hydraulics.

The comparison, given above, between results of the experiments of Mr. Yarnall and Mr. Cone, reveals a type of error which is much more difficult to eliminate than the erratic errors of a single series of experiments. Properly termed the "error of condition," it is made up of a number of smaller errors resulting from peculiar conditions in the channel of approach, from different materials used for the weir plate, from errors made in determining the angle of the notch and the zero of the hook gage, and in the calibration of the volumetric or weighing tanks. Since these errors are of different magnitudes and signs in different laboratories, it seems that the only way to obtain a formula for each particular shape of weir which will show small errors under all conditions, is to combine the results of all existing experiments, weighing each particular series of experiments according to the probable error which it indicates.

The staff of the Hydraulic Laboratory at Purdue University has been working on this problem in connection with semi-circular and triangular weirs and, although a formula has not yet been worked out, careful examination of the existing reliable data indicates that an equation may be derived which will fit most of the different sets of experiments with very small error.

Frederic G. Ely.[1] The weir equipment described by the author is now being used at the Richmond Station of the Philadelphia Electric Company as a test meter for calibration of various orifices employed in general plant metering service.

Orifice sections from the station piping are inserted in the inlet piping of the weir meter, and calibration points obtained throughout their working ranges. Weir measurements are made by the method described in the University of Pennsylvania tests, and the orifice indication of flow determined from fluid meter records, and from differential manometer readings of venturi head. The manometer, of special design, employs mercury as a fluid medium, and its tube is inclined at such an angle that a given differential head of water will produce a corresponding deflection of the mercury column. Both meter and manometer may be calibrated by means of vertical water columns connected to the line.

The orifice section, after careful inspection, is installed in the desired location in the test piping, meter connections properly made, and the weir, most suitable to the range of the orifice, installed in the tank. Calibration checks are made of the meter and manometer by the water-column method.

[1] Superintendent of Plant Efficiency, Philadelphia Electric Company, Philadelphia, Pa. Jun. A.S.M.E.

The zero correction for the external hook gage is found through the use of an auxiliary hook gage, mounted close to the weir plate, which permits accurate mechanical adjustment to the elevation of the crest. By using a calm water surface as a reference plane, both the auxiliary and the external gages are set to the same

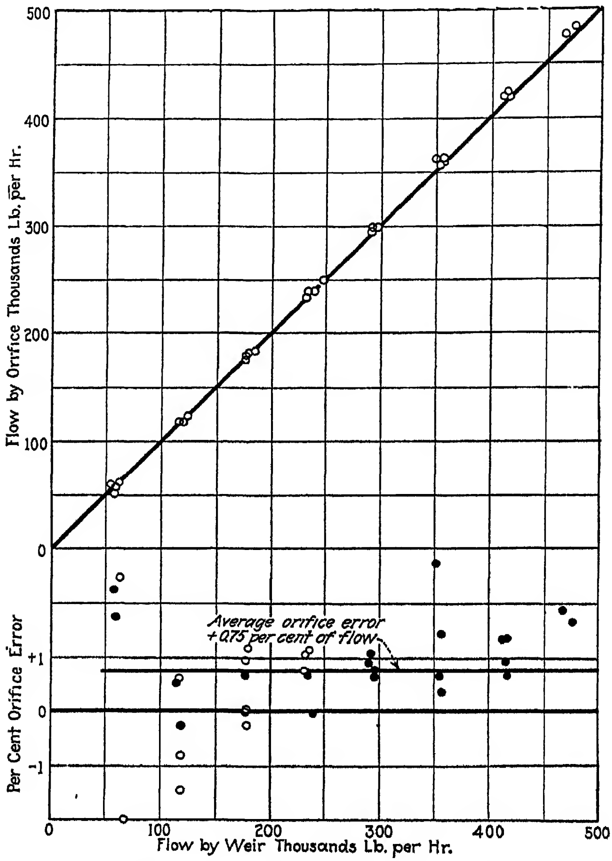


FIG. 13 TYPICAL CALIBRATION CURVE FOR AN ORIFICE

Test of Aug. 23, 1926. No. 10 condensate pump discharge orifice calibration. 8-in. line, horizontal flow orifice. Maximum capacity, 600,000 lb. per hr. at 90 deg. fahr. Calibration temperature, 87 deg. fahr. Open circle, 1/4 90-deg. weir; closed circle, 1/2 90-deg. weir.

elevation, and comparative readings taken to indicate any discrepancy of zero values.

Typical results of these orifice tests are shown in Figs. 13 and 14, and a more detailed description of the apparatus and methods employed may be seen in *Power,* Mar. 8, 1927, p. 352.

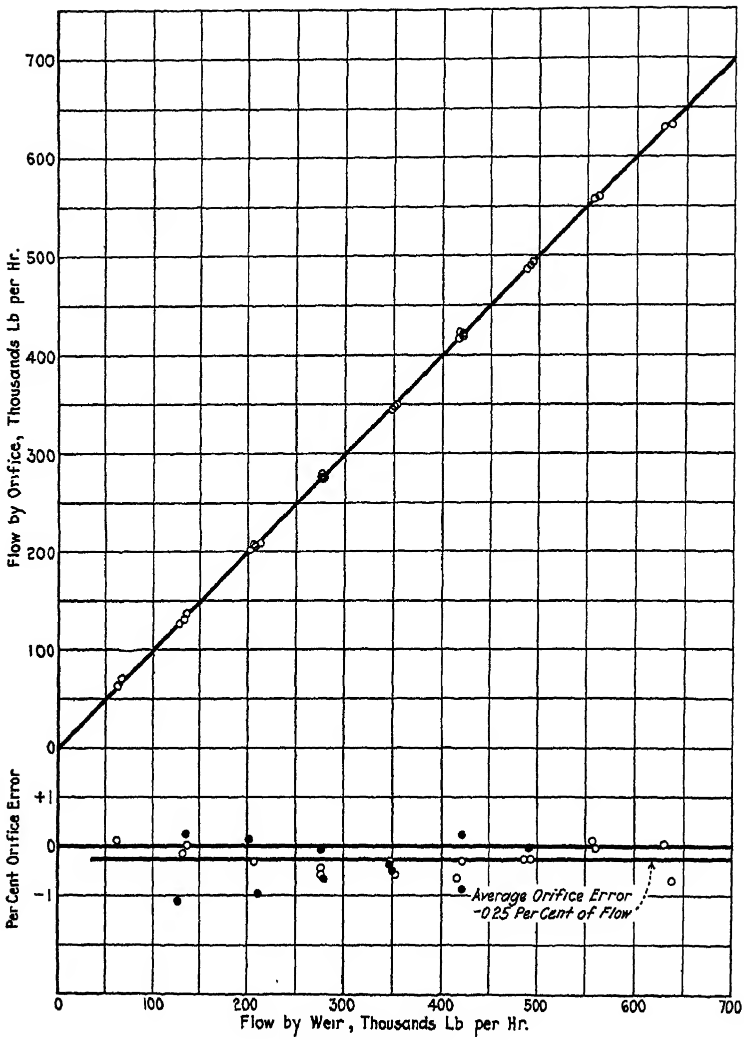


Fig. 14 Typical Calibration Curve for an Orifice

Test of Sept. 16, 1926. No. 10 third heater discharge orifice calibration. 10-in. line, vertical down-flow orifice. Maximum capacity, 700,000 lb. per hr. at 285 deg. fahr. Calibration temperature, 81 deg. fahr. Open circle, full 90-deg. weir; closed circle, 1/2 90-deg. weir.

The Author. In this paper description of the translating device or automatic recorder for continuously indicating the rate of flow was purposely omitted because the basis of any such device is the coefficient. We desired to give the engineering profession the advantage of these tests which give the coefficients for the higher heads; these data, so far as known, never heretofore have been published. As to the translating device, the author believes in the type having a long curve of flow wrapped around a drum. The curve or cam of such an instrument is plotted by using the coefficient values which are obtained in such tests as have been described. It is reasonably safe to say that the well-known Lea recording device, which has such a curve of flow wrapped around a drum, will give accuracies well within 1 per cent from zero up to the maximum capacity of the meter.

In regard to the relative merits of the orifice, the venturi tube and the V-notch weir, each one has its field. The V-notch weir method of measurement has an accuracy well within 1 per cent over its entire range from zero to maximum flow. The fact mentioned by Mr. Spitzglass and Professor Pardoe that those accuracies go over the entire range is a generally recognized advantage of the V-notch.

As to the mathematics of the subject, may we refer to the very able discussion which precedes these closing remarks. From the V-notch tests described in this paper and plotted in Fig. 5, the curve of coefficients for 90 deg. and half notch very definitely flatten out, so that the coefficients probably for 20-, 24-, and 30-in. heads will lie along a horizontal line.

In answer to the questions raised about Fig. 4: These velocities were calculated on the basis of maximum flow and conveniently plotted in a curve which graphically shows their variation as the water flows from the inlet to the discharge over the V-notch.

In answer to Professor Christie's question as to how the zero was established, in commercial practice the zero is usually established by gradually drawing off the water from a high level until it comes down to the zero pointer which has previously been mechanically set from the apex of the V-notch. That method, however, was not used in these tests, because it is not as accurate as the one followed. In the first place, we used a supplementary hook gage, placed not very far behind the V-notch on the upstream side. This supplementary gage, by means of a spirit level, was actually set at the apex of the V-notch. A zero reading (x) was then taken on its scale. The water was next run off about $\frac{1}{2}$ in. below the zero line or V-notch apex and a second reading (x') taken. Without any change in this water level a reading was also taken on the bay-window hook gage; the difference ($x'-x$) was added to the reading on the bay-window hook gage and the correct zero thus established. This, we believe, is a more accurate

method than that of trying to establish the water level at the exact apex of the V-notch, since it eliminates the difficulty with the meniscus mentioned by Professor Christie.

In regard to viscosity, in commercial practice we try to establish fairly constant viscosity by introducing a heating coil at the bottom of the weir tank so that constant temperature is maintained. In cases of volume measurement the instrument is calibrated in volume rather than in weight, and a correction for the changes in density and viscosity must be made by the user.

The third point raised by Professor Christie — how close can the discharge water level come to the apex of the V-notch — we find that in practice it is not safe to have the water level come nearer than the value of H. If it is a 14-in. discharge over a 90-deg. weir, the water level in the discharge tank should not approach nearer to the apex than 14 in. We believe that some tests have been made where the water level was brought very close to the apex of the V-notch with very little effect on the discharge, but we have no positive information from our own tests.

No. 2026

MEASUREMENT OF STATIC PRESSURE

By Carl J. Fechheimer,[1] East Pittsburgh, Pa.

Non-Member

The paper describes a new instrument for measuring static pressures in air-flow determinations. Made in the form of concentric brass tubes, the outer of which is ¼ in. diameter, the instrument is easily introduced into air ducts through small openings, such as a bolt hole. Pressure is communicated to manometers through two holes, one to the inner tube and the other to the concentric space between the tubes, about 78.5 deg. apart. The instrument is held perpendicular to the flow in such a manner that the direction of flow bisects the angle between the axes of the holes. In this position, which can be determined by balancing the pressures on one manometer, the reading on a second manometer gives the static pressure. The instrument has less error in turbulent flow than other types.

THE usual way of measuring static pressure, in a stream which flows at a comparatively high velocity, is by means of a static plate, piezometer ring, or the static side of a pitot tube. In all three the direction of stream flow is parallel to the axis of the walls or of the tube. If the character of the flow is turbulent, but is made up of regularly defined systems of whirls, the flow adjacent to the walls is laminar, as had been proved experimentally.[2] With laminar flow, there are thin layers adjacent to the boundaries that are stationary, and consequently, if connected through a small hole to a manometer, that instrument reads the static pressure. There are cases, however, in which the changing character of the inner-wall surface causes the moving fluid to impinge thereon; again, it is not always convenient to arrange for using such a device and to provide for connection to a manometer. It usually is satisfactory, however, to provide for a hole large enough to pass a small metal tube through, a bolt often being removed temporarily. Especially if it is to be used for measurement of the static air

[1] Research Engineer, Power Engineering Department, Westinghouse Electric & Manufacturing Co.

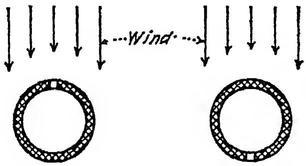
[2] On the Boundary Conditions of a Fluid in Turbulent Motion, by T. E. Stanton, Miss Marshall, and Mrs. Bryant. Proceedings, Royal Society, August 3, 1920.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

pressure in the end bell of an electrical machine, such as a turbo- or salient-pole alternator, this is a convenient method. But then, an ordinary tube is not satisfactory, as the reading on the manometer is partly due to static pressure and partly to velocity head Thus, a considerable error may be introduced because a fraction of the velocity head may be added to, or subtracted from, the static pressure.

2 It was once thought that a tube with very small holes, uniformly distributed all around, should read substantially correct, as for some holes the velocity head adds to, and for others it subtracts from, the static pressure, the net result being negligible influence of velocity head. It was recognized, however, several years ago, that such a device was not satisfactory, and, from some recent calculations, it appears that the reading is low by about 50 per cent of the velocity head.

3 The author developed a device for measuring static pressure, as described in the July, 1923, issue of the *Electric Journal* and in a paper on Performance of Centrifugal Fans for Electrical Machinery,[1] presented in 1924. That device is too cumbersome to use generally, and its readings may be affected by jets and the like. The device herein described was suggested by the curves in a paper of the Bureau of Standards.[2] It is believed that it can be used easily, and can be passed through a bolt hole and held in the stream. While there may be a small error due to the influence of velocity head, that error is considerably smaller than with any other device known to the author.



a b

FIG. 1A FIG. 1B

4 It is evident that if on a hole in a circular tube a fluid be allowed to impinge as in Fig. 1*a*, the reading on a manometer connected with the tube is nearly the static plus the velocity head. If the tube is turned through 180 deg., the reading is somewhat more than the static minus the velocity head. (Fig. 1*b*). It is evident that, somewhere between those two positions, there is some position for which the influence of velocity head is zero. If the tube can be turned through such an angle, the static pressure will be read. In the course of the work of Dr. Dryden at the Bureau of Standards, referred to above, he took observations of the sum of velocity and static heads on cylinders of various diameters, the cylinders being rotated about their axes, and a small hole was drilled in the wall of each through which the fluid entered. Those tests showed that the angle between the direction of stream and

[1] Trans., A.S.M.E., vol. 46 (1924), p. 287.

[2] Air Forces on Circular Cylinders, Hugh L. Dryden. Scientific Paper No. 394.

the position where the influence of the velocity head was zero was a little less than 40 deg. For smaller angles the influence of the velocity head was positive, and for larger angles it was negative. If the exact value of this angle (which we shall call the critical angle) were known, and if a tube with a small hole be placed in a stream with the hole at that angle to the stream, the reading on the manometer should be the static only.

5 The device as built consists of two small brass tubes (as shown in principle in Fig. 2), the diameter of the outer one being $\frac{1}{4}$ in., and the inner $\frac{1}{8}$ in., the smaller one going into the outer. The inner tube communicates with the outer surface through a small

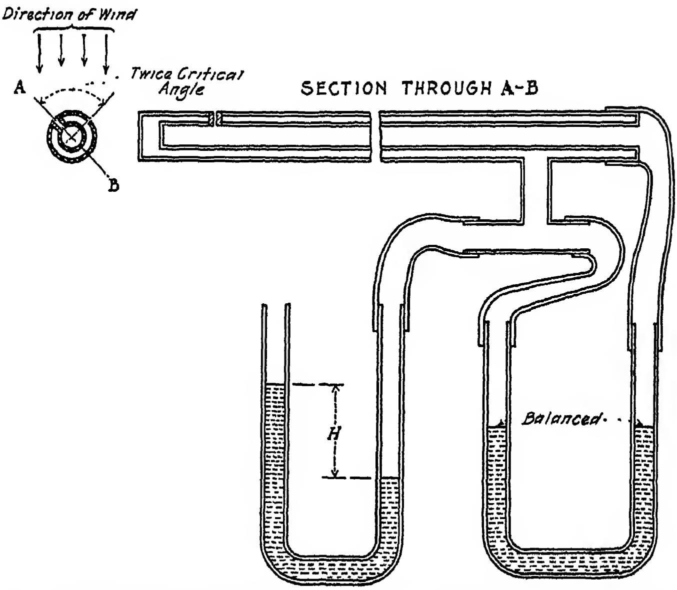


FIG. 2 MANOMETER CONNECTIONS FOR MEASURING STATIC PRESSURE

hole, and the annular space between the two tubes communicates with the outer surface by means of another small hole displaced from the first hole circumferentially by double the critical angle. The inner and outer tubes are then connected independently to the two sides of a U-shaped tube or other manometer, and when the device is in a moving stream, it is turned on its axis until the U-tube reads zero. Then the stream bisects the double-angle, so that if one side of the U-tube is then opened to the atmosphere, the static pressure only will be read. There is, of course, the possibility that the tube be turned 180 deg. from the impact side, and then the reading would be too low. To overcome this difficulty, it has been found advisable to use two manometers, one for balancing and one for reading the static pressure, a suitable T-connection

being used on one side for connecting to the second manometer. Then the manometer used for balancing would read zero for the two positions of the pressure tube, which are 180 deg. apart. The second manometer, which reads the pressure, would then record a high and a low value for the two balanced positions. The high value is the correct static pressure.[1]

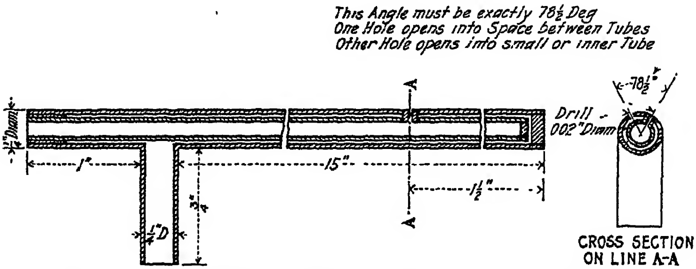


FIG. 3 BRASS STATIC-PRESSURE MEASURING INSTRUMENT

(Large tube ¼ in. outside diameter. Must be smooth and true on the outside. Small tube 0.125 in. outside diameter, 0.063 in. inside diameter. All joints must be air-tight.)

6 The tube as it was constructed is shown in Fig. 3. The small projection shown at the line *A-A* was silver-soldered into the small tube, and a hole was drilled into the outer tube of the size of the diameter of the projection. The small tube was then pushed into the large one, and after the projection was passed through the

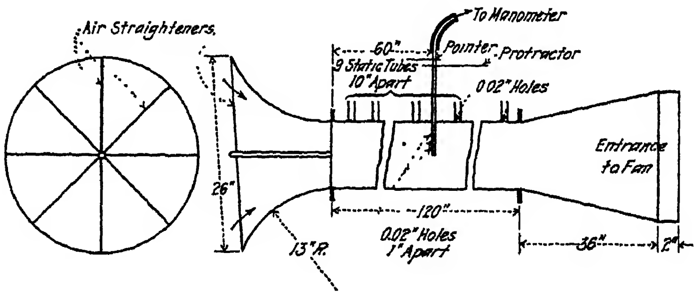


FIG. 4 EXPERIMENTAL AIR DUCT

(All but one hole was sealed in the small tube for a particular set of observations.)

hole in the outer tube, the projection was soft-soldered in place. The two holes of about 0.02 in. diameter were drilled 78½ deg. apart. Careful tests were made to insure that there was no leakage between the inner and outer chambers after the tube was completed. It was also found later that scratches and slight departures from a true cylindrical surface on the outside affected

[1] If the pressures are below atmosphere, the higher reading is the one that is numerically smaller.

the results. The tube was consequently ground, so as to make it a substantially perfect cylinder.

7 The tests made at the Bureau of Standards could not be used directly to determine the exact value of the angle between the two holes. These tests were made on cylinders much larger than those which we could use, and realizing that the angular position of the holes was of vital importance, tests were made to determine it. For this a long duct was constructed 8 in. in diameter with a bell-mouth entrance, and with air straighteners therein, as shown in Fig. 4. Small static pressure tubes were distributed throughout the length of this duct, and, with the

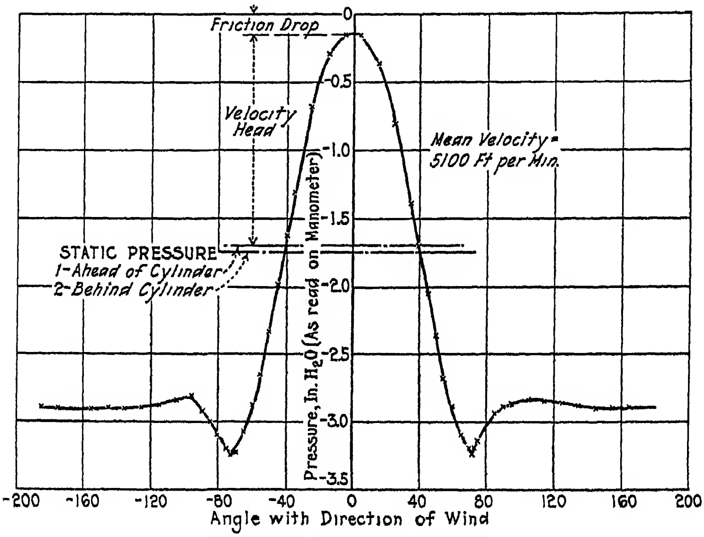


FIG. 5 PRESSURE AGAINST $\frac{3}{16}$-IN. CYLINDER PERPENDICULAR TO AIR STREAM IN 8-IN. DUCT

excellent entrance conditions producing a smooth flow, the static pressures were measured quite accurately along the duct. The air was drawn directly from the atmosphere into this duct. About half way down the duct, a small brass tube with holes of 0.02 in. diameter, arranged 1 in. apart, was inserted. A pointer was attached to this tube, which, in conjunction with a stationary protractor, enabled the observer to determine the angular position. The tube was connected with a suitable manometer, and readings were taken for various angular positions. This was repeated for a tube of slightly different diameter, and for one with a spherical cup at the end instead of a square end. It was found that the location of the hole axially, or the use of a spherical instead of a square end, had practically no influence upon the critical angle. A few of the results of these observations are plotted in

Figs. 5, 6 and 7. It is believed that these curves require no further explanation. Tests were also made in a duct $1\frac{1}{16}$ in. in diameter, but the results showed that the tube disturbed the flow so much

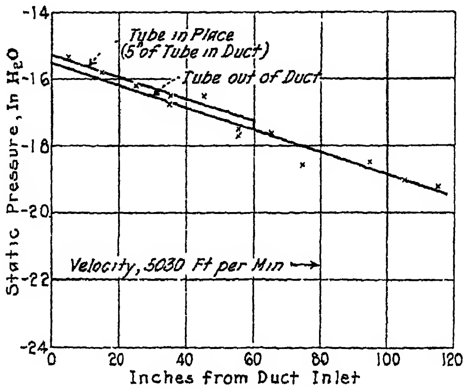


Fig. 6 Static Pressure Along 8-In. Duct, $\frac{3}{16}$-In. Cylinder Projecting 5 In. into Duct

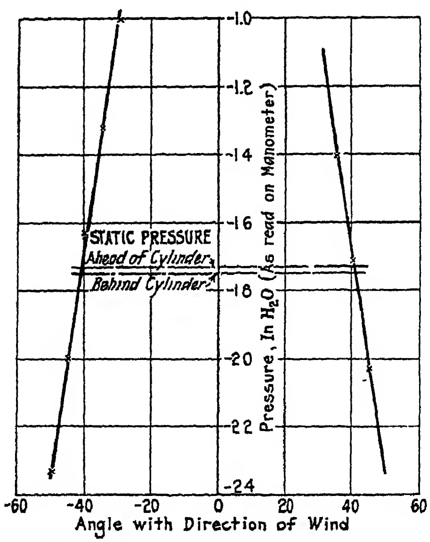


Fig. 7 Pressure Measured at 0.02 In. Diameter Hole 1 In. from End of a $\frac{3}{16}$-In. Diameter Cylinder and at the Center of an 8-In. Diameter Duct

(Mean velocity, 5020 ft. per min. Cylinder axis perpendicular to direction of the stream.)

that the static pressures, ahead and behind the cylinder, were considerably different. The device is not recommended for the measurement of static pressures in small ducts.

8 Tests were also made with the air issuing from an orifice into the atmosphere, in which case the static pressure was that of the atmosphere, which is usually taken as zero for reference. The results of this test are shown in Fig. 8. The two curves were taken with the axis of the tube normal to the air stream and at 45 deg. to the air stream. It will be seen that the angular position differs slightly for the two angles of inclination of the tube. When the device is used for measuring pressure in end bells of electrical machines, the direction of flow is not known. That is why it is important to recognize that slight errors may be introduced in the reading

9 The tube as constructed, in which the holes are 78½ deg. apart, was used to determine the magnitude of the error for

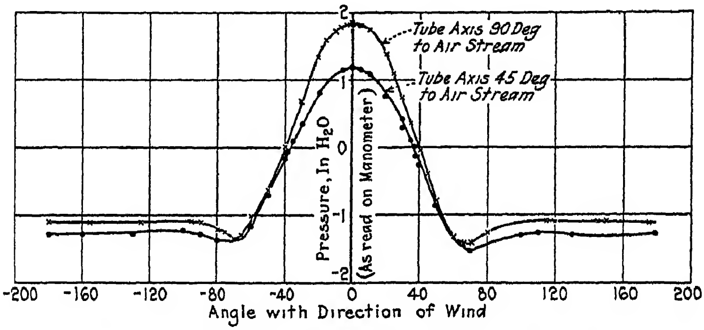


FIG. 8 PRESSURE DISTRIBUTION AROUND A ¼-IN. CYLINDER IN THE ATMOSPHERE

(Static pressure is zero for reference. Velocity, 5400 ft. per min.)

different angles of inclination with the air stream. These may be tabulated as follows:

Angle, deg.	Error, as per cent of velocity head
90	10% high
80	6¼% high
45	3% low
30	1½% low

10 The errors as reported are usually quite small. For instance, if in the end bell of a turbo-alternator the static pressure is 10 in. of water, and the velocity of the air adjacent to the tube is 5000 ft. per min., the error, if the tube is normal to the stream, is $0.1\left(\frac{5000}{4030}\right)^2 = 0.154$ in. water. This is only 1.54 per cent of the static pressure, which is a comparatively small error. As previously stated, with the tube that has been used before, the error was substantially 50 per cent of the velocity head. With most measurements, the error is less than indicated by these figures, for generally the tube is inclined with the air stream, in which case the percentage of error decreases.

11 When the stream is highly turbulent, it is difficult to state the static pressure. There are probable interchanges from static to velocity head, and vice versa, and it is therefore impossible to define the static pressure; consequently an accurate measurement cannot be made. In such cases the best that can be done is to standardize upon some method of measurement, which would serve as a basis for comparison between similar ducts, chambers, etc. It is believed that measurements with the tubes, as described herein, might serve for such basis, or at least until the character of turbulent flow is understood better than it is at present.

12 By attaching a pointer, or other suitable device, the tube may be used as a direction finder, and as such is sensitive and very accurate.

13 Mr. G. W. Penney conducted tests at the Westinghouse Laboratories and the value of his work is hereby acknowledged.

DISCUSSION

H. L. Dryden.[1] The author has adapted a device, used by wind tunnel experimenters for many years as a direction finder, to the measurement of static pressure in turbulent air streams. The errors of the instrument will probably never exceed 10 per cent of the velocity pressure and in most cases will be considerably less.

The angle between the two positions on the cylinder at which the pressure equals the static pressure varies with the size of the cylinder, with the distance from the end of the cylinder, and with the wind speed. The maximum included angle ever observed is 84 deg., although, theoretically, for extremely small cylinders at low wind speeds the angle could approximate 90 deg. This value of 84 deg. is nearly reached in Figs. 5 and 7 on a $\frac{3}{16}$-in. cylinder, 1 in. from the end. This value has also been found at Langley Field, Va., for the center of a 1-in. cylinder extending completely across the air stream. The lowest values obtained for cylinders not exceeding 1 in. in diameter are about 74 deg. although measurements on 1-in. cylinders inclined as in Fig. 8 are not available. Now the error in static pressure produced by an error of 5 deg. in the location of the holes is about 8 per cent of the velocity pressure, so that a tube with an angle between the holes of 80 deg. should not give an error greater than 8 per cent of the velocity pressure. For particular purposes a judicious selection of the angle would reduce the error. For example, if the stream were known to be nearly perpendicular to the cylinder, the use of holes 0.02 in. in diameter, 1 in. from the end of a $\frac{3}{16}$-in. cylinder, with an angle between the holes of 82 deg., would make the errors less than 5 per cent.

[1] U. S. Bureau of Standards, Washington, D. C.

As a matter of interest it may be stated that with cylinders 12 in. in diameter the included angle falls as low as 60 deg. The general variations may be summarized as follows:

a The angle is larger for holes in the center of a cylinder running completely across the stream than for holes near the end of a cylinder extending only to the center of the stream.

b The angle decreases with increasing diameter of the cylinder, other things remaining the same.

Care must be taken to keep the instrument small compared to the cross-section of the duct. Fig. 6 shows that even in an 8-in. duct, the flow is perceptibly affected by the introduction of a $\frac{3}{16}$-in. tube. With these considerations in mind, results of a very satisfactory degree of accuracy can be obtained.

J. M. Spitzglass.[1] The idea is prevalent that an opening facing the flow obtains the full velocity head plus the static pressure at the point of measurement. The difficulty has always been considered to be the determination of the true static pressure, which subtracted from the total, would give the correct velocity head at the given points. The author has found a method for determining the true static pressure. But how is the true velocity pressure to be determined, in view of the inference from Par. 4, that the total pressure on an opening facing the flow is not exactly the static pressure plus the velocity head?

We may infer, however, that this statement does not apply to impact tubes, where the opening is sharp edged against the flow. If such is the case a third opening would have to be provided in a separate tube, to obtain the true impact or total pressure. Could the author advise how the third tube is to be arranged so that its registration could be considered complementary to the registration of the static pressure, and at the same time not cause any disturbance by the proximity to the other tubes?

It is interesting to note, in Fig. 5, where the static pressure is shown against a $\frac{3}{16}$-in. cylinder, that the lowest pressure corresponds to twice the critical angle, and that the effect of suction at that angle is practically the same as the effect of impact when the opening is facing the flow. This being the case, a combined tube could be built so as to have the high- and low-pressure openings placed at the respective angles to produce the maximum differential. This would materially increase the accuracy of the readings and it would eliminate the necessity of determining the exact static pressure. That is, the tube would register, directly, twice the velocity head of the given flow.

[1] Vice-President, Republic Flow Meters Co., Chicago, Ill. Mem. A.S.M.E.

E. N. Fales.[1] Since care is specified in the construction and use of the instrument described by the author, there is a further use to which it may be put, namely, the measurement of velocity pressure. The writer suggests that an impact pressure orifice be located half way between the 78½-deg. holes, and offset axially if necessary to avoid interference. A passageway from the impact orifice to a third hose connection will enable the recording of impact, and, therefore, velocity pressures simultaneously with static pressures. The instrument thus modified would then be capable of measuring static pressure, velocity pressure, and direction. Precedent for such impact orifices as the above exists in a half-inch "integrating impact tube" developed for determining at one reading the average impact pressure along the diameter of the McCook Field 5-ft.-diameter wind tunnel. Impacts read on such a tube are reliable if the instrument is carefully handled, and oriented within 1 deg.

An instrument very similar to that of the author has been used in Germany by Dr. Karman and by Dr. Klemperer for precise determination of the flow-direction. The angular separation of the two holes was about 66 deg., chosen because the point of inflection of the angle-pressure curve occurred at 33 deg.

Regarding the axial position of holes mentioned in Par. 7, it is assumed that all the positions tested were reasonably far from the end of the tube. Figs. 9 and 10 show visually how the air flow changes near the end of a cylinder; the pressure changes would probably extend to a further distance from the end than the flow-line distortion in the photographs indicates.

Frank W. Caldwell.[2] It is known that the flow of air around a cylinder may change for different velocity or turbulence of the flow, or for different sizes of the model. Tests at McCook Field have shown this for spheres, airfoils, and struts; and tests at the Bureau of Standards have shown it for cylinders. Lacking information to the contrary, it would not be wise to assume that the point of zero static pressure always occurs at 39¼ deg. It is thought that for uses differing greatly from the specific cases discussed by the author, calibration of the instrument in a wind tunnel would be a good precautionary measure.

Regarding Fig. 6, in the McCook Field 5-ft.-diameter wind tunnel, similar conditions to those of Fig. 6 give different results. If a ½-in. diameter tube is inserted diametrically through the tunnel, the longitudinal pressure gradient upstream of the tube remains the same as when the tube is removed. But downstream of the tube, the negative pressure is greater when the tube is in

[1] Aeronautical Engineer, U. S. Air Corps, McCook Field, Dayton, Ohio.
[2] Chief, Propeller Branch, U. S. Air Corps, McCook Field, Dayton, Ohio.

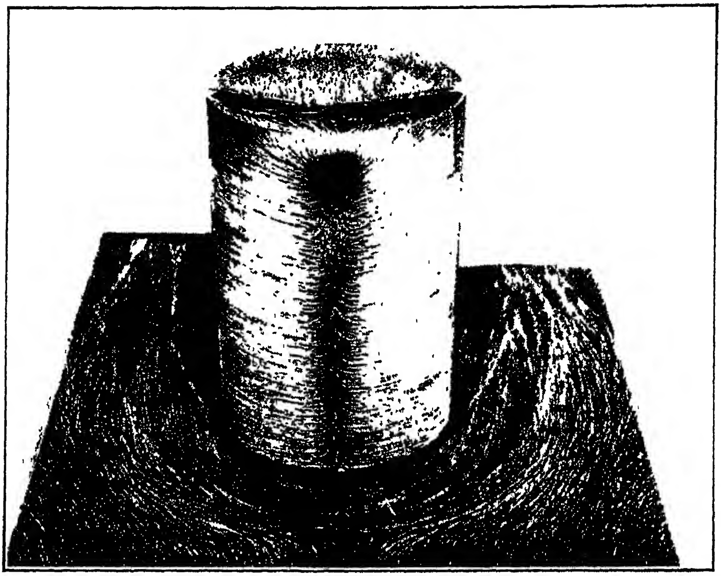


Fig. 9 Air Flow around Cylinder, Looking Down Wind
(4-in. cylinder, 6 in long Air speed, 30 m.p.h.)

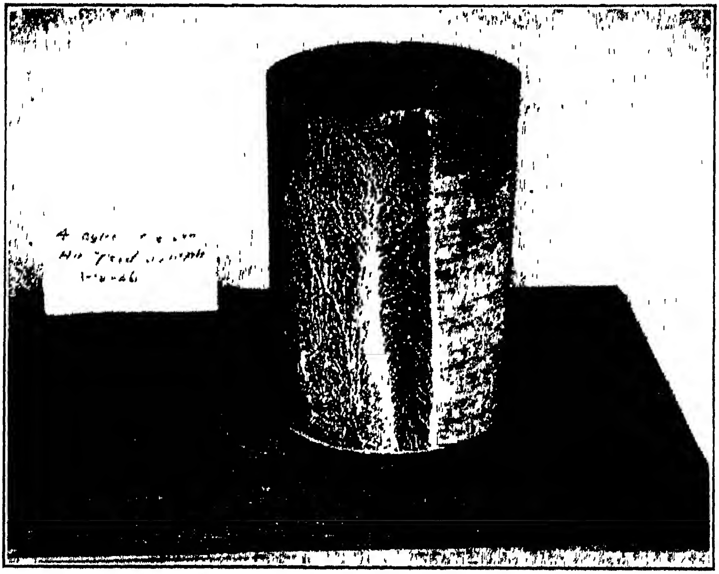


Fig. 10 Air Flow around Cylinder, Looking Across Wind
(4-in. cylinder, 6 in. long Air speed, 30 m.p.h.)

the tunnel than when out. Thus the curves for the two cases lie together upstream of the tube, and apart downstream. This is the reverse of the condition of Fig. 6.

Sanford A. Moss.[1] The statements and experiments of the author seem sound. The writer has made an instrument to the author's specifications and tested it in a jet discharged from a flow-measuring nozzle into the atmosphere, checking all the data in the paper. As the author points out, at various angles and positions the tube does not give exact static pressure, but a fairly close approximation to it.

Emphasis should be laid on the fact that any device which measures the force on a portion of a wall is subject to exactly the same errors due to the impact of stray jets as is an ordinary static hole in the same place.

G. W. Penney.[2] This static-pressure measuring tube has proved to be very valuable for measuring pressures in end bells and other similar places where the flow is very turbulent, so that ordinary methods are useless, and even in straight ducts it is more convenient than arranging special-pressure measuring holes, because this instrument can be inserted in a bolt hole.

It has been found that the instrument is very sensitive to small particles that may lodge in the holes where they will not be wiped off with a rag. Any slight scratch in the surface of the tube near the pressure-measuring holes will also disturb the flow and make the readings inaccurate. For this reason, the tube is usually checked before using by holding the tube at some point where an air stream is discharging from a duct into the atmosphere. If the tube is held a short distance from the point where the air leaves the duct, the static pressure will be atmospheric, but there will be considerable velocity head. If the tube reads atmospheric under these conditions, it is functioning correctly.

The Author. It is gratifying to note that the opinions of those who participated in the discussion have been in agreement regarding the practicability of the device described in the paper. There seems to be a belief that the double angle of 78.5 deg. may not be as accurate as it should be. Mr. Penney conducted a large number of tests, and an analysis of them seemed to indicate that the angle chosen was the correct one. It would be very valuable to us to secure more tests, in a wind tunnel, as suggested by Mr. Caldwell; and go further so as to cover cylinders of various diameters, at various air velocities, at different distances from the end of the

[1] Engineer, Thomson Research Laboratory, River Works, General Electric Co., West Lynn, Mass. Mem. A.S.M.E.

[2] Power Engineering Dept., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

tube, as implied by Dr Dryden. If such further tests are made, they should also include some with the axis of the tube at various angles with the direction of the flow of the fluid, and should be made in ducts of various diameters. At the time our tests were made, we felt that we had covered the cases sufficiently for determining the value of the critical angle. Our recommended value of 78.5 deg. is somewhat of a compromise, taking into account the various angles that might obtain between the tube axis and the direction of the stream; see Fig. 8. It is interesting to note that Dr. Moss made an instrument similar to ours, and checked our results.

The point raised by Mr. Fales, and also discussed by Mr. Spitzglass, regarding the use of an impact orifice, is of interest. One difficulty would lie in the uncertainty of the direction of flow of the fluid. It is evident from Fig. 8 that with the tube axis inclined at 45 deg. with the stream, the velocity pressure is considerably less than if it is at 90 deg., as one would expect. It is the opinion of the author that if the tube axis is normal to the stream. a small impact hole should give a means of measuring the velocity plus the static pressures. To answer Mr. Spitzglass, it is believed that a tube as suggested by Mr. Fales could be used satisfactorily, three manometers being employed. It would then be necessary, first, to rotate the tube on its axis until the balancing manometer for the static pressure read zero; and, second, to change the direction of the tube axis with the direction of the stream until a maximum reading is obtained on the impact manometer, checking back again on the balancing manometer to see that it read zero. The author has not tried this out, but perhaps others, who have use for such an instrument, might wish to do so.

It is believed that there should be further checking before use is made of the suggestion of Mr. Spitzglass that accuracy of the reading be increased by employing the lower cusps of the curve of pressure, which seems to occur at double the critical angle. The author believes that uncertainties are liable to be introduced which would more than offset the increased accuracy due to doubling the reading.

No. 2027

KINEMATICS OF CAMS, CALCULATED BY GRAPHICAL METHODS

By H. Schreck,[1] New York, N. Y.

Member of the Society

This paper presents the application of graphical methods for the calculation of velocity and acceleration of cams. The theory of the graphical methods is explained, its application to problems comprising various shapes of cams is presented, and changes in kinematic conditions due to variation in the shape of the cam are described. These results are shown in tables and plotted in graphs in order to present a comparison of the forces produced by the various cams. Particular care has been taken to describe the graphical methods in such a way that they can be readily employed in every-day work without referring each time to the theoretical analyses.

So far as the author is aware, no such demonstration and comparison of cams has thus far been published and it is his wish that this very simple graphical method may find wide employment with engineers who use cams in their work.

GRAPHICAL methods of cam calculation have been dealt with before by other engineers[2] and it remains for this paper to explain concisely the kinematic principle on which these authors have based their work and also to explain the use of these methods on actual examples.

Determination of Velocity

2 The principle employed is to reduce the various problems to the well-known problem of the four-link chain, the theory of which may be recalled briefly by the aid of a few illustrations.

3 Fig. 1 is a four-link chain with *a* as a crank revolving about point 1 at a constant speed, which we assume is known. Lever *c* is reciprocated about point 4 by means of link *b* and the system is held fixed by *d*, which is generally an engine frame.

[1] In charge of design, Combustion Utilities Corp.

[2] Prof. Poeschl, *Z. d. Oest. Ing. u. Arch. Vereines*, 1912; Prof. Koerner, *Z. d. Oest. Ing. u. Arch. Vereines*, 1915; and Prof. Poeschl, *Z. für angew. Mathematik u. Mechanik*, 1923.

Contributed by the Oil and Gas Power Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers. Revised and abridged. Complete copies are on file at the headquarters of the Society.

4 When the mechanism is in motion each point on each moving link will move at each instant at some velocity which has a definite relation to every other point of the system. The problem is to find this relation as regards certain specific points when the velocity of one specific point of the system is known, this latter, in our case, being point 2 revolving at a constant velocity v_1. Point 2 is common to links *a* and *b*, and in order to find the velocity of, say, the other end of link *b*, that is, of point 3, we make use of the theory of instantaneous centers. This theory says

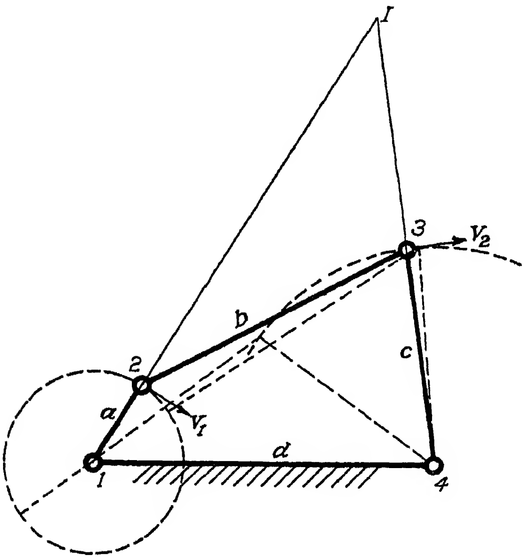


FIG. 1

that any body may be considered to rotate at any given instant about an axis which lies on the intersection of verticals to the instantaneous lines of motion of two or more points on the body. In the present instance two such verticals from points 2 and 3 intersect at *I* and this point *I* is known as an instantaneous center or axis. The velocity of any point on link *b* is then directly proportional to the distance of such point from this center *I*. Therefore, since the velocity v_1 of point 2 is known, in order to find the velocity of point 3 the velocity v_1 has to be multiplied by the ratio of the radii, and we obtain

$$v_2 = v_1 \frac{I\text{-}3}{I\text{-}2}$$

5 Fig. 2 is a modification of Fig. 1 and illustrates the well-known slider-crank chain on which the link *c* has been removed, or rather two members have been paired into a sliding block.

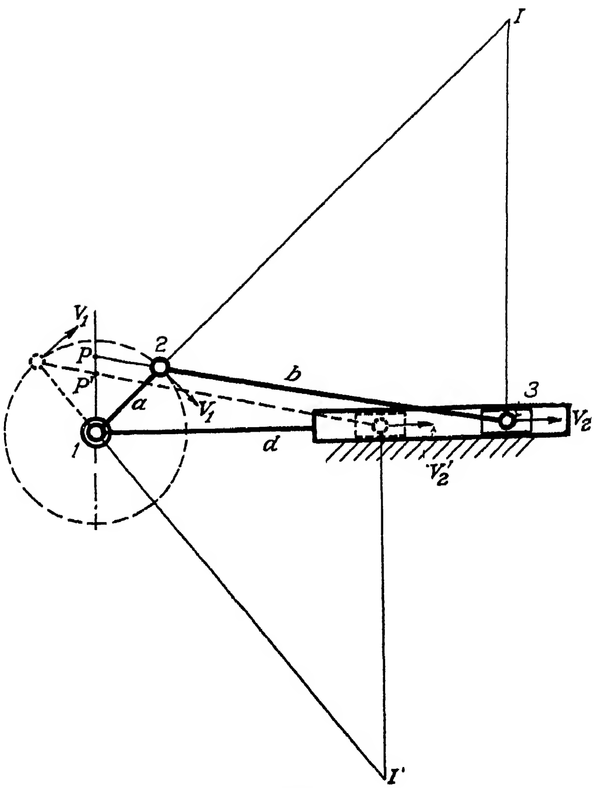


FIG. 2

6 The instantaneous center in this case is found by drawing the vertical on velocity v_1 and the vertical on the unknown velocity v_2. These lines intersect at I. The linear velocity of v_2 is then determined by the ratio of the radii from I. Therefore

$$v_2 = v_1 \frac{I\text{-}3}{I\text{-}2}$$

and triangles I-2-3 and P-1-2 being similar, also

$$v_2 = v_1 \frac{P\text{-}1}{1\text{-}2}$$

But

$$1\text{-}2 = \text{radius of crank} = r$$
$$v_1 = \text{velocity of crank} = \omega r$$

Then we find

$$v_2 = \omega r \frac{P\text{-}1}{r} = \omega \times P\text{-}1$$

which means, by extending the center line of connecting rod b which intersects the vertical at P (the vertical must be parallel to the vertical on the desired velocity v_2) and if the drawing is

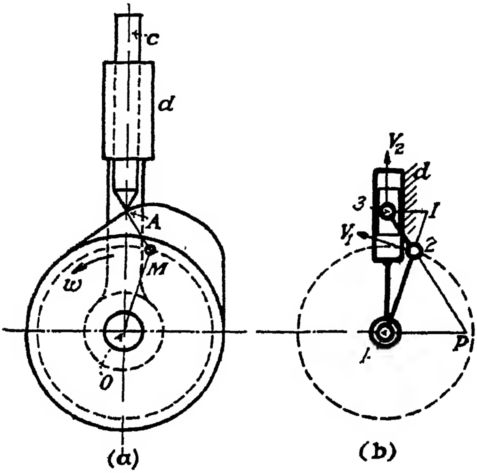


Fig. 3

made to scale, that the distance P-1 multiplied by the angular velocity ω of the crank gives the velocity v_2 of the crosshead for any given position.

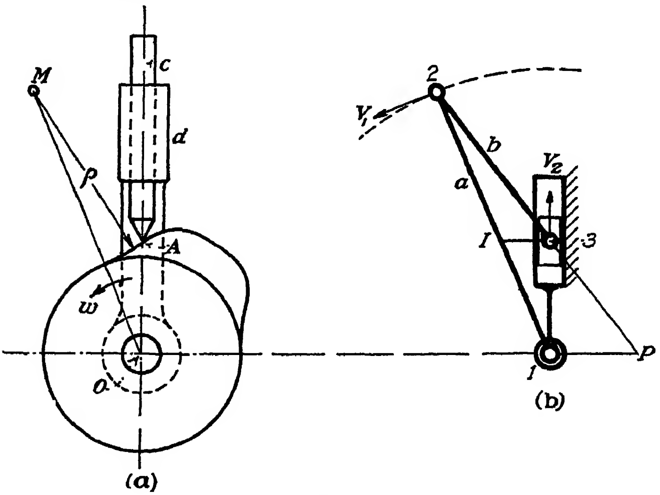


Fig. 4

7 The dotted lines in Fig. 2 show the crank in a different position. In this case the instantaneous center I' falls below the center line, but by the use of similar triangles it again can be proved that the velocity of the crosshead is $\omega \times P'$-1.

8 We shall now proceed to reduce the various parts of a cam motion into the respective system of a four-link chain. In Fig. 3 a slider c moves along the curved part of the cam. The center of radius of this curve is at M. Considering M as a point of the cam, it must revolve at the constant velocity v_1. Therefore, this point M shall be the point 2 revolving about 1 at velocity v_1. Point A is guided in block d, which is held fixed and is, therefore, point 3 of the chain. (Fig. 2.)

9 The solution of Fig. 3b is known and, therefore, if ω is the angular velocity of the cam, the velocity of the crosshead or slider is

$$v_2 = \omega \times P\text{-}1$$

10 Fig. 4 (a and b) represents the first part of a cam, which is in this case a concave curve of the radius ρ, the center of which lies at M. Point M revolves with the cam at the constant speed about O. We then have a crank r, the crank pin of which 2 travels about 1 at velocity v_1. The instantaneous center is I because I-2 is vertical on v_1, and I-3 is vertical on v_2. Then

$$v_2 = v_1 \frac{I\text{-}3}{I\text{-}2}$$

or on account of similarity of triangles I-2-3 and P-1-2

$$v_2 = v_1 \frac{P\text{-}1}{1\text{-}2} = \omega a \frac{P\text{-}1}{1\text{-}2} = \omega \times P\text{-}1$$

11 Fig. 5 (a and b) represents a convex curve with the radius ρ the center M of which revolves about the center of the cam at the constant velocity v_1. The instantaneous center is I because I-2 is vertical on v_1 and I-3 is vertical on v_2. The condition is then the same as in the previous case,

$$v_2 = v_1 \frac{P\text{-}1}{1\text{-}2} = \omega a \frac{P\text{-}1}{1\text{-}2} = \omega \times P\text{-}1$$

12 Fig. 6 (a and b) shows the slider working on the tangent part of the cam. In this case point A moves along a straight line, which is equivalent to the motion in a sliding block. This means that members a and b of the four-link chain have been paired together. This modification of the four-link chain is called the skew double-slider crank chain.

13 This particular case will be dealt with, together with its modification, as shown in Fig. 7 (a and b). This latter design differs from Fig. 6 only in that the path of motion does not go through the center of rotation or center of cam.

14 The case of the straight tangent part of the cam is a modification of Figs. 4 and 5, but with the radius ρ of infinite length.

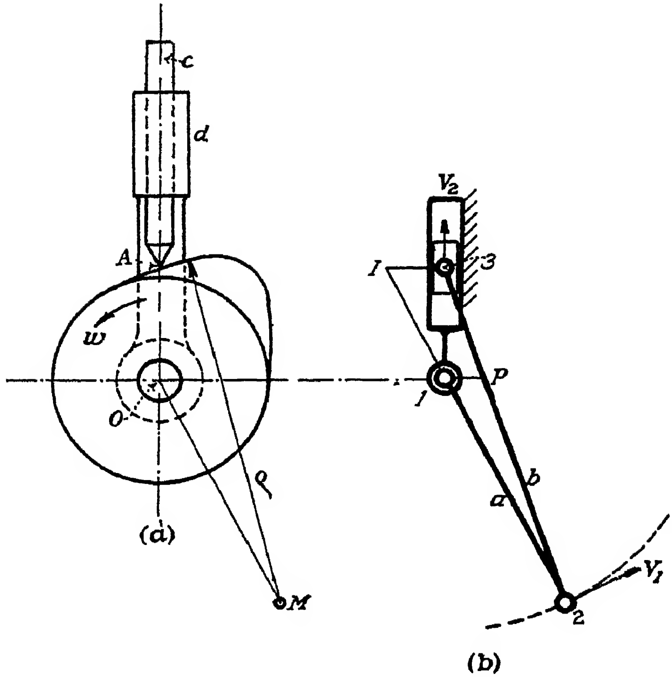


FIG. 5

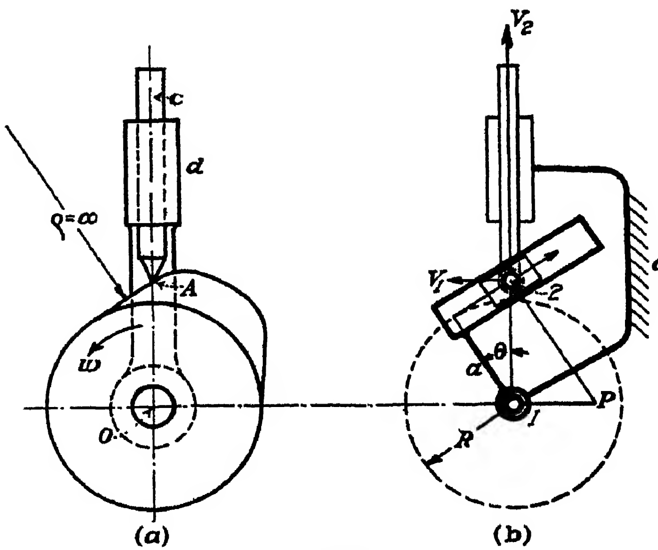


FIG. 6

By making this assumption the point P is easily found, as the intersection of the vertical on the direction of motion of the slider and the vertical (because the radius is infinite) on the sliding block. On the cam proper the latter is vertical on the tangent line of the cam.

15 It will now be proved that the assumption that radius ρ is infinite is correct, that is, referring to Fig. 4, to assume crank a (1-2) and coupler b (2-3) to be parallel.

16 Assuming the cam has moved through an angle θ, then the travel would be (from Fig. 6b)

$$s = \frac{R}{\cos\theta} - R = R\left(\frac{1}{\cos\theta} - 1\right)$$

Differentiating this equation and dividing both sides by dt

$$\frac{ds}{dt} = R\frac{d\theta}{dt}\left(\frac{\sin\theta}{\cos^2\theta}\right)$$

Then since velocity $v = ds/dt$, and angular velocity $\omega = d\theta/dt$, the velocity is

$$v_2 = \omega R\left(\frac{\sin\theta}{\cos^2\theta}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [1]$$

Also

$$1\text{-}2 = \frac{R}{\cos\theta}$$

and

$$P\text{-}2 = \frac{1\text{-}2}{\cos\theta} = \frac{R}{\cos^2\theta}$$

$$P\text{-}1 = P\text{-}2\sin\theta = R\left(\frac{\sin\theta}{\cos^2\theta}\right)$$

Substituting this expression in Equation [1] we obtain $v_2 = \omega \times P\text{-}1$, as was to be proved. We have, as in all other cases, again the relation that the distance P-1 multiplied by the angular velocity is the velocity of the slider at any point.

17 It will now be proved that the assumption of an infinite radius ρ is also correct if the path of motion does not go through the center of rotation, such as shown in Fig. 7. The lines of motion and velocity of problem of Figs. 6 and 7 have been combined in Fig. 8. The line A_0A_1 is the tangent part of the cam, and the slider has reached point A_1. The slider, corresponding to Fig. 6, slides on a line through the center of the cam and the slider, corresponding to Fig. 7, slides on the line ST. According to the proof presented in Par. 16 the velocity of the slider, when its direction passes through the center, is found by erecting a vertical in A_1 on A_0A_1 and a vertical in the center O on the direction of motion of the slider, intersecting at P_6 (the subscript 6 refers to Fig. 6). The velocity of

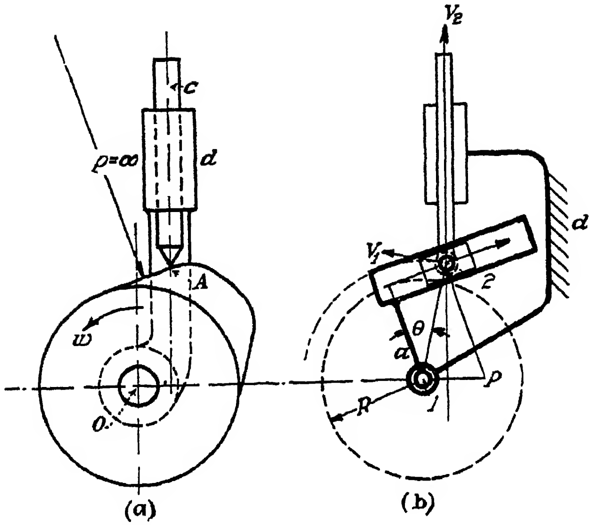


FIG. 7

A_1 is then $OP_6\omega$ when the slider goes through the center, as was proved in Par. 16.

18 For the slider traveling on line ST draw P_7 (referring to Fig. 7) in the same manner as before; that is, use the vertical

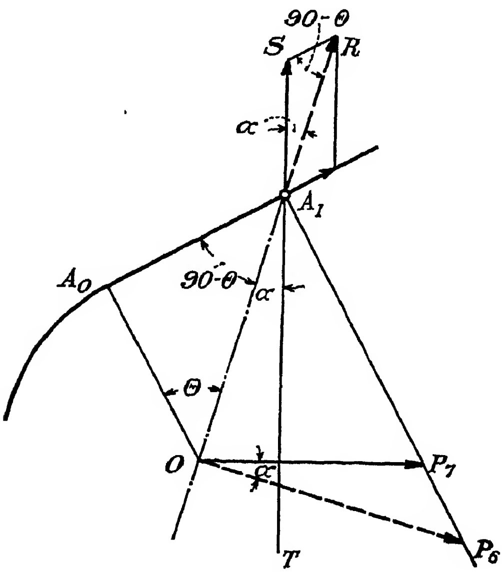
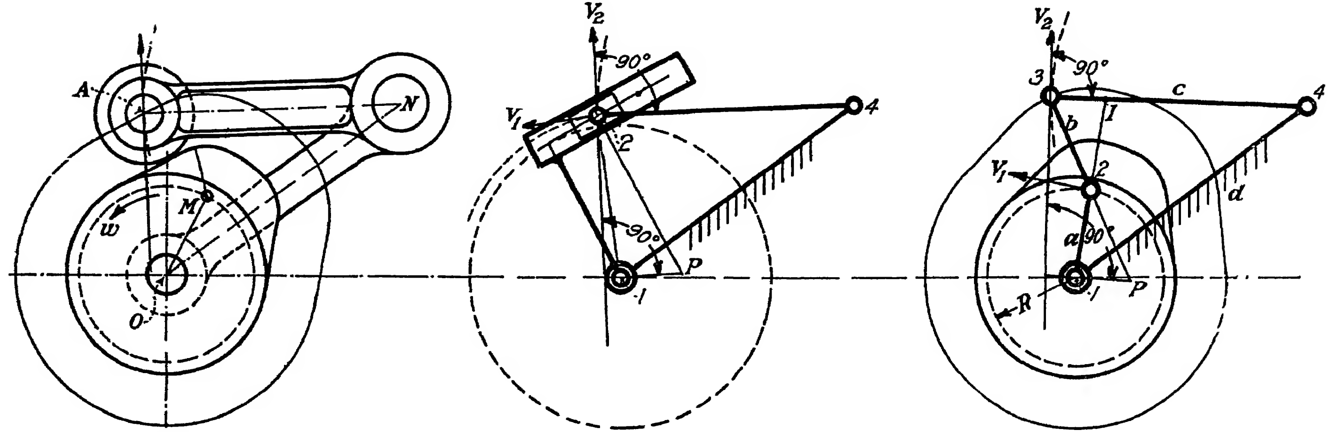


FIG. 8



(a) (b) (c)

FIG. 9

previously erected in A_1 on A_0A_1 and draw a vertical through the center O on the direction of motion. This line intersects at P_7. The velocity of A_1 is $OP_7\omega$ when the slider moves on line ST, which will now be proved.

19 Draw A_1R to scale as the increase in travel of the slider moving on the line OR at the known velocity $OP_6 \times \omega$. The motion of the slider Fig. 7 takes place on the line ST. Therefore, resolve the travel A_1R into its components and find the amount of travel along the line ST as A_1S. And since triangle OP_6P_7 is similar to triangle A_1SR, $\frac{OP_7}{OP_6} = \frac{A_1S}{A_1R}$ which indicates that the ratio of the velocities is equal to the ratio of the increase in travel of each case. Since it has been proved in Par. 16

that OP_6 is the correct value for velocity (Fig. 6) and since we have now proved that the ratio of OP_6: OP_7 is correct, it can be stated that OP_7 gives the correct value for the velocity of the slider when its path of motion goes on line ST, or as its application is shown in Fig. 7.

20 Fig. 9 (*a*, *b*, and *c*) shows a cam on which the motion does not take place on a straight line, but on an arc, which in this case is the circle described by the center of a roller at the end of a rocker arm. This motion is a special case of Fig. 7, in that the path of motion does not go through the center of rotation and, therefore, the velocity is found as before by erecting a vertical in 1 on the direction of motion v_2 of the roller. The direction of motion v_2 forms a right angle with lever 2-4, because it is the tangent at point 2 to the circle described by a radius 2-4. Since P-1 is also vertical on motion v_2, P-1 is parallel to lever 2-4. The intersection of the two verticals is P and the velocity at point 2 is $P\text{-}1 \times \omega$.

21 The motion on the curved part, or nose, of the cam is analyzed in Fig. 9*c*. It is a four-link motion with *a* as crank, *b* as coupler, *c* as lever, and *d* as frame. Proceed as in Fig. 1. Draw a vertical on direction of motion v_1, that is, extend the center line of the crank *a* over point 2 until it intersects the vertical on the direction of motion v_2 (which happens in this case to be the member *c* itself) in I, which is the instantaneous center. Therefore

$$v_2 = v_1 \frac{I\text{-}3}{I\text{-}2}$$

Now draw a parallel to lever 2-4 through 1 and extend 2-3 over 2 intersecting at P. Two similar triangles I-2-3 and P-1-2 have been formed and, therefore, the velocity of point 3 is

$$v_2 = v_1 \frac{P\text{-}1}{1\text{-}2}$$

and since $v_1 = \omega R$, and $R = 1\text{-}2$, it becomes

$$v_2 = \omega R \frac{P\text{-}1}{R} = \omega \times P\text{-}1$$

22 From these various cases, the following general rules can be drawn for finding the velocity at any point:

(*a*) The line P-1 is always a vertical on the path of motion through O

(*b*) P is the intersection of this vertical with a line which is either to be erected vertical in a point on the straight part of the cam or with a line which is to be drawn from

the center of the curve to a point on the curved part of the cam and extended to such intersection

(c) P-1 multiplied by the angular velocity $\omega = \frac{2\pi n}{60}$, is the velocity of the slider at any point. n = r.p.m. of the camshaft.

Acceleration

23 A method of finding acceleration or deceleration will be shown here for one case. For more complex cases, such as, for instance, in connection with the use of a rocker arm, the design and proof will be presented with the examples which are to follow.

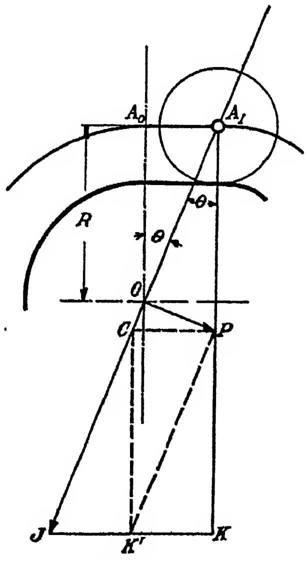


Fig. 10

24 The following proof is for the straight or tangent part of the cam. The correctness of the methods presented later on for the curved part of the cam can also be checked indirectly by the law that if a motion starts at zero velocity and ends at zero velocity, the area of acceleration plotted over time must be equal to the area of deceleration.

25 Let us now find the acceleration for the case shown in Fig. 6, that is, for the tangent part of the cam. The geometry of the case is shown in Fig. 10. The cam has moved through the angle θ and the roller has reached the point A_1. To find the acceleration we proceed as follows:

Draw A_1K perpendicular to A_0A_1 at A_1

Draw OP perpendicular to OA_1 at O

Make PK equal PA_1
Erect at K a perpendicular to A_1K
Extend path of motion OA_1 through O, intersecting at J.
Then if ω is the angular velocity, velocity of point $A = OP \times \omega$ and acceleration of point $A = OJ \times \omega^2$.

26 In Par 16 the velocity was calculated in Equation [1]

$$v = \omega R \frac{\sin\theta}{\cos^2\theta}$$

which can be transformed to

$$v = \omega R \frac{\tan\theta}{\cos\theta}$$

Differentiating this equation and dividing both sides by dt

$$\frac{dv}{dt} = \omega R \frac{d\theta}{dt}\left(\frac{1}{\cos^3\theta} + \frac{\tan\theta\sin\theta}{\cos^2\theta}\right)$$

Since acceleration

$$a = \frac{dv}{dt} \quad \text{and} \quad \omega = \frac{d\theta}{dt}$$

then

$$a = \omega^2 R\left(\frac{1}{\cos^3\theta} + \frac{\tan\theta\sin\theta}{\cos^2\theta}\right) \quad . \; . \; . \; . \; . \; . \quad [2]$$

From the geometry of the figure

$$OA_1 = \frac{R}{\cos\theta}$$

and

$$A_1P = \frac{OA_1}{\cos\theta} = \frac{R}{\cos^2\theta}$$

or

$$R = A_1P\cos^2\theta$$

This value substituted in Equation [2] gives

$$a = \omega^2 A_1P\cos^2\theta\left(\frac{1}{\cos^3\theta} + \frac{\tan\theta\sin\theta}{\cos^2\theta}\right)$$

$$= \omega^2 A_1P\left(\frac{2}{\cos\theta} + \cos\theta\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

From the geometry of Fig. 10

$$OA_1 = A_1P\cos\theta$$

Since, by construction,

$$A_1K = 2A_1P$$

then

$$A_1J = \frac{2A_1P}{\cos\theta}$$

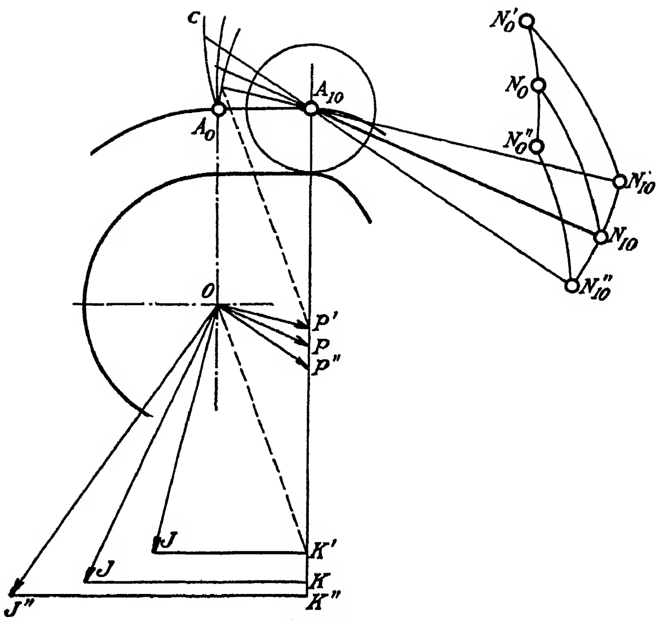


FIG. 11

and

$$OJ = A_1J - OA_1 = \frac{2A_1P}{\cos\theta} - A_1P\cos\theta$$

$$= A_1P\left(\frac{2}{\cos\theta} - \cos\theta\right) \quad \ldots\ldots \quad [4]$$

This value substituted in Equation [3] gives the acceleration

$$a = OJ \times \omega^2 \quad \ldots\ldots\ldots\ldots \quad [5]$$

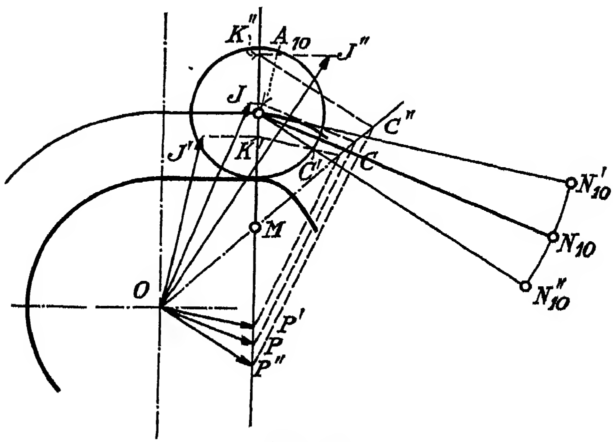


FIG. 12

27 Going back to Fig. 10, a slightly modified method for finding the acceleration OJ will be presented. This modification will also be used later. Instead of making $PK = PA_1$,

Erect a perpendicular to A_1P at P intersecting path of motion at C
Draw through C a line parallel to zero line of cam A_0O
Erect a perpendicular to OP at P, intersecting at K'
Draw a perpendicular to extension of A_1P through K'
Draw path of motion through O, intersecting $K'K$ at J.

From the geometry of the figure it can easily be seen that J lies at the same point if designed by either method.

Value of a Graphical Solution

28 The value of a graphical solution can best be shown by two illustrations, such as Figs. 11 and 12. The fulcrum of a rocker arm may lie for zero position at N_0, N'_0, or N''_0. Then, when the roller has moved to A_{10} the pivot will be at N_{10}, N'_{10}, or N''_{10}. For these three positions of the pivot it is easy to find, by means of a few lines, the velocity OP or the acceleration OJ (Fig. 11) and deceleration (Fig. 12).

29 Design of acceleration of point A_{10}, in Fig. 11, is as follows:

Strike radii about O through N_0, N'_0, N''_0
Strike radius with length of rocker arm N_0A_0 about A_{10}, intersecting at N_{10}, N'_{10}, and N''_{10}
Draw $N_{10}A_{10}$ and extend to C on circle about N_0 through A_0
Draw OP parallel to $N_{10}A_{10}$
Erect a perpendicular to A_0A_{10} at A_{10}, intersecting at P
Draw OK parallel to CP, intersecting at K
Erect a perpendicular to $A_{10}K$ at K
Draw OJ perpendicular to line $N_{10}A_{10}$ or to line OP, intersecting at J.

Then, if ω is the angular velocity, $OP \times \omega$ = velocity of point A_{10}, and $OJ \times \omega^2$ = acceleration of point A_{10}.

30 For the design of deceleration[1] of point A_{10} in Fig. 12, proceed as follows:

In the previous figure, location of points N_{10} were explained
Draw a line from A_{10} through M and extend over M
Draw OP parallel to $N_{10}A_{10}$, intersecting at P (OP is the same as in previous figure)
Draw PC parallel to OA_{10}
OC is a line starting at O and passing through M, which is the center of radius of the nose of the cam
Draw CK parallel to OP or to $A_{10}N_{10}$
Erect a perpendicular to MK at K

[1] See also Pars. 66, 67, and 74.

Draw line OJ parallel to direction of motion which lies on a perpendicular to OP and also to $A_{10}N_{10}$. Intersection is at J.

The same process can be followed for any point A on the curved part of the cam. Then, if ω is the angular velocity, $OP \times \omega$ = velocity of point A_{10}, and $OJ \times \omega^2$ = deceleration of point A_{10}.

Examples

31 The cams which are analyzed in this paper are chosen arbitrarily but are of the same characteristics as far as possible for the purpose of comparison. These characteristics are as follows:

Radius of base circle	2.25 in.
Radius of roller	0.75 in.
Radius of pitch circle	3.00 in.
Total angle of cam (except Figs. 19, 20, and 21)	112 deg.
R.p.m. of cam	250
Angular velocity ω	26.18 ft. per sec.
Angular velocity squared (ω^2)	685.4 ft.^{2} per sec.^{2}
Time of travel of each 6 deg. of cam	0.004 second

32 The first problem to be solved graphically will be that of a tangential cam with $\frac{3}{4}$-in. lift. Its solution is presented in Fig. 13. The lines OJ_0 to OJ_{10} represent the acceleration at the various stations up to point A_{10} and the lines OJ_{10} to OJ_{20} represent the deceleration. The acceleration is found as explained in Par. 25, and the deceleration is found, for instance, for point A_{11} as follows:

Draw $A_{11}O$ and extend over A_{11}

Erect a perpendicular to $A_{11}O$ at O

Draw a line through A_{11} and M, intersecting at P_{11}

Draw a parallel to $A_{11}O$ through P_{11}, intersecting at C_{11} on line from O through M, the center of radius of the rounding of the cam

Draw a parallel through C_{11} to OP_{11}, intersecting at K_{11}

Erect perpendicular to $P_{11}MK_{11}$ at K_{11}, intersecting the direction of motion OA_{11} at J_{11}.

Then, if ω is the angular velocity, $OP \times \omega$ = velocity of points A_1 to A_{20}, and $OJ \times \omega^2$ = acceleration of points A_1 to A_{10} and deceleration of points A_{10} to A_{20}.

33 The results thus obtained can be scaled in inches on the scale next to the drawing and have been compiled in Table 1. It will be noted that column 4 is the increase of speed during the interval of 6 deg., or 0.004 sec., and that from this increase the acceleration in column 5 has been calculated for the purpose of checking the graphically-found values of acceleration in column 7. The value in column 5 is the "mean acceleration" between two stations and, therefore, must lie between the values of instantaneous acceleration as scaled from the drawing at the various stations and as stated in columns 6 and 7.

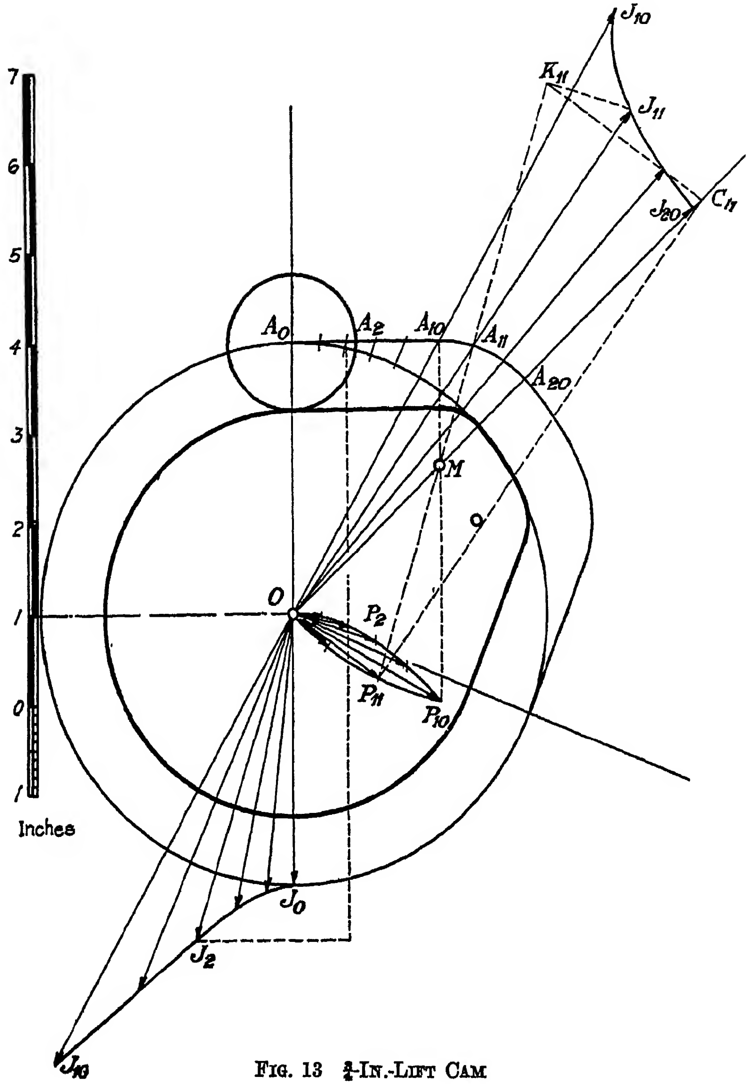


FIG. 13 ¾-IN.-LIFT CAM

34 In columns 8, 9, and 10 the travel is calculated from the velocity, graphically found, based on the condition, that if s is the travel during the time interval, the velocity is $v = s/t$ or $s = v \times t$. This procedure may be used as a check of the graphical method, or to figure the travel at the various points. The total should then be equal to the lift of the cam. The value of 0.7516 in. seems to be very gratifying, considering that we are dealing with a graphical method and all calculations are being made by slide rule.

TABLE 1 ¾-IN. TANGENTIAL CAM, FIG. 13[1]

1	2	3	4	5	6	7	8	9	10
	Velocity			Acceleration			Travel, in.		
			Checking calculation						
Deg.	Reading OP, in.	$v = OP \times \frac{\omega}{12}$ ft. per sec.	Diff. v' ft. per sec.	$\frac{v'}{t}$ ft. per sec.[2]	Reading OJ, in.	$OJ \times \frac{\omega^2}{12}$ ft. per sec.[2]	Due to velocity v $v\times12\times t$	Due to increase v' $\frac{v'}{2}\times12\times t$	Col. 8 + Col. 9 in.
0	0	0			3	171	0		
			0.69	$\times \frac{1}{0.004} = 172.5$				$\times0.004 = 0.0165$	0 0165
6	0.316	0.69			3.08	176	$\times0.004 = .0331$		
			0.732	$\times \frac{1}{0.004} = 183$				$\times0.004 = 0.0176$	0.0507
12	0.652	1.422			3.34	191	$\times0.004 = .0683$		
			0.814	$\times \frac{1}{0.004} = 203.5$				$\times0.004 = 0.0195$	0.0878
18	1.025	2.236			3.81	218	$\times0.004 = .107$		
			0.944	$\times \frac{1}{0.004} = 236$				$\times0.004 = 0\ 0226$	0.1296
24	1.46	3.18			4.57	261	$\times0.004 = .1525$		
			1.18	$\times \frac{1}{0.004} = 295$				$\times0.004 = 0.0284$	0.1809
30	2.00	4.36			5.76	329			
			....	....					
30	2.00	4.36			—7.65	—437			
			1.66	$\times \frac{1}{0.004} = 415$				$\times0.004 = 0.0398$	0.1693
36	1.24	2.70			—6.82	—390	$\times0.004 = .1295$		
			1.50	$\times \frac{1}{0.004} = 375$				$\times0.004 = 0.0360$	0.0936
42	0.55	1.20			—6.54	—374	$\times0.004 = .0576$		
			1.20	$\times \frac{1}{0.00322} = 373$				$\times0.00322 = 0.0232$	0.0232
46° 49′ 40″	0	0			—6.48	—370		Total	0.7516

[1] In columns 5, 8, and 9, only the values for t are entered. The values of v', $v\times12$, and $\frac{v'}{2}\times12$ are omitted, because v and v' can be read easily from columns 3 and 4.

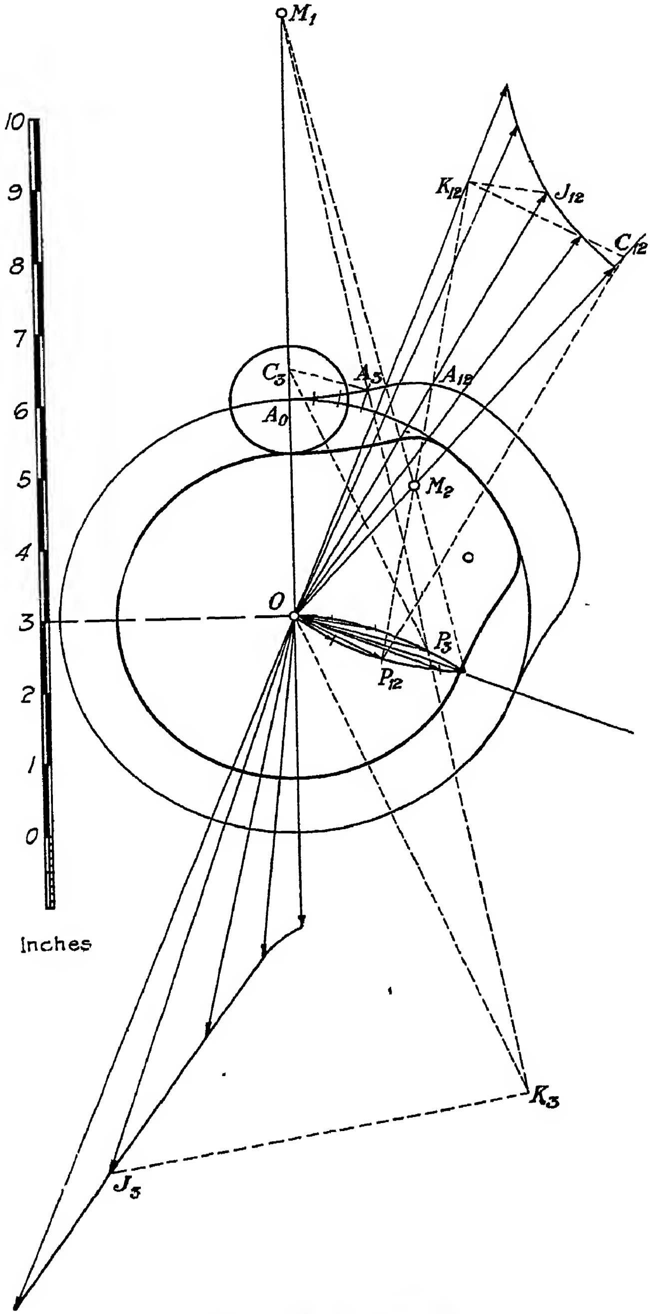


Fig. 14 $\frac{3}{4}$-In.-Lift Cam

35 Fig. 14 represents another cam of $\frac{3}{4}$-in. lift, starting with a concave curve, the center of which lies in M_1. The much larger values of acceleration and deceleration can be recognized at once. The acceleration in this case is found as follows:

Draw M_1A_3 and extend through A_3
Draw OP_3 perpendicular to OA_3 at O, intersecting at P_3
Draw A_3C_3 perpendicular to OA_3 at A_3
Connect P_3 and C_3 and draw a parallel through O to P_3C_3, intersecting at K_3
Erect perpendicular to A_3K_3 at K_3
Draw A_3O (which is direction of motion) and extend through O, intersecting at J_3.

With ω as angular velocity, $OP \times \omega =$ velocity of point A, and $OJ \times \omega^2 =$ acceleration of point A. The deceleration is found as described in Par. 32 or Fig. 13.

36 A convex cam of the same lift of $\frac{3}{4}$ in. is shown in Fig. 15. Acceleration is found the same way as described in Par. 35 and deceleration the same way as in Par. 32. The drawing shows clearly the advantages of this cam due to the small values of acceleration and deceleration.

37 Fig. 16 shows a cam operating a mushroom-type follower. In this case the graphical solution is extremely simple. Assume again that the cam is held fixed and that the center of the follower $F_0, F_1, F_2, \ldots F_{20}$ is moving around the cam.

38 The point of contact between the face of the mushroom and the cam is designated by the letter A and these points A_0, $A_1, A_2, \ldots A_{20}$ correspond to the respective position of the follower F with the same subscript. For example, A_2 is found by drawing a parallel to OF_2 through M_1, and A_{13} by drawing a parallel to OF_{13} through M_2.

39 To solve the problem we have, as before, to deal with the points of contact A, since they determine the lift of the follower. We shall then find acceleration and deceleration by using the method which was explained in Par. 27 and Fig. 10.

40 In the present case, for both acceleration and deceleration, OP (Fig. 10) and CP coincide, because the perpendicular to A_2P_2 at P_2 passes through O in Fig. 16. Also K' coincides with M_1 and M_2, respectively, because K' is the intersection of a perpendicular to OP, and a parallel to the zero line of the curve drawn through C or, in this case, since O and C coincide, drawn through O. Such lines drawn in Fig. 16 must intersect according to the geometry of the figure in M_1 and M_2 respectively.

41 The solution is as follows:

Acceleration. Draw vertical in O on direction of motion OF_2
Draw a normal to the tangent to the curve of the cam at A_2 which is line A_2M_1, intersecting at P_2
Extend F_2O through O, which is the direction of motion through O

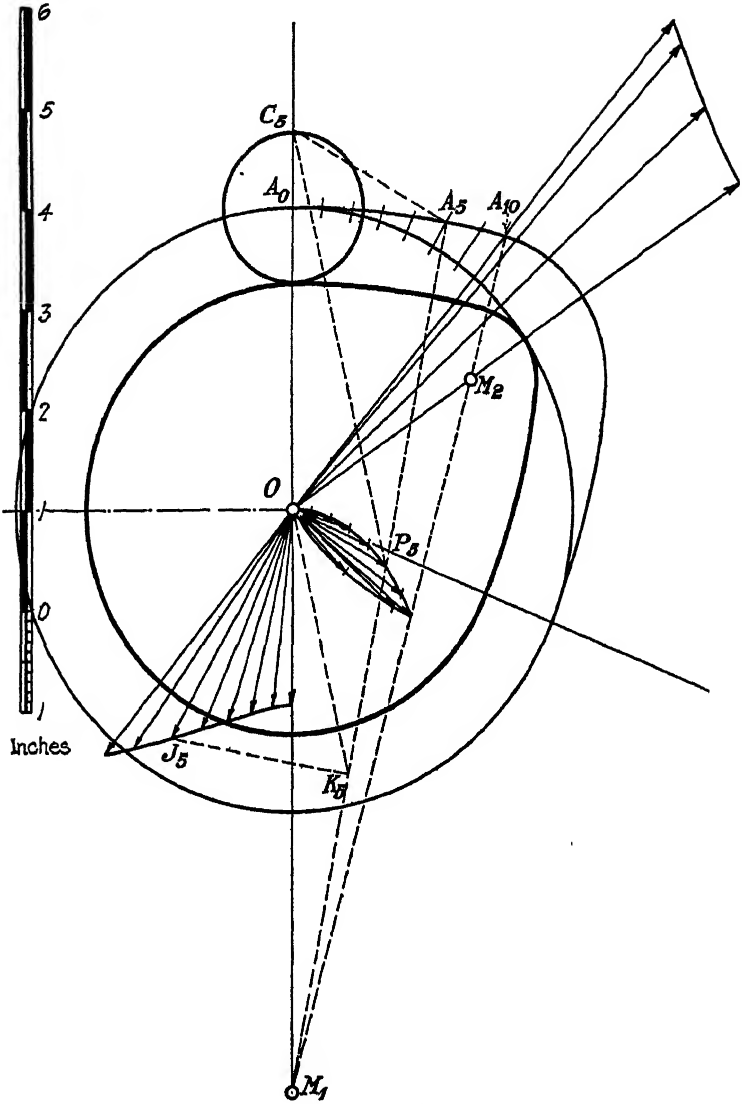


FIG. 15 $\frac{3}{4}$-IN.-LIFT CAM

Erect a perpendicular to $A_2P_2M_1$ at M_1 (identical with K'), intersecting at J_2.

Deceleration. Draw at O on direction of motion a perpendicular to OF_{13}

Draw a normal to the tangent to the curve of the cam at A_{13} which is line $A_{13}M_2$. Its extension is intersecting at P_{13}

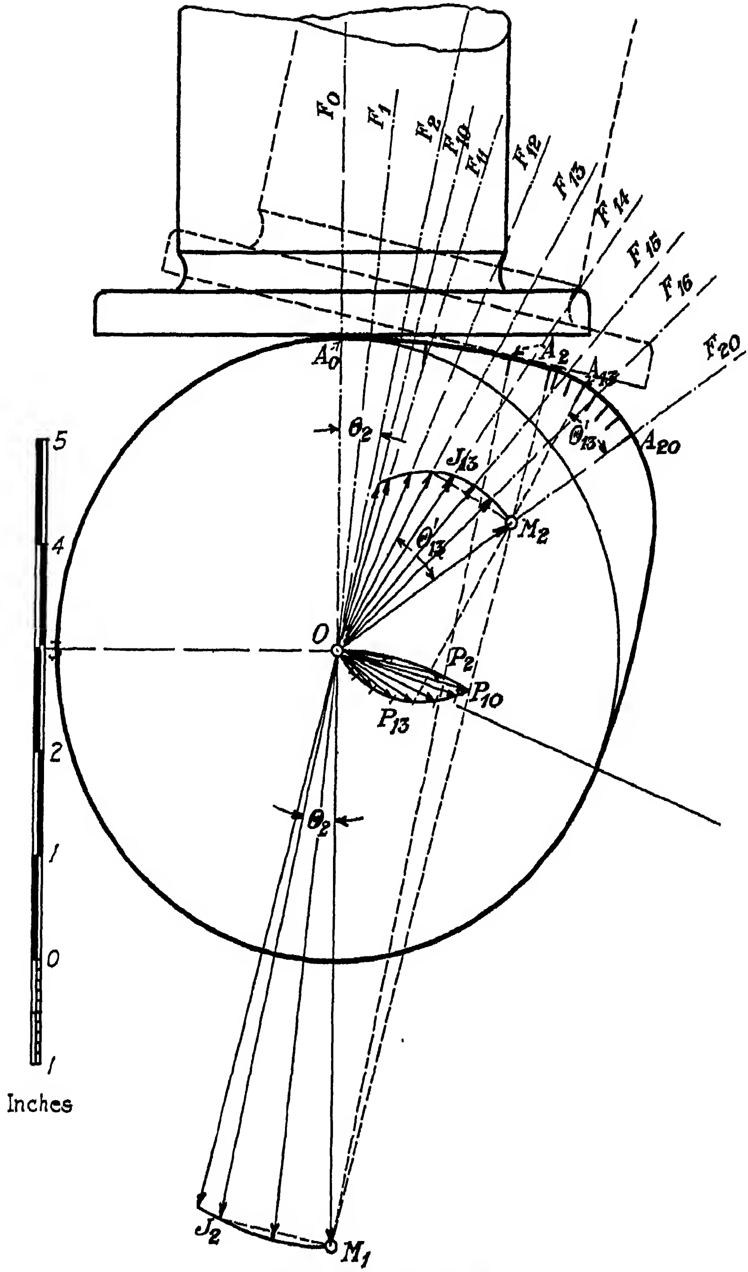


FIG. 16 ¾-IN.-LIFT CAM

Erect at M_2 (identical with K') a perpendicular to $A_{13}M_2P_{13}$, intersecting at J_{13}.

Then, with angular velocity $= \omega$, $OP \times \omega =$ velocity of point A, or position F, and $OJ \times \omega^2 =$ acceleration of point A, or position F.

42 This solution can be checked by the known formulas for acceleration and deceleration for this type of cam:

$$\text{Acceleration} = OM_1 \times \cos\theta \times \omega^2$$
$$\text{Deceleration} = OM_2 \times \cos\theta' \times \omega^2.$$

It will be noted that in this particular case the graphical determination as well as the calculation is very simple.

43 Attention is called to the rapid motion of the follower in the beginning of the lift, as indicated by the quick progress of point A on the cam surface, and it should be noted, also, how small the values of deceleration are.

44 The results of the various cams of $\frac{3}{4}$-in. and 1-in. lift are, for the purpose of comparison, drawn together in Fig. 17. In addition to acceleration and deceleration the travel for each cam has been computed and entered on this chart. This chart shows in an impressive manner the characteristics of the various cams. The concave cam e has the maximum acceleration and deceleration. It requires, therefore, the heaviest spring, and is the most unfavorable shape of cam. As is to be expected, this cam, in regard to lift is, and should be, the most favorable. The convex cam is, as to kinematic condition, the most favorable, and as to lift, the most unfavorable. The mushroom follower cam f is, in regard to lift, almost as good as the concave cam. It has, however, very small deceleration and will, therefore, require the lightest spring, but the acceleration at the beginning is the largest of all cams. Therefore, this cam may be the noisiest. The acceleration force is, however, reduced on this design; first, because this cam requires a lighter spring, and second, because the mushroom-type follower can be designed lighter in weight than the roller-ended follower, both of which reduce the acceleration force. This subject is considered further in connection with roller clearance in Pars. 54 and 55.

45 The lifts may be taken practically as representing the free area through the valves, and with this assumption, by planimetering the areas of valve lift of the various cams, the following ratios of areas have been found on the basis that the lift curve of the convex cam was assumed as unity.

a $\frac{3}{4}$-in. lift tangential cam with large nose........	1.21
b $\frac{3}{4}$-in. lift tangential cam with small nose........	1.28
c 1-in. lift tangential cam with small nose........	1.48
d $\frac{3}{4}$-in. lift convex curve cam	1
e $\frac{3}{4}$-in. lift concave cam............................	1.50
f $\frac{3}{4}$-in. lift mushroom-type cam....................	1.86

46 This ratio of opening, together with the kinematic condition illustrated in Fig. 17, demonstrates the advantages of one cam

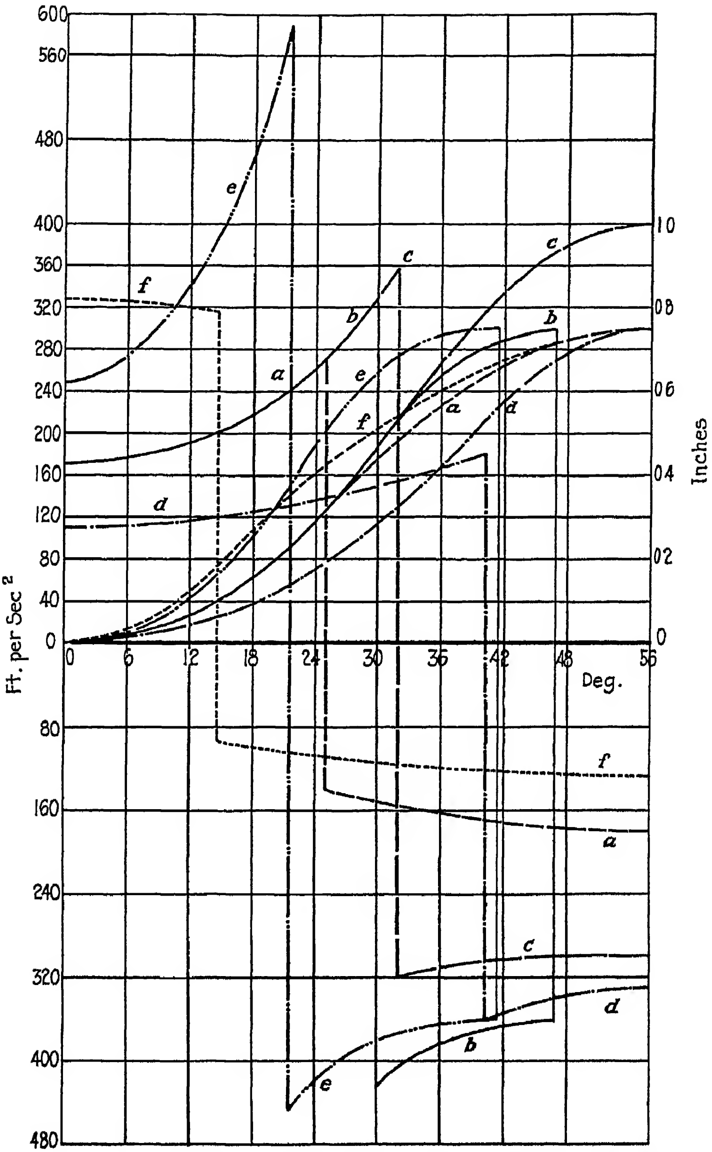


FIG. 17 TRAVEL AND ACCELERATION OF VARIOUS CAMS

a $\frac{3}{4}$-in.-lift, tangential cam, large radius nose, Fig. 13 (but no dwell on cam)
b $\frac{3}{4}$-in.-lift, tangential cam, small radius nose, Fig. 13.
c 1-in.-lift, tangential cam, small radius nose, Fig. 13 (but no dwell on cam)
d $\frac{3}{4}$-in.-lift, convex curve cam, Fig. 15
e $\frac{3}{4}$-in.-lift, concave-curve cam, Fig. 14
f $\frac{3}{4}$-in.-lift, mushroom-type cam, Fig. 16

against another. The deceleration of the ¾-in. large-radius-nose cam is low, but the opening ratio also is low. This ratio is very favorable for the ¾-in. concave cam and for the 1-in. tangential cam, but the kinematic conditions of the former require the heaviest spring of any cam, whereas the larger, lift cam requires a spring almost as light as the ¾-in. tangential cam. In other words, a concave cam may be replaced by a cam of higher lift and the same opening result but with a more favorable kinematic condition than a cam of lower lift.

Roller Clearance

47 In none of the calculations has it been considered that on the actual engine we must have clearance between cam and roller or mushroom tappet in order to take care of the expansion of the engine parts under operation. The clearance does not affect the calculations of the graphs.

48 The kinematic conditions of the convex cam in regard to initial speed are the most favorable ones; but there is a question as to whether these conditions are really so good under actual operating conditions. This question is worth studying since the opening ratio is low, and if not compensated by other advantages, the tangential cam might be preferable in many cases.

49 When the engine is warmed up, that is, in operating condition, the clearance between cam and roller or mushroom tappet may be assumed to be 0.004 in., which would mean on a tangential cam[1] of 3-in. pitch circle radius that the cam has turned through about 3 degrees from point A_0 to the point A'_0 before the roller actually touches the cam. Therefore, the follower does not start at zero velocity, but at 0.344 ft. per sec.[2] Allowing the same roller clearance for the convex cam, the cam will have rotated through 3 deg., 38 min., and 54 sec. before the roller touches. Then the initial velocity of the follower is 0.265 ft. per sec.[3] which figure is, of course, more favorable than on the tangential cam.

50 However, the force with which the cam will strike the follower, as well as the force with which the valve returns to its seat, depends also on the spring force. This force is a function of the deceleration which is, for the ¾-in. tangential cam, 141 ft. per sec.^{2} [2] and for the convex cam 362 ft. per sec.^{2} [4] The ratio of forces, when the cam begins lifting or at the seating of the valve, changes in direct proportion to the forces applied and as

[1] See Fig. 17 cam *a* (In original paper, cam Fig. 16).
[2] See Table 2 of cam *a* in original paper.
[3] See Fig. 15 — value interpolated from Table 5 of original paper.
[4] See Table 5 of original paper.

[Editor's Note: The original paper is on file at the headquarters of the Society and will be loaned upon request.]

the square of the velocity of these forces; therefore, the tangential cam is superior to the convex cam in the example by the ratio of

$$\frac{362}{141} \times \frac{0.265^2}{0.344^2} = \frac{1.51}{1}$$

This ratio will vary from case to case. However, it should be noted that the theoretical advantages of a convex cam in regard to a quietly-running gear will not always bear out in practice. As a matter of fact, the advantages of a convex cam will only be materialized if the roller clearance is reduced to an exceedingly small amount.

51 In order to ascertain the conditions for a cam of an actual engine, a cam of 135-deg. opening and 1-in. lift has been shown in Fig. 18 with three very slight variations in the contour of the cam. The outer line is an ordinary tangential cam; the middle line is a cam which starts as a convex cam and goes over into a tangent cam at A'_2, and finally the inner line, which is a true convex cam. The cams go over into the rounding of the nose at the points A_1, A_2, and A_3, respectively. Since the deceleration at this point determines the force of the spring, only this point of these cams will be investigated.

52 The acceleration for A_1 and A_2 is found as described in Par. 25, and for point A_3 in Par. 35. The deceleration for all three points is found as described in Par. 32.

53 The layout shows that the deceleration force of the tangential cam $-J_1$ is the smallest, and that of the convex cam $-J_3$ is the largest. Now, assume the same roller clearance as was assumed in Par. 48. Then the forces are in this case in favor of the tangential cam at the ratio of

$$\frac{4.1}{1.59} \times \frac{0.265^2}{0.344^2} = \frac{1.53}{1}$$

4.1 in. and 1.59 in. are the readings OJ from Fig. 18. This means that with a slightly better valve-lift ratio there will be in addition an improvement in quiet running in the ratio of 1.53 to 1 in the tangential as compared with the convex cam.

54 From the above calculation it is easy to judge how important it is to reduce the roller clearance to a minimum for a quiet running engine. The simplest method known for cutting down this clearance to a minimum is to reduce the conventional base circle, marked B', to a smaller base circle, marked B''. The line connecting the circle B'' with the tangent of the cam should be a circle in order that, if the roller touches before point A_0, the cam will become a case of a convex cam with its favorable accelerating condition. In this manner the actual roller clearance at A_0 can be reduced to a very small figure, and it can even be set to zero for a certain operating condition.

55 Referring back to Fig. 17 it will be noted that the valve-lift ratio of the mushroom-type follower is favorable, also the opening of the valve is very rapid and will, therefore, eliminate wire

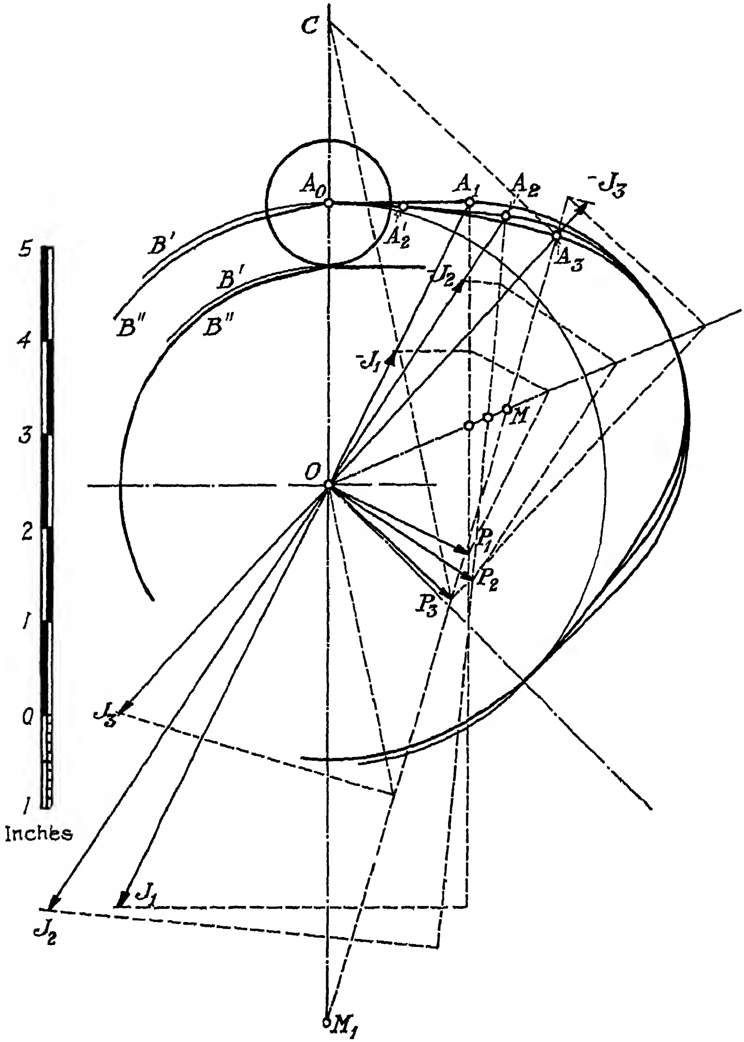


FIG. 18 1-IN.-LIFT CAM

drawing of the gases. The deceleration force is smaller than on any other cam. Therefore, the spring is relatively light, but the conditions due to necessary clearance come into play even more than with the tangential cam, because the initial speed at the

time when the cam actually begins to lift the tappet is more than twice that for the tangential cam. The reason for this is the very rapid opening at the beginning of the lift.

Cam of Low Lift and Quick Opening

56 In certain cases it is required that a cam be used with a small lift, but to open during a very short angle. This is, for example, the case in the operation of the fuel-pump plunger of solid-injection oil engines.

57 The cams of Figs. 19 and 20 have a lift of $\frac{5}{16}$ in. and an angle of opening of 25 deg.

58 The cam of Fig. 19 is not a true tangent cam, as it may appear at first glance, but starts as a concave cam with a radius equal to the radius of the roller R_2. This cam has no rounding at the nose. Therefore the center of the radius of the equidistant or pitch line lies on the curve of the cam in point M. The straight part of the cam is then a line laid through point M and tangent to the circle described by radius R_2.

59 The acceleration[1] in this case is determined as follows:

Draw a perpendicular OC to A_0A_{10} through O
Erect a perpendicular to OA_{10} (or OA_1, OA_2, etc.) at O
Draw a perpendicular to A_0A_{10} at A_{10} (or A_1, A_2, etc.), intersecting at P_{10}
Find point K either by making $PK = AP$, or by using the method described in Par. 35, Fig. 14.
Draw path of motion AO and extend through O
Erect a perpendicular to JK at K, intersecting at J.

With ω as angular velocity, $OP \times \omega$ = velocity of point A, and $OJ \times \omega^2$ = acceleration of point A.

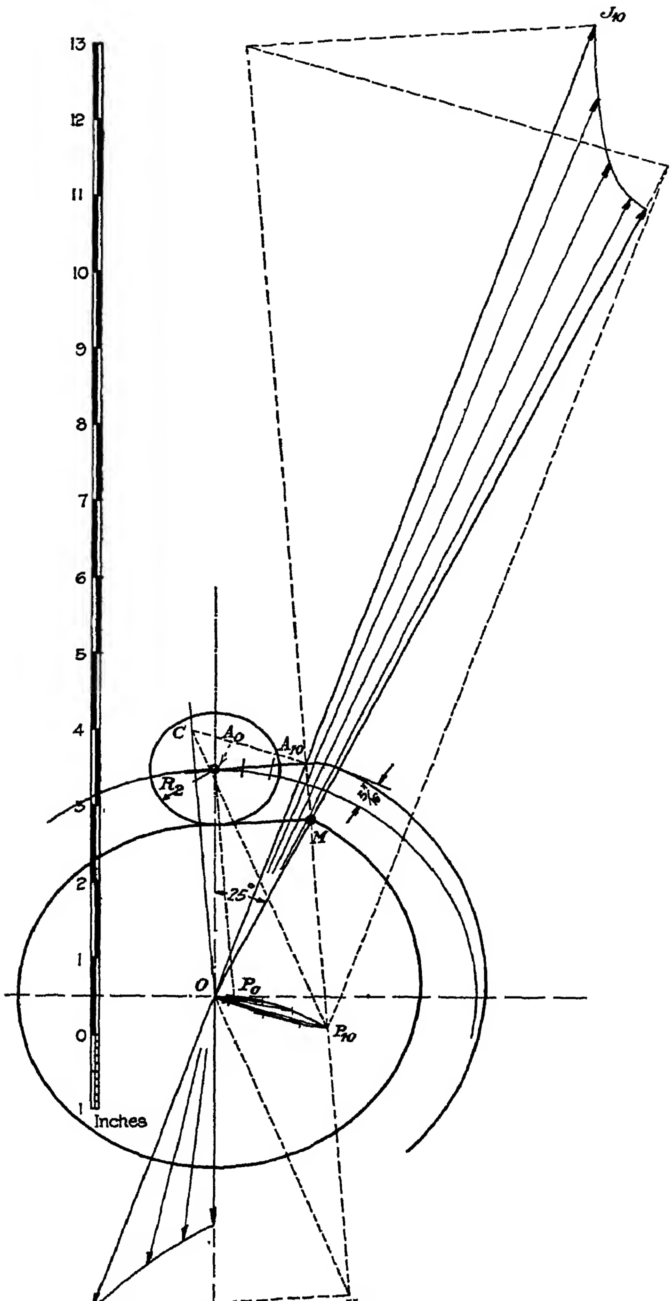
60 It should be noted that the velocity for A_0 is not zero but is equal to $OP_0 \times \omega$. The point P_0 is the intersection of the normal to A_0O at O and the parallel to OC through A_0, which is in accordance with the procedure described under Par. 59.

61 Considering that acceleration is equal to increase in velocity divided by time, and since at point A_0 the velocity is equal to $OP \times \omega$, and the time is zero, the acceleration at this point is infinite.

62 The deceleration is designed according to the method described under Par. 32, Fig. 13. Note the large values of deceleration which might be expected from a cam on which the rounding of a nose has come down to a point.

63 Fig. 20 is a cam which starts with a concave curve whose radius is equal to the radius of the roller R_2; it has no straight part

[1] The proof of this design is found in Pars. 29 to 31 of the original paper.



J_{10}
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1
Inches
C
A_0
A_{10}
R_2
M
25°
O
P_0
P_{10}
J_{10}
K_{10}

FIG. 19

and the concave curve passes into a large radius of the nose. This cam is interesting from the kinematic point of view. It starts with infinite acceleration at point A_0, and then passes over at the same point into deceleration, or in other words, there is no period of acceleration. The deceleration is designed as described under Par. 32 for Fig. 13. The deceleration is very small on account of a much larger radius than on the previous cams. This cam is very favorable as it requires a very light spring. However, it can be used only for a relatively small lift. If the lift is too large, the spring

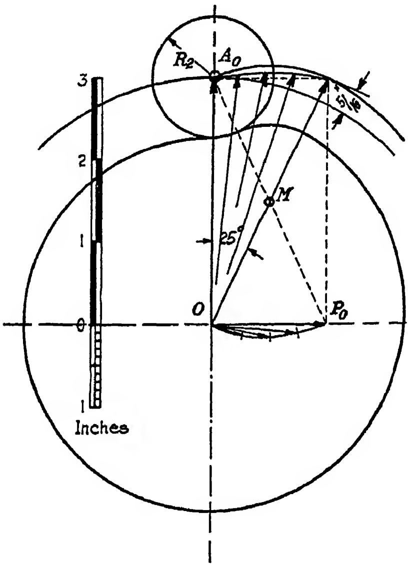


FIG. 20

required to keep the roller from bouncing and losing contact with the cam at point A_0 becomes too heavy.

64 A suggestion for a cam is offered in Fig. 21 (*a* and *b*), which is to employ the advantages of the most favorable cam and accept the least unfavorable conditions. In order to reduce the deceleration, which determines the force of the spring, the lift has been increased from $\frac{5}{16}$ in. to $\frac{3}{8}$ in., or by only $\frac{1}{16}$ in. Thereby, a larger radius for the rounding is possible and consequently the deceleration is cut down considerably. It will be noted that on this cam the requirement of lift of $\frac{5}{16}$ in. at 25 deg. has been retained and the larger angle is used only to take care of the larger radius at the nose.

65 The layout of this cam is drawn on an exaggerated scale in Fig. 21*b*. The problem may be considered as a tangential cam

whose radius of the pitch circle is R' and is working over the angle θ_{20}. However, the first part has been cut away here, and therefore the actual motion takes place through the smaller angle

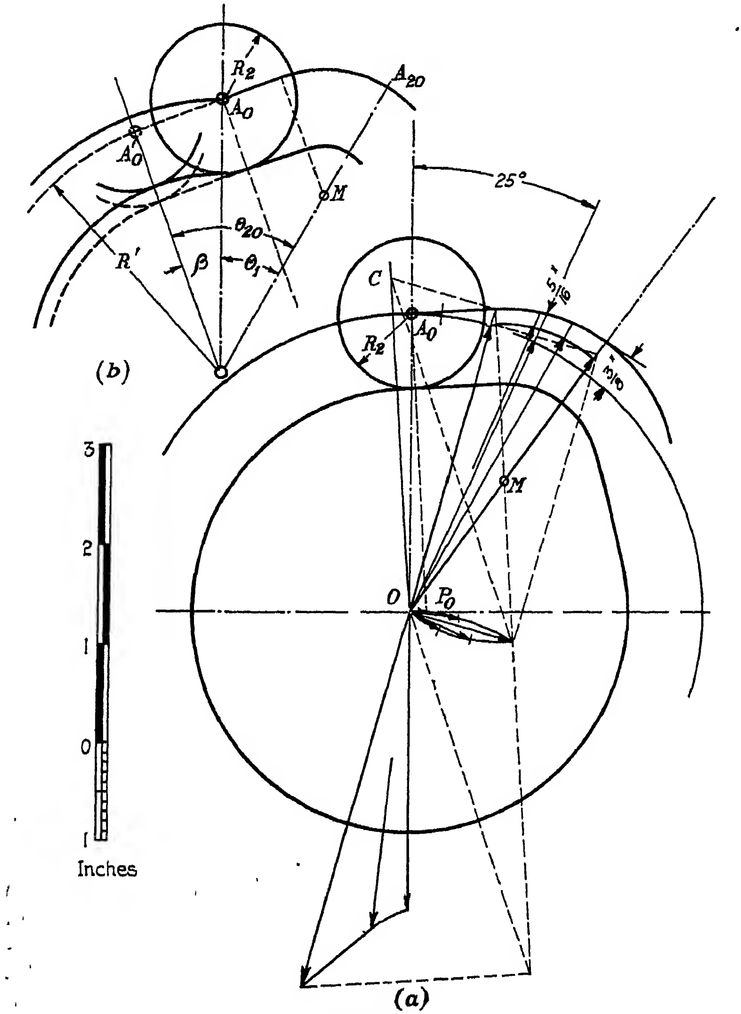


Fig. 21

θ_1. It will be noted at the break in the pitch line at A_0, which is characteristic for cams of Figs. 19 and 20, and which indicates infinite acceleration which was noted before.

66 The results of various cams of low lift and quick opening have been plotted in Fig. 22, which shows in an impressive

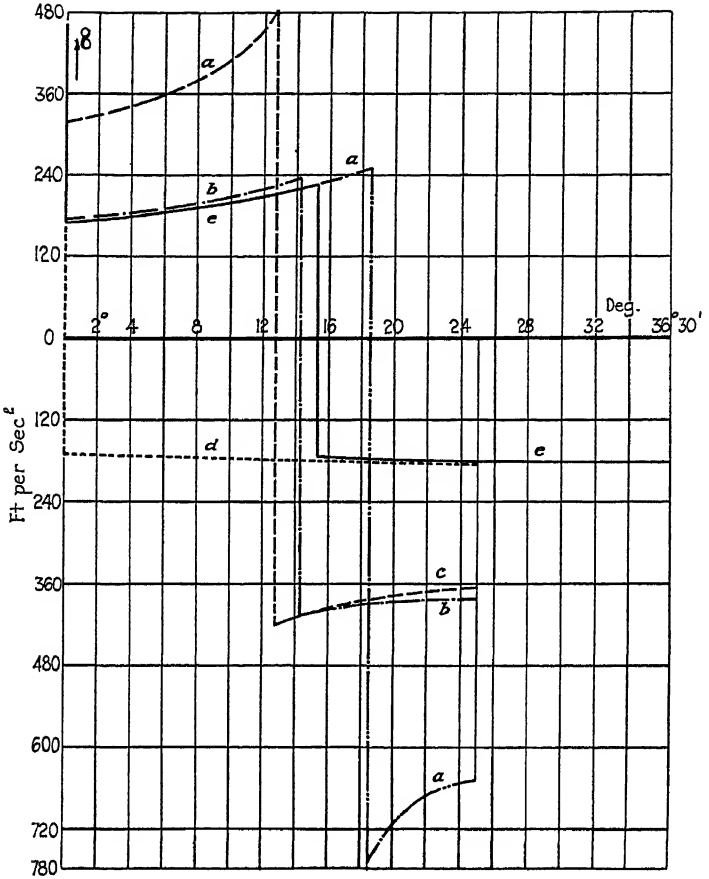


FIG. 22 ACCELERATION AND DECELERATION OF QUICK-OPENING CAMS

a Cam with no rounding at the nose (Fig. 19)
b Cam with small radius at the nose (Fig 25 of original paper)
c Cam with a concave curve (Fig. 26 of original paper)
d Cam with large radius at the nose (deceleration only) (Fig. 20)
e Cam with slightly increased lift (Fig. 21)

manner the great difference resulting from very minor changes in the shape of the cam. It also shows that the proposed cam has the least acceleration of any cam and the minimum value of deceleration of any of the cams investigated.

CAMS USED IN CONNECTION WITH A ROCKER ARM
OR
CAMS ON WHICH THE PATH OF MOTION DOES NOT GO THROUGH THE CENTER OF CAM

67 The arrangement of cam drive to be considered now is analyzed in Fig. 23. This cam has a 1-in. lift. The rocker arm

is represented by the line A_0N_0. It should be noted that the most favorable position for the pivot N would have been position N', but this point was displaced by $\frac{15}{16}$ in. for two reasons: (1) to make the design somewhat clearer for the case, so that the path of motion does not go through the center of the cam, and (2) to show how pronounced is the effect on the kinematic condition by such a small change in the design ($\frac{15}{16}$ in.) chosen here.

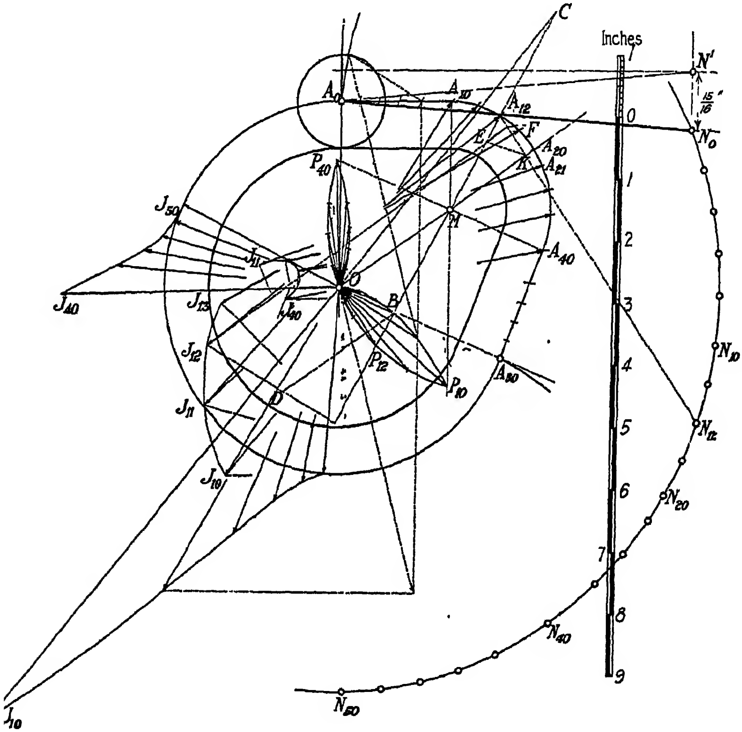


FIG. 23 1-IN.-LIFT CAM

68 For the calculation let the cam again be fixed and let the pivot N revolve at equal intervals about center O. The respective points are then found by striking a circle about the points N with N_0A_0. The acceleration is found as described in Par. 29 for Fig. 11. The deceleration is based on the theory developed by Professor Poeschl (see footnote, Par. 1) and is found in Fig. 23 in the following way:

Draw $A_{12}N_{12}$, intersecting OM at K

Draw a parallel to $N_{12}A_{12}$ through O

Draw $A_{12}M$ and extend over A_{12} and over M intersecting at P_{12}

Draw a parallel to $P_{12}K$ through O, intersecting at C
Make $A_{12}B$ equal MC
Draw and make BD parallel to and equal to MO
Erect a perpendicular to MBP_{12} through D
Draw a parallel through K to $N_{12}B$, intersecting $A_{12}M$ at E
Draw a parallel through E to OM, intersecting $A_{12}N_{12}$ at F
Erect a vertical on $A_{12}N_{12}$ in F, intersecting at J_{12}.

With angular velocity equal to ω, $OP \times \omega$ = velocity of point A, $OJ \times \omega^2$ = acceleration of points A_0 to A_{10}, and $AJ \times \omega^2$ = deceleration of points A_{10} to A_{20}.

69 In Fig. 23 the great difference of acceleration of OJ_{10} and OJ_{40}, and of the deceleration $A_{10}J_{10}$ and $A_{40}J_{40}$ should be noted. The two values of the former and the two of the latter would be the same if the path of motion went through the center of the cam in all positions. In this example, the value is in one case 90 per cent and in the other 60 per cent larger than its corresponding value. This large difference is due partly to the displacement of pivot N_0 by $\frac{1}{1}\frac{5}{6}$ in. and partly to the travel of point A on an arc, instead of on a straight line passing through the center of the cam in every position.

70 In order to check the correctness of the graphical method, this example is also calculated from point to point. The mathematical problem is shown for the straight part of the cam in Fig. 24 and for the nose in Fig. 25. These figures are shown somewhat distorted in order to have the angles show up better. The point N_0 is the pivot of the rocker arm for zero position. The angle N_0OA_0 is 66 deg., 40 min. As explained in Par. 68, pivot N is considered to revolve about O. Then as point N moves by increments of 6 deg., the angle μ will increase from 0 to 6 deg., 12 deg., and so on until μ = 31 deg., 4 min., 25.632 sec. when the roller is at the end of the straight part of the cam. This means that there resulted

$$\beta = 66 \text{ deg., } 40 \text{ min.} + 31 \text{ deg., } 4 \text{ min., } 25.632 \text{ sec.}$$
$$= 97 \text{ deg., } 44 \text{ min., } 25.632 \text{ sec.}$$

The design up to this point is shown in Fig. 24, and the calculation in Table 2, Part 1.

71 The calculation in Table 2 is carried one point beyond the last point in the drawing, that is to station β = 104 deg., 48 min., 51.264 sec., which was necessary in order to figure the acceleration for the last point (β = 97 deg., 44 min., 25.632 sec.). The acceleration is then figured from the increase in travel between 90 deg., 40 min.; 97 deg., 44 min., 25.632 sec.; and 104 deg., 48 min., 51.264 sec., or an equal increment of 7 deg., 4 min., 25.632 sec.

72 The calculations in Table 2, Parts 1 and 2, have been carried through in great length because they are intended to prove

that the graphical solution of velocity as well as acceleration is correct. Both of the tables have been calculated in two independent ways. First, the increase of the angle σ has been figured, and from it the travel on the arc of 6-in. radius, which is the length of the rocker arm. From this, travel, velocity, and acceleration have been estimated. Second, since it was previously proved that $OP \times \omega$ is the velocity, the distance OP has also been calculated, and, in turn, the acceleration estimated from this figure

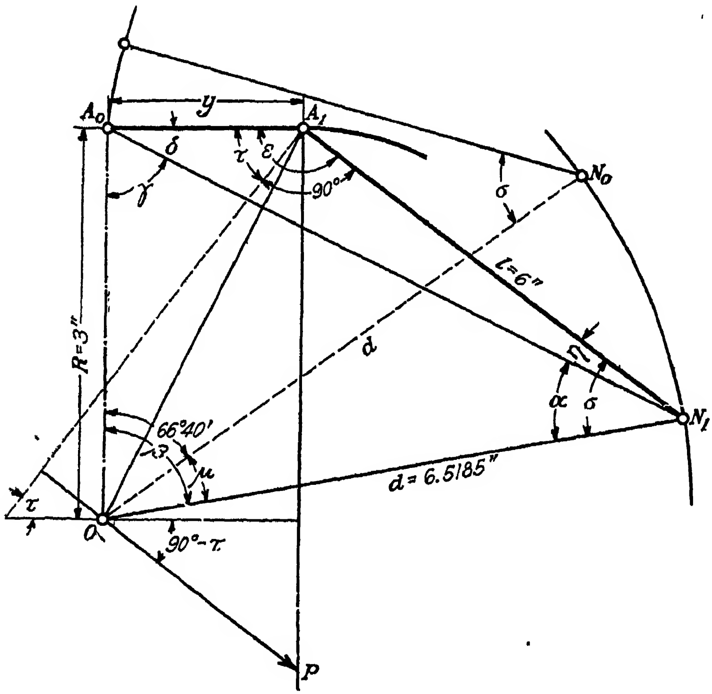


FIG. 24

as a second check. The two results of acceleration must check each other.

73 The point A will then continue to the curved part of the cam from A_{10} to A_{20} (Fig. 25), that is, until angle θ is 56 deg. In this position pivot N has turned to a position corresponding to point A_{20}. Then angle $\mu = 22$ deg., 42 min., 36.168 sec., a value which is estimated and also checked on the drawing. This angle or travel is divided into four equal parts, 5 deg., 40 min., 39.042 sec. for each interval. For positions A_{10} and N_{10} we then have

$$\alpha = 97 \text{ deg., } 44 \text{ min., } 25.632 \text{ sec.} - 56 \text{ deg.}$$
$$= 41 \text{ deg. } 44 \text{ min., } 25.632 \text{ sec}$$

and this angle increases from station to station by 5 deg., 40 min., 39.042 sec. A division of equal parts, that is, 5 deg., 40 min., 39.042 sec. instead of 6 deg., as in all other cases, has been chosen here in order that the values of calculation and graphical solution may be directly comparable, and this even division makes the calculation somewhat simpler. The mathematical problem is shown in Fig. 25 and its calculation in Table 2, Part 2.

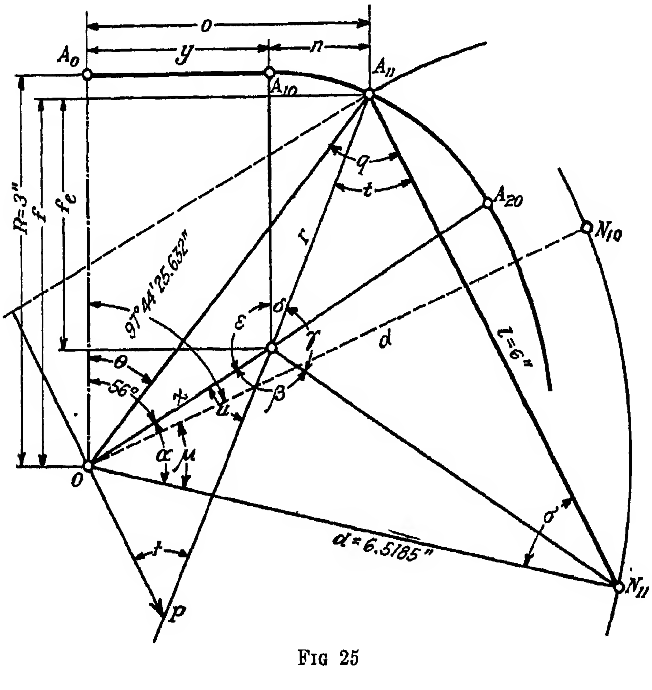


FIG 25

74 In Table 2, Part 1, as well as Part 2, should be noted in particular the agreement between the values calculated and those found graphically.

75 The graphical solution of Fig. 23 seems to be rather complicated, although when working with it it will be found rather simple in its application. However, for all purposes of a first layout or comparison of various cams, another graphical solution[1] which is simpler, even if not absolutely correct when the path of motion lies on an arc, is offered in Fig. 26. The cam has the same arrangement as that of Fig. 23.

[1] Prof. Dr. Ing. Poeschl, Lehrbuch d. Techn Mechanik, 1923, pp. 140-143.

TABLE 2 — PART 1 1-IN. TANGENTIAL CAM WITH ROCKER ARM, FIG. 23 CALCULATION OF TRAVEL, VELOCITY, AND ACCELERATION (STRAIGHT PART OF CAM) USING FIG. 24

$\tan\gamma = \dfrac{d \times \sin\beta}{R - d \times \cos\beta} = \dfrac{6.5185 \times \sin\beta}{3 - 6.5185 \times \cos\beta}$

$c = \dfrac{R \sin\beta}{\sin\alpha} = \dfrac{3 \sin\beta}{\sin\alpha}$

$\sin\epsilon = \dfrac{c \sin\delta}{l} = \dfrac{c \sin\delta}{6}$

$y = \dfrac{l \sin\eta}{\sin\delta} = \dfrac{6 \sin\eta}{\sin\delta}$

$OP = \dfrac{y}{\cos(90 - \tau)} = \dfrac{y}{\sin\tau}$

									7° 4′ 25.632″					7° 4′ 25.082″	
travel increment		6°		6°		6°		6°		6°		6°			
β	66° 40′		72° 40′		78° 40′		84° 40′		90° 40′		96° 40′		97° 44′ 25.632″		104° 48′ 51 264″
γ	86° 0′ 12.83″		80° 21′ 2.79″		74° 56′ 45 85″		69° 45′ 8.28″		64° 44′ 14.97″		59° 52′ 32.45″		59° 1′ 11.82″		58° 28′ 44.27″
$\gamma + \beta$	152° 40′ 12.83″		153° 1′ 2.79″		153° 36′ 45.85″		154° 25′ 8.28″		155° 24′ 14.97″		156° 32′ 32.45″		156° 45′ 37.452″		158° 17′ 36.584″
$\alpha = 180° - (\gamma + \beta)$	27° 19′ 47.17″		26° 58′ 57.21″		26° 23′ 14.15″		25° 34′ 51.77″		24° 35′ 45.03″		23° 27′ 27.55″		23° 14′ 22.548″		21° 42′ 24 466″
$\delta = 90° - \gamma$			9° 38′ 57.21″		15° 3′ 14.15″		20° 14′ 51.77″		25° 15′ 45.03″		30° 7′ 27.55″		30° 58′ 48.18″		36° 31′ 15.73″
log c			.80014985		.82075975		.84996705		.85777435		.87421405		.87701575		.894406072
ϵ			169° 50′ 39.33″		163° 21′ 4.88″		156° 29′ .13″		149° 9′ 36.42″		141° 14′ 2.96″		139° 44′ 5.27″		128° 56′ 31.47″
$\eta = 180° - (\epsilon + \delta)$	0		0° 30′ 23.46″		1° 35′ 40.97″		3° 16′ 8.1″		5° 34′ 38.55″		8° 38′ 29.09″		9° 17′ 6.55″		14° 32′ 12.8″
$\sigma = \eta + \alpha$	27° 19′ 47.17″		27° 29′ 20.67″		27° 58′ 55.12″		28° 50′ 59.87″		30° 10′ 23.58″		32° 5′ 56.64″		32° 31′ 29.098″		36° 14′ 37.266″
increase		9′ 33.50″		29′ 34.45″		52′ 4.75″		1° 19′ 23.71″		1° 55′ 33.06″		2° 21′ 5.52″		3° 43′ 8 168″	
increase in seconds		573.50″		1774.45″		3124.75″		4763.71″		6933.06″		8465.52″		13388.168″	
Travel on arc of 6″ radius $= \dfrac{12\pi \times \text{seconds}}{360 \times 60 \times 60}$ in.		.016682″		.051616″		.090895″		.13857″		.20167″		.24625″		.3894″	
time interval in seconds		.004		.004		.004		.004		.004		.00472		.00472	
Mean velocity $= \dfrac{\text{travel in inches}}{12 \times \text{time}}$ ft./sec.		.847		1.072		1.89		2.88		4 19		4 36		6.87	
velocity increase	.847		.725		.818		.99		1.31				2 51		
Time interval in seconds	.002		.004		.004		.004		.004				.00472		
Acceleration $= \dfrac{\text{velocity increase}}{\text{time}}$ ft./sec²	173		181		205		248		328				523		
Graphically determined value [1]	172		183		206		249		328				525		
$\tau = \epsilon - 90°$			79° 50′ 39.33″		73° 21′ 4.88″		66° 29′ 13″		59° 9′ 36.42″				49° 44′ 5.27″		
log y			.5003027 — 1		.8081363 — 1		9950282 — 1		.1355823				.2743275		
OP	0		.321482″		.67102″		1.07815″		1.59144″				2.4647″		
Velocity $v = OP \times \dfrac{\omega}{12}$ ft./sec.	0		.7		1.462		2.35		3.48				5.38		
Graphically determined value [2]	0		.697		˙1.46		2.34		3.44				5.37		
velocity increase (v)		.7		.762		.888		1.13			1.90				
time interval in seconds		.004		.004		.004		.004			.00472				
Mean acceleration $= \dfrac{\text{velocity increase}}{\text{time}}$ ft./sec.²		175		191		222		282			403				

[1] and [2] Graphically determined values of acceleration [1] and velocity [2] are values taken from Fig. 23

TABLE 2 — PART 2 1-IN. TANGENTIAL CAM WITH ROCKER ARM, FIG. 23 CALCULATION OF TRAVEL, VELOCITY, AND DECELERATION (CURVED PART OF CAM) USING FIG. 25

Travel increment			5° 40′ 39 042″		5° 40′ 39 042″		5° 40′ 39.042″		5° 40′ 39 042″	
	α	41° 44′ 25.632′		47° 25′ 4 674″		53° 5′ 43.716″		58° 46′ 22.758″		64° 27′ 1.8″
$\tan\beta = \frac{d \sin\alpha}{x - d\cos\alpha} = \frac{6\ 5185 \sin\alpha}{2.268566 - 6.5185 \cos\alpha}$	β	120° 52′ 51.51″		114° 3′ 6.53″		107° 31′ 19.309″		101° 16′ 13.149″		
$c = \frac{d \sin\alpha}{\sin\beta}$	log c	.7038556		.720651187		.787672207		.754629763		
$\cos\gamma = \frac{r^2 + c^2 - l^2}{2rc} = \frac{1.731434^2 + c^2 - 6^2}{2 \times 1.781484 \times c}$	γ	115° 7′ 16 2″		107° 11′ .238″		99° 30′ 6.68″		92° 1′ 55.168″		
	ϵ	124°		124°		124°		124°		
$\delta = 360° - (\gamma + \epsilon + \beta)$		0		14° 45′ 53 232″		28° 58′ 34.011″		42° 41′ 51.683″		
$e = r \cos\delta$	e	1.73144		1.6742682		1.51470132		1.2725079		
$f = R - r + e = 3 - 1.73144 + e$	f	3.0000		2.9428282		2.78326132		2 5410679		
$n = r \sin\delta$	n	0		.44024523		.83878788		1 17414149		
$0 = n + y$ (From Part 1, this table)	y	1.8807264		1.8807264		1.8807264		1.8807264		
	0	1.8807264		2.32097163		2.71951878		3.05486789		
$\tan\theta = \frac{0}{f}$	θ	32° 5′ 2.26″		38° 15′ 44.642″		44° 20′ 9.16″		50° 14′ 45.588′		56°
	$\alpha + 56° - \theta$	65° 39′ 23 372″		65° 9′ 20.032″		64° 45′ 34 556″		64° 31′ 47.17″		64° 27′ 1.8″
$m = \frac{f}{\cos\theta}$	log m	.54909968		.573794065		.59009292		59918067		.60206
$\sin\sigma = \frac{m \times \sin(\alpha + 56° - \theta)}{l} = \frac{m \times \sin(\alpha + 56° - \theta)}{6}$	σ	32° 31′ 28.82″		34° 32′ 10.1″		35° 55′ 5 15″		36° 43′ 9.5″		36° 58′ 32 29″
increase			2° 0′ 41.28″		1° 22′ 55.05″		48′ 4.35″		15′ 22 79″	
increase in seconds			7241.28″		4975.05″		2884 85″		922.79″	
Travel on arc of 6-in. radius $= \frac{12\pi \text{ sec}}{360 \times 60 \times 60}$ in.			.21064″		.144718″		.083923″		.0268429″	
time interval in seconds			.00379″		.00379″		.00379″		.00379″	
Mean velocity $= \frac{\text{travel in in.}}{12 \times \text{time}}$ ft./sec			4.65		3.195		1.85		.593	
velocity decrease				1 455		1 345		1 257		.593
time interval in sec.				.00379″		.00379″		.00379″		.001895
Deceleration $= \frac{\text{velocity decrease}}{\text{time}}$ ft./sec.[2]				384		356		332		318
Graphically determined value [1]		410		384		356		332		
$\sin q = \frac{d \sin\sigma}{m}$	q	81° 49′ 5.83″		80° 23′ 58.85″		79° 19′ 24.55″		78° 45′ 39.45″		
$t = q - [180° - (90° + \delta + 34°) - (56° - \theta)] = q + \delta - \theta$	t	49° 44′ 8.07″		56° 54′ 7.44″		63° 57′ 49.401″		71° 12′ 45 545′		
	$\mu = 56° - \delta$	56°		41° 14′ 6.768″		27° 1′ 25.989″		13° 18′ 8.317″		
$OP = \frac{\gamma \sin\mu}{\sin t}$	OP	2.4647″		1.785″		1.1472″		.55135″		
Velocity $v = \frac{OP}{12}\omega$ ft./sec.		5.38		3.9		2.5		1.203		0
Graphically determined value [2]		5 37		3.88		2.48		1.19		0
Velocity decrease (v)			1.52		1.4		1.297		1.203	
Time interval in sec.			.00379		.00379		.00379		.00379	
Mean deceleration $= \frac{\text{velocity decrease}}{\text{time}}$ ft./sec.[2]			398		370		342		318	

[1] and [2] See footnote Table 2, Part 1.

TABLE 3 1-IN. TANGENTIAL CAM WITH ROCKER ARM — FIGS 23 AND 26

	Velocity						Acceleration				
				Checking calculation			Fig. 23			Fig. 26	
Deg. Min. Sec.	Reading OP, in.	Estimated value * ft. per sec.	$OP \times \frac{\omega}{12}$ ft. per sec.	Diff v' ft. per sec.	$\frac{v'}{t}$ ft. per sec.²	Estimated value * ft. per sec ²	Reading OJ and AJ, in.	$\left.\begin{matrix} OJ \\ AJ \end{matrix}\right\} \times \frac{\omega^2}{12}$ ft. per sec ²	Estimated value * ft. per sec ²	Reading OJ, in	$OJ \times \frac{\omega^2}{12}$ ft. per sec.²
0	0	0	0				3.01	172	...	...	...
				$0.697 \times \frac{1}{0\ 004}$	$= 174$	175					
6	0.32	0.7	0.697				3.20	183	181	.	...
				$0.763 \times \frac{1}{0\ 004}$	$= 191$	191					
12	0 67	1.462	1.46				3 6	206	205		...
				$0.88 \times \frac{1}{0\ 004}$	$= 220$	222					
18	1.075	2.35	2.34				4 36	249	248	.	...
				$1\ 1 \times \frac{1}{0.004}$	$= 275$	282					
24	1.58	3 48	3.44				5.74	328	328	. .	...
				$1\ 93 \times \frac{1}{0\ 00472}$	$= 408$	403					
31- 4-25.632	2.46	5.38	5.37				9 20	526	533	...	...
				. ...	....	...					
31- 4-25 632	2 46	5.38	5 37				—7 13	—410	...	—7 95	—454
				$1.49 \times \frac{1}{0\ 00379}$	$= 393$	—396					
36-45-4.674	1.78	3 9	3 88				—6 71	—384	—384	—7 04	—402
				$1.4 \times \frac{1}{0\ 00379}$	$= 369$	—370					
42-25-43.716	1 14	2 5	2.48				—6 23	—356	—356	—6 32	—361
				$1\ 29 \times \frac{1}{0\ 00379}$	$= 340$	—342					
48- 6-22.758	0.545	1.203	1.19				—5 80	—332	—332	—5.78	—331
				$1\ 19 \times \frac{1}{0\ 00379}$	$= 314$	—318					
53-47- 1.8	0	0	0				..	.	...	—5 4	—309

* Estimated values taken from Table 2, Part 1 and 2 See also Par. 77

TABLE 3 (CONTINUED)

Deg. Min. Sec.	Velocity: Reading OP, in.	Velocity: Estimated value * ft. per sec.	Velocity: $OP \times \frac{\omega}{12}$ ft. per sec.	Checking calculation: Diff. v' ft. per sec.	Checking calculation: $\frac{v'}{t}$ ft. per sec.^{2}	Acceleration: Estimated value * ft. per sec.^{2}	Acceleration, Fig 23: Reading OJ and AJ, in.	Acceleration, Fig 23: $\left.\begin{matrix}OJ\\AJ\end{matrix}\right\} \times \frac{\omega^2}{12}$ ft per sec.^{2}	Acceleration: Estimated value * ft. per sec.^{2}	Acceleration, Fig. 26: Reading OJ, in	Acceleration, Fig. 26: $OJ \times \frac{\omega^2}{12}$ ft. per sec ^{2}
58-47- 1.8	0	.....	0				...	...	...	—5 62	—321
				$0.829 \times \frac{1}{0.002815}$	$= 295$	...					
58	0.88	.....	.829				—5.10	—292	...	—5 16	—290
				$1.121 \times \frac{1}{0.004}$	$= 280$	...					
64	0.89	... .	1.95				—4.65	—266	...	—4.72	—270
				$1.03 \times \frac{1}{0.004}$	$= 258$	...					
70	1.37	.....	2.38				—4.41	—252	...	—4 43	—253
				$1.52 \times \frac{1}{0.005975}$	$= 254$	...					
78-54-18	2.065	.....	4.50				—4.48	—256	...	—4 16	—238
				.....	....	...					
78-54-18	2.065	.....	4.50				4.82	276	.....	...	...
				$1.52 \times \frac{1}{0.006025}$	$= 252$	...					
88	1.37	.....	2.98				3 83	222	...	...	...
				$0\ 83 \times \frac{1}{0.004}$	$= 207$	...					
94	0.985	.....	2.15				3.44	197	...	...	...
				$0.755 \times \frac{1}{0.004}$	$= 189$	...					
100	0.64	.....	1.395				3 18	182	...	...	...
				$0.698 \times \frac{1}{0.004}$	$= 175$	...					
106	0.32	.....	.697				3.04	174	...	. .	...
				$0.697 \times \frac{1}{0.004}$	$= 174$	...					
112	0	.....	0				2 49	171	...	...	. .

* Estimated values taken from Table 2, Part 1 and 2 See also Par. 77.

76 The acceleration is found in the same way as in Fig. 23, but the deceleration is designed as follows:

Draw a parallel to $A_{13}N_{13}$ through O

Draw $A_{13}M$, extending through M, intersecting at P_{13}

Draw a parallel to $A_{13}O$ through P_{13}, intersecting at C

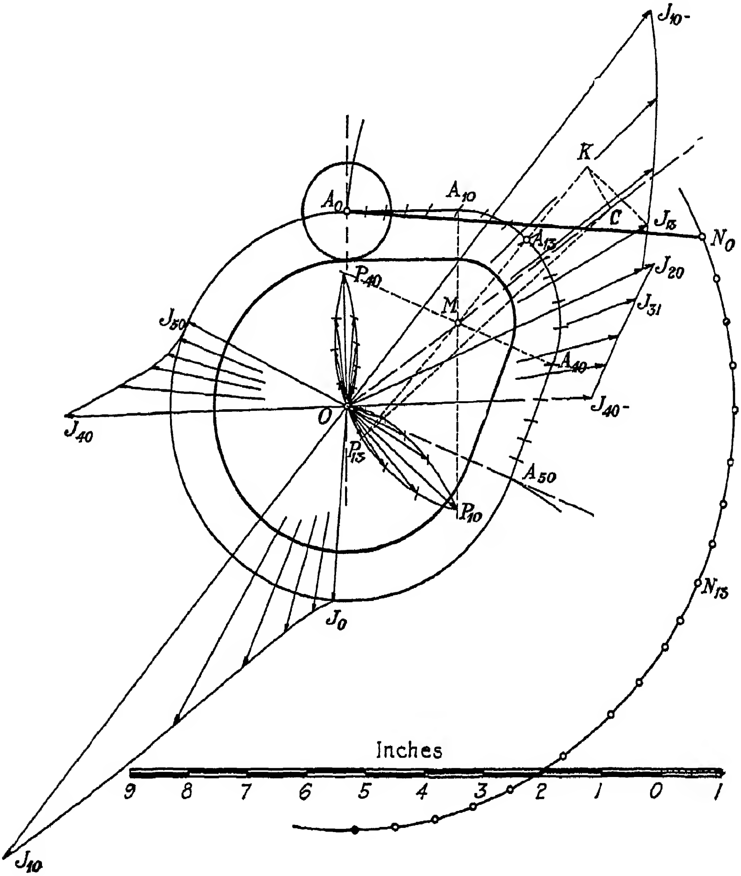


FIG. 26 1-IN.-LIFT CAM

Draw a parallel to $A_{13}N_{13}$ through C, intersecting at K on line $P_{13}A_{13}$

Draw a perpendicular to $A_{13}N_{13}$ through O (which is the direction of motion)

Erect a perpendicular to line $P_{13}A_{13}$ at K, intersecting at J_{13}.

With angular velocity $= \omega$, $OP \times \omega =$ velocity of point A, and $OJ \times \omega^2 =$ acceleration or deceleration of point A.

77 The results for this cam are shown in Table 3. In this table are entered the values as found by the graphical method shown in Fig. 23, and also as found by the simpler method shown in Fig. 26 (see last two columns of table). The values for acceleration for the latter case have not been repeated because these values are the same for both methods. The values of velocity and acceleration as found by calculation in Table 2 are also entered in this table for comparison. The difference of the values found by the two graphical methods will be noted to be very small, practically negligible for any preliminary calculation, and therefore the simpler method shown in Fig. 26 is close enough for the layout and comparison of various shapes of cams. However, when the final calculation of the valve spring is made, the analysis of Fig. 23 should be used.

78 In none of the graphical solutions is it possible to lay out the acceleration at point A_{20}, because the crank of the four-link chain is at dead center in this position. This point is, however, easily found by extending the curves beyond the points J_{13} and J_{31}.

79 It will be noted that there are two values for J_{20}, in Fig. 26, as there should be for this reason: Due to the location of point N_0 (Par. 67) the velocities from point A_0 to A_{20}, and from A_{80} to A_{60} differ, as is clearly expressed in the two different shapes of the velocity ellipses; OP_{10} is much larger than OP_{40}. Therefore, if the velocities are plotted in a graph against the time, the velocity curves will be different, and, since the acceleration can also be found as a function of the tangent laid on the velocity curve, the acceleration at point A_{20} must differ, depending on which branch of the velocity curve the tangent is laid.

80 For the purpose of comparison, the results of three tangential cams have been plotted in Fig. 27 for the complete angle of the cam. In Fig. 17 these values are plotted only for the points from A_0 to A_{20}, because the cams are symmetrical and the path of motion runs through the center of the shaft. Therefore, the curves for opening and closing are the same. This present figure is to illustrate the difference of kinematic conditions for a cam of a $\frac{3}{4}$-in. lift and one of a 1-in. lift. The latter is shown in two ways, that is, first, a cam on which motion is going through the center of the shaft, and second, a cam in connection with a rocker arm. All three cams have no dwell, and the other characteristics are the same as given in Par. 31.

81 In all the charts acceleration and deceleration have been shown as feet per second per second (ft. per sec.^{2}). Since the force of the spring is figured by the formula

$$F = \frac{W}{g} \times a$$

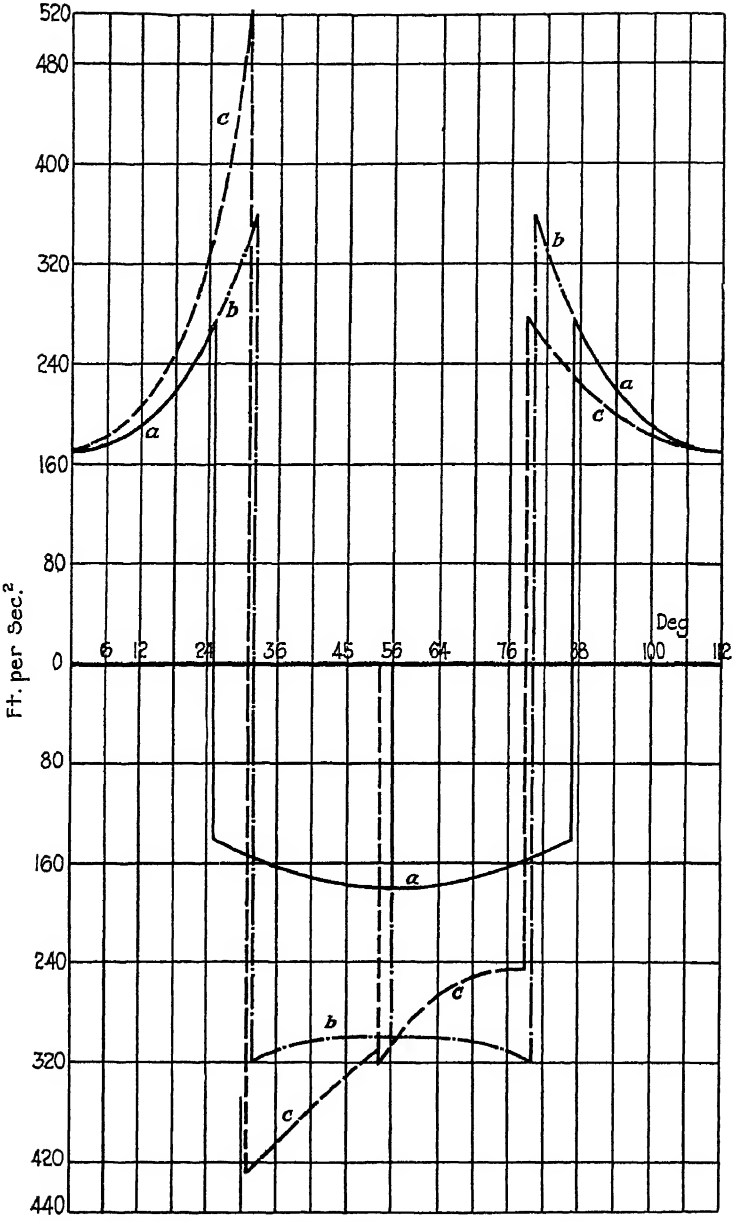


FIG. 27 ACCELERATION AND DECELERATION OF THREE TANGENTIAL CAMS

a ¾-in.-lift cam with straight-line motion
b 1-in.-lift cam with straight-line motion
c 1-in.-lift cam with rocker arm (Fig. 23)
(For calculation of cam *a* and *b* see original paper)

where W = weight of reciprocating parts in lb.
g = acceleration of gravity = 32.2
a = acceleration or deceleration

the force of the spring can be read from the same chart by changing the scale.

82 The kinematic condition of a tangential cam of $\frac{3}{4}$-in. lift with rocker arm[1] is plotted in Fig. 28. It may be assumed that this cam is, in proportion, good for a four-cycle Diesel engine operating at 500 r.p.m., or a camshaft running at 250 r.p.m., which

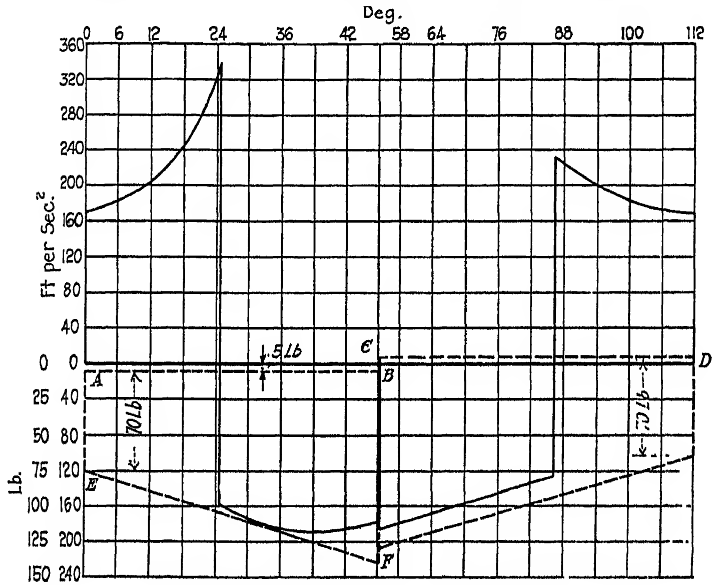


FIG. 28 $\frac{3}{4}$-IN. CAM WITH ROCKER ARM

would correspond to an engine of about 50 hp. per cylinder with a 10-in. bore and 12-in. stroke.

83 The weight of the reciprocating parts would be about as follows:

	lb.
Weight of valve	3.95
Weight of push rod	10.65
Weight of rocker arm (reduced to center line of motion)	3.75
Weight of $\frac{1}{2}$ spring	1.75
Total	20.10

It then follows that the scale of force on the chart becomes, say, for 40 ft. per sec.²

$$40 \times \frac{20.10}{32.2} = 25 \text{ lb.}$$

which means, therefore, that 40 ft. per sec.² scales also 25 lb. pressure.

[1] For calculation, see pages 59 and 60 of original paper.

84 By the addition of this scale the chart has become a diagram of forces at any position of the cam and the force of the spring can be plotted right over it. It is assumed that the friction of the valve stem and all joints will amount to approximately 5 lb. in this case. This force acts in the direction of the spring force while the valve is being opened and, therefore, reduces the force required for the spring, whereas, this force of friction is opposed to the spring force while the valve is closing and must then be subtracted from the actual force of the spring. In other words, the zero lines of the force of the spring are the lines *AB* and *CD* respectively. The line of spring force *EF* must always lie below the maximum line of the force due to deceleration of the masses.

85 However desirable it is to keep the spring as light as possible, there is one more factor in addition to kinematic requirements which on a four-cycle engine will control the size of the spring. This is the pressure below atmosphere in the cylinder during the suction stroke of the engine. This suction must not be allowed to draw the exhaust valve open, as it would mean noise when the valve returned to its seat. The sub-atmospheric pressure will depend on the type and speed of engine. In this case we may assume a negative pressure of 6 lb., and with a valve diameter of about $3\frac{3}{4}$ in., the requirements are as follows:

Area of valve	11 sq. in.
Suction on valve	$11 \times 6 = 66$ lb.
Friction	5 lb.
Minimum spring force required	$66 - 5 = 61$ lb.

The initial spring force assumed in Fig. 28 shows $75-5 = 70$ lb. tension when the valve is on its seat and, therefore, the spring is also strong enough to hold the valve closed during the suction stroke.

Suggestions

86 When laying out a new cam a study as suggested in Fig. 18 should be made. If a rocker arm is used, Figs. 11 and 12 will apply, and the laying out of the one point A_{10} is generally sufficient. For cams of low lift and quick opening a favorable design is suggested in Fig. 21. If a cam for a quick opening and larger lift is required, the mushroom-follower cam may be used to great advantage, provided the clearance is reduced to a minimum, as suggested in Fig. 18. It has been shown that with a properly laid-out cam the forces can often be cut in two. Further improvement can readily be added by reducing the weight of the reciprocating parts, which can be accomplished by the use of light-weight metal wherever possible.

87 The author desires to acknowledge and express his thanks for the courteous interest and suggestions which Prof. W. E. Goldsborough has extended and made from time to time in connection with this paper, and also to thank Mr. George Rathbun for his kindness in supplying the data used in Par. 83, which apply to an Ingersoll-Rand oil engine.

Conclusion

88 Particular attention is called to the conspicuous value of the graphical methods presented when they are applied to intricate problems which ordinarily require much time for calculation. It should be mentioned that these methods lend themselves equally well to the solution of the kinematics of a special cam, that is, levers with roller-path motion or wiper levers.

89 To the knowledge of the author, it is invariably stated in textbooks that transmission by a rocker arm affects very little the results arrived at either by formulas commonly known or by other widely known graphical methods. To clear up this statement let us refer to Fig. 26 in which the pivot of a rocker arm is displaced only $\frac{15}{16}$ in. (see also Fig. 23), with the result that both the maximum accelerations and decelerations vary about 1: 2, whereas the two values of acceleration OJ_{10} and OJ_{40} and deceleration $OJ_{10}-$ and $OJ_{40}-$, respectively, would be the same *if* the path of motion went through the center of the cam.

90 The result is so surprising that the example has been calculated all through from point to point in Table 2 — Parts 1 and 2. The author wishes to have the graphical method made so clear and its value so incontestably shown that it will find general use among engineers who have to solve or to analyze cam problems.

DISCUSSION

Carroll F. Merriam.[1] The method of the author is particularly adaptable for determining the motion, velocities, and accelerations produced by cams of easily manufactured profiles. At the same time it should be borne in mind that it is possible to develop a theoretically perfect cam from acceleration, velocity, and space versus time diagrams, which represent the results desired. Unfortunately the resulting profile will not necessarily be easily constructed in the shop, and it is, therefore, a question whether exact conformity to the desired motion is worth the added expense of manufacturing a more or less irregular shape. However, it is suggested that such an approach to the problem be used as a preliminary trial, from which a profile that is more practical from the manufacturer's standpoint may be selected. The method set forth by the author may then be followed very advantageously to check whether or not the shape selected is a sufficiently accurate approximation.

R. C. H. Heck.[2] The working relationships and constructions of this paper can be much more clearly and directly established

[1] Pennsylvania Water & Power Co., Baltimore, Md.

[2] Professor of Mechanical Engineering, Rutgers University, New Brunswick, N. J. Mem. A.S.M.E.

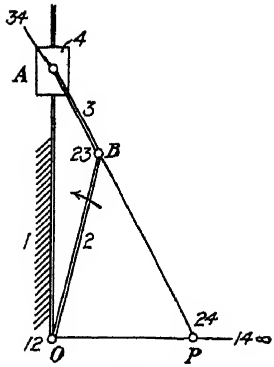


FIG. 29

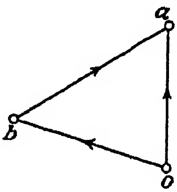


FIG. 30

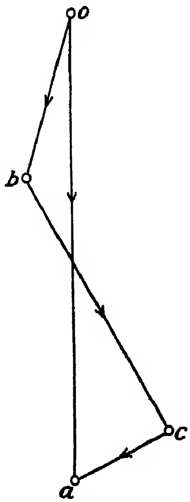


FIG. 31

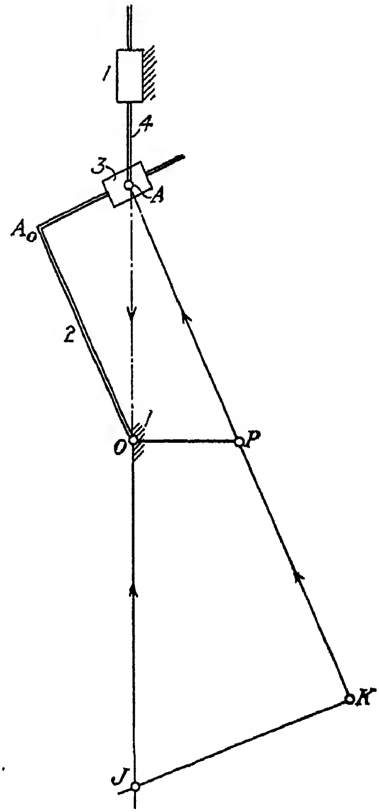


FIG. 32

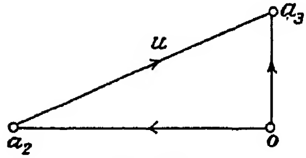


FIG. 33

by the methods of really graphical kinematics. Thus in Figs. 3 to 7 the point P is simply the relative instant center between the rotating member that represents the cam and the sliding member that represents the follower. Here in Fig. 29 (Fig. 3 of the paper) center 24, the point of common velocity between arm 2 and slide 4, lies on the three-center lines 12-14-24 and 23-34-24. In Figs. 9 (*b*) and (*c*) this center is at the intersection of lines 14 and $P3$, and $1P$ is a radius in proper ratio to 42 or 43.

The velocity diagram for Fig. 29 is given in Fig. 30. This triangle has two interpretations or two meanings. It is similar to OBP of Fig. 29 in the order oba, or with corresponding sides perpendicular to each other. Then on the plane of member 2, vector ob is radius $OB \times$ angular velocity ω_2, and vector oa is $OP \times \omega_2$, the velocity of point P, hence also of point A or any other point on slide 4.

Alternately, rod BA or 3 translates with and rotates about master point B, of which both the velocity v_b and the acceleration a_b are known. Point A has the vertical velocity ob, compounded of the velocity ob of point B and the component ba, due to the rotation of radius BA on B and perpendicular to that radius. The intersection at a makes the determination of both ba and oa. From linear velocity ba and radius BA can be found the angular velocity of rod BA.

The acceleration oa of point A, found in Fig. 31, is made up of three components. These are,

ob = acceleration of point B, equal to velocity $ob \times \omega_2$

bc = centripetal acceleration of A toward B, equal to velocity $ba^2 \div$ radius BA

ca = radius $BA \times$ angular acceleration α_3, primarily unknown, but determined, along with oa, by intersection a.

Consider now Fig. 6, outlined in Fig. 32 with the construction of Fig. 10 upon it. Velocity triangle oa_2a_3 in Fig. 33 is similar to OAP: vector oa_2 is the velocity of point A-on-2, oa_3 is the total velocity of point A-on-3 or of the slide, and a_2a_3 (the completing component from oa_2 to oa_3) is the relative velocity of A-on-3 upon member 2.

In Fig. 32 let $\omega_2 = 1$. Then AO can represent v_{a2} or oa_2 and also the acceleration $a_{a2} = v_{a2} \times \omega_2$, and AP will represent relative velocity u. Now the acceleration of point A-on-3 is compounded of a_{a2} or AO, the Corioli component $2u\omega$ or KPA (PA being $u\omega$ as well as u) and a_u or JK, the relative acceleration of point A-on-3 upon member 2. Intersection J determines the final acceleration JO of point A or slide 4, also the last component named.

With a mechanism like Fig. 9 (*b*) this final acceleration will have a fourth component, like ra in Fig. 31. With Fig. 9 (*b*) the construction in Fig. 31 is similarly elaborated, with an ra for member 4. For these methods see the writer's Mechanics of

Machinery, Kinematics and Dynamics, Sections 7 and 8. The derivations just made do not lend themselves readily to confirming calculations, but for the latter there is absolutely no need.

P. R. Hoopes.[1] While the author does not specifically limit his study to cams for internal combustion engines, it is apparent that the subject was developed with such cams in mind. The question of follower velocity and acceleration in cam mechanisms is, however, of practical importance in connection with all high-speed machinery. Outside the field of internal-combustion engines, it is rarely necessary to arrive at accurate quantitative figures for either velocities or accelerations of cam mechanisms, although it is always desirable to know the order of magnitude of these factors and to have available a practical method by which they can be calculated in those cases requiring precision.

As a purely practical matter we know that the actual accelerations and resulting forces do not conform to the theoretical analysis. The author correctly points out that initial clearance between the cam and its follower results in a theoretically infinite force which, in practice, is of relatively negligible importance. The same condition exists in the case of all strictly uniform-motion cams. The answer, of course, is that our materials are resilient and that actual forces may be vastly different from those indicated by analysis based on the assumption of perfect rigidity of the parts. Another disturbing factor is that of mechanical perfection of the cam itself. The slightest inaccuracy of the cam contour, or small defects in surface finish, are capable of producing a very considerable increase in the instantaneous velocities and forces.

The kinematic problem is not, of course, a new one; nor, if we neglect the practical considerations just noted, does it present any serious difficulties. For the three fundamental types of cam motion used in automatic machinery (that is, the uniform, the uniformly accelerated, and the harmonic) simple analytical methods are commonly employed. The general case, however, must be treated graphically, at least to the extent that an accurate layout of the cam, or preferably a space-time diagram of the motion, plotted in rectangular coördinates, must be available.

To attempt to handle the general problem directly from a drawing of the cam involves intricate draftsmanship, considerable expenditure of time, and somewhat unnecessary sacrifice of resulting accuracy. One of the factors which complicates a general graphical analysis of the motions directly from the cam drawing is that of the direction of the follower path. Cams for offset and swinging followers are obviously not polar space-time curves of the follower motions and cannot be so considered for the purpose of deriving velocity and acceleration diagrams.

[1] Consulting Mechanical Engineer, Hartford, Conn.

The rectangular space-time diagram of the motion is the only logical foundation for a generalized study of cam kinematics. It is easy to construct and to interpret, lends itself readily to graphical, mechanical, or analytical study, and automatically eliminates the necessity for considering the direction of the follower path in relation to the cam center. The methods of dealing with such diagrams are explained in the majority of elementary works on mechanics and graphical analysis published during the past fifty years. The plotting of the initial curve is purely drawing-board routine, and graphical differentiation by the construction of tangents is only a matter of reasonable care.

In the field of automatic machinery, where cams have their most varied applications, the construction of the rectangular space-time diagrams of the motions usually precedes the detailing of the cams, since it is upon the basis of these diagrams that the time chart of the machine is laid out. It is at this stage of the design that information regarding velocity and acceleration is necessary, and the simplest procedure consists in successive graphical differentiations of the curve, or the use of a mechanical differentiator.

A logical extension of the ordinary graphical methods of cam analysis was outlined by E. G. Coker in the Transactions of the Royal Society of Canada for 1903. This consisted in the application of Fourier's analysis to the rectangular space-time diagram of the cam, and the consequent derivation of an exact or approximate analytical expression for the motion, the first and second derivatives of which gave the velocities and accelerations for the case in question. Although interesting from the purely theoretical standpoint, this method is of doubtful practical value, since the same results can usually be secured by more elementary means.

The Author. Mr. Merriam states that the method presented is particularly adaptable for cams of easily manufactured profiles. However, it should be stated that the methods presented can be applied to cams of any shape. For example, assuming that we had a cam contour of any curve, we would resort to Fig. 14 or 15, draw a tangent on the arbitrary curve at any point or points, say, A_3 or A_5, and erect a vertical on this tangent in these points, intersecting at M_1. The procedure would then continue as explained in the paper.

Mr. Hoopes states that we know that the actual accelerations do not conform to the theoretical analysis. The author does not believe that many engineers will agree with this statement. Mr. Hoopes admits that the general case of a cam must be treated graphically. Why treat it at all if the results of the actual cam do not agree with the theoretical analysis? He states that offset and swinging followers are obviously not polar space-time curves and cannot be so considered. This is the very reason why this

paper presents a method of analyzing cam arrangements of such specification, and their solution is shown in Figs. 23 and 24, and proven in Table 2. These methods have not been published in elementary works or in the literature during the past fifty years. If Mr. Hoopes will study the paper he will see how simple the methods presented are and that they can be used for any arrangement of cams and followers, and he may then feel inclined to withdraw his remarks.

The author greatly appreciates Professor Heck's suggestion for solving the problem in another way. The purpose of the paper, however, was to present a method applicable to any form or arrangement of cam and which could be carried out very easily by any draftsman within a very limited time. From this point of view, let us compare the solution presented in Fig. 13 and the drawings required by Professor Heck in Figs. 29, 30, and 31 — and the latter figures are the layout for but a single point.

How the method suggested by Professor Heck will work out, if the path of motion does not go through the center of the cam, cannot be stated without a further investigation.

Fig. 32 is a true copy of Fig. 10 of the paper (the dotted lines in this figure apply to a special case and therefore is only a variation of the method), but with a different interpretation, and in that Professor Heck takes the opportunity to call the attention of the reader to his book on Kinematics.

Replying both to Mr. Hoopes and Professor Heck, the author would say that the paper presents first the simpler cases, which can readily be solved analytically or graphically in a number of ways, but this was for the purpose of leading up to problems involving rather intricate calculations, as can be judged from Figs. 24 and 25, and in particular from the calculation in Table 2 — Parts 1 and 2.

It was a great satisfaction to the author that at the meeting at which this paper was read, Mr. Attendu stated that the only difficulty which he had experienced on his high-speed oil engine was due to acceleration of the valves. The shape of the cams was changed and the trouble was eliminated. It seemed that a paper on kinematics of cams was very timely.

No. 2028

IDEAL GAS-ENGINE CYCLES

By Robert C. H. Heck,[1] New Brunswick, N. J.

Member of the Society

The purpose of this paper is to promote the use, with the internal-combustion engine, of the ideal performance as a standard of comparison, by making that standard almost as readily convenient as is the Mollier chart for the steam engine. This purpose is accomplished in two steps: (a) developing a working chart that combines in one the temperature-entropy diagram and a diagram of internal energy and total heat on entropy, in which any horizontal line is a line of constant temperature, constant energy, and constant total heat; and (b) showing by trial upon representative examples that this chart, based on the properties of an average gas mixture, gives essentially correct output for any working mixture within the range of gas-engine practice.

Avoiding the common type of chart, with its confusing multiplicity of curves, the present scheme employs the principle of moving curves, carried by templates. The chart carries one curve of constant-volume and one of constant-pressure heating, with horizontal scales of volume and pressure and vertical scales of energy and total heat or "enthalpy." These are so disposed that any problem can be solved by construction upon a separate working sheet, laid down across the chart, which will serve as a permanent record of the solution.

Important features of the paper are found in Figs. 9 *and* 10 *on the specific heats of working mixtures, Figs.* 11 *and* 12 *for the general idea of chart and method, in the summary of Otto-cycle determination, Par.* 73 *to* 76, *and in that of the Diesel-cycle determination, Par.* 85 *and* 86.

EVERY heat engine has two principal efficiencies; the absolute efficiency e_a, which is the ratio of the heat converted to the heat received; and the relative efficiency e_r, which is the ratio of the actual to the ideal or best possible performance.

2 In order that e_r may become known it is necessary to make the ideal performance definite, or to find the value of ideal efficiency e_i. This must be done by theoretical deduction for a scheme

[1] Professor of Mechanical Engineering, Rutgers University.

Contributed by Oil and Gas Power Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers. Revised; see Pars. 53 and 71, and closure, page 1072.

of operation which, within its limiting conditions, is free from all the secondary losses that are of the controllable type.

3 Let q be the heat input per unit of working medium and w the work output, expressed in heat units and distinguished as actual w_a and ideal w_i; then

$$e_a = \frac{w_a}{q}; \quad e_i = \frac{w_i}{q}; \quad e_r = \frac{e_a}{e_i} = \frac{w_a}{w_i} \quad . \quad . \quad . \quad . \quad [1]$$

4 For the steam engine the use of ideal performance as a standard of comparison has long been a routine matter; it is given by the Rankine or Clausius cycle, of which the output can readily be calculated, and is put into convenient practical form by the Mollier chart of total heat on entropy. With the gas engine the use of a similar standard has been retarded for two reasons: to calculate the dimensions of the ideal cycle is a difficult and laborious task, and with each different composition of gaseous mixture a different set of data enters the calculation. This paper presents a convenient and flexible method for getting numerical results, and shows how nearly representative are the results from a single, average gas composition.

5 As a short but comprehensive title the name " steam engine " is used to include the steam turbine as well as the piston engine. In like fashion, " gas engine " is used in the present paper for " internal-combustion engine," emphasizing the gaseous nature of the working medium rather than the physical form of the fuel. In the latter respect and from the viewpoint of the entrance of fuel into internal operation, the three kinds of fuel are gas, vapor, and oil.

6 The scheme of the paper is as follows:

A schedule of symbols, units, and ratios, with an incidental summary of simple thermodynamic principles

A statement and brief discussion of simplifying assumptions

The presentation of data as to the specific heat of gases

The properties of ten selected fuels and gaseous mixtures, to be studied as representative examples

The specific heats of the working mixtures

A graphical method for cycle dimensions, the first main object of the paper

Computational form of the graphical method

Comparison of results, correct and approximate methods

Method for the Otto cycle, by simple use of chart

Method for the Diesel cycle

An appendix showing the preliminary calculations on gaseous mixtures and completing the discussion of specific heat data.

Symbols, Units, Ratios

7 In the practical thermodynamics of the internal-combustion engine three systems of measurement are encountered, based upon different combinations of two temperature scales and two sets of units for pressure, mass, and volume. These are

The English-fahrenheit system, symbol *ef*
The English-centigrade system, symbol *ec*
The metric-centigrade system, symbol *mc*.

The English-fahrenheit system continues to prevail in the United States, but British engineers have largely gone over to the English-centigrade system.

8 For the primary quantities just mentioned the symbols and relations are as follows:

Temperature. The 1.8 ratio of the fahrenheit to the centigrade measurement of a temperature range holds also for heat or energy quantities per unit of mass.

t = thermometer reading; $T = t+459.6$ (fahr.)
T = absolute temperature; $T = t+273.1$ (cent.)

Results here shown, computed primarily in centigrade units, are based on $T_0 = 273$.

Pressure. Engineering pressure p and thermodynamic pressure P.

p in lb. per sq. in. (Eng.) or kg. per sq. cm. (met.)
P in lb. per sq. ft. (Eng.) or kg. per sq. m. (met.)
$P = 144p$ (Eng.) $P = 10{,}000p$ (met.)
$p_e = 14.223p_m$ $P_m = 4.882P_e$

Mass. The relative size of the pound and kilogram does not appear directly, because values of volume, energy, and entropy are regularly stated per unit of mass; but the ratio 2.2046 enters into the relations between more complex units and ratios.

Volume. Specific volume v is the volume of the unit mass, at any particular state as defined by pressure and temperature (or by any other two coördinates)

v in cu. ft. per lb. (Eng.) or cu. m. per kg. (met.),
$$v_e = 16.02v_m$$

9 In the characteristic equation or equation of state of any gas (for its ideal behavior)

$$pv = bT \quad \text{or} \quad Pv = BT \quad . \ . \ . \ . \ . \ . \ . \ [2]$$

the gas constant b or B has relative values as follows:

$B = 144b$ (Eng.) $B = 10{,}000b$ (met.)
$b_{ef} = 126.57b_{mc}$ $b_{ec} = 1.8b_{ef} = 227.83b_{mc}$
$B_{ef} = 1.8227B_{mc}$ $B_{ec} = 1.8B_{ef} = 3.281B_{mc}$

The general gas equation is

$$pv = \frac{r}{m} T \quad \text{or} \quad Pv = \frac{R}{m} T \quad . \ . \ . \ . \ . \ . \quad [3]$$

in which r or R is the general gas constant and m is the molecular weight of the particular gas. The absolute values are

		ef	*ec*	*mc*
With engineering pressure p,	$r =$	10.732	19.30	0.0847
With thermodynamic pressure P,	$R =$	1544	2779	847.1

Putting $V = mv$ in Equation [3] gives

$$Pmv = PV = RT \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad [4]$$

which is the equation of state for a mol of any gas. (The mol is the quantity of gas having a mass of m units.) Barring departures from ideal behavior the volume of the mol is the same for all gases, at the same pressure and temperature.

10 The product Pv may be considered a work quantity, implying the operation of a constant-pressure expansion at P from zero volume to v. Then Pv is directly in work units, either foot-pounds or meter-kilograms; but pv is in units of 144 ft-lb. or 10,000 m-kg.

11 Pressure-volume work is converted to heat units by factor A, the heat equivalent of work, this A being the reciprocal of J, the mechanical equivalent of heat. Besides the B.t.u. and the kilogram-calorie there is a third heat unit to be considered, the centigrade heat unit or c.h.u., which is the heat required to raise 1 lb. of water 1 deg. cent. Remembering that the c.h.u. is to be related to ft-lb., the three values of J are

	ef	*ec*	*mc*
Mechanical equivalent $J =$	777.6	1400	426.6

With engineering pressure p in product pv the effective value of factor A is not simply $1/J$ but is either $144/J$ or $10{,}000/J$, thus

	ef	*ec*	*mc*
Effective A with pv, $A' =$	1/5.4	1/9.72	23.44

12 For heat, energy, and entropy quantities the symbols, all per unit of mass, are

q = heat imparted or received
u = change of internal energy
i = total heat or "enthalpy," the heat added under constant pressure from a chosen initial temperature
ϕ = entropy change or entropy from a chosen origin.

As noted in Par. 8, a B.t.u. value per pound is 1.8 times the corresponding c.h.u. per pound or calories per kilogram. Entropy, a pure ratio, has the same value in all systems.

13 For specific heat, the symbols c_p and c_v are used for common values per unit mass and the symbols C_p and C_v for heat rate

per mol per degree. (The values are, of course, independent of the system.) For the ideal gas the difference $c_p - c_v$ is equal to AB, which may be called the specific heat for outer work under constant pressure. Between C_p and C_v there is the constant difference AR,

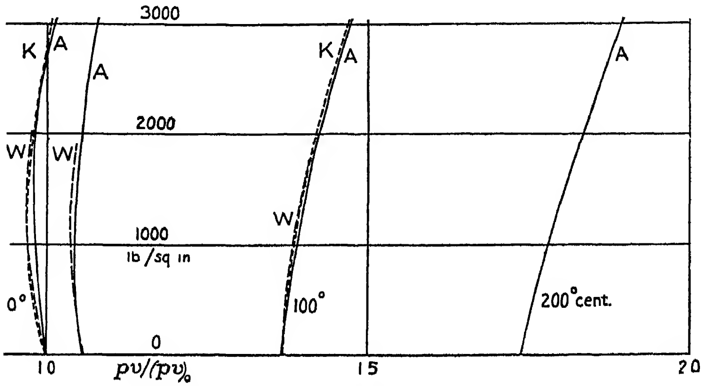


FIG 1 ISOTHERMS pv ON p FOR AIR
(A, Amagat; W, Witkowski; K, Keyes' equation)

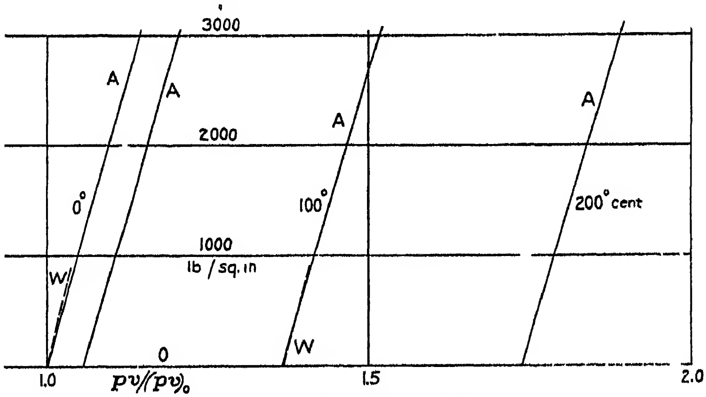


FIG. 2 ISOTHERMS pv ON p FOR HYDROGEN
(A, Amagat; W, Witkowski.)

which has the value 1.986 when found with any pair of the R and J values above, or in any system.

SIMPLIFYING ASSUMPTIONS

14 First, the gaseous medium, both before and after combustion, is assumed to follow volumetrically the simple ideal law $pv = bT$, which will be more nearly true for the mixture than for some of the separate ingredients. On this matter a few physical

data are presented by Figs. 1 and 2, which show isotherms in the plot of product pv (horizontal) on pressure p (vertical). (Note that the unit is $(pv)_0$, at zero deg. cent. and zero pressure.)

15 Under the ideal law the pv-p isotherms would be vertical straight lines. Departures from that form are due to two causes; namely, molecular volume and the contractive influence of molecular attractions. The first has a small effect directly proportional to pressure, the second is stronger toward the region of liquefaction. Over the range of exhibited observations air shows a contraction that persists but is rapidly decreasing (with rise of temperature), while in the case of hydrogen this effect has disappeared. The latter type of behavior is expressed by the equation

$$p(v-w) = bT \quad \text{or} \quad pv = bT + pw \quad . \quad . \quad . \quad . \quad [5]$$

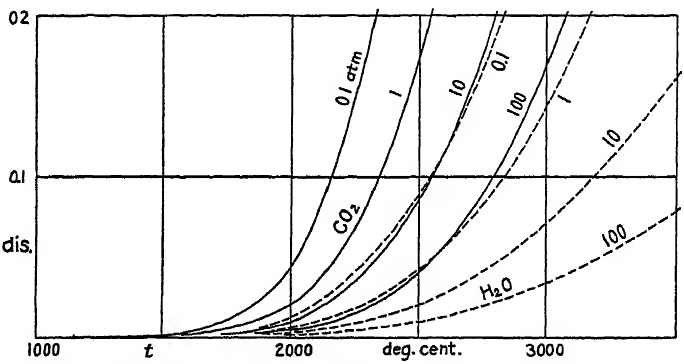


Fig. 3 Nernst's Data as to the Dissociation of CO_2 and H_2O

in which w is a small constant representing the combined volume of the molecules. Its presence causes the pv-p isotherms to slant toward the right as they rise.

16 Considering the comparatively low pressure range of gas-engine conditions — as compared with the height of Figs. 1 and 2 — and the fact that high pressures are associated with high temperatures, it is entirely proper to disregard the very small departures from the ideal law of Equation [2]. With equal propriety the small actual departures from Avogadro's law, as expressed by Equation [3], may be ignored; and in that formula it will be sufficiently accurate to use the simple integral values of molecular weight m, instead of the more precise values from the international atomic weights.

17 A very important consequence of the essential parallelism of the pv-p isotherms — or of essential conformity to either Equation [2] or Equation [5] — is that the specific heats c_p and

c_v do not vary with pressure (along the isotherm), however they may increase with temperature.

18 The second major assumption is that combustion is complete as a simple overall operation, and need not be considered in detail. This is carried so far as to consider even dissociation to be an element of actual as against ideal action. That is, we imagine a limiting combustion in which the total heating value becomes immediately active and raises the final gaseous products to the maximum temperature derivable from specific heat relationships. Of course, heat insulation is assumed to be perfect in the ideal case.

19 About dissociation — as an item of chemical equilibrium at high temperatures or of retarded combination — there is little quantitative information. Fig. 3 shows graphically the lower range of the small Nernst table which is generally quoted in reference books. The curves are lines of constant pressure, on a base of temperature, and the ordinate is the proportion or fraction of ultimate CO_2 or H_2O that remains separated into CO and O_2 or H_2 and O_2.

Specific Heat of Gases

20 Interest attaches to the molecular specific heat C and to the pound specific heat c, for constant pressure as C_p or c_p and for constant volume as C_v or c_v. For a gas of ideal volumetric behavior the constant-volume specific heat has a general meaning as the rate of change of internal energy u with temperature t or T in any action. These ratios of heat added to temperature change all vary with the temperature in a manner that can be accurately enough represented by the equations

$$C = A + BT + GT^2 \quad \ldots\ldots\ldots\ldots [6a]$$
$$c = \alpha + \beta T + \gamma T^2 \quad \ldots\ldots\ldots\ldots [6b]$$

21 Upon this subject the readiest sources of information are, the book The Specific Heat of Gases, by Partington and Shilling,[1] and an Investigation of the Maximum Temperature Attainable in the Combustion of Gaseous and Liquid Fuels, by Goodenough and Felbeck[2]; formulas from the latter are reproduced in Marks' Handbook, 1924 edition, page 378. Formulas from Goodenough's Principles of Thermodynamics, 1920, are also shown in Fig. 4 and one of them is used in the work of the paper.

22 In each section of Fig. 4 the full-line curve 1 represents the formula used in the work of this paper. The complete showing is as follows:

Section (a), a common value of C_p for the diatomic gases air, N_2, O_2, CO, and H_2, except that Partington and Shilling give the separate straight line 1′ for H_2.

[1] Benn, London, 1924.
[2] Bulletin 139, March, 1924, Engineering Experiment Station, University of Illinois.

Section (b), carbon dioxide, CO_2.

In both of these, curve 1 is from Partington and Shilling, 2 from Goodenough, and 3 from Marks' handbook.

Sections (c) and (c'), H_2O or highly superheated steam, at low pressure where temperature is low: curve 1, Goodenough; curve 2, Partington and Shilling; curve 3, Marks' Handbook. In separate

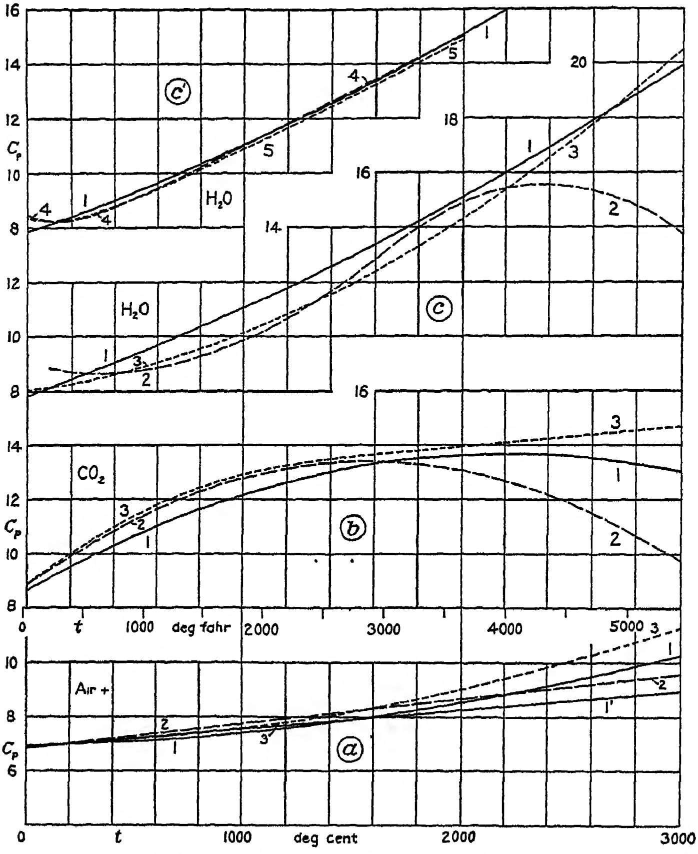


Fig. 4 Comparative Curves of Molecular Specific Heat C_p

(In every case, C_v is less by the constant 1 99)

section (c') curve 1 is compared with two equations for C_{p0}, the specific heat at zero pressure, as used in steam formulation. These are 4, Goodenough, and 5, Heck (1920).

23 For these curves most of the data lie below 2000 deg. cent. The few between 2000 and 3000 deg. are not very dependable and our quantitative knowledge of behavior in that region is as yet quite indefinite.

24 The type of curvature shown for CO_2 is well established, but that the specific heat drops off after reaching a maximum is

highly improbable. Curve 2 is out of the question; compound curve 3 is of best probable form; but single curve 1 (with only a little droop) is satisfactory. For a discussion of curves 3 in comparison with curves 1 — which latter are used in the calculations that follow — turn to the Appendix, Par. 105 to 108.

25 For the accepted curves, numbered 1, with the Partington and Shilling formulas correctly converted from t to T as base, the coefficients for Equation [6a] are as follows:

	A_p	A_v	B_c	B_f	G_c	G_f
(a) Air, N_2, O_2, CO	6 933	4.947	0	0	$0.0_6 31$	$0.0_7 957$
(b) CO_2	7.382	5.396	0 00506	0 00281	$-0.0_5 102$	$-0.0_6 312$
(c) H_2O	7.12	5.13	0.00256	0.00144	$0.0_6 405$	$0.0_6 125$

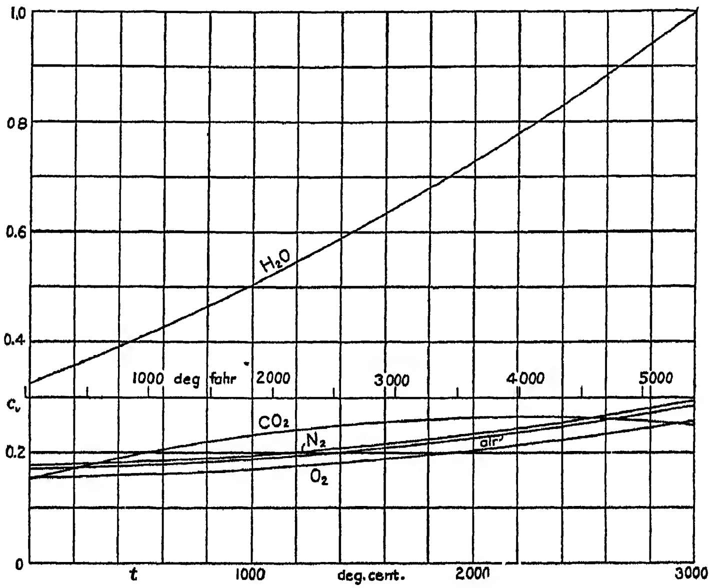


FIG. 5 CURVES OF c_v FOR THE FOUR COMBUSTION-PRODUCTS GASES

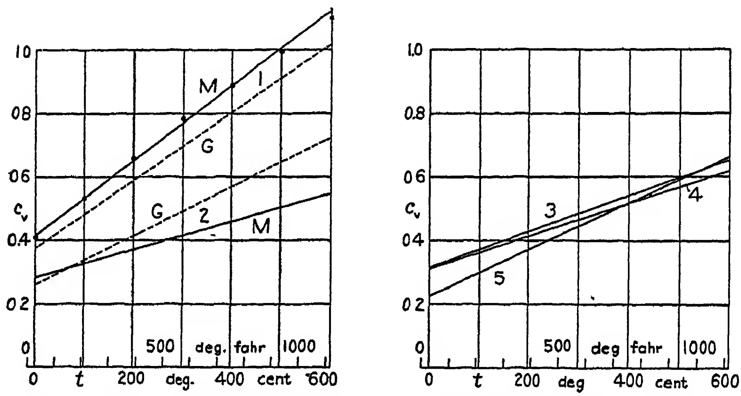
Coefficient A has different values for constant-pressure and constant-volume heating, but is the same with either temperature scale. The difference between A_p and A_v, or between the mol specific heats C_p and C_v, is 1.986 or 1.99 (see Par. 13). The two values of B and G are for the two temperature scales, with the ratios 1.8 and 3.24. Hydrogen is not stated here because that gas appears only as an ingredient in combustible mixtures, which are considered in Par. 36.

26 Further illustration is given in Fig. 5 of the four gases that enter into the products of combustion and therefore have to be considered over wide temperature ranges. Derived from curves

1 of Fig. 4 by change to C_v and division by the particular molecular weight m, these curves show the pound (or kilogram) values of c_v, the specific heat in constant volume or for internal-energy change. The outstanding fact is the strong influence that the presence of H_2O will have in raising the mean specific heat of a mixture above that for air alone. It should be noted that in this paper the "mean" specific heat is always the average for a mixture of gases, not the mean value over some temperature range.

27 The scanty information available as to the specific heat of the hydrocarbon gases and vapors is presented in Figs. 6 and 7, all in straight-line relationships of the form

$$c_v = \alpha + \beta T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [7]$$



FIGS. 6 AND 7 SPECIFIC HEAT c_v OF CERTAIN HYDROCARBON GASES

Here the full lines show the Goodenough and Felbeck formulas from Marks' Handbook. The two dotted lines show older formulas quoted by Goodenough from the handbook Hütte. Only for methane, gas 1, do Partington and Shilling give information as to the variation of specific heat with temperature. This is shown by the dots along the line 1. The gases given are,

1 Methane, CH_4
2 Ethylene, C_2H_4
3 Acetylene, C_2H_2
4 Ethane, C_2H_6
5 Benzene, C_6H_6.

The first two are the only hydrocarbons that appear in appreciable quantity in fuel gases, except ethane in some natural gases. The last, benzene, may throw some light upon the behavior of the other and heavier oil vapors. For a quantitative statement about these fuel ingredients see Par. 36.

Examples for Study

28 Everything up to this point has been preliminary or preparatory to the special problem of the paper. That problem cannot be solved by direct analysis, but must be handled by the method of trial solution, applied to representative examples. Covering the whole field of gas-engine practice, ten such examples have been chosen, namely, six fuel-gas mixtures, two limiting oil compositions and the two common alcohols — the last taken to be pure fuel, or without admixture of water.

29 For the fuel gases the examples given by Goodenough[1] are used, with one changed (producer gas) and one added (retort or coke-oven gas). Shown as to volume composition and relative volume by Table 1, these gases are as follows:

TABLE 1 FUEL-GAS COMPOSITIONS BY VOLUME, ETC.

Gas No. / Symbol	1 B. F.	2 Prod.	3 Wat.	4 C. W.	5 Nat.	6 Ret.
Hydrogen .... H_2	0 04	0.13	0.50	0.30	0.02	0.50
Carbon monoxide ... .CO	0 27	0.25	0.45	0.27	0.01	0.07
Methane ....CH_4	. . .	0.03	....	0.25	0.93	0.32
Ethylene ....C_2H_4	. . .	....	....	0 15	....	0.04
Carbon dioxide ....CO_2	0 10	0.05	0.03	0.01	0.01	0.03
Nitrogen ....N_2	0.59	0.54	0.02	0.02	0.03	0.04
Total	1.00	1.00	1.00	1.00	1.00	1.00
Mean molecular weight m_m ...	28.56	25.06	15.48	17.36	16.48	11.64
Specific volume relative to air..	1.014	1.157	1.871	1.668	1.758	2.489

1 Blast-furnace gas, the leanest of engine fuels
2 Producer gas, high in nitrogen and with but little hydrocarbon whatever the coal used
3 Water gas, of high-grade type
4 Carburetted water gas, the standard illuminant
5 Natural gas, nearly all hydrocarbon
6 Retort or coke-oven gas.

These are arranged in the order of increasing hydrogen content or of H_2O content in the products of combustion.

30 The mineral oils are adequately represented by two weight compositions for the pure combustible, namely,

7 Heaviest oils, $H_2 = 0.12$, $C = 0.88$
8 Lighter oils, gasolines, $H_2 = 0.16$, $C = 0.84$.

As regards hydrogen content these are extreme compositions, showing a bigger range of variation than is likely to be encountered; especially, 12 per cent of H_2 is less than is to be found in oil analyses. See Par. 101 in the Appendix.

31 The alcohols are

9 Ethyl or grain alcohol, C_2H_6O
10 Methyl or wood alcohol, CH_4O.

[1] See Principles of Thermodynamics, p. 297.

32 Besides composition by volume, Table 1 gives the mean molecular weight of the fuel gas and the ratio of its specific volume to that of air. Since volumes are inversely as molecular weights, this ratio is equal to m_a/m_m or $28.97/m_m$. The first two gases are near to air, but the others are very much lighter because of the content of H_2 and CH_4.

33 In Table 2 the ten examples are brought to weight measurement, with the pound of fuel as base, and are compared as to combustible content, air requirement and heating value. The first

TABLE 2 COMBUSTIBLE, AIR RATIOS, HEATING VALUES

	1	2	3	4	5	6	7	8	9
						Per lb. of fuel		Heating value	
Fuel	H_2	C in hydro-carbons	C in CO	Sum	Comb.	Air	Mixture	Fuel	mixture
1 B. F.	0.003	.....	0.113	0.116	0 268	0.860	1 860	1,303	701
2 Prod.	0.015	0.014	0.120	0.149	0 309	1.585	2.585	2,183	844
3 Wat.	0.065	.....	0.349	0.414	0.879	4 867	5.867	6,970	1188
4 C. W.	0.127	0.380	0.187	0 694	0.943	11 29	12.29	13,660	1112
5 Nat.	0.228	0.676	0.007	0.911	0.921	17.11	18.11	19,710	1038
6 Ret.	0.210	0.413	0.072	0 695	0.791	14.24	15.24	16,780	1100
7 Oil	0.120	0.880	.....	1.000	1.000	16.48	17.48	18,600	1058
8 Gaso.	0.160	0.840	.....	1,000	1.000	17 54	18.54	18,500	1000
9 Eth.	0 130	0.522	.....	0.652	1.000	13.56	14.56	11,200	769
10 Meth.	0.125	0.375	.....	0.500	1.000	9.75	10.75	8,400	781

TABLE 3 COMPARATIVE PROPORTIONS

	Column No.										
	1	2	3	4	5	6	7	8	9	10	11
	In products				Molecular weights		Relative volumes			Outer work rate	
	Volume		Weight								
Fuel	H_2O	CO_2	H_2O	CO_2	m_m	m_p	v_m	v_p	v_p/v_m	AB_m	AB_p
1 B. F.	0.024	0.218	0.014	0 306	28.69	31.31	1.010	0.926	0.916	0.0693	0 0635
2 Pro.	0.087	0.159	0.053	0 224	26.83	29.60	1.080	0.979	0 907	0 0740	0.0671
3 Wat.	0.159	0.153	0.099	0.233	25.14	28.95	1.152	1.001	0.868	0.0790	0.0686
4 C. W.	0.147	0.111	0.093	0.171	27.36	28.40	1.059	1.020	0 963	0.0726	0.0700
5 Nat.	0.166	0.084	0.108	0.133	27.75	27.78	1.044	1 043	0.999	0.0716	0.0715
6 Ret.	0.189	0.077	0.124	0.124	26.29	27 45	1.098	1 055	0.958	0.0756	0.0724
7 Oil	0.100	0.122	0.062	0.185	.....	29.06	....	0.996	.....	......	0.0683
8 Gaso.	0.123	0.108	0.078	0.166	30.00	28.59	0.966	1 013	1.049	0.0663	0.0695
9 Eth.	0.198	0.050	0.131	0.081	29.60	28.34	0.979	1 022	1.045	0.0671	0.0701
10 Meth.	0.195	0.065	0.128	0.105	29.10	27.92	0.995	1 038	1.042	0.0683	0.0712

three columns give the weight of hydrogen, of carbon in hydrocarbons, and of carbon in CO. These are summed for the combustible elements in column 4, then the total combustible in column 5 is greater by the oxygen in CO or in the alcohol. The air required per pound of fuel is given in column 6, calculated with 15 per cent excess for the gases and oils and with 50 per cent excess for the alcohols. Adding the one pound of fuel we get the total weight of mixture in column 7. The heating values, per pound of fuel and per pound of mixture, are given in columns 8 and 9.

34 An example showing the calculation of these and following quantities is worked in the Appendix.

35 The characteristics of the various gaseous media considered are further shown by Table 3. The first two columns give the volume of H_2O and CO_2 in one cubic foot of products, the next two the similar weights per pound of products. The rest of the table compares the precombustion mixture (subscript m) and the products of combustion (subscript p) as to mean molecular weight, volume relative to air, and outer-work rate AB under constant pressure. The ratio in column 9 indicates the shrinkage or swelling of specific volume that takes place during combustion. With all of the gas fuels there is a shrinkage, running as high as 13 per cent in one case; with the heavier liquid-fuel molecules

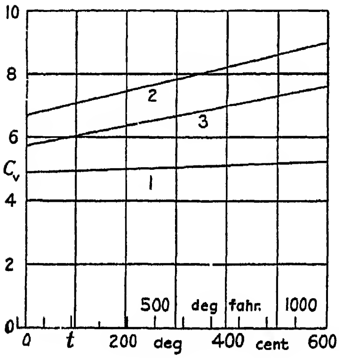


FIG. 8

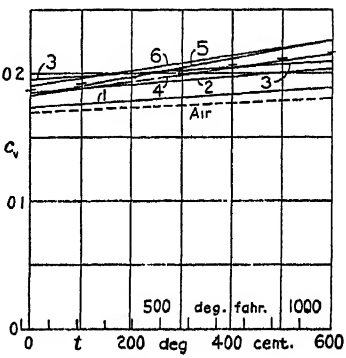


FIG. 9

FIG. 8 MOLECULAR C_v FOR THE THREE GASES OF FIG. 4

FIG. 9 POUND c_v FOR THE SIX COMBUSTIBLE MIXTURES OF GAS FUELS

there is an expansion running up to 5 per cent. This matter is more fully discussed in Pars. 100, 101, and 102 of the Appendix, and the investigation of its effect is begun in Par. 51.

SPECIFIC HEATS OF WORKING MIXTURES

36 The specific heats of the combustible mixtures are of practical interest only over the comparatively short temperature range of the adiabatic compression in Otto-cycle engines, which will not run above 600 deg. cent. Straight lines that agree closely with the curves of Fig. 4 over this range, giving

$$C_v = A + BT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [8]$$

are drawn in Fig. 8. For these and the two full-line curves of Fig. 6 the coefficients are (with H_2 included in the first group),

	Fig. 8			Fig 6	
	1	2	3	1	2
	Air, etc.	CO_2	H_2O	CH_4	C_2H_4
A	4.76	5.30	4.90	1.48	4.7
B	0.0_354	0.0038	0.0031	0.019	0.012

It must be remembered that, because of its small molecular weight, hydrogen has a very high "pound" specific heat; while CO_2, with a larger molecular weight, has a c_v not much above that of air, as shown in Fig. 5.

37 Applied to the six examples of fuel-gas mixtures, as illustrated in Par. 103 of the Appendix, these "mol" C_v's yield the "pound" c_v's shown by the lines in Fig. 9. Here we see two typical manners of variation of specific heat: in cases 1, 2, and 3, influenced by the presence of CO_2 and H_2, the lines have the same trend as that for air, running higher as H_2 is greater; case 5, determined by CH_4, shows a much steeper slope; while in cases 4 and 6 there is a combined influence, but with hydrocarbon predominating over hydrogen. The values of coefficients α and β for these six combustible mixtures are given in Table 4.

TABLE 4 SPECIFIC HEATS OF MIXTURES

Fuel	a_v	AB	a_p	β
1	0.1670	0.0693	0.2363	0.0_4250
2	0.1764	0 0740	0.2504	0.0_4310
3	0.1895	0.0790	0.2685	0.0_4226
4	0.1683	0.0726	0.2409	0.0_4533
5	0.1625	0.0716	0.2341	0.0_4722
6	0.1752	0.0756	0.2508	0.0_4577

These are coefficients for use in the specific-heat equation $c_v = \alpha + \beta T$, with T in deg. cent., for combustible mixtures of the six gas fuels in Table 1, with 15 per cent excess air.

38 The mean of lines 2 to 6 in Fig. 9 is represented by the series of cross marks on the ordinates. These lie in a straight line, the equation of which (with T in deg. cent.), is

$$c_v = 0.1745 + 0.00476T \quad . \; . \; . \; . \; . \; . \; . \; . \quad [9]$$

From Table 3 the mean value of AB for these five examples is 0.0745, therefore the specific heat of the average mixture under constant pressure is

$$c_p = 0.2490 + 0.00476T \quad . \; . \; . \; . \; . \; . \; . \quad [10]$$

39 In like manner the specific heats of the combustion products from the ten fuels in Table 1 are shown by the curves in Fig. 10. The numbering of these curves corresponds with that of the examples, except as to 9 and 10. The two alcohol products have specific heats so nearly the same that their curves are barely distinguishable when drawn with the scales of Fig. 10. Then curve 9 (which agrees with 6 over two-thirds of its temperature range) is a mean for the two alcohols with 50 per cent excess air,

and curve 10 is a similar mean with 15 per cent excess. The latter ratio is the one used with the gases and oils, but is too low for the assured combustion of alcohol. Coefficients for these ten curves

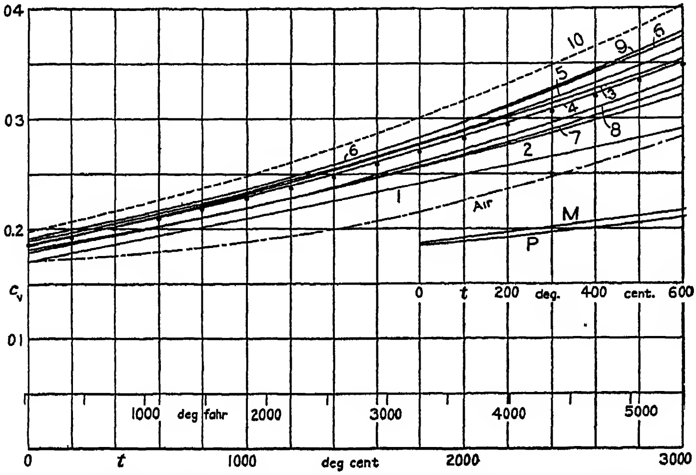


FIG. 10 POUND SPECIFIC HEAT c_v FOR THE PRODUCTS OF COMBUSTION OF THE TEN FUELS IN TABLE 1

of c_v, to be used in Equation [6b] with T in deg. cent., are given in Table 5.

TABLE 5 SPECIFIC HEATS OF THE PRODUCTS OF COMBUSTION

Fuel	a_v	AB	a_p	β	γ
1	0.1614	0.0635	0.2249	0.0_4872	0.0_8071
2	0.1700	0.0671	0.2371	0.0_4334	0.0_8396
3	0.1747	0.0686	0.2433	0.0_4410	0.0_8420
4	0.1770	0.0700	0.2470	0.0_4331	0.0_8623
5	0.1806	0.0715	0.2521	0.0_4310	0.0_8770
6	0.1829	0.0722	0.2551	0.0_4318	0.0_8820
7	0.1729	0.0683	0.2412	0.0_4302	0.0_8541
8	0.1721	0.0695	0.2416	0.0_4304	0.0_8623
9	0.1767	0.0700	0.2467	0.0_4267	0.0_8737
10	0.1793	0.0711	0.2504	0.0_4297	0.0_8776

40 The curves in Fig. 10 are to be studied in connection with the first four columns of Table 3 Curve 1, of unique type, shows the effect of very low H_2O and excessive CO_2. In the main group the influence of H_2O as indicated by Fig. 5 is strikingly apparent, the curves running higher as H_2O content is greater. Note the rather low position of the oil fuels, remembering that example 8 is more representative than 7.

41 The mean of examples 2 to 8, omitting blast-furnace gas and alcohol, is shown by the black-dot "curve" in Fig. 10, for which the equation is

$$c_v = 0.1760 + 0.0_433T + 0.0_86T^2 \quad . \; . \; . \; . \; . \quad [11]$$

also with T in deg. cent. By a further averaging of these seven

examples the mean value of AB or $c_p - c_v$ is found to be 0.0695. For reasons of convenience, and strongly justified as the scheme works out to final shape, the small departure from the dry-air value 0.0686 is disregarded and the latter is used. Then for the constant-pressure operation

$$c_p = 0.2446 + 0.0_433T + 0.0_86T^2 \quad . \; . \; . \; . \; . \quad [12]$$

42 The practical use of these specific-heat formulas is found in the calculation of entropy in constant-volume and in constant-pressure heating, of internal energy and of total heat. Changing to common logs by the factor 2.3026 and taking the state at 0 deg. cent. as the origin in each case, Equations [11] and [12] give

$$\phi_v = [9.60773]^* \log \frac{T}{273} + 0.0_433(T-273) + 0.0_83(T^2 - 273^2) \quad . \quad [13]$$

$$\phi_p = \phi_v + [9.19854] \log \frac{T}{273} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [14]$$

$$u = 0.1760(T-273) + 0.0_4165(T^2 - 273^2) + 0.0_82(T^3 - 273^3) \quad . \quad [15]$$

$$i = u + 0.0686(T-273) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [16]$$

An outstanding purpose of the paper is to obviate the necessity of calculating cycle dimensions by such relationships as

$$\alpha_p \log_e \frac{T}{T_1} + \beta(T - T_1) + \tfrac{1}{2}\gamma(T^2 - T_1^2) = AB \log_e \frac{p}{p_1} \quad . \; . \quad [17]$$

or

$$\alpha_v \log_e \frac{T}{T_1} + \beta(T - T_1) + \tfrac{1}{2}\gamma(T^2 - T_1^2) = AB \log_e \frac{v_1}{v} \quad . \; . \quad [18]$$

for the adiabatic operation and

$$q = \alpha(T - T_1) + \tfrac{1}{2}\beta(T^2 - T_1^2) + \tfrac{1}{3}\gamma(T^3 - T_1^3) \quad . \; . \quad [19]$$

for the heating operations, with T as unknown quantity in each case.

Graphical Method for Cycle Dimensions

43 The general idea of the method now to be presented is to combine in one the temperature-entropy diagram and a diagram of energy or of total heat on entropy — the last being equivalent in effect to the Mollier chart for the steam engine. The graphical apparatus is shown by Fig. 11 and its use is illustrated by Fig. 12. At first we assume the simple condition of like properties, thermal and volumetric, for the combustible mixture and the combustion products, using those of the average products as made definite in Par. 41. After describing this scheme of solution, which meets the

* This notation uses the bracketed log as symbol for the antilog.

first main purpose of the paper as stated in Par. 4, the effect of variations will be investigated and discussed.

44 The parts of Fig. 11 are as follows:

Curve V applies to constant-volume heating and curve P to constant-pressure heating. On the temperature base T at the left these show entropy ϕ_v or ϕ_p from Equations [**13**] and [**14**], to the

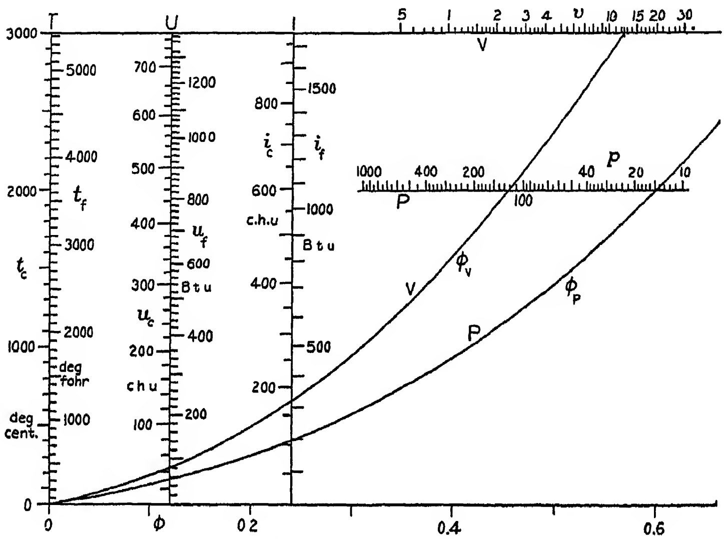


Fig. 11 Graphical Apparatus for Determining Ideal Gas-Engine Cycles

scale along the horizontal base line. These curves are drawn in zero position, or with the zero of entropy at the standard point where $t = 0$ deg. cent. or 32 deg. fahr. and $p = 14.7$ lb., the latter value characterizing the P curve. Then the V curve corresponds to the specific volume 12.38 cu. ft. For any other volume or pressure the curve is simply shifted horizontally according to the scale marked V or P. An accurately cut template of either curve is used instead of drawing a chart with a confusing multiplicity of curves.

45 Energy content u and total heat i, each per pound of medium and estimated from 0 deg. cent., are shown by the scales U and I. The fact that specific heat depends only upon temperature makes any horizontal line a line of constant u or of constant i.

46 Passage from one volume curve to another or from one pressure curve to another is most simply an operation of isothermal

expansion or compression. That is, the horizontal entropy distance between any pair of curves is

$$_1\Phi_2 = 2.3026AB \log \frac{v_2}{v_1} = 2.3026AB \log \frac{p_1}{p_2} \quad . \quad . \quad . \quad [20]$$

47 In Fig. 12 are reproduced only the needed vertical scales of t in deg. fahr. and u in B.t.u. Initial point A is located by drawing a pressure curve through $p_a = 14$ lb. and a temperature line at $t_a = 200$ deg. Adiabatic compression to $p = 120$ lb. is

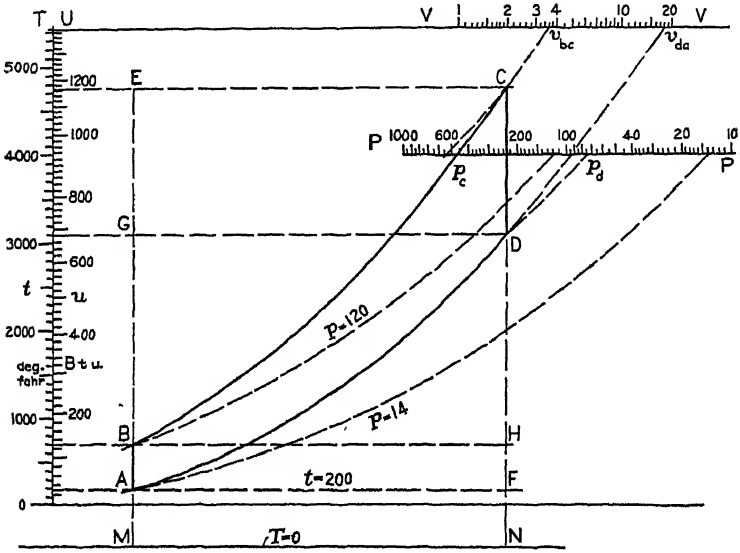


Fig. 12 Illustrative Determination of an Otto Cycle

defined by the intersection of vertical AB with the p_b curve at that pressure.

48 On energy scale U set off now the height BE of the heat of combustion q, conceived simply as heat added in constant volume to the pound of gaseous medium: Through B and A draw the constant-volume curves BC and AD; intersection C fixes the state at the end of ideal combustion with no escape of heat, then vertical CD shows adiabatic expansion and fixes fourth point D, from which the gas is taken to return to state A by constant-volume cooling.

49 The work output is either the difference between adiabatic works CD and AB or between heat received BE and heat rejected DF, all these vertical heights being read on the energy scale.

On this diagram the line of absolute zero is also drawn, in order to give an idea of the influence of absolute temperature upon efficiency.

50 Extension of the two V curves BC and AD to the V scale shows the constant specific volume of heating and of cooling. Drawing P curves from C and D to the P scale determines the pressures at those points of the cycle. In the same way, a pair of curves from any point on AB or CD will give v and p coördinates for a point on the adiabatic curve of a pressure-volume diagram.

51 For use with any other than exceptionally accurate and complete tests of combustion engines this graphical method of the single chart is sufficiently approximate for practical purposes, in spite of rather wide variation of the properties which are averaged in order to supply its working data. The important

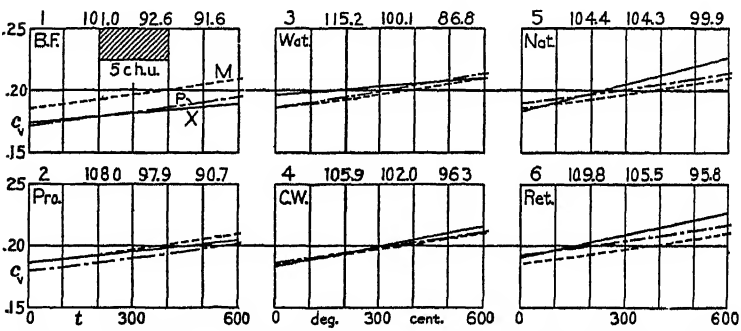


FIG. 13 COMPARISON OF SPECIFIC HEATS c_v OF MIXTURE AND PRODUCTS

influences causing theoretical efficiency to depart from values thus found are: specific heat of both precombustion mixture and products of combustion (Figs. 10 and 13), specific volume (columns 7 and 8 of Table 3), and the change of volume during combustion as expressed by the ratio in column 9 of Table 3.

52 The influences just named, for the six gaseous fuels of Table 1, are made graphic in Fig. 13, which is a group of individual comparison diagrams. In each case line X (full-line) for the mixture is from Fig. 9 and represents the particular case of Table 4. Curve P (dot-and-dash) for the products is from Fig. 10 and Table 5. Curve M (dotted and the same in all diagrams) represents the mean products, Equation [11]. In general, lines P and X keep fairly close together, showing that the change in specific heat due to the passage through combustion is comparatively small — at least so far as concerns the low range where the specific heat of the mixture is known and necessary. Volumetric information also appears in Fig. 13, each diagram carrying its three ratios from Table 3. Expressed as percentages, these ratios are the

volume relative to air of mixture and of products, and the ratio of products to mixture.

53 Beginning with Par. 51 the paper has been radically revised from the form in which it was first presented. The purposes of this change are, to make the presentation clearer and to eliminate an error due to not taking immediately into account the change of specific volume during combustion. The form and effect of that error are indicated in Par. 70.

Computational Form of the Graphical Method

54 In order to make an accurate numerical determination corresponding to Fig. 12 it is necessary to have a computed table

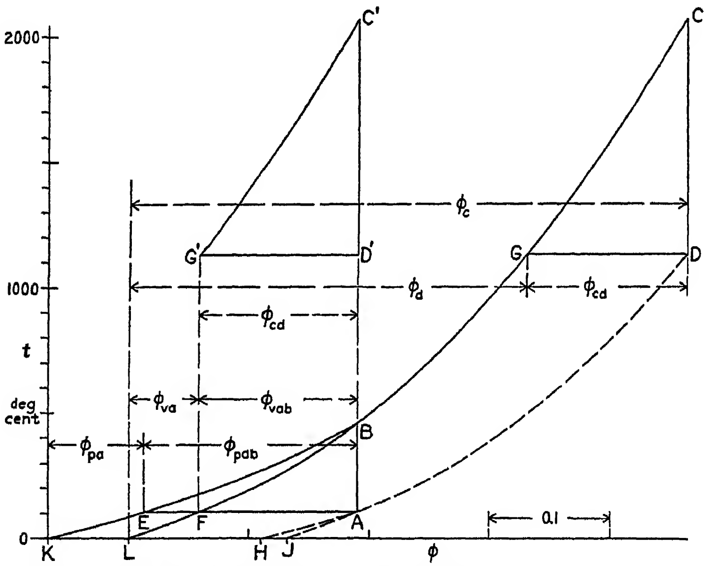


Fig. 14 Graphical Outline of Calculation of Otto Cycle

of the quantities given by Equations [13] to [16], whether with the coefficients belonging to the average mixture or with those belonging to a particular gas composition. Table 6 is from those equations as written, and has been plotted in the two curves and the u and i scales of Fig. 11.

55 The numerical method of getting cycle dimensions puts special emphasis upon the dimensions of the adiabatic compression and expansion; its first step is to relate the rise or fall of temperature to the volume ratio r_v or the pressure ratio r_p, as illustrated by Fig. 14. In terms of compression AB, ratio $r_v = v_a/v_b$, $r_p = p_b/p_a$, and $r_T = T_b/T_a$. Always these are to be taken in the direction that makes them greater than 1, not fractions, so that

operations upon their logarithms will involve only positive numbers. In terms of an expansion from state 1 to state 2,

$$\frac{p_1v_1}{p_2v_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{r_p}{r_v} = r_T \quad \text{or} \quad r_p = r_vr_T \quad . \quad . \quad . \quad [21]$$

which shows the general relation among these ratios.

56 Graphical construction for adiabatic compression AB, Fig. 14, begins with curve HA of a given initial pressure, or curve JA of a given initial volume, on either of which point A is located by the height of initial temperature t_a. From this first position the curve is shifted to the left, to a higher pressure along KB or a smaller volume along LB. Entropy distance AE or AF is found

TABLE 6 ENTROPY AND ENERGY VALUES FOR THE MEAN PRODUCTS, CENTIGRADE UNITS

t deg. cent.	ϕ_v	ϕ_p	u c.h.u.	i c.h.u.	t deg. cent.	ϕ_v	ϕ_p	u c.h.u.	i c.h.u.
50	0.0313	0.0429	9.3	12.7	1100	0.3261	0.4368	228.6	304.1
100	0.0584	0.0798	18.7	25.6	1200	0.3426	0.4582	252.1	334.4
150	0.0823	0.1123	28.2	38.6	1300	0.3582	0.4785	276.2	365.3
200	0.1038	0.1415	37.8	51.6	1400	0.3734	0.4979	300.7	396.7
250	0.1233	0.1679	47.5	64.7	1500	0.3880	0.5164	325.8	428.7
300	0.1412	0.1920	57.3	77.9	1600	0.4021	0.5342	351.4	461.1
350	0.1577	0.2143	67.2	91.2	1700	0.4158	0.5514	377.5	494.1
400	0.1731	0.2350	77.2	104.7	1800	0.4289	0.5680	404.2	527.7
450	0.1876	0.2544	87.3	118.2	1900	0.4417	0.5841	431.5	561.9
500	0.2013	0.2726	97.5	131.8	2000	0.4543	0.5997	459.6	596.7
550	0.2143	0.2898	107.8	145.5	2100	0.4666	0.6149	488.0	632.0
600	0.2265	0.3062	118.2	159.4	2200	0.4786	0.6298	517.1	668.0
650	0.2382	0.3218	128.7	173.4	2300	0.4904	0.6443	546.8	704.6
700	0.2495	0.3366	139.4	187.4	2400	0.5020	0.6585	577.2	741.9
750	0.2603	0.3508	150.1	201.6	2500	0.5131	0.6724	608.2	779.8
800	0.2705	0.3644	161.0	215.9	2600	0.5246	0.6861	639.9	818.3
850	0.2804	0.3775	171.9	230.3	2700	0.5357	0.6995	672.3	857.5
900	0.2901	0.3902	183.0	244.8	2800	0.5466	0.7127	705.4	897.5
950	0.2995	0.4024	194.2	259.4	2900	0.5574	0.7257	739.1	938.1
1000	0.3086	0.4142	205.6	274.2	3000	0.5681	0.7385	773.6	979.4

by means of Equation [20]. For the coefficient $2.3026AB$ or δ in that equation, since $AB = 1.986/m$, the general value is

$$\delta = 4.573 \div m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [22]$$

where m is the mean molecular weight of the gas mixture.

57 The scheme of calculation by means of Table 6 has the effect of referring the whole determination to a single curve, which is KB or LB for the adiabatic compression. With the temperature rise AB along either curve is associated the entropy change EA or FA belonging to an isothermal operation of the same pressure ratio r_p or volume ratio r_v. This is called ϕ_{ab} and distinguished as $EA = \phi_{pab} = \delta \log r_p$ or $FA = \phi_{vab} = \delta \log r_v$. The procedure is, then, read ϕ_{pa} or ϕ_{va} in Table 6 at temperature t_a, add ϕ_{pab} or ϕ_{vab} to get ϕ_{pb} or ϕ_{vb}, and read (interpolate) t_b. Taking energy u_b at t_b and subtracting u_a at t_a, the difference is equivalent to the outer work of compression.

58 As in Fig. 12, add heat of combustion q and get u_c. At the value of u_c interpolate in Table 6 for t_c and ϕ_c; distinguishing subscript v may now be omitted, since only volume ratio r_v is determinative of the adiabatic expansion CD, physically, graphically, and computationally. From ϕ_c subtract $\phi_{cd} = \phi_{vab}$, getting ϕ_d. Interpolate for t_d and u_d; then $u_c - u_d = u_{cd}$ is the work of adiabatic expansion. Finally, the net work in heat units is

$$Aw = u_{cd} - u_{ab} \quad \ldots\ldots\ldots\ldots \quad [23]$$

and the ideal thermal efficiency of the cycle is

$$e = Aw/q \quad \ldots\ldots\ldots\ldots \quad [24]$$

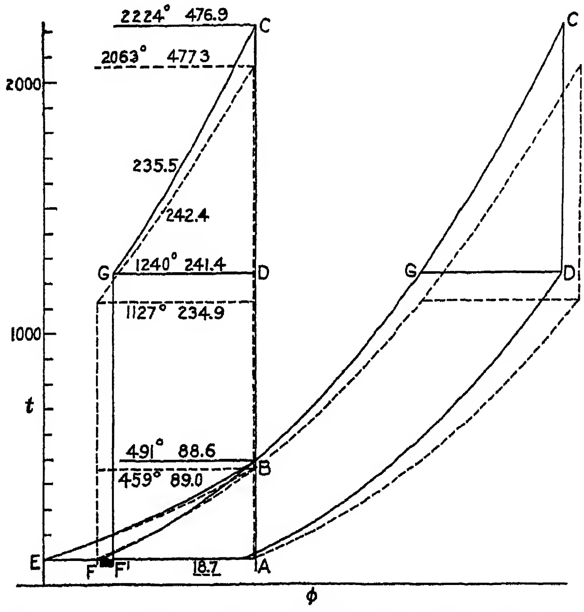


FIG. 15 THE CYCLE WITH A PARTICULAR GAS MIXTURE, IN ITS VARIATIONS

59 A final showing of the essential numerical relationship is made in Fig. 14 by moving expansion diagram CDG over to $C'D'G'$, with $C'D'$ above AB; this makes clear the governing influence of compression entropy ϕ_{vab}. The computations for Fig. 14 are given as a part of the example under Fig. 15, in Par. 61.

60 Computation of the cycle with a particular or actual gas mixture, taking into account all variations in ideal action, is illustrated by Fig. 15. For this there must be computed a table of ϕ_v, ϕ_p, and u for the combustible mixture, and one of ϕ_v and u for the products, as combined in Table 7; and the two values of

δ or 2.3026AB, with their ratio, are required. Having pressure ratio r_p as primary datum, the procedure is as follows:

Compute ϕ_{pab} or EA, add it to ϕ_a and get ϕ_{pb}

At ϕ_{pb} read t_b and u_b for the mixture

From the table read ϕ_{vab} or FA, then by the ratio δ_m/δ_p or v_m/v_p convert to $\phi_{vab} = F'A$ for the products: this can also be done by finding r_v by means of Equation [21], then applying Equation [20]

Having ϕ_{vab} for the products, this is also ϕ_{cd} and the dimensions of expansion CD are found just as under Fig. 14.

In Fig. 15 the dotted outline, reproduced from Fig 14, shows the same cycle (same in governing data) with mean products properties according to Table 6.

61 For the example of Figs. 14 and 15 the general data are: $t_a = 100$ deg. cent.; $p_a = 13.5$ and $p_b = 180$ lb. abs.; $r_p = 13.33$, $\log r_p = 1.1249$; gas 1 of Tables 1 and 3, with $AB = 0.0692$ and $\delta = 0.1593$ for the mixture and ratio $v_m/v_p = 1.091$; heat of combustion $q = 388.3$ c.h.u. In condensed outline, with first column for the actual composition (Fig. 15) and second column for the mean products (Fig. 14), the computation is as follows:

			Gas 1	Mean
1	Compression entropy... $\delta_m \log r_p = EA =$	ϕ_{pab}	0.1792	0.1777
2	Initial entropy	ϕ_{pa}	0.0764	0.0798
3	At end of compression.................	ϕ_{pb}	0.2556	0.2575
4	From Table 7 or Table 6...............	t_b	490.5	458.5
5	....	u_b	88.6	89.0
6	....	u_a	17.6	18.7
7	Work of compression......	u_{ab}	71.0	70.3
8	Temperature ratio .... $T_b \div T_a =$	r_T	2.047	1.961
9	Volume ratio .... $r_p \div r_T =$	r_v	6.520	6.799
10	By Equation [22]............ $\delta_p \log r_v =$	ϕ_{vab}	0.1297	0.1316
11	From Table 7 or Table 6...............	ϕ_{vb}	0.1845	0.1899
12	....	ϕ_{va}	0.0548	0.0584
13	Entropy FA for mixture...............	ϕ_{vab}	0.1297	0.1316
14	Ratio of specific volumes..............	v_m/v_p	1.091	1.000
15	Entropy $F'A$ for products........ $\phi_{vab} =$	ϕ_{cd}	0.1189	0.1317
16	Heat of combustion....................	q	388.3	388.3
17	Add to u_b, line 5, get...................	u_c	476.9	477.3
18	From Table 7 or Table 6..........	t_c	2224	2063
19	....	ϕ_c	0.4417	0.4621
20	Deduct ϕ_{cd}, line 15, get...............	ϕ_d	0.3228	0.3305
21	From Table 7 or Table 6...............	t_d	1240	1127
22	....	u_d	241.4	234.9
23	Deduct from u_c, line 17, get............	u_{cd}	235.5	242.4
24	From line 7, corrected for mean gas.....	u_{ab}	71.0	76.7
25	Net work or ideal output...............	Aw	164.5	165.7
26	Divide by input q, get efficiency........	e	0.424	0.426
27	Discrepancy, in output Aw, per cent....			+0.7

Here temperatures and energy quantities are in centigrade units.

62 The preceding schedule merely carries out the process described in Pars. 57 and 58 and in Par. 60, and needs but a few comments. Lines 8-10 find temperature and volume ratios, useful

for comparison with results from other data, and get volume-ratio entropy ϕ_{vab}; then lines 11-13 make a parallel determination of the last quantity: agreement of the two values is a useful check on computation. Between lines 13 and 15 lies the adjustment emphasized by the shaded block *FF′* in Fig. 15: 0.1297 from line 13 is divided by 1.091, to get effect of change in *AB* or δ. The heat of combustion q, line 16, is the full value belonging to the pure mixture, not considering the dilution by waste gases retained in the clearance space; for discussion of the latter refer to Par. 76. Lines 17 to 23 carry out the general scheme in straightforward way and lines 25-26 complete it and compare results.

63 The correction of u_{ab} in line 24 of the calculation with mean-gas properties calls for special explanation. In this case the specific volume of the combustible mixture is 1.091 times that of the products of combustion. Then if other conditions were equal, the compression of the mixture through a given ratio r_p would require 1.091 times as much work as the same compression of the products. To apply this idea to the approximate calculation with mean gas, take $u_{ab} = 70.3$ from line 7, multiply it by 1.091 and get the adjusted value 76.7 for line 24. Of course other things are not quite equal and the " correction " is only a rough one. It is finally justified only by its effect, which is not always so good as here.

64 This general scheme for finding output places all emphasis upon the two work quantities in compression and expansion, and upon their numerical difference or algebraic sum. An alternative method would be to find the heat rejected during the cooling operation *DA* and deduct it from received heat q. The first determination has the advantage that it goes around all questions such as are suggested by the failure of the cycle to close at *A* in Fig. 15. Those questions are interesting and one or two of them might slightly affect the temperature t_c at the end of combustion. Correct answers would lead to the same result by either path of approach, but it is much simpler not to go into this matter.

Comparisons of Results

65 The things thus far accomplished in this discussion are:

(*a*) Development of the graphical method of Figs. 11 and 12, effective chiefly for the approximate determination by use of the average gas mixture; unless a great deal of routine work is to be done the plotting of a chart like Fig. 11 or Fig. 16 is rather a large task.

(*b*) The establishment of numerical methods which, even when they include the preliminary computation of Table 7, are much more convenient than the direct use of Equations [**17**] to [**19**].

(*c*) The beginning of a comparison of the correct and approximate determinations, in one example, and the development of a major correction for the approximate result.

66 The next step is to try out the approximate as against the close method, using for this purpose eight of the ten examples in Tables 1 to 5. The ones omitted are No. 7, heavy oil, and No. 10, methyl alcohol. By means of condensed tabulations results will be compared, first as found exactly by the form in Par. 61, then

TABLE 7 QUANTITIES FOR BLAST-FURNACE GAS, NO. 6. CENTIGRADE UNITS

Mixture				Products					
t	ϕ_v	ϕ_p	u	t	ϕ_v	u	t	ϕ_v	u
100	0.0548	0.0764	17.6	900	0.2690	169.7	2000	0.4179	419.9
150	0 0771	0.1074	26.5	1000	0.2860	190.4	2100	0.4287	445.0
200	0 0971	0.1351	35 4	1100	0.3020	211.6	2200	0.4392	470.6
250	0.1152	0.1602	44.4	1200	0.3171	233.2	2300	0.4495	496.6
300	0.1318	0.1831	53.5	1300	0.3315	255.1	2400	0.4596	523.0
350	0.1470	0 2041	62.6	1400	0.3453	277.5	2500	0.4695	549.8
400	0 1612	0.2236	71.8	1500	0 3585	300.3	2600	0.4791	577.0
450	0.1745	0.2419	81.0	1600	0.3712	323 4	2700	0.4885	604.5
500	0.1869	0.2589	90 3	1700	0.3835	346 9	2800	0.4978	632.5
550	0 1987	0.2750	99.7	1800	0.3953	370.8	2900	0.5069	661.0
600	0 2098	0 2902	109.2	1900	0.4068	395.1	3000	0.5158	689.9

with a start from the same volume ratio r_v and from the same temperature range t_a-t_b.

67 In Table 8 the first exhibit is a comparison of energy u at 500 deg. cent., given first for the precombustion mixture and then for the products of combustion; in effect this makes numerical the comparisons in Fig. 13 and extends them to examples 8 and 9.

TABLE 8 COMPARISON OF GAS-MIXTURE PROPERTIES

No.	1	2	3	4	5	6	8	9	Mean
Fuel	B. F.	Prod.	Water	O. W.	Nat.	Ret.	Gaso.	Alco.	
500 u_m	90.3	95 0	100 6	98.0	100.3	102.5	98.7	96.7	97.5
500 u_p	90.5	94.3	98 5	98.0	99.5	101.0	96.2	96.4	97.5
1000 u_p	190.4	198 4	208.7	206.5	209.6	213.0	202.1	202.4	205.6
2000 u_p	419.9	440 3	468.9	461.9	469.6	478.8	449.8	450.4	459.5
3000 u_p	689.9	733.6	789 2	778.5	795.4	813.8	755.7	759.2	773.6
δ_m	0.1593	0.1679	0.1819	0.1672	0 1649	0.1738	0.1524	0.1545	0.1580
v_m/v_p	1.091	1.087	1 152	1.038	1.001	1.044	0.954	0.958	1.000

Next, by values of u at 1000, 2000, and 3000 deg., the specific heat of each products mixture is compared with that of the average products in the last column. Note how gases 1, 2, 8, and 9 lie below the mean in specific heat, while 4, 3, 5, and 6 run increasingly above it. Finally, the volumetric properties immediately used in the calculation of Par. 61 are given in the last two lines; for a fuller showing turn back to Table 3, in its last five columns.

68 Table 9 gives selected items from computations exactly like the one in Par. 61, starting with the same data $p_a = 13.5$ and $t_a = 100$; only in Example 2 do the p_b and r_p values not quite correspond with the 13.5 for p_a. The compression pressure p_b is a typical value for each kind of fuel. In each pair of lines after the first two, the first one gives values for the actual gas, and the

second for the mean gas, so that the two lines represent the two columns of the calculation in Par. 61. Touching upon only a few salient points, comment may be made as follows:

Lines 3 and 6: Larger specific volume, shown by δ_m in all but cases 8 and 9, means more work u_{ab} of compression, tending to raise t_b; lower specific heat tends to make t_b higher, higher specific heat to make it lower; except in cases 8 and 9 these influences so work together as to make t_b higher for the actual mixture than as computed for the mean products.

Lines 4 and 5: These comparisons are interjected in order to suggest the degree of difference that may result from change to equal r_v or equal r_T as primary datum.

TABLE 9 OUTLINED COMPUTATIONS OF THE EIGHT EXAMPLES, EACH FOR ACTUAL AND FOR MEAN COMPOSITION

	No.	1	2	3	4	5	6	8	9
1	p_b	180	150	90	100	135	135	100	162
2	r_p	13.33	11	6 67	7.41	10	10	7.41	12
3a	t_b	490.5	452 2	366.5	370 0	411.8	421 4	345.3	432.2
3b		458.8	424 0	340.3	357 0	407.2	407.2	357.0	439.4
4a	r_T	2.047	1.944	1.715	1.722	1.836	1 862	1.658	1.897
4b		1.961	1 869	1.644	1.689	1.824	1.824	1.689	1.910
5a	r_v	6.520	5.658	3.888	4.301	5.447	5.371	4 469	6.326
5b		6.819	5.887	4.055	4.386	5.484	5.484	4.386	6.283
6a	u_{ab}	71.0	67.2	53.5	52.8	62.7	66.1	48 2	64.4
6b		70.3	63.4	46.6	49.9	60.0	60.0	49.9	66.5
7	q	388.3	470	660	625	575	610	550	435
8a	u_c	476.9	555.6	733.2	696.4	656.3	695.5	617.1	518.1
8b		477.3	552.1	725.3	693.6	653.7	688.7	618.6	520.2
9a	t_c	2224	2416	2839	2760	2600	2672	2573	2239
9b		2063	2317	2859	2764	2642	2750	2533	2210
10a	t_d	1240	1450	2042	1892	1636	1717	1695	1253
10b		1127	1383	2022	1897	1677	1758	1699	1271
11a	u_{cd}	235.5	254.1	251 9	264.7	288.1	298.2	248 3	257.7
11b		242.4	255 6	259.7	262.9	282.3	295.6	241.2	251.1
12a	u_{ab}	71.0	67.2	53.5	52.8	62.7	66.1	48.2	64 4
12b		76.7	68.9	53.8	51.8	60.1	62.6	47.6	63.7
13a	Δw	164.5	186.9	198.4	211.9	225.4	232.1	200.1	193.3
13b		165.7	186.7	205.9	211.1	222.2	233.0	193.6	187.4
14	e	0.424	0.397	0.301	0.339	0.392	0.380	0.364	0.444
15	disc.	+0.7	−0.1	+3.8	−0.4	−1.4	+0.4	−3.2	−3.1

Line 9: Here specific heat has controlling influence, so that in cases 3 to 6, temperature t_c is less for the actual gas than for the mean gas.

Line 12: The first u_{ab} is repeated from line 6, the second is adjusted as explained in Par. 63.

Lines 13 and 15: Here is the real test of the approximate method; as in line 27, Par. 61, the discrepancy is expressed as a percentage of the actual output. Example 3 is anomalous, but fortunately that gas is not an engine fuel; the other gas-fuel examples show a discrepancy ranging up to 1 per cent, while the liquid fuels are determined about 3 per cent low by the approximate method.

Line 14: This efficiency is for the actual gas, line 13a.

69 The next condition to be considered is that in which volume ratio r_v is the same in both calculations, exact and approximate.

As a datum this is simpler than r_p, because r_v for the mixture need only be adjusted by v_m/v_p in order to become r_v for the products. For Table 10 the general data p_a, t_a and q are continued and r_v is taken at about the mean of the two values in Table 9. On the whole, for the gas fuels agreement is not quite so good as in the preceding set of calculations; in cases 4 and 6 it would really be a little better without the adjustment of u_{ab} by v_m/v_p. In the general run there is an average discrepancy of about −1.7 per cent, say 1 in 60.

TABLE 10 RESULTS FROM EQUAL VOLUME RATIO

	No.	1	2	3	4	5	6	8	9
1	r_v	6.5	5.7	4.0	4.3	5.5	5.5	4.4	6.3
2a	t_b	489.5	453.8	373.1	370.0	414.1	427.0	344.8	433.0
2b		447.3	416.2	337.3	352.6	408.0	408.0	357.8	440.0
3a	u_{ab}	70.8	67.5	56.7	52.7	63.2	67.3	47.6	64.6
3b		68.0	61.8	46.1	49.1	60.1	60.1	50.1	66.6
4a	u_c	476.7	555.9	736.4	696.3	656.8	696.7	616.5	518.3
4b		475.0	550.0	724.8	692.8	653.8	688.8	618.8	520.3
5a	t_c	2223	2417	2848	2760	2602	2675	2571	2240
5b		2055	2312	2857	2762	2643	2750	2588	2211
6a	t_d	1235	1448	2037	1893	1632	1708	1702	1256
6b		1140	1393	2028	1906	1675	1764	1696	1270
7a	u_{cd}	235.9	255.0	259.5	264.4	289.6	301.9	246.3	257.4
7b		237.0	250.3	257.2	259.6	283.6	294.0	242.6	251.3
8a	u_{ab}	70.8	67.5	56.7	52.7	63.2	67.3	47.6	64.6
8b		74.2	67.3	53.1	51.0	60.2	62.7	47.8	63.8
9a	Aw	165.1	187.5	202.8	211.7	226.4	234.6	198.7	192.8
9b		162.8	183.0	204.1	208.6	223.4	231.3	194.8	187.5
10	e	0.425	0.399	0.307	0.339	0.394	0.384	0.362	0.443
11	disc.	−1.4	−2.4	+0.6	−1.5	−1.3	−1.4	−2.0	−2.8

70 In Table 9, line 5, it is seen that with the same r_p there is a comparatively small difference between the two r_v's for a given gas, by the exact and approximate methods. In Table 11, starting with a given temperature range in each case, there appears a large reaction upon r_p and r_v. The effect of this relationship is shown by the results in lines 4 and 5, which are given without any of the intermediate steps in the calculation. The output values in line 4 are computed without paying any attention to differences in r_v, or with the tacit assumption that ϕ_{cd} is the same for the actual gas, line 4a, and for the mean gas, line 4b. Really and correctly taking the factor v_m/v_p into account, the actual gas yields the output in line 5a, from which line 4b is discrepant by as much as 10 per cent in the worst case. Dividing result 4b by the ratio

TABLE 11 THE EFFECT OF USING EQUAL TEMPERATURE RANGE IN THE ADIABATIC COMPRESSION

	No.	1	2	3	4	5	6	8	9
1	t_b	480	450	350	370	410	420	350	430
2a	r_p	12.68	10.89	6.09	7.41	9.90	9.92	7.63	11.70
2b		14.93	12.74	7.10	8.01	10.16	10.76	7.10	11.40
3a	r_v	6.28	5.62	3.65	4.30	5.41	5.35	4.57	6.20
3b		7.41	6.57	4.25	4.68	5.55	5.79	4.25	6.04
4a	Aw	177.4	208.4	229.7	219.9	222.7	242.2	192.9	182.4
4b		177.5	201.4	219.0	221.6	223.1	238.4	188.4	181.5
5a	Aw	161.6	186.4	198.1	211.9	222.1	231.4	201.9	190.8
5b		162.7	185.3	189.9	213.6	222.8	228.3	197.5	189.5

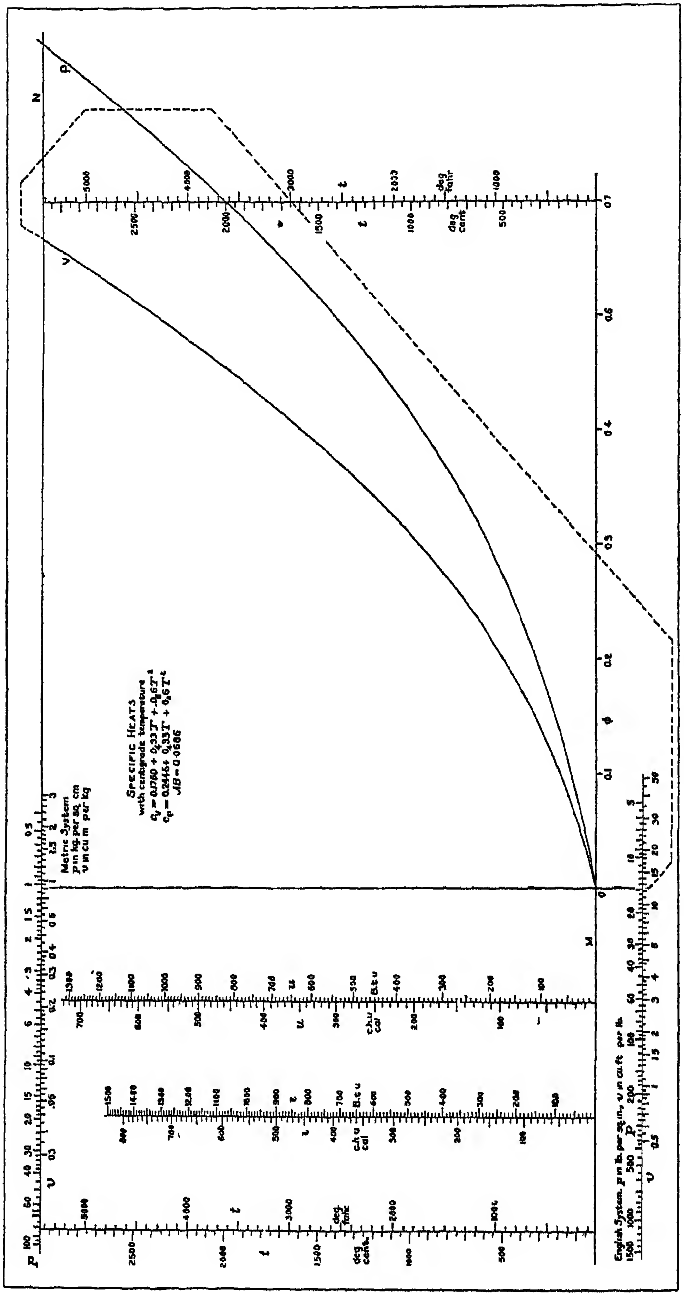


FIG. 16 CHART FOR FINDING IDEAL OUTPUT OF GAS-ENGINE CYCLES
(See Par 73 and footnote.)

v_m/v_p gives the adjusted values in line 5b, which are in good accord with 5a.

71 Failure to perceive this effect of specific-volume change was a serious error in a portion of the paper as originally presented, although that error had little if any effect upon the final method there presented. In any case, that final method has now been a good deal simplified in its exact form.

72 A good agreement between the approximate and the exact solution, as in most cases of Tables 9 and 10, means generally that the influences complicating real performance have an interplay that makes them more or less balance each other. In Table 11 we see how they may so interact as to cause a big discrepancy. The approximate solution is much more serviceable for finding output than for cycle dimensions.

Method for the Otto Cycle

73 The working chart for approximate determinations is pictured in Fig. 16, but in working form is nearly four times that size.[1] It is like Fig. 11 in general terms, with all of its curves and scales based on the properties of the mean products of combustion, as stated in Par. 41 and as computed (partly) in Table 6. The scales are so disposed that any problem can be constructed upon a separate sheet of paper, laid down from above across the space between the u scale and the right-hand scale of t, the template guide lines M and N being drawn across this problem sheet. Of course the chart is to be laid down on the drawing board, with use of the T-square for horizontals of constant t, u, or i. The complete form of one of the heavy-paper templates, that for curve V, is shown in dotted outline. Having the metric P and V scales laid out along the upper guide line (but to be projected to line M as needed), this chart will give results in any one of the three measurement systems named in Par. 7.

74 The graphical example in Fig. 17 is worked in English-fahrenheit units and starts with the data $p_a = 13.5$, $p_b = 80$, and $t_a = 150$. After drawing the two p-constant curves and locating compression AB, the two v-constant curves AD and BC are drawn. Of course, the volume values lettered on these curves, as read from the v scale of the chart, are only very roughly approximate for the actual mixture, with its specific-volume ratio 1.152. Adiabatic expansion CD is located just as in Fig. 12, by adding q or u_{bc} to u_b to get u_c.

75 In order to give an idea of probable accuracy by graphical determination, the following tabulation compares the results on Fig. 17 with those computed by means of a fahrenheit-unit Table 6:

	u_a	t_b	u_b	u_c	t_c	t_d	u_d	u_{cd}	u_{ab}'	Aw
Const.	20	520	91	1288	5100	3700	846	487	82	355
Calc.	22.1	516	92.2	1284.2	5115	3715	850.5	488.7	81.7	352

[1] Copies of the full-size chart, separately printed, may be obtained for a nominal charge, by addressing the Secretary of the Society.—*Editor*.

As might be expected, it appears that the construction can easily accumulate an error of the same order of magnitude as the discrepancies in Tables 9 and 10.

76 The increase of compression range *AB* to *AB′* represents the "correction" for a larger specific volume of mixture, as described in Par. 63. The dotted expansion line *C′D′* shows the determination when dilution of mixture by waste gases (in clearance space) is taken into account. As a fair approximation, assuming exact geometrical scavenging and that the influences of pres-

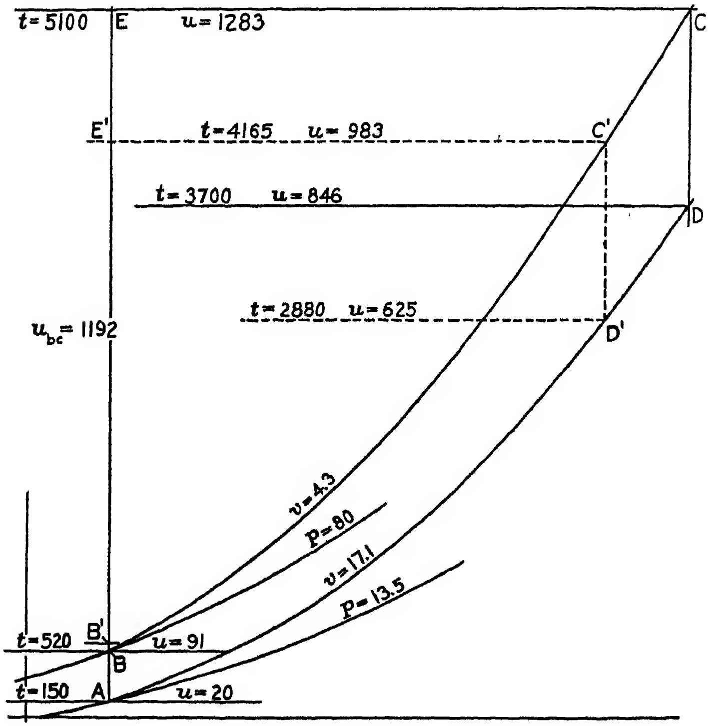


FIG. 17 CONSTRUCTION FOR THE OTTO CYCLE, GAS EXAMPLE No. 3

sure difference, temperature difference, and density difference balance each other, the weights of fresh mixture and retained gases are taken to be proportional to displacement volume and clearance volume. Using the volumes on Fig. 17, the ratio of these volumes is here $4.3 \div 17.1 = 0.25$. Its chief influence is to diminish the effective heating value, of the pound of actual mixture in the cylinder, from 1192 to $1192 \times 0.75 = 892$ B.t.u. This new value determines the height of line *E′C′* and the location of *C′D′*. A second effect is to increase slightly the specific heat and the relative volume of the mixture, in the direction of making the mixture more like the products of combustion in these properties.

Method for the Diesel Cycle

77 In discussing this cycle a purely constant-pressure combustion will be assumed, at the pressure p_b due to the adiabatic compression of the charge of air. That charge will be the full amount of air entering into the cycle, which means practically that the engine has solid injection, not air injection, of the fuel oil. Both physically and computationally, conditions are more complex than in the Otto cycle, because the compressed charge differs more widely from the products in specific heat and has a

TABLE 12 QUANTITIES FOR DIESEL-ENGINE MEDIA—CENTIGRADE UNITS

Air					Products				
t	ϕ_v	ϕ_p	u	i	t	ϕ_v	ϕ_p	u	i
50	0.0288	0.0403	8.6	12.0	1000	0.2948	0.4011	195.1	264.2
100	0.0537	0.0751	17.2	24.1	1100	0.3109	0.4224	216.3	292.3
150	0.0754	0.1054	25.8	36.1	1200	0.3260	0.4424	237.9	320.8
200	0.0947	0.1324	34.5	48.2	1300	0.3404	0.4613	259.8	349.6
300	0.1280	0.1789	51.8	72.4	1400	0.3542	0.4793	282.1	378.9
400	0.1561	0.2181	69.3	96.8	1500	0.3673	0.4965	304.7	408.4
500	0.1806	0.2520	87.0	121.3	2000	0.4263	0.5727	423.8	562.0
600	0.2022	0.2820	104.8	145.9	2100	0.4371	0.5864	448.7	593.8
700	0.2218	0.3090	122.8	170.8	2200	0.4476	0.5998	474.2	626.2
800	0.2396	0.3333	141.0	195.9	2300	0.4579	0.6128	500.1	659.0
900	0.2560	0.3460	159.4	221.2	2400	0.4679	0.6254	526.4	692.2
1000	0.2713	0.3769	178.1	246.7	2500	0.4777	0.6378	553.1	725.9

different mass, and because the ratio of adiabatic expansion differs from that of compression by an amount that is dependent upon several variable conditions of working.

78 For air, evaluating Equation [6b] according to the particular constants from Par. 25, with centigrade temperature,

$$c_v = 0.1708 + 0.0_7107T^2, \quad c_p = c_v + 0.0686 \quad . \; . \; . \quad [25]$$

Assuming as fuel a pure hydrocarbon with the weight composition hydrogen 0.15 and carbon 0.85, the specific heats of the products, with 50 per cent of excess air, are

$$c_v = 0.1740 + 0.0_4239T + 0.0_8382T^2, \quad c_p = c_v + 0.0691 \quad . \quad [26]$$

In the gas equation $pv = bT$, with p in pounds per square inch, T in degrees centigrade, and for v in cubic feet per pound, the values of constant b are,

$$\text{For air and for mean products, } b = 0.6664 \quad . \; . \quad [27a]$$

$$\text{For actual products, } \quad b = 0.6713 \quad . \; . \quad [27b]$$

A comparison of values of energy u, like that in Table 8, gives the following:

	$t = 500$	1000	1500	2000	2500
Air	87.0	178.0	275.9	383.3	502.9
Actual products	93.8	195.0	304.7	423.8	553.1
Mean products	97.5	205.6	325.8	459.6	608.2

79 Tables like Nos. 6 and 7, for the two gas compositions of this example, are combined in Table 12.

80 A double example will now be given, starting first with the same pressure range, then with the same volume range, in adiabatic compression AB. With the fuel composition stated in Par. 78 the weight ratio is 22.02 lb. of air to 1 lb. of oil, or the air compressed for 1 lb. of total medium is 0.9566 lb.; this number is the mass ratio r_m. General data are, $p_a = 13.5$ lb. abs., $t_a = 75$ deg. cent., $p_b = 600$ lb. abs. in case (a), $r_{vab} = 16$ in case (b). In either case the first column is for actual compositions, Table 12, the second is an approximate calculation by means of Table 6. Instead of being made continuous, as in Par. 61, this outlined calculation is broken into sections, with explanations as needed.

81 In the adiabatic compression, with $r_p = 600/13.5 = 44.44$, the entropy ϕ_{pab} is $0.1580 \times 1.6478 = 0.2604$ for both columns of case (a); with $r_v = 16$ the similar common value is

$$\phi_{vab} = 0.1580 \times 1.2041 = 0.1903$$

for case (b). Working at first with the full pound of air (as to entropy values), the two calculations are,

	Case (a)		Actual	Mean
1	Initial entropy	ϕ_{pa}	0.0577	0.0614
2	Entropy of pressure range	ϕ_{pab}	0.2604	0.2604
3	Entropy at B	ϕ_{pb}	0.3181	0.3218
4	Temperature at B	t_b	737.5	650.0
5	Temperature ratio T_b/T_a	r_T	2.904	2.652
6	Volume ratio, by Equation [21]	r_v	15 31	16.76

	Case (b)		Actual	Mean
1	Initial entropy	ϕ_{va}	0.0414	0.0449
2	Entropy of volume range	ϕ_{vab}	0.1903	0.1903
3	Entropy at B	ϕ_{vb}	0.2317	0.2352
4	Temperature at B	t_b	755.6	637.2
5	Temperature ratio	r_T	2.973	2.616
6	Pressure ratio	r_p	47.57	41.85
7	Pressure at B	p_b	642.2	564.9

In case (a) the difference in specific heat causes a large difference in temperature t_b, with incidental inequality of volume ratio; then equalizing the volume ratio makes the temperature difference yet larger in case (b).

82 From here on the two calculations follow the same schedule and can be carried forward in parallel columns. All the energy quantities belonging to the adiabatic compression (next to be given) are read from the table for the pound of medium, then diminished by the factor r_m or 0.9566; practically, from each is deducted the fraction 0.0434 of itself. Besides compression work u_{ab} we now need enthalpy or "total heat" i_b, also the "outer

work" $APv = i_b - u_b = o_b$; the last is for use in finding the work of constant-pressure expansion BC.

Adiabatic compression		Case (a) Actual	Case (a) Mean	Case (b) Actual	Case (b) Mean
8 Enthalpy at B.....	i_b	172.4	165.9	176.8	162.4
9 Energy at B.......	u_b	124.0	123.1	127.1	120.5
10 Energy at A.......	u_a	12.3	13.4	12.3	13.4
11 Compression work..	u_{ab}	111.7	109.7	114.8	107.1
12 Outer work at B....	o_b	48.4	42.8	49.7	41.9

83 As concerns physical data, the most uncertain point in the whole determination is the change of enthalpy, at constant pressure p_b and temperature t_b, from r_m lb. of air to 1 lb. of air plus vaporized oil. Lacking definite knowledge of the thermal properties of average Diesel-engine oil, the approximation will here be made of simply adding the heat of combustion q (itself no more than an average value) to enthalpy i_b of the compressed charge (line 8 above) and taking this to be i_c at the end of combustion. Per pound of total medium or per 0.0434 lb. of oil, the heating value here used is 435 c h u. Continuing the calculation,

Constant-pressure combustion		(a)		(b)	
13 Enthalpy at C..........i_b+q =	i_c	607.4	600.9	611.8	597.4
14 Temperature at C............	t_c	2142	2012	2156	2002
15 Energy at C	u_c	459.2	462.9	462.6	460 2
16 Outer work at C	o_c	148.2	138.0	149.2	137.2
17 Work of expansion....$o_c - o_b$ =	o_{bc}	99.8	95 2	99.5	95.3

84 The increase of volume from B to C is due to increase of mass, change of specific volume through combustion (here very small), and rise of temperature. The two volumes, per pound of total medium, are

$$v_b = \frac{r_m b_m T_b}{p_b}, \qquad v_c = \frac{b_p T_c}{p_c} \quad . \; . \; . \; . \; . \quad [28]$$

and since $p_b = p_c$, the volume ratio is

$$r_{vbc} = \frac{1}{r_m} \frac{b_p}{b_m} \frac{T_c}{T_b} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [29]$$

Here the subscripts m and p on gas constant b refer to mixture and products, as in Table 3. With the values of b as in Par. 78 and of r_m as in Par. 80, the coefficient of the temperature ratio is,

For actual gases, 1.0530
For the mean gas, 1.0455.

Dividing r_{vbc} into total r_v, line 6 in case (a), primary datum in case (b), we get the ratio r_{vcd} of adiabatic expansion CD, from which comes the entropy distance that appears as EF or GD in Fig. 18. Using this scheme, which goes around the impossibility of representing on the T-ϕ diagram the changes in mass and

volume that take place through combustion, the calculation is completed as follows:

	Adiabatic expansion, etc.		(a)		(b)	
18	Temperature ratio	T_c/T_b	2.390	2.476	2.363	2.499
19	Volume ratio, range BC....	r_{vbc}	2.517	2.589	2.488	2.613
20	Volume ratio, range CD....	r_{vcd}	6.082	6.474	6.431	6.123
21	Entropy change	ϕ_{vcd}	0.1247	0.1282	0.1285	0.1243
22	Entropy at C.........	ϕ_{vc}	0.4415	0.4557	0.4430	0.4545
23	Entropy at D..............	ϕ_{vd}	0.3168	0.3275	0.3145	0.3302
24	Temperature at D.........	t_d	1139	1108	1124	1125
25	Energy at D..............	u_d	224.7	230.6	221.5	234.5
26	Work of expansion. $u_c - u_d =$	u_{cd}	234.5	232.3	241.1	225.7
27	From line 17..............	o_{bc}	99.8	95.2	99.5	95.3
28	Gross work		334.3	327.5	340.6	321.0
29	Negative work, line 11......	u_{ab}	111.7	109.7	114.8	107.1
30	Net work or output........	Aw	222.6	217.8	225.8	213.9
31	Efficiency	e	0.512	0.500	0.519	0.491
32	Discrepancy, per cent......			−2.4		−5.3

In case (*a*) the actual output is fairly well represented by the approximate result which depends wholly upon the mean-gas properties in Table 6. In case (*b*) the degree of approximation is poor. Here, as with the Otto cycle, a good agreement (when obtained) is due to a balancing of disturbing influences.

85 The final graphical construction, made by means of the chart pictured in Fig. 16, is shown in Fig. 18, for case (*a*) of the preceding example. It is simpler in idea than the calculation there made and gives a better balance of disturbing influences and a closer result. All of the graphical work is done as if for a full pound of medium in the cylinder, with the coördinates of point B corresponding to that assumption; and the smaller mass during compression is taken into account only in diminishing work u_{ab} to the value in line 11, Par. 82, by the use of mass factor r_m. In a condensed outline, showing only results, computation of this construction for the two cases gives the following:

	Graphical Determination		(a)	(b)
1	One-pound enthalpy at B............	i_b	173.4	169.8
2	Add q, get enthalpy at C............	i_c	608.4	604.8
3	Temperature at C....................	t_c	2032	2023
4	Energy at C.........................	u_c	469.0	466.1
5	Temperature at D....................	t_d	1117	1121
6	Energy at D.........................	u_d	232.6	233.4
7	Constant-pressure work	o_{bc}	94.7	94.9
8	Adiabatic-expansion work	u_{cd}	236.4	232.7
9	Gross out-stroke work................		331.1	327.6
10	Compression work, line 11, Par. 82....	u_{ab}	109.7	107.1
11	Net work	Aw	221.4	220.5
12	Exact result, line 30, Par. 84..........		222.6	225.8
13	Discrepancy, per cent.................		−0.5	−2.4

86 The working scheme, with p_a and p_b as data, is then to construct Fig. 18 as shown and find output Aw from the scale readings recorded on that diagram. Practically, the construction

need not extend to the left of line AB, this compression being equally well determined by "triangle" $AB'B$. Volume curve CE is put in rather to illustrate a step in the computation than because it is needed graphically. With the volume ratio as primary datum the proper procedure is to make the "actual" computation of case (b), Par. 81, then go to the chart with the value of p_b thus found.

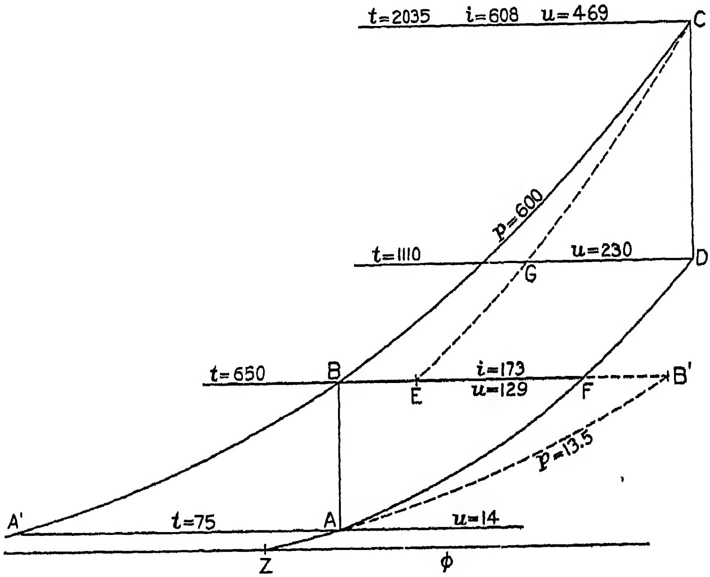


FIG. 18 CONSTRUCTION FOR THE DIESEL CYCLE

Computation of output from the readings on Fig. 18 may be slightly shortened, as follows:

$$\begin{aligned} Aw &= (i_c - u_c - i_b + u_b) + (u_c - u_d) - r_m(u_b - u_a) \\ &= (i_c - i_b) - (u_d - u_b) - r_m(u_b - u_a) \quad . \; . \; . \; . \; . \; . \quad [30] \\ &= 435 - 101 - 110 = 224 \end{aligned}$$

This result is a little large, chiefly because point D reads low as compared with its computed height.

87 Because the best mixture proportion for the Diesel engine is subject to much less variation than in the wider class of Otto-cycle engines, the "exact" properties in Table 12 are very fairly representative for general use. As with the Otto cycle, it is to be understood that the construction in Fig. 18 is claimed to be a good approximation only as to overall output, not as to the details of the cycle and the coördinates of its "points."

APPENDIX

Computations on Gas Mixtures

88 The constants for dry air as here used are as follows:

Ingredient	Volume		Weight	
O_2	0.209	1.00	0.231	1.00
$N_2+Ar+CO_2$	0.791	3.78	0.769	3.33
	1.000	4.78	1.000	4.33

Mean molecular weight $m = 28.97$

Gas constant $B = 53.3$, $b = 0.37$

$AB = c_p - c_v = 0.0686$

Standard volume $v_0 = 12.38$

The argon in air is 1 per cent of the total volume, but in present calculations this is disregarded and all of the inert matter is counted as nitrogen, with molecular weight 28. The amount of atmospheric CO_2 is, of course, very minute.

89 The "lower" heating values, in B.t.u. per pound (not including the latent heat of the steam in products of combustion) are

Hydrogen ....H_2 = 52,920
Carbon monoxide ....CO = 4,363
Methane ....CH_4 = 21,670
Ethylene ....C_2H_4 = 20,420
Oil ....all grades = 18,500
Ethyl alcohol ....C_2H_6O = 11,200
Methyl alcohol ....CH_4O = 8,400

90 As an illustrative example, take gas No. 6 from Table 1; then Table 8 shows the calculation of every derived quantity in Tables 1

TABLE 13

Gas No. 6	Column 1: Volume	2: m	3: Weight	4: Weight	5: H_2	6: C in hydrocarbons	7: C in CO	8: Q
H_2	0.50 ×	2 =	1.00	0.086	0.086	.....	.....	52,920
CO	0 07 ×	28 =	1.96	0.168	.....	.....	0.072	8,552
CH_4	0.32 ×	16 =	5.12	0.441	0.110	0.331	.....	110,950
C_2H_4	0 04 ×	28 =	1.12	0.096	0.014	0.082	.....	22,870
CO_2	0.03 ×	44 =	1.32	0.113	.....	.....	.....	......
N_2	0.04 ×	28 =	1.12	0.096	.....	.....	.....	......
	1.00	m_m =	11.64	1.000	0.210	0.413	0.072	195,292

and 2 except the air ratio (for the latter see Par. 100). Steps in the operation are as follows:

91 Consider the unit volume to be one mol, so that each fraction in column 1 of Table 13 is part of a mol of the particular ingredient. Multiply each partial mol by its molecular weight m, getting the weights in column 3, which sum up to the weight of a mol of mixture or the mean molecular weight. Division of each partial weight in column 3 by the sum gives the weight composition in column 4. In columns 5 to 7 the combustible elements are apportioned in order to get the exhibit values in the first part of Table 2.

92 The work in columns 4 to 7 of Table 13 is decidedly special, not serving any useful purpose in further calculations. Instead of finding the composition of a unit of weight it is better to apply the pound

heating values of the ingredients to the weights in column 3, thus getting the heating values per mol of fuel. Then the division

$$q = Q \div m = 195{,}292 \div 11.64 = 16{,}780 \text{ B.t.u.}$$

yields the heating value per pound

93 With a gas fuel the proper procedure is to figure all combination relationships in terms of volume. Ratios are shown by the chemical equations

$$\left.\begin{aligned} 2H_2+O_2 &= 2H_2O \\ 2CO+O_2 &= 2CO \\ CH_4+2O_2 &= CO_2+2H_2O \\ C_2H_4+3O_2 &= 2CO_2+2H_2O \end{aligned}\right\} \quad \ldots\ldots\ [31]$$

remembering that coefficients or numbers of molecules represent volumes. The first two operations show a shrinkage equal to one-half of the combustible or the whole of the oxygen, the other two show parity of volume.

TABLE 14

	1		2		3			
Gas No. 6	mol ratio	O_2	H_2O	CO_2	Mixture		m	Weight
H_2	0 50 × ⅓ =	0.25	0 50	....	H_2	0.50	2	1 00
CO	0 07 × ½ =	0.035	....	0.07	CO	0 07	28	1.96
CH_4	0.32 × 2 =	0.64	0 64	0 32	CH_4	0 32	16	5.12
C_2H_4	0 04 × 3 =	0.12	0 08	0.08	C_2H_4	0.04	28	1.12
	net O_2 =	1.045	....	0.03	CO_2	0.03	44	1.32
	add 15%	0.157	1.22	0.50	O_2	1.202	32	38 46
	total O_2	1.202	O_2	0.157	N_2	4.584	28	128.35
	3.78 O_2 = N_2	4.544	N_2	4.584		6.746		177.33
	air	5.746		6 461		m_m = 26 29		

94 In the first section of Table 14 we start with the combustible ingredients of the fuel gas, apply to each its oxygen ratio, get the net and total oxygen and the associated nitrogen from the atmosphere. All quantities are in terms of the mol of original gas as unit.

95 In section 2 of Table 14 the products of combustion are made definite, being sure to include in CO_2 and N_2 the amounts originally in the fuel; only the excess oxygen appears here. Note the contraction from 6.746 mols of mixture to 6.461 mols of products, equivalent to one-half of H_2+CO and here 4.08 per cent of the mixture volume.

96 Finally, for the 6.746 mols of mixture the same operation is performed as that in the first part of Table 13, but now the mean molecular weight must be found by the division $177.33 \div 6.746 = 26.29$, of total weight by number of mols.

97 In order to get the air ratio, as given in Table 2, take the weight of the mol of fuel gas, here 11.64 from Table 8, and divide it into the total weight of mixture, here 177.33. This gives 15.24 lb. of mixture or 14.24 lb. of air per pound of fuel.

98 Table 15 shows calculations on the products of combustion, paralleling the last portion of Table 14, getting volume and weight compositions per unit and working out the set of constants for this gas. Note that as a check on computation the total weight per mol of original gas, here 177.33, must be the same for mixture and for products.

99 With a liquid fuel it becomes preferable or even necessary to work from the weight composition. Section 1 of Table 16 starts with gasoline, example No. 8, from Par. 30, applies the weight ratios

for O_2, then parallels Table 14 in finding the air required. Section 2 first shows the weights of products per pound of fuel, then converts this to a weight composition per pound of products. Physically, the next step would be to multiply each partial weight by the proper value

TABLE 15

Gas No. 6	1 mol	2 m	3 Weight	4 Volume	5 Weight	6 Constants
H_2O	1.22	18	21 96	0.189	0.124	$m = 177\ 33 \div 6.461 = 27.45$
CO_2	0 50	44	22.00	0.077	0.124	$B = 1544 \div m = 56.25$
O_2	0.157	32	5 02	0.024	0.028	$b = 10\ 723 \div m = 0.3906$
N_2	4.584	28	128.35	0.710	0.724	$AB = 1\ 986 \div m = 0\ 0723$
	6 461		177.33	1.000	1.000	

of $B = R/m$ and add up to the mean B for the mixture of gases. The more convenient procedure is to omit the factor R or 1544, divide each weight by its m and sum the column. The reciprocal of the sum is the mean molecular weight m_m, multiplying it by 1544 gives the gas constant B of the mixture, and by dividing it into the partial values in column we get the volume composition set off as section 3.

TABLE 16

1				2			3
Gas No. 8	Weight Ratio	O_2	Product	Weight	Weight	m	Volume
H_2	$0.16 \times 8 =$	1 28	H_2O	1.44	$0.0777 \div 18 =$	.00432	0.123
C	$0\ 84 \times 8/3 =$	2 24	CO_2	3.08	$0\ 1662 \div 44 =$	.00378	0.108
	net $O_2 =$	3.52	O_2	0.53	$0.0285 \div 32 =$	.00089	0.026
	add 15%	0.528	N_2	13.49	$0.7276 \div 28 =$	.02599	0.743
	total O_2	4.048		18.54	1.0000	.03498	1.000
	$3.33\ O_2 = N_2$	13.493			$m_m = 1 \div .03498 = 28.59$		
	air	17.54			$B = 1544 \times .03498 = 54.00$		

100 Lacking a single, definite form of molecule — because they are mixtures of various ingredients — the oil vapors do not lend themselves to close volumetric treatment. The weight composition $H_2 = 0.16$, $C = 0.84$, is exactly given by heptane, for which the combining equation is

$$C_7H_{16} + 11O_2 = 7CO_2 + 8H_2O, \qquad \text{ratio } 1.25$$

This shows a volume increase of three times the fuel vapor or 3/11 of the oxygen used. Taking this ingredient as a mean for the oil, computation by the method of Tables 14 and 15 gives a mean molecular weight of 30.00 for the combustible mixture and for the products of combustion the same value as found in Table 16. Note how the large addition of N_2 and excess O_2 cuts the expansion ratio from 1.25 for the active ingredients to the overall 1.049 in Table 3.

101 The other oil composition used as example 7, namely, $H_2 = 0.12$, $C = 0.88$, is not realized by any petroleum ingredient, although some reported oil analyses approach this low content of hydrogen. The idea is to show the extreme limits of composition for this class of fuels.

102 For the alcohols the combining equations are,

$$C_2H_6O + 3O_2 = 2CO_2 + 3H_2O, \qquad \text{ratio } 1.25$$
$$2CH_4O + 3O_2 = 2CO_2 + 4H_2O, \qquad \text{ratio } 1.20$$

also showing an expansion from mixture to products due to the break-up of heavy fuel molecules.

103 To get specific heat c_v for the combustible mixture of gas No. 6, start in Table 17 with the mol values from section 3 of Table 14, lumping together the volumes of H_2, CO, O_2 and N_2, since these have a common specific heat per mol. In column 2 enter the values of coefficient A and in column 4 those of B, from Par. 36. Multiplying each mol value

TABLE 17 COMBUSTIBLE MIXTURE OF GAS NO. 6

1		2	3	4	5
		A	AV	B	BV
Diatomic gases	6 356	4 76	30.255	0.0_354	0.00343
CO_2	0.03	5.3	0 159	0.0038	0.00011
CH_4	0.32	1 48	0.474	0.019	0.00608
C_2H_4	0.04	4 7	0.188	0 017	0.00060
mol	6.746		31.076		0.01022
lb.	177 33		$a = 0.1752$		$\beta = 0.0_4577$

by its A and B gives the products AV and BV in columns 3 and 5, which used as coefficients in Equation [8] would give the heat capacity of 6.746 mol or 177.33 lb. of the mixture. Division by the total weight 177.33 yields the pound values of a and β, to be used in Equation [7].

104 Provided that the composition per unit volume has been found, it is a little more convenient to start with this rather than with mols per mol of fuel gas. Thus column 1 of Table 18 is from column 4 of

TABLE 18 PRODUCTS OF COMBUSTION OF GAS NO. 6

	1	2	3	4	5	6	7
		A	Av	B	Bv	G	Gv
H_2O	0.189	5.13	0.970	0.00256	0.000484	0.0_6405	0.0_60765
CO_2	0.077	5.40	0.416	0.00506	0.000390	-0.0_5102	-0.0_60785
$O_2 + N_2$	0.734	4.95	3.633			0.0_631	0.0_82275
mol	1.000		5.019		0.000874		0.0_82255
lb.	27.45		$a = 0.1829$		$\beta = 0.0_4318$		$\gamma = 0.0_8820$

Table 15. Then the simple mean molecular weight serves as divisor for each of the three sums of products.

Comparison of Specific Heat Formulations

105 The curves numbered 3 in Fig. 4 are the same in Marks' Handbook and in the Goodenough and Felbeck bulletin, except as to a small difference in the one for H_2O. In order to see what would be the effect of using those curves instead of the curves 1 that gave Table 5, a new computation has been made for the ten examples in Tables 1 to 3, with some changes in the air ratios. The values now taken are,

For the six gases, Nos. 1 to 6, the same.......... 15 per cent
For the light oil, No. 8......................... 20 per cent
For the heavy oil and the alcohols, Nos. 7, 9, 10.. 50 per cent

106 A comparison of results, in specific heat c_v, is shown by Fig. 19. Middle section (a) gives average curves, first reproducing as 1 the curve of Equation [11] or the mean curve of Fig. 10 for seven examples. The new mean for all ten examples appears as 1′. Curve 2, representing the Goodenough and Felbeck formulas, runs closely with 1 over the useful range up to about 2200 deg. cent., then takes a higher trend.

107 Results for two single examples are also shown by Fig. 19, at (b) for blast-furnace gas No. 1 and at (c) for retort gas No. 6. Out of the whole group of ten these have extreme proportions as to relative content of CO_2 and H_2O in their products of combustion, also lowest and highest specific heat respectively. Here curve 1 is that of the paper, from Fig. 10, curve 2 is by the Goodenough and Felbeck formulas.

108 Turning back to Fig. 4, it is to be noted that over the lower two-thirds of the temperature range curve 1 runs well below curve 3

for CO_2, while for H_2O this relationship is reversed. In curve groups (*a*) and (*c*) of Fig. 19 these two influences closely balance each other, but in group (*b*) the greater weight of CO_2 makes curve 2 lie above 1 The general agreement shown in section (*a*) of Fig 19 makes the chart

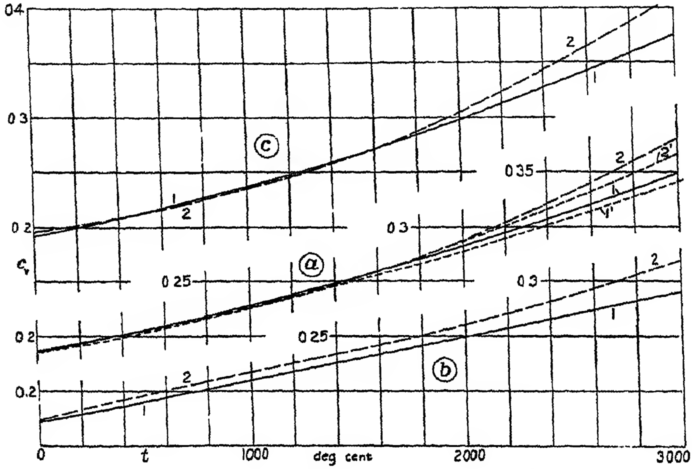


FIG. 19 COMPARISON OF SPECIFIC HEATS OF MIXTURES BY THE TWO SETS OF FORMULAS

of this paper and its numerical expression by Table 6 represent effectively either set of specific heat formulas.

109 For the curves in section (*a*) of Fig. 19, with centigrade temperature, the equations are,

$$1 \quad c_v = 0.1760 + 0.0_4 33T + 0.0_8 6T^2 \quad \ldots \quad [32]$$

$$1' \quad c_v = 0.1749 + 0.0_4 316T + 0.0_8 59T^2 \quad \ldots \quad [33]$$

$$\left.\begin{array}{ll} 2a & c_v = 0.1793 + 0.0_4 258T + 0.0_8 850T^2 \text{ to } T = 1611 \\ 2b & c_v = 0.1998 + 0.0_4 008T + 0.0_7 1634T^2 \text{ above } 1611 \end{array}\right\} \quad \ldots \quad [34]$$

$$2' \quad c_v = 0.17856 + 0.0_4 2454T + 0.0_7 1T^2 \quad \ldots \quad [35]$$

These are compared numerically in Table 19, by giving values of c_v at intervals of 500 deg. cent. The last formula and curve, numbered 2′, is a close approximation to two-part curve 2 by means of a single curve, put through the points at 0, 1000, and 2000 deg. as given in the table.

TABLE 10 COMPARISON OF AVERAGE FORMULAS FOR SPECIFIC HEAT c_v

Temp. cent.	0	500	1000	1500	2000	2500	3000
Curve 1	0 1855	0 2051	0.2277	0.2534	0.2820	0 3136	0.3483
1′	0.1840	0.2029	0.2247	0.2495	0.2772	0.3079	0.3415
2	0.1870	0.2044	0.2261	0 2520	0.2861	0.3277	0.3776
2′	0 1860	0.2035	0.2260	0.2535	0.2860	0.3235	0.2080

110 To facilitate investigatory work, by making it easier to compute a table like No. 7 or No. 12 for any particular gas mixture, the temperature functions to be used in Equations [13] to [16] are given in Table 20, for the centigrade temperature scale. These are as follows:

$\Delta T^2/10^6 = (T^2 - 273^2)$ pointed off six places to the left
$\Delta T^3/10^9 = (T^3 - 273^3)$ pointed off nine places to the left
$\log r_T = \log (T/273)$.

TABLE 20 TEMPERATURE FUNCTIONS TO COMPUTE ENTROPY AND ENERGY

t	25	50	75	100	125	150	200	250
$\Delta T^2/10^6$	0.0143	0.0298	0.0466	0.0646	0.0839	0.1044	0.1492	0.1990
$\Delta T^3/10^9$	0.006	0.013	0.022	0.032	0.043	0.055	0.085	0.123
$\log r_T$	0.03806	0.07304	0.10542	0.13555	0.16372	0.19018	0.23870	0.28234
t	300	350	400	500	600	700	800	900
$\Delta T^2/10^6$	0.2538	0.3136	0.3784	0.5230	0.6876	0.8722	1.0768	1.3014
$\Delta T^3/10^9$	0.168	0.221	0.284	0.442	0.645	0.901	1.215	1.594
$\log r_T$	0.32199	0.35833	0.39186	0.45202	0.50485	0.55195	0.59444	0.63314
t	1000	1100	1200	1300	1400	1500	1600	1700
$\Delta T^2/10^6$	1.5460	1.8106	2.0952	2.3998	2.7244	3.0690	3.4336	3.8182
$\Delta T^3/10^9$	2.043	2.568	3.176	3.872	4.662	5.553	6.550	7.660
$\log r_T$	0.66867	0.70151	0.73204	0.76057	0.78734	0.81254	0.83638	0.85897
t	1800	1900	2000	2200	2400	2600	2800	3000
$\Delta T^2/10^6$	4.2228	4.6474	5.0920	6.0412	7.0704	8.1796	9.3688	10.6380
$\Delta T^3/10^9$	8.888	10.240	11.723	15.110	19.085	23.702	29.008	35.052
$\log r_T$	0.88044	0.90090	0.92044	0.95706	0.99084	1.02218	1.05140	1.07879

With these factors and a calculating machine it does not take long to compute a complete table for a given gas mixture. The odd-hundred values between 2000 and 3000 deg. can easily be interpolated.

DISCUSSION

F. O. Ellenwood.[1] While the writer agrees with the author regarding the desirability of some short method of making the necessary calculations of cycles involving variable specific heats of the mixture of fuel and air, he is not convinced that the author's scheme is entirely satisfactory.

The ratio of the thermal efficiency of the actual and ideal engines is a true measure of the engine efficiency only when both of these thermal efficiencies have been accurately determined. A variation of 5 per cent in the ideal cycle efficiency may easily change the engine efficiency from 10 to 11 per cent. Thus if an internal-combustion engine has a thermal efficiency of 30 per cent and the corresponding ideal cycle efficiency is 40 per cent, the engine efficiency is 30/40 or 75 per cent. If, however, the cycle efficiency is 35 per cent the engine efficiency becomes 30/35 or 85.7 per cent instead of 75 per cent.

Evidently the proposed scheme of using the templates is intended to be much more convenient than using the thermodynamic equations, and it is implied that the accuracy of results will be satisfactory. To the writer it seems that for commercial engineering applications it will be far simpler and much more accurate to prepare for each fuel used a family of curves that show the ideal cycle efficiencies for the various ratios of compression and whatever percentages of excess air that may be involved. Such curves probably will give results that are more accurate than the method outlined in this paper, and after they are once calculated their use is far simpler than the proposed scheme. It is true that the preparation of a comprehensive set of

[1] Professor of Heat Power Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

curves for general use involves a large amount of labor, but when it is once done correctly, the results are available for indefinite use without further delay.

As evidence of how conveniently the ideal cycle efficiencies may be plotted, curves *A, B, C,* and *D* in Fig. 24 are shown for the Otto cycle. These curves show the efficiencies obtained by the long method of calculation with the variable specific heats of Goodenough and Felbeck, whose values are probably the most reliable of any now available.[1] The compression ratio is one of the vital factors in these efficiencies and it is therefore plotted as

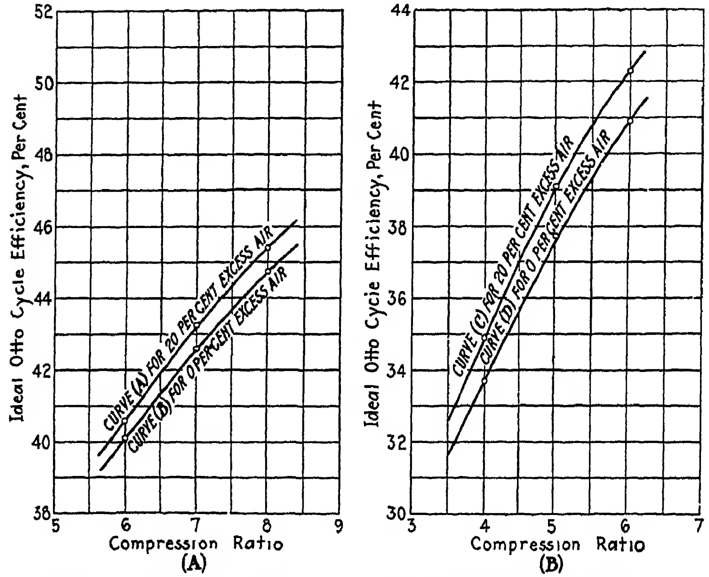


FIG. 24 CURVES OF IDEAL CYCLE EFFICIENCIES FOR OTTO CYCLE
A, results with blast-furnace gas
B, results with gasoline

one coördinate and may be extended over whatever range is needed. The other factor is the mixture ratio or the percentage of excess air, because it is this ratio that determines the specific heat of the mixture for any specified temperature. In Fig. 24 the curves have been drawn for zero and 20 per cent excess air but the range can be extended as needed. For the Diesel cycle the percentage of excess air needs to cover a much wider range than for the Otto cycle, because at light loads the Diesel engine uses a very high ratio of air to fuel.

From Table 10 and Par. 85 of the revised paper, it appears that the ideal cycle efficiency of the Otto and Diesel engines using

[1] Bulletin 139, University of Illinois, 1924.

different fuels with a small amount of excess air may be calculated by the approximate method of the author to within about 2 per cent of the correct result. For the Diesel under light loads the air-fuel ratio will be widely different from the cases worked out, and the approximate method may, therefore, be more in error than is shown in Par. 85. The error involved may not be objectionable in many applications, and if appreciable time is saved by using this approximate method, it becomes justified in such cases. However, it seems to the writer that this approximate method itself will probably be found to involve too much labor to make it generally applicable. Approximate methods are usually successful chiefly because of their extreme simplicity and convenience.

H. A. Everett.[1] The need for a standard is obvious to all who have done any work in fundamentals. The only standard for comparing internal-combustion engine performance that we have to date that has had any wide acceptance has been the old "Air Standard" proposed in 1905 by a committee of the Institution of Civil Engineers of Great Britain. At that time they encountered the same difficulties that we are now meeting. These were overcome by making five sweeping assumptions: First, the material should have instantaneous action (for example, the combustion should be complete and instantaneous); second, there should be no transfer of heat during the expansion and compression; third, for any given pressure and temperature there was no change in the specific volume of the fluid; fourth, there was no change in the chemical constitution of the fluid; and, fifth, there was no change in specific heat.

For any prospective criterion the first and second assumptions can still be retained, and should be, because they are optimum conditions. The desirability of discarding the third and fourth assumptions, involving, respectively, no change in specific volume and no change of chemical characteristics, is still a debatable point. The author considers it important to take cognizance of both of these effects, and does so, but at the expense of considerable increase in the complexity of the solution. Personally the writer prefers to retain these simplifying assumptions, even though it involves a slight departure from the ideal cycle using actual gas characteristics, because of the enormous simplification of computation. The fifth assumption of the old Air Standard, namely that relating to the variation of specific heats, is without any question the most erroneous of all, and cannot be dropped from consideration.

[1] Professor of Thermodynamics, Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

The suggestion that the author makes of using an average gas as typical, or sufficiently typical, to serve as a standard for all conditions, is open to possible pitfalls in conditions arising that may depart, perhaps radically, from the average that he suggests.

It is true that there is an appreciable divergence between the ideal efficiency, considering the working substance as air and as an actual mixture as suggested by the author, but does not the added simplicity resulting from assuming air as the working fluid, provided its variation in specific heat be taken into account, make it a standard preferable to that proposed? It involves no selective judgment and yet gives an ideal efficiency not much divergent from that obtained by considering the actual mixture or the hypothetical mixture proposed by the author.

To illustrate the magnitude of the differences when the ideal efficiency is figured by these different standards the following example has been computed. For a five to one compression ratio the efficiency according to the old Air Standard is 47.5 per cent (this with constant specific heat); the efficiency according to the air standard with variable specific heat is 42.6 per cent; and the efficiency taking the actual mixture in detail is 37.1 per cent. For the details of this comparison see paper[1] by the writer entitled A Temperature-Entropy Diagram for Air and the Diatomic Gases O_2, N_2, and CO.

The Author. As to accuracy, according to the set of representative examples in Tables 9 and 10, the ideal efficiency of the Otto cycle, with best conditions of working (as to air proportions), ranges from 0.30 to 0.45. The chart, carefully handled, will determine such an efficiency within the range from +0.005 to −0.015 with respect to the correctly computed value. This will be true even if no correction is made for the change in specific volume through combustion; and unless the gas fuel is unusually high in H_2 and CO that correction may very well be omitted. Computation of the chart determination, using Table 6 as shown in Par. 61, will considerably shorten the range of error, by eliminating graphical inaccuracies, and is not at all a complex operation.

As to complexity, the method of the computed table, as here proposed, is nothing else than the easiest way of applying and satisfying general equations [17] to [19]. What Fig. 14 really does is to give a physical meaning, in terms of entropy quantities, to the two members of either Equation [17] or [18]. The author has done quite enough work by the direct method — which involves simplification and trial solution — to realize very fully how much easier the new method is.

[1] *Mechanical Engineering,* vol. 48, no. 112, Nov. (Part 2) 1926, p. 1329.

Wishing to construct such curves as are shown by Professor Ellenwood, the author would proceed as follows, after making the preliminary calculations on the gas mixture:

By means of a chart determination get the approximate temperature height at each of points *B*, *C*, and *D* — unless such knowledge was already established by previous calculations

With the help of Table 20 compute a few sets of values in the region of each point

Compute by the form in Par. 61 (Otto cycle) or that in Pars. 81 to 84 (Diesel cycle).

No. 2029

PROPERTIES OF BOILER TUBING AT ELEVATED TEMPERATURES DETERMINED BY EXPANSION TESTS

By A. E. White,[1] Ann Arbor, Mich.
Member of the Society

and

C. L. Clark,[2] Ann Arbor, Mich.
Non-Member

The object of this investigation was to determine safe working loads for low-carbon-steel seamless tubing at elevated temperatures. Present-day boiler practice employs temperatures varying from 550 to 750 deg. fahr., and pressures of around 350 lb. per sq. in. There is an increasing tendency, however, to increase both temperature and pressure, but little is known of the properties of metals at elevated temperatures, particularly when temperatures are maintained for long periods of time.

The investigation recorded in this paper sets forth the preliminary findings on 0.13 carbon-steel tubing when loaded at temperatures of 900, 1000, 1250 and 1500 deg. fahr.

Introduction

PRESENT tendencies are daily calling for metal with high stability at elevated temperatures. Information in this field is most limited. The engineering profession has a vast amount of data with respect to the properties of metals at atmospheric temperatures, but little, if any, upon the properties of metals at elevated temperatures. Such work as has been done in the range of elevated temperatures has been in the nature of short-time tests. Even in this test field only a few of the possible metals and alloys have been studied.

[1] Professor of Metallurgical Engineering and Director of Department of Engineering Research at the University of Michigan.

[2] Holder of The Detroit Edison Fellowship in Metallurgy at the University of Michigan.

Contributed by the Power Division and presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

2 Efforts have been made to develop short-time tests which would give an indication of the stability of metal under prolonged heating. A number of investigators have arrived at the conclusion that metal is stable at all loads which are under the proportional limit for the temperature used, irrespective of the length of time at which the metal is held at the given temperature under the given load. The findings of the authors do not agree with this observation; at least in so far as temperatures of 1250 and 1500 deg. fahr. are concerned. They are inclined to believe that the observation is not correct for lower temperatures, although they have not sufficient data at this moment to support their contention. They are working on this phase of the problem, however, and hope to have further data within a year.

3 Possibly, however, the differences of opinion with respect to the use of the proportional limit between those who claim it as a criterion of stability and the authors is only a matter of accurate determination; that is, if a longer time were taken to determine the proportional limit, it is possible that the generalization as to its being a criterion of stability might be correct. The authors are inclined to doubt this possibility, as their proportional limit values are in general agreement with the findings of other investigators and they fail to discover any relationship between stability and proportional-limit value.

4 The paper is given with due appreciation of the fact that the data are incomplete, but because of the importance of the subject the authors feel warranted in presenting it at this time. Future studies contemplate analyzing the problem more fully, covering further study of plain carbon and alloy steels, as well as other alloys at various high temperatures. Attention is specifically called to the fact that, in the opinion of the authors, no design involving the use of materials at elevated temperatures should be based on short-time, proportional-limit values.

5 The data in this paper have been confined to 0.13 per cent carbon steel. The determinations were made on tubing. The results show that at 1500 deg. fahr. a load of 595 lb. per sq. in. would ultimately result in failure, although the short-time test showed the metal to have a proportional limit of 1880 lb. per sq. in. Failure would be expected at a load under 595 lb. per sq. in., although no data are available on this point, except that a tube with a load of 106 lb. per sq. in. did not expand after 540 hours at heat, although it was so badly oxidized that with continued heating it would ultimately have failed from this cause.

6 The short-time, high-temperature tests at 1250 deg. fahr. gave for the 0.13 carbon steel a proportional limit of 3050 lb. per sq. in. The long-time tests indicated that loads under 213 lb. per sq. in. could not successfully be applied over any long period of time, in view of the fact that a tube subjected to 213 lb. per sq. in. manifested marked expansion at the end of 482¾ hours.

7 The data for 1000 and 900 deg. fahr. are not complete, and no conclusions can be drawn. The information is presented because it may be of some service at this time to designing engineers.

Apparatus Used

8 The apparatus used in this work is simple in principle, but many difficulties, due to the temperatures and pressures employed, were encountered before it could be made to operate successfully.

9 The set-up in each case consisted of a pressure accumulator, a valve arrangement, a test specimen in the shape of a seamless

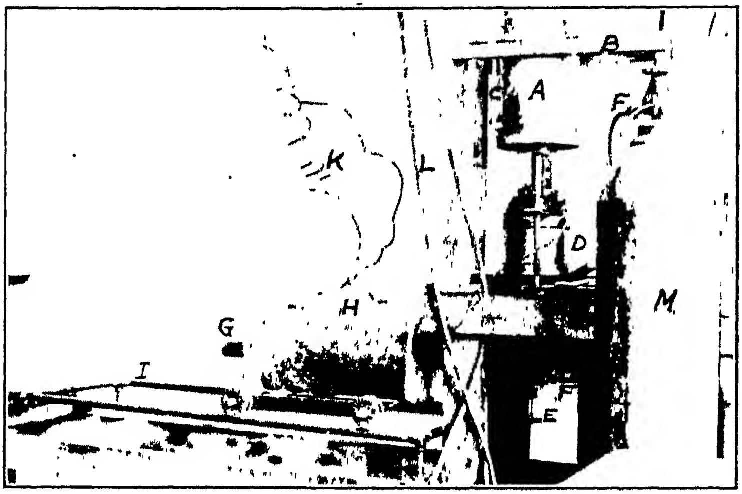


Fig. 1 General Arrangement of Expansion Test Apparatus

A—Water tank; B—Water tank support and guide; C—Water tank gage; D Pressure accumulator; E—Valve stem; F—Nitrogen tank leads; G—End of test specimen; H—Electric furnace; I—Furnace tracks; K—Electric wires to furnace; L—Water-cooling jacket tubes; M—Nitrogen tank.

steel tube 30 in. long, and a movable furnace mounted on a track. The relative positions of these parts are shown in Fig. 1.

10 A sketch of the pressure accumulator is given in Fig. 2. An accumulator consisted essentially of two pistons, one working against oil and the other against nitrogen gas. The small pistons exposed to the oil and to the nitrogen had a diameter of 1 in. in each case, while the size of the large pistons working against the oil varied in the different accumulators used, one being 6 in., one 4 in., and one 2.5 in. in diameter. Therefore, any load applied to these accumulators was multiplied 36, 16, and 6.25 times, respectively.

11 Leather gaskets were used on the surfaces of the pistons exposed to the oil and these worked very well, only a small amount

of oil leaking through. Much trouble, however, was experienced in making the pistons working against nitrogen leak proof. A leather gasket was quickly hardened by the hot gas and then ceased to function. The trouble was finally overcome by the use of a glycerine trap, as shown in Fig. 2. The main function of the glycerine was to keep the leather gaskets soft and permit them to

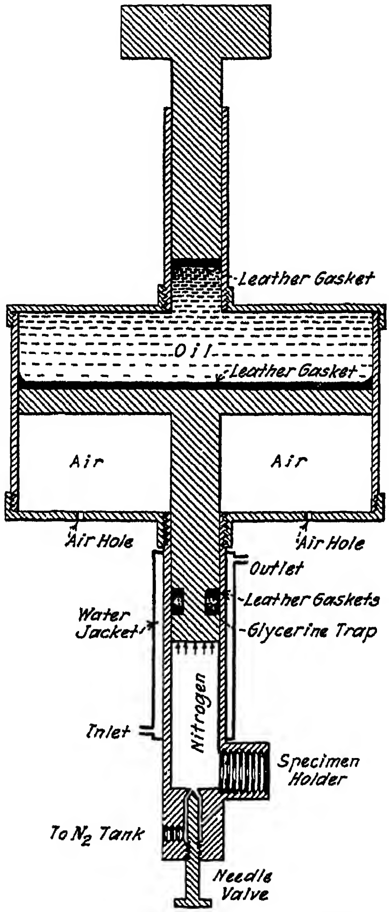


Fig. 2 Pressure Accumulator

function properly. Glycerine was used mainly because it is not as likely to cause corrosion or deposit a residue as some of the other substances that might have been employed. The water jacket was added so as to reduce thermal expansion to a minimum, as well as to prevent the glycerine from vaporizing.

12 After trying various valve arrangements, it was found that the needle valve gave the most satisfactory results. The tip of the

stem was hardened steel, while the seat was ordinary steel, a combination which permitted the stem to be forced into the seat. This type of valve has given no trouble whatsoever.

13 Fig. 3 shows the specimens used in both the tensile and pressure tests. The only trouble found in the use of this type of pressure specimen was in making the cap leak-proof. Various leak-proof mixtures, such as white lead, litharge and glycerine were applied, but all proved useless at the temperatures and pressures used. Cast iron caps were welded on, but they became porous during the welding process. The difficulty was finally solved by brazing on steel caps.

14 In using the tensile-test specimen, steel plugs were inserted in those parts of the specimen held by the jaws, so as to prevent crushing. A Riehle tensile machine of 50,000 lb. capacity was used in all the tensile tests.

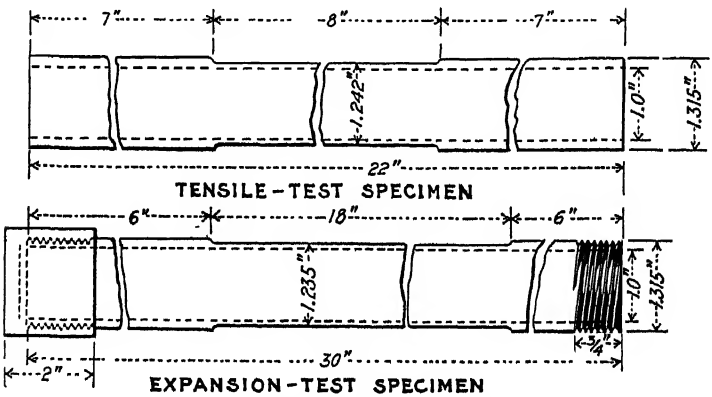


Fig. 3 Types of Specimens Used

15 The type of furnace used in both the tensile and rupture tests is shown in Fig. 4. The furnaces differed in size only. Each consisted of an iron core with an outer steel shell. The heating element consisted of two coils of chromel A-14-gage wire connected in parallel. The coils were separated from each other and from the iron core by thin layers of alundum cement. Sil-o-cel was used as the insulating material.

16 For low-pressure rupture tests, the specimen was first brought to the required temperature and nitrogen introduced until the desired pressure was obtained. The pressure was measured by means of a graduated tank placed on the upper piston of the accumulator. Water was used for balancing the pressure, as this allowed more accurate balancing than could be obtained with solid weights. For the higher pressures, lead weights also were added,

but the final adjustment of load was obtained with water. By keeping the piston balanced, the pressure was accurately measured at all times.

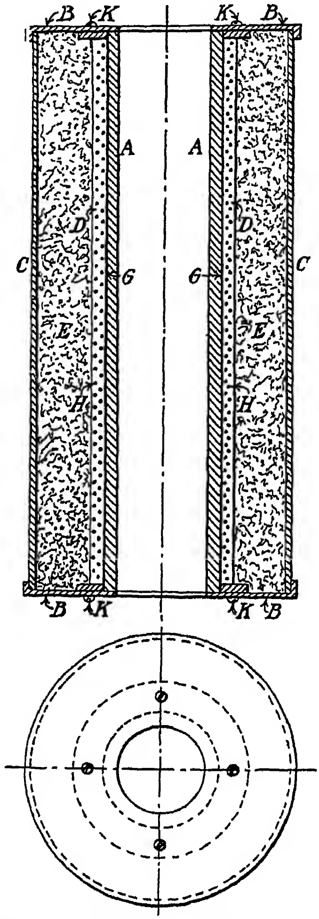


Fig. 4 Type of Furnace Used

A—Inner iron tube; B—Sheet-steel end plates, $\frac{3}{16}$ in.; C—Sheet-steel outer tube, $\frac{3}{16}$ in.; D—Alundum cement; E—Sil-o-Cel; G—Asbestos covering; H—Chromel A-14-gage wire, resistance, 0.1625 ohm per ft.; K—End-plate screws.

17 In order to convert the load from lb. per sq. in. of interior surface to tensile stress in the tube walls, the following relations were used:

Tension force in tube

$$= \frac{(\text{lb. per sq. in. of interior surface}) \times (\text{diam. in in.})}{2}$$

Tension force in lb. per sq. in.

$$= \frac{\text{tension force in tube}}{(\text{thickness of tube in in.}) \times (\text{unit length in in.})}$$

18 After the specimen had attained the right temperature and pressure, the temperature was held constant and the diameter of

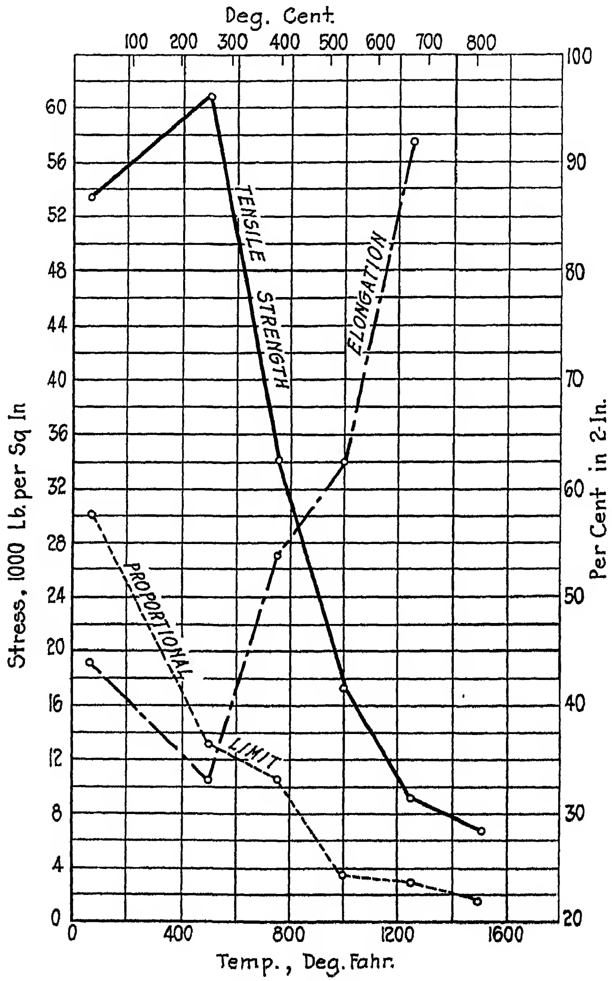


FIG. 5 PROPERTIES AT ELEVATED TEMPERATURES OF 0.13 CARBON STEEL. SHORT-TIME TENSILE TESTS

the tube measured at regular intervals. This was accomplished by running the furnace away from the heating position and using calipers on the tube. In case the diameter was enlarged, the run was considered complete. Otherwise, the run was continued until

enlargement or actual rupture occurred, or, for various reasons, the test was discontinued.

Results of Tests

19 *Chemical Analyses.* Upon chemical analysis, the steel tubing was found to have the following composition:

Carbon	0.13 per cent
Manganese	0.29 per cent
Sulphur	0.044 per cent
Phosphorous	0.010 per cent

20 The microstructure of the tubing in the condition in which it was received is shown in the photomicrographs of Fig. 6, and the structure is seen to be that of a typical low-carbon pearlitic steel. The crystals are fairly uniform in size and shape and equi-axed, indicating the metal to be in a fully annealed condition.

21 *Short-Time Tensile Tests.* The tubing was subjected to tensile tests at temperatures of 70, 500, 750, 1000, 1250, and 1500 deg. fahr. The shape of the specimen used was as shown in Fig. 3. The results obtained are given in numerical form in Table 1 and in graphical form in Fig. 5.

22 From the curves, it can be seen that the tensile strength increases up to 500 deg. fahr. and then decreases rapidly with further increases of temperature. The proportional limit decreases

TABLE 1 SHORT-TIME TENSILE TESTS AT ELEVATED TEMPERATURES

Temp., deg. fahr.	Proportional limit, lb. per sq. in.	Tensile strength, lb. per sq. in.	Elongation, per cent in 2 in.
70	30,500	53,520	43.75
500	13,612	62,215	33.50
750	10,800	34,030	54.00
1000	3,750	17,370	62.50
1250	3,050	8,860	92.00
1500	1,880	7,000	*

Note. Test specimens used as shown in Fig. 3.
* Elongation not determined, as all gage marks were destroyed at this temperature.

continuously as the temperature is raised. The elongation decreases to 500 deg. fahr. and then increases rapidly. The reduction of area could not be determined because of the shape of the specimen used.

23 These tests are in close agreement with the work of other investigators when determining the properties of low-carbon steel by short-time tests at elevated temperatures. The metallographical structures for each of the samples after test are given in magnifications of 100 and 500 in the photomicrographs of Figs. 6 to 12, inclusive. Although pains were taken to examine the specimens at the point of rupture, yet the sections from the specimens broken at elevated temperatures are little, if any, different from the sections obtained from the sample tested at atmospheric temperature.

24 *Expansion Tests.* The expansion tests record the results of the authors' findings at 1500, 1250, 1000 and 900 deg. fahr. The program calls for other tests at these temperatures, but with other

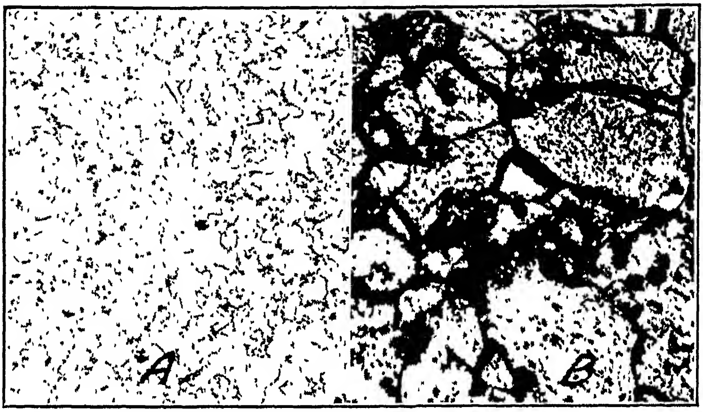


FIG. 6 TUBING AS RECEIVED, ETCHED WITH 4 PER CENT HNO_3 IN ALCOHOL
A—Magnified 100 times; B—Magnified 500 times.

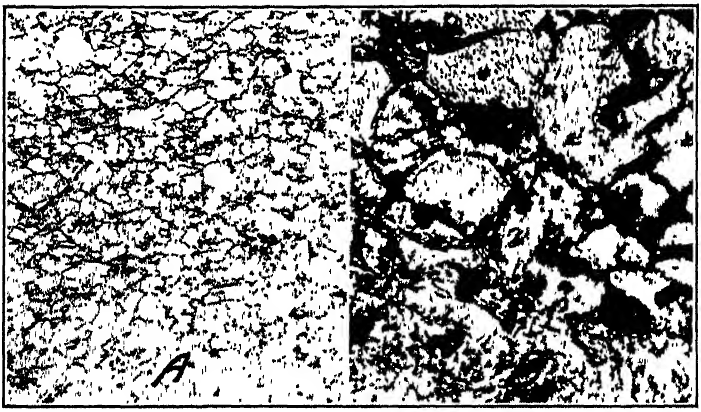


FIG. 7 SHORT-TIME TENSILE TEST, 70 DEG. FAHR. (21 DEG. CENT.). ETCHED WITH 4 PER CENT HNO_3 IN ALCOHOL
A—Magnified 100 times; B—Magnified 500 times.

loads than those employed; for tests at other temperatures, and for tests on other materials. The work, therefore, has really just started. The data obtained to date, however, are so vital and of such moment at the present time that they are presented in spite of the fact that they are most preliminary in nature.

25 *Expansion Tests at 1500 Deg. Fahr.* Six tests at 1500 deg. fahr. are submitted. The data are given in Table 2 and in Fig. 13. Two of the tests were run at loads below the proportional limit determined in the short-time tests. Both of these show decided expansion at loads under the proportional limit. The test employ-

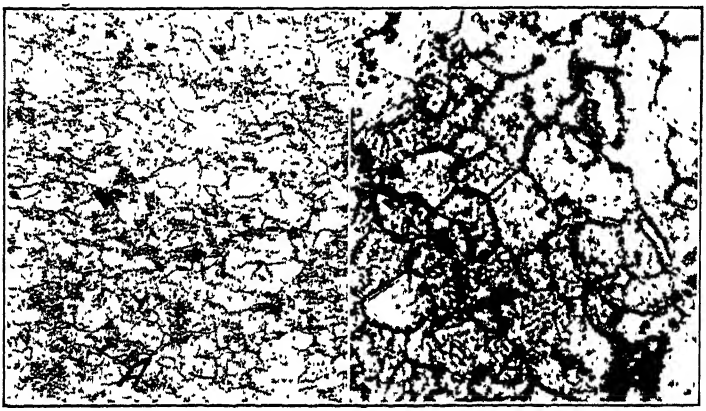


FIG. 8 SHORT-TIME TENSILE TEST, 500 DEG. FAHR. (260 DEG. CENT.). ETCHED WITH 4 PER CENT HNO_3 IN ALCOHOL
A—Magnified 100 times; B—Magnified 500 times.

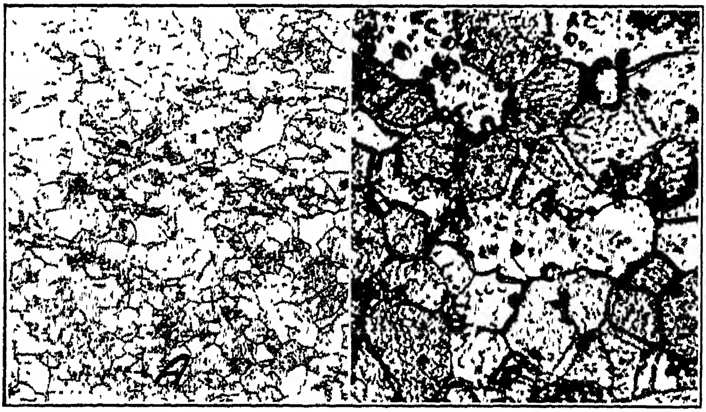


FIG. 9 SHORT-TIME TENSILE TEST, 750 DEG. FAHR. (399 DEG. CENT.). ETCHED WITH 4 PER CENT HNO_3 IN ALCOHOL
A—Magnified 100 times; B—Magnified 500 times.

ing a tensile load of 106 lb. per sq. in. showed no expansion after 540 hours of heating, but in time the tube would have failed from oxidation.

26 *Expansion Tests at 1250 Deg. Fahr.* Considerable work was done at 1250 deg. fahr., eleven tests in all being run. Although

in the short-time, high-temperature test the proportional limit was determined as 3050 lb. per sq. in., yet the expansion tests showed decided enlargement of the tubes with loads as low as 213 lb. per sq. in. Seven of the tests, with loads ranging from

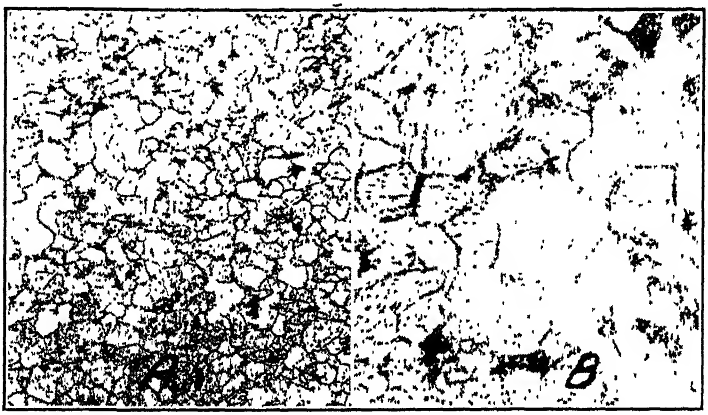


Fig. 10 Short-Time Tensile Test, 1000 Deg. Fahr. (538 Deg. Cent.). Etched with 4 Per Cent HNO_3 in Alcohol
A—Magnified 100 times; B—Magnified 500 times.

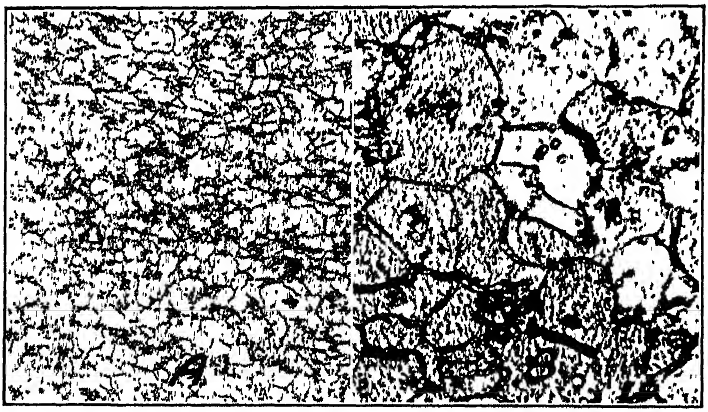


Fig. 11 Short-Time Tensile Test, 1250 Deg. Fahr. (675 Deg. Cent.). Etched with 4 Per Cent HNO_3 in Alcohol
A—Magnified 100 times; B—Magnified 500 times.

2834 down to 213 lb. per sq. in., were under that of the short-time, proportional-limit value for 1250 deg. fahr., yet all showed expansion and all of the tubes would undoubtedly have failed had the test been of longer duration.

27 The data are given in Table 3 and in Fig. 14. Attempts were made to convert these data into a mathematical expression.

TABLE 2 RESULTS OF EXPANSION TESTS AT 1500 DEG. FAHR. (815 DEG. CENT.)

Specimen No.	Lb. per sq. in., interior surface	Tensile stress, lb. per sq. in.	Diam. of tube, in. Initial	Final	Time for expansion, hr.
9	665	2834	1.235	1.400	0.75
7	634	2696	1.235	1 314	1.50
17	466	1982	1.235	1.295	2.00
13	300	1276	1.235	1.261	3.00
14	140	595	1.235	1 268	10.00
26	25	106	1 235	.....	*

* Not expanded after 540 hr., very badly oxidized.

TABLE 3 RESULTS OF EXPANSION TESTS AT 1250 DEG. FAHR. (677 DEG. CENT.)

Specimen No.	Lb. per sq. in., interior surface	Tensile stress, lb. per sq. in.	Diam. of tube, in. Initial	Final	Time for expansion, hr.
16	1030	4383	1.235	1.250	2.50
15	750	3192	1.235	1.250	4.00
10	665	2834	1.235	1.300	5.00
12	614	2611	1.235	1.368	18.00
19	600	2582	1.235	1.287	19.25
11	550	2350	1.235	.....	*35.00
20	477	2024	1.235	1.255	61.25
21	355	1510	1 235	1.261	139.00
22	250	1064	1.235	1.264	208 50
23	130	553	1.235	1.266	285.50
24	50	213	1 235	1.262	482.75

* Tube removed at end of 35 hr. unexpanded.

FIG. 12 SHORT-TIME TENSILE TEST, 1500 DEG. FAHR. (815 DEG. CENT.). ETCHED WITH 4 PER CENT HNO_3 IN ALCOHOL

A—Magnified 100 times; B—Magnified 500 times.

This phase of the work was done by S. Vesselowsky, of The Detroit Edison Company. One expression, derived directly from the data, is as follows:

$$S = 200 + 2800e^{-0.75t}$$

In which

S = stress in lb. per sq. in.
e = Naperian base
t = time in 100 hr.

Another was worked out on the basis of the data in accordance with a mathematical interpretation of a supposed molecular

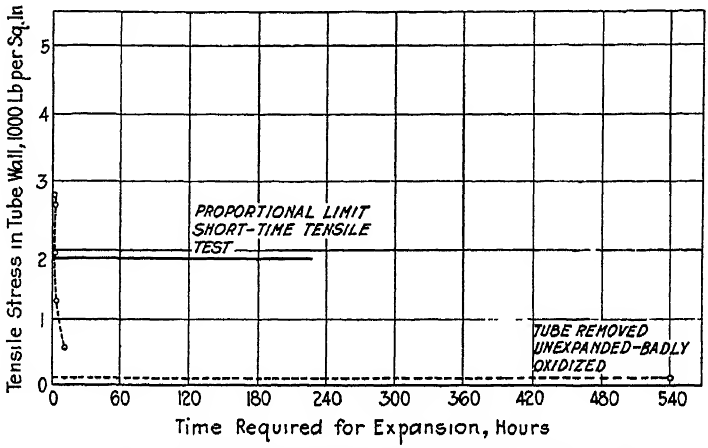


FIG. 13 EXPANSION TESTS ON 0.13 CARBON STEEL AT 1500 DEG. FAHR. (815 DEG. CENT.)

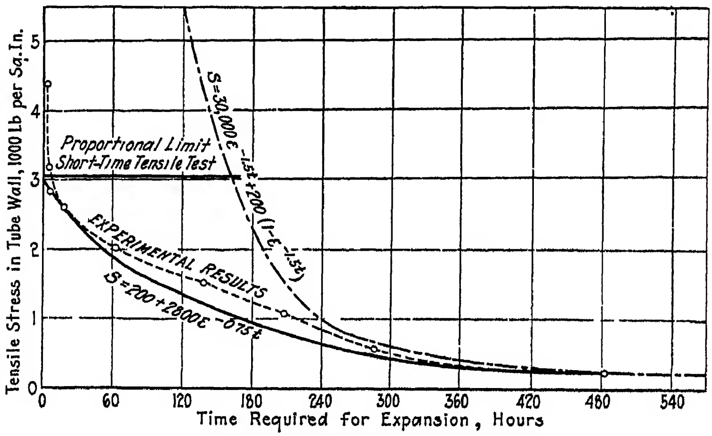


FIG. 14 EXPANSION TESTS ON 0.13 CARBON STEEL AT 1250 DEG. FAHR. (675 DEG. CENT.)

transformation going on in the material under the influence of high temperature. It follows:

$$S = 30{,}000e^{-1.5t} + 200(1 - e^{-1.5t})$$

28 The curve derived from a plotting of the data checks closely with the curve derived from the use of the first formula. The curve drawn from the equation setting forth the supposed

molecular transformation checks with the curve drawn from the data with heating exposures of 200 hours or greater.

29 *Expansion Tests at 1000 Deg. Fahr. and 900 Deg. Fahr.* Some preliminary tests were run at 1000 deg. fahr. and 900 deg.

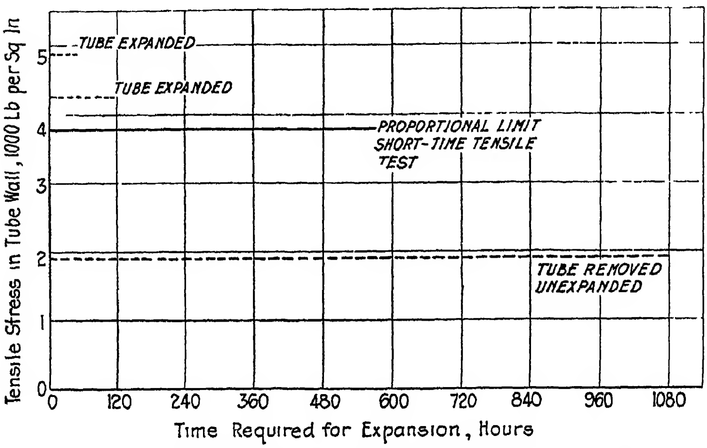


Fig. 15 Expansion Tests on 0.13 Carbon Steel at 1000 Deg. Fahr. (538 Deg. Cent.)

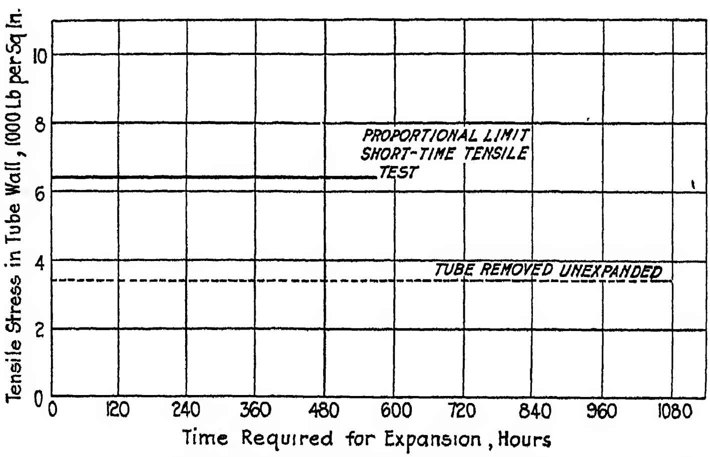


Fig. 16 Expansion Tests on 0.13 Carbon Steel at 900 Deg. Fahr. (482 Deg. Cent.)

fahr. The data are not complete and further tests at these temperatures are in progress. These findings appear to indicate that between 1000 deg. fahr. and 1250 deg. fahr. there is a sharp decrease in strength.

30 The data are given in Table 4 and in Figs. 15 and 16.

TABLE 4

Results of expansion tests at 1000 deg. fahr. (538 deg. cent.)

Specimen No	Lb per sq. in., interior surface	Tensile stress, lb. per sq. in	Diam of tube, in Initial	Final	Time for expansion, hr
18	1150	4893	1.235	1.260	43.25
27	450	1914	1.235	.....	*
28	1000	4250	1.235	1.260	100.00
	Results of expansion tests at 900 deg. fahr. (482 deg. cent.)				
29	800	3404	1.235	.....	*

* Not expanded after 1080 hours.

Conclusions

31 Possibly one of the most significant features of the test is the fact that, for temperatures of 1500 and 1250 deg. fahr., at least, *the proportional limit, as ordinarily determined, was found not to be the criterion of stability.* The authors are under the impression that it will not be the criterion for stability at other temperatures. These statements are made with due caution, appreciating the fact that more careful methods of obtaining proportional limits may make more justifiable the claims of those who hold that loads under the proportional limit for any given temperature may safely be used. Yet, the authors would point out that their proportional-limit findings check quite closely those of other investigators whose works are recognized as of a high degree of accuracy. Therefore, it is suggested for engineering design that, until more data are available, no regard be given to the present-day proportional-limit values for elevated temperatures, or if used, then only with an adequate factor of safety.

Acknowledgment

32 The authors desire to express their appreciation to The Detroit Edison Company for the coöperation of its staff and for the funds which made possible the study. Particularly do they desire to express their gratitude for the encouragement and ever-ready counsel given by C. F. Hirshfeld, Chief of Research, and J. W. Parker, Chief Engineer of the Company. Attention is likewise directed to the mathematical work of S. Vesselowsky.

DISCUSSION

H. J. French.[1] The reiteration of the fact that roughly determined proportional limits are not a criterion of load-carrying ability is important, but the authors have by no means demonstrated the absence of a relation between load-carrying ability and accurately determined proportional limits.

[1] Senior Metallurgist, Bureau of Standards, Washington, D. C.

Data given in Table 1 show that in the range of 1000 to 1250 deg. fahr., where the tensile strength of the 0.13 per cent carbon steel drops about 50 per cent, the drop in proportional limit as determined by the authors is only from 3750 to 3050 lb. per sq. in. This lack of appreciable change in numerical values of the proportional limit, as the temperature is raised from 1000 to 1250 deg. fahr., is so at variance with what can justly be expected from a large mass of available data on low carbon steels, that the reaction on reading this paper is that the authors' determinations of the proportional limits at temperatures of 1250 deg. fahr. and above must be highly inaccurate.

It is fairly well recognized that roughly determined proportional limits will differ more widely from values of load-carrying ability, as the temperature at which the tests are carried out is increased. Even the authors' proportional limit determinations at 1000 and 900 deg. fahr. and the incomplete pressure tests of the tubes recorded in Fig. 15 and 16 fall in line with the view which has been expressed by Lynch, McVetty and Mochel, Malcolm, the writer, and others, to the effect that "creep" occurs at stresses above the proportional limit, and does not occur at stresses well below, providing always that the proportional limit is determined with suitably sensitive equipment.

If the authors will furnish to the writer samples of the grade of steel which they have used in the experiments described in this paper in such a form that accurate determinations of the proportional limits can be made with the Tuckerman-Martens' extensometer at the Bureau (it is well known that accurate proportional limits are exceedingly difficult to obtain on specimens in the form of thin wall tubes), he will be glad to make such determinations at 1250 deg. fahr. and possibly also at some higher temperature close to 1500 deg. fahr.

Being unable to accept all of the proportional limit values given by the authors in this paper, the writer also is unable to accept the implied conclusions to the effect that the authors are under the impression that the proportional limit is not a criterion of stability at any temperature between that of the room and 1500 deg. fahr.

E. C. Wright.[1] The type of test outlined by the authors appears to be particularly valuable because it approximates service conditions and it is hoped that the further tests will give more complete data.

In considering the experimental results submitted, it is evident that there are several features which are questionable and that the authors have reached conclusions which are not warranted by experimental data. The main deduction which the authors

[1] Metallurgical Engineer, National Tube Co., Ellwood City, Pa.

make is that the proportional limit is not a safe maximum stress at which metal can be used at elevated temperatures, and while the writer does not feel that this proposition is sufficiently established, it is not apparent that the authors' data refute this idea. Proportional limit determinations require the most careful experimental procedure, and the gaging of the test piece cross-section to enable unit stress calculations must be very precise. While the authors do not specify whether the tubes used were of the welded or seamless (weldless) variety, it is assumed that the latter is the more probable. As the proportional limit determinations are made on full sections it is certain that no accurate unit stress calculations can be made on such test sections on account of the eccentricity of the metal wall in such products. The original tubing used in these tests has the dimensions of what is called extra heavy pipe, the average commercial dimensions being 1.315 in. O.D. by 0.179 in. wall thickness. In this size, the seamless tubes are made cold drawn and then given a 700 deg. cent. anneal. The commercial tolerance on this wall thickness, however, is plus or minus 10 per cent, and tubes made to this average size by the weldless mill will thus vary from 0.161 to 0.197 in. in wall thickness. Turning these tubes down to an outside diameter of 1.242 in. for the gage length of the test specimen would merely accentuate the variation in eccentricity if the tube were chucked to the outside diameter in the lathe, and would not greatly remedy this variation if the tube were chucked to the inside diameter. It seems hardly possible that the authors have overlooked this condition in preparing their test specimen, but as no mention is made of this very necessary correction in the paper, it is necessary that this point be raised. Certainly no accurate proportional limit determinations can be made on sections which are not exactly concentric.

The foregoing statements apply likewise to the calculations of the fiber stress in the metal wall from the gas pressures of the various tests. If the tubes were exactly concentric, at a pressure of 100 lb. per sq. in. the fiber stress as calculated by the formula used by the authors would be as follows:

$$\text{Tensile stress} = \frac{\text{Pressure} \times \text{I.D.}}{2 \times \text{wall thickness}} = \frac{100 \times 1}{2 \times 0.235/2}$$
$$= 425 \text{ lb. per sq. in.}$$

If the original tube had the maximum allowable eccentricity of 10 per cent the wall thickness would vary between

$$\left(\frac{0.315}{2}\right) - \left(\frac{1}{10} \times \frac{0.315}{2}\right) \quad \text{and} \quad \left(\frac{0.315}{2}\right) + \left(\frac{1}{10} \times \frac{0.315}{2}\right),$$

that is, from 0.1395-in. wall to 0.1705-in. wall. Turning the tube outside diameter to 1.235 in. merely takes 0.04 in. off of the wall thickness and leaves the minimum wall at 0.0995 in. and the maximum at 0.1305 in. Thus, at the pressure of 100 lb. per sq. in.

the fiber stress at the light portion of the wall would be 502 lb. per sq. in., a variation of 17.6 per cent between the maximum and average fiber stress.

Moreover, the formula used for transposing the static pressures into fiber stresses is known to be inaccurate in tubing of a high ratio of wall thickness to outside diameter. In short tube lengths, such as were used, there is also a longitudinal tension in the tube, due to the end pressures. Two other formulas which are used commonly for heavy wall tubing are those of Barlow, where

$$\text{Fiber stress} = \frac{\text{Pressure} \times \text{O.D.}}{2 \times \text{wall thickness}} \qquad \text{or } F = \frac{P \times D}{2t},$$

and Clavarino, where

$$f = p\,\frac{(13D_1^2 + 4D_2^2)}{10(D_1^2 - D_2^2)}$$

in which

$$D_1 = \text{outside diameter}$$
$$D_2 = \text{inside diameter}$$

Barlow's formula usually gives fiber stresses which are slightly high, while Clavarino's formula gives results which are more nearly theoretically correct.

Referring to Table 3, at a pressure of 1030 lb. the stress calculated is 4383 lb. per sq. in., while at this pressure Barlow's formula yields 5420 lb. per sq. in., and Clavarino's formula gives 4675 lb. per sq. in. Here is a variation of 23.6 per cent, using Barlow's formula, and 6.65 per cent, using Clavarino's formula As the latter formula includes consideration of the elasticity of the material and the effect of end pressures, it is probably more nearly correct than the other two formulas, but actual bursting tests indicate that the calculated stresses are slightly low. Apparently the authors' stress calculations are about 8 to 10 per cent too low. These figures, of course, are based on an average or uniform wall thickness throughout the tube.

Another feature of the data which is lacking is the apparent oversight of the authors in neglecting to consider the normal thermal expansion of the tube. Referring again to Table 3, it will be noted that tubes are considered as having expanded due to the stress when the outside diameter at 677 deg. cent. is 1.250, 1.255, 1.261, 1.262, etc. At this temperature the room temperature outside diameter of 1.235 would expand to 1.247, and the method described for gaging would hardly detect variations of 0.003 to 0.008 in. Furthermore, the scaled surface of these tubes, especially in the long extended tests, would render the detection of such slight gage variations highly questionable.

This type of testing is similar to the work which Dickenson[1] has carried out in England, although his test stresses were over

[1] Dickenson, J. H. S., Jour. Iron & Steel Inst., vol. 106, p. 103.

the proportional limit at the temperature of the test. On the other hand, there are several boilers operating at 300 and 400 lb. pressure with tube sizes of 4-in. O.D. and 0.185-in. wall in which the fiber stresses vary between 3000 and 4300 lb. per sq. in. in the tubes, and reach 3700 to 4000 lb. per sq. in. in the 2-in. O.D. by 11-gage superheated tubes. Although the operating temperatures are below those of the authors' tests, the temperature of these tubes may exceed 400 deg. cent. when somewhat scaled on the inside, and these tubes are quite generally giving satisfactory service at much longer periods than those indicated in this paper. In this respect these experimental data are actually contradictory to established engineering facts.

No. 2030

THE EMERGENCY STOPS OF THE GEARLESS TRACTION ELEVATOR AT THE TERMINAL LANDINGS

PERFORMANCE AND LIMITATION OF THE EMERGENCY TERMINAL STOPPING DEVICES; CALCULATION OF THE REQUIRED OVERHEAD CLEARANCES

By F. Hymans,[1] New York
Member of the Society

The present paper is confined to an investigation of the oil buffer and the limit switch.

Part I is a short description of these and allied devices.

Part II gives of a theory of the oil buffer, developed by the author, that part which bears on the subject matter of this paper, namely the complete calculation of the properties of a given buffer if struck by a free weight at a given speed. It is shown that this theory applies when a buffer is suitable for service with an elevator. On the other hand it is a fact, not as yet fully recognized, that a buffer satisfactory when engaged by a free weight may be wholly objectionable if the same weight instead of being free is the car or counterweight of an elevator.

In Part III a very useful and simple concept is introduced under the name of "the equivalent system," by means of which the problems in Part IV are quickly disposed of with no greater difficulties than the application of first principles.

Part IV is divided into four sections.

The first is an investigation of the stop of an elevator when a limit switch is opened. It is found that the limit switch alone is inadequate as an emergency terminal stopping device unless the top and bottom clearances are exorbitant. The second is an investigation of the stop of an elevator when brought about by the action of the buffer alone. It is shown that an intimate relation exists between the buffer and the particular elevator it is to serve and two requirements are deduced which a suitable buffer must satisfy. One of these relates to the force it must exert on the member engaging it and the other to the stroke. It is further shown that a well-designed buffer is always

[1] Mechanical and Electrical Engineer, Otis Elevator Company.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

an excellent terminal stopping device for the descending member of the elevator but not, in general, for the ascending member, unless the buffer stroke and the top clearance are considerable. This part also includes a discussion of the present buffer testing in the field and in particular of what is involved if such tests are conduced at governor tripping speed—as some codes now specify—with buffers of the usual stroke.

Having to abandon the accepted notion that in the oil buffer and in the limit switch there are two independent emergency terminal stopping devices, each capable of bringing the elevator to a safe stop within the usual clearances, there is next investigated the case when both of these devices become simultaneously effective. The results are gratifying although they plainly indicate that the coöperation of both devices is necessary. They further show that the location of the limit switches is a matter of major importance.

The third section investigates the stop of the elevator with buffer and limit switch coöperating in the usual sequence of practice.

One of the outstanding results of Part IV is the proof that the required amount of top clearance can be readily calculated for any given emergency. Vice-versa, it is shown that a given top clearance provides for certain definite emergencies only. The author shows how to calculate the required top clearance and, upon a consideration of the accumulated experience, specifies the emergency upon which the calculation is based. It is shown in Part IV that the selection of the buffer stroke not only depends on the speed but also on the particular details of the elevator and location of the limit switches. The author further discusses which of a number of buffers of the same stroke but with different graduations is suitable for service with a given elevator.

Since the emergency stops at high speed are generally associated with slack hoist rope, Part V develops the formulas for determining the stresses upon the subsequent fall of the ascending member to pick up the slack. In it will be found the reason why present methods of field testing are so frequently accompanied by a failure of the shaft of the machine and often make a re-alignment necessary.

PART I

THE gearless traction machine for elevators with speeds of 450 ft. per min. or greater, an early form of which is shown in Fig. 1, was introduced in 1904. It comprised a low-speed 6- or 8-pole motor coupled directly to a grooved driving sheave and a brake pulley. Having very little field regulation, the control of the motor was by resistances in series and parallel with the armature. This arrangement had the disadvantage that the low speeds so obtained varied greatly with load as well as the direction of motion and made an accurate stop difficult. Although resistance control has not been abandoned altogether, the past few years have

seen an extensive application of the well-known variable-voltage system of control. This system is by no means new in the elevator art, having been employed as early as 1893, but because of it and the micro-drive, recent improvements in service and increase in speed have become possible.

2 Fig. 2 shows the general arrangement of a gearless traction elevator with micro-drive. This is a combination of a machine of

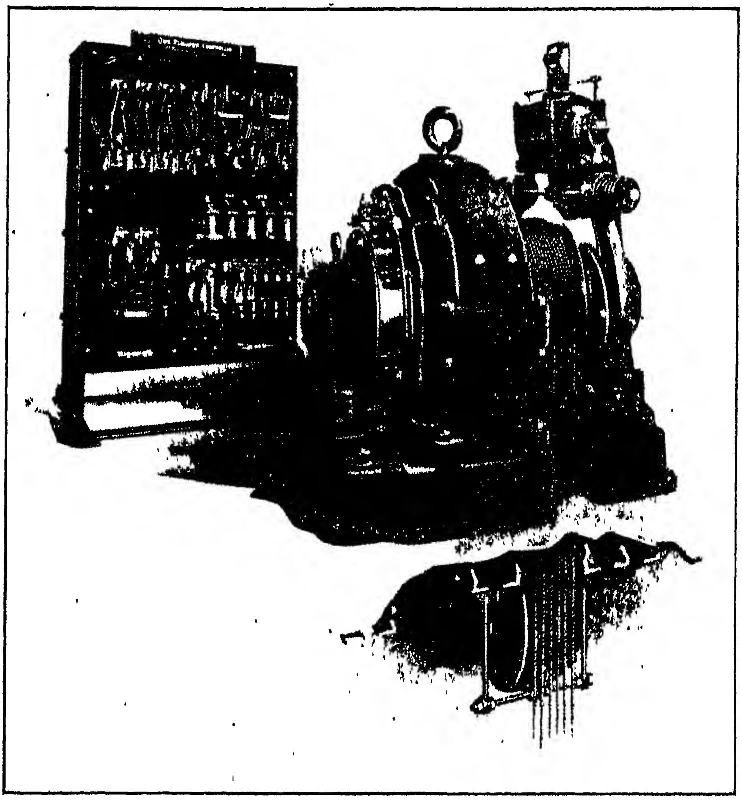


Fig. 1 Early Type of Machine with Resistance Controller

the type of Fig. 1 with an auxiliary worm-gear machine to which the load is transferred at low speed shortly before the stop. The latter brings the car automatically to a stop level with the landing and maintains it there during loading and unloading.

3 The roping will be best understood from a consideration of Fig. 3 (*a* and *b*). The driving and secondary sheaves are provided with concentric grooves whose profile is the arc of a circle of a radius slightly larger than the rope. The ropes are four to six in number and their sizes $\frac{1}{2}$ in., $\frac{5}{8}$ in., or $\frac{3}{4}$ in. As Figs. 2 and 3 show,

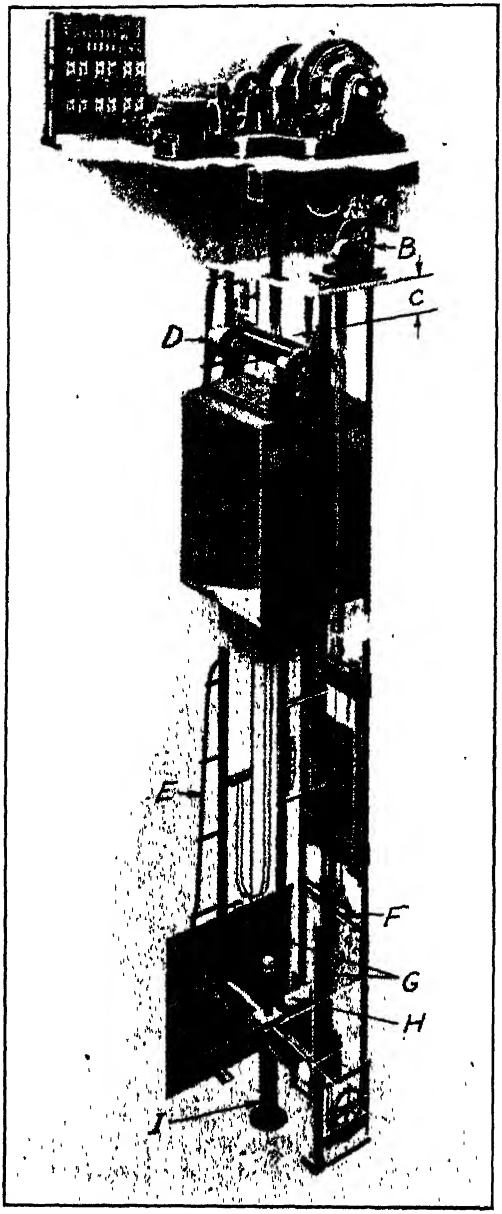


Fig. 2 General Arrangement of Gearless Traction Elevator with Micro-Drive Machine

A — Hoist ropes; B — Governor; C — Top clearance for car; D — Normal terminal stopping device; E — Lower stopping cam; F — Counterweight buffer; G — Compensating ropes; H — Tension weight and sheave; I — Car buffer.

the weight of these ropes is compensated by other ropes which run from the bottom of the car to a tension sheave in the pit and up to the counterweight. The compensating ropes and the control cables which hang in a loop from the middle of the hatch to the car, balance the weight of the hoisting cables. In the present paper the weight of the control cables will be neglected and it will be

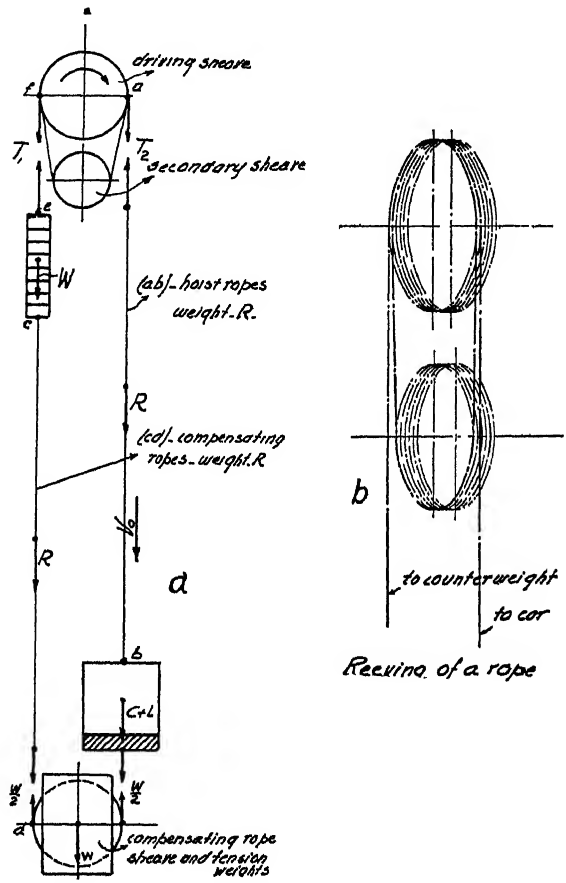


FIG. 3 (a) DIAGRAM OF ROPING; (b) REEVING OF A ROPE

assumed that the weight per ft. of the compensating ropes is precisely the same as that of the hoist ropes.

4 Fig. 3*b* shows diagrammatically the reeving of one of the hoist ropes. It runs up from the car and over a groove of the driving sheave to the secondary sheave, then up again along the next groove of the driving sheave to the counterweight. The

contact between rope and driving sheave thus consists of two part wraps, each usually in the neighborhood of 180 deg.

5 The arrangement just described is the usual one, although the number of part wraps and the profile of the groove can be chosen to suit any condition. The characteristic feature, however, is that the friction or traction at the contact between ropes and driving sheave is the only coupling between the machine and the elevator system comprising car, counterweight, and ropes. The

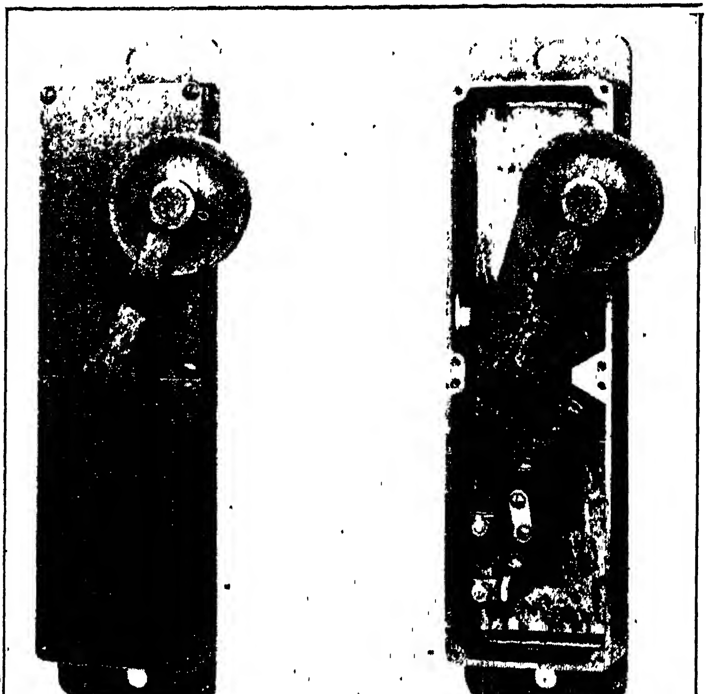


FIG. 4 DOUBLE-POLE LIMIT SWITCH

force exerted by the machine on this system or by the system on the machine can never exceed this friction. While this is an advantage, it is also a disadvantage, for it implies a limited control of the elevator system. This becomes apparent in connection with emergency stops, which take place according to definite laws, which must be known in order to provide the greatest amount of safety.

6 The devices which normally stop the car at the terminal landings are subject to frequent operation, wear, and tear, and faults may develop in them as well as in the controller operating

with them, in which case the emergency terminal stopping devices must be relied upon to bring the elevator to a safe stop. It is these devices, which are usually limit switches and oil buffers, with which this paper is concerned

Limit Switches

7 The object of the limit switch, Fig. 4 — or the final terminal stopping device as it is called in the A.E.S.C. elevator code — is to stop the machine and to prevent further operation in either direction if the car over-runs a terminal by more than a fixed amount. A cam on the car engages the roller of the limit switch, opens the contacts and holds them open as long as cam and roller are engaged. In this condition the energy supply of the motor is cut off and normal service is restored by manipulation of the controller by hand until the cam is freed from the limit switch. Over-run of the terminal landings from one cause or another cannot wholly be avoided and a certain amount of over-run is normal for the particular speed and type of control employed. The over-run is comparatively large with resistance control because of the lack of steps of low speed independent of the load. For the variable-voltage control, on the other hand, having as many definite steps of low speed as may be desired, it is comparatively small. However this may be, the opening of a limit switch causes a temporary shut-down of the elevator and an interference with normal service of the building. The following investigations in Part IV will show that it is essential that the limit switches be brought into action at the earliest possible moment, unless special devices — of which one is described in Par. 132 — are employed.

8 While the limit switch is called an emergency terminal stopping device, it is in reality only an auxiliary, for it requires the coöperation of a number of contactors on the controller to bring the machine to an emergency stop. It is therefore not sufficient to consider the arrangement of the limit switch alone, but also the design of the contactors from the point of view of their ability to respond promptly and without fail in an emergency.

9 The most important of these contactors is the so-called potential switch, which is normally closed, and which is arranged to cut the power supply to the motor when the circuit through its coil is opened by one of the limit switches. To meet the requirements of an emergency safety device the wiring should be so arranged that grounds or short circuits either manifest themselves immediately or else do not interfere with a proper functioning. In addition the switch must open quickly when its circuit is interrupted, for which purpose there should be available an ample mechanical force operating with a magnetic circuit which is rapidly demagnetized.

Oil Buffers

10 Oil buffers are not only important terminal safety devices but also, on account of the complete absence of parts and adjustments which may get out of order, they are as nearly foolproof as any apparatus can be made. The car buffer shown in Figs. 2 and

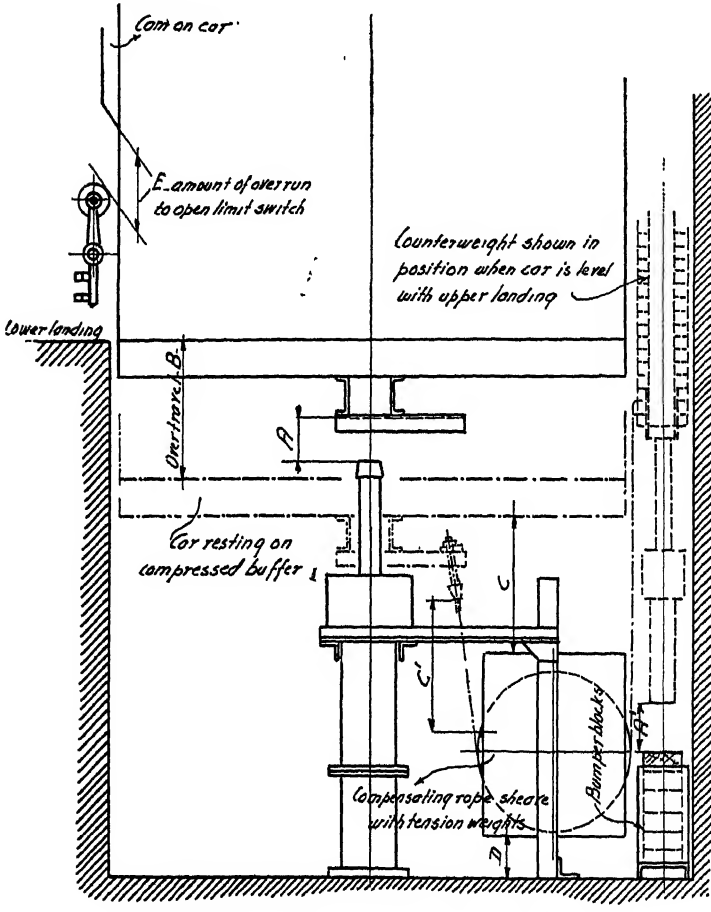


Fig. 5 Car at Lower Landing — Arrangement of Pit

5 is situated in the pit, ready to stop the car in the event of an over-run of the bottom landing of an amount A. Its design is shown in Fig. 6. A piston 1 works in an inner cylinder 2, provided with small perforations (not shown) through which the oil is forced into the outer cylinder 3, whence it rises into the chamber 4 provided with baffles to prevent the oil from splashing out. Piston and piston rod are made as light as possible and the spring 6 serves

to maintain them in their highest position or to return them to it after an engagement.

11 To eliminate the impact when the car, moving at high speed, strikes the buffer, the following arrangement is made: Just above the small holes for the efflux of oil from the cylinder 2, a row of large holes 5 is provided, so that the piston can travel the distance a with practically no resistance of the displaced fluid. On top of the piston rod is a spring 7 which the car compresses solid and which during this process accelerates piston and rod to the speed V_0 of the car. The effective stroke of the buffer is s and it begins after the piston has moved through the distance a to close off the large holes 5.

12 In the usual arrangement shown in Fig. 5 the car is allowed to over-run the bottom landing by the amount A before the buffer is engaged. The bottom over-travel B, which is the distance the car can travel below the lower landing until it has fully compressed the buffer is, therefore:

B = nominal buffer stroke ($a+s$ of Fig. 6) plus A plus the amount to compress the accelerating spring (7 of Fig. 6) plus ½ in. to compress the rubber block at the top of the piston rod.

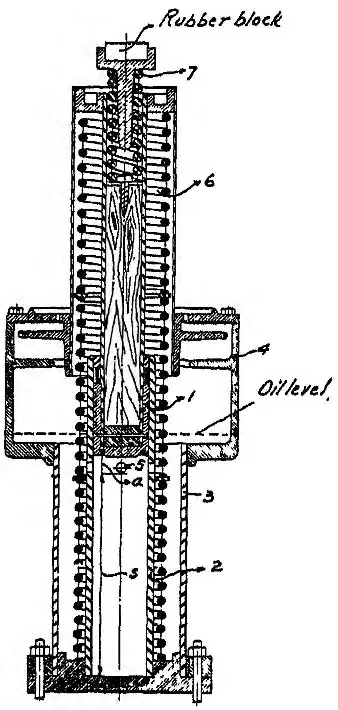


Fig. 6 Car Buffer

13 There are two reasons why the distance A should be small: (1) the buffer should be brought into action as early as possible in case of an emergency such as the loss of control at high speed; (2) when the car is a distance A below the bottom landing, the top clearance for the counterweight, that is the distance available for its stop in an emergency, will have been reduced by the same amount. Good present practice is therefore to allow for A of 6 to 12 in. It is true that occasional over-runs of the bottom landing may quite normally be larger than 6 to 12 in. and so cause a partial engagement of the buffer. But in such cases the buffer is struck at a much reduced speed and offers a resistance not noticeable in the car. Furthermore, the elevator is not temporarily shut down.

14 Upon the selection of a buffer of a suitable stroke and the distance A, it follows from the above formula for B that the bottom over-travel is completely determined. There is, therefore, in the author's opinion, no good reason why some elevator codes should set a minimum for it, in particular if to comply with them a distance A much greater than 6 to 12 in. must be allowed. It serves no useful purpose, besides making the pit deeper than necessary.

15 With further reference to Fig. 5, the clearance C or C' — whichever happens to be the smaller — must be made sufficient for the vertical displacement of the compensating-rope sheave, since such displacement will occur upon an engagement of the buffer at high speed (see Part IV). The clearance D of 10 in. or more is to provide for a reasonable amount of stretch of the ropes before the tension sheave will bottom.

16 In the usual arrangement the counterweight buffer is suspended from, and is part of, the counterweight (Fig. 2), in which case the design is in accordance with Fig. 7. The plunger 1 is fastened to the counterweight from which the body 4 of the buffer and the cylinder 2 are suspended by means or rods 3. The buffer body is further provided with extensions which carry guide shoes 5 sliding on the counterweight rails. In Fig. 5 the counterweight and the buffer are shown in the positions they occupy when the car is level with the top floor. If the car over-runs the latter by the amount A' the buffer cylinder is made to land on suitable bumper blocks, while the plunger (1 of Fig. 7) continues downward with the counterweight and so displaces the oil in the buffer cylinder through slots or grooves (6 of Fig. 7). The operation of the buffer, therefore, begins with the sudden arrest of the motion of the comparatively heavy buffer body and cylinder. To cushion the resulting shock on the foundation or on the supports of the bumper blocks, it is highly desirable to provide a small measure of elasticity by means of springs or a pad of cork.

17 In high-rise elevators, with correspondingly long ropes, there is a considerable permanent stretch (6 to 8 in. per 100 ft. of hoist rope) of which fortunately the greater part develops during the first week or so of service. As the car is bound to travel between the two terminals, all of this stretch becomes manifest on the counterweight side. The distance A' of Fig. 5, therefore, becomes steadily smaller until the counterweight buffer cylinder strikes the bumpers every time the car is at the top landing. For this reason the bumper is made of wooden blocks held in a frame so that the blocks may be removed as required.

18 The conditions as a rule make it necessary to be niggardly with the clearance A' of Fig. 5, for whatever it is, it is an encroachment on the available top clearance of the car. This is obvious, since in an emergency no action of the counterweight buffer results until the car is at a distance A' above the top floor. Even for

high-rise elevators the allowance for A' seldom exceeds 12 in., which means that this clearance and the 12 to 16-in. adjustment provided by the bumper blocks is soon taken up by the stretch of the hoist ropes. If therefore the clearance A' is to be maintained, it is necessary that the ropes be cut and shortened once or twice after their installation until the permanent stretch has been taken out. If to avoid this the ropes about to be put in place should be cut shorter than required by the amount they are expected to stretch, say 30 in. in a certain case, it will mean that with the ropes just installed there will be no action of the counterweight buffer until the car is A' plus 30 in. above the upper landing. At the same time, when the car is level with the bottom landing the counterweight will be 30 in. nearer to the upper limit of the hatchway than the designer intended.

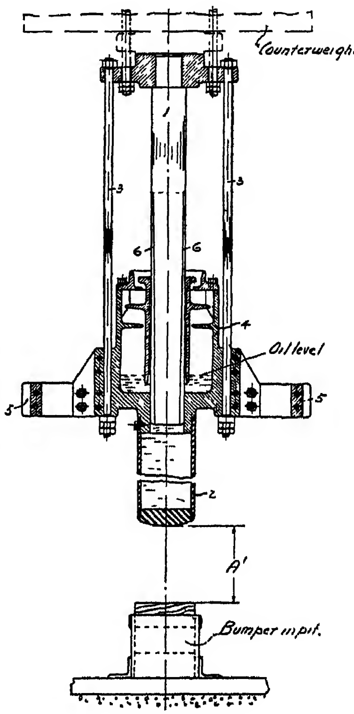


Fig. 7 Counterweight Buffer

19 There is no danger in this procedure when it concerns a new installation, in which case the elevator remains in the hands of the erector for a time sufficient to develop nearly all of the expected amount of permanent stretch. It is different, however, with the reroping of an existing elevator which must be placed in public service as soon as the job is done. In this case the 30 in. that the ropes have been cut too short are in effect an encroachment on the top clearances for car and counterweight, which can ill be afforded.

20 It is easy to provide against this situation. It is obviously always possible to so arrange the counterweight that its top clearance will permit the loss of the 30 in., assumed in the argument above, just after reroping without inviting an element of danger. It is further easy to arrange for a temporary extension of the bumper blocks so that there will be no longer the encroachment on the top clearance for the car, referred to above. At any rate, the minimum permissible top clearance for the counterweight and the maximum allowance for the clearance A' are two important

dimensions which should be given on the layout for the guidance of the engineer in charge of the elevator installation.

21 The practice of having the buffers engaged upon an over-run of the terminal landing arose in connection with resistance-controlled elevators. At present, however, with the nearly universal use of the variable-voltage system of control, the approach to the terminals under normal conditions is at a definite slow speed. This being the case, there can be no objection to striking the buffer 12 in. or more before the car reaches either of the terminals, for there will be practically no reaction to the low striking speed of the buffer, nor will it be associated with undesirable noise. An arrangement as described constitutes a welcome extension of the top clearances for car and counterweight in case of an emergency. It requires, however, for the counterweight a buffer of the type of Fig. 6 with the blocks for the compensation of the rope stretch fastened to it. This is due to the fact that, with a buffer of the type of Fig. 7, compressed 12 in. or more when the car is level with the top, the heavy buffer body will suddenly be picked up at rather high speed after a corresponding travel when the car is started on the downward journey. The shock thus caused, while in no way severe, is nevertheless unpleasantly felt in the car.

22 Mention must also be made of the automatic terminal stopping device. Originally conceived as an emergency device to bring the elevator to a terminal stop independently of the operator, it has since long lost this distinction, functioning, as aptly termed in the A.E.S.C. elevator code, as a normal terminal stopping device on which the operator absolutely relies. It is essentially a duplicate of the master switch with contacts in series with it and likewise carried on the car (Fig. 2). It is operated by cams located near the terminal landings in such a manner that the various contactors on the control panel, necessary to slow down the elevator car as it approaches the terminals and to stop it there, are opened in proper sequence.

23 The car of each elevator is always equipped with a safety device, which is in effect a brake, acting on the guide rails. It is brought into action by a governor (Fig. 2), driven by the car, at predetermined over-speeds which range from 40 per cent above normal for car speeds up to 500 ft. per min. down to 20 per cent for car speeds of 800 and 900 ft. per min. These are the governor tripping speeds, the highest speeds the elevator can attain, and because of them it is held in two or three elevator codes that all emergencies must be assumed to occur at these speeds. The examples in Part IV have been worked out on this basis in order to show what it means should these emergencies actually happen at these speeds.

PART II

THEORY AND CALCULATION OF OIL BUFFER

24 The following notation applies to the equation developed in this section. Let

C = the weight in lb. of the empty car

L = the weight in lb. of the load

Q, Q_c = the load in lb. to be stopped by the buffer. The notation Q_c will be used for a load brought to rest with constant retardation

t = the time in seconds

V_0 = the initial speed in ft. per sec. at which the load strikes the buffer

v = the speed in ft. per sec. of the load at the time t

S = the effective buffer stroke in ft., also the distance at the time ($t = 0$) of the buffer piston from the terminal position (Fig. 8*b*)

s = the distance in ft. of the piston from 0 (Fig. 8*b*) at the time t

$S-s$ = the displacement in ft. of buffer and the load at time t

p_0 = the initial fluid pressure in lb. per sq. in. in the buffer cylinder at time ($t = 0$), when the load strikes the buffer at speed V_0

p = the fluid pressure in lb. per sq. in. at time t

p_{av} = the average fluid pressure in lb. per sq. in. averaged over the effective buffer stroke S

p_t = the terminal fluid pressure in lb. per sq. in., i.e., the pressure when the buffer has completed its stroke

A = the area in sq. in. of the buffer piston

$P = Ap$ = the hydraulic force in lb. exerted on the buffer piston. The direction of P which is also the direction of the acceleration impressed on the buffer and its load will be counted positive

$a = dv/dt$ = the acceleration in ft. per sec. per sec. impressed on buffer and load by the force P at time t

a_c = constant acceleration, ft. per sec. per sec.

a_0 = the initial acceleration in ft. per sec. per sec. or the acceleration at time ($t = 0$)

a_{av} = the average acceleration in ft. per sec. per sec. averaged over the effective buffer stroke

a_t = the terminal acceleration in ft. per sec. per sec. or acceleration when the buffer has completed its stroke

g = the acceleration of gravity, 32.16 ft. per sec. per sec.

Y = the total number of holes through which the fluid escapes when the buffer is at the beginning of its effective stroke

$y =$ the number of holes through which the fluid escapes at the time t when the displacement of the piston is $S-s$

$c =$ the coefficient of efflux of the fluid through the openings provided for it. The pressure in lb. per sq. in. to discharge the fluid at a certain velocity through the openings will be c times the square of the velocity.

In acknowledgment of the custom which designates a body as being retarded when its motion is one wherein the speed diminishes, the word "retardation" will frequently be used instead of the more proper "acceleration."

25 Before proceeding with the subject matter of this section, it is necessary shortly to establish certain facts which will be found more fully discussed in Part IV. For this purpose, refer to Fig 17, which shows the descending counterweight striking the buffer at speed V_0. When this occurs, the ascending car is close to the limit of travel and must be brought to rest in the shortest possible distance. This requires that the retardation impressed by the buffer on the counterweight be such that the hoist ropes at b are immediately relieved of all tension through the formation of a small amount of slack as shown in the figure. If this is the case, counterweight and buffer after their engagement move as if they are totally independent of the elevator, and the load on the buffer is

$$Q = W+\tfrac{1}{2}w-w' \quad \ldots\ldots\ldots\ldots \quad [1]$$

In this equation W is the weight of the counterweight, which normally includes the weight w' of the counterweight buffer body and cylinder (4 and 2 of Fig. 7). In action, however, w' is stopped by the bumper blocks in the pit and only the remainder $(W-w')$ has to be taken care of by the buffer.

26 Similarly, under the same condition but now with reference to Fig. 18, which shows the car buffer in action, the buffer load is

$$Q = C+L+\tfrac{1}{2}w \quad \ldots\ldots\ldots\ldots \quad [1a]$$

The slack hoist rope mentioned above will be associated with a vertical displacement of the compensating rope tension sheave and weight w. Under proper conditions, however, this displacement is small so that for all practical purposes the effect of w on the system is that of two constant forces $\frac{1}{2}w$, as shown in Figs. 17 and 18.

27 In both cases Q in Equations [1] and [2] comprises a weight which is $(C+L)$ for the car buffer and $(W-w')$ for the counterweight buffer and a force $\frac{1}{2}w$. As a rule the latter is a small fraction of Q and hereafter — without appreciable error — $\frac{1}{2}w$ will be dealt with as if it were a weight and not a force.

28 The buffer problem, when slack rope actually develops — and only then — may be stated as follows: A free weight Q strikes

a buffer at speed V_0; design the buffer in such a manner that the weight is brought to rest in a prescribed manner. Or, more general: A buffer is struck at speed V_0 by a free weight Q; determine the motion of the weight.

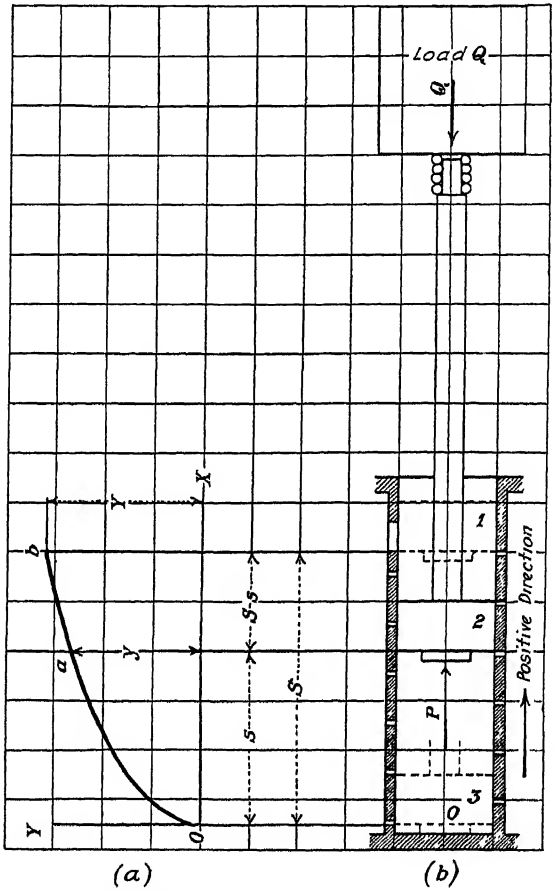


FIG. 8 (a) DIAGRAM SHOWING EFFECT OF WEIGHT Q STRIKING BUFFER AT SPEED V_0; (b) DIAGRAMMATIC ARRANGEMENT OF WEIGHT AND BUFFER

29 It is the latter problem, in particular, for which the formulas given below will be derived, and by means of them the performance of a buffer may be calculated for any given load and striking speed. But it must be borne in mind that the performance so calculated is strictly that of a buffer and a free weight, and this performance will only then be the same in actual elevator service if the slack hoist rope mentioned in Par. 25 actually develops.

For the means of ascertaining the latter, the reader is referred to Part IV.

30 For the purpose of calculation, reference will be made to Fig. 8*b*, which shows diagrammatically the essential parts of the buffer and a weight Q striking at speed V_0. The origin of the system of reference axes is taken at O, corresponding to the terminal position 3 of the piston. The initial position of the piston is at 1, when it has just closed the large holes mentioned in Par. 11. At this time the small spring on top of the piston rod has been compressed solid and both weight and piston have the speed V_0. The effective buffer stroke is S.

31 With the piston at 1, a distance S above O, oil escapes through all of the small holes — Y in number — below it. In Fig. 8*a*, lay off on OX the distance S and perpendicularly to it a line representing Y, which locates a point b. In position 2 the piston is s ft. away from O and the number of holes below the piston through which oil is discharging is now y. Accordingly, lay off s on OX and y at right angles to it and so obtain another point a. Repeating this process for each position of the buffer piston within the effective stroke, the curve Oab, Fig. 8*a*, is obtained, the so-called graduation, which determines, as presently will be seen, the characteristics of the buffer.

32 The weight of the piston and piston rod may be assumed as balanced by the return spring 6 of Fig. 6. Further, if their mass is neglected the error of considering $\frac{1}{2}w$ in Equations [1] and [1a] as a weight instead of a force is practically compensated for. Also the leakage between piston and cylinder for a clearance not exceeding 0.004 in. may be neglected.

33 Let s and t define the position of the piston with respect to O, Fig. 8*b*, with y the number of holes for the efflux of fluid and v the speed of the piston. The efflux area will be fy and the velocity with which oil is discharged through it $(Av)/(fy)$.

34 Accordingly the fluid pressure p at the time t will be expressed by

$$p = c\left(\frac{Av}{fy}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

Since this development deals with a line of buffers, all having the same size piston and using the same grade of oil,

$$C = \frac{cA^2}{f^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2a]$$

whence Equation [2] appears in the form

$$p = C\left(\frac{v}{y}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2b]$$

At the time $(t = 0)$ the speed was V_0, the pressure p_0 and the number of holes discharging oil Y, whence from Equation [2b]

$$p_0 = C\left(\frac{V_0}{Y}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{2c}]$$

and combining Equations [**2b**] and [**2c**]

$$p = \frac{Y^2}{V_0^2}\, p_0 \frac{v^2}{y^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{2d}]$$

35 Before proceeding with the problem in hand, it is well to consider it in a general way. The average fluid pressure (averaged over the effective stroke of the buffer) follows at once from the general energy equation, according to which

$$\begin{bmatrix}\text{work done by fluid}\\ \text{pressure}\end{bmatrix} = \begin{bmatrix}\text{kinetic energy}\\ \text{of load}\end{bmatrix} + \begin{bmatrix}\text{work done by}\\ \text{load}\end{bmatrix}$$

$$Ap_{av}S \qquad = \qquad \frac{Q}{2g}V_0^2 \qquad + \qquad QS$$

whence

$$p_{av} = \frac{Q}{A}\left(\frac{V_0^2}{2gS} + 1\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

In the particular case where there is a load Q_c for which the fluid pressure remains constant it is obvious that the initial pressure p_0 and the average pressure p_{av} are the same. Substituting therefore in Equation [3] ($p_{av} = p_0$) and further the value for p_0 from Equation [**2c**],

$$\frac{Q_c}{A}\left(\frac{V_0^2}{2gS} + 1\right) = \frac{CV_0^2}{Y^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{3a}]$$

According to this equation the properties of a given buffer (S and Y given) may be such that there are loads Q_c which will be brought to rest with constant fluid pressure and therefore also with constant retardation, and if so, there will be only one such load Q_c for each striking speed V_0. Or in other words, if such a buffer is struck at one and the same speed V_0 by various loads Q, there will be only one of them which will come to a stop with constant pressure and retardation.

36 Now assume that by means of Equation [3] p_{av} is calculated for a given buffer and striking speed V_0 when engaged by three different loads $Q' < Q_c < Q''$ and lay off the three values p_{av} as lines parallel to the line representing the effective stroke S in Fig. 9. According to Equation [**2c**] the initial pressure p_0 for a given Y depends only on V_0, and as the latter has been assumed to be the same for all three buffer loads it follows that the pressure curves for all must start at the same point a of Fig. 9. If the graduation is arranged so as to give constant fluid pressure when engaged by Q_c, the indicator card will be the straight line ab. The pressure diagram for Q', however, must begin at a but must otherwise have the average pressure indicated for it. It follows

therefore that it must be some curve such as ab', sloping down from a.

37 Similarly the pressure curve for Q'' must be one like ab'' sloping upward from point a in order that it may have the average associated with it. It will thus be seen that the retardations will no longer be constant for loads Q' and Q'' and that these will vary in accordance with the fluid pressure. The average retardation

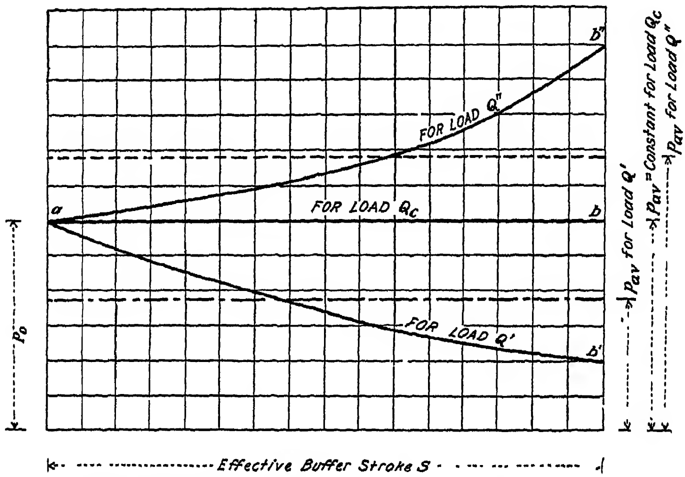


FIG. 9 PRESSURE DIAGRAMS FOR LOADS $Q'<Q_c<Q''$ AT THE SAME STRIKING SPEED V_0

a_{av}, however, averaged over the effective stroke, will be the same for any load so long as the striking speed remains the same. For

$$a = \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds} = v\frac{dv}{ds}$$

whence

$$a_{av} = \frac{I}{S}\int ads = \frac{I}{S}\int_0^{V_0} vdv = \frac{V_0^2}{2S} \quad . \; . \; . \; . \; . \quad [4]$$

which shows in the well-known manner that the average retardation depends only on the speed when S is given.

38 Returning to the problem in hand, the hydraulic force P exerted on the piston at the time t is

$$P = Ap$$

Because of this the load Q receives the acceleration a in the direction of P, Fig. 8b, and counting this direction positive the equation of motion is

$$Ap - Q = \frac{Q}{g}a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

whence

$$a = g\left(\frac{Ap-Q}{Q}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5a]$$

39 The relation between the pressure p, speed v, and the number of holes y through which oil is discharged at the time t is given by Equation [**2d**]. All that is needed for the integration of Equation [**5**] is the graduation, i.e., the equation of curve (Oab), Fig. 8a. The most important of these curves is the parabola, whose general equation when symmetrical with OX and passing through O is

$$y^2 = ks \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [6]$$

in which k is a constant. Since the curve must pass through point b, Fig. 8a, whose coördinates are $s = S$ and $y = Y$, it is found upon substitution in Equation [**6**] that $k = Y^2/S$ and Equation [**6**] becomes

$$y^2 = Y^2 \frac{s}{S} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [6a]$$

Eliminating now Y^2/y^2 between Equations [**6a**] and [**2d**],

$$p = \frac{p_0}{V_0^2} \frac{S}{s} v^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [7]$$

40 For constant fluid pressure, p must remain equal to p_0, and since the retardation in that case will also be constant, $s/S = v^2/V_0^2$. It will be seen that these satisfy Equation [**7**] and hence a parabolic graduation has the property that at the striking speed V_0 there is always a load Q_c — and as we have seen, only one such load — which will be brought to rest with constant retardation and fluid pressure.

41 For arbitrary loads Q and striking speeds V_0 neither the pressure nor the retardation will be constant and the properties of a buffer with parabolic graduation are obtained by the integration of Equation [**5**] in connection with Equation [**7**]. For this purpose, write for a of Equation [**5**]

$$a = \frac{dv}{dt} = \frac{I}{2} \frac{d}{ds} v^2$$

whence

$$pA-Q = \frac{Q}{2g} \frac{d}{ds} (v^2)$$

while the differentiation of Equation [**7**] with respect to s results in

$$\frac{d}{ds} v^2 = \frac{V_0^2}{p_0 S}\left(p+s\frac{dp}{ds}\right)$$

Eliminating $\frac{d}{ds}v^2$ between these equations,

$$pA-Q=\frac{QV_0^2}{2gp_0S}\left(p+s\frac{dp}{ds}\right) \quad \ldots\ldots \quad [8]$$

Before integrating Equation [8] it can be simplified by introducing the average retardation as given by Equation [4]. Substitute $S = V_0^2/2a_{av}$ and write it in the form

$$\frac{ds}{s}=\frac{Qdp}{\left(\frac{g}{a_{av}}Ap_0-Q\right)p-\frac{g}{a_{av}}Qp_0} \quad \ldots\ldots \quad [8a]$$

For $(t = 0)$, $(s = S)$ and $(p = p_0)$. Hence integrate Equation [8a] between the limits $(s = s)$ and $(s = S)$ on the left and between the limits $(p = p)$ and $(p = p_0)$ on the right, which gives

$$\log\frac{s}{S}=\frac{Q}{\frac{gAp_0}{a_{av}}-Q}\log\frac{\left(\frac{gAp_0}{a_{av}}-Q\right)p-\frac{gQp_0}{a_{av}}}{\left(\frac{gAp_0}{a_{av}}-Q\frac{a_{av}+g}{a_{av}}\right)p_0} \quad \ldots \quad [9]$$

in which either natural or common logs may be used.

42 Attention is called to the fact that the quantities p_0/a_{av} in Equation [9] are constants for a given buffer, because the initial pressure p_0 according to Equation [2c] will be proportional to the square of the initial speed V_0 while this is also the case for the average acceleration a_{av}.

43 It is of particular interest for practical purposes to know the terminal pressure p_t. Since it is the pressure p in Equation [9] at the end of the stroke for $(s = 0)$, it will be that pressure p for which the numerator of the fraction of the logarithm becomes zero, whence

$$p_t=\frac{gQp_0}{gAp_0-a_{av}Q} \quad \ldots\ldots\ldots \quad [10]$$

As stated before, the quantity p_0/a_{av} is a constant for a given buffer, and since the striking speed V_0 does not appear in Equation [10], a buffer with parabolic graduation has the remarkable property that the terminal pressure p_t depends only on the load Q and is otherwise independent of the striking speed.

44 Equation [9] also shows at what load Q_c the pressure and retardation will be constant. Evidently it is that load Q for which the denominator of the fraction of the logarithm becomes zero. Or, it is that load Q in Equation [10] for which p_t is equal to p_0. Either method gives

$$Q_c=\frac{gAp_0}{g+a_{av}} \quad \ldots\ldots\ldots \quad [10a]$$

which is also identical with Equation [5], since for constant retardation ($a_{av} = a$).

45 There is further a limiting load for which the terminal pressure p_t, no matter what the striking speed is, becomes infinite. This, according to Equation [10], will be the case when

$$Q = \frac{gAp_0}{a_{av}}$$

46 To complete the equations the initial and terminal accelerations impressed on the load Q must be deduced. The former is obtained from Equation [5a] by writing a_0 for a and p_0 for p, and the latter by replacing a by a_t, p by p_t, and substituting the value of p_t from Equation [10]. This gives

$$\left.\begin{aligned} a_0 &= \frac{g}{Q}(Ap_0 - Q) \\ a_t &= g\frac{Q}{\frac{gAp_0}{a_{av}} - Q} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad [11]$$

Also the terminal acceleration for a given buffer with parabolic graduation depends only on the load Q and is independent of the striking speed.

47 The use of the equations developed above is best shown by an example, as follows:

Example 1: The effective stroke of the buffer $S = 4$ ft. and the area of piston $A = 15.78$ sq. in. The buffer is provided with a parabolic graduation with $Y = 34$ and the coefficient $C = 4250$. The striking speed is 16 ft. per sec.

a What is the load Q_c which will be brought to rest with constant retardation and pressure, and what is the magnitude of the pressure and retardation?

b Determine the pressure diagrams, the speed-displacement, and acceleration-displacement curves for buffer loads of 4900 and 9500 lb. respectively

c What is the force exerted by the buffer on the foundation or supports?

Regardless of what Q is, by Equation [4]

$$a_{av} = \frac{16^2}{2 \times 4} = 32 \text{ ft. per sec. per sec.}$$

which shows that the average retardation in the present example is very nearly gravity. Equation [2c]

$$p_0 = \frac{4250 \times 16^2}{34^2} = 941 \text{ lb. per sq. in.}$$

which is the magnitude of the initial pressure.

Solution (*a*): The load Q_c brought to rest with constant retardation follows from Equation [3a] or [10a]. From the latter

$$Q_c = \frac{32.16 \times 15.78 \times 941}{32.16 + 32} = 7400 \text{ lb. (approx.)}$$

The retardation impressed on Q_c and the fluid pressure being constant are of course equal to a_{av} and p_0 above.

Solutions (*b*): The pressures p for various positions of the buffer as defined by s are calculated from the Equation [9] for $Q = 4900$ and 9500 upon substitution of the above values for a_{av} and p_0. Thence, with p known for each s the corresponding velocity v and retardation a follow from Equations [7] and [5a].

48 The results of these calculations are shown in Table 1. The plot of the values of p with respect to the piston displacements

TABLE 1

$\frac{s}{S}$	Displacement $S-s$ in per cent of buffer stroke S	Fluid pressure p $Q = 4900$	Fluid pressure p $Q = 9500$	Velocity v $Q = 4900$	Velocity v $Q = 9500$	Acceleration a $Q = 4900$	Acceleration a $Q = 9500$
1.0	0	$p_0 = 941$		$v_0 = 16$		$a_0 = 65\ 5$	$a_0 = 18\ 2$
0 9	10	850	983	14.4	15.52	56.0	20.32
0.8	20	770	1037	12.9	15.0	47.5	23.3
0.7	30	695	1075	11 5	14.3	39.2	25.0
0 6	40	633	1120	10.17	13.55	33 4	27.8
0.5	50	581	1177	8.88	12.65	28.0	30.8
0.4	60	535	1240	7.6	11.7	23.2	34.1
0.3	70	505	1300	6 43	10.3	20.1	37 4
0.2	80	480	1380	5 1	8.65	17.5	41.1
0.1	90	465	1470	3.56	6.32	16.1	46.6
0 0	100	$p_t = 463$	$p_t = 1670$	0.0	0.0	$a_t = 15.75$	$a_t = 56.0$

$(S-s)$ in per cent of S results in the pressure diagram, Fig. 10, wherein Curve 1 is the pressure for the load $Q = 4900$, the straight line 2 for a load of 7400, and Curve 3 for a load of 9500 lb. Curves like these are also obtained from the ordinary buffer tests wherein a weight is made to strike the buffer at the desired velocity while the oil pressures are recorded by a steam-engine indicator. It will be seen that the above pressure curves have the shape already predicted for them in Par. 48. It should also be noted that the load Q cannot be very much higher than the load $Q_c = 7400$ lb., brought to rest with constant retardation, without a considerable terminal pressure. Curves 1, 2, and 3 are the velocity-displacement curves; Curve 2 is the well-known parabola associated with constant retardation ($a_c = 32$ ft. per sec per sec.) of the load $Q_c = 7400$ lb., according to the formula

$$v = V_0 \sqrt{\frac{s}{S}}$$

Curves 1″, 2″, and 3″ are the retardation-displacement graphs, Curve 2″ being the straight line representing the constant retardation of the weight $Q_c = 7400$ lb. It will be seen that a load $Q < Q_c$

will have the pressure and retardation greatest at the beginning of the buffer stroke, while for loads $Q > Q_c$ they will be largest at the end.

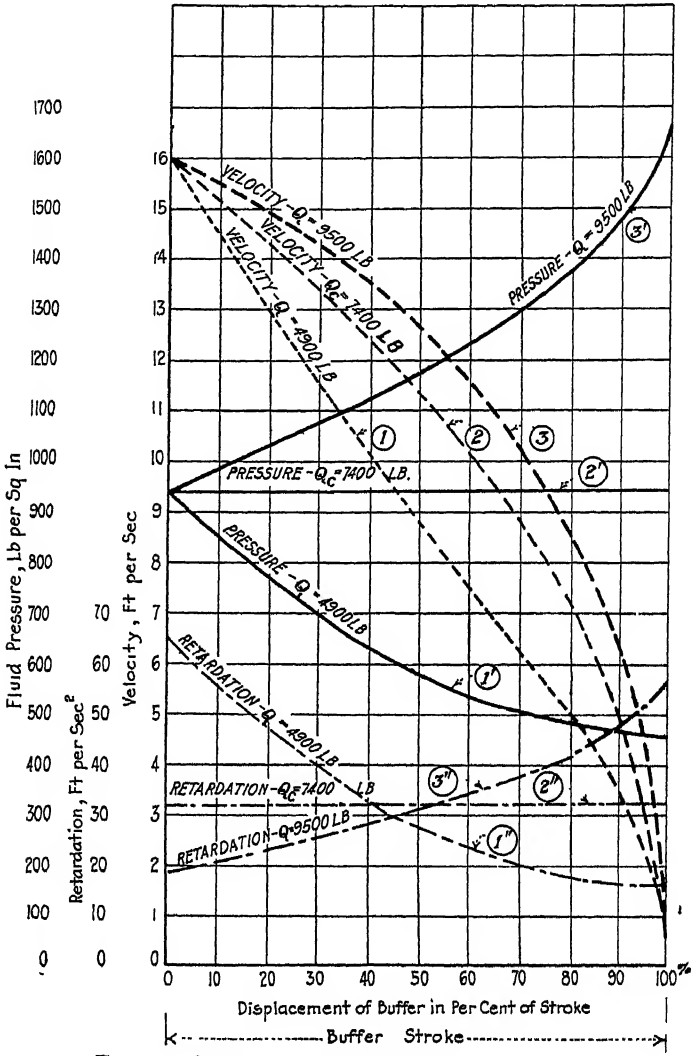


FIG. 10 CHARACTERISTIC CURVES OF OIL BUFFER

49 *Solutions* (*c*): The force exerted by the buffer on the foundation is the hydraulic force $P = Ap_{max}$. The maximum pressure for loads 4900 and 7400 lb. is according to the foregoing $p_0 = 941$ lb. per sq. in.; hence the force exerted on the buffer foundation will be $15.78 \times 941 = 14{,}850$ lb. For the load $Q = 9500$

lb. the greatest pressure (see Table 1) is $p_t = 1670$ lb. per sq. in.; hence the force exerted on the foundation will be

$$15.78 \times 1670 = 26{,}400 \text{ lb.}$$

50 *Example 2:* For the buffer of Example 1, determine the speed-time curves.

51 The speed-time curves, as Part IV will show, are most important in the determination of the suitability of a given buffer for actual elevator service. For the load $Q_c = 7400$ the matter is very simple, since it comes to rest with the constant retardation $a_c = 32$ ft. per sec. per sec. For the other loads, however, the

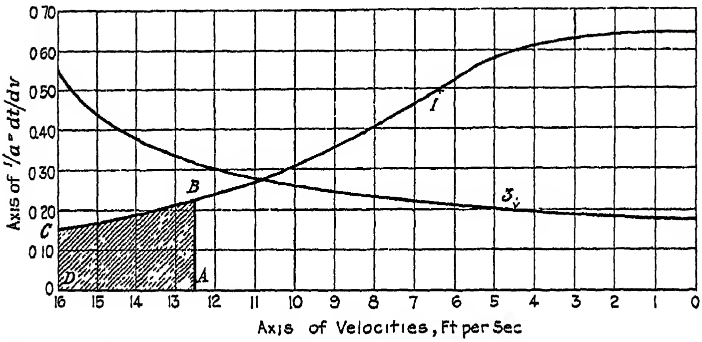


FIG. 11 PLOT OF $1/a = dt/dv$ WITH RESPECT TO v FOR THE DETERMINATION OF SPEED-TIME CURVES

mathematical derivation of the speed-time curves meet with grave difficulties, for which reason the simple graphical process[1] below will be followed. From Curves 1 and 1″ and 2 and 2″ the acceleration ($a = dv/dt$) associated with any given value of the speed v for loads of 4900 and 9500 lb. is known. Therefore for each of these loads a curve can be constructed in which the values of v are the abscissas and the *inverse* of the corresponding values of a, i.e. ($1/a = dt/dv$), are the ordinates. These curves are 1 and 3 of Fig. 11.

52 Considering more particularly Curve 1, it is evident that the area $ABCD$ will be expressed by

$$\int_v^{V_0} \frac{1}{a}\, dv = \int_v^{V_0} \frac{dt}{dv}\, dv = \int_v^{V_0} dt = \Big|_x^{V_0} t$$

and is therefore the time which has elapsed when the speed has changed from V_0 to v. Or, since time is counted from the be-

[1] This process is more in general the graphical integration of the differential equation $dy/dx = \phi(y)$. It is extremely useful in various problems of mechanics and electricity. See for example St. Germain in the *Revue generale de l'Electricite* of 1914, also Lindquist and Yearsly; Transient Performance, A.I.E.E. 1924.

ginning of the effective buffer stroke, the above area is the time at which the speed is v. Thus by means of an ordinary planimeter the time t for any value of v can be determined and their plot results in the speed-time curves of Fig. 12. For the load of 7400 lb. which comes to rest with constant retardation $a_c = 32$ the speed-time curve is the straight line 2.

PART III

DYNAMICS OF THE EMERGENCY STOPS OF THE TRACTION ELEVATOR

The Equivalent System

53 The dynamics considered in this section are those of the emergency stops which are always associated with slip of the hoisting ropes with respect to the driving sheave of the machine.

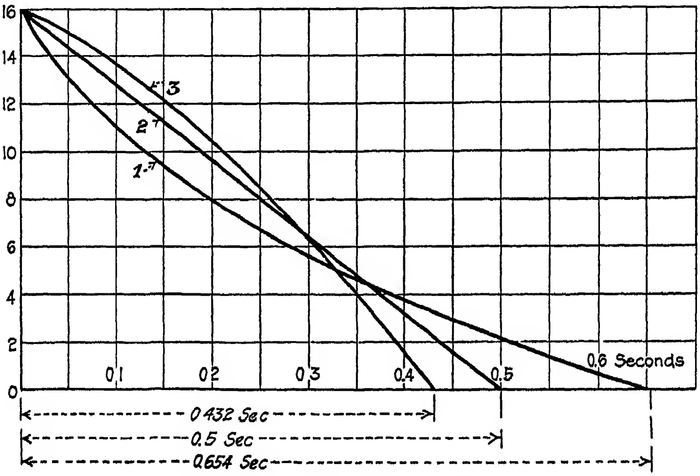


Fig. 12 Speed-Time Curves

Curve (1) Speed-Time Curve $Q = 4900$ lb.
Curve (2) Speed-Time Curve $Q = 7400$ lb.
Curve (3) Speed-Time Curve $Q = 9500$ lb.

The calculations in Parts IV and V will be greatly simplified by the consideration of the following fundamental problem. In Fig. 13a let 1 be a sheave which for simplicity will be considered locked so that it cannot rotate. Masses M and m are suspended from the sheave by ropes which are assumed to be without weight and elasticity. At the time ($t = 0$) forces P and p are applied and cause motion in the direction of P. The object is to determine this motion.

54 Since, by hypothesis, the sheave remains stationary, the motion of the system will be associated with the slip of the ropes. If then at the time t the rope tensions at a and c are T_1 and T_2, the motion of the portion abc of the rope will be due to these

forces and the friction or traction which it experiences in slipping over the sheave. Under all circumstances the mass is negligible and for practical purposes, therefore, T_1, T_2, and the friction just mentioned are in equilibrium. The friction on the portion *abc* of the rope is necessarily opposite to the motion. It evidently assists T_1 and hence T_2 must balance both T_1 and the friction, and is the larger of the two tensions. According to the theory of equilibrium of a perfectly flexible and non-extensible band in contact with a sheave there exists at the point of slip or while slipping the following relation between T_1 and T_2:

$$T_2 \doteq \alpha T_1 \quad \ldots\ldots\ldots\ldots \quad [12]$$

in which α, the so-called traction relation, is a quantity greater than unity.

55 Taking the direction of motion as indicated in Fig. 13*a* as positive, and applying Newton's law to M and m,

$$P - T_2 = Ma; \qquad T_1 - p = ma \quad \ldots\ldots \quad [13]$$

in which a denotes the acceleration which is common to M as well as m. Eliminating T_1 and T_2 from Equations [12] and [13],

$$a = \frac{P - \alpha p}{M + \alpha m} \quad \ldots\ldots\ldots\ldots \quad [14]$$

56 Consider now the simple system of Fig. 13*b* which comprises two masses whose aggregate is $(M + \alpha m)$ and two forces whose resultant is $(P - \alpha p)$. It is seen that the acceleration impressed on this system is exactly as expressed by Equation [14]. It is concluded therefrom that the system, Fig. 13*b*, is the dynamic equivalent of the actual system, Fig. 13*a*.

57 Upon a closer inspection of Fig. 13*b* it will be noted that the equivalent system is built up as follows: To the right of a vertical line which represents the vertical through the center of the driving sheave all masses and forces are situated on the right of the driving sheave (Fig. 13*a*) as they actually are. On the left of the above line, however, the masses and forces are situated on the left of the driving sheave, but now taken at α times their actual value. The side having the factor α is evidently the side of the smaller rope tension T_1. These are the rules to determine the system dynamically equivalent to that of Fig. 13*a*. And the introduction of this simple artifice permits dealing with the problems of Parts IV and V with a simplicity and clarity not otherwise possible.

58 The assumption, made above, that sheave 1 of Fig. 13*a* remains stationary, restricts in no way the general validity of the results, which apply as long as the circumferential speed of the sheave is different from the rope speed. As to the weight of the ropes, only that of the very negligible portion *abc* must be neglected; of the remainder, the inertia is merged with M and m, and the weights with P and p.

PART IV

THE EMERGENCY STOPS OF THE TRACTION ELEVATOR

THE POTENTIAL SWITCH STOP. THE LIMIT SWITCH AS AN INDEPENDENT EMERGENCY TERMINAL STOPPING DEVICE

59 With reference to Fig. 3 the following symbols are used:

C = the weight in lb. of the car
L = the weight in lb. of the load

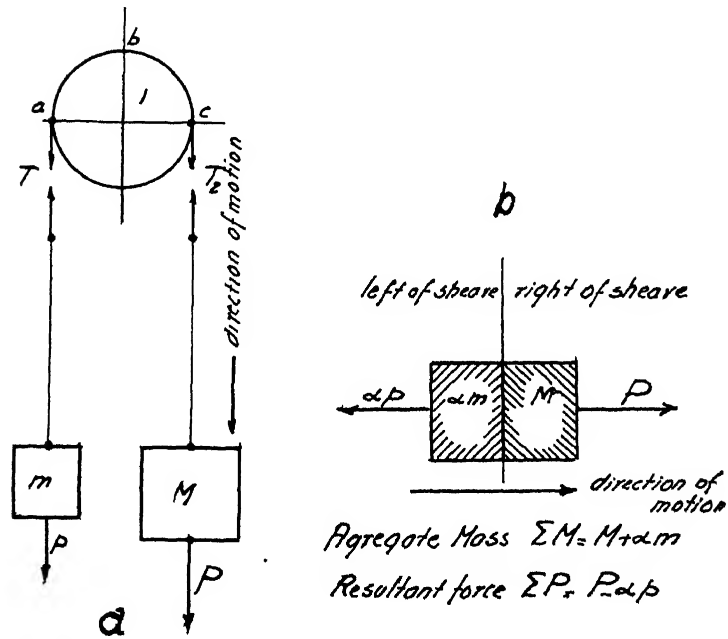


FIG. 13

W = the weight in lb. of the counterweight
w = the combined weight in lb. of the compensating sheave and tension weights
R = the weight in lb. of hoist and compensating ropes
V_0 = the initial speed of the elevator in ft. per sec.
s_d and s_u = the stopping distance in ft. when the stop occurs while the elevator is descending or ascending respectively
g = the acceleration in ft. per sec. per sec. of gravity = 32.16

a = the acceleration in ft. per sec. per sec. (or taken negative the retardation) impressed on the elevator
α = the traction relation (see Part III).

For all practical purposes the length of the ropes for the calculation of R may be taken equal to the rise. In the case of complete compensation, here assumed, the weight of the compensating ropes will also be R. Also, for the purposes of this chapter, the ropes may be considered to be non-elastic. Mechanical friction of car or counterweight in the guides will be neglected.

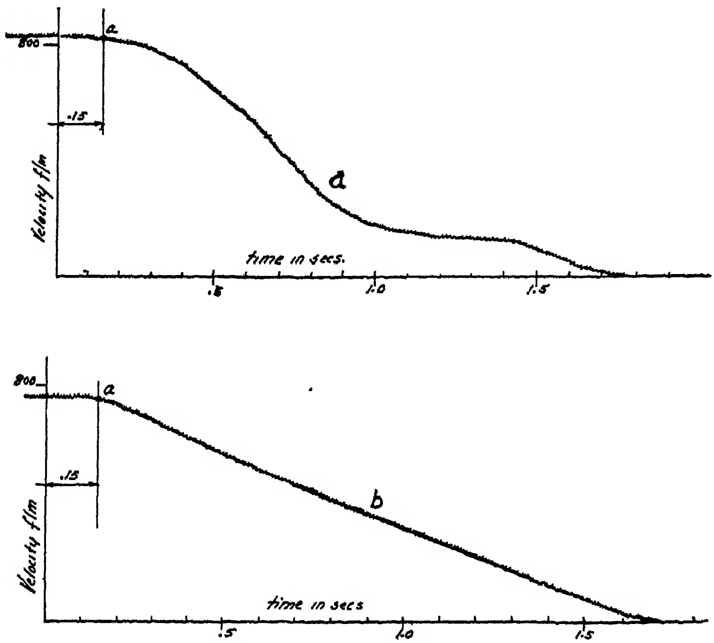


FIG. 14 VELOCITY-TIME GRAPH OF AN ACTUAL POTENTIAL-SWITCH STOP
(a) Circumferential velocity of driving sheave — loaded car descending.
(b) Velocity of car descending with full load.

60 A potential-switch stop is defined as the stop of the elevator brought about solely by the opening of the potential switch. Accordingly a potential-switch stop will take place upon the opening of:

(*a*) the safety switch in the car
(*b*) the contactor controlled by the governor
(*c*) the limit switches

and the object is to determine the stopping distance of the elevator under various conditions.

61 The velocity-time graphs of an actual potential-switch stop are shown in oscillograms Figs. 14*a* and *b*, the former being

the circumferential velocity of the driving sheave and the latter the car speed. Both tests were made with the loaded car descending at full speed (approximately 800 ft. per min.), and the zero of time is the instant at which the circuit through the coil of the potential switch is interrupted. It will be seen that the response is not instantaneous, for there is a lapse of about 0.15 sec. until at point a a change in speed begins. After this there is a rapid deceleration of the machine, associated with slip of the hoist ropes, evidenced by the fact that the velocity of the latter is larger than the circumferential speed of the driving sheave. From the straight inclined portion of Fig. 14b, characteristic of a constant retardation, it is seen that the traction or friction experienced by the slipping ropes is for all practical purposes independent of the velocity of slip.

62 Now referring to Fig. 3, let the loaded car be moving downward at speed V_0 when one of the emergencies mentioned above arises. The car has been shown at or near the bottom landing and the calculations are for this position. It will be understood, however, that with complete compensation of the hoist ropes the results of the calculation will be wholly independent of the position of the car in the hatch. Since the rotational inertia of the compensating rope sheave, which is very small, is neglected, the effect it has on the system will comprise two forces $\frac{1}{2}w$ as shown. The stop of the car occurs, as mentioned, 0.15 sec. after the emergency arises, and since it is wholly associated with slip of the ropes, the dynamics of Part III apply and the first object is to determine the equivalent system. For this purpose, it is noted that during the stop the speed of the hoist ropes in the direction of the arrow, Fig. 3, is larger than the circumferential speed of the driving sheave. It follows that the friction experienced by the slipping rope will assist T_1. This makes T_1 smaller than T_2 and hence according to the rules of Part III the system dynamically equivalent to that of Fig. 3 will have the forces and masses on the left of the driving sheave multiplied by α, while those on the right remain as they actually are. The equivalent system is shown in Fig. 15.

63 The direction of the acceleration impressed on this system will be opposite to the direction of V_0 and will be counted positive, whence

$$\Sigma M = \frac{C+L+\alpha W+(\alpha+1)R}{g} \quad . \quad . \quad . \quad . \quad . \quad [15]$$

$$\Sigma P = -(C+L)+\alpha W+(\alpha-1)R+(\alpha-1)w/2 \quad . \quad . \quad [15a]$$

and the acceleration

$$a = \frac{\Sigma P}{\Sigma M} = \frac{-(C+L)+\alpha W+(\alpha-1)R+(\alpha-1)w/2}{C+L+\alpha W+(\alpha+1)R}\, g \quad . \quad [16]$$

The latter is evidently constant and hence the stopping distance will be

$$s_d \text{ (full load down)} = \frac{V_0^2}{2a} = \frac{V_0^2}{2g} \times \frac{C+L+\alpha W+(\alpha+1)R}{-(C+L)+\alpha W+(\alpha-1)R+(\alpha-1)w/2} \,. \quad [17]$$

When a potential-switch stop occurs while the car is ascending, the desired stopping distance follows immediately from Equation [17] by writing W for $(C+L)$ and vice versa, whence

$$s_u \text{ (full load up)} = \frac{V_0^2}{2g} \times \frac{W+\alpha(C+L)+(\alpha+1)R}{-W+\alpha(C+L)+(\alpha-1)R+(\alpha-1)w/2} \,. \quad [17a]$$

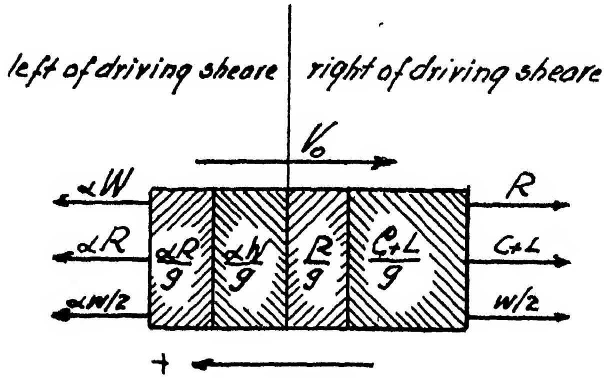


FIG. 15

To obtain the stopping distances associated with a potential switch stop when the car is empty, place $(L = 0)$ in Equation [17] and [17a] and obtain

$$s_d \text{ (no load down)} = \frac{V_0^2}{2g} \, \frac{C+\alpha W+(\alpha+1)R}{-C+\alpha W+(\alpha-1)r+(\alpha-1)w/2} \quad [17b]$$

$$s_u \text{ (no load up)} = \frac{V_0^2}{2g} \, \frac{W+\alpha C+(\alpha+1)R}{-W+\alpha C+(\alpha-1)R+(\alpha-1)w/2} \quad [17c]$$

64 These are the shortest possible stopping distances associated with a potential switch stop and it must be borne in mind that they are figured on the supposition that slip of the ropes takes place during the entire effective stop. This in turn requires that the dynamic braking torque exerted by the motor and that due to the mechanical brake must at all times be sufficient to produce the slip. From equations [17], [17a], [17b], and [17c]

it follows that the greatest stopping distance will occur when the fully loaded car is descending. They also show that the stopping distances can be decreased by

1 An increase in the traction constant α, which can be obtained by providing the driving sheave with undercut grooves, Fig. 6. It is impossible, however, to go far in this direction without inviting rapid wear of the grooves

2 An increase in the tension weights w. Neither can this expedient be employed to a sufficient extent materially to affect the stopping distance. It has the disadvantage of increasing the tension in the hoist ropes without benefit of it in normal operation and of increasing the load on the buffer (Equation [**1**]).

To both s_d and s_u must be added the distance traveled by the elevator at full speed during the time which elapses from the opening of the circuit through the potential-switch coil to the instant the actual stop begins.

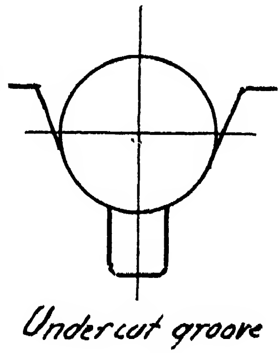


Fig. 16

65 *Example 1:* The details of an elevator are: $L = 2500$ lb., $C = 4500$ lb., $W = 5500$ lb., $w = 800$ lb., the weight of the six $\frac{5}{8}$-in. hoist ropes with which the elevator is equipped is 3.72 lb. per ft., the rise is 400 ft., whence $R = 400 \times 3.72 = 1490$ lb. and the traction constant $\alpha = 2$. Calculate the stopping distances associated with a potential-switch stop when the actual stop begins 0.15 sec. after the emergency occurs under the following conditions:

a The emergency takes place when the fully loaded car travels at normal speed ($V_0 = 13.33$ ft. per sec., 800 ft. per min.) downward. Determine the stopping torque which must be exerted by the machine on the driving sheave in order that slip of the ropes may actually occur during the entire stop

b The emergency takes place when the fully loaded car travels at normal speed in the upward direction

c The emergency happens while the empty car travels at normal speed in the downward direction

d The emergency occurs while the empty car travels upward at normal speed.

66 *Solution (a):* From Equation [**17**]

$$s_d \text{ (full load down)} = \frac{13.33^2}{64.32} \times \frac{4500+2500+2\times 5500+3\times 1490}{-(4500+2500)+2\times 5500+1490+400} = 10.53 \text{ ft.}$$

Adding $0.15\times 13.33 = 2$ ft., the distance traveled at full speed for 0.15 sec., the total s_d (full load down) $= 10.53+2 = 12.53$ ft. The torque necessary to cause slip is figured as follows: Considering that portion of the system comprised between points a and b of Fig. 3, the resultant force on it is $T_2-(C+L+R+w/2)$ and the mass $(C+L+R)/g$. The resultant force must be of such magnitude as to impress on the above mass the acceleration a of Equation **[16]**, whence by Newton's law:

$$T_2-(C+L+R+w/2) = \frac{C+L+R}{g}\cdot a$$

so that

$$T_2 = (C+L+R)\,\frac{a+g}{g} + w/2 \quad . \; . \; . \; . \; . \; . \quad [18]$$

Furthermore, T_2 in the present case being larger than T_1, according to the laws of traction

$$T_2 = aT_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [18a]$$

For the above example we obtain from Equation **[16]**

$$a = \frac{-(4500+2500)+2\times 5500+1490+400}{4500+2500+2\times 5500+3\times 1490}\, g = 0.262g$$

and substituting in Equation **[18]**

$$T_2 = (4500+2500+1490)1.26+400 = 11{,}120 \text{ lb.}$$

and finally by Equation **[18a]**

$$T_1 = 11{,}120/2 = 5560 \text{ lb.}$$

Therefore the torque exerted by the elevator system on the machine is

$$(T_2-T_1)\times\begin{bmatrix}\text{radius of}\\ \text{driving}\\ \text{sheave}\end{bmatrix} = 5560\times\text{radius of driving sheave}$$

67 It is concluded from this that in order to have slip actually occur so as to have the shortest possible stopping distance under the given conditions, the average of the combined dynamic braking torque plus that of the mechanical brake (averaged over the whole stopping distance or any part thereof) must be larger than $5560\times$ radius of driving sheave. This torque is quite considerable and is best expressed in terms of the so-called full-load torque on the machine. The latter is the static torque with fully loaded car and is, in the present example,

$$(C+L-W)\times\begin{bmatrix}\text{radius of}\\ \text{driving}\\ \text{sheave}\end{bmatrix} = 1500\times\text{radius of driving sheave}$$

Thus the average stopping torque that must be exerted on the driving sheave must not be less than $5560/1500 = 3.7\times$full-load torque.

68 As is known, the field of the gearless-traction-machine motor is permanently excited and a potential-switch stop therefore normally occurs with full field excitation. Under these conditions the motor is easily able to exert the stopping torque required to cause the ropes to slip. On the other hand the controller is so arranged that the potential switch also drops out upon a failure of the field current. A stop under these conditions takes place with a failing field and the stopping torque of motor and brake will no longer be sufficient to cause slip of the ropes during the entire stop.

69 The rise of the elevator will be found to be of little influence on the stopping distance. For example, should the rise be so small that the weight of the ropes may be neglected, from Equation [**17**] with ($R = 0$)

$$s_d \text{ (full load down)} = 11.53 \text{ ft.}$$

that is only 1 ft. shorter than found for 400-ft. rise.

70 *Solution* (*b*). In this case, from Equation [**17a**]

$$s_u \text{ (full load up)} = 6.36 \text{ ft.}$$

and adding 2 ft. for the distance traveled at full speed for 0.15 sec.

$$\text{total } s_u \text{ (full load up)} = 8.36 \text{ ft.}$$

71 *Solution* (*c*). Employing Equation [**17b**] and adding 2 ft. as before,

$$\text{total } s_d \text{ (no load down)} = 6.57+2 = 8.57 \text{ ft.}$$

72 *Solution* (*d*). From Equation [**17*c***] by adding 2 ft.

$$\text{total } s_u \text{ (no load up)} = 9.77+2 = 11.77 \text{ ft.}$$

Stopping Distance in Regular Service

73 The stopping distances calculated above are also a measure for the stopping distances in normal service. The latter must occur without slip, and as it is in practice impossible so to control the machine that the ropes — during the stop — are just below the point of slip, a normal stopping distance associated with a car-switch stop about 40 to 50 per cent in excess of the stopping distance of a potential-switch stop must be figured on.

THE LIMIT SWITCH AS AN INDEPENDENT EMERGENCY TERMINAL STOPPING DEVICE

74 Let us now turn our attention more particularly to the limit switches and determine their requirements if they are to function independently of all other devices as emergency terminal stopping devices. For this purpose assume that an elevator is equipped with this device alone and that a complete failure of the operating and normal terminal stopping devices occurs. The car then proceeds up or down as the case may be until it opens the top or bottom limit switch and there results a potential-switch stop initiated at a point above the upper or below the lower terminal landing.

75 In case of such an emergency with the elevator of Example 1, Par. 74, *at normal speed* the results already obtained can immediately be used. In that example it was found that a potential-switch stop at normal speed for full load downward and no load upward required stopping distances of 12.53 and 11.77 ft., respectively. Hence if the limit switches are to function as entirely independent emergency terminal stopping devices, there will be necessary at normal speed

$$\left.\begin{array}{l}\text{a bottom over-travel of......12.53 ft.}\\ \\ \text{a top clearance for the car of..11.77 ft.}\end{array}\right\} + \left\{\begin{array}{l}\text{the amount of over-}\\ \text{travel by the car to}\\ \text{open the limit switches.}\end{array}\right.$$

76 This section could be concluded with a few remarks concerning the results just obtained were it not for the fact that some codes, and amongst them the A.E.S.C. Code in rule 222*c*, require that the bottom over-travel and top clearance for the car shall be sufficient for an emergency terminal stop by the limit switches alone *at governor tripping speed.* The meaning of this is best shown by an example.

77 *Example 2:* For the elevator of Example 1 (Par. 74), for which the governor tripping speed is 960 ft. per min. or 20 per cent in excess of normal speed, calculate the stopping distances for a potential-switch stop with no load upward and full load downward at the tripping speed of the governor. As before 0.15 sec. will be allowed for the lag of time before the stop becomes effective. With $V_0 = 960/60 = 16$ ft. per sec., from Equations **[17]** and **[17c]** with the addition of $0.15 \times 16 = 2.4$ ft. for the distance traveled at full speed for 0.15 sec.

$$\text{total } s_u \text{ (no load up)} = 14.06 + 2.4 = 16.46 \text{ ft.}$$
$$\text{total } s_d \text{ (full load down)} = 15.15 + 2.4 = 17.55 \text{ ft.}$$

Hence the requirements at governor tripping speed are

$$\left.\begin{array}{l}\text{a bottom over-travel of......17.55 ft.}\\ \\ \text{a top clearance for the car of..16.46 ft.}\end{array}\right\} + \left\{\begin{array}{l}\text{the amount of over-}\\ \text{travel by the car to}\\ \text{open the limit switches.}\end{array}\right.$$

78 The conclusions to be drawn from the above are:

Limit switches can function as independent emergency terminal stopping devices at the expense of an exorbitant increase in the customary top and bottom clearances.

Limit switches alone are inadequate to stop the elevator in the emergency of complete loss of control at the terminal landings at either normal or governor tripping speeds within the customary clearances at top and bottom.

The Plain Buffer Stop

79 In the following equations let

S_c = the stopping distance in ft. when the car is the ascending member

S_w = the stopping distance in ft. when the counterweight is the ascending member

A_c = the acceleration, or taken negatively, the retardation in ft. per sec. per sec. impressed on the car

A_w = the acceleration in ft. per sec. per sec. impressed on the counterweight.

The symbols C, W, R, V_0, and α are defined in Par. 68.

80 A plain buffer stop is here defined as the stop of the elevator when brought about solely through the engagement of the oil buffer by the descending member of the elevator. It is therefore also the stop which occurs upon complete and simultaneous failure of:

1 The normal terminal stopping device
2 The limit switches
3 The auxiliary emergency stopping device (see note below rule 222*c* of A.E.S.C. elevator code) if one is furnished.

It is not necessary to dwell at this time on the probability of these devices failing simultaneously because of the buffer test prescribed by various elevator codes. The A.E.S.C. Code, rule 200*f*, for example specifies, "Buffers shall be tested by running into them at governor tripping speed," which obviously cannot be done unless the devices under (1) and (3) above are put out of commission. The rule, just cited, does not say what to do with the limit switches at the time of test. However, it is not unusual to conduct buffer tests in the field with the limit switches short-circuited. Therefore, even if it never occurs afterward, at the time of the tests a plain buffer stop is made.

81 In the plain buffer stop it is possible to distinguish : (1) the counterweight buffer stop when the car is the ascending member and it is the counterweight which engages the oil buffer, and (2) the car buffer stop when the counterweight is the ascending member of the elevator. While the former will be considered in

detail it will be understood that all deductions and conclusions apply fully to the latter as well.

82 *Counterweight Buffer Stop with Car Ascending.* A diagram of the configuration of the elevator with the counterweight buffer just striking the bumper blocks is shown in Fig. 17*a*. All parts at this instant have the initial velocity V_0 in the direction of the arrows. Since the various stopping agencies mentioned in Par. 80, at least at the time of the field test, are out of commission, a plain buffer stop is characterized by the fact, which should be kept in mind, that the machine keeps on running at full speed in the direction of the arrow during the entire stop.

83 The action of the counterweight buffer begins at the instant the cylinder strikes the bumper blocks in the pit and on account of the retardation impressed on the counterweight the hoist ropes *ab* will be relieved of at least part of the tension due to it. This causes slip of the ropes relative to the driving sheave, and it is in this manner that the buffer initiates the retardation of the ascending car.

84 A buffer suitable for actual elevator service must satisfy two requirements. The first is: The force which the buffer impresses on the counterweight must be of such magnitude that the tension at the end *b* of the hoist ropes is zero during the entire stop. This important requirement is obvious. For, when a plain counterweight buffer stop is about to take place the ascending car is already a certain distance above the top terminal, and in view of the limited clearance above it, it must be brought to a stop in the shortest possible distance. This is only accomplished when there is no tension at the counterweight end of the hoist ropes during the stop of the car. It is, of course, impossible so to control the motion of the counterweight buffer that the ropes just remain taut and at the same time have zero tension at *b*. In practice, then, this first requirement cannot be satisfied unless a small amount of slack hoist rope is permitted to develop as shown at *b* of Fig. 17*a*.

85 It will be demonstrated later that it is desirable to have the buffer bring the counterweight to rest shortly before or at the instant the car attains zero speed in the highest point of ascent. But with slack rope the car immediately falls an amount equal to the slack. This is not a free fall, for the falling car must drag the hoist ropes over the driving sheave which is still running in the direction of the arrow of Fig. 17*a*. In doing so the tractive effort due to the weight of the ropes *ab* retards the fall. Nevertheless, unless the amount of slack is small, the falling car will have attained considerable speed at the instant the ropes become taut and the resulting shock will cause excessive stresses. (See Part V.)

86 The amount of slack hoist rope under the conditions of Par. 94, neglecting the elasticity of the ropes, will be the difference

between the stopping distance of the ascending car and the stroke of the counterweight buffer. This introduces the second requirement, which is: The stroke of the counterweight buffer must be shorter than the stopping distance of the ascending car by an

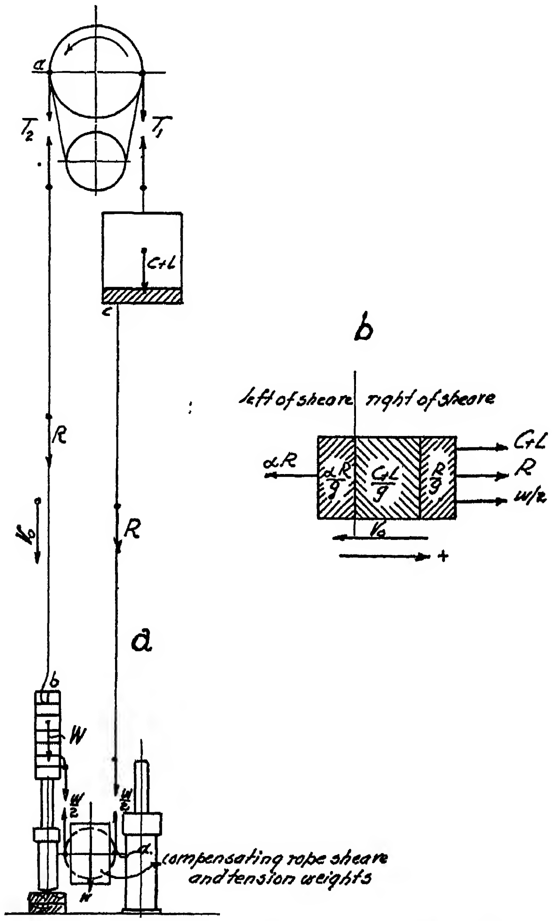


FIG. 17 (a) COUNTERWEIGHT-BUFFER STOP—CAR ASCENDING; (b) DYNAMIC EQUIVALENT OF SYSTEM COMPRISING ROPES (*ab*), CAR AND COMPENSATING ROPES (*cd*)

Weight of hoist ropes (*ab*) = *R*. Weight of compensating ropes (*cd*) = *R*.

amount which is small enough to permit the subsequent fall of the car to pick up the corresponding amount of slack hoist rope without undue stresses in the apparatus.

87 The above analysis discloses the following interesting points:

a The function of the counterweight buffer is promptly to initiate the retardation of the ascending car

b The counterweight buffer can prolong the stopping distance of the ascending car, but there is a fixed minimum beyond its control. This minimum depends on the details of the elevator and is associated with slack rope (zero tension) at the counterweight end of the hoist ropes

c A suitable counterweight buffer, because of the formation of slack rope as indicated in Fig. 17*a*, splits the elevator during the stop in two independent systems; one comprising the ascending car and the other the counterweight as the chief member

d Whether or not a counterweight buffer is suitable for elevator service cannot be determined without knowing the details of the motion of the ascending car; for it is the latter which determines what force the buffer must exert on the counterweight so that the hoist rope does become slack and what the stroke of the buffer should be.

88 Returning to the problem in hand, the present object is to determine the stopping distance of the system which comprises the ascending car and the parts moving with it, such as the hoist ropes *ab* and the compensating ropes *cd*, Fig. 17*a*. Since the amount of slack rope developed during the stop should be very small there will be only a small vertical displacement of the compensating-rope tension sheave, whose effect on the above system is for all practical purposes has been shown to be a force $\frac{1}{2}w$.

89 To determine the system dynamically equivalent to the system just named, it is necessary to know which of the tensions T_1 and T_2 is the smaller. Evidently it is T_2 for the machine keeps on running at full speed while the velocity of the car decreases so that the friction experienced by the hoist ropes at their contact with the driving sheave acts in a direction to assist T_2. Thus the equivalent system (Fig. 17*b*) will be obtained by taking forces and masses situated at the left of the driving sheave at α times their actual value and those on the right as they are. Choosing in Fig. 17*b* the direction of $(C+L)$, which is the direction of the acceleration impressed on the equivalent system as positive,

$$\left.\begin{array}{l} \Sigma M = \dfrac{(C+L)+(\alpha+1)R}{g}; \quad \Sigma P = C+L-(\alpha-1)R+w/2 \\ \text{whence} \\ \qquad A_c \text{ (car fully loaded)} = \dfrac{C+L-(\alpha-1)R+w/2}{C+L+(\alpha+1)R}\, g \end{array}\right\} \quad [19]$$

90 The acceleration is evidently constant and hence the stopping distance of the car is:

$$S_c \text{ (car fully loaded)} = \frac{V_0^2}{2A_c} = \frac{V_0^2}{2g}\,\frac{C+L+(\alpha+1)R}{C+L-(\alpha-1)R+w/2} \quad [19a]$$

When the ascending car is empty put $L = 0$ in [**19a**] and obtain:

$$A_c \text{ (car empty)} = \frac{C-(\alpha-1)R+w/2}{C+(\alpha+1)R}\, g \quad \ldots\ldots \quad [\mathbf{19b}]$$

$$S_c \text{ (car empty)} = \frac{V_0^2}{2A_c} = \frac{V_0^2}{2g}\,\frac{C+(\alpha+1)R}{C-(\alpha-1)R+w/2} \quad . \quad [\mathbf{19c}]$$

91 These equations show that the stopping distance of the ascending car is greatest when the plain counterweight buffer stop takes place with the car empty; and the smaller C is — all other conditions remaining the same — the larger is S_c. Also the greater R is, that is the greater the rise is — all other conditions remaining the same — the greater will be S_c. Indeed S_c for the ascending empty car (Equation [**19c**]) becomes infinite when

$$C-(\alpha-1)R+w/2 = 0$$

When this relation exists the counterweight buffer has lost even the capacity to initiate the retardation of the ascending empty car, for ($S_c = \infty$) denotes evidently nothing but that the car continues to ascend at undiminished speed as long as the machine rotates. Writing the above limiting condition in the form

$$C+R+w/2 = \alpha R$$

it is seen that it expresses that the weight of the hoisting ropes *ab*, Fig. 17*a*, is such that it balances at the point of slip the weight of the empty car, the weight of the compensating ropes *cd*, and the pull on the latter due to the tension sheave and weights. While the action of the buffer always reduces the maximum tractive effort the machine can exert, the remainder is sufficient to maintain the motion of the ascending car at full speed. There is no fear of meeting this situation in practice at the present time, for buildings are not nearly high enough. Nevertheless, the tractive effort which remains after the engagement of the buffer is by no means negligible, even where the rise of the elevator is moderate, and for this reason the stopping distance of the ascending member at a plain buffer stop is considerable.

92 *Car Buffer Stop — Counterweight Ascending.* The position of the elevator during the plain car buffer stop when the counterweight is the ascending member is shown in Fig. 18*a*, and under the conditions of zero tension at *b*, as assumed for the counterweight buffer stop, the desired stopping distance of the ascending counterweight will be found by replacing $(C+L)$ in Equations [**19**] and [**19a**] by W. This gives

$$A_w = \frac{W-(\alpha-1)R+w/2}{W+(\alpha+1)R}\, g \quad \ldots\ldots\ldots \quad [\mathbf{20}]$$

$$S_w = \frac{V_0^2}{2A_w} = \frac{V_0^2}{2g}\,\frac{W+(\alpha+1)R}{W-(\alpha-1)R+w/2} \quad \ldots \quad [\mathbf{20a}]$$

93 These equations show that the stopping distances decrease upon a decrease in the traction constant a, which is just the opposite of what was found for the potential-switch stop. It is further seen that the stopping distances increase with an increase in the weight

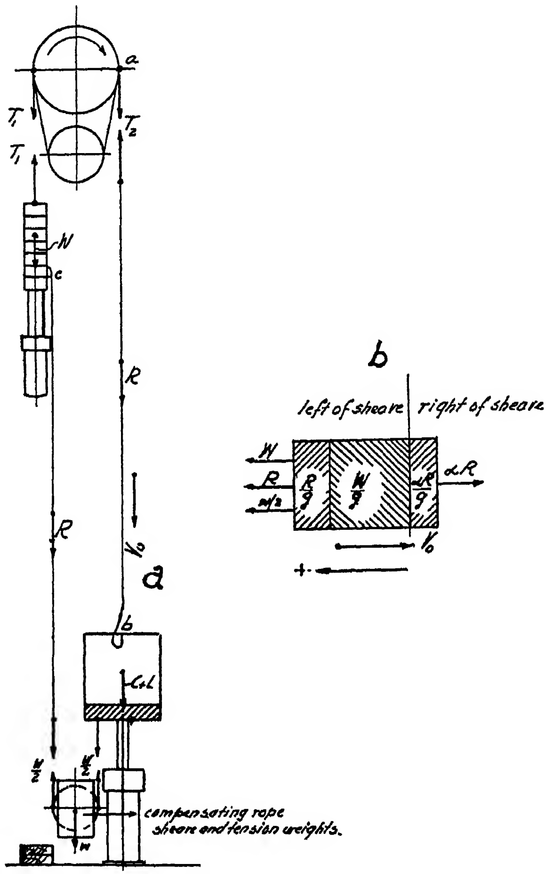


FIG. 18 (a) CAR-BUFFER STOP — COUNTERWEIGHT ASCENDING; (b) DYNAMIC EQUIVALENT OF SYSTEM COMPRISING ROPES (ab), COUNTERWEIGHT AND COMPENSATING ROPES (cd)

Weight of ropes (ab) = R. Weight of compensating ropes (cd) = R.

of the ropes, whereas the stopping distances associated with a potential stop are practically independent of it.

94 *Special Case.* The tractive effort due to the weight of the hoist ropes is negligible for very small rises and the stopping distances of the ascending member under such condition will be obtained by the substitution of $(R = 0)$ in Equations [**19a**],

[**19c**], and [**20a**]. This stopping distance will be found in all cases to be slightly in excess of $V_0^2/2g$, that is slightly in excess of the gravity height corresponding to the speed V_0. When the rise of the elevator is small, the elasticity of the ropes will also be small and only a very little slack rope can be permitted to develop during the stop. Under these conditions therefore requirement (2), Par. 95 will be satisfied by arranging the buffer stroke S according to the formula

$$S = V_0^2/2g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [\mathbf{21}]$$

This is the formula which has been employed in the calculation of the stroke of the car buffer ever since the introduction of the gearless traction machine. For reasons unknown to the author the stroke of the counterweight buffer is frequently taken at from 10 to 25 per cent less. The fact is, however, that in this type of elevator the weight of the ropes, even for comparatively moderate rises, is far from negligible and the use of Equation [**21**] as a standard formula to determine the buffer stroke is not justified. There is only one rational method of determination and that is to base it on the stopping distance of the ascending member and the amount of slack hoist rope which can be allowed.

95 *Examples 3 and 3a.* The details of the elevator are the same as in Example 1, Par. 74. Calculate and discuss

a A plain counterweight buffer stop with the ascending car loaded as well as empty for a rise of 400 and 200 ft. respectively

b A plain car buffer stop for a rise of 400 and 200 ft.

The buffers in all cases are struck at governor tripping speed ($V_0 = 16$ ft. per sec.).

96 *Solution* (*a*). For a rise of 400 ft. as in Example 1, Par. 74, ($R = 1490$ lb.) and from Equations [**19**], [**19a**], [**19b**], and [**19c**], when ascending fully loaded

$$A_c = \frac{4500+2500-1490+400}{4500+2500+3\times1490}\,g = 0.515g$$

$$S_c = \frac{16^2}{2\times0.515g} = 7.74\text{ ft.}$$

and when ascending empty

$$A_c = \frac{4500-1490+400}{4500+3\times1490}\,g = 0.38g$$

$$S_c = \frac{16^2}{2\times0.38g} = 10.48\text{ ft.}$$

For a rise of 200 ft. ($R = 745$ lb.), when ascending fully loaded

$$A_c = \frac{4500+2500-745+400}{4500+2500+3\times 745}\,g = 0.72g$$

$$S_c = \frac{16^2}{2\times 0.72g} = 5.53 \text{ ft.}$$

and when ascending empty

$$A_c = \frac{4500-745+400}{4500+3\times 745}\,g = 0.615g$$

$$S_c = \frac{16^2}{2\times 0.615g} = 6.5 \text{ ft.}$$

Solution (*b*). For a rise of 400 ft.

$$A_w = \frac{5500-1490+400}{5500+3\times 1490}\,g = 0.442g$$

$$S_w = \frac{16^2}{2\times 0.442g} = 9.0 \text{ ft.}$$

and for a rise of 200 ft.

$$A_w = \frac{5500-745+400}{5500+3\times 745}\,g = 0.665g$$

$$S_w = \frac{16^2}{2\times 0.665g} = 6.0 \text{ ft.}$$

The Oil Buffer as an Independent Emergency Terminal Stopping Device

97 When the car is the ascending member the stopping distances when empty have been shown to be 10.48 ft. for the 400-ft. rise and 6.5 ft. for the 200-ft. rise. These are the greatest stopping distances at a plain counterweight, buffer stop, and if such a stop is to be provided for, it will be necessary to arrange the top clearances for the car accordingly. Attention is called to the very considerable difference in the above two stopping distances brought about only by a difference of 200 ft. in the rise.

98 Assume in each case that the car is 1.0 ft. above the top floor when the counterweight buffer lands on the bumper blocks in the pit and allow a clearance of 0.5 ft. above the car in the highest point of the ascent, the top clearances for the car for a plain counterweight buffer stop at governor tripping speed will be:

$$10.48+1.0+0.5 = 11.98 \text{ ft. for 400-ft. rise}$$
$$6.5\ +1.0+0.5 = 8.0 \text{ ft. for 200-ft. rise.}$$

Both of these, and in particular the first, exceed the customary allowance of practice which rarely is more than 7.0 ft. for elevators of the normal speed of examples 3 and 3*a*.

99 Equation [**19a**] can be written in the form

$$S_c \text{ (empty car)} = \frac{V_0^2}{2g} \cdot \frac{1+(a+1)R/C}{1-(a-1)R/C+w/2C}$$

and as w is always small as compared with C, the stopping distance of the ascending empty car at a plain counterweight stop depends practically altogether on the ratio R/C. In Example $3a$ the latter was $745/4500 = 16.55$ per cent. Thus from the point of view of the required top clearance for the car: (1) The customary top clearances for the car are inadequate for a counterweight buffer stop from governor tripping speed when the weight of the hoist ropes is about 17 per cent or more of the weight of the empty car; (2) The counterweight buffer alone as an independent emergency terminal stopping device is inadequate to stop the car from governor tripping speed within the customary top clearances unless the weight of the ropes is approximately less than 17 per cent of the weight of the car. From these it follows that unless the weight of the ropes is comparatively small relative to the weight of the car, a plain counterweight buffer stop at governor tripping speed, and in case of high-rise elevators even at normal speed, must not be permitted to occur. That is to say, other devices in addition to the counterweight buffer must be employed.

100 In further explanation of these statements it must be emphasized that the buffer, irrespective of the rise of the elevator, and when properly designed, is always an excellent terminal stopping device for the descending member of the elevator which engages it. The inadequacy of the buffer in certain cases is only with respect to the ascending member. It must also be borne in mind that the stopping distances calculated above are the very minimum that can be realized. The buffer cannot possibly reduce them, but, on the contrary, will increase them if incorrectly designed.

101 The additional devices mentioned under (3) in Par. 108 are solely needed to reduce the stopping distance of the ascending member, and as such — as is shown later — limit switches will be found generally sufficient for the purpose.

The Buffer Stroke

102 In view of the above conclusions there is little profit in discussing what stroke the buffers of the elevators of Examples 3 and $3a$ should have. It is more instructive to discuss a plain buffer stop at governor tripping speed with buffers of a stroke according to Equation [**21**]. In the present examples this stroke would be

$$S = 16^2/2g = 4.0 \text{ ft. (approx.)}$$

103 The amount of slack rope — neglecting the small elastic contraction of the hoist ropes when relieved of the tension normally

due to the member which engages the buffer — will be the difference between the stopping distances of the ascending member and the stroke of the buffer. For the elevators of Examples 3 and 3*a*, with the plain buffer stop at governor tripping speed, the amount of slack hoist rope is as follows:

Ascending car fully loaded.	7.74 — 4 = 3.74 ft.	for 400-ft. rise
	5.53 — 4 = 1.58 ft.	for 200-ft. rise
Ascending car empty.	10.48 — 4 = 6.48 ft.	for 400-ft. rise
	6.5 — 4 = 2.5 ft.	for 200-ft rise
Counterweight ascending.	9.0 — 4 = 5.0 ft.	for 400-ft rise
	6.0 — 4 = 2.0 ft.	for 200-ft. rise

It is through these distances that the ascending member must fail to pick up the slack.

Buffer Tests in the Field

104 Buffer tests in the field are frequently so conducted that the result is a plain buffer stop. For elevators of Examples 3 and 3*a* such tests at governor tripping speed and with the customary buffers will be associated with the amounts of slack hoist rope given in Par. 112. They are very considerable for the high-rise elevator and it is plain that the fall of the ascending member through the given distances will set up dangerous stresses in all parts of the apparatus. For a rise of 200 ft. the falling distance is less than half as large. But also the elasticity of the ropes will be only half the amount of the high-rise elevator and the stresses will therefore still be considerable. For the calculation of these stresses the reader is referred to Part V.

105 It is fortunate that most codes require buffer tests at normal speed only; but even then with moderate- and high-rise elevators considerable amounts of slack rope develop.

106 Of course there must be slack rope at a buffer test to make sure that the buffer has been subjected to the full load and impact of the member engaging it, but the slack should be limited to an amount which will keep the machine, its supports, ropes, car, and counterweight frame free from the excessive stresses that now frequently are imposed upon them. It is a crude performance and, as will be shown later, wholly unnecessary.

Simultaneous Potential-Switch and Buffer Stop

107 The symbols used in the following development will be the same as in Par. 88 except that stopping distances and retardations will be denoted by small letters, s_c, s_w, a_c, a_w, instead of S_c, S_w, A_c, and A_w. In Par. 87 it has been stated that limit switches alone are never adequate emergency terminal stopping devices with respect to the customary top clearances for the ascending member, as well as to the customary bottom over-travels for the descending member of the elevator. In Par. 109 it has been shown

on the other hand that the oil buffer alone — provided it is well designed — is always an excellent device to stop the descending member in case of an emergency, but it falls short as a stopping device for the ascending member with respect to the customary top clearances unless the ratio R/C is very small. It is proper now to investigate the conditions if both of these devices become operative at the same instant. For this purpose it will be assumed that the car is ascending at speed V_0 when the normal terminal stopping device fails. The car over-runs the top landing, opens the upper limit switch and thereby initiates a potential-switch stop, which becomes effective say 0.15 sec. later. At the very same instant the counterweight buffer strikes the bumper blocks and there occurs what will be termed a simultaneous potential-switch and buffer stop. The position at the instant the stops begins is again that of Fig. 17*a*, except that in the present case the driving sheave of the machine is quickly brought to a stop by the opening of the potential switch.

108 As to the buffer, it is obvious that requirements 1 and 2 of Par. 93 and 95 also apply here for the same reasons. Also the formation of slack hoist rope at *b*, Fig. 17*a*, splits the elevator during the stop into two independent systems, and the present object is to determine the stopping distance of one of these, comprising the hoist ropes *ab*, the ascending car and the compensating ropes *cd*. Now the fact that the driving sheave is quickly brought to rest means that the direction of slip of the hoist ropes with respect to the driving sheave will just be the opposite of the direction of slip in Par. 98. We shall therefore obtain the desired equations from those of [**19a**], [**19b**], [**19c**], [**20**], and [**20a**] by replacing α by the reciprocal $1/\alpha$. Thus, when the car is the ascending member,

$$a_c \text{ (car loaded)} = \frac{\alpha(C+L)+(\alpha-1)R+\alpha w/2}{\alpha(C+L)+(\alpha+1)R}\, g \quad \ldots\ldots \quad [22]$$

$$s_c \text{ (car loaded)} = \frac{V_0^2}{2a_c} = \frac{V_0^2}{2g}\cdot\frac{\alpha(C+L)+(\alpha+1)R}{\alpha(C+L)+(\alpha-1)R+\alpha w/2} \quad [22a]$$

$$a_c \text{ (car empty)} = \frac{\alpha C+(\alpha-1)R+\alpha w/2}{\alpha C+(\alpha+1)R}\, g \quad \ldots\ldots\ldots \quad [22b]$$

$$s_c \text{ (car empty)} = \frac{V_0^2}{2a_c} = \frac{V_0^2}{2g}\cdot\frac{\alpha C+(\alpha+1)R}{\alpha C+(\alpha-1)R+\alpha w/2} \quad \ldots \quad [22c]$$

and when the counterweight is the ascending member

$$a_w = \frac{\alpha W+(\alpha-1)R+\alpha w/2}{\alpha W+(\alpha+1)R}\, g \quad \ldots\ldots\ldots \quad [23]$$

$$s_w = \frac{V_0^2}{2a_w} = \frac{V_0^2}{2g}\cdot\frac{\alpha W+(\alpha+1)R}{\alpha W+(\alpha-1)R+\alpha w/2} \quad . \quad [23a]$$

109 The important difference with the previous results is that here, since $\alpha > 1$, an increase in R, the rise of the elevator, will not indefinitely increase s_c or s_w. Indeed for $(R = \infty)$, Equations [22a], [22c], and [23a] give

$$s_c = s_w = \frac{\alpha+1}{\alpha-1}\,\frac{V_0^2}{2g}$$

and with $\alpha = 2$

$$s_c = s_w = \frac{3V_0^2}{2g}$$

which shows that for a simultaneous potential-switch and buffer stop the stopping distance of the ascending member cannot exceed 3 times the gravity height corresponding to V_0.

110 *Example 4.* For an elevator of the same details as in Example 1, Par. 74, determine for a simultaneous potential-switch and buffer stop at governor tripping speed ($V_0 = 16$ ft. per sec.) (*a*) the stopping distance when the car fully loaded, as well as empty, is the ascending member, and (*b*) the stopping distance when the counterweight is the ascending member.

111 *Solution* (*a*). Applying Equations [22], [22a], [22b], and [22c] for a car ascending fully loaded

$$a_c = \frac{2(4500+2500)+1490+800}{2(4500+2400)+3\times 1490}\,g = 0.863g$$

$$s_c = \frac{16^2}{2\times 0.863g} = 4.61 \text{ ft.}$$

and for a car ascending empty

$$a_c = \frac{2\times 4500+1490+800}{2\times 4500+3\times 1490}\,g = 0.84g$$

$$s_c = \frac{16^2}{2\times 0.84g} = 4.73 \text{ ft.}$$

Comparing these with the results of a plain buffer stop ($S_c = 7.74$ and 10.48 ft. respectively) it is seen that the combination of a potential-switch and buffer stop to become effective simultaneously brings about a very considerable and acceptable reduction in the stopping distance of the ascending car. Indeed the reduction is such that top clearances for the car according to good present-day practice or as allowed by the A.E.S.C. Code, rule 103, are more than ample.

112 *Solutions* (*b*), *Counterweight Ascending.* Applying Equations [23] and [23a]

$$a_w = \frac{2\times 5500+1490+800}{2\times 5500+3\times 1490}\,g = 0.86g$$

$$s_w = \frac{16^2}{2\times 0.89g} = 4.63 \text{ ft.}$$

as compared with 9.0 ft. in the case of a plain car buffer stop (Example 3).

113 In the above example it was assumed that both the potential switch and the buffer stop become effective at the same instant. This is a condition which cannot always be realized in practice. However, the conclusion can be drawn that reasonable stopping distances of the ascending member can be obtained only under the combined action of limit switches and buffers and not — as shown before — by either of them alone. In actual practice the buffer is generally engaged first and the limit switch then opened, so that the potential-switch stop is delayed with respect to the buffer stop. This condition will be dealt with later, but it is plain that the greater the delay, the more we approach the unsatisfactory conditions of a plain buffer stop, and the positioning of the limit switches thus assumes an importance not generally realized.

The Buffer Stroke

114 Examples 4 and 3 concern elevators of precisely the same details, but in the former buffer and limit switches bring about the stop, and in the latter the buffer alone. According to requirement 2, Par. 95, the car, buffer stroke must be determined in relation to the stopping distance of the ascending counterweight, which is 4.63 ft. in Example 4 and 9.0 ft. in Example 3. That is to say, in one case a car buffer with a stroke of about 4.0 ft. is required and in the other case one of nearly 9.0 ft. stroke. Thus the buffer stroke cannot be determined according to an invariable formula but depends on the stopping agencies which come into play.

Calculation of the Required Top Clearance for the Car on the Basis of a Delayed Potential-Switch and Buffer Stop

115 The purpose of the top clearance and bottom over-travel is to provide a run-way for the free play of the emergency stopping devices in the case of loss of control at the terminal landings. The bottom over-travel is immediately determined upon the selection of a car buffer of the proper stroke and the amount of over-run below the bottom terminal before the car strikes the buffer.

116 Of much greater importance is the top clearance, and it is therefore one of the objects of this paper to place the determination of it on a rational basis. Before such calculation is possible, however, the contingencies to be provided against and the terminal stopping agencies to be employed must be agreed upon. As to the former the author proposes for the present to assume a complete failure of the operating and normal terminal stopping devices associated with an over-speed equal to the tripping speed of the governor. The emergency terminal stopping devices here to be considered will be the buffers and limit switches, under the distinct

understanding that these devices coöperate with each other in stopping the elevator in the sequence dictated by the layout. If so, all the preliminary data for the calculations have already been derived, for an emergency terminal stop under the usual conditions of practice consists partly of a plain buffer stop and partly of a simultaneous potential-switch and buffer stop.

117 Since the calculation of the required top clearance for the counterweight is largely a repetition of what follows, attention will be confined to the determination of the top clearance for the car.

118 The situation is best understood with the aid of Fig. 19 which contains the velocity-time diagram of the stop of the ascending car. Zero of time is the instant at which the plain counterweight buffer stop begins with the cylinder of the counterweight buffer striking the bumper blocks (Fig. 5). When this occurs the car is A' feet above the top floor and the speed — for the duration of the plain buffer stop — decreases from the initial car speed V_0 according to the straight line $ABCD$ with the constant retardation A_c as expressed by Equation [19] or [19b] Note that the tangent of angle $CDF = A_c$. When the car is E' feet above the top floor it opens the upper limit switch. It takes x seconds for the car to travel the distance $(E'-A')$ and at the end thereof the car speed has decreased to v_1. As has been shown, there is a certain lag of time of, say, n seconds before the opening of the limit switch becomes effective in the rapid deceleration of the machine. It follows therefore that the plain buffer stop lasts $(x+n)$ seconds, at the end of which the car speed has decreased to v_2. From this instant on until the car attains zero speed the combined effect of buffer and limit switch is felt, that is a combined potential-switch and buffer stop with the constant retardation a_c of Equation [21] or [21b], and the velocity of the car decreases according to line CE (tangent of angle $CEO = a_c$). As the retardations are constant, the problem is simple.

119 The time x to travel the distance $(E'-A')$ at constant retardation A_c follows from

$$V_0x-A_cx^2/2 = E'-A' \quad . \; . \; . \; . \; . \; . \; . \quad [24]$$

and the speed v_1 of the car after x seconds is

$$v_1 = V_0-A_cx \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [24a]$$

The total duration of the plain buffer stop being $(x+n)$ seconds, the velocity v_2 at the end is

$$v_2 = V_0-A_c(x+n) \quad . \; . \; . \; . \; . \; . \; . \quad [24b]$$

and the distance S_c traveled during the $(x-n)$ seconds is

$$S_c = \frac{V_0+v_2}{2}(x+n) \quad . \; . \; . \; . \; . \; . \; . \quad [24c]$$

At the end of $(x+n)$ seconds the combined buffer and potential-

switch stop begins with the initial velocity v_2 and the constant retardation a_c. Hence the stopping distance s_c will be

$$s_c = \frac{v_2^2}{2a_c} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{24d}]$$

in time

$$t = v_2/a_c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{24e}]$$

The total stopping distance Σ_c of the ascending car is therefore

$$\Sigma_c = S_c + s_c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{24f}]$$

and the total time required for it

$$\tau = x + n + t \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\mathbf{24g}]$$

120 *Example 5.* For the elevator of Example 1, Par. 74, calculate for a counterweight buffer stop with delayed potential-switch stop from governor tripping speed (a) the stopping distance of the ascending car, and (b) the required top clearance for the car, under the following conditions: The elevator is equipped with variable-voltage control and is of the micro-drive type. For this reason the normal amount of over-run at the terminal landings is small and the upper limit switch will be set to be opened by the car when it is 1.25 ft. above the top landing. The counterweight buffer is arranged to strike the bumper blocks in the pit when the car is 1 ft. above the upper terminal. In the present example then $E' = 1.25$ and $A' = 1$. The governor is set to trip at a speed of 16 ft. per sec., whence $V_0 = 16$.

121 *Solution (a), Ascending Car Fully Loaded.* In Examples 3 and 4 there have already been found for the given conditions

$$A_c = 0.515g \quad \text{and} \quad a_c = 0.863g$$

Otherwise these retardations must be figured from Equations [**19**] and [**22**]. Thus from Equation [**24**]

$$6x - 0.515gx^2/2 = 1.25 - 1 = 0.25$$

whence

$$x = 0.0157 \text{ sec.}$$

For the time n which has to elapse between the opening of the upper limit switch and the effect in the rapid deceleration of the machine 0.15 sec. will be allowed, whence

$$x + n = 0.0157 + 0.15 = 0.1657$$

and by Equation [**24a**]

$$v_1 = 16 - 0.515g \times 0.0157 = 15.74 \text{ ft. per sec.}$$

by Equation [**24b**]

$$v_2 = 16 - 0.515g \times 0.1657 = 13.26 \text{ ft. per sec.}$$

by Equation [24c]

$$S_c = \left(\frac{16+13.26}{2}\right) 0.1657 = 2.421 \text{ ft.}$$

by Equation [24d]

$$s_c = \frac{13.26^2}{2\times 0.863g} = 3.17 \text{ ft.}$$

by Equation [24e]

$$t = 13.26/0.863g = 0.478 \text{ sec.}$$

by Equation [24f]

$$\Sigma_c = 2.42+3.17 = 5.59 \text{ ft.}$$

by Equation [24g]

$$\tau = 0.0157+0.15+0.478 = 0.644 \text{ sec.}$$

The loaded car therefore comes to rest in 5.59 ft. of which 2.42 ft. are consumed by the plain buffer stop and 3.17 ft. by the simultaneous buffer and potential-switch stop.

122 *Solution (a), Ascending Car Empty.* The retardations A_c and a_c may be obtained from Equations [**19b**] and [**22b**]. In the present example, however, the results of Examples 3 and 4 can be employed, wherein

$$A_c = 0.389g \quad \text{and} \quad a_c = 0.84g$$

Thus by Equation [**24**]

$$x = 0.0157 \text{ sec.}$$
$$x+n = 0.1657 \text{ sec.}$$

by Equation [24a]

$$v_1 = 15.81 \text{ ft. per sec.}$$

by Equation [24b]

$$v_2 = 13.98 \text{ ft. per sec.}$$

by Equation [24c]

$$S_c = 2.48 \text{ ft.}$$

by Equation [24d]

$$s_c = 3.63 \text{ ft.}$$

by Equation [24e]

$$t = 0.518 \text{ sec.}$$

by Equation [24f]

$$\Sigma_c = 2.48+3.63 = 6.11 \text{ ft.}$$

by Equation [24g]

$$\tau = 0.0157+0.15+0.518 = 0.684 \text{ sec.}$$

Thus the ascending empty car comes to rest in 6.11 ft.

123 *Solution (b).* Calculations show, as was to be expected, that the greater stopping distance of the ascending car occurs at no load, and the top clearance for the car must, of course, be arranged accordingly. Since the stop begins when the car is 1 ft. above the top landing, the minimum top clearance for it must be

$$6.11+1.0 = 7.11 \text{ ft.}$$

For the present elevator the A.E.S.C. Code, rule 103, allows a top clearance for the car of 8 ft., which still allows a margin of 0.89 ft.

The Selection of a Suitable Buffer

124 The next problem is the selection of a suitable counterweight buffer. If one of the type of Fig. 6 is employed the load on the buffer, according to Equation [**1a**], will be

$$Q = 5500+800/2 = 5900 \text{ lb.}$$

Or, if one of the type of Fig. 7 is used and the weight of the buffer cylinder and body is estimated at 500 lb., the buffer load according to Equation [**1**] is

$$Q = 5500+800/2-500 = 5400 \text{ lb.}$$

These will be the loads which the buffer will have to bring to a stop, provided that slack hoist rope at *b*, Fig. 17, actually develops from the very instant the buffer comes into action. It is also only under this proviso that the stopping distances of 5.59 and 6.11 ft. will be realized. Obviously if the condition of slack rope is obtained with the shorter of the two stopping distances it will also be the case with the longer. The former occurs when the car is fully loaded and it will therefore be supposed that the broken line *ACE* of Fig. 19 is the speed-time curve of the ascending fully loaded car as it comes to a stop. Obviously, the area comprised between *ACE* and the axes of reference is the stopping distance of the ascending car when fully loaded; this distance in the above example was calculated to be 5.59 ft.

125 According to Par. 95, the stroke of the buffer must be smaller than the stopping distance of the ascending car, and if there is to be slack rope at all, the difference between the two must at least be equal to the elastic contraction of the hoist ropes when they are relieved of the tension normally due to the weight of the counterweight. For six ⅝-in. ropes this contraction per 1000 lb. of weight and per 100 ft. of rope is approximately 0.138 in. The normal tension due to the counterweight is equal to the weight plus half the weight of the tension sheave for the compensating ropes, or 5500+800/2 = 5900 lb., and since the rise of the present elevator is 400 ft., the elastic contraction will be

$$5.9\times 4\times 0.138 = 3.25 = 0.271 \text{ ft.}$$

If an allowance of 0.8 ft. or nearly three times the theoretical amount is made, the stroke of the counterweight buffer will be

$$5.59-0.8 = 4.79 \text{ ft.}$$

126 Having determined the stroke of the buffer, the next object is to select a suitable graduation for it. It is supposed that a

number of parabolic graduations are available and that the properties of each of them have been determined, by the means developed in Part II, when the buffer is struck at speed $V_0 = 16$ ft. per sec. by a free weight, Q, equal to the load the buffer must stop under the proviso set forth in Par. 134. Most important among these properties is the speed-time curve associated with each of the graduations, of which, according to Par. 95, only such come into consideration as stop the weight Q in a time equal to or smaller than the time τ, Fig. 19, which the ascending loaded car requires to come to a stop. The speed-time curves; AB_1E_1 and AB_2E_2 of the type of Curve 3, Fig. 12; the straight line AE_3 characteristic of a constant retardation; and the curve AB_4E_4 of

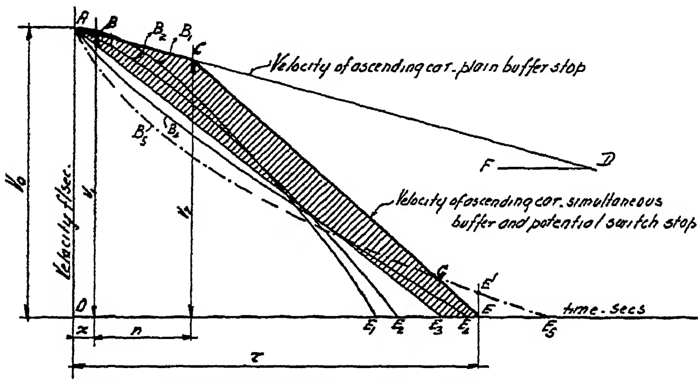


Fig. 19 Velocity-Time Diagram of the Stop of the Ascending Car

the type of Curve 1, Fig. 11a, are drawn in Fig. 19, and OE_1, OE_2, OE_3, OE_4 are the times in which the free weight Q comes to rest. To realize the condition of slack hoist rope from the very beginning of the buffer action it is obvious that the buffer must cause the speed of the counterweight to decrease from the initial speed V_0 more rapidly than the reduction in speed experienced by the ascending car. Hence, one of the criteria of the suitability of a graduation is that the initial retardation impressed on a free weight Q, equal to the weight which the buffer has to stop in actual elevator service, must exceed the initial retardation suffered by the car. In the present example the initial retardation impressed on the loaded ascending car is $A_c = 0.515g$, and if a buffer of the type of Fig. 6 is employed under the counterweight, the load Q to be stopped by it in actual elevator service is 5900 lb. Hence, only such graduations are suitable in which a retardation larger than $0.515g$ is impressed on a free weight of 5900 lb. at the instant it strikes the buffer at the initial speed of 16 ft. per sec. Furthermore, only such graduations come into consideration as will stop

the above free weight of 5900 lb. in times equal to or less than 0.644 sec., which, according to our calculations, is the time required for the loaded car to come to a stop. In general this means that the speed-time curves associated with desirable graduations must terminate at points between O and E, Fig. 19, and thus the following conclusion is drawn: Graduations suitable for use in actual elevator service must have their speed-time curves, with the exception of point A, Fig. 19, wholly contained within the speed-time curve of the ascending car. According to the foregoing, then, the graduation associated with curve AB_1E_1, Fig. 19, should not be used with a speed-time curve for the ascending car according to $ABCE$, although it may be a very desirable one under other conditions.

127 Applying the above criterion it is found that among the suitable graduations is the one whose speed-time curve is the straight line AE_3; and the area AE_3OA contained between it and the reference axes is, of course, the travel of the buffer, that is, the buffer stroke of 4.79 ft. of the above example. Similarly the area $ACEOA$ is the stopping distance of 5.59 ft. of the loaded ascending car. The difference between these areas, the hatched portion $ACEE_3A$, is evidently the total amount of slack hoist rope which will develop at the instant the car comes to rest in the highest point of ascent. In the present example, Par. 133, the total calculated amount of slack is 0.8 ft., neglecting the elastic contraction of the ropes. Now the speed-time curves AB_2E_2, AB_4E_4 are by hypothesis associated with a buffer of the same stroke as is curve AE_3, and hence the area comprised between either of these curves and the reference axes is the same. From this follows: All graduations with which a counterweight buffer of a given stroke is provided, having for the load they are to stop speed-time curves which, with the exception of a common starting point A, corresponding to the initial speed V_0, lie wholly below the speed-time curve associated with an emergency terminal stop of the ascending car, will develop the same amount of slack hoist rope when the latter comes to rest in the highest point of ascent.

128 Between the graduations of which AB_2E_2, AE_3, AB_4E_4 are the corresponding speed-time curves there is therefore no choice in so far as the total amount of slack hoist rope developed at the stop is concerned. Nevertheless AB_4E_4 is preferred. The reason is that the ropes have to contract before they can be relieved of the tension due to the weight of the counterweight. The quicker, therefore, the reduction in speed of the counterweight just after the buffer comes into action, the more will requirement 1, par. 93, be realized. From this it follows that speed-time curve AB_4E_e is the most desirable of all here considered.

129 The present buffer can, of course, be provided with a graduation such that the speed-time curve will be AB_5E_5. The

first part of this curve shows a very desirable rapid decrease in the speed of the counterweight and yet it is thoroughly objectionable. Since by assumption the buffer stroke associated with it is as before 4.79 ft., the area contained between AB_5E_5 and the reference axes will be the same as that contained between AE_3 and the axes. But at the instant E when the ascending car attains zero speed the amount of slack hoist rope will now be represented by the area $ACGB_5A$ minus $EE'GE$. This is obviously more than the cross-hatched area of Fig. 19 which represents in magnitude the amount of slack rope which will develop with either of the graduations associated with curves AB_2E_2, AE_3, or AB_4E_4. Hence the requirement (2) of Par. 95, that the buffer must bring the counterweight to rest before or at the instant the ascending car comes to rest.

Buffer Tests in the Field with Limit Switches Intact

130 Neglecting the elastic contraction of the hoist ropes, the amount of slack will be the difference between the stopping distance of the ascending car and the stroke of the counterweight buffer. In the above example this is:

$5.59-4.79 = 0.8$ ft. when the ascending car is fully loaded
$6.11-4.79 = 1.32$ ft. when the ascending car is empty.

Had a counterweight buffer of 4-ft. stroke been used in accordance with the best present-day practice, the corresponding figures should have been:

$5.59-4.0 = 1.59$ ft. with the ascending car fully loaded
$6.11-4.0 = 2.11$ ft. with the ascending car empty.

These are also the amounts of slack rope which develop at a counterweight-buffer test at the tripping speed of the governor if the upper limit switch is left intact. The fact that slack rope develops implies that the buffer has been subjected to full load and impact and thus a perfect buffer test in the field can be accomplished without the short-circuiting of the limit switches. By leaving the limit switches intact excessive amounts of slack rope and the excessive stresses set up in all parts when the ascending member falls and picks up the slack are avoided (see Par. 113). The practice of short-circuiting the limit switches should, therefore, be abandoned, for it is not sensible to expose the apparatus to excessive stress for the sake of a buffer test when it can just as well be made without doing so.

Conclusions

131 The foregoing shows that the top clearance for the car can be determined by calculation, and that the amount depends in the

first instance on two important points, (a) the particular emergency which is to be provided against, and (b) the emergency terminal stopping agencies which are to be brought in action upon the failure of the normal terminal stopping device. After this the factors determining the required amount of clearance at the top are:

1 The weight of the car, load, counterweight and ropes
2 The layout in so far as it gives the angle of contact between ropes and driving sheave, that is, the traction constant
3 The speed at which the emergency happens.

If the terminal stopping agencies referred to under (*b*) are buffers and limit switches, it has been found that the latter alone are never adequate protection against exceeding the limits of the travel unless there is, because of special conditions, an abnormal amount of over-head room. The same applies to oil buffers except when the weight of the cables is negligible as compared with that of the car. The important conclusion is therefore drawn that the safety at the terminal landings requires the functioning of both of these devices. If so, the required top clearance further depends on:

1 The type of control, which in turn determines
2 The location of the limit switches, that is, the distance *E* the car can travel beyond terminals before engaging the limit switches
3 The time which elapses before the opening of the limit switches becomes effective in the rapid retardation of the machine.

132 Under all considerations it is absolutely necessary that the machine be capable of exerting the retardation torque on the driving sheave which will cause the cables to slip. One of the most important points concerns the location of the limit switches; and in order to get the greatest benefit from them, they should be opened upon as small an amount of over-travel of the car as is possible. A solution of this problem requires that the elevator determine for itself what is or what is not inconsequential over-run. This can be accomplished as follows: Fig. 20 shows diagrammatically a car flush with the bottom landing; a cam 1 on it adapted to strike the roller of limit switch 2 upon an over-travel *E*. Coöperating with the limit switch 2 is a cylinder 3, having a piston 4 held in the position shown by a spring 5. The cylinder 3 is connected with the working cylinder of the buffer and is therefore subject to the pressure within it. If this pressure is high enough it will overcome the resistance of the spring 5 and the rod of piston 4 will push the limit switch open. The initial pressure in the buffer cylinder varies as the square of the speed with which the buffer is struck by the car and this speed is in turn a measure of whether or not the particular failure, which caused the over-run,

is harmless. Thus by the simple adjustment of the spring 5 the speed can be set so that an opening of the limit switch will occur coincidentally with the striking of the buffer by the car and without waiting for sufficient over-run to develop to cause the cam 1 to engage the limit switch. If, upon an operation of the device, the elevator comes to rest with an over-travel smaller than E, the pressure in the buffer sinks to zero, whereupon the spring 5 will force the oil in cylinder 4 back into the buffer. This permits the usual spring on the limit switch to close its contacts again, whereby

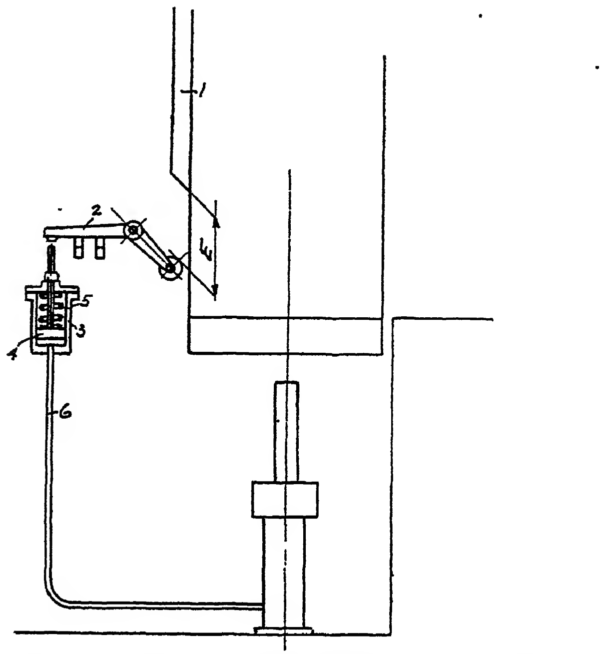


Fig. 20 Hydraulic Speed-Responsive Device to Stop Machine

the elevator is restored to operative conditions. Only when the over-travel of the car is E or more will the limit switch be held open by the cam, thereby indicating that a more or less serious failure of the normal terminal stopping device has occurred, which should be investigated. With this device, then, the limit switches can be set so that there is not the faintest possibility of interference with normal service; at the same time it makes the location of the limit switches entirely independent of the type of control.

133 In a discussion of the particular emergency on which the calculation for the top clearance should be based, it is instructive to tabulate the results already obtained in Examples 2, 3, and 4.

Common in the three cases of Table 2 is the assumed complete failure of the operating and normal terminal stopping devices together with an over-speed of 20 per cent, equal to the governor tripping speed. In Case 2 there has also been assumed the failure of the limit switch, and in Case 3 that of the buffer.

134 The emergency terminal stopping devices serve the purpose of protecting the elevator against loss of control at the terminals. Such loss of control has happened and it is reasonable, therefore, to provide against a complete failure of the normal terminal stopping device. But is it reasonable to assume simultaneously with it a failure of either the limit switches or the buffers?

135 To answer this question it is necessary to turn to practice, for, when there are thousands of elevators in daily operation over periods of time varying from yesterday to more than 20 years ago, the lack of safety from one cause or another would have become manifest long ago.

TABLE 2

Details of elevator: Weight of car = 4500 lb, weight of load = 2500 lb, weight of counterweight = 5500 lb., rise = 400 ft., number and size of ropes, six $\frac{5}{8}$-in., normal speed = 800 ft per min., governor tripping speed = 960 ft. per min.

Kind of emergency stop	Brought about by	Top clearance for car, calculated for ascending car empty
Case 1. Buffer stop with delayed potential-switch stop, Example 5.	Complete failure of normal terminal stopping device at 20 per cent over-speed.	8 ft.
Case 2. Plain buffer stop, Example 3.	Complete failure of normal terminal stopping device and failure of limit switch at 20 per cent over-speed.	11.98 ft.
Case 3. Potential-switch stop, Example 2.	Complete failure of normal terminal stopping device and failure of oil buffer at 20 per cent over-speed	16.46 ft. plus over-travel to open limit switch.

136 Case 3 of Table 2 can be disposed of first, for the evidence against it is clear. It assumes the simultaneous failure of the normal terminal stopping device and of the oil buffer, and the stop of the elevator is therefore brought about by the limit switches alone. The assumption of these simultaneous failures is, on the face of it, unreasonable, for the oil buffer is beyond question the most reliable terminal stopping device, while the extensive experience referred to has proved that the normal terminal stopping device is a close second to it. However, it has been shown that the limit switches alone are inadequate to stop an elevator from normal as well as from governor tripping speed within the customary clearances at the top and the over-travels at the bottom. In view of the abundant experience, these elevators may be called safe, and the inevitable conclusion is reached that Case 3 must be eliminated from the range of possibilities.

137 The situation in Case 2 is similar. Here is assumed the simultaneous failure of the normal terminal stopping device and

of the limit switches, which result in a plain buffer stop. It has been shown, however, that the usual top clearances are inadequate for this particular emergency at governor tripping speed for elevators of a rise of roughly 200 ft. and over. Thus again accumulated experience with such elevators eliminates also Case 2, at least at governor tripping speed.

138 This leaves Case 1, and a similar investigation forces the conclusion that even the failures assumed for it are too severe; that is to say, if a complete failure of the normal terminal stopping device is assumed, it is excessive to assume in addition as large an over-speed as the governor tripping speed. Indeed, the governor tripping speed is the highest possible speed the elevator can attain, notwithstanding which it has no place in the present problems unless there exist a connection between it and the safety of the elevator at the terminals.

139 There is obviously no justification to reckon with considerable over-speeds at the terminals unless it is a fairly common experience to have them occur in the regular service between floors, which, however, is not the case. Furthermore, if the adjustment of the governors of all elevators of the type here considered is changed — a matter of a few turns of a nut — so that they trip at a higher speed, will the safety of these elevators at the terminals be decreased and will there be accidents at these points where there were none before? Evidently not. The facts in the matter are that if complete failure of the normal terminal stopping device is assumed, a situation is assumed which is highly unlikely to occur. The author therefore proposes that the top clearances for the car be figured on the following uniform basis: The top clearance for the car shall be calculated for a complete failure of the normal terminal stopping device at an over-speed of 10 per cent above normal, with buffers and limit switches operative in the sequence permitted by the conditions.

140 Finally, a word about buffers: Various codes require the submission of buffers to test by a competent authority and authorize a range of loads and speeds for them. The author suggests that the design of a buffer depends on load and speed alone only when the former is a free weight. For service with an elevator, however, the stroke as well as the graduation depends on a complete knowledge of the motion of the ascending member of the elevator, and this in turn depends on a host of factors which vary from case to case and of which the approving authority has no fore knowledge. To determine the suitable range for free weights is of little service since it has been amply shown that a buffer may be eminently satisfactory for a free weight and yet wholly unsuitable if the same weight is either the car or the counterweight of an elevator.

PART V

SLACK HOIST ROPE

141 From the foregoing it is apparent that the emergency stops of the gearless traction elevator when they occur at high speeds are nearly always associated with slack hoist rope and followed by a fall of the ascending member through a corresponding distance. In the present section the forces and stresses associated with it will be determined, so that the designing engineer may know the magnitude of the forces which the apparatus may be called upon to sustain and also for the purpose of acquainting those who conduct field tests with just what the present crude methods involve. To prevent needless duplication only the case of the ascending car will be presented. There are three distinct stages in the present problem.

142 In the first the counterweight has come to rest on the fully compressed buffer, Fig. 21*a*, and the car is at zero speed in the highest point of ascent. The car falls H feet to pick up the slack; H being the difference between the stopping distance of the car and the stroke of the counterweight buffer minus the elastic contraction of the ropes. The object is to determine the speed v_c of the car.

143 It is immaterial in the present case whether the machine continues to run in the direction of the arrow or comes to rest; the falling car in either case drags the hoist ropes over the driving sheave and the moving system comprises the hoist ropes *bacd* — of which the portion *cd* will be neglected — the loaded car, and the compensating ropes *ef*, while the effect of the tension sheave with its weights will be sufficiently approximated by the two forces $\frac{1}{2}w$ as shown. Employing the equivalent system, the acceleration impressed on the falling car is found to be:

$$\left.\begin{aligned} a_c &= \frac{(C+L)-(\alpha-1)R+w/2}{C+L+(\alpha+1)R}\,g \\ \text{whence} \qquad v_c &= \sqrt{2a_cH} \end{aligned}\right\} \quad \ldots\ldots \quad [25]$$

At the instant the car attains the speed v_c the ropes are just taut with zero tension at b, Fig. 21*a*. The tension T_2 at a on the other hand must be such as to impress on the rope ab the upward acceleration a_c, and hence

$$T_2 = R(1+a_c/g)$$

The average tension T_{av} in the rope ab or the actual tension in the center cross-section is one-half of the sum of the tensions at a and b and the magnitude is therefore $\frac{1}{2}T_2$. However, as the quantity (a_cR/g) is very small as compared with the tensions developed later, only a negligible error is committed by placing

$$T_{av} = R/2 \quad \ldots\ldots\ldots \quad \mathbf{[25a]}$$

144 The second period begins with the hoist ropes just taut with the velocity of the car and the average tension in the ropes *ab* as expressed by Equations [25] and [25a]. For the calculation of this and the following periods the elasticity of the system which principally resides in the hoist ropes *ab* can no longer be neglected. Fig. 21*b*, therefore shows the position of the system at the end of the first and at the beginning of the second period of motion, with the ropes *ab* replaced by a massless spring of equal elastic properties. The tension in this spring will be the same as the actual tension in the center section of the ropes *ab*, while the mass of the ropes is considered by concentrating one-half of their weight at each of the ends as shown in Fig. 21*b*.

145 During this second period the car keeps on falling until the tension T in the spring reaches the value

$$T = W+R/2+w/2 \quad . \; . \; . \; . \; . \; . \; . \quad [25b]$$

at which time it balances the forces acting on the counterweight and associated parts. Any increase in the spring tension above this value will cause an upward movement of the counterweight.

146 The system here under consideration comprises the weight $R/2$ on which the spring tension T acts, a portion of the rope, shown as *b′acd*, whose weight is again neglected, the loaded car, the compensating ropes, and the force $\frac{1}{2}w$ at the end. The equivalent system is shown in Fig. 21*b′*, from which the resultant force is

$$\left.\begin{aligned} &\Sigma P = C+L-\left(\frac{\alpha}{2}\quad 1\right)R+w/2-\alpha T \\ &\text{and the aggregate mass is} \\ &M_c = \frac{C+L+\left(\frac{\alpha}{2}+1\right)R}{g} \end{aligned}\right\} \quad . \; . \; . \; . \quad [26]$$

During the present period the car travels the distance s'_c as indicated by the dotted line in Fig. 21*b*, and the speed decreases from v_c at the beginning of the period to v'_c at the end. Therefore, according to the energy equation

$$\tfrac{1}{2}[v'^2_c - v_c^2]M_c = \text{work done by external forces} \quad . \; . \quad [27]$$

147 The external force as expressed by Equation [26] has a constant component

$$P = C+L-\left(\frac{\alpha}{2}-1\right)R+w/2 \quad . \; . \; . \; . \; . \quad [28]$$

and a component $-\alpha T$ in which T varies from the value of Equation [25a] at the beginning of the period to that of Equation [25b] at the end. The mean spring tension T_{av} during this period is half the sum of the two, or

$$T_{av} = \frac{W+R+w/2}{2} \quad . \; . \; . \; . \; . \; . \; . \quad [28a]$$

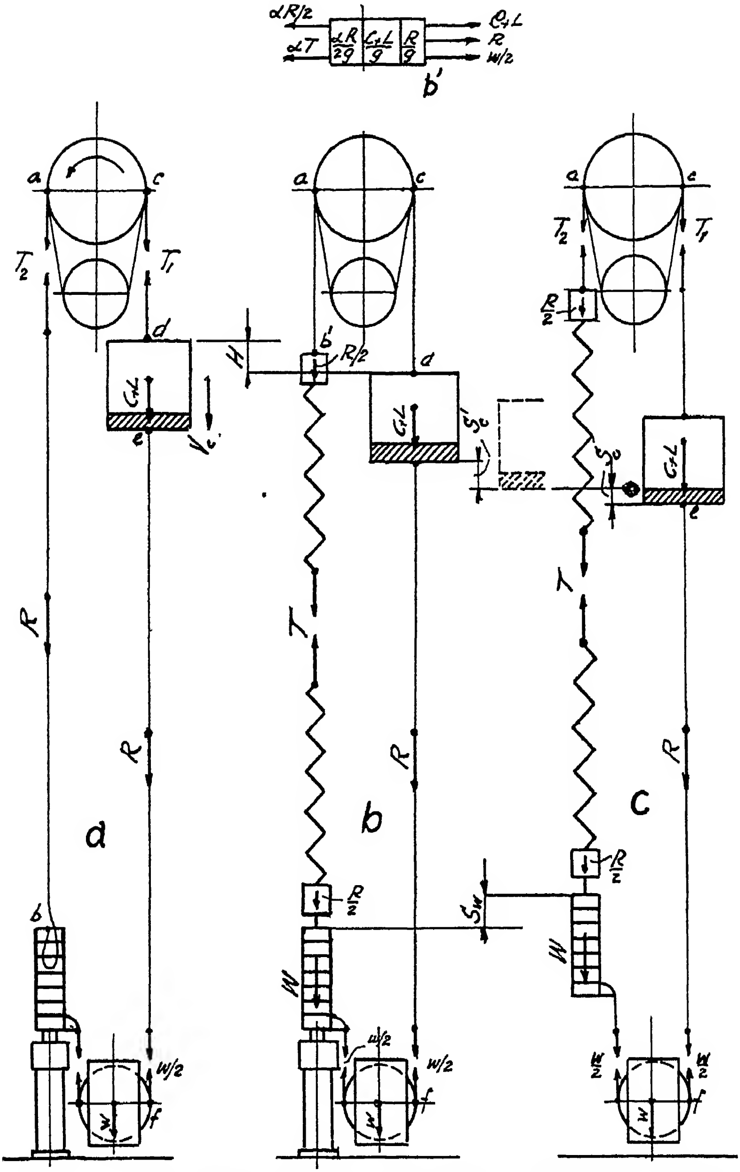


FIG. 21 THREE STAGES IN EMERGENCY STOP OF GEARLESS TRACTION ELEVATOR

(a) Car falling to pick up slack.
(b) Hoist ropes just taut.
(c) Counterweight pulled upward.

The average external force during this period is therefore $(P-\alpha T_{av})$ and if s'_c is the displacement of the car,

$$\text{the work done by external forces} = (P-\alpha T_{av})s'_c \quad [\mathbf{28b}]$$

148 But if s'_c is the displacement of the car while the counterweight remains at rest, it is also the increase in elongation of the spring, Fig. 21*b*, when the tension increases from that of Equation [**25a**] at the beginning of the period to that of Equation [**25b**] at the end. Thus if by f in ft. is denoted the elongation of the hoist ropes (or of the spring by which they have been replaced) subjected to a tension of 1 lb.,

$$\text{tension [25b] at end of period minus tension [25a] at beginning} = s'_c/f$$

whence

$$W+w/2 = s'_c/f \quad \ldots\ldots\ldots \quad [\mathbf{28c}]$$

149 The work done by the external forces in the present period may now be obtained by substituting in Equation [**28b**] for P from Equation [28], for T_{av} from Equation [**28a**] and for s'_c from Equation [**28c**], and with it from Equation [**13**]

$$\tfrac{1}{2}[v'^2_c - v_c^2]M_c = f(W+w/2) \left[C+L-\alpha W/2-(\alpha-1)R-\left(\frac{\alpha}{2}-1\right)w/2\right] \quad [\mathbf{19d}]$$

Since in this equation v_c is known, v'_c can be determined from Equation [25].

150 The third period begins with the car falling at speed v'_c, the counterweight still at rest and the spring or rope tension as expressed by Equation [**26b**]. Rewriting this equation by adding to T the suffix 0

$$T_0 = W+R/2+w/2 \quad \ldots\ldots\ldots \quad [\mathbf{29}]$$

to indicate that T_0 is the initial spring or rope tension of the third period. Continued downward motion of the car now causes motion of the counterweight upward, and at the time t the displacement of the car, measured from the position at the end of the second period, will be s_c, while the counterweight has suffered a displacement s_w. As before the spring tension during this period will be denoted by T.

151 The elongation of the spring associated with a displacement s_c of the car and s_w of the counterweight is $s_w - s_c$ and the corresponding tension

$$T = \frac{s_c - s_w}{f} \quad \ldots\ldots\ldots \quad [\mathbf{29a}]$$

By parting the spring as shown in Fig. 21*c* and applying at each of the ends the tension T there are two systems for which to

determine the equation of motion. One of the systems comprises part of the massless spring, the weight $R/2$, the ropes *acd* whose weight is neglected, the loaded car and the compensating ropes. The equivalent system is the same as shown in Fig. 21*b'*, for which the resultant force and the aggregate mass is given in Equation **[26]**. The equation of motion of this system, using Newton's notations for time derivatives, is therefore

$$\Sigma P = M_c \ddot{s}_s \quad \text{. . . . . [30]}$$

According to Equation **[26]** it is possible to write for ΣP

$$\Sigma P = gM_c - \alpha R + w/2 - \alpha T$$

whence Equation **[30]** becomes

$$gM_c - \alpha R + w/2 - \alpha T = M_c \ddot{s}_c \quad \text{. . . . . [30a]}$$

152 The other system comprises part of the spring $R/2$, and the counterweight, for which the equation of motion evidently is

$$T - R/2 - W - w/2 = \frac{W + R/2}{g} \ddot{s}_w$$

or placing

$$M_w = \frac{W + R/2}{g} \quad \text{. . . . . [30b]}$$

it is possible to write in the form

$$T - gM_w - w/2 = M_w \ddot{s}_w \quad \text{. . . . . . [30c]}$$

Differentiating Equation **[29a]** twice with respect to time gives

$$f\ddot{T} = \ddot{s}_c - \ddot{s}_w$$

and eliminating $\ddot{s}_c$ and $\ddot{s}_w$ from this equation, and Equations **[30a]** and **[30c]**

$$fM_cM_w\ddot{T} + (\alpha M_w + M_c)T = 2gM_cM_w - \alpha M_w R + (M_c + M_w)w/2$$

of which the solution is

$$T = A \sin pt + B \cos pt + D \quad \text{. . . . . [31]}$$

in which

$$p = \sqrt{\frac{\alpha M_w + M_c}{fM_cM_w}} \quad \text{. . . . . [31a]}$$

and

$$D = \frac{2gM_cM_w - \alpha M_w R + (M_c + M_w)w/2}{\alpha M_w + M_c} \quad \text{. . . [31b]}$$

and A and B are the constants of the integration.

153 The constants are determined as follows: For $t = 0$, $T = T_0$; T_0 is expressed by Equation **[29]** but with reference to Equation **[30b]** it can be written in the form

$$T_0 = gM_w + w/2$$

and the substitution of $t = 0$ and $T = T_0$ in Equation [31] gives

$$B = gM_w + w/2 - D \quad . \; . \; . \; . \; . \; . \; . \; . \quad [31c]$$

154 Next it is known that at the time $t = 0$ the velocity of the car was v'_c and that of the counterweight was zero, whence

$$(\dot{s}_c)_{t=0} = v'_c \quad \text{and} \quad (\dot{s}_w)_{t=0} = 0 \quad . \; . \; . \; . \quad [32]$$

Differentiating Equations [29a] and [31] with respect to time,

$$f\dot{T} = \dot{s}_c - \dot{s}_w \quad \text{and} \quad \dot{T} = Ap\cos pt - Bp\sin pt \quad . \quad [32a]$$

Substituting $t = 0$ we obtain because of Equation [32]

$$f(\dot{T})_{t=0} = v'_c \quad \text{and} \quad (\dot{T})_{t=0} = Ap$$

whence

$$A = v'_c/fp \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [32b]$$

From Equation [31] it further follows that the maximum spring tension, which is also the maximum tension in the center cross-section in the ropes on the counterweight side of the driving sheave, is

$$T_{max.} = \sqrt{A^2+B^2} + D \quad . \; . \; . \; . \; . \; . \; . \quad [33]$$

which is now fully determined.

155 With reference to Fig. 21*c*, it will be seen that the rope tension T_2 at *a* differs from the spring tension T only by the amount necessary to accelerate the weight $R/2$ at the top of the spring. Neglecting this results in an ample degree of accuracy

$$T_2 = T \quad \text{and hence} \quad (T_2)_{max.} = (T)_{max.} \quad . \; . \; . \quad [33a]$$

$(T_2)_{max.}$ is the greatest rope tension in the immediate neighborhood of the driving sheave (point *a*) on the counterweight side of it. On the car side (point *b*) however, since the car still drags the hoist ropes over the driving sheave,

$$(T_1)_{max.} = \alpha(T_2)_{max.} = \alpha T_{max.} \quad . \; . \; . \; . \quad [33b]$$

156 The greatest force $F_{max.}$ which is exerted on the shaft of the machine by the rope tensions $(T_1)_{max.}$ and $(T_2)_{max.}$ is also of interest. When the secondary sheave is vertically below the driving sheave as assumed in Fig. 21*c*, this evidently is

$$F_{max.} = (T_2)_{max.} + 2\sqrt{\alpha}(T_2)_{max.} + (T_1)_{max.} \text{ (approx.)}$$

or because of Equations [33a] and [33b]

$$F_{max.} = (1 + 2\sqrt{\alpha} + \alpha)T_{max.} \quad . \; . \; . \; . \; . \; . \quad [33c]$$

157 *Example.* For the elevator of Example 1, Par. 74, the amount H of slack rope at a counterweight buffer stop with the ascending car empty is 1, 2, 3, 4, and 5 ft. respectively. Determine the greatest rope tensions and the force on the machine shaft upon the subsequent fall of the car through the given distances.

158 *Solution.* The calculations will be carried out in detail for $(H = 2)$, and the first object is to determine the quantity f. The total iron cross-section of six $\frac{5}{8}$-in. ropes is 0.87 sq. in. and the modulus of elasticity of a rope can be taken at $(E = 10^7$ lb. per sq. in.). The length of the ropes in the present example is 400 ft., hence their elongation for a tension of 1 lb. is

$$\frac{400}{0.87 \times 10^7} \text{ ft.} = 4.6 \times 10^{-5} \text{ ft.}$$

159 While nearly all of the elasticity of the system resides in the hoist ropes there is some of it in the machine shaft, machine supports, car, and counterweight frames, which will more than amply be covered by an allowance of 10 per cent of the elasticity of the ropes. Hence

$$f = 1.1 \times 4.6 \times 10^{-5} = 5.05 \times 10^{-5}$$

From Equation [**25**] with $L = 0$

$$a_c = \frac{4500 - 1490 + 400}{4500 + 3 \times 1490} g = 0.38g$$

and with $H = 2$

$$v^2_c = 2 \times 0.38g \times 2 = 48.8$$

whence

$$v_c = 7.0 \text{ ft. per sec. (approx.)}$$

Thus when the falling car has picked up all of the slack and the ropes are just taut again the speed will be 420 ft. per min.

160 From the second of Equations [**26**] with $(L = 0)$

$$M_c = \frac{4500 + 2 \times 1490}{g} = 233$$

and from Equation [**28d**] with $v_c^2 = 48.8$

$$\frac{233}{2}[v'^2_c - 48.8] = 5.05 \times 10^{-5} (5500 + 400)(4500 - 5500 - 1490)$$

whence

$$v'_c = 6.5 \text{ ft. per sec.}$$

This is the speed of the falling car at the instant the counterweight begins to move upward.

161 In addition by Equation [**30a**]

$$M_w = \frac{5500 + 745}{g} = 194$$

by Equation [**31a**]

$$p = \sqrt{\frac{2 \times 194 + 233}{5.05 \times 10^{-5} \times 233 \times 194}} = 16.5$$

by Equation [**31b**]

$$D = \frac{2g \times 233 \times 194 - 2 \times 194 \times 1490 + (233+194)400}{2 \times 194 + 233} = 4000 \text{ lb.}$$

by Equation [31c]

$$B = g \times 194 + 400 - 4000 = 2645 \text{ lb.}$$

by Equation [32b]

$$A = \frac{6.5}{5.05 \times 10^{-5} \times 16.5} = 7800 \text{ lb.}$$

by Equation [33]

$$T_{max.} = \sqrt{7800^2 + 2645^2} + 4000 = 12{,}240 \text{ lb.}$$

by Equation [33a]

$$(T_2)_{max.} = 12{,}240 \text{ lb.}$$

TABLE 3

Amount of slack H, ft	1 0	2.0	3.0	4.0	5.0
Speed of falling car, ropes just taut.					
v_c, ft. per sec.	4 94	6 99	8.56	9.88	11.0
Ft. per min.	296.4	419.4	513.6	592.8	660.0
Rope tension on car side of driving sheave.					
$(T_1)_{max}$ lb.	19,500	24,480	28,300	31,600	34,600
Per cent of normal	305	383	443	495	540
Rope tension on counterweight side of driving sheave.					
$(T_2)_{max}$ lb.	9,750	12,240	14,150	15,800	17,300
Per cent of normal	132	166	192	214	234
Force on machine shaft.					
F_{max} lb.	56,800	71,500	82,500	92,000	101,000
Per cent of normal	206	260	300	334	367

by Equation [33b]

$$(T_1)_{max.} = 24{,}480 \text{ lb.}$$

by Equation [33c]

$$F_{max.} = (1+2\sqrt{2}+2)12{,}240 = 71{,}500 \text{ lb.}$$

162 It is instructive to compare the greatest values of T_1, T_2, and F with the magnitude of these quantities in normal service. For the empty car these are evidently:

$$T_1 = C+R+w/2 = 4500+1490+400 = 6390 \text{ lb.}$$
$$T_2 = W+R+w/2 = 5500+1490+400 = 7390 \text{ lb.}$$

For the force F on the shaft of the machine in normal service it is customary to take

$$F = 2T_1 + 2T_2 = 27{,}560 \text{ lb.}$$

although it is actually somewhat smaller.

163 A comparison of the values of the rope tensions and the force on the machine shaft associated with 2 feet of slack rope with their magnitudes in normal service, results therefore in:

$$(T_1)_{max.} = 3.83 \text{ times normal}$$
$$(T_2)_{max.} = 1.66 \text{ times normal}$$
$$F_{max.} = 2.6 \text{ times normal}$$

164 It will be seen that as moderate an amount of slack as 2 ft. for ropes 400 ft. long will cause very considerable stresses even if the falling member is the empty car.

165 The results of calculations with the various amounts of slack rope specified in the above example are given in Table 3.

166 Table 3 is an eloquent argument for the selection of a buffer stroke of sufficient length so that only a small amount of slack will develop at the test. In it will also be found the reason for the necessity of re-aligning the machine after a field test or for finding a shaft of a machine fractured. It also shows that field tests which needlessly develop considerable amounts of slack ropes should be dispensed with. Amongst these is the test of the car safety with empty car as now required by one or two of the elevator codes.

DISCUSSION

Bassett Jones.[1] The problem of buffer and safety function is essentially a problem both of acceleration and of impact. The problem of acceleration has never been properly solved, first, because the effect of acceleration (and retardation) on the passenger's body, if it be not entirely ignored, is generally a matter of assumption, and secondly, because no satisfactory instruments for measuring and recording acceleration are available.

Impact is and always has been — even in our most recent works on applied dynamics — purely a matter of thumb rule. The equations of impact are highly complicated differential equations, of which no workable solution has been published. Mr. David Lindquist has succeeded in generalizing the graphical method used in Transient Performance of Electric Elevators,[2] leading to a solution of the impact problem, and opening the subject to practical use in design. Attention is drawn to the importance of this generalized graphical method which, it would seem, is a real contribution to applied mathematics.

It is necessary to appreciate the fact that the real problem in elevator design lies in the hoistway and not in the hoisting equipment. To drive this point home to several manufacturers of electrical equipment who have ventured into the elevator field, it is emphasized that there are a number of excellent sources of supply for motors and controllers, but very few sources of supply for hoistway work to which it is safe to trust the lives of the public.

It is for this reason that papers like Mr. Hyman's, dealing with the little understood problems of the hoistway, are of prime importance. The writer knows of no other discussion of the subject that even begins to compare with that of the author.

[1] Meyer, Strong, and Jones, Inc., Engineers, New York, N. Y.
[2] Jl. A.I.E.E., 1924.

The writer has to offer one fundamental criticism of the paper. While it is true that the elevator industry in general has insufficient knowledge of the fundamental mechanical problem, it is also true that there is practically no general working knowledge of the problem which, after all, must determine the satisfactory operation of such purely mechanical devices as buffers and safeties. This problem is the passenger. The passenger's psychology and his body are of equal importance in determining how buffers and safeties should act, not to make emergency stops comfortable, but to make them safe. Injuries to passengers may be, and often are, primarily due to the passengers' reaction to the mechanical behavior of the elevator.

The author's analysis is incomplete in that the car used in his diagrams should contain an added mechanism, namely, the delicate mechanism of the passenger's body, which must be so studied that the rules governing the safe application of the forces of acceleration, retardation, and impact to it are, first, known, and second, understood. Then only are we able to determine how safeties and buffers should act, and design them accordingly.

Specifically it may be pointed out that from this point of view, constant acceleration or retardation is entirely undesirable. It follows that parabolic graduation of the buffer action is not suitable. In fact, the graduation should be such as to produce a constant rate of change in acceleration (or retardation). That is to say, from the point of view of the mechanics of the passenger's body it is the *third* derivative of distance with respect to time that is of importance. Furthermore, the passenger can stand a much higher retardation than acceleration. Extreme peaks of retardation, if of short enough duration, are not noticed by him. The maximum retardation or acceleration of any material duration that he can endure depends upon how the acceleration or retardation is built up and reduced. The solution of this problem, when found, may set up the data for a radical redesign of both safeties and buffers, or, indeed, for the entire elevator mechanism.

In the final paragraph of his paper the author remarks that the requirement in the A.E.S.C. Code for the light load test of safeties should be dispensed with. The writer entirely disagrees with this. This requirement was written into the code for the purpose of adequately safeguarding the passenger, and to enforce a redesign of safeties and their associated mechanism. The reason is quite simple. Safeties, as now commonly designed, if set, as they must be, to stop and hold the fully loaded car from exceeding governor tripping speed, will stop the empty car almost dead in its tracks. Many injuries to passengers have resulted. Obviously, from the passenger's viewpoint it is quite as necessary to give him a chance for his life when he is alone in the car as when he has company. True, as the author points out, the light load test is very severe on the present type of elevator mechanism, but, also, it is quite as

severe a test of the passenger's mechanism. The light load test is the proper criterion of safety under emergency stopping conditions. This initial requirement presents a problem to designers that must be solved by them.

THE AUTHOR. A point in Mr. Jones' discussion which merits special mention is his statement to the effect that the emergency stopping devices of high-speed elevators present problems which are theoretically as well as practically difficult of solution. The author's paper is concerned with the motion of the car or counterweight as a whole, which is also the motion chiefly responsible for the stresses and strains set up in a passenger's body. The absolute motion of any point of the car or of a passenger's body, on the other hand, consists of two components. One of them is what is above called the motion of the car as a whole, which is more correctly defined as the motion of the center of gravity of the material points which comprise the car and its passengers. The other is a transient, a free vibration which observation indicates to be rapidly damped. Of importance are therefore only the first few cycles of this transient and these superposed on the motion of the car as a whole give rise to what Mr. Jones calls "peaks of retardation of small duration" which he claims are not injurious to the passengers. This is also the author's opinion and it follows that it is sufficient to base the estimate of the stress and strain in a passenger's body on the motion of the car as a whole only. This motion is completely expressed by equations for the various emergency stops considered in the paper and it is entirely in the designer's hand to so dimension the apparatus that no injury to a passenger results.

Why, of all the possible ways to bring an elevator to a safe stop, only that one is correct in which the motion is one with a constant time rate change of retardation, is a question Mr. Jones fails to support with arguments.

A potential switch stop of a gearless traction elevator, for example, has never injured a passenger and is not even unpleasant. Yet it is a motion with substantially a constant retardation, which Mr. Jones particularly condemns.

No. 2031

THE LUBRICATION OF WASTE-PACKED BEARINGS

By G. B. Karelitz,[1] East Pittsburgh, Pa.
Non-Member

Since there is little published information in the technical literature on the mechanism of waste-packed bearings, the following paper based on results of an investigation on this subject made by the Westinghouse Electric and Manufacturing Company will be of interest.

The article discusses the feeding of oil through waste and the existence of a load carrying oil film in such bearings as essential for their proper performance. It gives observations on the friction and temperatures of these bearings and on the importance of proper packing to insure sealing the window by oil-saturated waste, and also discusses the existence of a critical oil lift at which the seal is broken. The reason for occasional end wear and scoring of the ends of a shell during the running-in period of service is given.

THE small space available in a car truck excludes the possibility, with the present arrangement of motors and gears, of using a bearing well sealed against end leakage with "perfect" lubrication in the construction of railway motors. Moreover, the constant increase in the power of railway motors with the consequent growth in size and speed has led to more severe conditions in the bearings. Unless a radical change is made in the general construction of railway motors, such as the application of roller bearings, or of double-universal-joint propelling shafts,[2] etc., the use of waste-packed bearings of the sleeve type will have to be continued. So far no adequate solution of the bearing problem has been found that would replace these waste-packed bearings which are simple and rugged, in spite of the fact that these features are partially counterbalanced by the dependence of their performance on proper packing. It appears to the author that for

[1] Research Department, Westinghouse Elec. & Mfg. Co.
[2] Tried in Germany with motors mounted on the car frame.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers.

a long period yet to come railway motors will be built with waste-packed bearings. Both the maintenance and design of these will have to be improved with respect to high loads and high speeds required by present practice, and it is of practical importance to have data on the mechanism of lubrication in such bearings.

2 This paper gives notes on the performance of waste-packed bearings, which resulted from an investigation undertaken by the

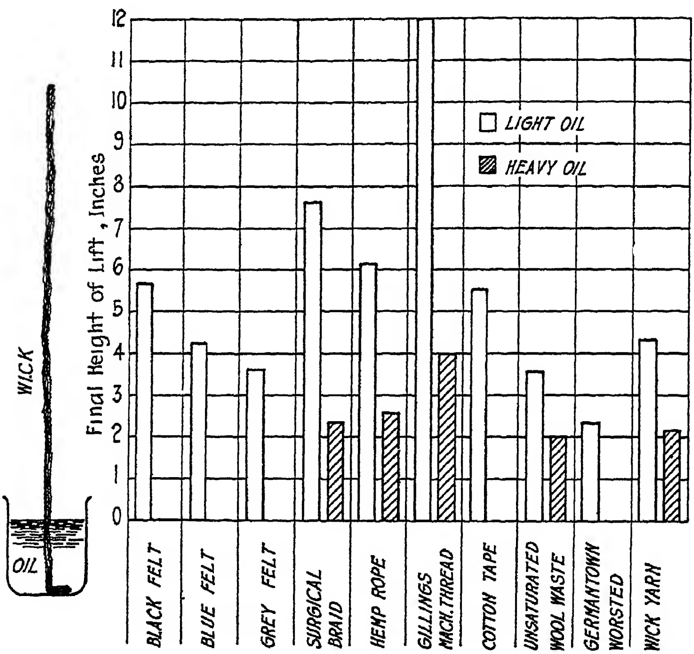


Fig. 1 Height of Lift for a Light Oil in Various Materials

Westinghouse Electric and Manufacturing Co. to study the general principles underlying the operation of such bearings.

3 It will be noted that this paper does not discuss such questions as the comparative value of different shell and journal materials or the performance of different oils as lubricants. These questions can be solved only by a long series of statistical experiments conducted under laboratory and service conditions. The present experiments were made on several 5 by 9-in. split bronze axle bearings using the bearing testing machine which is described in the Appendix.

Oil-Lifting Capacity of Waste

4 In general, a waste-packed bearing consists of a shell, either solid or split, which is pressed or clamped tightly in the surrounding housing. The shells are provided with windows into which waste is packed. Oil is supplied to the bearing by the waste which communicates with an oil reservoir and lifts it to the journal by capillary action. For satisfactory performance a bearing requires that a certain minimum of oil should be supplied to it and that the window should be reasonably well sealed by the oil-saturated waste.

5 The amount of oil carried through a wick, as well as the height to which it may be lifted, depends greatly on the kind of

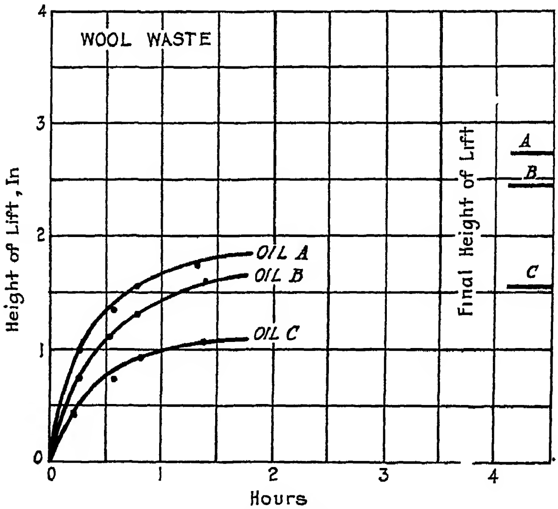


Fig. 2 Height of Lift against Time for Wool Waste

waste and oil[1] used and on the temperature of the oil. When a long wick made of several strands of a given waste material is dipped into oil the capillary action causes the oil to rise to a certain height. This depends on the surface tension of the oil and its adhesion to the material of the wick. The lift is an increasing function of surface tension and adhesion. Another factor influencing the height of lift is the size of the microscopic channels in the waste through which the oil flows. The smaller these are, the higher is the lift.

[1] See the article by C. Bethel, Packing Railway Motor Bearings for Oil Lubrication; *Electric Railway Journal*, April 19, 1924.

6 Fig. 1 gives the height of lift[1] for a light air-compressor oil (specific gravity 0.872, viscosity 53 sec. at 100 deg. cent.) in different materials. This also is shown for some materials with a heavy cylinder oil (specific gravity 0.905, viscosity 630 sec. at 70 deg. cent.).

7 When a dry wick is dipped into oil the height of lift changes rapidly at first, then the rate of rise decreases, the lift finally attaining a constant value after a day or two. Fig. 2, plotted

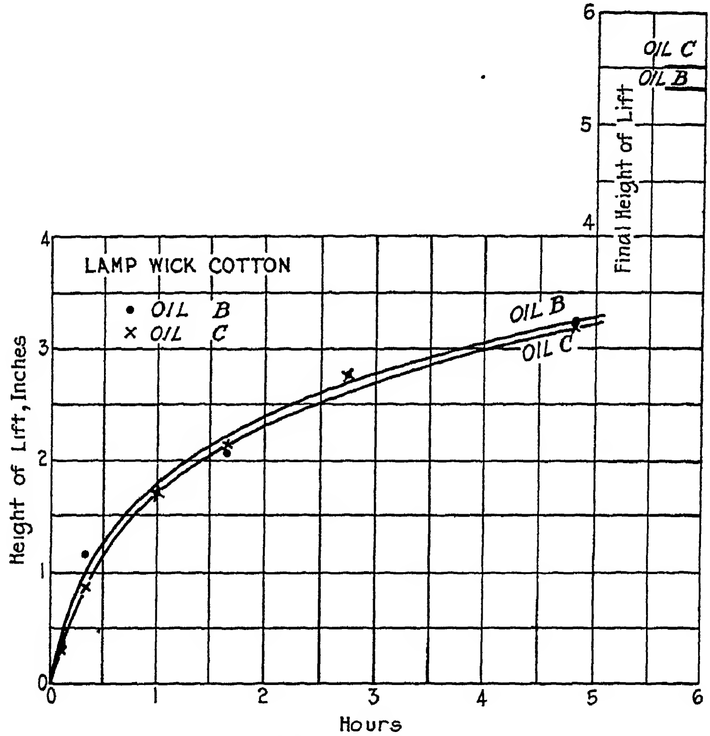


FIG. 3 HEIGHT OF LIFT AGAINST TIME FOR LAMP-WICK COTTON

from test results obtained by the author, shows the height of lift against time for wool waste with three oils at a room temperature of approximately 30 deg. cent.

Oil	Description	Viscosity, sec. 100 deg. fahr.	130 deg. fahr.	210 deg. fahr.
A	Medium machine oil	229	116	52
B	Heavy machine oil	625	273	68
C	Summer car oil	690	307	73

[1] Data obtained by B. F. James, formerly of Westinghouse Electric & Manufacturing Co.

8 Oils A and B were submitted as overhead distillates of an asphalt-base crude, the sample C as a residue oil of a paraffin base crude. Fig. 3 shows the same data for strands of lamp-wick cotton, with oils B and C.

9 The ultimate heights of lift for higher temperatures are slightly lower, since the surface tension of oils decreases slowly with an increase in temperature. This explains the fact that in waste-packed bearings the oil level rises in the oil chamber after the bearing warms up, the difference in the cold and hot oil level being observed to be from $\frac{1}{4}$ to $\frac{1}{2}$ in.

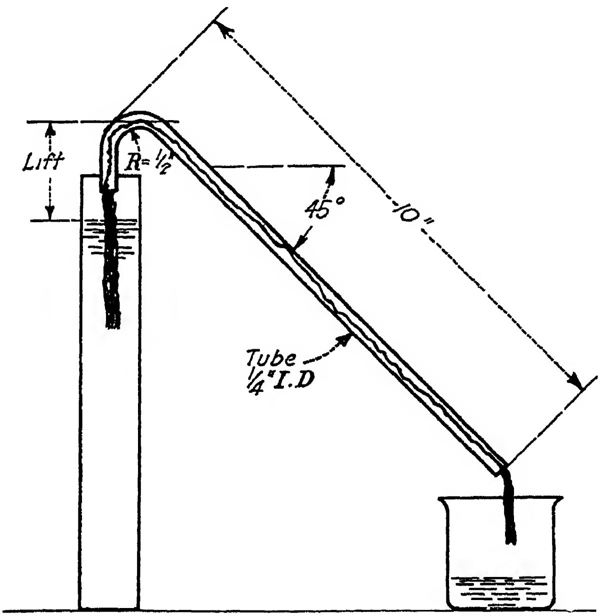


Fig. 4 Arrangement for Determining Siphon Action of Wick

10 It has been found that the amount of oil a wick of a given size and of different material is able to syphon over with an arrangement similar to the one shown in Fig. 4 is not correllated with the heights given in Fig. 2. The gravity and capillary forces acting upon the oil and causing it to flow, work against the internal friction or viscosity of the oil and against the friction between oil and waste. It is quite natural that, in some materials with high lifting capacity, this friction is considerable, due to the small capillary channels, which will cause a small flow of oil. The commonly used materials when graded for their ability to transfer oil come in the following order: lamp wick, cotton waste, felt, wool waste; the lamp wick being the best and the wool the worst.

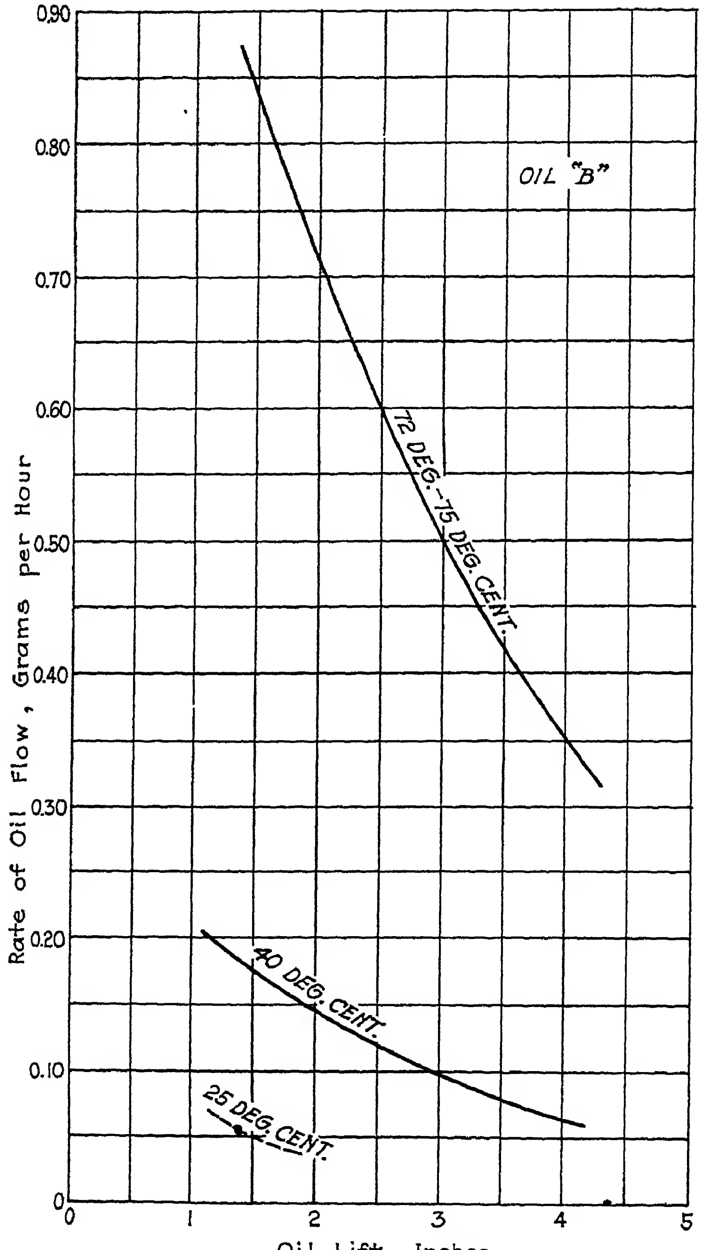


FIG. 5 FLOW OF OIL THROUGH A COTTON WICK FOR OIL B

Cotton materials, so superior in lifting and feeding capacity, are unsuitable as packing for bearings because they glaze readily, and, having no inherent elasticity, the positive contact with the journal is easily destroyed by vibration and jarring of the car on the track. The only material successfully used as packing has been wool waste; when, however, the mechanical properties are of secondary importance, as in auxiliary oil drippers, syphon feeds, etc., lamp wick or cotton thread should be used.

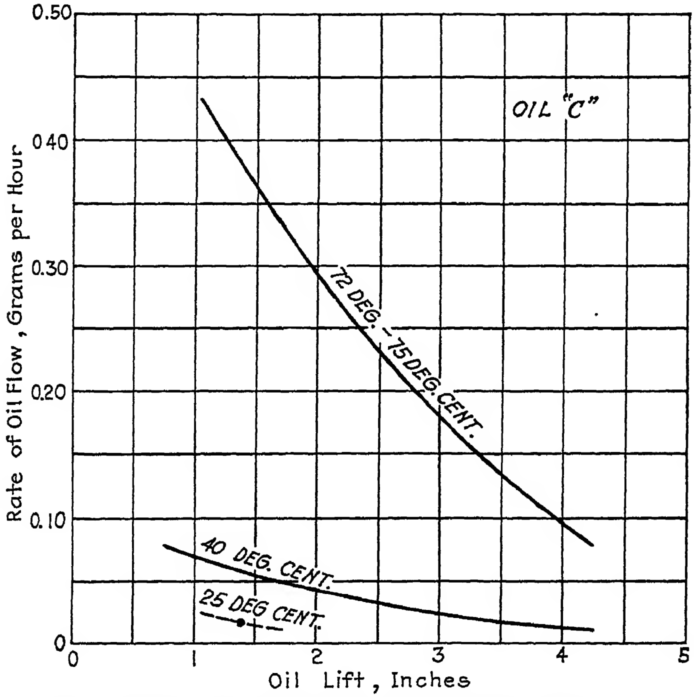


FIG. 6 FLOW OF OIL THROUGH A COTTON WICK FOR OIL C

11 At high temperatures, while the surface tension, capillary action, and specific gravity change but slightly, the viscosity of oil decreases many times; consequently, the flow of oil increases very rapidly with rise of temperature.

12 Figs. 5 and 6 show the flow of oil through a cotton wick for oils B and C respectively. The wicks were made of 26 strands of lamp-wick cotton; their weight was 0.182 gr. per in.[1] The geometrical dimensions of the wick are shown on Fig. 4. Although Fig. 3 shows no appreciable difference in the static heights of lift for B

[1] It was found that the amount of oil fed is roughly proportional to the cross-section of wick, all other conditions being equal.

and C, and the viscosities of the two oils are quite close to each other, the amounts of oil fed differ very much. Probably this may be explained by the fact that oil C had in it insoluble matter which might clog up the minute channels in the cotton. In both cases the temperature is the most decisive factor influencing the flow of oil. The above consideration explains the phenomenon that

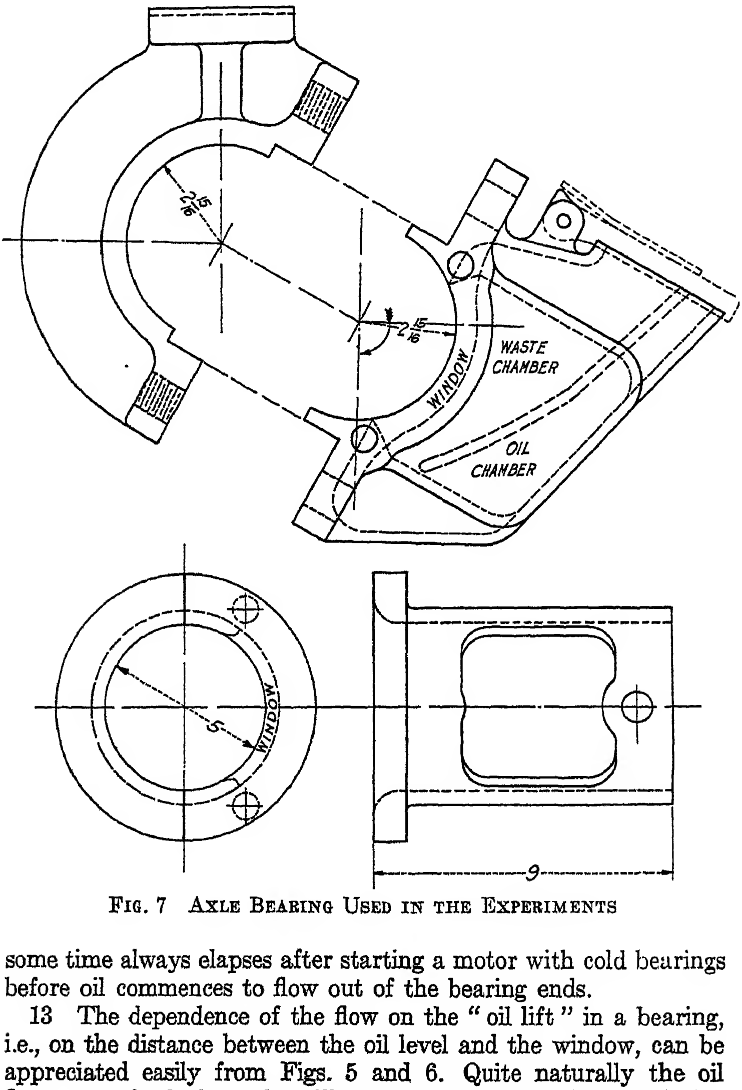


Fig. 7 Axle Bearing Used in the Experiments

some time always elapses after starting a motor with cold bearings before oil commences to flow out of the bearing ends.

13 The dependence of the flow on the "oil lift" in a bearing, i.e., on the distance between the oil level and the window, can be appreciated easily from Figs. 5 and 6. Quite naturally the oil flows more freely just after filling the bearing, this flow gradually decreasing in quantity toward the end of the oiling period. This

is illustrated in the following experiment with a well run-in axle bearing shown on Fig. 7: The bearing 5 in. in diameter by 9 in. long, belonged to a 60-hp. railway motor. It was run at 350 r p.m. with a total load of 1120 lb. This corresponded to service conditions at 32 m p.h., assuming 33-in. wheels. Oil C was used for lubrication. The bearing was run for a certain time, then stopped and cooled. Fig. 8 shows the relationship between oil lift, oil consumption, and the mileage run after oiling. The points on the curves were taken at the end of each run. After 6250 miles the friction became large and irregular and the temperature rose sharply, indicating that the window had become unsealed and that no adequate oil film was building up. Evidently, further running without reoiling would have injured the bearing.

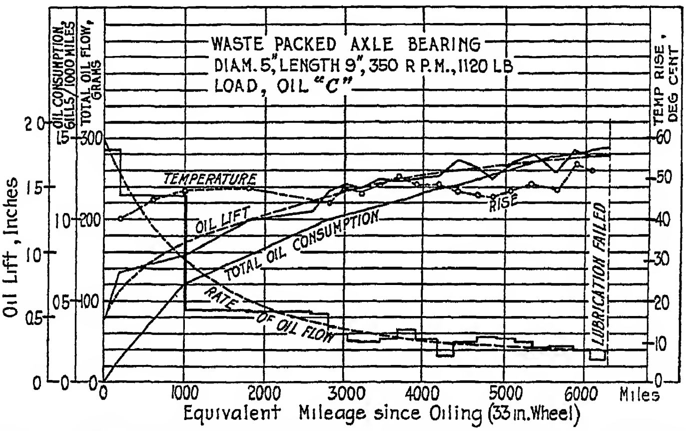


FIG. 8 RELATION BETWEEN OIL LIFT, OIL CONSUMPTION, AND MILEAGE

14 It will be noticed later that the performance of a waste-packed bearing practically does not depend on the amount of oil flowing through it, provided the amount of oil supplied to the journal is sufficient to keep the bearing clearance full. Therefore, it appears that the high rate of flow during the first 1500 miles after reoiling leads to a waste of oil. Of the 300 grams lost by the bearing during the whole period at least 100 grams could be saved if a constant, adequate flow and constant oil lift were maintained. Nevertheless, filling the oil chamber to a high level while reoiling is necessary, otherwise there is no assurance that the waste, which is usually fairly dry at the end of the oiling period, will be saturated quickly enough to provide a good seal for the window. In the new type of "oil-sealed" motor housings the oil lift is automatically maintained at a predetermined value and service experience has shown a corresponding increase of the oiling periods for these bearings.

15 The phenomena observed on the axle bearing under test are qualitatively the same as those which would be found in an armature bearing. However, the oil flow and the critical oil lift in an armature bearing will be greater, corresponding to its higher peripheral speed. For a given mileage the ratio of oil consumption is about three to one.

The Oil Film in Waste-Packed Bearings

16 When a journal comes to rest, part of the oil stored in the clearance flows out of the bearing at the ends. A certain part of it is kept inside by the capillary action between the oil, shell, and journal and collects at the bottom of the bearing. In the case of axle bearings where the shell rests on the journal the capillary action keeps oil also at the line of contact on the top of the bearing.

17 After the journal is started this oil lubricates the bearing even during the first moments of rotation. The starting torque of the journal corresponds to a coefficient of friction of from 12 to 18 per cent. The running friction drops instantly to a value corresponding to a coefficient of 6 to 8 per cent, provided the speed of rotation is not too low. Under proper conditions the journal wipes off the waste in a few revolutions a sufficient amount of oil to fill the whole clearance, while the oil is also gradually squeezed out toward the ends of the bearing.

18 In the tests the very sensitive recording torsiometer used with the testing machine (see the Appendix) showed erratic fluctuations in friction whenever the lubrication of the journal failed over a part of its length. On many occasions, due to some disturbance in the bearing, such as dirt or slush in the clearance, lack of proper lubrication could be observed until the bearing was warmed up sufficiently to produce a more ready flow of oil, reëstablishing the oil film. The friction in the bearings with an incomplete oil film was so high that they could not run for a long time under such conditions without being overheated. A bearing which runs satisfactorily in service must necessarily have a clearance completely filled with oil.

19 The performance of a waste-packed bearing, therefore, is subject to a large extent to the same hydrodynamical laws which govern the operation of "perfectly" lubricated bearings, although the oil films in these two types of bearings have different characteristics.

20 The action of the oil film in a bearing with perfect lubrication, that is, one running in an oil bath or with an unlimited or large supply of oil, is well described in the literature.[1] The

[1] H. A. S. Howarth, A Graphical Study of Journal Lubrication, Parts I to III. Trans., A.S.M.E., 1923-1925.

G. B. Karelitz, Charts for Studying the Oil Film in Bearings. Trans., A.S.M.E., 1925.

L. Guembel — E. Everling, Reibung und Schmierung im Maschinenbau (Friction and Lubrication of Machinery), Berlin, M. Krayn, 1925.

boundary layers of oil are kept by adhesion to the surfaces of the shell and journal. A relative motion of oil layers is thus created inside the clearance, the velocity gradient across the small clearance, from journal to shell, being very high. Due to the viscosity of the oil, the slipping of oil layers over each other results in forces of two kinds:

a Friction forces resisting the motion of the journal and measurable in terms of torque on the shaft

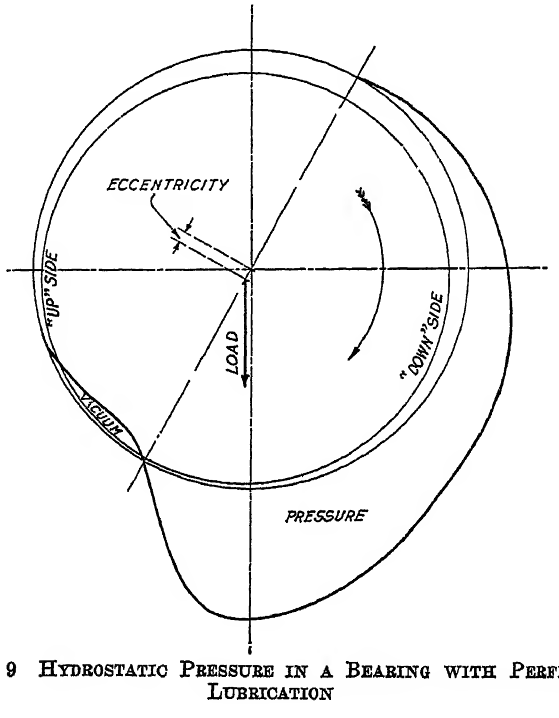


FIG. 9 HYDROSTATIC PRESSURE IN A BEARING WITH PERFECT LUBRICATION

b Hydrostatic pressure, sufficient to carry the load on the journal, separating the journal and shell by a steady oil film. The pressure is built up on the " down " side of the journal (for a downward load), the " up " side having a vacuum (Fig. 9).

21 In a bearing of finite length the pressure at the ends is necessarily equal to atmospheric pressure, and the hydrostatic pressure must vary in the longitudinal direction as well as along the circumference. The vacuum on the up side is small, as it cannot exceed the vapor pressure of the oil at the bearing temperature. A pressure gradient from the center of the bearing toward both ends causes an axial flow of oil through the clearance, the bearing acting thus as a pump. " Perfect " lubrication is maintained when

the supply of oil to the bearing is not less than the pumping capacity of the bearing under the given conditions of load, oil, temperature, and speed. This is the case with oil-ring, oil-chain, or forced-feed bearings.

22 On the other hand, the amount of oil supplied by the waste is negligible when compared with the pumping capacity of the bearing. Therefore, assuming that a journal-carrying oil film were established in some way, oil would flow out of the bearing ends until the journal seated itself on the shell. Boundary lubrication[1] would exist along the contact area between journal and bearing. After such a contact is established, the oil flow through the bearing is very small, the oil being kept inside the clearance partly by the capillary action of the clearance and partly by the light vacuum created on the up side.

23 The shape of the oil film in a waste-packed bearing is determined only by the machined clearance of the bearing. On the other hand, in a perfectly lubricated bearing, the oil film adjusts itself to each condition of service. The equilibrium conditions for these bearings are usually such that the maximum pressure of the oil is about three times the average pressure on the projected area of the bearing. It is known that this pressure depends on the eccentricity of the journal in respect to the shell, increasing very rapidly when the eccentricity nears half of the clearance, that is, when the journal nears contact with the shell. Therefore, the pressures in oil films of waste-packed bearings may be found greatly exceeding those in perfectly lubricated bearings. In the 5 by 9-in. axle bearings tested by the author a total load of 1200 lb. at 450 r.p.m. gave a pressure of 900 to 1000 lb. per sq. in. on a gage located at the mid-length of the bearing, 30 deg. behind the load line with respect to the direction of rotation. The nominal pressure with reference to the projected area was only 30 lb. per sq. in. (considering the bearing 8 in. long due to the radius of the bearing collar).[2]

24 In contrast with the perfectly lubricated bearing, the load in a waste-packed bearing is now shown to be carried partly by hydraulic pressure and partly by the contact area. The percentage of the total load carried by each can be estimated by friction consideration. Let P_1 represent the part of load carried by the oil film, and P_2 the part carried by the contact area, then $P = P_1+P_2 =$ the total load on bearing.

25 The coefficient of friction in the oil film may be taken equal to 0.01, corresponding to the high-viscosity oil used in waste-packed bearings. The coefficient of friction in the contact area

[1] A clear picture of this type of lubrication, when the oil film is too thin for hydraulic phenomena to take place, has not been given to date.

[2] Compare with the article of Erich Schulze, Studien über Achslager für Fahrzeuge (A Study of Car Axle-Bearings); Verkehrstechnik, June 25, 1926.

is near 0.12, for conditions of boundary lubrication. The overall coefficient of friction was observed to be of the order of 0.03. Then

$$0.01P_1+0.12P_2 = 0.03P = 0.03(P_1+P_2)$$

or

$$0.09P_2 = 0.02P_1; \qquad P_1 = 4.5P_2$$

26 Although the oil film carries most of the load, the contact area creates most of the friction:

Friction in oil film is $0.01P_1 = 0.045P_2$
Friction in contact area is $0.12P_2$

The above shows that the viscosity of the lubricant is, in waste-packed bearings, as important as its oiliness. Since these bearings run normally at a temperature near 80 deg. cent., a high viscosity at room temperature is essential.

27 The load-carrying ability of the oil film is so important that the design and manufacture of bearings should be such as not to interfere under any conditions with its being formed and remaining uninterrupted.

28 The window in a bearing must be located along the circumference in a way to provide an ample angle between the window and the load line under any conditions of service, in order that an efficient load-carrying oil film may be established.

29 For instance, the test bearings, when loaded as shown in Fig. 7, and run in the indicated direction, showed at different loads and speeds a friction torque of from 1.4 to 2.6 times lower than the torque for the opposite direction. (An analysis of the forces acting in a railway-motor and gear system shows that axle bearings when rotating in the unfavorable direction are usually loaded but lightly. If heavy loads should occur while running in this direction, high temperatures should be expected.)

30 The usual construction of railway motors is such that under all possible conditions of service the window is in the vacuum zone of the oil film. This helps the waste to feed oil to the journal. But the packing must be made carefully in order not to impair the vacuum which is partly responsible for keeping the oil inside the clearance. The packing must cover the whole window, so that it is completely sealed by the oil-saturated waste. The easiest way to pack with this in view is by placing a wick, long enough to reach the bottom of the oil well, across the window and filling the waste chamber behind the wick with waste. This must be tamped very tight so as to force the wick against the journal and to prevent any loosening of the waste in service due to vibration and jarring.[1]

[1] See article by C. Bethel, loc. cit.

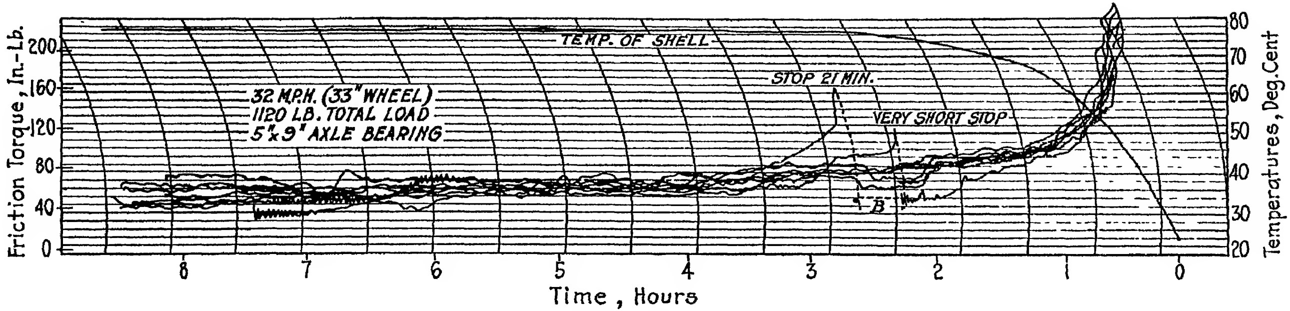


Fig. 10 Friction Records Taken from Experimental Axle Bearing

Friction and Temperatures

31 *Influence of Temperature on Friction.* In the tests it was observed that after starting the journal, under various conditions of load and speed the friction decreased considerably as the bearing warmed up. No friction change occurred after a steady temperature of the bearing was reached. Fig. 10 shows a series of friction records taken on the experimental axle bearing under 1200 lb. load at 350 r.p.m. with different oil levels. The bearing was well run in. The average growth of the temperature with time is also plotted in the figure.

32 It will be noticed that a coefficient of friction of 0.01 corresponds to a friction torque $0.01 \times 1200 \times 2.5 = 30$ in-lb. The coefficient of friction varies therefore from 0.075 when the bearing is cold to approximately 0.0175 when the bearing is hot.

33 Since the coefficient of friction under boundary conditions of lubrication due to oiliness of the lubricant has been found to vary slightly with change of temperature,[1] the variation in friction as found is therefore to be ascribed to the decrease in viscosity of the oil with temperature rise. A typical relation between friction and viscosity is

[1] Paul Woog. Mesure de frottement onctueux (Measurement of Greasy Friction). *Comptes Rendus,* vol. 180, pp. 1824-1826.

given in Fig. 11. The curve was derived from the known relation between the viscosity and temperature for oil C, used in this experiment, and from the observed friction-temperature function.

34 This change in viscosity of the oil no doubt affects the load-carrying capacity of the oil film so that the distribution of the load and friction between film and contact area varies with temperature. An increase in viscosity relieves the contact area, and the distribution of friction should be of a character as represented by the dotted line in Fig. 11. No reliable data on the exact shape of this line are available.

35 It follows from the above that a coefficient of friction cannot be given for a certain waste-packed bearing unless its temperature is specified.

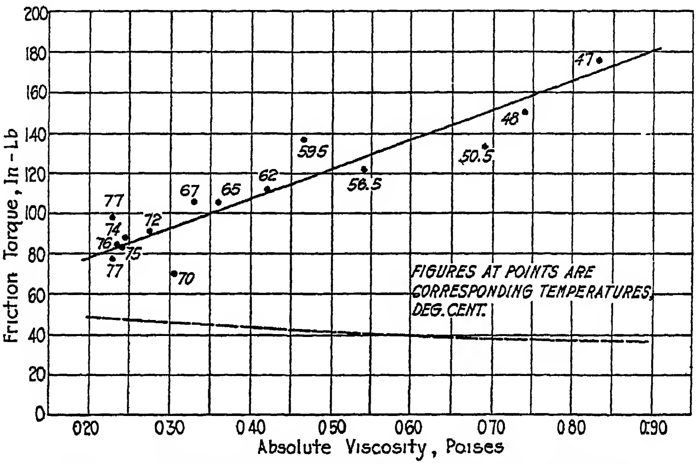


FIG. 11 RELATION BETWEEN FRICTION AND VISCOSITY

36 *Influence of Oil Lift.* The amount of oil the waste can supply is not sufficient at the best to produce a change in the friction, this amount being too far below that necessary to bring about a state of perfect lubrication. The performance of the bearing is practically independent of the oil flow, provided the clearance is full of oil. This can be seen from Fig. 8 where temperatures are given for different oil lifts after steady conditions were attained. It will be noted that the bearing ran distinctly warmer only during the last 200 miles of the oiling period, when conditions grew unsteady and the clearance was not completely filled with oil.

37 *Temperatures at Different Loads and Speeds.* A series of experiments was made with varying loads and speeds until steady conditions of temperature and friction were attained. The temperature rise of the bearing is given in Figs. 12 and 13. These

show that speed has a considerably more pronounced influence on the temperature than the load.

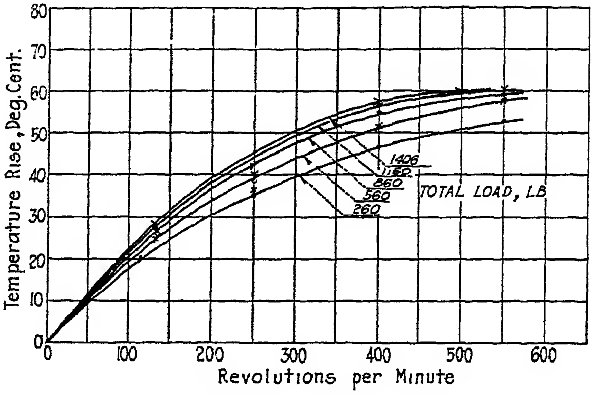


FIG. 12 TEMPERATURE RISE IN BEARING

38 The final temperature attained is determined by the equilibrium of heat generated in the bearing and dissipated through radiation. The generated heat is proportional to the work of the friction forces, $H_g = K \frac{T}{r} V$, where

H_g = the rate of heat generation
K = a coefficient of proportionality
T = the friction torque on journal
r = the radius of the journal
V = the peripheral speed.

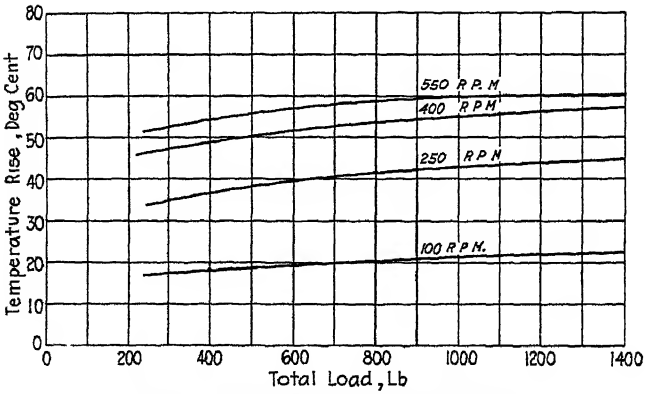


FIG. 13 TEMPERATURE RISE IN BEARING

39 The dissipated heat may be presented as $H_d = C(\Delta t)^m$ where H_d is the rate of heat dissipation, m and C are coefficients,

constant for a given bearing, Δt is the temperature rise of the bearing. At steady conditions

$$K\frac{T}{r}V = C(\Delta t)^m$$

or

$$(\Delta t)^m = \frac{K}{Cr}TV$$

40 The shape of the oil film being constant in a waste-packed bearing (the temperature distortion not being taken into consideration), the liquid friction is proportional to the speed, and the torque T is an increasing function of both load and speed. The temperature rise being a function of the product TV, the stronger influence of the speed as compared with that of the load is evident. The relation between temperature, load, and speed is complicated by the fact that the torque T, depending greatly on the viscosity of the oil in the oil film, is a decreasing function of the temperature of the bearing. This accounts for the flat shape of the curves in Figs. 12 and 13.[1]

41 *Limiting Pressures and Speeds.* A formula which would give the interrelation between all factors in a waste-packed bearing would be very complicated and its use would require a preliminary determination of many constants dependent on details of the bearing construction. Simple formulas used frequently (for instance the formula for limiting loads and speeds, $PV = C$, where P and V are pressure per square inch of projected area and the peripheral velocity, respectively, and C a constant) can serve only for expressing in an approximate way results of experience, covering a more or less limited range of conditions.

42 *Distribution of Temperatures in the Bearing.* Fig. 14 shows the distribution of temperatures in the bronze shell of the experimental bearings, as measured by thermocouples imbedded into the shell at the middle. The difference between the top and bottom for steady conditions was 14 deg. cent., and this difference was still higher shortly after the bearing was started. There was also a variation of temperature along the journal of 2 to 5 deg. when the conditions were stable. When the oil film was disturbed for some reason, a sharp rise of temperature at one end or the other was observed following an increase in friction recorded by the torsiometer. No doubt this was due to interruption of the oil film at that end. After restitution of the lubrication the temperature would quickly even out along the journal.

[1] It should be noted that the pressures and velocities in these experiments were quite low when compared with those met in armature bearings of railway motors. The shape of similar curves for higher speed and pressure might be different, and further tests are being planned.

43 Temperatures at the center line of the shaft at mid-length of the bearing were recorded by means of a thermometer inserted into an axial hole drilled into the shaft. The thermometer rotated with the journal. It invariably registered a temperature approximately 4 deg. cent. higher than the temperature of the shell at the hottest spot. It is reasonable therefore to assume that the temperature of a journal is the same as the average temperature of the oil film.

Critical Oil Level

44 In order to demonstrate the absolute importance of maintaining a good oil film throughout the whole bearing, experiments

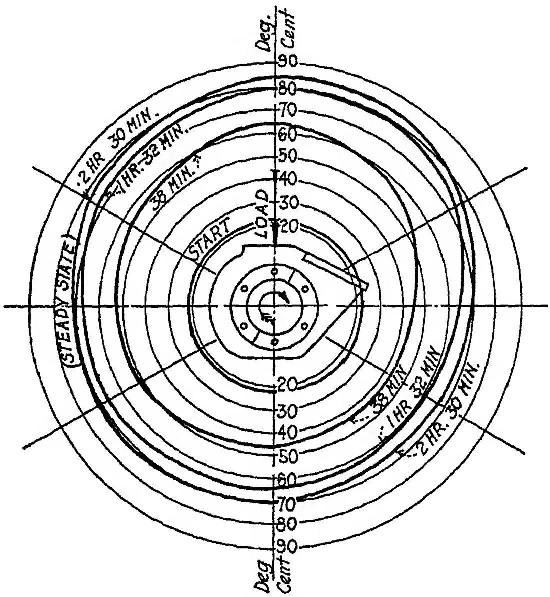


Fig. 14 Distribution of Temperatures in Bronze Shell of Experimental Bearing

were made repeatedly to observe the influence of lowering the oil level in the oil chamber.

45 It was found that after reaching a certain oil lift ($1\frac{1}{2}$ to $1\frac{3}{4}$ in. for this bearing, compare also with Fig. 8), the load-carrying oil film was ruptured. This rupture can be explained on the basis that the capillary action of the waste was not sufficient to hold at such an oil-lift enough oil to seal the window and to supply to the bearing the very small amount of oil necessary to replace the loss through end leakage or evaporation, thereby allowing air to enter the vacuum zone of the clearance.

46 Fig. 15 gives the friction and temperature during a typical experiment of this kind. By pumping the oil out of the oil well

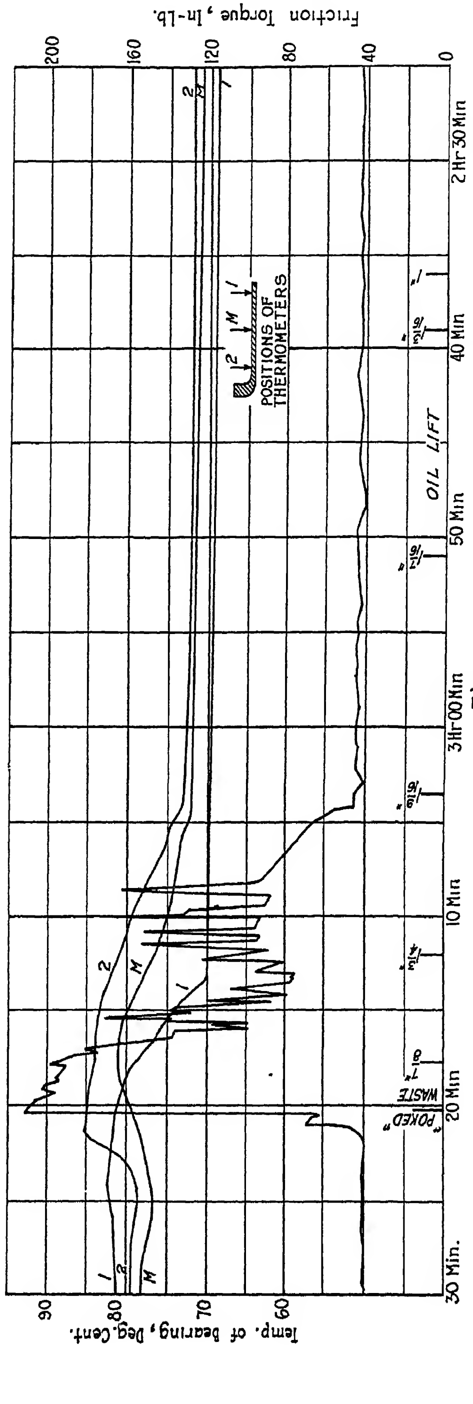


Fig. 15 Friction and Temperature Variations Due to Ruptured Oil Film

in small quantities the oil lift was gradually increased. When it reached $1\frac{9}{16}$ in. a rapid increase in friction occurred. The erratic friction record indicates a ruptured oil film. At this point the friction torque reached 120 in-lb., as compared with 46 in-lb. shown before the disturbance. The temperature curves show that the oil film was ruptured at No. 2 end of the bearing. A further increase in oil lift, to $1\frac{3}{4}$ in., caused another sudden change in friction torque, bringing its value to 200 in-lb. The temperature record shows that this was due to the additional rupture of the film at No. 1 end. The oil well was then refilled with oil to an oil lift of $\frac{7}{8}$ in., but the friction did not change. A vigorous "poking" of the waste with a packing iron reëstablished the oil film and the friction dropped suddenly to the original value.

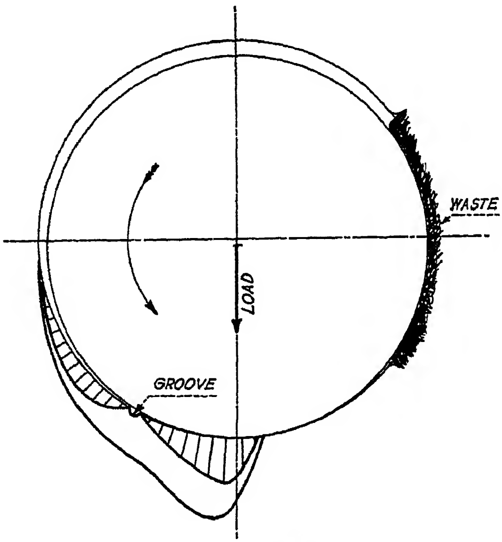


FIG. 16 EFFECT OF OIL GROOVES ON OIL FILM

GROOVING OF BEARINGS AND FINISH OF SURFACES

47 The design of grooves in a waste-packed bearing must take into consideration the proper formation of an uninterrupted oil film. The location of grooves in a pressure zone is detrimental to the performance of a bearing which is illustrated in Fig. 16. The load-carrying oil film, being broken into two parts, loses part of its carrying capacity, thus throwing more load on the contact area and increasing the friction considerably.

48 Grooves must be well chamfered or rounded, otherwise the sharp corner will disrupt the continuity of the oil film. In addition, sharp edges on the grooves may scrape the oil from the journal. It was noticed that, with the bearing at rest and well

cooled, no oil could be found in the grooves after carefully dismantling the bearing. This is due to the capillary action of the clearance being sufficient to draw the oil out of the grooves. They cannot therefore serve as reservoirs for oil in the bearing while it is at rest.

49 It is the author's opinion that, in general, the usefulness of grooves in waste-packed bearings for oil transmission along the bearing is doubtful, and that when properly designed and manufactured a waste-packed bearing will function just as satisfactorily without grooves.

50 Regarding the nature of the bearing surface, there is very little oil in the clearance after the bearing is started, and it takes a certain time until a complete oil film is built up (see the friction record on Fig. 10 where a very short stop was made). It is therefore very important to finish the surfaces of the journal and shell as well as possible, this being particularly essential for the journal. The work and expense of polishing the journal surface is generally repaid by increasing the life of bearings and by a decrease in bearing troubles.

Running-In Period of Service

51 It was observed that, as a rule, new waste-packed bearings, while being run-in, wear from the ends to the middle. Fig. 17, shows clearly a typical example of this. Quite often, on opening bearings after several hours of run, it is found that the shell is scored at the ends, yet after cleaning and scraping, the bearings give satisfactory service. An investigation of this phenomenon showed this to be the result of a distortion due to uneven heating of the shell. The bearing bends in such a manner as to prevent the middle part of the shell from coming into contact with the journal. The journal rides on both ends of the shell, provided there is no misalignment between the bearing and shaft. Figs. 18 and 19 show the arrangement of experiments carried out to determine the cause of such a distortion and to measure the magnitude of it. The 5-in. by 9-in. axle bearing was used with the load pressing down on the bearing as shown. Angle bars 1 and 2 were clamped tightly to a rib of the housing and the collar of the shell, and a mirror extensometer 3 was used to measure the change in distance between the top ends of these angle bars. The knife edges of the extensometer were held between the top of bar 1 and a spring strap 4 bolted to the bar 2, so that any relative movement in the angle bars resulted in a minute turning of the mirror observed by means of a telescope and scale 5. The geometrical relations in the set-up were such that the actual deflection of the angle bar tops in inches was equal to 0.00074 of the change in reading of the scale. Thermocouples were inserted into the

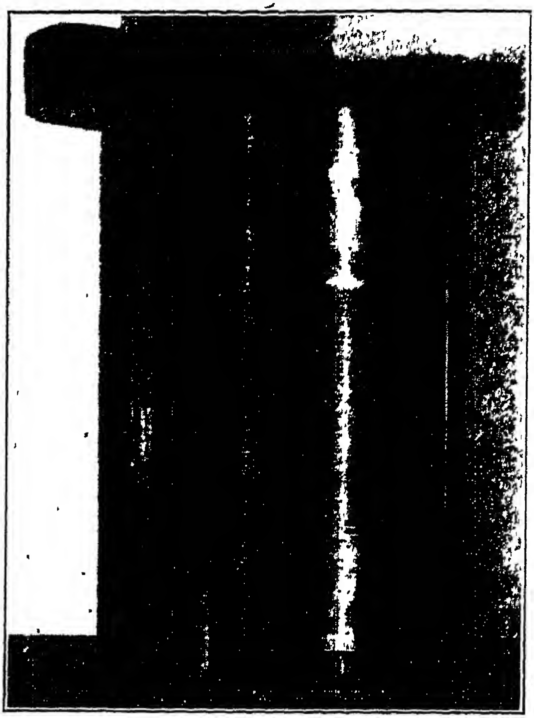


Fig. 17 Showing Wear at Ends of a New Bearing

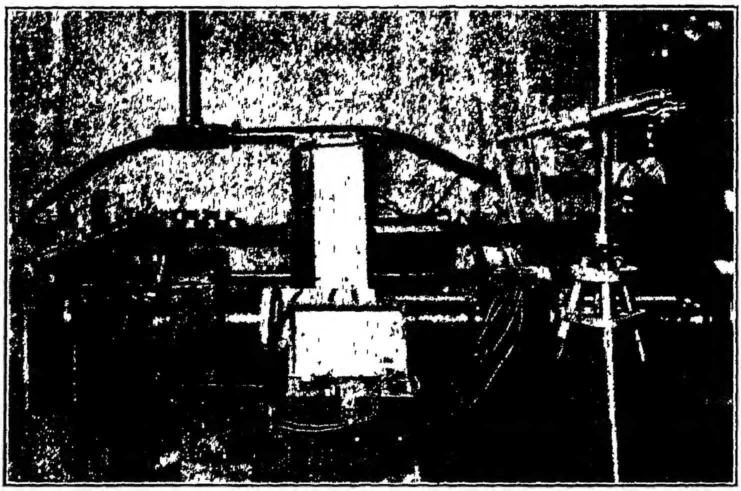


Fig. 18 Arrangement of Apparatus Used in Experiments

shell of the bearing in the positions shown on Fig. 20. The leads to these may be seen also in Fig. 19.

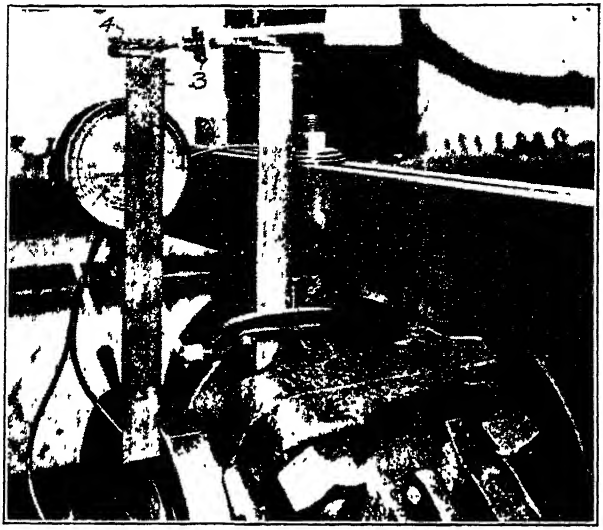


FIG. 19 APPARATUS USED IN EXPERIMENTS

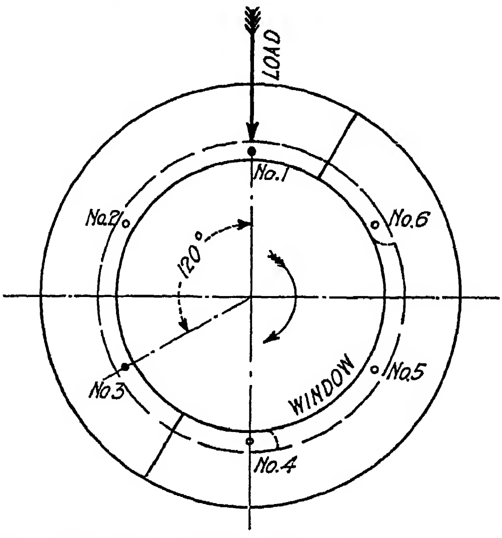


FIG. 20 LOCATIONS OF THERMOCOUPLES IN BEARING SHELL

52 Loading and unloading the bearing at rest and while running failed to produce any appreciable change in the reading of the extensometer. Fig. 21 shows measurements taken during one of

the runs. The change in scale reading is plotted against the time from the starting of the bearing. The change was corrected for

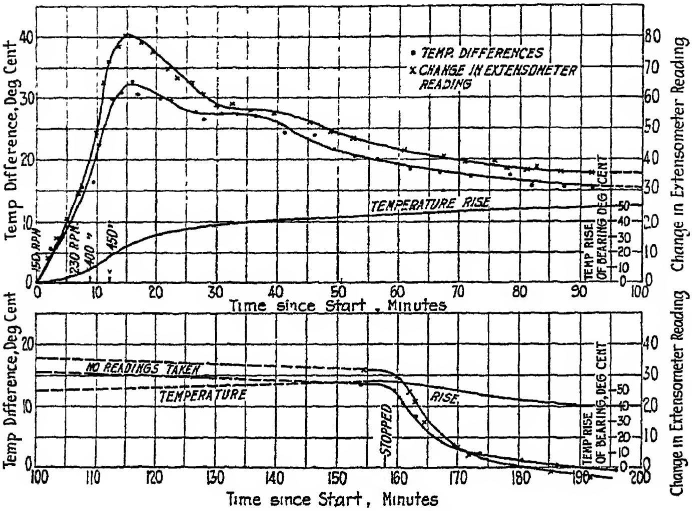


Fig. 21 Extensometer Readings

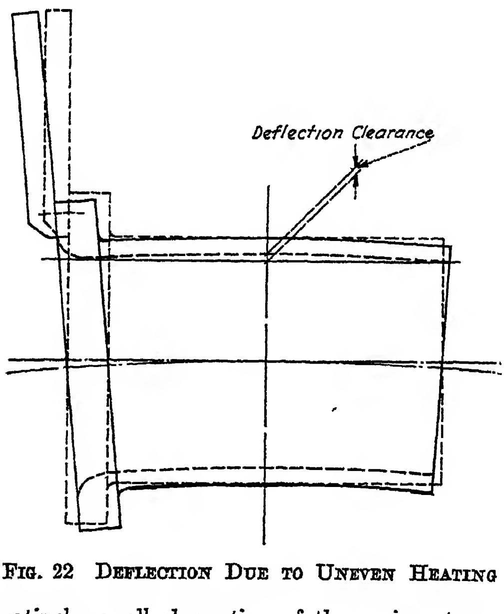


Fig. 22 Deflection Due to Uneven Heating

the comparatively small elongation of the spring-strap due to its slight heating. The temperature rise of the bearing with respect to the ambient air and the difference between temperatures regis-

tered by the thermocouples No. 1 and No. 3 are plotted. The perfect correlation between the deflection of the bearing and this difference shows clearly that the shell bends under the influence of an uneven heating of the top and bottom of the bearing (Fig. 22). A simple estimate of the amount of bending to be expected for a certain temperature difference, the coefficient of linear expansion for the material being known, gave results checking closely with the observed values. It was found that the deflection-clearance for the middle-sized bearing used was of the order of 0.0013 in. at the moment of maximum distortion, and 0.0005 in. corresponded to steady conditions with a temperature difference of top and bottom of approximately 13 deg. cent.

53 Such a temperature variation being inherent for the operation of waste-packed bearings, it is evident that a wear of the order of 0.001 in. at the ends must take place before the journal can seat itself uniformly over the whole length of the shell, under running conditions. Before this occurs, load is carried only by the ends of the shell, which accounts for the occasional scoring of the bearings at these places. This fact should be considered in the design of these bearings.

Thickening of the Outflowing Oil

54 It was noticed that the oil C passing through the bearing and dripping out of its ends was considerably more dense than fresh oil put into the oil chamber. The density seemed to increase inversely with the rate of flow, the oil attaining a jelly-like state at a rate of flow of one drop in several minutes. The reason for this is not known exactly. Distillation of the lighter constituents from the oil cannot alone account for it. Samples of a commercial car oil both before and after passing through a bearing, were examined. Distillation tests under vacuum (1 to 2 mm. pressure) gave the boiling temperatures of both oils after definite fractional distillation[1] as shown in Table 1.

TABLE 1

Part of oil distilled off, per cent	Boiling temperatures in vacuum, deg. fahr.	
	Fresh oil	Used oil
0	250	372
5	316	474
10	362	508
15	388	528
20	406	557
25	428	...
30	445	...
35	462	...
45	494	...
55	536	...
65	590	...
75	624	...
85	692	...
95	710	...

[1] This investigation was made by Mr. Wilharm of the Chemical Section of the Westinghouse Research Department.

55 The temperature of 372 deg. fahr. for the used oil would indicate, when compared with data for fresh oil, a loss of approximately 13 per cent due to distillation while passing the bearing. If 20 per cent of this oil is distilled off, the remainder should correspond to $0.80 \times 0.87 = 0.70$ of the new oil, or what remains after 30 per cent of it is distilled off. Yet the boiling temperatures of the two remainders are 557 deg. fahr. and 445 deg. fahr. There is evidently some oxidation of the oil taking place, and the influence of this on the lubricating action of the oil will be worth while investigating.

Conclusion

56 It will be seen from this paper that maintaining an efficient load-carrying oil film, even in a waste-packed bearing, is of extreme importance. This implies that the bearing must be

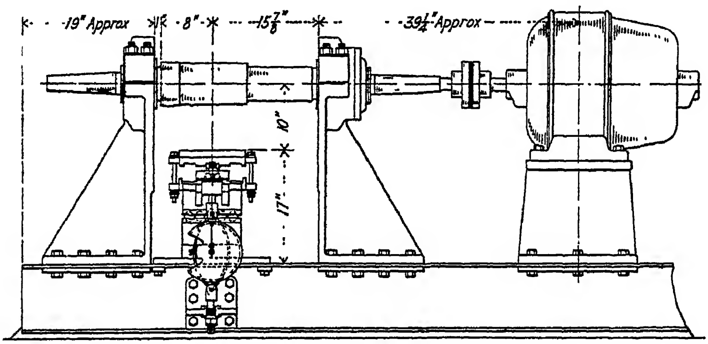


Fig. 23 Arrangement of Bearing Testing Machine

properly packed, so that the waste covers the entire window while being kept closely pressed against the shaft. In addition, the existence of a critical oil level in waste-packed bearings is established, and to insure having this oil film the oil level must be kept above the critical value.

57 In order to obtain satisfactory service and long life from waste-packed bearings it is absolutely important that attention to the general principles stated in this paper be observed by the service as well as by the design engineer.

APPENDIX

DESCRIPTION OF THE TESTING MACHINE

58 The general arrangement of the bearing testing machine is shown in Fig. 23. A test journal is mounted on self-aligning roller bearings in two pedestals and a test bearing may be placed on the table underneath the journal and pressed against it with a force of 12,500 lb. In addition, the shaft has on both ends tapered extensions on which sleeves with tapered bores may be slipped, thus forming

two additional journals on which axle bearings or similar types requiring low loads may be tested. The journal shaft is driven by a directly

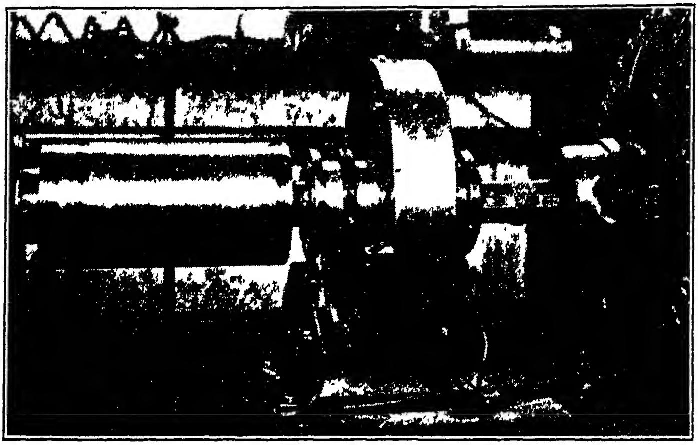


Fig. 24 Torsiometer Used in Experiments

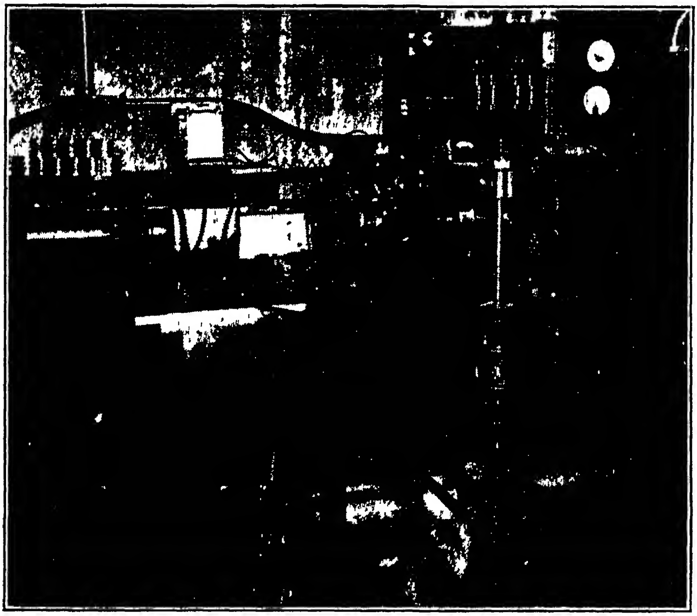


Fig. 25 Testing Equipment with Axle Bearing Mounted for Test

connected 12-hp. motor through a flexible coupling which can be replaced by a torsiometer-coupling.

59 The test bearing table is supplied with two straight ball bearings acting as knife edges, one parallel and the other normal to the center

line of the journal. Such a construction enables a uniform pressure of the journal against the shell along the bearing, while by wedging up the table it permits a study of the effects of misalignment. The ball bearings allow the bearing to adjust itself freely to the journal in order that an oil film may build up identical with the film in a fixed bearing and a running loaded journal.

60 The leverage on the fulcrum of the table is 2½ to 1; a 5000 lb. dynamometer serves to adjust the load on the bearing.

61 The torsiometer measures the total friction torque on the test shaft, from which the friction of the experimental bearing is obtained by deducting the friction in the roller bearings. The friction in these auxiliary bearings is small when compared with the friction in the test bearing and is known for different loads with a fair degree of accuracy, so that the errors in the observed values of friction are not large.

62 Fig. 24 shows the torsiometer used in the experiments with axle bearings. The rotation is transferred from the motor to the test shaft through a spiral spring. The driving disk carries an insulated coil, and the driven sleeve an adjustable arm with a brush holder, the whole making a rotating potentiometer. A constant voltage is applied to the ends of the coil and a recording voltmeter records the voltage drop between one end of the coil and the sliding brush. The position of this brush is a function of the torque. Careful calibration shows a fair straight-line relation between torque and voltage reading. The centrifugal force on the spring involves a certain correction for speed; this was found by experiment.

63 The motor is of the variable-speed type, 220 volts, 600 to 1800 r.p.m. Flexible control is obtained by (1) usual regulation of the field current and (2) supplying the armature current from a motor-generator set, the output voltage of which is regulated by varying the field current of the generator. In addition to this, an armature control rheostat is installed. A variation of speed from 6 to 2200 r.p.m. is obtainable.

64 A photograph of the equipment with an axle bearing mounted on one of the sleeves is shown in Fig. 25. In the foreground is a table with a potentiometer and a contactor used with the thermocouples. The recording voltmeter giving a record of the friction torque is seen on the shelf.

DISCUSSION

L. K. Sillcox.[1] Our experience indicates that complete saturation of wool waste requires soaking for a period of approximately 48 hr. at a temperature above 75 deg. fahr. In present-day service we are operating passenger locomotives 7000 miles between inspections, and 5000 miles in freight service. We have found the objective in maximum utilization of motive power very much restricted, because of the constant attention required by bearings as at present constructed on electric locomotives.

The author, in Par. 49, gives as his opinion that oil grooves are not needed to obtain good lubrication, with which the writer agrees, because we feel that oil grooves are more or less of a starting condition. They soon become more or less filled with foreign particles as the bearing advances in service.

[1] General Superintendent of Motive Power, Chicago, Milwaukee & St. Paul Ry., Chicago, Ill. Mem. A.S.M.E.

In Par. 51, reference is made to newly packed bearings, and the information is quite correct. We run all of our armature bearings a certain length of time before installing them in our running gear. The armature is mounted in the case and the bearing is run with 220 V. load, and as the author states, it is noticeable that the newly packed bearings, if they run warm, will show evidence of this at the outer end, and after cleaning and scraping there is no further trouble.

The matter of handling waste-packed bearings is something which needs to be given systematic and methodical attention as to the length of time that waste is left in service, and also as to the time that the waste is loosened up in the bearing. We have obtained the best results from loosening the waste in armature and suspension bearings about every eight or ten thousand miles. This does not mean repacking, but moving the waste so as to get the glazed portion away from the bearing. Nearly every paper one reads on waste-packed motor bearings brings out the desirability of getting the longest stranded waste possible, entirely free from any short ends, and a specification for wool waste for armature and suspension bearings should so state. The usual railroad specification shows a small percentage of short ends and strands, and these are the portions which most often give trouble en route.

Practical experience has shown that common car oils are not suitable for high-speed bearings like an armature bearing. What is needed is an oil with a high surface tension, that is, one not thinned out readily by heat.

The title of the paper suggests other features that might be given consideration supplementary to the specific outline of the subject, as handled by the author. The following thoughts interlock with the development of the subject of lubrication of waste-packed bearings.

In all the studies that we have made on this subject, it is shown that between the brass and the journal there is an actual oil pressure which can be indicated by a pressure gage. Also, that after a bearing has been once successfully started, it is possible to make the brass on a journal work so nicely that there should be absolutely no metallic contact between the journal and the brass, the whole of the weight being borne by the oil. The important practical inference is that it is actually possible so to lubricate a bearing (once its operation is successfully established) that not only would metallic friction be altogether eliminated, and thereby the amount of power lost by friction be reduced, but metallic wear and tear also would be eliminated. However, this is an ideal which is difficult of attainment because there seems to be an initial metallic contact before lubrication begins, except in rare cases. Even then, internal fluid friction may result because of this lack of metallic contact, by reason of the grade of lubricant em-

ployed to maintain a static oil film on the journal. Lubrication results from the heat generated, causing the oil to flow.

Under ordinary circumstances there is not much difficulty in carrying out the lubrication of a journal from beneath; however, care is needed with an exceedingly well fitting journal because of the difficulty in getting oil in between. It is generally recognized that the oil acts as a lubricant by merely furnishing molecules rolling in between the two surfaces, and unless the molecules can get in between, there is no possibility of lessening the friction. Any attempt to oil, from the top, a bearing under heavy pressure is impracticable.

Another consideration which ought to be clearly understood is the difference between the load a journal can carry when the load is continuously on it, as for instance in the journals of railway cars, and when the load is intermittent. In railway work, the average static load is 450 lb. per sq. in. This figure is obtained by assuming an angle of 57 deg. on either side of a line passing through the center of the bearing surface and the center of the cross-section of the journal. If the length of the bearing surface is 8 in., the area would be 34 sq. in. on a 5 x 9 A.R.A. journal bearing. The radius of the bearing surface is $2\frac{17}{32}$ in., and the radius of the journal is $2\frac{16}{32}$ in. This only permits of a theoretical line contact, but factors of pressure and flow of metal in the bearing produce nearly complete contact in a comparatively short period of time.

The distribution of weight quoted above is 450 lb. per sq. in., assuming a 40-ton car loaded to capacity. If maximum conditions encountered in service, such as sharp curves, low rail joints, frogs, switches, etc., are considered, the estimated distribution of weight would be approximately 735 lb. per sq. in. These figures do not estimate impact values that might run as high as 1500 lb. per sq. in.

Using the figures given in Circular No. 101 of the U. S. Bureau of Standards, page 133, it is found that the permanent deformation at 70 deg. fahr. of a cylinder of babbitt, $1\frac{1}{4}$ in. in diameter by $2\frac{1}{2}$ in. high, having the composition of the present A.R.A. specification for babbitt, would be 0.004 in. at 1000 lb. Resolving this figure to the same terms as the distribution of weight, it can be stated that a deformation of 0.004 in. would be developed, with a static load of 814 lb. per sq. in. The Brinell hardness of this bearing metal at 70 deg. fahr. is 19.5. At 212 deg. fahr. the Brinell hardness is reduced to 8.6. Assuming that there is a definite relation between Brinell hardness and permanent deformation, the actual deformation under a load of 814 lb. per sq. in. at 212 deg. fahr. would be 0.0172. Therefore the difference in radius between the bearing surface and the journal is sufficient to take care of the average wear of the babbitt surface — there is no "appreciable" deformation of the bearing surface to conform to the radius of the axle.

The average operating temperature of car bearings is approximately 125 deg. fahr. The deformation or hardness of the bearing metal under operating conditions would be, therefore, in between the figures given for 70 deg. fahr. and 212 deg. fahr. Any "seating" of the axle in the bearing probably is due to wear and to a small amount of "flow" of the crystals on the surface of the babbitt.

When the motion is reciprocating, so that the load is alternately relieved from the journal, as with crank pins, much higher loads may be applied than those considered for the car journal. However, the journals on a railway car differ from ordinary practice in that they are in a state of vibration which probably allows the oil to pass freely around.

It is well to give consideration to definitions of some of the terms referred to in the paper. Friction must be separated into solid friction, fluid friction, and internal friction of fluids. Results of experiments which we have considered indicate that friction, when the rubbing surfaces are kept well separated by the lubricant, is more dependent upon the physical characteristics of the lubricant than upon the nature of the solids carrying the load. There seems to be a combined friction of the particles in the lubricant, usually expressed as "absolute" viscosity, and on the moving surfaces in contact with the lubricant. With constant pressure and temperature, it is dependent upon the extent of surface in contact, and varies in direct proportion. According to T. C. Thomsen, the frictional resistance with fluid friction is:

(*a*) Independent of the pressure between the surfaces
(*b*) Increased with speed of rubbing surfaces
(*c*) Increased with area of rubbing surfaces
(*d*) Independent of the condition of the rubbing surfaces, or the materials of which they are composed, so long as they are definitely separated
(*e*) Depends entirely on the viscosity of the lubricant at the working temperature of the oil film.

Absolute viscosity is "the force which will move a unit area of plane surface with unit speed relative to another plane surface from which it is separated by a layer of the liquid of unit thickness."

As the resistance of lubricated surfaces is made up of the resistance of the particles of the lubricant, it is evident that any influence that will change its fluidity will also effect the frictional resistance.

Increase of temperature, increasing the fluidity (or viscosity) causes a decrease in the coefficient of friction. If F is the frictional resistance, and P the total pressure between rubbing surfaces, then the coefficient of friction, C, may be expressed as

$$C = \frac{F}{P}$$

The density of a lubricant in the metric system means the weight of one cubic centimeter, as compared with the weight of one cubic centimeter of water at 4 deg. cent. It is proportional to the specific gravity, but has no direct bearing on the lubricating value of a lubricant. The condition to be attained is that where the viscosity of the oil is such at the working temperature that it will be sufficient to keep the solids from contact under the pressure which must be sustained.

Where friction is produced it is important to distinguish between the two conditions to which the two sets of laws apply; in one it is a solid against a solid, the particles of each interlapping and causing resistance by the efforts of the particles of one metal to tear away those of the other. Where lubrication is introduced it is intended that the two solids shall be separated by a film of the lubricant, generally a liquid. In this latter case the resistance assumes the nature of the laws of fluids, and consists of the friction of the particles of the lubricant and that of the solid against the fluid, forming a combined resistance, the percentage of each of the whole retardation depending upon the nature of the lubricant and the metal surfaces. As long as the metals are prevented by the lubricant from coming in contact, it is found that friction is dependent upon the fluidity of the lubricant, and varies with changes of this fluid condition, decreasing with a higher temperature and increasing with a less degree of heat.

If we assume that the lubricant, in all cases, prevents any contact of the metal surfaces, the condition then stands between the laws of solid friction on one hand — friction independent of the surfaces in contact, but dependent upon the total pressure — and the laws of friction of liquids on the other, that friction is independent of the pressure per unit of surface and increases as the square of the velocity. From such studies as we have been able to make, this intermediate condition has been found to be, when stated in a general way, that the coefficient of friction decreases with an increase of pressure, although the total resistance rises directly but not proportionately with the higher unit pressures, and increases with the velocity, although not as rapidly as its square. It is also found to be dependent upon the extent of surface in contact.

It should be remembered that with high pressures, such as those obtained before seizure takes place, the film of oil separating the bearing and journal has been found to be very small. This would indicate the result that may be expected with a bearing where the irregularities or projections on the surface of the journal or bearing are greater than the thickness of the film of oil used to separate them, producing when in motion a rapid and detrimental abrasion of the metals with a marked increase in the friction. The other extreme, of having the surfaces too highly polished, must not be selected, as it has been found that a moderately rough machined

surface will carry something like seven times greater pressure before seizing than can be obtained from highly polished surfaces.

When starting with a new bearing, the surface in contact is much less than when it has worn down to a point where the whole arc of contact has been obtained. This is one of the conditions which must be met, for with the irregularities of the parts accompanying the distribution of the load it is found, excepting with the so-called soft-bearing metals, that heating will almost invariably result if the bearing is fitted to the journal throughout the whole arc which it is capable of including, because there seems to be a binding action on the journal. If the bearing is so fitted as to allow a small amount of motion, the wear will take place in a manner consistent with the alignment of the journal box.[1]

In Par. 33 reference is made to the "oiliness" of the lubricant. This term is frequently misused, as a satisfactory definition for this property has not been developed. All that we know of this property is that two oils having similar physical and chemical properties, and the same absolute viscosities, appear to have different coefficients of friction. The reason for this condition has not been explained.

Frequent reference is also made to surface tension of lubricants. The standard definitions for surface tension do not satisfy all of the problems encountered in lubrication. The gradual acceptance of the theories explaining the properties of colloids will necessitate material change in the ideas of surface tension as applied to lubricants. The surface molecules exert the same sphere of influence as all the other molecules in the liquid, and the equilibrium between the surface of the liquid and the air or metal in contact with the liquid should absorb the surplus energy that may exist at the surface. The nature of the lubricant varies the surface tension according to the number of "unsatisfied" atoms in the molecule.

Allen F. Brewer.[2] In Par. 12 the probability of insoluble matter clogging the minute channels in cotton wicking is mentioned. This is for a so-called straight mineral residual car oil. It would be interesting to know the author's opinion as to the possible action of certain compounded steam-railway-car oils, under such conditions of test. In other words, would the presence of such a component as lead-fish-oil soap have any material effect upon the capillarity of cotton wicking?

[1] In practice "bright spots" frequently appear on bearing surfaces due to the film of lubrication being broken by foreign material in the lubricant or by excessive static or initial pressure. After this metallic contact is made, considerable difficulty is experienced in service in attempting to re-establish the film in this area.

[2] Mechanical Engineer and Editor of *Lubrication,* The Texas Company, New York, N. Y. Mem. A.S.M.E.

Conditions in actual operation are frequently akin, both in steam and electric railway service, especially with the advent of the gas-electric railway car. It would seem advisable, therefore, to extend the author's studies to such car oils as are commonly used in journal boxes in the former service.

Another point of inquiry is the matter of adequate lubrication as brought out in Par. 11. The author states that bearings have been noticed to run warmer due to lack of oil flow until oil and waste become heated up. Would it not be possible for such conditions to give rise to glazing of the waste in the bearing window? Observation of certain instances of the latter has led to the impression that lack of lubrication in the face of higher temperatures may have been the responsible factor. Certainly if this is the case the adoption of "oil-sealed" housings, as mentioned in Par. 14, is a decided advancement toward the attainment of more effective lubrication in such bearings.

The Author. Replying to Mr. Brewer's questions, the author has not available complete data concerning the influence of soap on the capillarity of waste. There is no doubt, however, that oil fed through waste contains less soap than it originally had. For instance, a certain grade of compounded oil showed, when centrifuged,

(1) Originally 6 per cent of precipitated matter, approximately
(2) When fed at room temperature through heavy long wool strands, 3 per cent of precipitated matter, approximately
(3) When fed at room temperature through cotton waste, 2.75 per cent of precipitated matter, approximately.

Another grade of compounded oil showing originally $6\frac{1}{2}$ per cent of precipitated matter was applied in a waste-packed armature bearing. The used oil, after passing through the bearing, showed $5\frac{1}{2}$ per cent of precipitated matter. Taking into consideration that about 15 per cent of the oil evaporates while passing the bearing, the oil fed by the waste had on the average $5\frac{1}{2} \times 0.85 = 4\frac{1}{2}$ per cent of precipitated matter, or 0.7 of the original amount.

If there should be any clogging of the waste channels by solid particles at low temperatures, the soap liquefies at from 40 to 45 deg. cent., and above this temperature no clogging will take place. Glazing of waste cannot therefore be attributed to starting conditions, as suggested by Mr. Brewer. In the author's opinion it is due to dirt penetrating into the bearing. In so far as the waste is clean and not completely dry, it will not glaze at temperatures up to 150 deg. cent. The introduction of the "oil-sealed" housing, which provides a constant oil lift in the bearing, will no doubt assist considerably in eliminating glazing.

The author fully agrees with Mr. Sillcox on the benefits of proper maintenance, proper inspection, and careful running-in of a new or rebabbitted bearing. In fact, most railway-motor-bearing troubles may be traced to faulty maintenance.

With regard to Mr. Sillcox's remark that common car-axle oil is not suitable for armature bearings, it should be noted that the methods of lubrication in the two instances differ considerably. In axle boxes it is important that the oil be well suspended in the waste, while oil carried by the journal through the brass returns largely into the box. In armature bearings the oil once passed through the bearing is lost, and the lubrication depends on the ability of the waste to carry over a sufficient amount of oil from the oil well to the window.

As shown in the paper, in waste-lubricated bearings it is impossible to avoid altogether metallic friction, and most of Mr. Sillcox's remarks dealing with conditions of a complete oil film could not be applied safely to this type of bearings.

Should a complete oil film exist in a bearing, separating completely the lubricated surfaces, then only internal fluid friction will take place. The work of Tower, Reynolds, Mitchell, Harrison, Kingsbury, Lasche, and others has shown quite convincingly that the layer of liquid lubricant in contact with the metal surface holds fast to it, only internal friction having place. The conception of small globules of oil rolling like balls between the two surfaces of metal, though imaginatively descriptive, does not appear to convey the viscous action of a lubricant as clearly as the conception of layers of a lubricant sliding over each other.

The remarks of Mr. Sillcox that a moderately rough machined surface will carry something like seven times the pressure than may be obtained from highly polished surfaces is very interesting. Recent bearing practice appears to abandon this viewpoint; considerations of proper lubrication and experience show that the journal cannot be too highly polished.

No. 2032

HEAT TRANSFER FOR FORCED FLOW OF AIR AT RIGHT-ANGLES TO CYLINDERS

By E L. Chappell,[1] Cambridge, Mass.
Non-Member

and

W H. McAdams,[2] Cambridge, Mass.
Member of the Society

For air flowing outside and at right-angles to single cylinders, published data are available covering extreme ranges of 120-fold for the coefficient of heat transfer, 250-fold for mass velocity, 1800-fold for diameter, and over 2-fold for film temperature in degrees fahrenheit absolute. Of the theoretical equations proposed by Boussinesq and King, that of the latter is far more in accord with these data. At the lower temperatures, the theoretical King equation gives predicted values in surprisingly good agreement with the data, in view of the rather questionable assumptions made during its derivation. The empirical equations of King for wires and of Rice for pipes agree well with the experimental values from which they were derived, but are not satisfactory when extrapolated to conditions for which they were not designed.

The method of plotting suggested by Davis has considerable promise, as does the rather complicated method of Rice involving the mean diameter of the effective film. No simple method was found for perfect correlation of the exact effect of each of the various factors. However, an empirical equation was developed and is recommended for practical use. For the high temperatures, where the exact effect of temperature is uncertain, the correction term for temperature was

[1] Research Associate, Research Laboratory of Applied Chemistry, Department of Chemical Engineering, Massachusetts Institute of Technology.

[2] Associate Professor of Chemical Engineering, Massachusetts Institute of Technology.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers, 29 West 39th Street, New York, at a Joint Session with the American Society of Refrigerating Engineers. Program arranged by the Committee on Heat Transmission, National Research Council. Published as the 15th contribution of the Heat Transmission Program of the Department of Chemical Engineering, Massachusetts Institute of Technology, Chem. Eng., Serial No. 175.

so chosen as to predict low rather than high values of the coefficient of heat transfer. The empirical equation involving the various factors is plotted in Fig. 8, thus making it unnecessary to substitute the values of the various factors in the recommended equation.

Data for staggered pipes are rather meager, but for a given mass velocity, diameter, and film temperature, the coefficients are somewhat higher than for single pipes. A simple method is suggested for estimating the increase due to staggering.

Introduction

IN ORDER to design many types of heat-transfer apparatus, it is necessary to be able to make a reasonably accurate estimate of heat transfer between cylinders and air or other gas flowing at right angles to heated or cooled cylinders, often arranged in the form of a bank of staggered pipes. Illustrations in commercial practice include air heaters, economizers, heat exchangers, heating coils used in the refining and cracking of petroleum, and the like. Such data also are necessary for the important problem of the calculation of the true temperature of a gas from the reading of a thermocouple or thermometer placed in a gas stream where the gas and walls are at different temperatures (1).[1] Similar data are used by the electrical engineer, for example, in the design of hot-wire anemometers, and in wind-tunnel work.

2 In reviewing the literature on this subject, much of which appears in publications not familiar to most engineers who work in these fields, it was found that a number of equations had been proposed for the estimation of the coefficient of heat transfer. Some of these were based entirely on theory, others were designed to fit particular data covering a rather narrow range of variation in gas velocity and pipe diameter, and still others were offered as of general applicability.

Object

3 The purpose of this investigation was two-fold: (1) to make previous data readily available in a common set of units; (2) to compare, and if possible to correlate, the available data, so that reasonable estimates could be made for conditions not yet investigated.

General Equations Previously Proposed

4 Several equations, based upon theoretical considerations and certain assumptions, have been proposed. Unless such equations are checked by experimental data over a wide range of all factors involved, the assumptions must be regarded as limitations to the equations.

[1] This and subsequent reference numbers correspond to those listed in the bibliography at the end of the paper.

5 *Boussinesq.* In 1905, Boussinesq(2) derived an equation which is frequently mentioned in the literature. This equation is based upon the differential equation for heat conduction proposed by Fourier(3) in 1820. The fluid was assumed to be incompressible and frictionless, and the effect of variation in physical properties of the fluid with temperature was neglected. Since the fluid was assumed to be incompressible, for the sake of consistency the specific heat C_v is taken at constant volume rather than at constant pressure.

6 At a definite fluid velocity and temperature, the fluid film upon a given cylinder was assumed to be of uniform thickness, which is contrary to experimental evidence later obtained by three independent investigations cited below. Making other assumptions as to the relative magnitude of certain terms, integration gave the following:

$$\frac{hD}{k} = 211\sqrt{\frac{Du\rho C_v}{k}} \quad \text{. . . . . [1]}$$

A complete table showing the meaning and units of all symbols is given under Nomenclature, at the end of the paper.

7 *Russell.* In 1909, in an excellent mathematical article on heat transfer, Russell(4) rederived the Boussinesq equation. Stress was laid on the limitations, it being emphasized that Equation [1] should hold for incompressible fluids only, thereby excluding gases. However, it was admitted that the equation should not be exact for liquids, because the effect of viscosity was ignored in the derivation.

8 On the other hand, Russell states that the relative effect of both velocity and diameter on the coefficient of heat transfer, as called for by the Boussinesq equation, is in line with experimental data on the forced convection of heat from small wires to air. This statement is sometimes misinterpreted by readers to the effect that the Boussinesq equation had been verified for wires. The Boussinesq equation is sometimes(6) referred to as the Russell equation.

9 *King.* In 1914, King(5) presented an excellent paper on the forced convection of heat from small cylinders to gases. In the theoretical part of this paper, King derived an equation from the same premises as Boussinesq, but the derivation was mathematically more exact. However, this equation, like that of Boussinesq, is also based on the assumptions of uniform film thickness around the wire, and assumes the fluid to be incompressible and frictionless. The theoretical equation is as follows:

$$\frac{hD}{k} = 3.81 + 166\sqrt{\frac{Du\rho C_v}{k}} \quad \text{. . . . . . [2]}$$

According to King, this equation should not hold when the product $Du\rho$, or Dv, is less than 0.000018. However, in practically all cases, except for the combination of a wire of extremely small diameter and a very low velocity, the value of $Du\rho$ does exceed this value. It should be noted that the equations of both Boussinesq and King are based wholly on theory and certain assumptions, and yet the quantitative relations between the various factors are predicted.

10 King represented his experimental data on small wires by the following empirical equation:

$$h = [0.0553(1+88.9D)(1+0.000663\Delta t)/D] + 1.968(1+0.000044\Delta t)\sqrt{\frac{u}{D}}$$

11 *Langmuir*. During 1912 and 1913, Langmuir presented several valuable papers(6, 7, 8) on both free and forced convection of heat, the 1913 paper(6) being the most complete. In treating free convection, Langmuir calculated the effective thickness of film necessary to account for the observed heat loss as due to conduction through the film. He found that the coefficient of heat transfer increased with temperature at the same rate as the true average specific thermal conductivity of the film. While for free convection this concept of a constant film thickness was satisfactory, for forced convection Langmuir(6) stated: "For forced convection, that is, loss of heat in strong air currents, the film theory does not seem to apply. On the other hand, an equation derived by A. Russell seems to agree well with the experimental data available." The Boussinesq or Russell equation is given above as Equation [1]. Langmuir calculated the ratio of Kennelly's 1909 forced-convection data(9) for three sizes of small copper wire to that called for by Equation [1], assuming the specific thermal conductivity k for air as 0.0162, with C_v equal to 0.17, and obtained an average ratio of $\frac{5}{8}$. Making a similar comparison based on some rather incomplete forced convection data published by Carpenter(10) for two-inch steam pipe, the ratio was found to vary from $\frac{7}{8}$ to 1. Therefore, Langmuir recommended Equation [1] for forced convection, with the understanding that it be multiplied by a factor of $\frac{5}{8}$ for small wires, the factor approaching unity for "large cylinders."

12 *Davis*. In 1922, Davis(11) attempted to modify Boussinesq's equation to allow for the effect of fluid viscosity, z. Unlike Boussinesq and King, Davis did not integrate and get a definite equation, but did obtain a grouping of variables, as shown by the following equation:

$$\frac{hD}{k} = f_1\left(\frac{Du\rho}{z}\right)f_2\left(\frac{cz}{k}\right) \quad . \; . \; . \; . \; . \; . \; . \quad [3]$$

where f_1 and f_2 are two functions to be determined experimentally. It is interesting to note that Equation [3] is the same as the original theoretical Nusselt equation(14) for gases flowing in turbulent motion inside pipes. For gases, cz is directly proportional to k, and the factor of proportionality is constant for permanent diatomic gases. Hence for air, Equation [3] cannot be distinguished from either of the following:

$$\frac{hD}{k} = f_3\left(\frac{Du\rho}{z}\right) \qquad [3a]$$

or

$$\frac{hD}{k} = f_4\left(\frac{Du\rho c}{k}\right) \qquad [3b]$$

Based on his experimental data for gas flowing in turbulent motion inside pipes, using an equation of the form of Equation [**3b**], Nusselt obtained

$$\frac{hD}{k_p} = 27.1\left(\frac{Du\rho C_p}{k_g}\right)^{0.786} \qquad [3c]$$

However, Royds(20) has shown that Nusselt's data may be correlated by the simple equation for gases inside pipes having an inside diameter of 0.868 inches:

$$h = 17.5 C_p v^{0.786} \qquad [3d]$$

13 In a 1921 paper(12), Davis shows that certain data for air flowing outside and at right-angles to single wires may be plotted, as suggested by Equation [**3a**], with satisfactory results. However, the equation of the curve representing these data was not given.

14 Davis(11) states that the forced convection data of Kennelly, Worthington, and Malone(13) between wires and water show satisfactory accord with the data for air.

15 *Rice.* For gases flowing at right-angles to single cylinders, Rice(13) in 1923 employed equations of the Nusselt or Davis type. Based on the data of Hughes(15) for the transverse flow of air over pipes having diameters of 0.164, 0.319, 0.76, and 1.99 in., Rice obtained:

$$\frac{hD}{k_f} = 62.2\sqrt{\frac{Du\rho_f}{z_f}} \qquad [4a]$$

The empirical constant of 62.2 naturally depends on the values used by Rice for k_f, z_f, and ρ_f. For air, with these values expressed as a power function of the geometric mean of the absolute temperatures of the pipe surface and the main body of the air, Rice obtained the equivalent of the following:

$$h = \frac{0.60\sqrt{u\rho T_g/D}}{T_{geom.}^{\;0.123}} \qquad [4b]$$

16 Like the Boussinesq equation, Equation [**4b**] states that h is directly proportional to the square root of $u\rho/D$.

17 Based on the data for air of King(5) and of Kennelly(9, 22), both for small wires, Rice obtained an expression for the thickness B of uniform film equivalent in thermal resistance to the actual film:

$$\frac{B}{D} = 0.0143 \left(\frac{z_f}{Du\rho_f}\right)^{\frac{2}{3}} \quad . \; . \; . \; . \; . \; . \; . \quad [\mathbf{5a}]$$

where B is in feet. Equation [**5a**] must be used in conjunction with the logarithmic mean of the areas of the film and of the pipe itself.[1] Expressing these relations as the coefficient of heat transfer per unit area of wire surface h, one obtains:

$$\frac{hD}{k} = \frac{10.43}{\log_{10}[1+0.343(z_f/Du\rho_f)^{\frac{2}{3}}]}$$

18 *Non-Uniformity of Film Thickness for Transverse Flow.* From the modern point of view, the film on the surface of a solid in contact with a moving fluid is defined as that zone in which the velocity is relatively small compared with that of the main body of the fluid. That this film is not of uniform thickness, is indicated by three different types of experiments.

19 For the transverse flow of air around a single unheated pipe, Morris(16) shows that the lines of equal velocity in the film were not concentric with the pipe, the thickness being much greater on the downstream than on the upstream side. These velocity contours were obtained by a hot-wire anemometer.

20 Interesting results were obtained by Rubach(17), who photographed the disturbances caused by the movement of an unheated rod through a bath of liquid. As the velocity was increased, violent eddy currents were set up on the downstream side.

21 Stanton(18) reports experiments by Jakeman, who determined the relative heat loss from an electrically-heated metal strip attached to a cylindrical rod of ebonite, the axis of the strip being parallel to the center-line of the rod. When the rod was

[1] When a small wire is subjected to a moderate gas velocity the effective thickness of the film, while very small, is large compared with the diameter of the wire. This is equivalent to a small pipe insulated with lagging of considerable thickness, in which case it is important to calculate the heat conduction through the insulator allowing for the mean cross-section of the covering. Hence Rice uses the logarithmic mean diameter of the effective film rather than the diameter of the wire itself.

This method of attack is obviously sound, and from the academic viewpoint it offers the most promise in correlating such data. However, it complicates the calculations, and involves a method not familiar to engineers. For these reasons this method was not used herein. Failure to do so accentuates the curvature of the plots of h versus v for the wires. See Fig. 4.

so turned that the air stream impinged directly on the metal strip more heat was lost than when the rod had been rotated 180 degrees. These results were reversed when using a prism. It was noted that the heat loss from the back side was practically the same for both shapes, and the heat loss from the front of the prism was only about one-half that from the front of the cylinder. Experiments of the same general significance are reported by Reiher(24).

22 These three experiments on cylinders show that the fluid film is not of uniform thickness, indicating that the film is thinner on the side facing the stream than on the reverse side.

Discussion of Experimental Data for Single Cylinders and Comparison with Previous Equations

23 The data covering the widest range of numerical values of the coefficient of heat transfer were obtained on single cylinders. These data are therefore considered first, after which the data on staggered pipes are discussed.

24 *Factors Pertaining to the Pipe.* The factors which may affect the coefficient of heat transfer may be grouped as those pertaining to the pipe and those referring to the gas. The work of previous investigators indicates that, other things being equal, the coefficient of heat transfer is an inverse function of the outside diameter of the cylinder.

25 It has long been suggested that the condition of the surface of the cylinder might possibly have an appreciable effect upon the result. Since proper data for flow at right-angles to cylinders are not available to throw light on this question, in the treatment of the data presented no allowance was made for this factor. However, it is appreciated that surface condition should have an effect if the surface of the cylinder were not clean. In this connection, the experiments of Parsons and Harper(19) show that, for gases inside pipes, surface conditions affect heat transmission. Expressed as percentage change of the heat transfer from the original drawn-brass tube, their results were as follows:

Surface lightly smoked, 10 per cent decrease
Surface scratched with sandpaper, 4 per cent increase
Highly polished, 17 per cent increase.

However, the percentage effect of the insulating layer of foreign material is not so marked as in the case of heat transmission for water flowing inside pipes(21), due to the relatively greater thermal resistance of the gas film as compared with the water film.

26 *Factors Pertaining to the Gas.* This group of factors includes gas velocity and the physical properties of the gas, such as specific thermal conductivity, viscosity, and specific heat. While these physical properties depend on the gas involved, as well as

the temperature, for the data considered herein, which apply only to air, these physical properties may be taken care of by some function of temperature, as is done by Rice(13). In the range of pressures ordinarily encountered these properties for a given gas are independent of pressure and are a function of temperature only, and are usually plotted against absolute temperature.

27 In résumé, in considering the experimental data for the flow of air at right-angles to single cylinders, the following factors

TABLE 1 KING'S DATA FOR $D = 0.00115$ IN.

Heat transfer from single cylinder to air at 60 deg. fahr.

Film temp., deg. fahr. abs.	751	836	956	1032	1158
v lb. per sec. per sq. ft.	h	h	h	h	h
0.043	98	106	122	132	134
0.117	127	136	169	157	165
0.435	198	205	220	232	236
0.816	242	253	272	284	287
1.05	265	275	302	308	313
1.64	316	328	350	356	363
2.20	356	370	385	404	409

TABLE 2 KING'S DATA FOR $D = 0.00602$ IN.

Heat transfer from single cylinder to air at 62 deg. fahr.

Film temp., deg. fahr. abs.	711	890	1071	1242	1410
v lb. per sec. per sq. ft.	h	h	h	h	h
0.0435	34.1	38.4	41.2	43.9	48.9
0.103	45.5	50.7	53.5	56.9	61.3
0.206	57.3	64.0	64.0	67.5	71.3
0.413	73.5	80.5	81.5	86.3	90.7
0.707	95.5	100.5	103.3	106.2	113.0
1.15	115.7	118.2	120.7	124.6	130.0
1.51	123.5	132	134.6	138.2	143.6
2.01	143.4	148	150.5	154.5	160.0

TABLE 3 KENNELLY'S DATA FOR $D = 0.0045$ IN.

Heat transfer from single cylinder in air at two atmospheres; gas pressure = 2 atm. absolute

v lb. per sec. per sq. ft.	h	v lb. per sec. per sq. ft.	h
3.55	258	9.65	422
4.43	289	10.7	438
5.25	315	10.9	448
6.13	339	3.52	255
7.14	362	2.61	221
7.92	382	1.51	179
8.78	403	0	52

should affect the coefficient of heat transfer: gas velocity, pipe diameter, and temperature. For gases flowing in turbulent motion inside pipes, Nusselt has shown both from theory and ample experimentation that linear velocity and gas density have equal effect on the coefficient of heat transfer. Therefore the product of linear velocity and gas density is used rather than linear velocity, this product being called mass velocity. Throughout this paper, mass velocity v is expressed in pounds per second per square foot of cross-sectional area, and is obtained by multiplying together the two following factors: linear velocity u in feet per second and gas density ρ in pounds per cubic foot. Coefficients of

heat transfer h between cylinder and gas are expressed as British thermal units per hour per square foot of cylinder surface per degree fahrenheit difference in temperature between the surface of the cylinder and the gas. The outside diameter D of the cylinder is expressed in inches. The arithmetic mean of the temperatures of the gas and the surface of the cylinders is expressed in degrees fahrenheit absolute; this is called "film temperature," T_f.

28 *Experimental Data on Single Cylinders.* The three major sources of comparable data are those of King, Kennelly, and Hughes, the first two applying to wires and the third to pipes.

29 *King.* In 1914 King(5) observed the heat loss from small hot wires mounted on a fork rotating about its horizontal axis in the air of a room which averaged approximately 63 deg. fahr. in

TABLE 4 HUGHES' DATA FOR $D = 0.169$ IN. AND $D = 0.319$ IN.

$D = 0.169$ in.		$D = 0.319$ in.			
v lb. per sec. per sq. ft.	h	v lb. per sec. per sq. ft.	h	v lb. per sec. per sq. ft.	h
0 0	7.35	0.0	4.7	2.72	21.1
0 83	17.6	1.92	11.7	2.85	20.7
1.07	17 8	1.76	15.9	2.90	21.8
1.38	20.5	2.00	17.7	2.87	21.4
1.55	20.0	2.07	17.8	2 90	21.3
1.87	22.4	2.11	18.0	2.95	22.3
1.94	23.2	2.13	17.9	3.00	22.2
2.05	23.5	2.29	19.1	3.10	23.0
2 18	24.6	2.49	19.0	3.23	22.7
2.19	25.5	2.37	19.1	3.27	23.0
2.41	27.1	2.56	19.9	3.32	23.5
2.48	26.0	2.70	20.0	3.50	24.7
2 82	28.5	2.65	20.3	3.52	24.3
2.86	28.9	2.73	20.2	3.52	24.8
3.08	30.4				
3 14	31.2				
3 27	31.1				

temperature. The relative velocity between the wire and the air was calculated from the speed of rotation and the radius of gyration, a small correction being applied for the swirl set up by the rotating arms. The power dissipated from the wire was measured by a wattmeter, and the temperature of the wire was calculated from its known resistance. Due to the high values of h obtained with these very small wires, the heat radiated from the wires, even at the highest temperatures, was small compared with the total loss of heat. The results given herein are based on the total heat loss with King's corrections for radiation. The temperature of the air did not rise appreciably and as a result the problem of the measurement of true gas temperature fortunately is not involved in the interpretation of these data.

30 Ten different platinum wires were used, ranging in diameter from 0.001115 to 0.00603 in. King gave tables of data only for the largest and smallest wires, and these are the results treated in this paper, Tables 1 and 2.

31 *Kennelly and Sanborn.* In 1914 these observers determined the heat loss from a platinum wire 0.0045 in. in diameter, mounted on a fork rotating inside a closed tank of air at about 68 deg. fahr. Data were obtained at nine different air pressures ranging from 0.45 to 4 atmospheres. The data were given in the form of curves, except the data for two atmospheres, total pressure, which are used herein. No correction for radiation was necessary, as the estimate for radiation amounted to only one per cent of the total heat loss. The data at the different pressures support the use

TABLE 5 HUGHES' DATA FOR $D = 0.76$ IN. AND $D = 1.99$ IN.

Heat transfer for single cylinder to air at 59 deg. fahr.

$D = 0.76$ in.		$D = 1.99$ in.	
v lb. per sec. per sq ft.	h	v lb. per sec per sq ft.	h
0.0	2.08	0	1.1
0.59	5.93	0.547	3.45
0.77	6.96	0.835	4.20
0.89	7.27	0.942	4.57
1.03	8.03	1.21	5.47
1.04	8.00	1.24	5.76
1.11	8.46	1.39	6.25
1.38	9.70	1.44	6.26
1.67	10.8	1.60	6.70
1.85	11.0	1.66	6.86
1.86	11.2	1.83	7.40
2.01	12.2	1.86	7.60
2.08	11.9	1.97	7.83
2.10	11.9	2.18	8.47
2.28	12.6	2.20	8.23
2.36	13.2	2.32	8.50
2.38	13.4	2.41	8.85
2.40	13.2	2.43	9.03
2.40	13.2	2.47	9.20
2.60	13.6	2.55	9.40
2.65	14.1	2.77	9.70
2.77	14.9	2.86	9.98
2.78	14.9	2.55	9.93
2.80	14.6	2.90	10.2
2.92	15.2	3.02	10.4
3.03	15.3	3.15	10.9
3.10	15.8	3.22	10.7
3.18	16.0	3.25	11.1
3.12	16.1	3.29	11.2
3.28	16.3	3.42	11.5
3.30	16.3	3.59	11.8
3.35	16.5		
3.29	16.6		
3.48	17.4		
3.50	16.8		
3.63	17.5		

of mass velocity, except the data for pressures below 0.5 and above 2 atmospheres. The data used are shown in Table 3.

32 *Hughes.* In 1916, Hughes(15) studied the heat loss from a single copper pipe with an air stream impinging at right angles to the axis of the pipe. Air at atmospheric temperature and pressure was drawn by a suction fan through a horizontal tunnel ten feet long and three by three feet in cross-section, through which projected a single vertical copper pipe supplied with dry saturated steam at atmospheric pressure. The rate of heat transfer over ten-minute intervals was determined by measuring the rate of flow of condensate from a trap at the bottom of the copper

pipe. The average velocity of approach of the air was determined by exploration by means of a Pitot tube. While Hughes did not tabulate the temperature of the steam or the temperature of the air, it would appear from the article that the temperature difference may be taken as 153 deg. fahr. without introducing a serious error. Hughes calculated the heat loss by conduction and convection by deducting from the total loss an estimate of the heat lost by radiation. This correction was not large except at the lowest velocity for the largest pipe, where it was 20 per cent. The results tabulated herein are based on the corrected values of Hughes. Four different single copper pipes were used, having outside diameters ranging from 0.169 to 1.99 in. See Tables 4 and 5.

33 *Comparison of Various Equations with Data.* In these data the extreme range of variations of the several factors is as follows:

	Lowest	Highest
Coefficient of heat transfer, h	3.5	410
Outside diameter of wire, D	0.00112	1.99
Mass velocity, v	0.043	10 9
Mean absolute temperature of film, T	595	1410

With this mass of data covering such an enormous percentage variation in the various factors, it was felt that a fair test could be made of the validity of the various equations which had been proposed. Referring to these equations, it is found that in all but the Davis equation, hD, for a given temperature, should be a linear function of $\sqrt{Dv}$. Further, in such cases, the exact values of the coefficients of proportionality were specified.

34 Therefore, it is of importance to determine two points: (1) whether or not the relationship is linear, and (2) how closely, on a percentage basis, the experimental points lie to the theoretical lines. The exact location of the theoretical line depends on the numerical values adopted for the physical properties, such as specific thermal conductivity and specific heat. The values employed are as follows:

$$c_v = 0.17 \text{ B.t.u. per deg. fahr. per lb.}$$

From Marks (23)

$$k = 0.0122(1+0.00132t), \qquad \text{where} \qquad t = \text{deg. fahr.}$$

However, the adoption of these values for the physical properties of air has no bearing whatsoever on the failure of the experimental data to give a linear relationship, affecting only the second comparison mentioned above.

35 *Comparison of Various Equations with Hughes' Data for Single Pipes.* Fig. 1 shows Hughes data for four sizes of copper pipe, ranging from 0.169 to 1.99 in., the film temperature in all cases being practically constant at 135 deg. fahr. or 595 deg. fahr.

absolute. On rectangular coördinates, the product hD is plotted as ordinates versus the square root of Dv as abscissas. The experimental points do not lie on a straight line, but the curvature is not serious. The theoretical King equation plotted on Fig. 1 runs high compared with the data at low values of $\sqrt{Dv}$, and low at high values of $\sqrt{Dv}$, the maximum deviation being of the order of plus 20 per cent. The theoretical Boussinesq equation, also plotted on Fig. 1, runs higher than the points at all values of $\sqrt{Dv}$. Recalling that the derivation of the Boussinesq equation was mathematically less exact than the derivation for the theoretical King equation, it is not surprising to find that the Boussinesq equation is less satisfactory than the theoretical King equation.

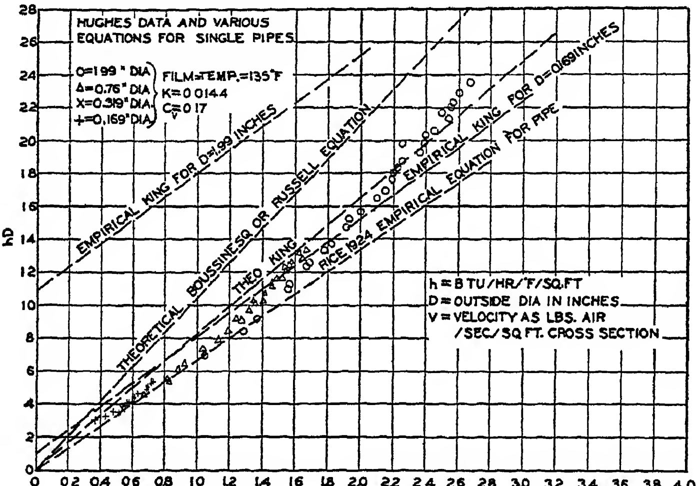


FIG. 1 COMPARISON OF EQUATIONS OF KING, BOUSSINESQ, AND RICE WITH DATA OF HUGHES FOR PIPES

As suggested by Langmuir, the value of hD computed from the Boussinesq equation could be made to check the experimental values by multiplying the theoretical value by a fraction less than unity. The necessary fraction for these data would vary between $\frac{2}{3}$ and $\frac{1}{3}$.

36 The empirical King equation has also been plotted on Fig. 1 for the two extreme diameters. For the largest diameter the empirical King equation gives predicted values at least four times as high as the data, whereas for the lowest diameter the empirical King equation gives values corresponding rather closely to the experimental values. This indicates that King's diameter correction becomes less reliable the greater the diameter, as would be expected from the fact that this empirical equation was obtained by King for very small wires. The empirical equation for pipes

published by Rice in 1924 is also plotted on Fig. 1, and this line is seen to check well with the data at low values of $\sqrt{Dv}$, while at the highest values of $\sqrt{Dv}$ the predicted coefficients are approximately 75 per cent of the observed values.

37 *Comparison of Various Equations with King's Data for Single Wires.* Fig. 2 shows the experimental data of King: A, for a wire having a diameter of 0.001115 in. and a film temperature of 291 deg. fahr. or 751 deg. fahr. absolute, and B for a wire having a diameter of 0.00602 in. and a film temperature of 251 deg. fahr. or 711 deg. fahr. absolute. These data are plotted on rectangular coördinates with the product hD as ordinates

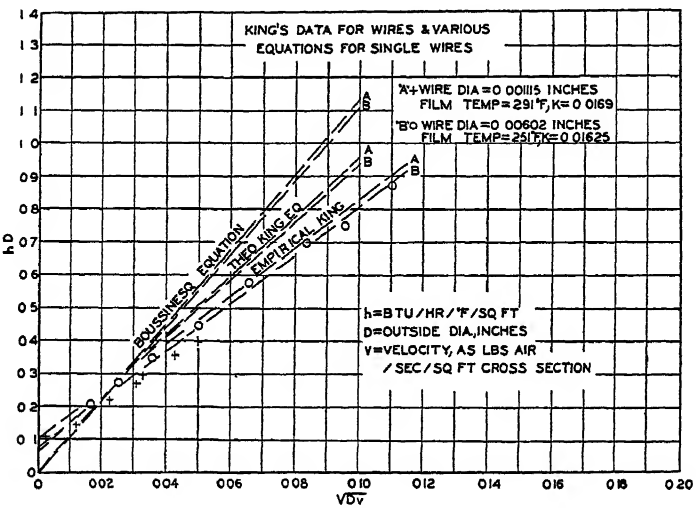


FIG. 2 COMPARISON OF EQUATIONS OF KING AND BOUSSINESQ WITH DATA OF KING FOR WIRES

versus the square root of Dv as abscissas. The experimental points fall satisfactorily on a straight line. The theoretical King equation is found to run higher than the experimental points A, the maximum deviation being plus 15 per cent. The Boussinesq equation, also plotted on Fig. 2, runs low at the small values of $\sqrt{Dv}$ and high at the larger values of $\sqrt{Dv}$. In order to make the Boussinesq equation fit the data for A, it would be necessary to multiply the theoretical values by $1\frac{1}{3}$ at the lowest values of $\sqrt{Dv}$ and by $\frac{3}{4}$ at the high values of $\sqrt{Dv}$. According to Langmuir (6) a value of $\frac{5}{8}$ should suffice for wires.

38 The empirical King equation for wires is also plotted on Fig. 2 against King's data for wires, and as expected the empirical equation fits the points. The empirical Rice equation for pipes

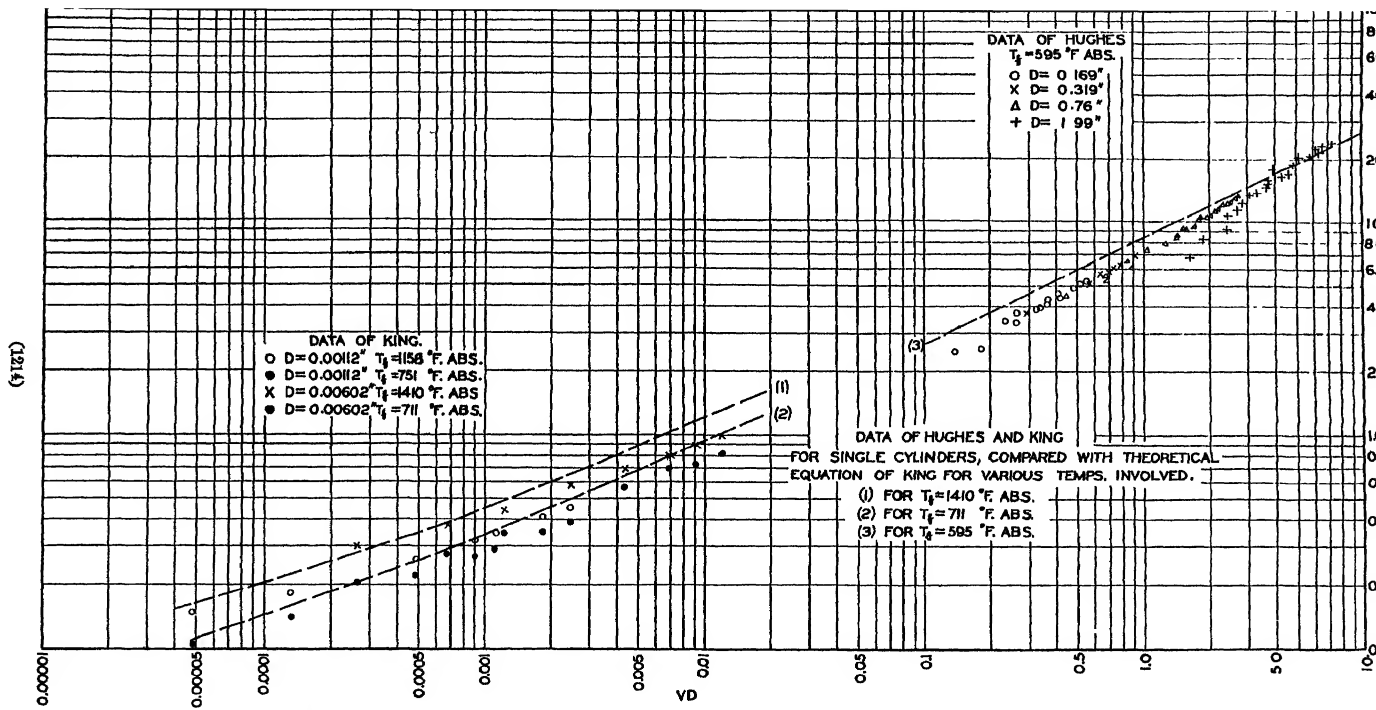


FIG. 3 COMPARISON OF THEORETICAL EQUATION OF KING WITH DATA OF HUGHES FOR PIPES, AND KING FOR WIRES

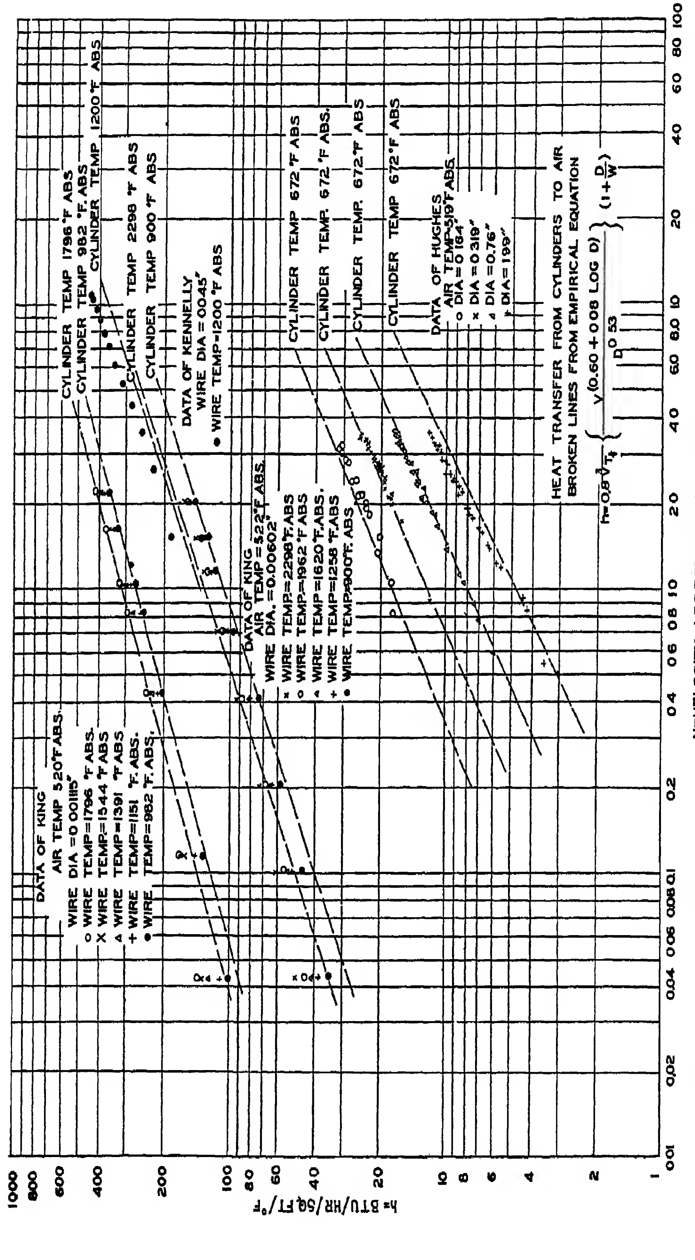


FIG. 4 COMPARISON OF RECOMMENDED EQUATION FOR SINGLE CYLINDERS WITH DATA OF KING, KENNELLY, AND HUGHES, AT VARIOUS VELOCITIES AND TEMPERATURES

is not shown on Fig. 2 because it was not designed to apply to wires.

39 According to the Boussinesq and Rice formulas, h is a power function of v, the exponent on velocity being specified as $\frac{1}{2}$. When h is plotted versus v on log paper, this calls for a slope of $\frac{1}{2}$. Fig. 4 is such a plot, and while it brings out other points discussed later, it shows that the velocity exponent varies from 0.36 to 0.62, depending on the diameter of the cylinder.

40 In the Davis equation, the function connecting hD and Dv is not specified. Fig. 3 shows a plot, on logarithmic coördinates, of hD versus Dv, based on the data of Hughes for all four sizes of pipe at a film temperature of 595 deg. fahr. abs. and the data for King's smallest wire at a film temperature of 751 deg. fahr. abs

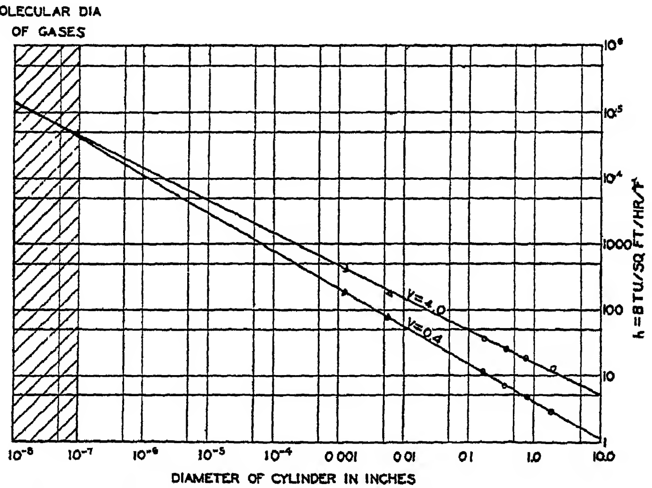


FIG. 5 EFFECT OF D UPON h, AT DIFFERENT VELOCITIES

This plot shows that these data could be coördinated approximately by a curve concave upwards. However, since the curve is not straight on logarithmic paper, the equation would not be of the convenient exponential type. Unless the equation were determined, the plot would not be well suited to the extrapolation which the engineer may be forced to make. On this same plot are shown curves representing the theoretical equation of King for the various film temperatures involved. As before, it is found that this equation gives values surprisingly close to the observed values, in view of the many assumptions made in the derivation. While this method of plotting offers promise, yet it is felt that another method is more satisfactory.

41 Fig. 4 shows all of the data plotted on logarithmic paper, with h as ordinates versus v as abscissas, each group of points applying to a definite cylinder diameter and wire temperature.

While curves drawn through a majority of the groups would be slightly concave upwards a straight line approximately satisfies each group of points. While such lines are not shown on Fig. 4, it can be seen that each line would have a different slope, the slope increasing from 0.36 to 0.62, with increase in diameter. In other words, for these data, while h is approximately a power function of the velocity, the exponent on the velocity term depends on diameter.

42 Fig. 5, prepared from Fig. 4, shows h versus D for each of two velocities, again on logarithmic paper. Straight lines fit these points very well, showing that h is a power function of D, but that the exponent on diameter depends on velocity. These facts suggest an equation which allows the exponent on velocity to vary with diameter, and vice versa, which led to the following equation:

$$h = \frac{6.72v^{(0.60+0.08 \log D)}}{D^{0.53}} \quad . \; . \; . \; . \; . \; . \; . \quad [5]$$

which applies at a film temperature of 595 deg. fahr. abs. It is interesting to note that the lines on Fig. 5 intersect at a value of h of 70,000 at a cylinder diameter of 4×10^{-8} in., which is in the range of the diameter of the individual molecules of gases.[1]

43 *Effect of Temperature.* In order to determine the effect of temperature, King's data were used, because when his wire temperature was varied, air velocity and pipe diameter were kept constant. King's values of h were therefore plotted versus T_f, the arithmetic mean of the temperature of the gas and that of the surface of the cylinder, expressed in deg. fahr. absolute. Fig. 6 shows the data for King's smallest wire, with h plotted on logarithmic paper versus T_f, each curve being for a definite velocity. Fig. 7 shows the data for King's largest wire plotted similarly. In all cases h increased as T_f increased, although the exponent on film temperature was found to vary with velocity, increasing as velocity decreased. In order not to complicate the equation, a

[1] Consider a cylinder having a diameter of the order of magnitude of one molecule of air. Fig. 5 indicates that h is about 70,000 and is independent of gas velocity, as shown by the intersection of the curves for different velocities at a common value of h. The same thing is shown by Equation [5] when $0.60+0.08 \log D$ equals zero. The possible significance of the above value of h may be seen from the following considerations: If the heat transfer, from one air molecule to another, takes place through a layer of air whose thickness is the mean free path of the molecule, h would be obtained by dividing the specific thermal conductivity of air, k, by the mean free path. For room temperature k is 0.0133, and Dushman(29) gives the mean free path as about 3.1 times 10^{-7} ft. The corresponding value of h is then $0.0133/3.1 \times 10^{-7}$, or about 43,000.. The latter value is of the same order of magnitude as the extrapolated value of 70,000 for a cylinder of molecular diameter.

convenient mean value of ⅓ was assigned to the exponent on film temperature, giving the general empirical equation for single pipes:

$$h = \frac{0.8\sqrt[3]{T_f}\,v^{(0.60+0.08 \log D)}}{D^{0.53}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [6]$$

On Figs. 6 and 7 the dotted lines are based on Equation [6], in which the correction term for temperature was selected so as to predict at the high temperatures, values of h lower than the data, rather than higher.

44 The dotted lines in Fig. 4 are calculated from this equation, and it is seen that the lines fit the points quite satisfactorily,

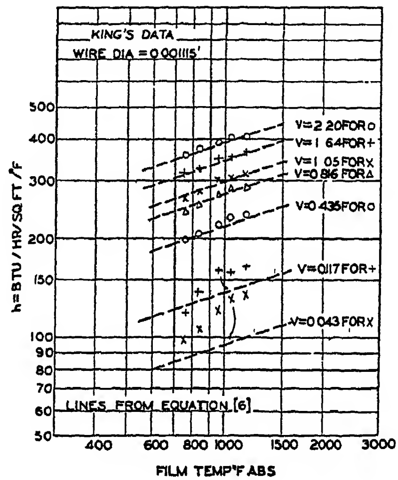


Fig. 6 King's Data for 0.00112-Inch Diameter, Showing Effect of T_f upon h, at Different Velocities. Dotted Lines Show Recommended Equation

except those at very low velocities and high temperatures, where the equation is on the safe side, as is desirable. This empirical equation is complex, due to the fact that the relations between the various factors are far from simple. Fortunately, for a given diameter and temperature it simplifies to an equation of the type $h = bv^m$. Furthermore, as shown by Fig. 8, the general equation may be plotted in such a way that one can read h directly from the plot, for any diameter and velocity. The plot is based on a standard film temperature of 595 deg. fahr. absolute, or 135 deg. fahr. For other temperatures, the small correction plot A immediately gives the factor by which h at the standard film temperature should be multiplied in order to apply at other film temperatures.

45 *Effect of Increased Turbulence Due to Obstructions.* Very recently Reiher(34) published data showing the effect of various

obstructions upon the coefficients for single pipes. The heat transfer was determined from hot air to cold water, the air flowing outside and at right angles to one or more vertical pipes placed in a wind tunnel 6.4 in. wide and 15.8 in. high. The hot air was recirculated by a blower and approached the test pipes over a number of distributing vanes. The temperature of the air was measured by a grid of platinum resistance wire placed across the air stream at a section between the distributing vanes and the pipes. The mean temperatures of the pipes were read from thermocouples placed at various points on the outer surfaces of the pipes. The heat picked up by the water in the pipe, minus

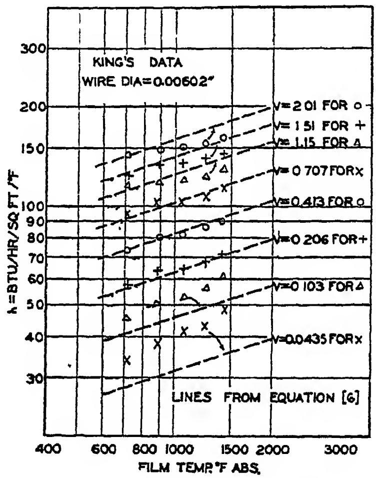


FIG. 7 KING'S DATA FOR 0.00602-INCH DIAMETER, SHOWING EFFECT OF T_f UPON h, AT DIFFERENT VELOCITIES. DOTTED LINES SHOW RECOMMENDED EQUATION

the calculated radiation from the walls of the tunnel to the pipe, gave the rate of heat flow from gas to pipe. Reiher's data for single pipes are given in Table 6, while his data for multiple pipes appear in Table 7.

46 In Fig. 9, curve C shows Reiher's data for the air flowing through the thermocouple grid or network of wires, before striking the single pipe, while curve D shows the relation predicted from Equation [6] for these conditions, namely, a pipe 0.591 in. in diameter with a film temperature of 744 deg. fahr. absolute. For any one velocity in this range, it is noted that curve C gives coefficients roughly 1.5 times those predicted for the unobstructed pipe. Curve A shows Reiher's data for a single pipe where the approaching air flowed, not only through the thermocouple grid, but also through a bank of staggered pipes to further increase the

NOTE TO FIG. 8:

Plot of the equation $h = 0.8\sqrt[3]{T_f}\,\dfrac{v^{(0.60+0.08\log D)}}{D^{0.53}}\left(1+\dfrac{D}{W}\right)$

For a single cylinder at 212 deg. fahr. in air at 59 deg. fahr. film temperature = 135 deg. fahr. (595 deg. abs.)

For other film temperatures multiply h read from chart by factor read from A

For pipes in rows (staggered) multiply reading (corrected for temperature) by $\left(1+\frac{D}{W}\right)$

h = B.t.u. per hr. per sq. ft. of pipe surface per deg. fahr. difference between cylinder and air

T_f = arithmetic mean temperature of pipe and air stream in deg. fahr. abs.

D = diameter of cylinder in inches

v = mass velocity through minimum free area as lb. per sec. per sq. ft. of free area = velocity

W = distance in inches between centers of pipes in first row

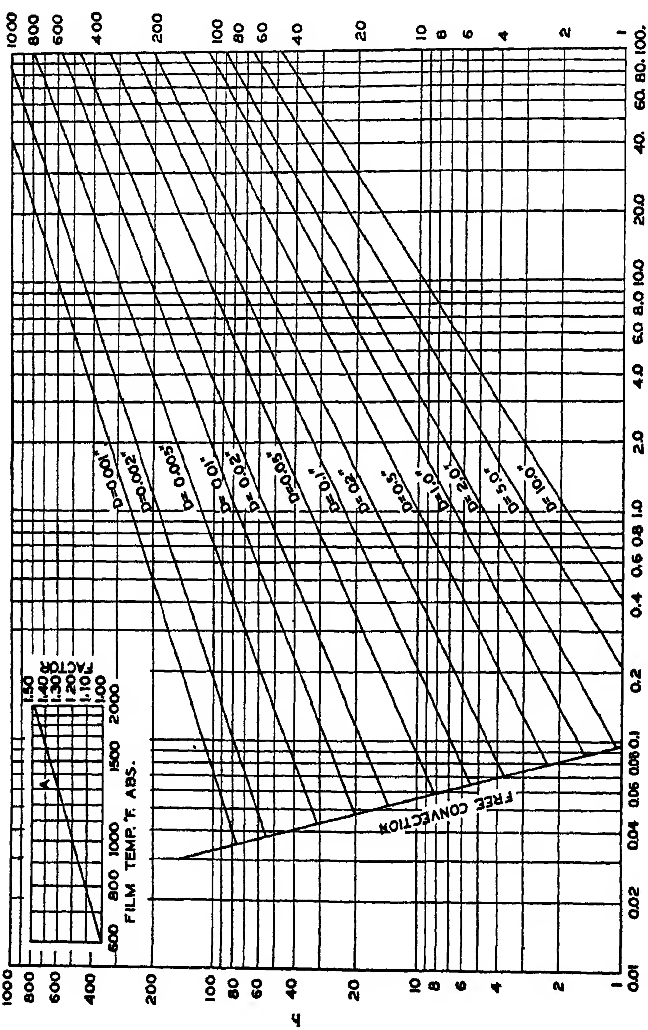


FIG. 8 CHART FOR PREDICTING HEAT TRANSFER RATES—FLOW OF AIR AT RIGHT-ANGLES TO CYLINDERS

(See Par. 48 for caution regarding use of Fig. 8 for gas flowing at low velocity over large pipes.)

turbulence. Curve A was obtained at an average film temperature of 628 deg. fahr. abs., and curve B shows the same data extra-

TABLE 6 REIHER'S DATA FOR SINGLE PIPES; AIR FLOW NORMAL TO AXIS; AIR BEING COOLED, WATER INSIDE PIPE

v = lb. per sec. per sq. ft. free area of cross-section
h = B.t.u. per hr. per sq. ft. per deg. fahr. temperature difference

Air temperature of 489 deg. fahr., film at 744 deg. fahr. abs.

Iron pipe, 1.102 in. diameter

v	h
0.393	6.17
0.493	7.18
0.577	7.89
0.597	7.96
0.693	8.75
0.380	6.15
0.479	6.95
0.535	7.12
0.596	7.60
0.757	8.83

Copper pipe, 0.709 in. diameter

v	h
0.361	7.57
0.453	8 52
0.531	9.12
0.578	9.55
0.638	10.05
0.688	10.61

German-silver pipe, 0.591 in. diameter

v	h
0 369	8.20
0.452	9.53
0.551	9 83
0.565	10.15
0.586	10.72
0 663	11.97
0.355	8.07
0.444	9.13
0.512	9.81
0.539	10.15
0.630	11.15

Iron pipe, 0.394 in. diameter

v	h
0.521	11.70
0.353	9.53
0.475	11.15
0.573	12.26
0.638	12.80
0.669	13.45

Brass pipe, 0.181 in. diameter

v	h
0.518	16.52
0.351	13.27
0.473	15.70
0 572	16.90
0.634	18.10
0.667	19.2

polated to a film temperature of 744 deg. fahr. abs. by means of the cube-root relation. Upon comparing B and C, both for the same diameter and film temperature, one finds that the adding

TABLE 7 REIHER'S DATA FOR AIR COOLERS; AIR FLOW NORMAL TO AXIS: WATER IN PIPES

Average for 5 rows deep, staggered, 7 pipes wide, brass pipes, 0.591 in. outside diameter
Minimum free area = 35 per cent

v	h	Film temp., deg. fahr.
2 01	24.5	220
1.536	19.4	220
1 357	17.8	220
1.448	17.15	137
1 79	19.95	137
2.01	21.65	137
2.20	23.15	137
2.41	25.1	137
1.563	18.15	112
1.904	20.1	112
2.12	21.9	112
2.33	23.9	112
2.70	27.4	112

Average for 5 rows deep, not staggered, 5 pipes wide, 0.591 in. outside diameter
Minimum free area = 53 per cent

1.165	13 05	275
0.927	10.84	275
0.611	8.74	275
1.334	14.07	206
1.12	11.00	206
0.745	8.55	206
0.902	8.56	129
1.315	11.67	129
1.63	13.45	129

Average for 5 rows deep, staggered, 5 pipes wide, brass pipes, 0.591 in. outside diameter
Minimum free area = 48 per cent

0.790	12.4	284
1.028	13.06	284
1.186	15.27	284
1.340	16.1	284
1.51	17.65	284
1.455	17.1	284
0.89	13.04	230
1.066	14.66	230
1.325	15.80	230
1.391	17.45	230
1.660	18.7	230
1.013	12.15	165
1.282	14.87	165
1.552	16.9	165
0.945	14.96	128
2.08	19.4	128

TABLE 8 PRIVATE COMMUNICATION

Heat transfer — Staggered pipes to air
Tube diameter = 0.275 in., spaced on 0.51-in. centers
Bank 9 tubes (3.15 in.) deep
Cooling surface per sq. ft. of frontal area = 5.48 sq. ft.
Free area = 65 per cent of frontal area

Velocity of approaching air, m.p.h.	v in free area, lb. per sec. per sq. ft.	h B.t.u. per sq. ft. per hr. per deg. fahr.
45	7.7	78
90	15.5	107
112	19.2	125

of the staggered pipes further increased the coefficient for the single pipe about 30 per cent. From these data one would expect that staggered pipes would give a higher coefficient than a single pipe subjected to the same air velocity.

47 *Procedure for Gases Other Than Air.* In order to estimate coefficients for gases other than air it is suggested that one adopt the usual expedient of multiplying the coefficient for air by the specific heat of the gas, and dividing by the specific heat of air.

48 *Free Convection.* Where there is no positive circulation of the air by mechanical means, such as a fan, the gas velocities are due to differences in gas density owing to differences in temperature. Under such conditions it is customary to express the varia-

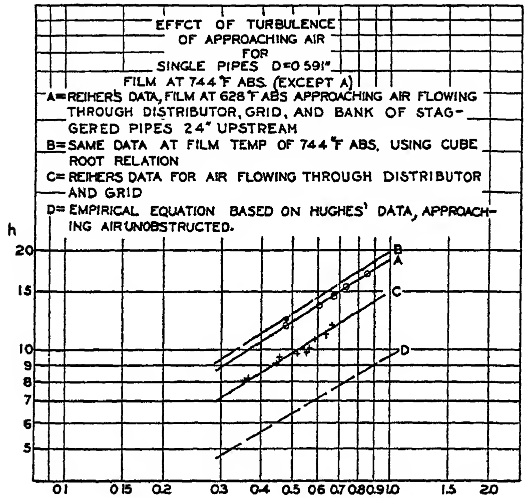


FIG. 9 EFFECT OF TURBULENCE OF APPROACHING AIR, ON HEAT TRANSFER RATES FOR SINGLE PIPES, DATA OF REIHER

tion in the coefficient of heat transfer as a function of the temperature difference between gas and pipe. Since the value of h should be plotted versus Δt rather than v, these plots and equations cannot be conveniently extrapolated beyond certain low velocities. Some of the experimenters, whose data have been plotted, measured the value of h where there was no forced circulation. The approximate locations of these free convection points have been indicated on Fig. 8 by cutting off the curves of h versus v at the respective values of h corresponding to these free convection values. As shown by Fig. 8, a pipe having an outside diameter of one inch gives a free convection h of about 1.6 at a mass velocity of 0.077. Should a smaller velocity of forced flow be used with this pipe, h would not decrease with v, but would depend on Δt, and could be estimated by means of

methods(25) not considered in this paper. For large pipes the location of the free convection range is particularly uncertain. Hence, when working at low velocities, free convection data(30) should be consulted, as well as the forced convection curves of Fig. 8; the higher value of h obtained in these two ways should be employed.

Staggered Pipes

49 So far the discussion has been limited to single pipes and wires. Fig. 10 shows h versus v for staggered pipes, based on data obtained from four sources. The data of Carrier(26) and both sets of the Sturtevant(27) data, all for air heaters, give curves

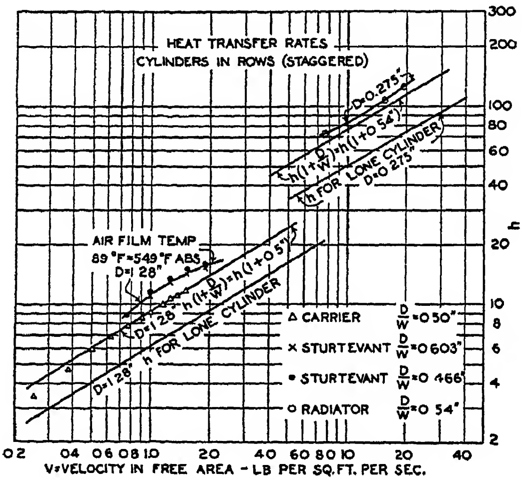


Fig. 10 Transfer Rates for Staggered Pipes (Carrier, Sturtevant, and Airplane Radiator Data) Compared with Rates for Single Pipes

concave downwards, while the curve marked "radiator"(28) has the reverse shape. These data are easily accessible and are therefore not tabulated, except those for the radiator given in Table 8.

50 Using the general equation for single cylinders, curves were plotted for each of these cases, giving the straight lines marked "h for lone cylinder." Since at a given velocity the data for the staggered pipes show higher coefficients than the equation for the lone cylinder, the difference was attributed to the fact that the pipes were staggered, thereby causing more violent eddy currents for a given air velocity than with the single cylinder.

51 Since the data now available are inadequate to properly determine the correction for staggering, for the present it is

recommended for staggered pipes that the results for single cylinders be multiplied by the correction term $\left(1+\frac{D}{W}\right)$, where W is the average distance in inches between the center-lines of pipes in the rows arranged at right angles to the direction of flow of the gas. On Fig. 10 the lines marked $h\left(1+\frac{D}{W}\right)$ were calculated in this way, and it is seen that on the whole these lines satisfy the various sets of experimental data. As is desirable, the correction term $\left(1+\frac{D}{W}\right)$ was selected so that the predicted line would tend to be low rather than high when compared with the points.

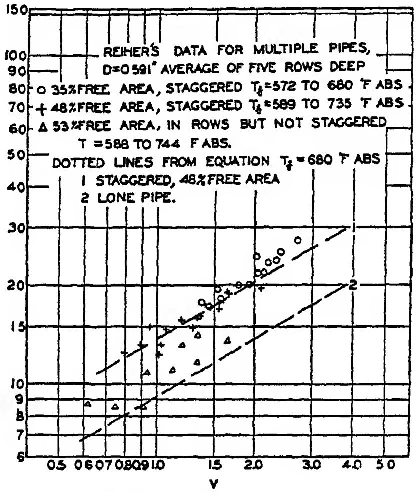


FIG. 11 REIHER'S DATA FOR MULTIPLE PIPES, COMPARED WITH RECOMMENDED EQUATION

52 Fig. 11 shows Reiher's data(24) for multiple pipes 0.591 in. in diameter, and five rows deep. The circles and crosses both apply to runs on staggered pipes, the circles applying to 35 per cent free area and film temperatures of 572 to 680 deg. fahr. abs., while the crosses refer to runs with 48 per cent free area and somewhat higher film temperatures. The dotted line marked 1 is that predicted for 48 per cent free area and a film temperature of 680 deg. fahr. abs., and it checks reasonably well with the experimental points.

53 The triangles of Fig. 11 shows Reiher's data for a heater five rows deep, but with the pipes all in line instead of staggered, so that the air may pass freely through the clear area without being forced to flow around the pipes. As would be expected,

this arrangement gave lower coefficients than the design in which the pipes were staggered. On Fig. 11, curve 2 is that predicted for lone unobstructed pipes by Equation [6].

54 Upon comparing the triangles of Fig. 11 with curve C of Fig. 9 one notices that Reiher's five-row heater with the pipes in line, or not staggered, gives somewhat lower coefficients than the single pipe preceded by the thermocouple grid.

55 *Use of Fig. 8 for Predicting h.* The general equation for staggered pipes is as follows:

$$h = \frac{0.8\sqrt[3]{T_f}\,v^{(0.60+0.08\log D)}}{D^{0.53}}\left(1+\frac{D}{W}\right) \quad . \ . \ . \ . \quad [7]$$

However, Fig. 8 allows one readily to determine h without recourse to Equation [6] for single cylinders. The value for staggered pipes is most easily calculated by multiplying h for a single pipe by the factor $\left(1+\frac{D}{W}\right)$, thereby making it unnecessary to substitute values in Equation [7].

56 For example, assume that one wishes to estimate the coefficient of heat transfer between gases and a pipe having an outside diameter of four inches, the average film temperature being $\frac{800+1400}{2}+460 = 1560$ deg. fahr. abs., for a mass velocity of 1.0 lb. of gas per sec. per sq. ft. of free area, the pipes being on 8-in. centers, and staggered. The gases have the same specific heat as air.

57 From Fig. 8, the value of h for a single cylinder four inches in diameter, and at the standard film temperature of 595 deg. fahr. abs., would be about 3.2. As shown by curve A, Fig. 8, in order to make this value of h apply at a film temperature of 1560 deg. fahr. abs., it must be multiplied by 1.38, giving

$$h = (3.2)(1.38) = 4.4$$

But

$$1+\frac{D}{W} = 1+\frac{4}{8} = 1.5$$

Hence the previous value of h must be multiplied by 1.5, giving $(4.4)(1.5) = 6.6$.

58 With a temperature difference between gas and pipe of 600 deg. fahr., the heat flow by "conduction and convection" would be $600(6.6) = 3960$ B.t.u. per hr. per sq. ft. of pipe surface.

59 If these pipes were exposed to walls warmer than the temperature of their own surfaces, an additional amount of heat would flow by radiation, which should be calculated separately in the usual way(25), and added to the above value.

Nomenclature

(For recommended equation)

D = outside diameter of pipe, in inches

h = coefficient of heat transfer between gas and pipe surface, as B.t.u. per hr. per sq. ft. of pipe surface per deg. fahr. mean difference in temperature between gas and pipe

v = mass velocity of gas as lb. per sec. per sq. ft. of free area of cross section = $u\rho$

T = arithmetic mean temperature of gas film, as deg. fahr. abs.

W = distance in inches between center-lines of adjacent pipes in a row taken at right-angles to air flow

(Additional symbols for other equations)

B = effective thickness of uniform circular film calculated to make the heat flow by conduction alone, equal to the heat flow actually obtained. For a film whose thickness is a very small fraction of the diameter of the cylinder $B = k_{av}/h$. In general, the relation between B, h, and k_{av} is

$$h = \frac{2k_{av}}{2.3D \log_{10}\left(\frac{2B+D}{D}\right)}$$

C_p = specific heat of gas at constant pressure, as B.t.u. per lb. per deg. fahr.

C_v = specific heat of gas at constant volume, taken as 0.17 B.t.u. per lb. per deg. fahr.

k = specific thermal conductivity of gas, as B.t.u. times ft. of thickness per sq. ft. per deg. fahr. difference per hr.

k_{av} = true mean value of k obtained by integrating the equation for k between the temperatures of gas and pipe

k_f = value of k at film temperature, T_f

k_g = value of k at gas temperature

k_p = value of k at pipe temperature

ρ = density of gas as lb. per cu. ft. at temperature and pressure used for calculating u

ρ_f = gas density as lb. per cu. ft. at geometric mean film temperature

T_g = temperature of main body of gas, as deg. fahr. abs.

$T_{geom.}$ = geometric mean temperature of gas film, as deg. fahr. abs.

t = temperature in deg. fahr.

Δt = mean temperature difference between main body of gas and outer surface of pipe, as deg. fahr.

u = relative linear velocity of gas and pipe, as ft. per sec. at actual temperature and pressure

z = viscosity of gas in centipoises. This appears in the theoretical equation of Davis, but it is not specified whether z should correspond to the temperature of the main body of the gas or the gas film itself

z_f = viscosity of gas at geometric mean film temperature, as centipoises

BIBLIOGRAPHY

1 Haslam and Chappell, *Ind. & Eng. Chem.*, vol. 17 (1925), p. 402.

2 Boussinesq, *Comptes Rendus*, vol 133 (1905), p. 257, also *Journal de Mathématique*, vol. I (1905), p. 285.

3 Fourier, Memoires de l'Académie (1820), p. 507.

4 Russell, Proc. of Phys. Soc. of London, vol. 22 (1909–1910), p. 432.

5 King, Phil. Trans., Roy. Soc. (London), Series A, vol. 214 (1914), p. 273.

6 Langmuir, Trans. Am. Electrochem. Soc., vol. 23 (1913), p. 299

7 Langmuir, *Phys. Rev.*, vol. 34 (1912), p. 401.

8 Langmuir, Trans., A.I.E.E., vol. 31 (1912), part 1, p 1229

9 Kennelly, Wright and Van Bylevelt, Trans., A.I.E.E., vol. 28 (1909), Part 1, p. 363.

10 Carpenter, Rolla C., Heating and Ventilating Buildings, Wiley & Sons, New York, Edition 5, 1911.

11 Davis, *Phil. Mag.*, vol. 44 (1922), p. 940.

12 Davis. *ibid*, vol. 41 (1921), p. 899.

13 Rice, Trans. A.I.E.E., vol. 42 (1923), p. 1288. See also *Ind. & Eng. Chem.*, vol. 16 (1924), p. 454.

14 Nusselt, Mitteilungen über Forschungsarbeiten, Heft 80–96 (1910).

15 Hughes, *Phil. Mag.*, vol. 31 (1916), p. 118.

16 J. T. Morris, *Engineering* (London), Aug. 8, 1913.

17 Rubach, Forschungsarbeiten, *V.D.I.*, Heft 185 (1916), p. 1.

18 T. E. Stanton, Tech. Report, U. S. Advisory Committee for Aeronautics, 1912–1913, p. 45.

19 U. S. Bureau of Standards, Tech. Paper No. 211 (1922), p. 326.

20 Heat Transmission, R. Royds, Constable & Co., London (1921), p. 127.

21 Orrok, Trans. A.S.M.E., vol. 32 (1910); Frost and Manley, M.I.T. Undergraduate Chem. Eng. Thesis, 1921.

22 Kennelly and Sanborn, Proc. Am. Phil. Soc., vol. 53 (1914), p. 55.

23 Marks' M. E. Handbook, First edition, McGraw-Hill Book Co. Inc., New York (1916), p. 306.

24 Reiher, Mitteilungen über Forschungsarbeiten, Heft 269 (1925).

25 Principles of Chemical Engineering, Walker, Lewis and McAdams, McGraw-Hill Book Co. Inc., New York, Feb. 1923, p. 118-177.

26 Carrier, Trans. A.S.M.E., vol. 33 (1911).

27 Based on tables given in the 1917 edition of Data Book for Engineers and Architects, B. F. Sturtevant Co., Hyde Park, Mass.

28 Private communication.

29 Smithsonian Physical Tables, 1920, p. 399, Smithsonian Institution, Washington, D. C.

30 Heilman. *Ind. & Eng. Chem.*, May 1924, p. 445.

DISCUSSION

B. N. Broido.[1] While the authors have clearly stated that this paper is limited to heat transmission by conduction and convection, when dealing with heat transmission from air to metal cylinders, particularly when the air is at elevated temperatures, one can hardly differentiate the heat transmitted by radiation from that transmitted by convection and conduction. This is especially important when dealing with flue gases in a boiler, oil heater, etc., where the gases contain a considerable amount of CO_2 or water vapor.

In a paper by the writer[2] there was shown the influence of the heat absorbed by radiation from gases. Coincidentally, an example given by the writer was similar to that mentioned by the authors. In this example, the total heat transmission is very close to the figures which would be obtained if calculated in accordance with the curve given in the paper. It is shown, however, that a considerable part of this heat is transmitted by radiation. This would rather indicate that there is a difference in the total heat transmission, depending upon the constituents of the gases besides their specific heat, particularly with higher temperature differences, while in accordance with the authors one would conclude that the heat transmission, all other conditions being the same, depends only on the specific heat of the gases.

In Par. 47 the authors suggest that in order to estimate the coefficient for gases other than air, the usual expedient be adopted of multiplying the coefficient for air by the specific heat of the gas divided by the specific heat of air. Tests conducted to determine the heat transfer from superheated steam to pipes containing water have shown that the heat transmission from superheated steam to the metal is much higher, as compared with air, than the difference in their specific heats.

According to Par. 51 the heat transmission with staggered pipe is higher than with a single pipe, and the results for single cylinders are to be multiplied by $\left(1+\frac{D}{W}\right)$. First of all the term $\frac{D}{W}$ takes care of the distance between the pipes in a plane at a right angle to the flow of the gases. The heat transmission, however, also will vary considerably with the distance between the pipes parallel to the flow of the gases. The distance between the pipe rows for which this correction term applies should be stated. In accordance with the data from Reiher,[3] mentioned by

[1] Chief Engineer, Industrial Dept., Superheater Co., New York, N. Y. Mem. A.S.M.E. Deceased, February 10, 1927.

[2] Radiation in Boiler Furnaces, Trans. A.S.M.E., vol. 47 (1925), p. 1123.

the authors, this correction will apply when the tubes are so staggered that a line drawn between two tubes on two successive rows will be at 60 deg. to the center line of each row.

In comparing the heat transmission of tubes as regards their location, the authors stated a method of determining the heat transmission in staggered tubes, as compared with single cylinders. Nothing was mentioned of the effect of the heat transmission when the tubes are located in straight rows, or one behind the other, with respect to the direction of the gases. The above-mentioned tests of Reiher have shown the difference between straight rows and staggered rows. It can hardly be assumed that tubes located in staggered rows will actually have a considerably higher heat transfer than a single tube, and the correction term mentioned by the authors will rather apply as comparing straight rows with staggered rows but not with single tubes. In view of the fact that the term $\left(1+\frac{D}{W}\right)$ may influence the heat transmission as much as 50 per cent, this point is a most important one. It would therefore be of considerable interest if the authors would state clearly whether the curve shown in Fig. 11 applies to single tubes or to tubes set in straight rows.

In considering velocity, or mass velocity, of gases parallel to a pipe, it is clearly understood what velocity means. In working, however, with gases flowing at right-angles to a cylinder, and particularly when there is a whole row of tubes or cylinders, the velocity changes, of course, while flowing over the cylinder. It will be the maximum in a plane at right angles to the flow of the gases and through the diameter of the tubes. Slightly above or below that, the area increases and the velocity decreases. The difference in standard apparatus may be as high as 33 per cent or more, which has, of course, a considerable influence upon the final result that will be obtained in predicting the heat transmission.

The Authors. Mr. Broido correctly pointed out that the authors should have included a statement that heat can be transferred by radiation, not only from solid to solid, but also from gases to solid. However, for the data analyzed by the authors, the influence of gaseous radiation was negligible. A paper dealing with gaseous radiation is soon to be published by the Department of Chemical Engineering of the Massachusetts Institute of Technology.

In accordance with the suggestion of Mr. Broido, the authors are glad to define the arrangement of pipes in the multiple-pipe heaters. Consider any two adjacent rows of pipes in parallel planes at right-angles to the direction of air flow. The distance

between the center-lines of these two rows is L inches, and the distance between the center-lines of adjacent pipes in the same row is W inches.

The following table shows these distances for the various heaters.

Heaters used by	W	L	L/W
Reiher	1 69	1.46	0 863
Carrier	2.63	1.88	0 715
Sturtevant	2 75	2.38	0 866

If the pipes are arranged as equilateral triangles,

$$L^2 = W^2 - (W/2)^2$$

whence $L/W = 0.866$. From the values of L/W given in the table one sees that the pipes of Reiher must have been arranged as equilateral triangles, which agrees with the statement of Mr. Broido. The Sturtevant pipes must have been arranged similarly.

Fig. 10 indicates that staggered pipes give values of h about 50 per cent higher than a single pipe, for a given air velocity and temperature. The same thing is shown by the circles and crosses on Fig. 11. The triangles on Fig. 11, for Reiher's multiple pipes arranged in line, i.e., not staggered, lie between the points for staggered pipes, Curve 1 of Fig. 11, and the line for a single pipe, Curve 2, Fig. 11.

All mass velocities for the air heaters were reported in pounds of air per second per square foot of free area taken in a plane passing through the line of centers at right angles to the flow of air. In the normal type of heater, this free area is sometimes called the minimum free area, and the corresponding mass velocity might be termed the maximum.

No. 2033

HEAT TRANSMISSION FROM CONDENSING STEAM TO WATER IN SURFACE CONDENSERS AND FEEDWATER HEATERS

By W. H. McAdams,[1] Cambridge, Mass.
Member of the Society

and

T. K. Sherwood,[2] and R. L. Turner,[3] Cambridge, Mass.
Non-Members

New experimental data are presented for single-tube water heaters supplied with exhaust steam. These, together with various data from the literature, are analyzed by the graphical method of E. E. Wilson, by means of which the overall resistance to heat transfer, 1/U, may be successfully resolved into its component parts. Furthermore, by employing a coefficient of 2000 for the steam side, and by using the equation of McAdams and Frost for the water-side coefficient, it is possible to predict the value of U for various water velocities and temperatures. These predicted values compare closely with published test data, both for vacuum condensers and feedwater heaters operated with exhaust steam, covering a range of sizes from 80 to 50,000 sq. ft. For abnormal amounts of air or scale, proper allowances should be made by applying the resistance concept.

The equation previously published for the heat transfer on the water side is found to be satisfactory for vacuum condensers and exhaust heaters. However, in analyzing laboratory data for single-tube apparatus heated by steam materially above atmospheric pressure, it was found that this equation gave unduly conservative results, the

[1] Associate Professor of Chemical Engineering, Massachusetts Institute of Technology.

[2] Research Associate, Department of Chemical Engineering, Massachusetts Institute of Technology.

[3] Instructor, Department of Chemical Engineering, Massachusetts Institute of Technology.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers, 29 West 39th Street, New York, at a Joint Session with the American Society of Refrigerating Engineers. Program arranged by the Committee on Heat Transmission, National Research Council. Published as the 14th contribution of the Heat Transmission Program of the Department of Chemical Engineering, Massachusetts Institute of Technology, Chemical Engineering, Serial No. 174.

discrepancy increasing with increase in steam pressure. This may be due to the boiling of the water on the inner wall of the pipe, to the evolution of dissolved air, etc. Fortunately, since commercial tubular apparatus are usually operated either under vacuum or with exhaust steam, this complication is of minor importance in plant practice.

Introduction

IN BOTH surface condensers and feedwater heaters, the heat of the condensing steam is first transferred to the metal, then conducted through the tube wall, and finally absorbed by the water. In order to compare the performances of various types of such apparatus, it is customary to employ the rate of heat flow per unit of temperature difference between steam and water per unit area of condensing surface. This capacity factor is called the *overall* coefficient of heat transfer, the term overall indicating that the temperature difference is based on the temperatures of the heating and cooling mediums, i.e., that of the steam and water. In a relatively few instances the tube temperatures have been measured, making it possible to base the temperature difference on the temperatures of the tube and the water, giving a so-called *water-side* coefficient. Since the tube wall must obviously be colder than the steam, the temperature difference from tube to water is always less than from steam to water; hence the water-side coefficient is always greater than the overall coefficient.

2 The overall coefficient is the value desired in comparing different designs, and in determining the result of changes in operating conditions. In attempting to correlate these overall coefficients, the majority of engineers have failed to take advantage of the fact that the overall resistance to heat flow, i.e., the reciprocal of the overall coefficient, is numerically equal to the sum of the individual series resistances, namely, the resistance on the steam side, that of the wall itself, and that on the water side. As a result, many equations have been proposed which are not suited to the extrapolation which often becomes necessary. But when the overall resistance is broken up into its component parts, each part may then properly be studied to determine the effect of various factors, such as water or steam velocity, temperature, etc. Furthermore, the general equations for the individual resistances so determined may be recombined to predict the result of combinations of conditions not yet investigated. Hence it is clear that much is gained by breaking up the overall resistance into its parts and by analyzing the separate resistances.

3 While the determination of tube temperatures is to be encouraged, especially on experimental apparatus, such measurements are not conveniently made on commercial installations. Even in the usual case where tube temperatures are not available, it will be shown that, by a method of plotting, individual resist-

ances may be determined from the observed overall resistances, thereby gaining the advantages mentioned above.

Object

4 The purpose of this paper is (1) to resolve observed overall resistances into their component parts and to analyze the results, and (2) to compare the water-side coefficients so obtained with values determined by direct measurement of tube temperatures. In addition, directions are given for predicting the overall coefficient for various cases which arise in practice.

Basis of Method of Plotting

5 In calculating the overall coefficient of heat transfer, engineers use the following form of Newton's law for steady conditions:

$$Q = UA(\Delta T)_{av.} \quad \ldots\ldots\ldots\ldots \quad [1]$$

where Q = rate of heat flow at right angles to the condensing surface, B.t.u. per hr.

A = square feet of surface on the vapor side

$(\Delta T)_{av.}$ = logarithmic mean difference[1] in temperature between steam and water = $(x-y)/2.3 \log (x/y)$, where x is the difference between steam and water temperatures at the water inlet, and y is a similar difference at the water outlet

U = overall coefficient of heat transfer from steam to water, expressed as B.t.u. per hr. per sq. ft. of condensing surface per deg. fahr. mean temperature difference between steam and water.

6 Since the rate of heat flow is the same through the various resistances in series, it can be shown that

$$R = \frac{1}{U} = r_v + r_t + r_s + r_w \quad \ldots\ldots\ldots \quad [2]$$

where R = overall thermal resistance per unit area of condensing surface = reciprocal of overall coefficient of heat transfer = $1/U$

r_v = individual thermal resistance on vapor side, per unit area of condensing surface

r_t = individual thermal resistance of the tube, per unit area of condensing surface

r_s = individual thermal resistance of any scale or other deposit on either or both surfaces of the tube, per unit area of condensing surface

r_w = thermal resistance on the water side, per unit area of condensing surface.

[1] This logarithmic mean is based on the assumption that the overall coefficient is constant throughout the apparatus. Although this assumption, and hence the logarithmic mean difference, is not strictly accurate, it is commonly accepted and is employed in this paper.

7 It is well known that an increase in water velocity, other things being equal, decreases the thickness of water film inside the pipe, and hence decreases the thermal resistance on the water

TABLE 1 DATA OF SCHULMAN AND SILVERMAN(1)[1]

Single standard ¼-inch wrought-iron pipe, 102 in long. Parallel flow of steam and water.

Run	Inlet water temp., deg. cent.	Outlet water temp., deg. cent.	Inlet vapor temp., deg. cent.	Outlet vapor temp., deg. cent.	Lb. water per hr.	V, ft. per sec.	U
1	14.6	57.9	110.5	104.1	1212	7.66	959
2	14.7	56.2	109.2	103.1	1242	7.85	945
3	14.8	70.2	109.8	104.4	664	4.20	757
4	14.8	65.9	108.2	103.4	777	4.91	806
5	14.8	62.6	107.7	103.2	864	5.46	813
6	15.3	79.4	107.0	103.5	375	2 37	589
7	15.4	54.9	105.4	102.2	1290	8.16	958
8	15.5	60.6	105.0	102.2	948	6.00	852
9	15.2	65.8	107.3	103.0	744	4.70	781
10	15.6	87.7	112.2	107.6	288	1.82	531
11	16.2	90.7	112.6	107.6	246	1.55	497
12	16.2	84.4	112.4	107.6	351	2 22	578
13	16.3	82.8	112.6	107.4	390	2.47	613
14	16.6	81.2	112.2	106.8	402	2.54	608

TABLE 2 DATA OF WISHNEW(2)

Single-tube condenser, parallel flow of water and vapor.
Brass tube, 0.494 in. I. D., 0 673 in O. D.
Heating surface, 47.75 in long. Steel jacket, 1 61 in. I. D.

Run	Inlet water temp., deg. cent.	Outlet water temp., deg. cent.	Inlet vapor temp., deg cent.	Outlet vapor temp., deg. cent.	Lb. water per hr.	V, ft. per sec.	U
1	19.5	34.2	101.0	99.0	2180	7.29	638
2	19.5	34.5	100.3	98.5	1990	6.64	586
3	19.5	39.2	100.5	98.0	1314	4.40	525
4	19.5	37.6	100.8	99.0	1492	5.00	546
5	19.5	35.2	100.6	98.4	1794	6.00	561
6	19.5	48.6	100.4	98.2	600	2.00	364
7	19.5	40.3	101.1	99.0	1122	3.76	480
8	19.5	36.5	100.0	98.0	1778	5.95	610
9	19.5	34.5	101.0	98.5	2052	6.86	606
10	19.5	35.5	100.8	99.5	1746	5.84	589
11	19.5	35.2	101.3	100.0	2004	6.70	611
12	19.5	35.4	101.0	100.0	1968	6.60	773
13	19 5	54.6	101.3	99.0	435	1.46	355
14	19.5	55.6	103.5	101.0	444	1.49	376
15	19 0	37.8	100.6	98.0	1580	5.30	606
16	19.1	34.2	101.0	98.5	2012	6.76	598
17	19.1	35.4	106.4	104.0	2103	7.05	630
18	19.5	34.7	102.0	100.0	2130	7.15	649

side, i.e., $r_w = 1/f(v)$ where $f(v)$ is some function of the water velocity. Hence, from Equation [2],

$$\frac{1}{U} = r_v + r_t + r_s + \frac{1}{f(v)} \quad \ldots\ldots\ldots \quad [3]$$

8 A given percentage of change in water velocity ordinarily produces a greater percentage of change in the overall resistance than an equal percentage change in any other factor. Further-

[1] For this and other references see bibliography at end of paper.

more, water velocity may be varied over a greater percentage range than any other factor. Hence, if the sum of the first three resistances remains substantially constant, a plot of $1/U$ as ordinates versus $1/f(v)$ as abscissas should give an approximately straight line, the ordinate corresponding to the zero value of $1/f(v)$ being equal to $r_v+r_t+r_s$ for foul pipes, and to r_v+r_t for clean pipes. Since r_t can be calculated from the known values of the thickness and conductivity of the tube wall, r_s can be determined for clean

TABLE 3 DATA OF BRAY AND SAYLER(3)

Single-tube condenser constructed of a copper tube, 0.494 in I D, 0.674 in. O. D., placed concentrically in a standard 1.5-in. steel jacket. Heating surface, 49 1 in. long. Runs 9-18 made with condensing benzol, using counter-current flow of cooling water and vapor.

Run	Inlet water temp., deg. cent.	Outlet water temp., deg. cent	Inlet vapor temp., deg. cent.	Outlet vapor temp., deg. cent.	Lb. water per hr.	V, ft. per sec.	U
9	3.9	29.6	80.0	80.0	267	0 90	153
10	3.9	29.1	80.8	80.0	281	0.95	157
11	4.1	35.2	82.8	81.7	233	0.78	164
12	8.0	49.6	80.0	79.7	100	0.33	120
13	6.2	45.4	79.7	79.7	120	0.40	126
14	6.0	39.5	79.7	79.7	146	0.49	123
15	6.1	30.1	79 1	78.9	243	0.81	135
16	7.2	24.8	79.3	79.1	410	1 37	161
17	6.6	20.0	79.3	79.3	675	2 27	192
18	6.4	21.2	80.4	80.0	657	2.21	205

Runs 32-45 made with steam, using parallel flow of cooling water and vapor.

Run	Inlet water temp., deg. cent.	Outlet water temp., deg. cent	Inlet vapor temp., deg. cent.	Outlet vapor temp., deg. cent.	Lb. water per hr.	V, ft. per sec.	U
32	6.3	80.1	100.7	100.7	153	0 51	320
33	6.0	84.4	100.2	100.3	137	0.46	340
34	4.7	70.1	99 2	99.8	395	1.32	645
35	4.8	72.9	99.6	100.0	335	1.12	585
36	4.6	48.7	100.6	100.6	875	2.93	752
37	4.7	46.1	100.7	97.4	919	3.08	741
38	4.5	45.9	100.6	100.6	980	3.29	769
39	4.4	48.0	101.2	101.2	859	2.88	715
40	4.4	47.8	101.2	101.2	865	2.90	715
41	4.5	50.1	101.5	101.5	834	2 80	735
42	4.2	50.0	103.3	103.3	861	2.89	741
43	4.2	48.3	105.8	105.8	904	3.03	725
44	4.0	37.9	103.3	103.3	1280	4.29	741
45	4.6	42.3	99.4	100.8	986	3.31	781

pipes. Wilson(4) was apparently the first to employ this method of plotting heat-transfer data; since then it has been used by Haslam, Ryan, and Weber(5) in analyzing data for the diffusion of matter through gas and liquid films.

9 From direct measurements of the water-side resistance, it has been shown(6) that[1] $r_w = 1/bv^{0.8}$. While the empirical factor b is a constant for a given set of conditions, its numerical value

[1] This relation was derived for the turbulent flow of water inside pipes, and hence would not be expected to apply at velocities so low that turbulent flow was not obtained. Fortunately commercial condensers and feedwater heaters employ turbulent flow.

TABLE 4 DATA OF BALDWIN AND SHERWOOD(7)

Single-tube condenser constructed of a copper tube, 0.494 in. I. D., 0.674 in. O. D., placed concentrically in a standard 1.5-in. steel jacket. Heating surface, 49.1 in long. Parallel flow of steam and water.

Run	Inlet water temp., deg. cent.	Outlet water temp., deg. cent.	Inlet steam temp., deg. cent.	Outlet steam temp., deg. cent.	Lb. water per hr.	V, ft. per sec.	U
52	18.5	37.8	97.0	98.5	1520	5.10	621
53	18.5	38.1	97.8	98.9	1484	4.98	613
54	35.6	52.5	97.5	99.2	1500	5.04	689
55	40.0	55.8	97.5	99.5	1489	4.99	685
56	41.3	56.8	97.4	99.5	1487	4.99	694
57	48.6	62.6	97.6	99.3	1491	5.00	720
58	60.8	71.8	98.1	99.6	1495	5.01	746
59	66.1	75.8	98.3	100.2	1495	5.01	755
60	69.0	78.0	98.1	99.8	1479	4.96	790
61	71.5	79.8	98.1	99.8	1481	4.98	781
62	72.3	80.3	98.0	100.0	1467	4.92	765
63	66.7	76.3	98.2	99.9	1460	4.90	751
64	53.4	66.5	98.0	99.9	1436	4.81	708
65	41.8	57.3	97.8	99.6	1454	4.81	681
66	44.9	60.0	97.9	99.9	1453	4.89	665
67	35.2	52.1	97.7	99.7	1463	4.91	663
68	20.2	40.4	97.4	99.4	1450	4.86	634
69	19.7	41.4	99.7	100.6	1468	4.93	688
70	20.0	41.3	99.4	100.2	1514	5.09	689
71	24.4	45.6	98.9	99.6	1441	4.84	704
72	26.9	47.6	99.1	99.5	1441	4.85	716
73	40.7	58.9	99.1	100.0	1441	4.85	750
74	47.5	63.6	99.5	100.5	1429	4.79	765
75	56.5	70.1	99.5	100.5	1452	4.88	797
76	68.7	78.1	98.8	99.8	1432	4.81	835
77	72.7	81.4	99.0	99.6	1440	4.84	839
78	78.1	85.0	98.9	99.6	1449	4.86	839
79	80.2	86.5	98.9	98.9	1466	4.91	884
80	66.5	76.6	98.7	99.5	1466	4.91	806
81	52.7	67.1	99.2	100.0	1470	4.94	818
82	23.6	58.0	99.7	100.4	384	1.29	335
83	20.4	45.5	100.2	102.0	925	3.11	508
84	37.7	58.3	99.9	101.9	930	3.13	561
85	55.5	71.4	101.8	104.0	956	3.21	574
86	54.5	69.4	98.6	100.9	964	3.24	560
87	70.6	80.7	99.0	101.4	975	3.28	595
88	50.1	61.5	98.2	100.0	1810	6.08	702
89	81.6	89.7	98.2	100.5	502	1.68	444
90	73.3	85.0	97.7	100.1	491	1.65	436
91	46.1	66.1	97.9	100.1	625	2.09	458
92	20.5	50.2	97.7	99.9	618	2.07	430
93	44.8	65.9	98.5	100.7	639	2.14	451
94	64.8	78.7	98.0	100.7	630	2.11	475
95	18.2	37.2	98.6	100.7	1707	5.73	664
96	29.0	46.1	98.6	100.2	1660	5.56	675
97	33.2	49.6	98.9	100.7	1689	5.66	696
98	56.7	67.9	98.6	100.3	1692	5.69	748
99	62.0	72.0	98.3	100.3	1692	5.69	778
100	64.7	74.2	98.9	100.8	1685	5.66	771
101	17.3	34.8	98.2	100.1	1905	6.40	670
102	29.7	45.4	98.5	100.5	1902	6.39	709
103	52.7	64.3	99.2	100.8	1920	6.45	790
104	62.0	71.2	98.9	100.5	1920	6.45	790
105	56.6	66.3	98.7	100.5	2304	7.75	864
106	49.3	60.0	98.9	100.6	2312	7.76	806
107	31.5	45.3	98.9	100.5	2330	7.82	765
108	16.7	32.8	98.7	100.0	2330	7.82	736
109	18.1	33.0	98.8	100.6	2992	10.05	764
110	60.9	69.3	99.2	101.4	2575	8.65	900
111	43.5	54.9	99.2	101.1	2625	8.82	867
112	45.5	54.2	98.2	99.6	3925	13.18	1054
113	27.4	38.2	99.2	100.0	3960	13.30	939
114	16.4	28.3	98.8	100.3	3970	13.32	902
115	29.2	68.6	97.6	100.0	316	1.06	385
116	34.4	73.7	97.5	99.8	317	1.06	404
117	48.0	80.5	97.7	100.1	325	1.09	478
118	69.6	88.5	98.0	100.5	319	1.10	464
119	80.8	92.4	98.2	100.5	311	1.04	444
120	89.5	95.8	98.7	100.8	292	0.98	395

depends on factors such as temperature and the dimensions of the tube. Hence Equation [3] becomes

$$\frac{1}{U} = r_v + r_t + r_s + \frac{1}{bv^{0.8}} \quad . \; . \; . \; . \; . \; . \; . \quad [4]$$

On a plot of $1/U$ as ordinates versus $1/v^{0.8}$ as abscissas, the slope of the line equals $1/b$, and the value of b so determined from the overall coefficient can be compared with the value of b determined from direct measurement.

Overall Coefficients of Heat Transfer in Single-Tube Apparatus

10 Tables 1 to 5 show the data of Silverman and Schulman(1), Wishnew(2), Bray and Sayler(3), Baldwin and Sherwood(7), and Whitaker(8), obtained in these laboratories in testing five experimental single-tube water heaters. In Table 6 are summarized the results of five other investigations on single tubes, three on water heaters, by Clement and Garland(9), Webster(10), and VanMarle(11), also experiments on surface condensation under vacuum by Orrok(12) and Hoefer(13).

11 The experiments of Orrok are the most extensive and were made with great care. Fig. 1 shows a plot of Orrok's data for a single-tube condenser of new Admiralty metal. The ordinate at the zero abscissa is 0.00040, which equals $r_v + r_t + r_s$. The tube had a conductivity of 63, an outside diameter of 1.00 in., and an inside diameter of 0.902 in., and hence had a wall thickness of 0.00408 ft. Therefore, the value of r_t was $\frac{0.00408(1.00)}{63(0.951)} = 0.000068$. Assuming no scale on the new tube, the value of

$$r_v = 0.00040 - 0.000068 = 0.000332$$

or 1/3010, and the corresponding vapor-side coefficient, u_v, is 3010. The slope of this curve is 0.00373, which equals $1/b$; hence b equals 268, and the empirical equation for the water-side coefficient under these conditions is $u_w = 268v^{0.8}$. To summarize, one finds from these data for U and the corresponding water velocities, the following individual coefficients, expressed as B.t.u. per hr. per deg. fahr., per sq. ft. of condensing surface:

$u_v = 3010$ for the steam side
$u_t = 14{,}700$ for the tube
$u_w = 268v^{0.8}$, where v is the water velocity in ft. per sec.

The correct empirical equation for these particular conditions is therefore

$$\frac{1}{U} = 0.00040 + \frac{1}{268v^{0.8}} \quad . \; . \; . \; . \; . \; . \quad [4a]$$

Such form[1] of equation is quite different from the type $U = av^n$, which is so frequently employed in attempts to correlate U and v.

TABLE 5 DATA OF WHITAKER(8)

Single copper tube, 0.494 in. I. D., 0.673 in. O. D., counter-current flow of steam and water. Condensing area, 0.823 sq. ft.

Run	Inlet water temp., deg. cent.	Outlet water temp., deg. cent.	Inlet steam temp., deg. cent	Outlet steam temp., deg. cent.	Lb. water per hr.	V, ft. per sec.	U
1	10.9	81 5	101.1	101.1	326	1.09	607
2	10 9	78.9	100.6	100.6	396	1.33	681
3	11 2	72.5	102.2	101.1	453	1.52	635
4	11.2	73.0	101.1	101.1	457	1.53	647
5	11.7	82.8	112.2	101.1	190	0.64	342
6	11.7	91.3	101.1	102.2	140	0.47	370
7	11.7	92.4	102.2	103.3	112	0.38	301
8	11.7	91.4	100.6	101.7	108	0 36	271
9	12.0	93.4	110.6	101.1	82.2	0.28	234
10	12.0	94.0	100.6	101.1	76.5	0.26	244
11	12.0	94.5	100.6	101.1	60.7	0 20	202
12	12.3	88 8	111.1	100.6	120	0.40	277
13	12.3	86.4	100.6	100 6	156	0 52	347
14	12.3	85.7	100.6	100.6	197	0 66	432
15	12.3	84.3	100.6	100.6	241	0.81	494
16	12.0	79.7	100.0	100 6	322	1 08	558
17	12.3	74.0	101.1	102.8	265	0.89	559
18	12.3	87.7	105.6	101.1	197	0.66	440
19	12.3	86.8	101.1	101 1	213	0.72	460
20	12.3	82.3	101.1	101.1	293	0 98	552
21	12 3	84.7	101.1	101.1	259	0 87	530
22	12.6	86 5	102.8	100.6	198	0.66	433
23	12.8	89.8	101.7	101.7	158	0.53	326
24	12.8	82 3	100.6	100.6	179	0.60	367
25	12.8	81 6	102.8	101.1	263	0.88	478
26	12.8	47 4	99.4	99.4	526	1.77	699
27	12.8	47.7	101.7	100.0	725	2.44	704
28	13.0	47.4	100.0	100.0	690	2.32	703
29	13.0	47.7	102.8	100.0	691	2.32	694
30	13.0	48 3	102.2	102.2	701	2.36	713
31	13.2	48.9	101.7	102 2	659	2.21	674
32	13.3	49.7	100 6	102.2	623	2.09	672
33	13.4	51.7	102.2	102.2	659	2.21	667
34	13.0	49.4	101.7	101.1	529	1.78	688
35	13.1	56.4	102.8	102.8	537	1.80	644
36	13.4	54 3	101.7	102.8	610	2.05	683
37	13.5	53.2	101.7	101.7	593	1.99	646
38	13.6	56.9	102.2	102.2	523	1.76	639
39	13.7	58.5	102.2	101.7	474	1.59	609
40	12.7	60.1	102.2	101.7	426	1.43	580
41	13.7	59.3	105.6	101.1	446	1.50	580
42	13.7	61.0	108.9	101.1	454	1.41	614
43	13.7	63.0	108.9	101.1	386	1.30	558
44	13.6	66.7	107.2	101.7	343	1.15	455
45	13.7	69.7	107.2	100.6	301	1.01	542
46	13.7	67.3	105.6	100.6	321	1.08	541
47	13.8	73.5	101.7	101.1	256	0.81	539
48	13.8	66.0	109.4	100.6	283	0.92	448
49	13.7	64.2	109.4	101.1	331	1.11	482

12 Fig. 1 also shows Orrok's data, using a tube of Admiralty metal taken from an old condenser, and as expected, the overall resistance for a given water velocity is greater than for the new tube. It will be seen that the lines for these two tubes are parallel,

[1] It is interesting to note that the equation employed by Carrier (14)

$$\frac{1}{U} = 0.000394 + \frac{1}{393v}$$

is similar in form to Equation [4a], except that Carrier employed v instead of $v^{0.8}$.

indicating that the thermal resistance of the scale was independent of water velocity. At the zero abscissa the difference in ordinates is the scale resistance, or 0.00092−0.00040 = 0.00052 = 1/1920, which corresponds to an apparent scale coefficient of 1920 based

TABLE 6 SUMMARY OF RESULTS OF FIVE INVESTIGATIONS ON SINGLE TUBES

Observer	Tube material	Tube dimensions, in I. D.	Tube dimensions, in O. D.	Ratio of total length to I. D.	Range of water velocity, ft per sec.	Range of feed-water temperature, deg. fahr.
Clement and Garland	Steel	0.985	1.253	105	0.4 –17 6	57–68
Orrok	Admiralty Metal	0 902	1.00	72	0.8 –11 3	46–55
VanMarle	Brass	0.65	0.75	185	0.82–21.02	56–72
Hoefer	Brass	0 789	0.985	131	0.15– 6.04	52–69
Webster	Copper	0.50	0 875	82	1.2 –14.4	42–49

on the condensing surface, or since the inside diameter was 90.2 per cent of the outside diameter, the actual scale coefficient was 1920/0.902 = 2130 based on the water-side area.

13 As shown below, an equation for r_w on the water side, based on *direct measurements* of r_w, by other experimenters, gives

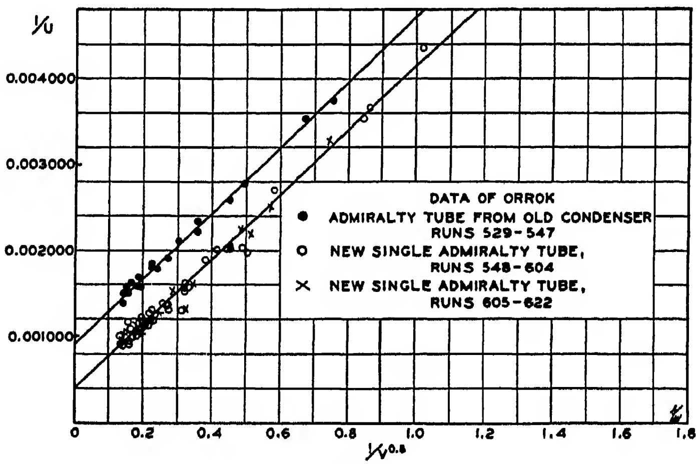


FIG. 1

$r_w = 1/Bv^{0.8}$, where the value of B depends on the dimensions of the apparatus and the temperature. Where scale is absent, $r_s = 0$, and hence the predicted relation for clean pipes would be

$$\frac{1}{U} = r_v + r_t + \frac{1}{Bv^{0.8}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

By assuming r_v as 1/3000, by calculating r_t from the known thickness and conductivity of the metal, and by calculating B from the equation previously published(6) for r_w, the predicted

relation between $1/U$ and $1/v^{0.8}$ is obtained. The predicted values have been plotted on Fig. 2, together with the original data for

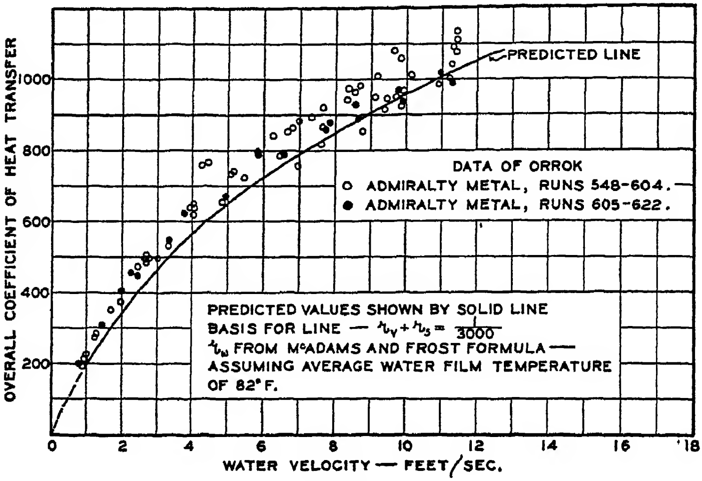


FIG. 2

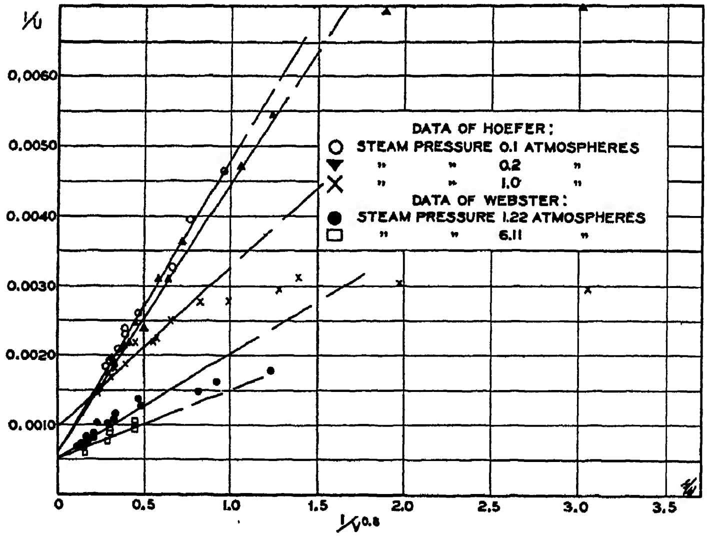


FIG. 3

With regard to lack of linear relation at low water velocities (high values of the abscissas) see Par. 15.

the new tube, giving a comparison between the predicted and actual values of U for the various water velocities.

14. In the preceding discussion it was assumed that the relation between $1/U$ and $1/v^{0.8}$ was linear, and this was justified by

Orrok's data. Inspection of Equation [4] shows that this relation could remain exactly linear only if both r_v and $1/b$ were constant, or if a decrease in r_v was exactly offset by an increase in $1/b$.

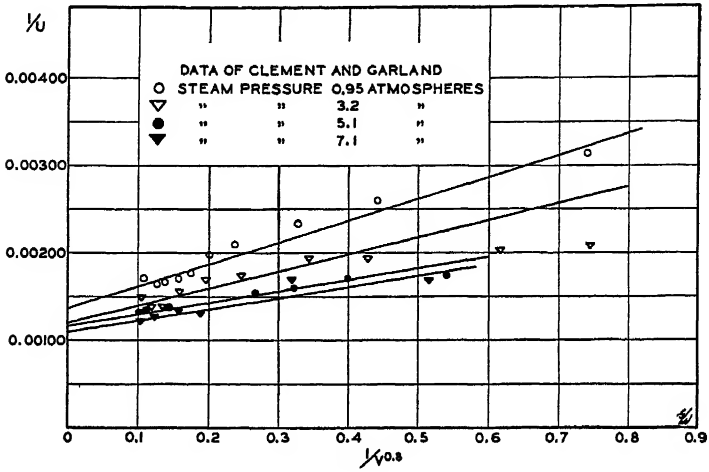


FIG. 4

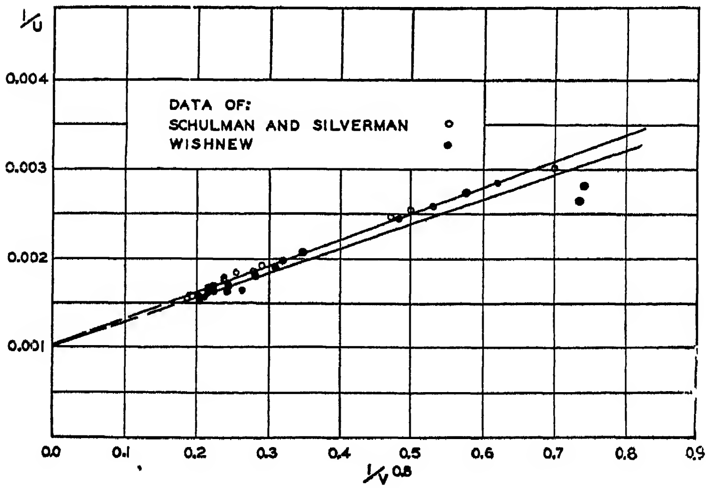


FIG. 5

As water velocity is increased, the water temperature is lowered and $1/b$ increases, but at the same time r_v decreases, due to higher steam velocity. The steam velocity increases with water velocity, due both to decreased overall resistance and increased temperature difference from steam to water at high water velocities. The

predicted curves shown on the plots are based on an average film temperature for the conditions involved on the water side. On Fig. 2 the runs made at the higher temperatures are represented by circles, while the dots apply to the runs at the lower temperatures. As expected, for a given water velocity, the high-temperature runs give higher coefficients than those at low temperature.

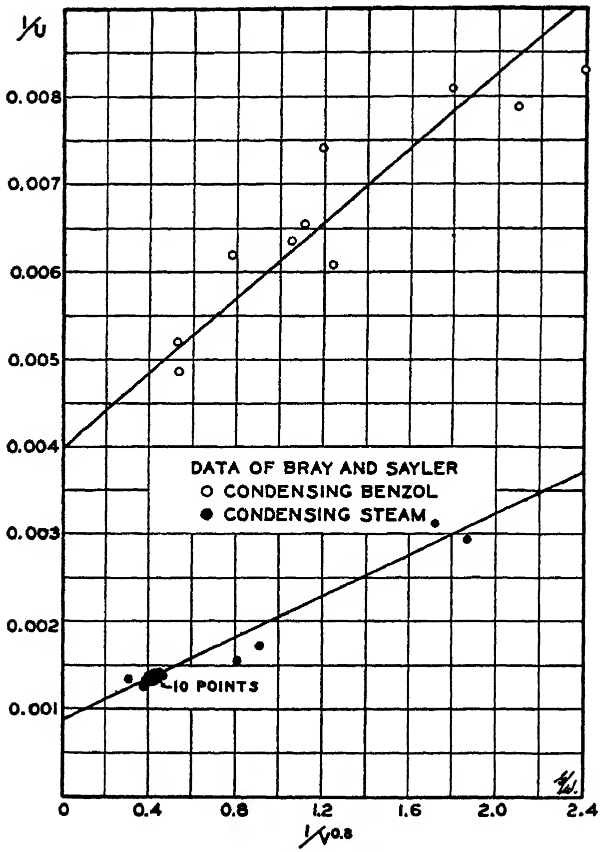


FIG. 6

15 Fig. 3 shows the data of Hoefer(13) plotted with $1/U$ as ordinates versus $1/v^{0.8}$ as abscissas. Here the linear relation is not satisfactory at the low water velocities, but such low velocities are seldom, if ever, used in commercial installations. However, at abscissas corresponding to values of v greater than 1 ft. per sec., straight lines may be drawn. At, say, $1/v^{0.8} = 0.5$, the overall resistance $1/U$ is greater for the steam at 0.1 atmosphere and least when the steam pressure is 1.0 atmosphere. This again

illustrates the decrease in overall resistance due to the increase in water temperature caused by elevating the steam pressure. The same thing is shown by the data of Webster(10), which are also plotted on Fig. 3.

16 Fig. 4, the plot of the data of Clement and Garland(9), shows a similar effect of steam pressure.

17 Fig. 5 shows a plot of the data of Silverman and Schul-

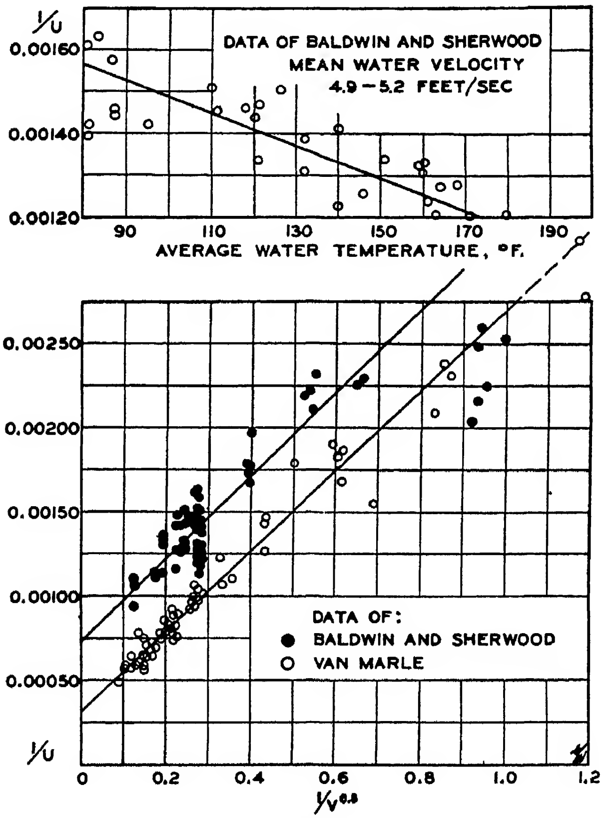


FIG. 7

man(1) and of Wishnew(2). In both cases the water velocities exceeded one foot per second, and the points are well represented by straight lines.

18 Fig. 6 shows a plot of the data of Bray and Sayler(3). It is interesting to note the large increase in the intercept due to the substitution of benzol vapor for steam. These data are well represented by straight lines.

19 Fig. 7 shows the results of Baldwin and Sherwood(7) and VanMarle(11), both using steam at substantially atmospheric

pressure, the points falling on straight lines. Baldwin and Sherwood's points at abscissa 0.27 deviate considerably, due to the effect of water temperature, which was deliberately varied by the use of a preheater. In the upper part of Fig. 7 these points have been replotted versus water temperature. This greatly reduces the deviations noted in the lower part of the plot, where the effect of water temperature was ignored.

20 Fig. 8 shows the data of Whitaker, giving a straight line even at the low water velocities at the right-hand end of the plot.

Intercepts

21 Table 7 summarizes the values of the combined resistances of the vapor side and scale, r_v+r_s, and also shows $\frac{1}{r_v+r_s}$. As

TABLE 7 SUMMARY OF VALUES OF COMBINED RESISTANCES OF VAPOR SIDE AND SCALE

	Metal	Thermal conductivity,[1] English units k	$r_v + r_s$	$\frac{1}{r_v + r_s}$
Bray & Sayler	Copper	220		
Steam			0.00083	1200
Benzol vapor			0.00400	250
Baldwin & Sherwood	Copper	220	0.00069	1450
Whitaker	Copper	220	0.00067	1500
Webster	Copper	220	0.00044	2270
Orrok	Admiralty	63	0.00033	3010
Alberger Heater	Muntz	64	0.00060	1670
Hoefer	Brass	63	0.00053	1890
Wishnew	Brass	63	0.00100	1000
VanMarle	Brass	63	0.00025	4000
Clement & Garland	Steel	25	0 00070	1430
Schulman & Silverman	Wrought iron	40	0 00083	1200
Allen				About 1400

[1] $\frac{\text{(B.t.u.) (ft. thickness)}}{\text{(hr.) (sq. ft.) (deg. fahr.)}}$

previously stated in discussing Fig. 1, these values of r_v+r_s were obtained by reading the intercept, $r_v+r_t+r_s$, and deducting the value of r_t calculated from the conductivity and dimensions of the tube. These resistances range from 1/1000 to 1/4000, important factors probably being differences in concentrations of air in the steam and the presence of various amounts of scale and rust. A resistance of 1/2000 is equivalent to a coefficient of 2000 B.t.u. per hr. per sq. ft. of condensing surface per deg. fahr. drop through the combined resistances on the vapor side and of the scale itself. As shown below, an average value of r_v+r_s of 1/2000 is satisfactory for design purposes.

Multi-Tubular Apparatus

22 Fig. 9 shows the data on an 80-sq. ft. Alberger(15) water-heater, and it is seen that a straight line fits the data remarkably

well. Fig. 10 shows the same data plotted in the usual way, with U versus v, and a predicted line, based on $r_v + r_s = 1/2000$, and

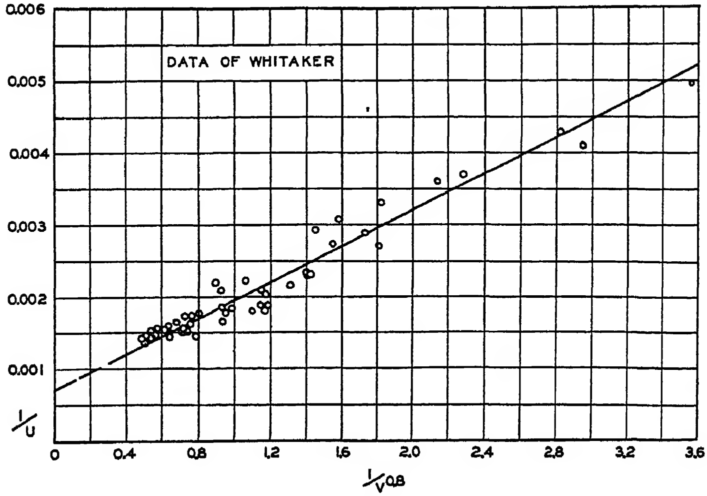


Fig. 8

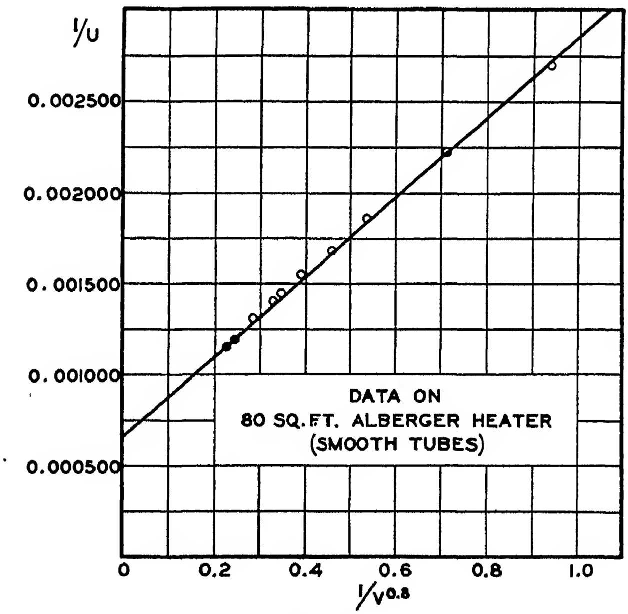


Fig. 9

upon the water-side equation previously published. The agreement between the data and predicted relation is most satisfactory.

23 Fig. 11 shows Allen's(16) data on a 300-sq. ft. condenser, plotted with $1/U$ versus $1/v^{0.8}$. The predicted lines A and C are based on the extreme temperatures involved in these tests,

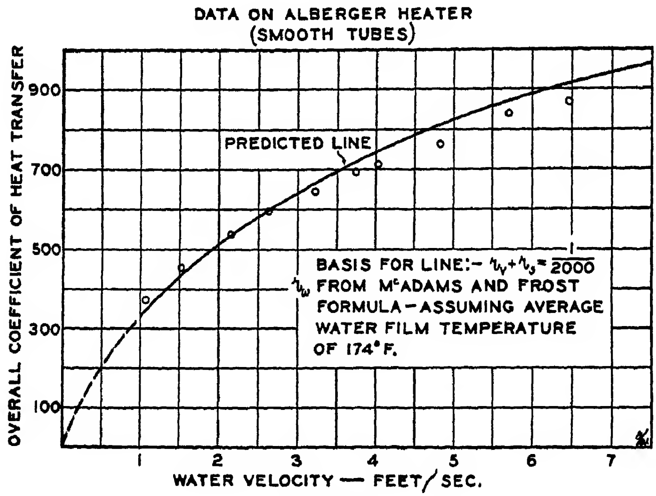


FIG. 10

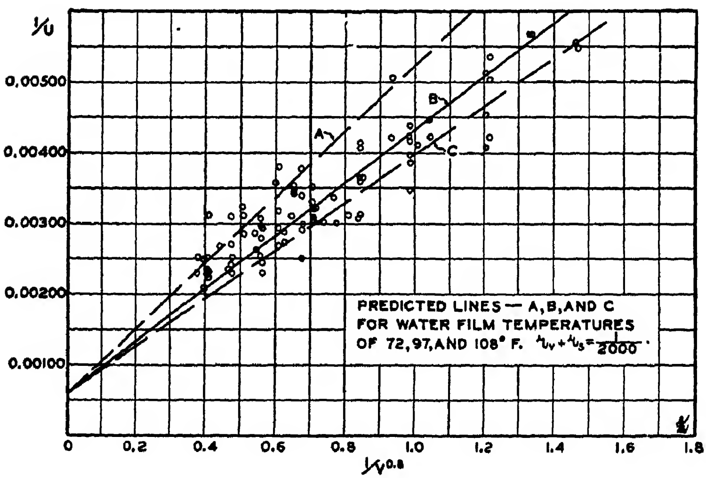


FIG. 11

while the predicted line B is based on a rough average temperature for the entire series.

24 Fig. 12 shows the same items plotted with U versus v, giving reasonable agreement between the observed and predicted coefficients.

25 The points on Fig. 13 show the coefficients and corresponding water velocities obtained in tests (17) upon four large power-plant condensers, ranging from 22,000 to 50,000 sq. ft. The arrows

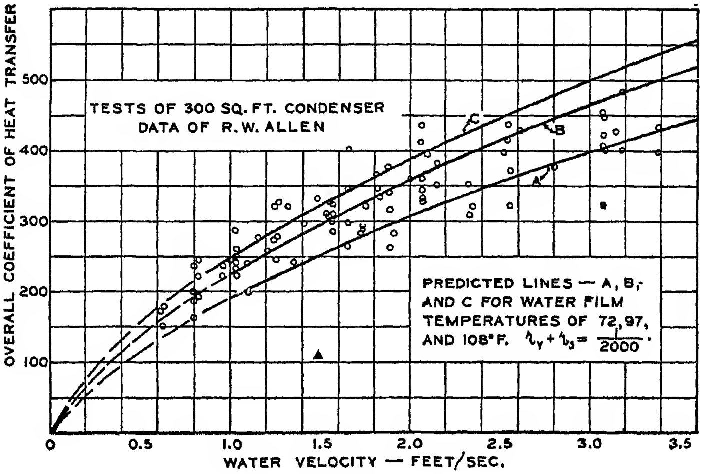


Fig. 12

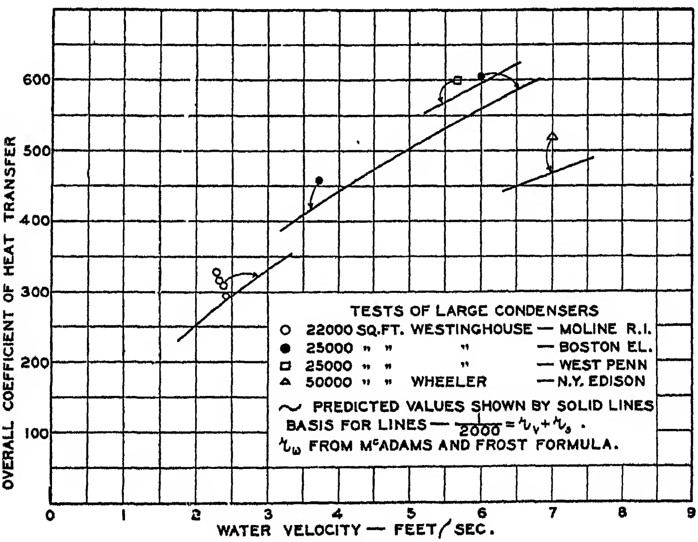


Fig. 13

point to the lines predicted for each of the four condensers. Excellent agreement is found between the observed and predicted values, the predicted values being on the conservative side, as is proper.

26 In calculating the coefficients plotted in Fig. 13, each temperature difference was based on a steam temperature corresponding to the vacuum employed. This gave results slightly different from those tabulated in Ref. 17, which were apparently calculated by some other method. The difference is not serious, as shown by Table 8.

TABLE 8

Condenser	B t u. per hr. per deg fahr. mean diff. per sq ft. of condenser surface (1) Based on vacuum	(2) As reported. Ref. 17	Ratio (1)/(2)
Moline, R I.	291	313	0 93
Moline, R I.	309	333	0.93
Moline, R. I.	315	338	0 93
Moline, R I.	326	350	0.93
Boston Elev.	456	456	1.0
Boston Elev.	612	612	1.0
West Penn	600	600	1.0
N. Y. Edison.	522	598	0.87

Method of Predicting Overall Coefficient of Heat Transfer, U

27 In the foregoing plots, curves were shown for the predicted relation between U and water velocity v. These predicted curves were based on Equation [6].

$$\frac{1}{U} = r_v + r_s + r_t + \frac{1}{Bv^{0.8}} \quad . \; . \; . \; . \; . \; . \; . \quad [6]$$

wherein all resistances are per unit area of condensing surface. Ordinarily $r_v + r_s$ was taken as 1/2000. The value of r_t was calculated from the known conductivity and dimensions of the tube, i.e.,

$$r_t = \frac{a}{k}\left(\frac{D_0}{\frac{D_0+D}{2}}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \quad [6a]$$

where a is the thickness of the tube wall in feet, D_0 and D are the actual outside and inside diameters of the tube in inches, and k is the specific thermal conductivity of the metal as (B.t.u.) (ft.)/(sq. ft.) (hr.) (deg. fahr.). The values of k employed are shown in Table 7.

28 The value B was predicted from the following equation of McAdams and Frost(6):

$$B = \frac{(D)138}{(D_0)D^{0.2}}\left(1 + \frac{50}{r}\right)(J_f)^{0.8} \quad . \; . \; . \; . \; . \quad [6b]$$

where r is the ratio of the length of tube in inches to the inside diameter in inches, and J_f is the fluidity corresponding to the average temperature of the film of water on the water side. Fig. 14 shows a plot of $J_f^{0.8}$ versus film temperature.

29 To illustrate the method of predicting B, consider the following case: Assume a clean tube of Admiralty metal heated by steam condensing under vacuum at a temperature of 126 deg. fahr., and with cooling water at an average temperature of 56 deg. fahr. passing through the tube. The tube is 65 in. long, with 1.00 in. outside diameter, and 0.902 in. inside diameter. What water velocity must be used in order to obtain an overall coefficient of heat transfer U of 500 B.t u. per hr. per deg. fahr. overall difference per sq. ft. of condensing surface?

D_0 = actual outside diameter = 1.00 in.
D = actual inside diameter = 0.902 in.
a = tube thickness = 0.049/12 = 0.00408 ft.
k = specific thermal conductivity of tube = 63

$$r_t = \frac{a}{k}\left(\frac{D_0}{\frac{D_0+D}{2}}\right) = \frac{0.00408(1.00)}{63(0.951)} = 0.000068$$

Steam temperature = 126 deg. fahr.
Average water temperature = 56 deg. fahr.
Total temperature drop, steam to water = 70 deg. fahr.

Since $U = 500, \quad R = 1/500 = 0.0020.$

Assume $r_v + r_s = 1/3000,$

$$r_v + r_s + r_t = \frac{1}{3000} + 0.000068 = 0.000401$$

The drop from steam to the water wall is then $\frac{0.0004}{0.0020}(70) = 14$ deg. fahr., leaving $70-14 = 56$ deg. fahr. drop on the water side. The temperature of the water film is taken midway between that of the inner wall of the tube and that of the water. This film temperature is then easily obtained by adding to the water temperature one-half of the drop on water side, giving $56 + \frac{56}{2} = 84$ deg. fahr. From Fig. 14, $J_f^{0.8}$, corresponding to 84 deg. fahr., is found to be 1.15. Since the tube is 65 in. long, $r = 65/0.902 = 72$ diameters and $D^{0.2} = (0.902)^{0.2} = 0.98$.

By Equation [**6b**],

$$B = \left(\frac{D}{D_0}\right)\left(\frac{138}{D^{0.2}}\right)\left(1+\frac{50}{r}\right)(J_f^{0.8})$$

$$= \left(\frac{0.902}{1.00}\right)\left(\frac{138}{0.98}\right)\left(1+\frac{50}{r}\right)(1.15) = 248$$

By Equation [**6**], the predicted relation between U and v for these conditions is

$$\frac{1}{U} = 0.000401 + \frac{1}{248v^{0.8}}$$

This was based on $U = 500$, whence $v = 3.17$ ft. per sec.

30 For other water velocities the water temperature would rise a different amount and hence would have a different mean temperature, thereby somewhat changing the film temperature. This would give a value of B somewhat different from 248. Such a change in the predicted relation due to temperature changes was brought out in Pars. 19 and 23.

31 Using the predicted values of B and $r_v + r_s = 1/2000$, Figs. 10, 12 and 13 show that the predicted curves of U versus v check reasonably well with the test results on six different pieces of apparatus, containing from 80 to 50,000 sq. ft. of condensing surface.

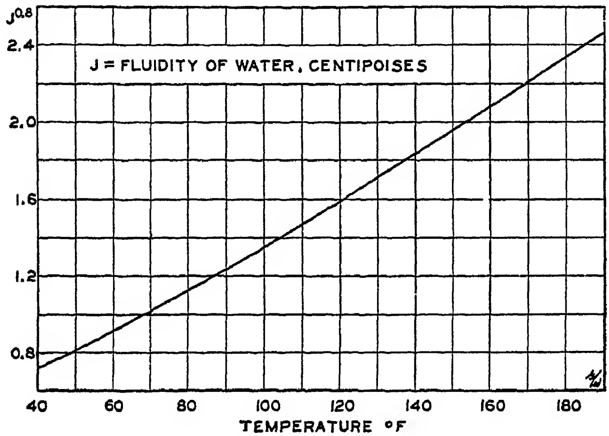


FIG. 14 PLOT OF $J_f^{0.8}$ VERSUS FILM TEMPERATURE IN DEG FAHR.

(Illustrative problem solved in Par 29 gives the method of computing the film temperature.)

ANALYSIS OF OBSERVED RESULTS ON WATER SIDE

32 As stated previously, the slopes of the plots of $1/U$ versus $1/v^{0.8}$ are $1/b$. This is the observed water-side resistance, per square foot of condensing surface, when the water velocity is one ft. per sec. In order to base calculations per square foot of surface on the water side, the value of b is multiplied by the ratio of D_0/D, i.e., $b' = b(D_0/D)$. The observed individual coefficient of heat transfer per unit area of condensing surface u'_w equals $b'v^{0.8}$. Table 9 shows the observed values of both b and b', and it is seen that the values of b' range from 297 to 1783.

33 In the equation $u'_w = B'v^{0.8}$, the predicted value of B' is given by the equation (6) of McAdams and Frost:

$$B' = \frac{138\left(1 + \frac{50}{r}\right) J_f^{0.8}}{D^{0.2}}$$

Hence, in order to compare the observed value of b' with the predicted values, one should plot the group $(b')(D^{0.2})/\left(1+\frac{50}{r}\right)$ versus J_f. If the observed values check with the predicted values the experimental points so plotted should fall on the curve whose equation is

$$\frac{B'D^{0.2}}{\left(1+\frac{50}{r}\right)} = 138J_f^{0.8}$$

Fig. 15 shows such a plot, and it is found that the experimental values check well with the predicted curve so long as the steam

TABLE 9

	Observed water-side coefficient[1] for a water velocity of one foot per second	
	b (per sq. ft. of condensing surface)	b' (per sq. ft. of surface on water side)
Wishnew	362	480
Silverman and Schulman	338	507
Bray and Sayler (steam)	844	1147
Bray and Sayler (benzol)	467	636
Baldwin and Sherwood	404	551
Whitaker	800	1089
Clement and Garland, 0.95 atm.	397	506
Clement and Garland, 3.2 atm.	508	646
Clement and Garland, 5.1 atm.	746	951
Clement and Garland, 7.1 atm.	770	981
Van Marle	424	489
Webster, 1.22 atm.	671	1182
Webster, 6.11 atm.	1020	1783
Orrok	268	297
Hoefer, 0.1 atm.	240	299
Hoefer, 0.2 atm.	261	326
Hoefer, 1 0 atm	439	548
Alberger heater	455	525
Allen	280	320

[1] B t u. per hr per sq. ft. per deg. fahr. average temperature difference from inner wall of tube to water.

is not above atmospheric pressure. However, as the steam pressure increases the deviation of the points from the line increases, the line giving extremely safe values at the high steam pressures. The predicted relation was based(6) on direct measurements of the water-side coefficients of heat transfer by the use of thermocouples attached to the tube wall. However, the constants of the predicted relation were evaluated by the use of data obtained with steam not materially above atmospheric pressure. Fig. 15 shows that this predicted relation breaks down at the high steam pressures. This may well be due to the fact that at the high steam pressures the tube becomes so hot that the water boils on the inner wall of the tube. Under such conditions the film thickness would be abnormally decreased, giving a corresponding increase in b'. It is well known that the evolution of dissolved gas from the water

becomes rapid at the high temperatures, further complicating the problem.

34 To illustrate the effect of high pressure, the same data have been replotted in Fig. 16 versus steam pressure in atmospheres, and the ordinate is seen to increase as the steam pressure is increased. In Fig. 16 the effect of variation in water temperature is ignored, except as depending on steam pressure. For high steam pressures one might prefer to obtain b' from Fig. 16, rather than to employ the method previously shown to be satisfactory for

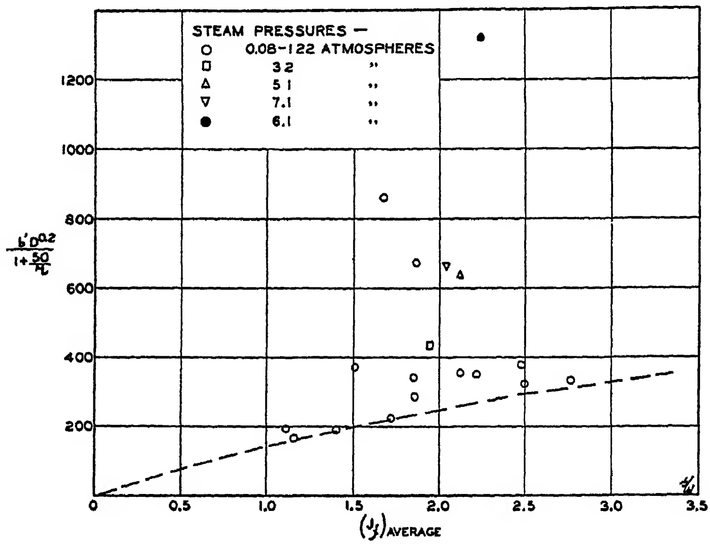


FIG. 15

steam not above atmospheric pressure. Other methods for correlating such data immediately suggest themselves. However, most commercial tubular apparatus for condensing steam are operated with steam under vacuum in power-plant work, and with exhaust steam in feed-water heaters. Hence a further study of the discrepancies brought out by Figs. 15 and 16 is mainly of academic interest.

35 The standard experimental data for the condensation of steam under vacuum are those of Orrok. In order to determine how the water-side coefficients determined by these tests compare with those predicted by the McAdams and Frost equation, the procedure was as follows: From Fig. 1 the observed value of $r_v+r_s+r_t$ was found to be 0.00040. This was deducted from the observed overall resistance for each run, giving the observed value

of r_w for each run. The corresponding reciprocal value, multiplied by the proper diameter ratio, gave the observed coefficient of heat transfer on the water-side. These values are shown plotted in Fig. 17 versus water velocity. The line, also drawn on Fig. 17, is based on the equation of McAdams and Frost, using 86 deg. fahr. as the average temperature of the film on the water side. It is seen that the predicted values check satisfactorily with the experimental values, and it should be noted that the water velocity varied over thirteen-fold.

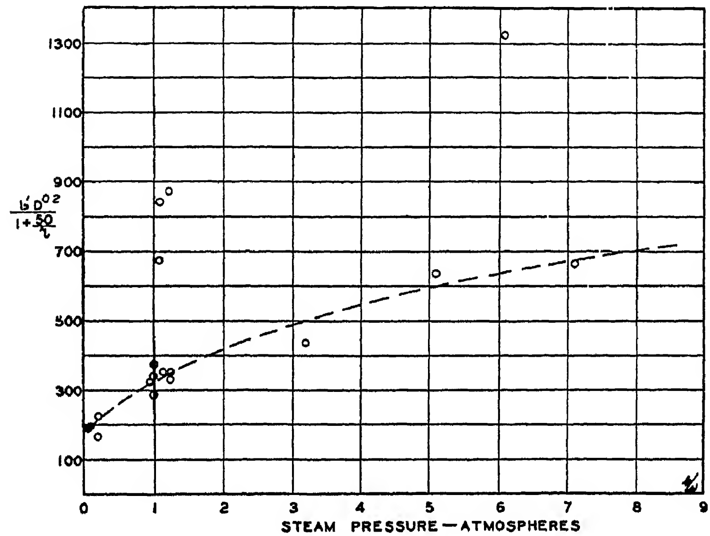


Fig. 16

Acknowledgment

36 The authors wish to acknowledge their appreciation of the work of Messrs. Baldwin, Bray, Sayler, Silverman, Schulman, Wishnew, and Whitaker, who obtained the experimental data given in Tables 1 to 5. Thanks are also due D. J. VanMarle, of the Buffalo Foundry and Machine Co., for permission to quote his data, as given in Fig. 7.

Table of Nomenclature

A = area of surface on which vapor condenses, sq. ft.
a = thickness of the tube wall in feet
B = predicted factor in the equation $u_w = Bv^{0.8}$
B' = predicted factor in the equation

$$u'_w = B'v^{0.8}; \quad B' = B\frac{D_0}{D}$$

b = an empirically-determined factor in the equation

$$u_w = bv^{0.8}$$

b' = an empirically-determined factor in the equation

$$u'_w = b'v^{0.8}; \quad b' = b\,\frac{D_0}{D}$$

D_0 = outside diameter of pipe, inches
D = inside diameter of pipe, inches

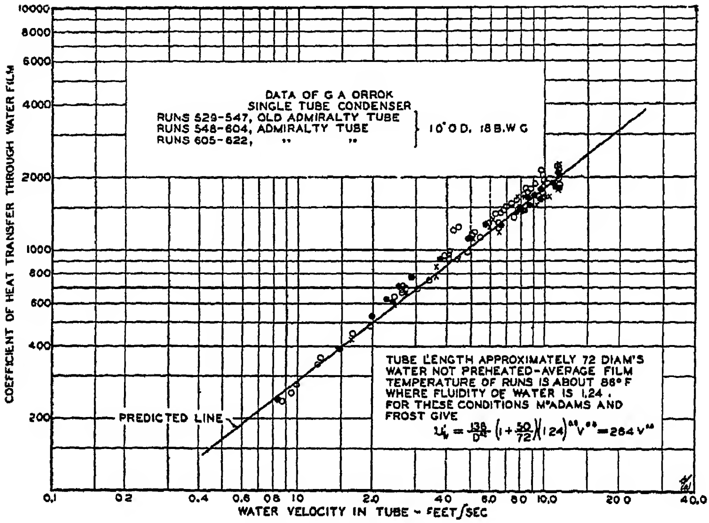


FIG. 17

Tube length, approximately 72 diameters; water not preheated; average film temperature of runs about 86 deg. fahr. where fluidity of water film is 1.24. For these conditions McAdams and Frost give

$$u_w = \frac{138}{D^{0.2}}\left(1+\frac{50}{72}\right)\left(1.24\right)^{0.8}V^{0.8} = 284V^{0.8}$$

J_f = fluidity of film on water side, reciprocal centipoises. For values of $J_f^{0.8}$, see Fig. 14

k = specific thermal conductivity of tube material, expressed as B.t.u. times ft. per sq. ft. per hr. per deg. fahr. drop through the metal wall; see Table 7 for values of k

Q = rate of heat flow, as B.t.u. per hour

r = length of tube in inches, divided by inside diameter in inches

R = overall thermal resistance per unit area of condensing surface, equals reciprocal of overall coefficient of heat transfer $= 1/U = r_v + r_t + r_s + r_w$

r_s = individual thermal resistance of any scale or other deposit on either or both surfaces of the tube, per unit area of condensing surface $= 1/u_s$

r_t = individual thermal resistance of the tube, per unit area of condensing surface $= 1/u_t$

r_v = individual thermal resistance on the vapor side, per unit area of condensing surface = $1/u_v$

r_w = thermal resistance on water side, per unit area of condensing surface = $1/u_w$

u_t = individual coefficient of heat transfer through the metal tube, expressed as B.t.u. per hr. per sq. ft. of condensing surface, per deg. fahr. mean difference between temperance of outer and inner metal surfaces = $1/r_t$

u_v = individual coefficient of heat transfer from steam to condensing surface, expressed as B.t.u. per hr. per sq. ft. of condensing surface, per deg. fahr. mean difference between temperature of steam and outer metal surface = $1/r_v$

u_w = individual coefficient of heat transfer from metal surface to water, expressed as B.t.u. per hr. per sq. ft. of condensing surface, per deg. fahr. mean difference between the temperatures of the inner surface of the tube and the main body of the water = $1/r_w$

u'_w = individual coefficient of heat transfer from metal surface to water, based on the inner surface of the tube; $u'_w = u_w D_0/D$

U = overall coefficient of heat transfer from steam to water, expressed as B.t.u. per hr. per sq. ft. of condensing surface per deg. fahr. mean difference between temperatures of steam and water = $Q/A(\Delta T)_{av.}$

v = average water velocity in tubes, ft. per sec.

$(\Delta T)_{av.}$ = logarithmic mean difference in temperatures between steam and water = $(x-y)/2.3 \log (x/y)$, where x is the difference between steam and water temperatures at the water inlet, and y is a similar difference at the water outlet.

BIBLIOGRAPHY

1 Schulman and Silverman, M.I.T. Undergraduate Chemical Engineering Thesis, 1922.

2 Wishnew, M.I.T. Undergraduate Chemical Engineering Thesis, 1921.

3 Bray and Sayler, M.I.T. Chemical Engineering Thesis, 1923.

4 E. E. Wilson, Trans. A.S.M.E., vol. 37 (1915), p. 47.

5 Haslam, Ryan, and Weber, *Industrial and Engineering Chemistry*, vol. 15, no. 11 (1923), p. 1105.

6 W. H. McAdams and T. H. Frost, *Refrigerating Engineering*, vol. 10, no. 9 (1924), p. 323.

7 Baldwin and Sherwood, M.I.T. Chemical Engineering Thesis, 1924.

8 Whitaker, M.I.T. Chemical Engineering Thesis, 1925.
9 Clement and Garland, Bulletin 40, Engineering Experiment Station, University of Illinois (1909).
10 Webster, Trans. Institute Engineers and Shipbuilders in Scotland, vol. 57 (1913), p. 58.
11 D. J. VanMarle, Buffalo, N. Y., by private communication.
12 G. A. Orrok, Trans. A.S.M.E., vol. 32 (1910), p. 1773.
13 Hoefer, reported by Josse in "Mitteilungen aus dem Maschinenlaboratorium der Techn. Hochschule zu Berlin" Heft V.
14 W. H. Carrier, *Journal* A.S.M.E., vol. 33, no. 12 (1911), p. 1073.
15 D. K. Dean, Boston, Mass., privately communicated.
16 R. W. Allen, Proceedings Institute of Civil Engineers, Feb. 28, 1905.
17 Report of Prime Movers Committee, National Electric Light Association, 1921.

DISCUSSION

THEODORE BAUMEISTER, JR.[1] The use, by the authors, of thermal resistances instead of, or to supplement, the overall coefficients of heat transmission appeals strongly to the writer. This method of analyzing data on heat exchange, and also for the application of data, has long been advocated and taught by Dr. Charles E. Lucke of Columbia University as a very useful tool, and as one which should lead to much valuable knowledge of the phenomena. The substitution of the method of thermal resistances for the more usual overall coefficients of heat transfer has many features which catch the spirit of scientific inquiry and which, if properly applied, will lead to other data and results of engineering utility.

The first and most important advantage lies in the additive property of thermal resistances, a feature which is not to be found in the usual heat transfer coefficients. In mathematical form it is expressed by the authors in their equation

$$R = \frac{1}{U} = r_v + r_t + r_s + r_w \quad . \; . \; . \; . \; . \; . \; . \; [2]$$

If, instead of the component thermal resistances given in the right hand member of Equation [2], the corresponding individual coefficients of heat transfer, U_v, U_t, U_s, U_w were known, these latter could not be added together directly to give the overall coefficient, U. To get this overall coefficient, the relations would be

$$R = \frac{1}{U} = \frac{1}{U_v} + \frac{1}{U_t} + \frac{1}{U_s} + \frac{1}{U_w}$$
$$= \frac{U_tU_sU_w + U_vU_sU_w + U_vU_tU_w + U_vU_tU_s}{U_vU_tU_sU_w} \quad . \; . \; [2a]$$

or

$$U = \frac{1}{R} = \frac{U_vU_tU_sU_w}{U_tU_sU_w + U_vU_sU_w + U_vU_tU_w + U_vU_tU_s} \quad . \; [2b]$$

[1] Instructor, Mechanical Engineering Department, Columbia University, New York, N. Y. Jun. A.S.M.E.

To use either Equation [**2a**] or Equation [**2b**] it would be necessary to handle much awkward mathematics, which could be readily avoided by Equation [**2**]. Further, the entire heat-exchange process cannot be so conveniently split up for individual consideration of each element of the problem. This additive feature is a decided merit in favor of the method of thermal resistances. A similar method is to be found in electrical engineering practice where much use is made of electrical resistances and little use is made of electrical conductivities. This case is one of direct analogy, and the advantages which prevail in the one prevail in the other.

The second advantage which the writer wishes to note is the direct comparability of thermal resistances as found on similar and dissimilar apparatus. For example, the results as found on the steam side of one feedwater heater can be used to check the results found on the steam side of another feedwater heater. Likewise, they can be checked against the thermal resistance of the equivalent steam films on condensers, fuel-oil heaters, steam-air heaters, evaporators, etc., and much valuable information obtained as to the details of heat transmission by condensing steam to metal when considering or studying steam-side conditions. By such and similar comparisons, which are innumerable, it is possible to have a rational basis of analysis for all heat-transmission problems and to scrutinize the details of any heat-transfer process with especial reference to film conditions, equivalent film thickness, and the relation to flow.

Having thus indicated the desirability of the method of thermal resistances the writer wishes to commend the authors for an analysis along these lines and to ask them some questions.

First, to what extent do the values of the exponent of velocity as given in Equation [4] and the values of the coefficient B as found from Equation [**6b**] check the work of the several investigators, who give heat-transfer rates as quite different functions of velocity, dependent on the authority? The values are as follows:

Ser	$U = 520\sqrt[3]{V}$
Josse	$U = 487\sqrt[3]{V}$
Weighton	$U = 430\sqrt[3]{V}$
Hepburn	$U = 419\sqrt{V}$
Hageman	$U = 282\sqrt{V}$
Stanton	$U = 340\sqrt[3]{V}$
Joule	$U = 315\sqrt[3]{V}$
Allen	$U = 220\sqrt{V}$
Clement	$U = 270\sqrt[3]{V}$
Orrok	$U = 308\sqrt{V}$

These equations have been conveniently collected by Mr. Orrok.[1] Certain of the data have been used by the present authors, and as

[1] Transmission of Heat in Surface Condensation, Geo. A. Orrok. Trans. A.S.M.E., vol. 32 (1910), p. 1139.

these relations are often quoted the question arises as to the agreement between these results and those as predicted by the authors' methods

Second, are the relations listed in the previous paragraph true when the velocity of flow is zero? That is, when $V = 0$, will $U = 0$? Also, what is the effect of critical velocities?

Third, how would the value of the coefficient B and the exponent, 0.8, be affected by modification of flow circumstances? For example, would the equations of the authors hold for cases of flow within a corrugated tube; for flow in helical or spiral tubes; for flow on the outside of tubes instead of on the inside of tubes; for flow transversely across a tube bank, or longitudinally along a tube bank; or for flow within the annular space between two concentric tubes?

Fourth, what is the definition of fluidity, J_f, and what is the relation between it and the variables of which it is a function?

Fifth, exception might be taken to the closing sentences of Par. 34. The authors should extend the scope of their investigations to include heating by high-pressure steam.

JAMES M. TAGGART.[1] In Par. 9 the formula $r_w = 1/bV^{0.8}$ is given. A footnote states that this formula is correct only for turbulent flow. Since the relation is applied to velocities as low as one foot per second it appears that this velocity is considered turbulent. Normally, so low a velocity would not be so considered. It seems that this point needs further explanation. It is common practice to produce or increase the turbulence of water flow in heater tubes and heat interchangers by bending or corrugating the tube, or by placing therein wire spirals or twisted plate. These devices produce turbulence at a much lower lineal velocity than is possible with straight clear pipe. It is often claimed that the form of turbulence so produced is more effective for increasing heat transfer than a simple increase in water velocity; that is, for the same head loss, the coefficient of heat transmission would be larger.

In determining the total coefficient of heat transfer in condensers, there has been a tendency during recent years to stress the importance of the thermal resistance at the steam surface. It is claimed, and to some extent proved, that this resistance, or the value called r_v in the paper, depends primarily on the presence or absence of an air film over the tube surface. It is also claimed that the presence or absence of this air film depends largely on the direction and velocity of the steam flow. At one time it was believed that this resistance varied mainly with the amount of the water covering over the tubes, due to the condensation and drippage.

[1] Consulting Engineer, New York, N. Y. Assoc-Mem. A.S.M.E.

In Table 7, a series of tests is given, with their derived values for thermal resistance of the steam surface and scale. These values vary from 0.00025 to 0.001. It is assumed that the variance is due to air content of the steam and to a varying scale. It is improbable that in tests made to determine the heat transfer through tubes there would be any perceptible scale. Any normal air content in the steam would not account for the variation. It is possible that some of the variance may be due to difference in the line of steam flow, and condensation removal. It certainly appears misleading to assume as a basis for any calculations an average of the results of the tests given. The attempt to prove the general validity of the formulas by using such an assumption, and comparing the computed results with those obtained from several condenser tests, is not conclusive to any one familiar with the conditions prevailing in condensers.

G. L. KOTHNY.[1] The authors' exposition of the agreement between the observed data and the predicted values would clearly indicate the possibility of predetermining with fair accuracy the required cooling surface in heat-exchange apparatus. However, two very important factors which influence the overall heat-transfer coefficient, and which should also be correlated, have not been discussed or taken into consideration.

One of these factors is the vapor velocity. The other is the presence of non-condensable gases in the steam to be condensed. It would be interesting if the authors would enlarge their paper by giving some data regarding the mass velocity of the vapors to be condensed, and the ratio of pounds of non-condensable gas per pound of steam to be condensed for the average value of $r_v + r_s = 1/2000$. If such data are not available it is suggested that additional experiments be made to determine the effects of these two variables.

In Par. 28 the value of B is given by means of Equation [**6b**]. In the text book Principles of Chemical Engineering, of which Mr. McAdams is co-author, this formula reads:

$$B = 204 \frac{J^{0.794}}{D^{0.206}}$$

While it is admitted that this latter formula is of earlier date, it would be interesting to know what led to its change, and particularly why the ratio of the length of the tube to the inside diameter was brought into it.

From the new formula [**6b**] it is evident that the individual coefficient of heat transfer from metal surface to water u_w will be larger if the ratio of the length of the tube to the diameter is

[1] Executive Engineer, C. H. Wheeler Mfg. Co., Philadelphia, Pa. Mem. A.S.M.E.

smaller. That is, with shorter tubes a better individual heat-transfer coefficient from metal surface to water will be obtained. It would be interesting to know if this conclusion is based upon experimental data.

C. F. Kayan.[1] The authors are to be commended on the use of the resistance idea — the only proper method of analyzing heat transfer data, and as valuable here as is Ohm's law in electrical circuits.

The material and ideas in the paper, however, are much along the same lines as those presented at the June, 1915, meeting of the Society by Lieut. E. E. Wilson.[2] As a result of the demonstration by Dr. C. E. Lucke of Columbia University that heat transfer was essentially a subject bound up rigidly with gas- and liquid-film resistances, Lieutenant Wilson showed that the coefficient of heat transfer varied inversely with velocity raised to the 0.8 power, and that the viscosity of the water was likewise an important factor. This method of treatment now is being accepted as the proper method of attack in heat-transfer problems.

It is regretted, however, that the chemical engineers writing this paper have not thought it advisable to use fahrenheit temperatures in conjunction with their use of pounds of water and other American engineering units, or even metric units exclusively, rather than to introduce a hybrid system of units involving degrees centigrade, pounds of water, feet per second, etc.

The thermal conductivity of tube material in Orrok's work is taken as 63, which corresponds with the value given in Marks' Handbook for brass at 63 deg. fahr. From this value, the vapor-side resistance has been deduced. Although here the sum of the vapor, tube, and scale resistance is low, and hence of no great account, in some cases the vapor resistance may be of greater weight. An example of this would be heat transfer from a vapor condensing to a liquid boiling. Exception is taken, therefore, to the value of 63 for the conductivity, first because conductivity is itself a function of temperature, and secondly because the conductivity of brass is variable, being dependent on the chemical composition of the alloy. Since even a slight variation in the components may throw the assumed value entirely off, and since a variation in temperature causes a variation in conductivity, it is evident that care must be exercised in accepting values. The variation of conductivity with temperature in some brasses may be as much as 25 per cent between 32 and 212 deg. fahr.

The authors refer to tests on power-plant condensers as checking predicted values quite acceptably. In the experience of the writer

[1] Assistant Instructor, Dept. of Mechanical Engineering, Columbia University, New York, N. Y. Jun. A.S.M.E.

[2] A Basis for Rational Design of Heat Transfer Apparatus, Trans. A.S.M.E., vol. 37 (1915), p. 47.

it is difficult to use ordinary condenser tests in comparison with calculated values, primarily because the velocity of the circulating water is in doubt, and secondly because the temperature on the vacuum side also is in doubt. In proportion as the amount of air present in the steam increases, so will the temperature decrease, inasmuch as the partial pressure of the water vapor in the mixture is reduced. Hence, in the travel of the steam from condenser inlet to air outlet, we often have a wide variation in temperature, which if properly allowed for (the amount of the allowance depending on the design of the condenser) would tend to raise the value of the overall coefficient of heat transfer.

As far as water velocity is concerned, this is, in essence, a function of the pressure difference between the tube sheets, or better still, between the ends of the various tubes. The pressure over the ends of the tubes in any one water box varies from a minimum at a point nearest the inlet of the water box to a maximum at a point farthest away from the inlet, due to impact on the far side of the box with a subsequent right-angle turn of the stream of water. Consequently, with unequal pressure differences, we get unequal velocities through the tubes, and though for any one pass we apply an average velocity, the character of heat transfer as a function of velocity throws the result in doubt. In addition to the foregoing, the tubes nearest the air outlet are given over more to heat transfer from air to water rather than from steam to water. That is, the major resistance has switched over from the water side to the "steam side," so that any attempt to consider the condenser as a whole and to obtain an overall condenser value for the heat-transfer coefficient is a method full of pitfalls.

Finally, exception is taken to the statement that heaters operating with steam at higher pressures than exhaust are primarily of academic interest. Attention is called to the fact that in many modern power plants steam turbines are bled, with the result that we have feedwater heaters operating on relatively high-pressure steam, i.e., from 50 to 100 lb. per sq. in.

D. K. Dean.[1] The authors followed the method of E. E. Wilson in plotting the inverse of the heat transfer against the inverse of the 0.8 power of velocity. In so doing have they endeavored to eliminate other exponential functions of the velocity? There was some criticism of Mr. Wilson's work, and as a result of that criticism he replotted his data using the first power of the velocity as the controlling factor and found that they agreed just about as well. It would seem, therefore, that before one finally accepts the 0.8 power of the velocity as being the controlling factor in

[1] Research Engineer, Wheeler Condenser and Engineering Co., New York, N. Y. Mem. A.S.M.E.

heat transfer, the other exponential functions of the velocity should be eliminated.

THE AUTHORS. Mr. Baumeister inquires as to the agreement between Equation [4] of the authors and certain empirical formulas tabulated by Mr. Orrok. In applying Equation [4] the authors assume that the sum of the resistances, r_v, r_t, and r_s, is substantially constant,[1] and plot $\frac{1}{U}$ versus $\frac{1}{V^{0.8}}$. Upon plotting a given series of data in this way, if a straight line be obtained, the equation holds for the operating conditions involved, thus determining the corresponding constants, $r_v+r_t+r_s$, and b.

Of the data corresponding to the ten formulas listed by Mr. Baumeister, the last three were considered in the paper, namely, those of Allen, Clement, and Orrok, representing both the square-root and cube-root relations. Figs. 11, 4 and 1 show these data, and in each case straight lines were obtained, thus justifying Equation [4]. Table 7 gives the intercepts obtained, and Table 9 shows the corresponding values of b.

Mr. Baumeister also inquires as to the agreement between the *observed* value of b and B, where B is the value *predicted* from Equation [6b] based on direct measurement of the water-side resistances in experiments made independently. This question is answered by Figs. 2 and 12, which show good agreement between the *observed* and *predicted* values. Of course, no agreement is expected between the exponent of 0.8 in Equation [4] and the exponents of ½ and ⅓ in the formulas cited by Mr. Baumeister, since as pointed out at the end of Par. 11, the forms of these two types of equation are entirely different. Nothing has delayed progress in this field more than attempts to correlate overall heat transfer data by assuming an exponential relation between U and water velocity, i.e., by using $U = av^n$.

Mr. Baumeister asks whether or not U would be zero at zero water velocity, and inquires as to the effect of critical velocity.

Certainly as v approaches zero, U remains finite. As shown by the footnote to Par. 9, Equation [4] does not apply at velocities below the critical value, as the flow is no longer turbulent. Since zero velocity is below the critical value, the same applies at zero velocity. When the velocity exceeds the critical value, the flow is turbulent and these equations apply for the types of tubes tested.

For a given water velocity, Mr. Baumeister inquires as to the quantitative effect of changing the degree of turbulence by using corrugated tubes, spiral coils, etc. The data of the authors apply only to turbulent flow inside straight pipes of constant diameter,

[1] See also Par. 14.

such as those of drawn brass, copper, and steel. Hence, this question is beyond the scope of this paper. It is generally agreed that anything which increases the useful turbulence on the water side will decrease the thermal resistance, r_w. The authors have under way an experimental study of heat transfer in annular spaces.

Regarding the term J_f, this represents the fluidity of the water film, expressed in reciprocal centipoises, and is a function of temperature. As explained in Par. 29, the film temperature is taken midway between the temperature of the inner wall of the tube and that of the main body of the water. Inasmuch as the equation for B calls for $J_f^{0.8}$, for convenience this value has been plotted versus temperature, rather than J_f itself.

Mr. Taggart notices that the authors call a velocity of 1 foot per second turbulent, and disagrees with this. The critical velocity is that velocity at which viscous flow changes into turbulent motion, as water velocity is increased. According to the data[1] of Stanton and others, the critical velocity for the isothermal flow of water at 68 deg. fahr., through a 1-in. copper tube, varies from 0.15 to 0.45 ft. per sec. When the tube is being heated, as in a condenser, there is every indication that the critical velocity would be even smaller. Regarding the corrugated pipe, this point has already been taken up in reply to Mr. Baumeister.

As to whether or not the thermal resistance on the steam side is so important that it should be specially investigated, as a matter of fact, in many commercial condensers, the resistance on the water side, due to the rather low water velocity, and the large resistance due to scale or slime forms the bulk of the whole resistance. Small corrections for variations on the steam side, therefore, are not of primary interest in practical work.

As to the values of $r_v + r_s$ of Table 7, Par. 21, Mr. Taggart feels that the differences should not be attributed to variations in scale or air, and that there would be no perceptible scale. In the laboratory tests made by the authors the formation of some sort of deposit, scale, slime or otherwise, on the water side was found to be of considerable importance, making it necessary to adopt a uniform procedure of cleaning in order to be able to duplicate results. In fact, in other experiments not given herein, it was found possible, by very careful cleaning, to obtain values of b much larger than those given, hence we cannot agree with Mr. Taggart as to the relative unimportance of scale or slime. To determine the effect of scale in a given case, the best procedure is to plot the test data as in Fig. 1. Where air content is kept uniformly small, it obviously would have little effect. As to the resistance offered by the film of condensate, it is recognized that this depends on the temperature of the condensate, the rate of steam flow, and other factors. Where water velocity is varied in a given set of data,

[1] *Jour. Ind. Eng. Chem.*, vol. 14, no. 2 (Feb., 1922), pp. 105-119.

as for the bulk of the results presented herein, it is unnecessary to speculate about $r_v + r_s$, as this term is directly determined from the intercept when plotting $1/U$ versus $1/V^{0.8}$. However, in the case of the isolated tests of Fig. 13, where only one water velocity was available, the recommended method of plotting could not be used. In order to get a rough check on the accuracy in predicting B, from Equation [**6b**], it was necessary to assume some value for $r_v + r_s$. This was taken as 1/2000, and gave conservative results. As a matter of interest, the value of 1/2000 is a conservatively high resistance according to many of our unpublished tests where this resistance was measured by thermocouples. No doubt plant conditions might be so unfavorable as to give higher resistances, but for the tests of the large condensers reported in Fig. 13, the value of 1/2000 worked well.

Considering the discussion of Mr. Kothny, the effect upon r_v of non-condensable gas was mentioned in Par. 21, and of steam velocity in Par. 14; both effects are further discussed in preceding paragraphs of this closure.

The 1923 formula, $B = 204J^{0.8}/D^{0.2}$, allows only for fluidity and diameter, while in Equation [**6b**], published in 1924, the constant of 204 is replaced by the term $138(1+50/r)$. It should be noted that when $r = 100$ diameters, the latter gives $138(1.5) = 207$, substantially agreeing with the first form determined for tube lengths of 100 diameters. Equation [**6b**], as shown by reference 6 of the bibliography, is based on data where r changed over a considerable range, although the larger part of the data were obtained for $r = 100$.

In regard to the discussion of Mr. Kayan, as to the choice of units, it should be noted that all coefficients of heat transfer were reported in the standard English engineering units. The test data of the authors and co-workers were expressed in the units used when making the measurements, which happened to involve temperatures in degrees centigrade, because of the available thermometers, those graduated in degrees centigrade were more suited to the purpose than those calibrated in degrees fahrenheit. The use of centigrade temperatures in no wise complicates the calculations of the values of U, because the same conversion factor of 1.8 appears in *both* numerator and denominator of U, and therefore drops out of the calculation.

The authors agree[1] with Mr. Kayan that the specific thermal conductivity of a given metal is subject to variation. For the Admiralty metal tube of Par. 29, the tube resistance was less than 4 per cent of the total thermal resistance. However, our most recent work has been done with copper, where the thermal conductivity is so high that the uncertainties mentioned by Mr. Kayan

[1] See p. 181 of Principles of Chemical Engineering, Walker, Lewis and McAdams, McGraw Hill Book Co., Inc., 1923.

become of little importance. This matter is exhaustively discussed in reference 6 of the bibliography.

Mr. Kayan feels that, in a large multi-tubular condenser, the average water velocity is in doubt, due to unequal flow through the different tubes. In spite of this, the points on Fig. 13, for large multi-tubular condensers, check reasonably well with the curves predicted from tests on single-tube condensers.

Replying to the points brought up by Mr. Dean, regarding the ability of certain data to conform to both the first power and

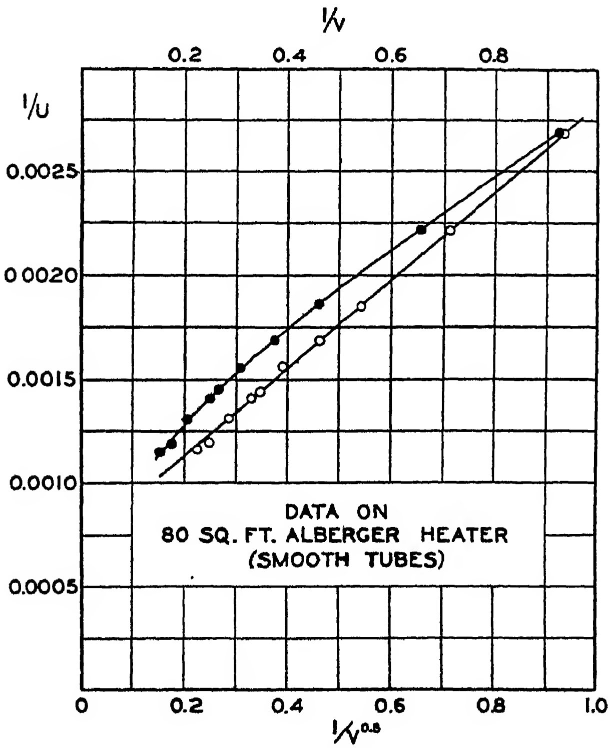


Fig. 18

the 0.8 power velocity with apparently equal facility, it must be borne in mind that over a limited range of velocity, the graphs of an 0.8 power function and a first power function are nearly coincident, hence the agreement.

The authors were aware of the discussion that followed Mr. Wilson's paper, but in plotting test values of U, as recommended by Equation [4], they prefer to use $1/v^{0.8}$ for several reasons. First, it works well, as shown by Figs. 1, 3, 4, 5, 6, 7, 8, 9, and 11. Second, the exponent of 0.8 is confirmed by plotting, on logarithmic

paper, the values of r_w measured directly by use of thermocouples; see reference 6 of the bibliography. Third, the special analysis of Orrok's data, Fig. 17, further corroborates the 0 8 power.

Fig. 9 is a plot of test data submitted by Mr. Dean, and an excellent linear relationship is obtained when using $1/v^{0.8}$. Fig. 18 shows the same data plotted against $1/v$, as suggested by Mr. Dean, and also against $1/v^{0.8}$, as recommended by the authors. It is seen that the plot of $1/U$ versus $1/v^{0.8}$ gives the better linear relationship.

No. 2034

HEAT TRANSFER THROUGH INSULATION IN THE MODERATE- AND HIGH-TEMPERATURE FIELDS: A STATEMENT OF EXISTING DATA

By L. B. McMillan,[1] New York, N. Y.
Member of the Society

The purpose of this paper is to set forth briefly the state of existing knowledge on heat transfer through insulating materials in the moderate- and high-temperature fields as indicated by the literature on the subject; to show in what respects this knowledge is insufficient; and to point out the directions in which future research work is most urgently needed.

This is not a review of the literature in the sense of attempting to abstract various articles and statements. No such attempt has been made, but after a comprehensive survey of the literature, the endeavor has been made to present a complete picture of the information available and to point out what is lacking. In addition the effort is made in some cases to supply needed information not now available in the literature. In this connection the method of determining mean temperatures, and the economic data in particular, are entirely new and original. Furthermore, the whole subject is presented from a new point of view.

The particular field of this research is the literature referring to heat transfer through insulation in the temperature range between the refrigeration field on the one hand and the refractories field on the other, with the specific exception of literature pertaining to heat transfer through building materials, which is covered by another National Research Council Heat Transfer Committee assignment, and which will be reported on separately by another author. Still other assignments cover the literature of heat transfer in the refractories field and in the refrigeration field. These likewise are being reported on by other authors.

[1] Chief Engineer, Johns-Manville Corporation.

Presented at the Annual Meeting, New York, December 6 to 9, 1926, of The American Society of Mechanical Engineers, at a joint session with the American Society of Refrigerating Engineers. Program arranged by the Committee on Heat Transmission, National Research Council.

Definitions and Equations

THE fundamental theory of heat transfer in the steady state (uniform temperature conditions maintained at both warmer and cooler surfaces) has been well-known throughout the past century by the leading authorities on heat transfer. Briefly, it is based on the same conception as Ohm's law; namely, that flow varies directly as the potential and inversely as the resistance. On the other hand, the lack of such thorough understanding of basic principles by many who have written on the subject is responsible for the state of the literature which, by some, is considered chaotic. The author is not inclined to such a gloomy view of the entire literature, however. It is true that in referring to the literature one must be able to differentiate between the sound and the unsubstantial, but this is also true of the literature of any subject, where it is as voluminous as in this case.

2 *Case I. Simplest Heat Transfer Equation, Flat Surfaces.* The resistance to heat transfer is dependent upon the thickness, x, and conductivity, k. It varies directly as the thickness and inversely as the conductivity, and is equal to x/k. Therefore, the simplest case of heat transfer through a material having flat surfaces is represented by the equation,

$$U = \frac{t_1 - t_2}{\frac{x}{k}} \quad \text{. . . . .} \quad [1]$$

in which U is the overall rate of heat transfer in B.t.u. per sq. ft. per hr., t_1 is the temperature of the warmer surface and t_2 is the temperature of the cooler surface.

3 *Conductivity.* Thermal conductivity is defined as rate of heat transfer in one direction (perpendicular to an area) per unit area, per unit temperature differential per unit thickness, per unit time (B.t.u. per sq. ft., per degree temperature difference between surfaces per 1 in. thickness, per hr.).

4 Conductivity is a specific property of a material. It is not dependent on the area, thickness or shape of the material. It is a *rate,* not a *quantity.* The total quantity of heat transmitted is dependent upon the area, shape and length of path (thickness of material), but conductivity is not. Conductivity is dependent upon temperature, but this is also true of other specific properties of material, density for example. This relationship of conductivity and temperature will be discussed in detail later in this paper.

5 The phrase, "*in one direction,*" has been added to the usual definition of conductivity for two reasons. First, because some materials, wood for example, may have different conductivities in different directions; and second, and more particularly, to warn the unwary to take account of conditions involved on curved sur-

faces in the portion of the equation dealing with the shape of the path and not in the conductivity itself.

6 *Case II. One Material and One Surface Resistance. Flat Surface.* Often the temperature of the cooler surface is not known, the known temperatures being those of the warmer surface, t_1, and of the air surrounding the cooler surface, t_a. Then the rate of heat transfer is given by the equation

$$U = \frac{t_1 - t_a}{\frac{x}{k} + \frac{1}{c}} \quad \ldots\ldots\ldots\ldots \quad [2]$$

in which c is the rate of heat transfer from outer surface to air.

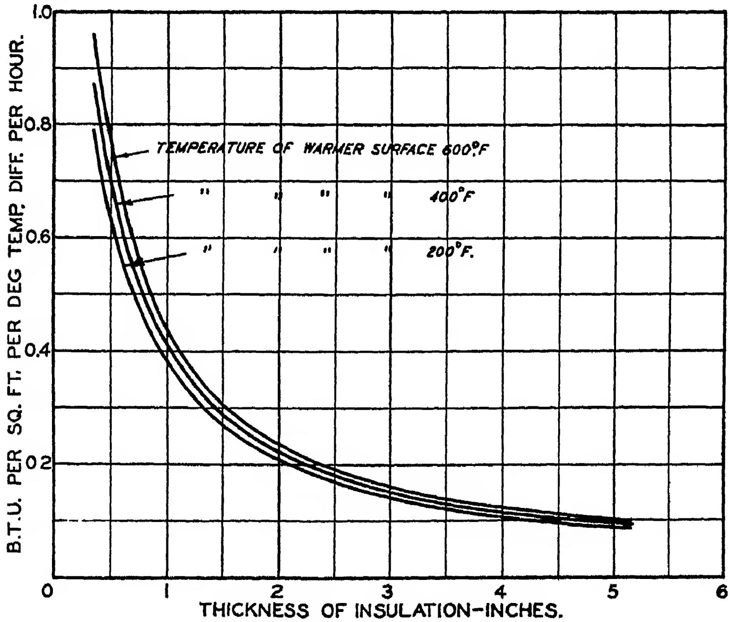


FIG. 1 EFFECT OF THICKNESS ON RATE OF HEAT TRANSFER. FLAT SURFACES

7 The manner in which rate of heat transfer varies as the thickness of the insulation is increased is illustrated in Fig. 1, which is based on the conditions of Case II. It will be noted that the transfer through material 2 in. thick is somewhat greater than half of that through material 1 in. thick. The reason for this is clearly apparent in Equation [2]. Doubling the thickness doubles the value of the term x/k, but does not double the value of the term $1/c$. Consequently, since the value of the denominator in the case of 2-in. thickness is somewhat less than double the value for 1-in.

thickness, the value of U for 2-in. material must be somewhat more than one-half of its value for 1-in. material.

8 *Surface Resistance.* The term x/k represents the resistance of the material in question, and the term $1/c$ represents the *surface resistance.* The lack of thorough understanding of the effect of surface resistance is perhaps responsible for more confusion in the literature of heat transfer than any other single item. The failure to separate the effects of *surface resistance* from those of *insulation resistance* is the cause of most of the conflicting conceptions of conductivity.

9 In some cases surface resistance is the controlling factor in total rate of heat transmission. For example, in the case of a $\frac{1}{4}$-in. thickness of a metal having a conductivity of 2500 B.t.u. per sq. ft., per deg. temperature difference between surfaces per 1-in. thickness, per hr., and with a temperature difference of 1 deg. fahr. between surfaces, the rate of heat transfer per sq. ft. per hr. would be $\frac{1}{(0.25/2500)} = 10{,}000$ B.t.u. If, however, the temperature difference between the warmer surface and still air surrounding the cooler surface is 1 deg. fahr., and the rate of heat transfer from surface to air is 2.0 B.t.u. per sq. ft. per deg. temperature difference per hr., the total heat transfer per sq. ft. per hr. would be

$$\frac{1}{(0.25/2500)+\frac{1}{2}} = \frac{1}{0.0001+0.5} = 1.9996 \text{ B.t.u.}$$

(or for all practical purposes = 2.0 B.t.u.)

In this case the surface resistance is 99.98 per cent of the total, and the resistance of the metal is practically negligible. In such a case a variation of 100 per cent in the conductivity item would have no appreciable effect on the total.

10 On the other hand, in the case of well-insulated surfaces, surface resistance is a very small part of the total. For example, in the case of a 10-in. thickness of a material having a conductivity 0.25 B.t.u. per sq. ft., per deg. temperature difference per 1-in. thickness, per hr., and with a temperature difference of 1 deg. fahr. between surfaces, the rate of heat transfer would be $\frac{1}{(10/0.25)}$ = 0.025 B.t.u. per sq. ft. per hr. If the temperature difference between the warmer surface and still air surrounding the cooler surface is 1 deg. fahr., and the rate of heat transfer from surface to air is 2.0 B.t.u. per sq. ft. per deg. temperature difference per hr., the rate of heat transfer would be

$$\frac{1}{(10/0.25)+\frac{1}{2}} = \frac{1}{40+0.5} = 0.0247 \text{ B.t.u. per sq. ft. per hr.}$$

In this case the surface resistance is only 1.25 per cent of the total.

11 In the first example, neglecting to take account of surface resistance would make the result 5000 times too large. In the second example, the surface resistance influences the result only to the extent of about $1\frac{1}{4}$ per cent. These are representative of the two extremes, but actual cases will be found in practice over this entire range, and at times even more extreme cases will be encountered.

12 This illustrates the importance of surface resistance. The matter would be greatly simplified if surface resistance were a relatively constant quantity, but it is not. It is influenced by the temperature of the surface, the temperature of surrounding objects, the nature of the surface, the position of the surface, the velocity of air currents over the surface, and by many other causes. Fortunately for the solution of insulation problems the value of surface resistance is usually small as compared with the insulation resistance — usually less than 25 per cent, and frequently less than 10 per cent of the insulation resistance. Therefore, with even approximate data on surface resistances, such problems may be solved with a very satisfactory degree of accuracy.

13 But the case of heat transfer from bare surfaces at high temperatures and at varying air velocities is quite another matter. It would be difficult to find a more interesting or more fruitful field for physical research. But if such research is to be of permanent value it must be directed along other lines than those generally followed in the past. It must take into account the absolute temperatures of the surface and surrounding objects and it must take separately into account the effects of radiation and convection, and the effect, on the latter, of air velocity, and extent and position of the surface. The work of Langmuir[1] is an excellent illustration of the direction in which to proceed, but the work must be carried much farther if the results are to be of real service in the engineering field.

14 *Case III. One Material, Two Surface Resistances, Flat Surface.* When the temperature of neither the inner nor outer surface is known, and the known temperatures are those of the air on either side of the insulation, the equation for heat transfer is

$$U = \frac{t_0 - t_a}{\frac{1}{c_1} + \frac{x}{k} + \frac{1}{c_2}} \quad \ldots\ldots\ldots \quad [3]$$

in which t_0 is the temperature of the air on the warmer side, c_1 is the rate of heat transfer from air to surface on the warmer side, and c_2 is the rate of heat transfer from cooler surface to surrounding air.

15 The inside surface resistance, $1/c_1$, is not used when the temperature of the warmer surface is known. Also, its magnitude

[1] Trans., American Electrochemical Society, vol. 23.

is often negligible where effective insulation is placed directly against a heated surface the temperature of which is known. However, it is included in these general equations in order that it be not neglected in cases where it should be taken into account.

16 *Case IV. Two or More Materials, Flat Surface.* So far, only one material has been considered. If heat must flow successively through two or more different materials the equation takes the form

$$U = \frac{t_0 - t_a}{\frac{1}{c_1} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \ldots + \frac{1}{c_2}} \quad . \; . \; . \; . \quad [4]$$

in which x_1, x_2, x_3, ... etc., are the thickness of the various materials, and k_1, k_2, k_3, etc., are the conductivities of the respective materials.

17 Inspection of Equations [1] to [4] shows that they are all of the same form. When heat must flow successively through various elements of the construction, in order to write the equation

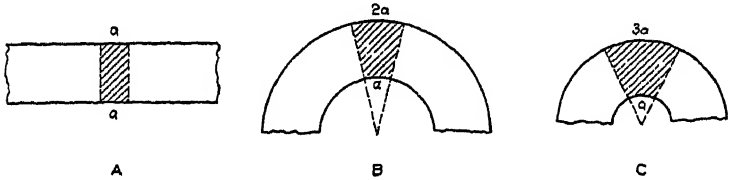


FIG. 2 AREAS OF PATHS FOR HEAT FLOW THROUGH INSULATIONS ON FLAT AND CYLINDRICAL SURFACES

for heat transfer, all that is necessary is to add to the equation an item representing the resistance of each successive element.

18 *Cylindrical Surfaces.*[1] Except on flat surfaces, the internal resistance of a material does not vary directly as the thickness. In the case of cylindrical surfaces, increasing the thickness supplies additional resistance through which the heat must flow, but at the same time increases the area of the path through which the heat may flow. This is illustrated in Fig. 2, which shows the areas of paths for flat and cylindrical surfaces. It is clearly apparent from this diagram that the heat transfer per unit of area of inner surface will be greater for insulation on a curved than on a flat surface, and that the smaller the radius of curvature, the greater will be the rate of heat transfer per unit of inner surface area.

19 *Case V. One Material—Cylindrical Surfaces.* The rate of heat transfer per hour, per square foot of outer surface, through a single layer of uniform material on a cylindrical surface, when the

[1] Equations of the type of [5] to [8a] inclusive were outlined by the author in a discussion in Journal Am. Soc. Heat. & Vent. Engrs., July, 1920, p. 571.

temperatures of the two surfaces are known, is given by the equation

$$U_2 = \frac{t_1 - t_2}{\dfrac{r_2 \log_e \dfrac{r_2}{r_1}}{k}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

in which t_1 and t_2 are the temperatures of the warmer and cooler surfaces, respectively, r_1 is the external radius of the pipe or cylinder and r_2 is the radius of the outer surface of the insulation.

20 The loss per square foot of pipe surface is

$$U_1 = \frac{r_2}{r_1} \times U_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5a]$$

in which U_1 and U_2 represent the rates of heat transfer per hour per square foot of pipe surface and outer surface of insulation, respectively.

21 Equation [5] might have been written to give the transfer per square foot of pipe surface directly, by substituting $r_1 \log_e \frac{r_2}{r_1}$ where $r_2 \log_e \frac{r_2}{r_1}$ appears. However, the reason for giving preference to the form of equation which results in loss per square foot of outer surface will be apparent in connection with subsequent equations involving surface resistance. The surface resistance most frequently applies to that surface; therefore it is more convenient to calculate the rate of transfer per square foot of outer surface. Then the loss per square foot of pipe surface is easily calculated from U_2 by means of Equation [5a].

22 It will be noted that the expression $r_2 \log_e \frac{r_2}{r_1}$ occupies the same position in Equation [5] that the thickness, x, occupies in Equation [1]. In fact, this expression is frequently referred to, for convenience, as "equivalent thickness." It is numerically equal to the thickness of material on a flat surface which would be required to give the same rate of heat transfer as that per square foot of outer surface of insulation on the cylinder or pipe. With this fact in mind, a like relationship will immediately be apparent between Equations [6], [7] and [8], which follow, and the corresponding equations for flat surfaces applying to similar conditions. Therefore, these equations are not nearly so formidable as they appear. In fact using values of $\log_e \frac{r_x}{r_1}$ shown in Fig. 3, the solution becomes very simple. The term r_x represents the outer radius of any layer of insulation, and in the case of a single material, $r_x = r_2$.

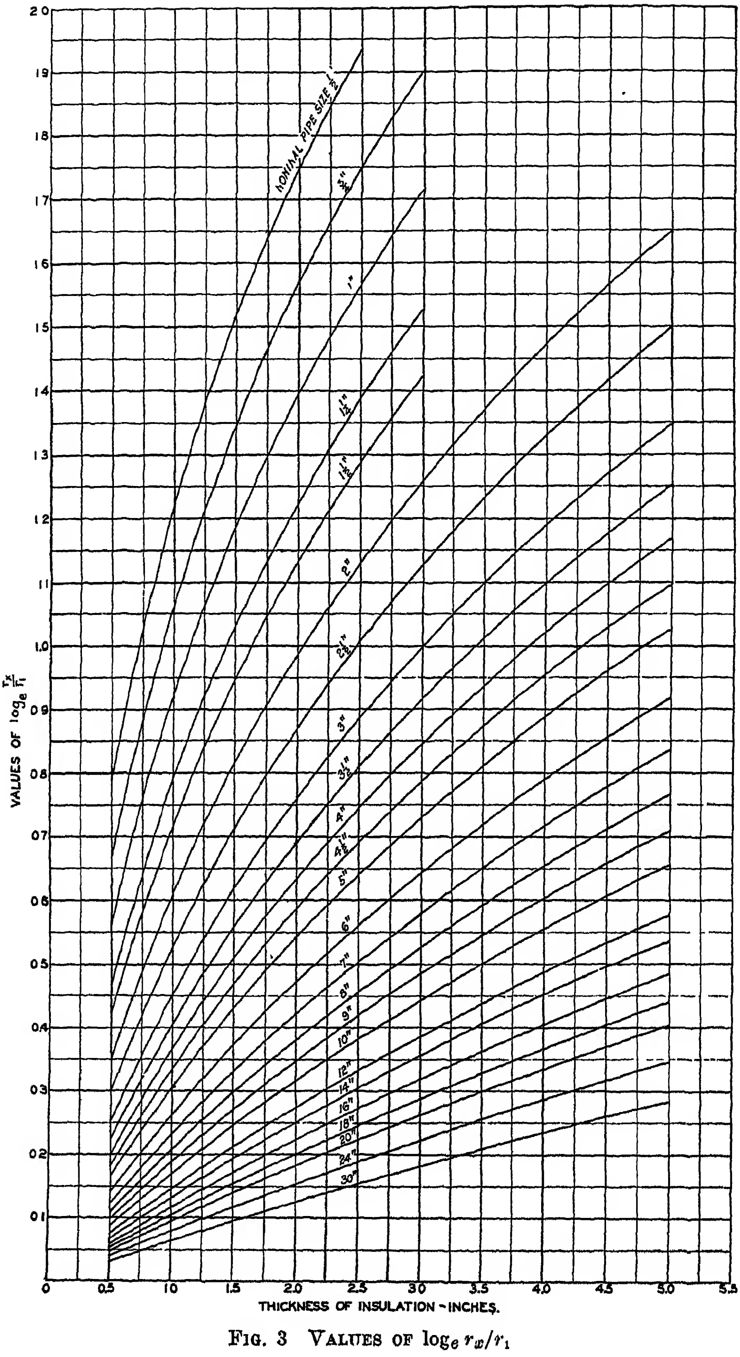


Fig. 3 Values of $\log_e r_x/r_1$

23 *Case VI. One Material, Cylindrical Surface. One Surface Resistance.* When the temperature of the cooler surface is not known the equation becomes

$$U_2 = \frac{t_1 - t_a}{\frac{r_2 \log_e \frac{r_2}{r_1}}{k} + \frac{1}{c}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; [6]$$

In the preceding equation all terms have the same significance as in Cases II and V, respectively. To obtain rate of heat transfer per unit of pipe surface use Equation [5a].

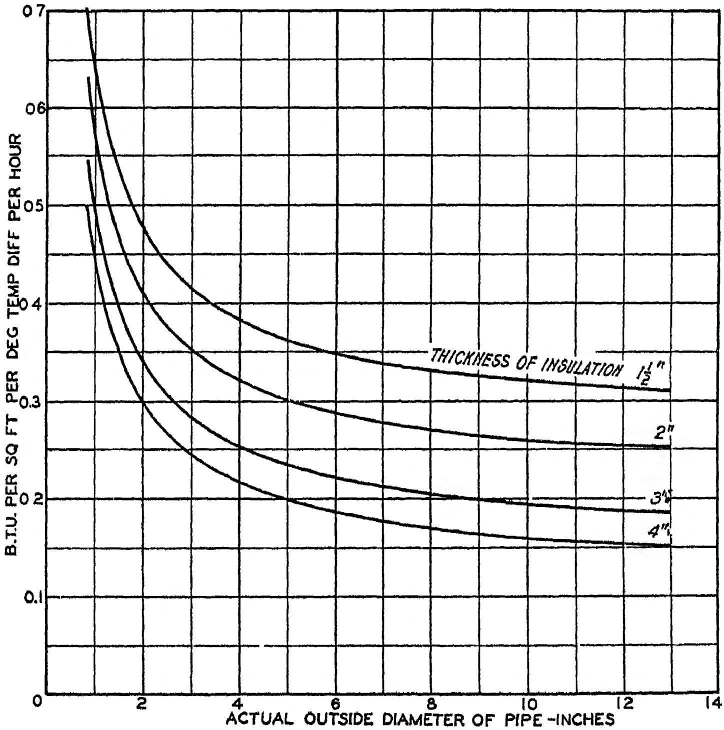


FIG. 4 VARIATION WITH PIPE SIZE OF RATE OF HEAT TRANSFER THROUGH A GIVEN THICKNESS OF INSULATION

24 The effect of pipe size on the rates of heat transfer through insulation per square foot of pipe surface under the conditions of Case VI, is illustrated in Fig. 4. It will be noted that the rate of heat transfer through 2-in. thick insulation on ½-in. pipe is more than twice as great as that through the same thickness of insulation on 12-in. pipe.

25 *Case VII. One Material, Cylindrical Surface, Two-Surface Resistances.*

$$U_2 = \frac{t_0 - t_a}{\frac{r_2}{r_1} \times \frac{1}{c_1} + \frac{r_2 \log_e \frac{r_2}{r_1}}{k} + \frac{1}{c_2}} \quad \ldots\ldots \quad [7]$$

in which all terms have the same significance as in Case III and V respectively. As before $U_1 = \frac{r_2}{r_1} \times U_2$.

26 *Case VIII. Two or More Materials, Cylindrical Surfaces, Two Surface Resistances.*

$$U_s = \frac{t_0 - t_a}{\frac{r_s}{r_1} \times \frac{1}{c_1} + \frac{r_s \log_e \frac{r_2}{r_1}}{k_1} + \frac{r_s \log_e \frac{r_3}{r_2}}{k_2} + \frac{r_s \log_e \frac{r_4}{r_3}}{k_3} + \cdots + \frac{1}{c_2}} \quad . \quad [8]$$

$$U_1 = \frac{r_s}{r_1} \times U_s \quad \ldots\ldots\ldots\ldots \quad [8a]$$

in which r_s is the radius of the outer surface of insulation and U_s is rate of heat transfer per hour per square foot of this surface. Other terms have the same significance as in previous equations.

27 Fig. 3 may be used for the evaluation of the logarithmic terms in combinations of materials, as well as for single materials. In the case of two materials

$$\log_e \frac{r_3}{r_2} = \log_e \frac{r_3}{r_1} - \log_e \frac{r_2}{r_1}$$

Therefore, in such cases, the values of the logarithmic terms are readily found by reading from the chart the value of the logarithmic term corresponding to the entire thickness and subtracting from this the value of the logarithmic term corresponding to the thickness of the first layer. Both of these values may be read from the chart, and their difference referred to above is the value of the logarithmic term for the second layer.

28 *Temperature at Any Point.* The temperature drop through each element of the construction bears the same ratio to the total temperature drop as the resistance of that element bears to the total resistance. Therefore, since the terms in the denominator of each of Equations [2], [3], [4], [6], [7] and [8] are the respective resistances to heat flow offered by the individual elements in the construction, this form of equation lends itself very conveniently to the calculation of the temperature at any point in the construction.

29 *Small Cylinders.* In the case of small cylinders, such as wires, the increased area of path for heat flow through covering

material often overbalances the added resistance through which the heat must flow. In such a case, applying material on the surface of the cylinder will increase the loss until the thickness has been built up to the point where the outside radius

$$r_2 = \frac{k}{c} = R_s k \quad \text{[9]}$$

after which the addition of still more material will decrease the rates of heat transfer. (See Appendix 2 for the proof of this equation.) If the radius of the cylinder itself is greater than k/c, the application of covering material will decrease the rate of heat transfer

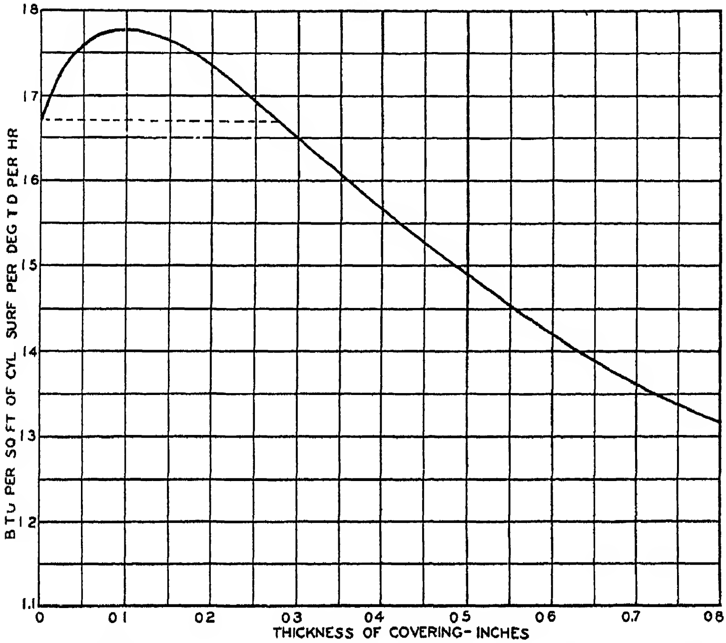


FIG. 5 VARIATION OF HEAT TRANSFER WITH THICKNESS OF COVERING MATERIAL ON A SMALL CYLINDER

from the outset. For example, if the conductivity of the covering material k is 0.2 B.t.u. per sq. ft., per deg. temperature difference per 1 in. thickness, per hr., and the rate of heat transfer from surface to air c is 1.67 B.t.u. per sq. ft. per deg. temperature difference per hr., the loss will be decreased from the outset by the application of covering in all cases where the outside radius of the cylinder is greater than 0.30 in.

30 A typical example of the variations of rate of heat transfer as the thickness of covering material is increased on a very small cylinder is illustrated in Fig. 5. This is based on applying material

having a conductivity of 0.5 B.t.u. per sq. ft., per deg. temperature difference per 1 in. thickness, per hr., on a cylinder with a diameter 0.4 in. ($r_1 = 0.2$ in.) and on a rate of heat transfer from surface to air of 1.67 B.t.u. per sq. ft. per hr. It will be noted that the maximum loss occurs when 0.10 in. of material has been added ($r_2 = 0.30$ in.) and that the loss is equal to that from bare surface when the thickness has been increased to 0.285 in. ($r_2 = 0.485$ in.).

31 For a given size of cylinder, the loss will be decreased from the outset by the application of insulation pro-

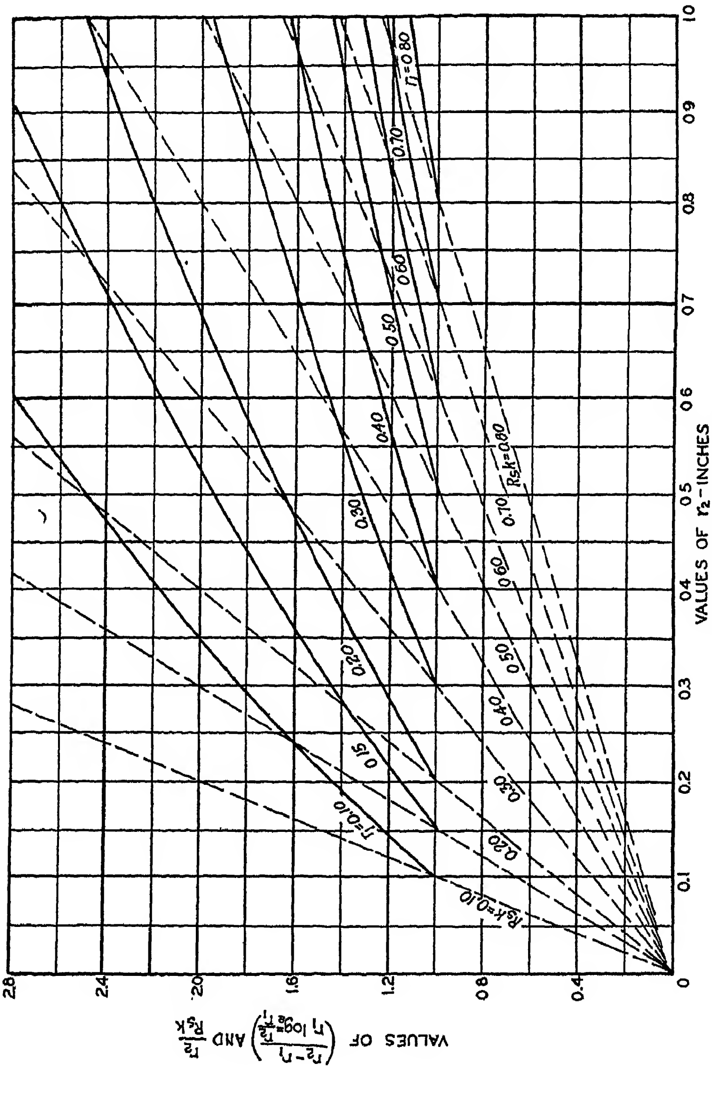


Fig. 6 Chart for the Solution of Equation [10]

vided $k < cr_2$. In the case of a 1 in. pipe, 1.315 in. actual outside diameter, and if $c = 1.67$, the loss will be decreased from the outset if the conductivity is less than 1.10 B.t.u. per sq. ft., per deg. temperature difference per 1 in. thickness, per hr. The conductivities of most good insulating materials are less than 1.0 B.t.u. per sq. ft., per deg. temperature difference per 1 in. thickness, per hour.[1] It is obvious therefore, that the loss will be increased by the application of covering material *only on extremely small cylinders,* if the material applied has low conductivity, and that the application of sufficient additional material will decrease that loss.

32 The thickness at which the loss through material on a small cylinder is equal to the loss from bare surface, and beyond which the application of additional material will show a saving as compared with bare surface loss, may be determined from the equation

$$\frac{r_2}{R_s k} = \frac{r_2 - r_1}{r_1 \log_e \dfrac{r_2}{r_1}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [10]$$

This equation may be solved through the use of Fig. 6, in which values of each side of the equation have been plotted against r_2 for different values of $R_s k$ and of r_1, respectively. With a given set of values for R_s, k and r_1, the loss through the covering will be the same as that from a bare surface at the value of r_2 located by the intersection of the line for the appropriate value of $R_s k$ with the curve for r_1. If these do not intersect, the indication is that r_1 is sufficiently large for the application of the given material to decrease the loss from the outset.

33 While this discussion of losses from small cylinders is of great importance in connection with electrical insulation on wires, it is of little practical importance in connection with heat insulation. However, the matter has been covered fully in order to show just what determines the upper limit of diameters on which these peculiar effects will be found.

Conductivity — A Function of Temperature

34 Conductivity has already been defined at the outset of this paper, and was there shown to be a specific property, not dependent upon the area, shape or thickness of the material. It has long been recognized that the conductivities of insulating materials are higher at the higher temperatures. It is reasonable that this should be so; because, due to the porosity of their structure, heat is transferred within these small spaces by radiation, convection and conduction, in addition to that conducted by the solid particles in

[1] If the covering material is concrete ($k = 6.258$ B.t.u. per sq. ft., per deg. temp. diff. per 1 in. thickness per hour) applied on a 3-in. pipe ($r_1 = 1.75$ in.) and $c = 2.0$, the loss will be a maximum when $r_2 = 3.12$ in., or after a thickness of 1.37 in. of concrete has been applied.

actual contact, and both radiation and convection increase more rapidly than the first power of the temperature.

35 It is well-known that insulating materials owe their insulating value to the porosity referred to above, and often it is said that the air spaces are responsible for the low conductivity. That is true as far as it goes, but a more complete statement of the situation would be that it is the multitude of surface resistances at the boundaries of the air spaces which give the material its resistance to heat flow. Because of the close proximity of the warmer and cooler walls of these spaces, the magnitude of each of these surface resistances is naturally small as compared with the resistance at an outer surface, yet because of their multitudinous number the sum of all of these small resistances may result in a total resistance of relatively high magnitude. The solid material between the spaces may be relatively a good conductor of heat, yet if the physical structure of the particles be such as to provide a sufficient number of spaces with their corresponding surface resistances, the resulting product may be a very good insulation. This is illustrated by the fact that the conductivity of magnesium carbonate in the form in which it is used as an insulation is about one-thirtieth of that of the same material in solid form. An even more striking example is the case of carbon, which in the form of graphite has a conductivity of more than two hundred times that of carbon in the form of lampblack.

36 The effect of temperature on conductivity has been taken into account by different investigators in various ways. The four most common methods of expressing conductivity are as follows:

a As a function of temperature difference between surfaces

b As a function of the temperature of the warmer surface

c As a function of temperature difference between warmer surface and room temperature

d As a function of mean temperature.

37 Since conductivity is a function of temperature and not of temperature difference, method *a* is subject to serious errors. The conductivity of a material with one surface at 100 deg. fahr., and the other at 200 deg. fahr., will be much lower than the conductivity of the same material with one surface at 500 deg. fahr., and the other at 400 deg. fahr., even though the temperature difference in each case is the same.

38 Method *b* is subject to objections of much the same character; because, while the temperature of one face is fixed, varying the temperature of the other face would cause wide variations in conductivity.

39 Method *c* has been found to be reasonably satisfactory so long as transfer through a single material only was involved; because, since one end of the temperature difference is the room temperature, which varies over only a relatively small range, the

mean temperature is fixed between comparatively close limits. However, it is not fixed absolutely, and a further objection is that, in the case of heat transfer through two or more materials in series, this method furnishes no means of evaluating the conductivity of the inner layer or layers.

40 Therefore, since conductivity is a function of temperature, the correct basis of expressing its value is in terms of conductivity at the mean temperature of the material (method *d*). This mean temperature is the arithmetical mean of the temperatures of the two surfaces and is not the temperature at the physical center of the material.

41 Carl Hering,[1] and L. L. Barrett,[2] have shown that where the curve of conductivity with respect to temperature is a straight line, the average conductivity for the entire thickness of material under consideration is equal to the true conductivity at the arithmetical mean of the two surface temperatures. The author has shown[3] that this relationship applies with a satisfactory degree of accuracy to conductivity curves of considerable curvature, as well as to straight lines. The average conductivity is equal to the true conductivity at the temperature

$$T = \sqrt[n]{\frac{T_2^{\,n+1} - T_2^{\,n+1}}{(n+1)(T_1 - T_2)}} \quad . \; . \; . \; . \; . \; . \; . \quad [11]$$

but the arithmetical mean of the surface temperatures closely approximates the value of this expression for a considerable range of values of *n*. See Appendix 3 for proof of this proposition. This proof also shows that the relationship holds for either flat or curved surfaces.

42 Conductivities expressed in this way may be used in computations involving a given layer of material regardless of whether it is used alone or in combination with other layers of material. All that is necessary is that the mean temperature of the layer of material be known. In the past it has usually been necessary to arrive at this mean temperature by the "cut and try" method which was quite tedious. However, in his own work the author has used for several years past a method whereby the mean temperature of each layer of material in combinations of two materials may be determined graphically with a highly satisfactory degree of accuracy. This method makes the computations for combinations of two materials extremely simple and may be extended to minimize the "cut and try" required for any number of materials. However, usually there are not more than two materials of major insulating value in a given construction.

[1] Trans. American Electrochemical Society, vol. 21, p. 520.
[2] Trans. A.S.M.E., vol. 44, p. 315.
[3] *Mechanical Engineering,* Oct., 1924, p. 603.

43 The method is illustrated in Fig. 7, which is based on the conductivities of Superex[1] and 85 per cent Magnesia, shown in Fig. 8. Briefly, the chart is based on plotting the temperature gradients through the two materials, using a scale of thickness for

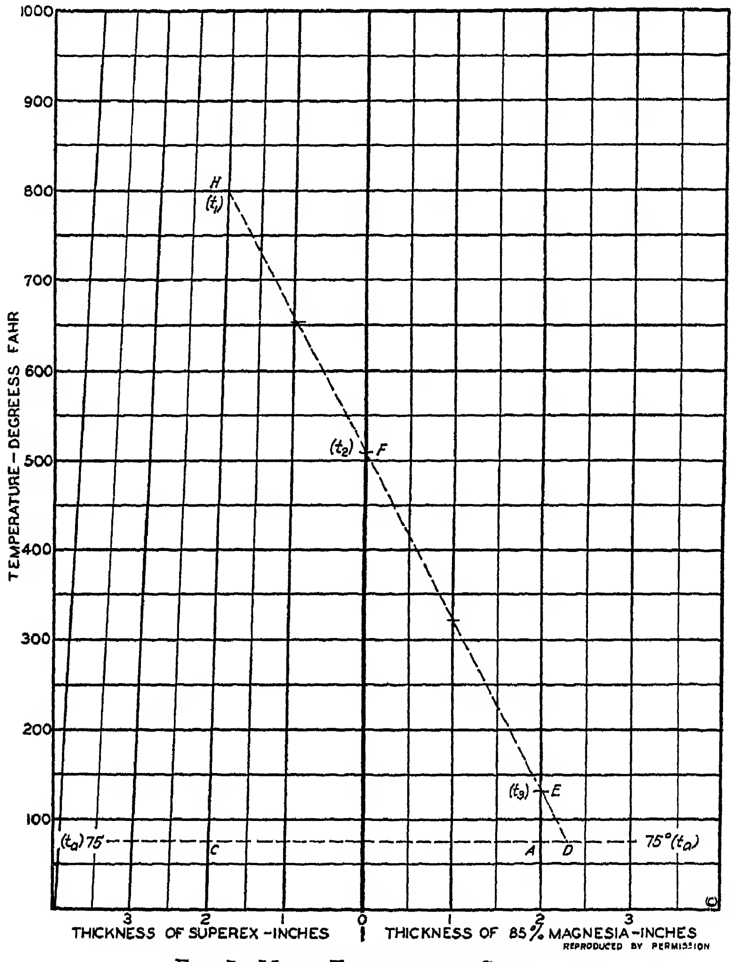


FIG. 7 MEAN TEMPERATURE CHART

one of the materials sufficiently different from that of the other so that, when plotted, the temperature of the warmer surface of the first material, the temperature of the surface of contact between the two materials and the temperature of the cooler surface of the

[1] A recent development in high temperature insulating material for temperatures from 500 to 1500 deg. fahr.

second material will all lie on the same straight line. Obviously if the conductivities of the materials are different, these points will not lie on a single straight line when plotted to the same scale of thickness. However, if the scales of thickness for the two materials be made in inverse proportion to their conductivities (scale for first material equals k_2/k_1 times the scale for second material) the three points will lie on the same straight line.

44 The construction of this chart is illustrated in Fig. 9, where the actual temperature gradients plotted to a uniform scale of thickness are indicated by the solid curved lines. It has already been shown that the average conductivity for each layer is the conductivity at the arithmetical mean of the temperatures of the two surfaces, and is not the conductivity at the physical center of

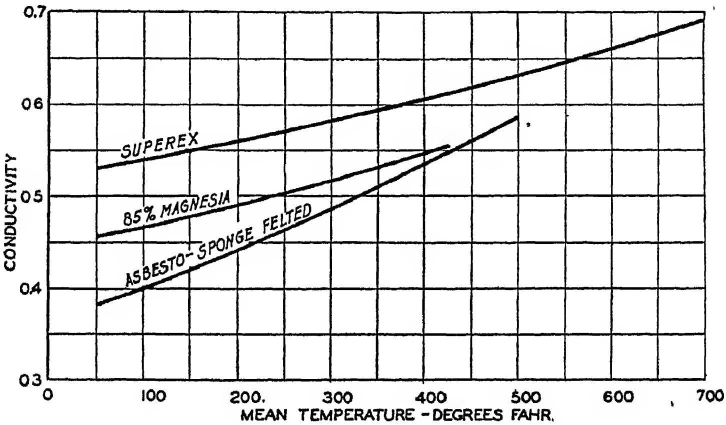


FIG. 8 CONDUCTIVITIES OF ASBESTO SPONGE FELTED, 85 PER CENT MAGNESIA AND SUPEREX

the material. Therefore, the actual shapes of the gradients between *EF* and *FG* are mainly of academic interest, and the locations of the points *E, F* and *G* are the vital considerations in the solution. The dotted line *CH* represents the thickness of the first material equal to *OA*, but plotted to a scale which will cause the three controlling temperatures to fall on a straight line.

45 The scale of thickness for the second material is laid out uniformly on the right of a base line *OO*, which represents the dividing line between the two materials. The scale for the first material along a horizontal line representing room temperatures is laid out to the left of the base line, using a scale k_2/k_1 times that used for the first material, $OC = OA \times \frac{k_2}{k_1}$. However, this is the only line on which the mean temperatures for both materials are the same; therefore, to determine the proper scale at higher

temperatures, the relative mean temperatures for several values of t_1 must be calculated and points located as at H. The distance OH and OA are inversely proportional to the conductivities of the respective materials at their mean temperatures.

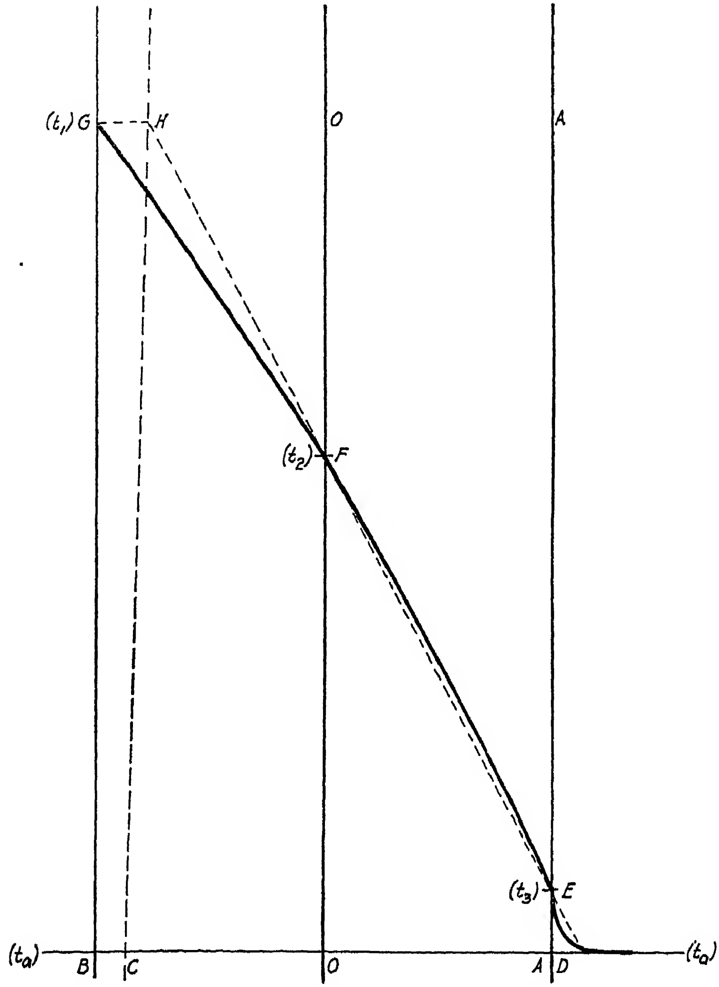


FIG. 9 TEMPERATURE GRADIENTS THROUGH SUPEREX AND 85 PER CENT MAGNESIA

46 It is evident that for combinations of thicknesses other than those for which the point H was located, the mean temperatures of the respective layers, and consequently the conductivities, will not be the same. However, in a given case, both temperatures will be higher, or both will be lower, so that variations are not

cumulative and in actual use, errors due to the slight variations in these actual values are surprisingly small. For example, varying the proportions of the materials 100 per cent from those on which the chart is based, results in a difference of less than 0.5 per cent between actual values of conductivities and those at the mean temperatures as determined from the chart.

47 In order to use the chart, Fig. 7, for two materials and one surface resistance, R_s, locate point H at the point on the chart representing the thickness of the first material and temperature at its warmer face. Locate point A at the point representing the thickness of the second material and room temperature. Lay off distance $AD = R_s k_2$. Draw the straight line HD. The intersection of this line with the base line OO gives the temperature at the boundary between the two materials. The intersection with the line representing the outer surface of the second material gives the temperature of this surface.

48 Where more than two materials are involved, each material, other than the two for which the chart is laid out, is provided for in the solution in the same manner as was illustrated above for surface resistance. The chart applies equally well to insulation on pipes and on flat surfaces. The only difference is that in the case of pipe insulation, "equivalent thicknesses" (values of $r_s \log_e \frac{r_2}{r_1}$, etc.) are used instead of actual thicknesses.

49 The slope of the line HD is a function of rate of heat transfer from the outer surface to air. Since surface resistance is a variable which may, for a given set of conditions, be expressed as a function of the same rate of heat transfer, this relationship forms the basis for a method which long use has demonstrated to be highly convenient for eliminating the "cut and try" operation in connection with surface resistance. This method consists of placing the base line of a protractor on the line HD and reading the surface resistance directly from suitable scales inscribed on the arc of the protractor.

50 Still another convenient short cut in the use of the chart is based on the fact that the mean temperatures of the two layers may be read directly from the chart at the mid-points of the lines EF and FG respectively. Consequently, the use of proportional dividers set in the ratio of 2 to 1 makes it possible to obtain the required mean temperatures directly and instantly, without the necessity of reading and averaging the respective surface temperatures.

51 The straight line HD on the chart, Fig. 7, illustrates the method of determining the temperature at the respective surfaces of 2 in. thick Superex and 2 in. thick 85 per cent Magnesia, where the temperature of the warmer surface is 800 deg. fahr., and the room temperature is 75 deg. fahr. The distance AD ($= R_s k_2$)

may, for a given combination of materials and for still air conditions, be taken as a constant without introducing appreciable error. For the combination of materials to which Fig. 7 applies this constant is 0.3 in. of the second material.

52 If for any particular solution, the results obtained from such a chart are not considered sufficiently accurate, these results may be used as the first trial solution, and the final solution may then be accomplished with any desired degree of accuracy. It will

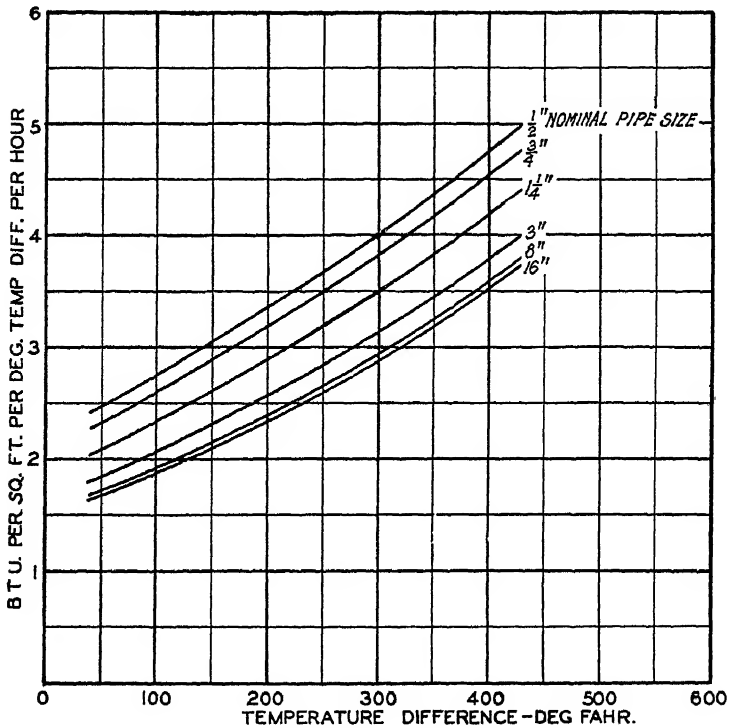


Fig. 10 Bare Pipe Losses — Paulding

generally be found, however, that the corrections required in this final solution are extremely small, so small, in fact, as to be rarely of practical importance.

53 Concluding this phase of the discussion, it may be said that there is no urgent need for an extended program of research devoted to further determination of conductivities of commercial insulating materials in the moderate and high temperature field. However, it cannot be too strongly urged that, in such results as are presented in the future, the mean temperature of the determination should be stated, and above all, the value reported as con-

ductivity should actually be conductivity and not some odd combination of true conductivity with other, and extraneous, effects.

Surface Effects

54 It has already been pointed out that the investigation of surface resistance offers a most fruitful field for further research. This does not mean that information on the subject is wholly lacking. In fact, sufficient data are available for the solution of most

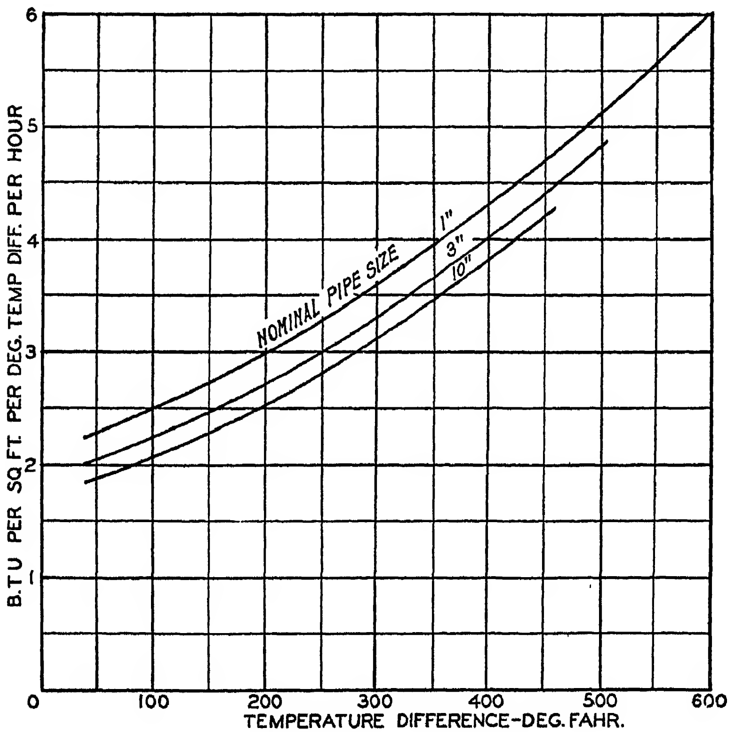


Fig. 11 Bare Pipe Losses — Heilman

insulation problems with a highly satisfactory degree of accuracy. However, the deficiency of the work in this field (and the author makes no exception of his own earlier published results in this connection) is that results presented are overall coefficients, which make no provision for the separate effects of radiation and convection. One of these effects, radiation, is a function of the difference of the fourth powers of the absolute temperatures, while the other, convection, is generally considered to be a function of temperature difference. Therefore, while the results may be

applied with assurance of accuracy to conditions closely approximating those under which they were obtained, they may not be extended very far beyond the range of actual experimentation.

55 The important effects of absolute temperature and of air velocity are too often neglected in this connection. It is generally conceded that the rates of heat transfer from cylindrical surfaces of small diameter, under still air conditions, are greater than from those of larger diameter. Paulding's[1] deductions, Fig. 10, indicated a rather wide variation in this regard. Actual experiments by

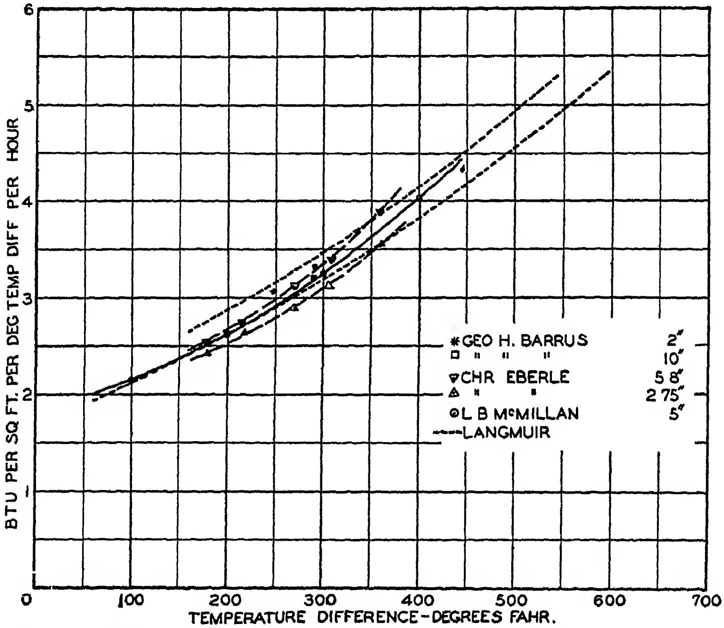


FIG. 12 RATE OF HEAT TRANSFER FROM METAL SURFACE TO AIR — BARRUS, EBERLE, McMILLAN AND LANGMUIR

Heilman,[2] Fig. 11, showed a much smaller range of variation and the composite chart,[3] Fig. 12, shows a close grouping of results for various diameters. In fact, it is apparent in all three charts that, except for the very small pipe sizes, smaller than 3 in., the differences due to pipe size are not large.

[1] Practical Laws and Data on the Condensation of Steam in Covered and Bare Pipes. Published by D. Van Nostrand Co., New York, 1904.

[2] Trans. A.S.M.E., vol. 44, p. 301.

[3] G. H. Barrus, Trans. A.S.M.E., vol. 23, p. 791.

Chr. Eberle, Mit. über Forschungsarbeiten, Verein Deutscher Ing., heft 78.

L. B. McMillan, Trans. A.S.M.E., vol. 37, p. 941.

Langmuir, calculated from Equations [13] and [14].

56 An interesting feature of Eberle's tests, shown in Fig. 12, is that the pipe of smaller diameter is shown to have the lower rates of loss. However, the air temperature, and consequently the absolute temperatures of both the pipe and its surroundings, were higher in the series of tests on the larger pipe. This undoubtedly accounts in a large measure for the peculiarity of the results.

57 As will be shown later, a relatively low air velocity will effect a relatively large increase in heat transfer from a surface. It is evident, therefore, that the effects of absolute temperature and of air velocity may completely overshadow the effect of pipe size. All results on bare surface losses are to be considered as

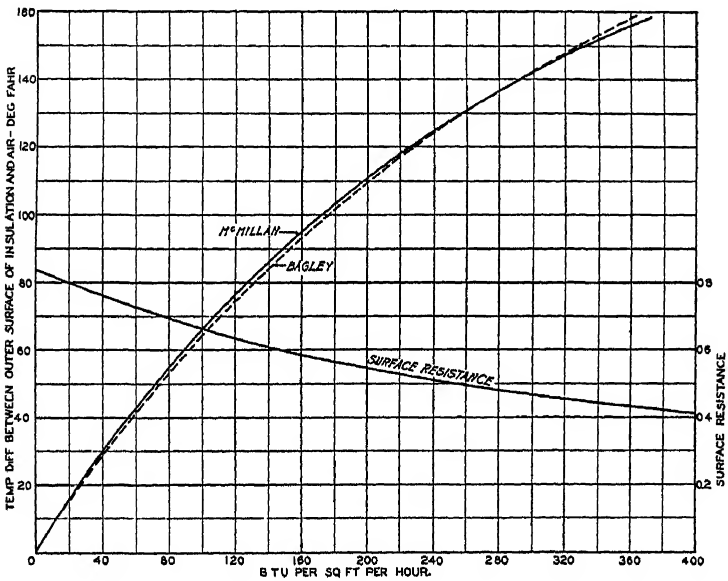


FIG. 13 RATES OF HEAT TRANSFER FROM CANVAS-COVERED INSULATION SURFACES TO AIR

fairly convenient approximations which will serve until more scientifically complete data are available.

58 The existing information on rates of heat transfer from insulation surfaces to surrounding air is in much the same state as that outlined above for bare surfaces. Fig. 13 shows results of experiments by the author[1] and by Bagley.[2] For the relationship between temperature difference, surface to air, $t_1 - t_a$, diameter D,

[1] Trans. A.S.M.E., vol 37, p. 956.
[2] Trans. A.S.M.E., vol. 40, p. 675.

and rate of heat transfer from surface of insulation to surrounding air, h, Heilman[1] gives the equation

$$t_s - t_a = \frac{272.5h}{h + \dfrac{564}{D^{0.19}}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [12]$$

However, this equation is admittedly empirical and may not be used for diameters much beyond the range of his experiments. Where the surface resistance is relatively a small part of the total resistance, as it always is in the case of effectively insulated surfaces, the use of any one of these three methods permits the calculation of total rate of heat transfer to an accuracy of about 1 to 2 per cent, which is as exact as the conductivity values themselves, but even at that the use of these approximations must sooner or later give way to the use of more scientifically accurate data. At that time, charts very much like those now in use will be required for specific conditions, but that will be in the nature of particularizing from the general instead of generalizing from the particular, which is the tendency of present methods.

59 *Effect of Air Velocity on Heat Transfer from Surface to Air.* The principal need for more complete data on surface losses is that, until these are accurately expressed in terms of their component parts, *radiation* and *convection*, and until the effect of air velocity on the latter has been accurately determined, no satisfactory statement of the effect of air velocity on heat losses from surfaces may be obtained. The work of Langmuir[2] is outstanding in this connection. For radiation he uses the Stefan-Boltzmann equation, which, in engineering units, may be expressed as

$$W_R = 0.178E\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \quad . \; . \; . \; . \quad [13]$$

and for convection, he has developed the equation, which for still air under average conditions may be expressed as

$$W_C = 0.296(T_1 - T_2)^{\frac{5}{4}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [14]$$

The total rate of heat transfer is represented by the sum of these. He shows that convection is increased by air circulation, according to the equation

$$W_{CV} = W_C\sqrt{\frac{V + 68.9}{68.9}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [15]$$

The increases in heat transfer due to air velocity calculated from these equations, at various surface temperatures, and with an air temperature of 80 deg. fahr., are shown in Fig. 14. The actual rates of heat transfer, under the same conditions, are shown in Fig. 15.

[1] Trans. A.S.M.E., vol. 44, p. 308.
[2] Trans. American Electrochemical Society, vol. 23.

60 In the above equation, W_R and W_C represent, respectively, the rates of heat transfer by radiation and by convection under still air conditions. W_{CV} represents rate of heat transfer by con-

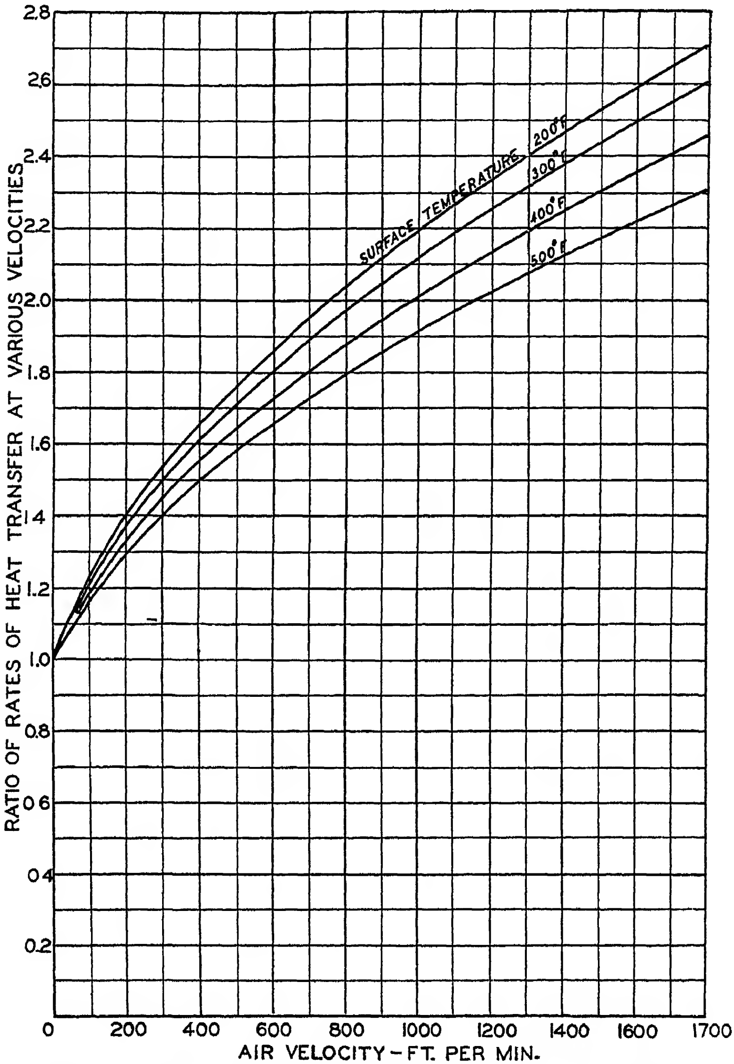


FIG. 14 RELATIVE RATES OF HEAT TRANSFER AT VARIOUS AIR VELOCITIES, BASED ON LANGMUIR'S EQUATIONS

(Rate of heat transfer under still-air conditions considered as unity.)

vection at any air velocity. All are expressed in B.t.u. per square foot per hour. E is the emissivity coefficient which for black body conditions is unity. T_1 and T_a represent the absolute temperatures

of the hot surface and of air, respectively, and T_2 represents the absolute temperature of surrounding objects to which heat is radiating. V represents the velocity of air flowing over the surface, expressed in feet per minute.

61 While Langmuir's analysis of the problem is very convincing, it is probable that the effect of air velocity is greater than

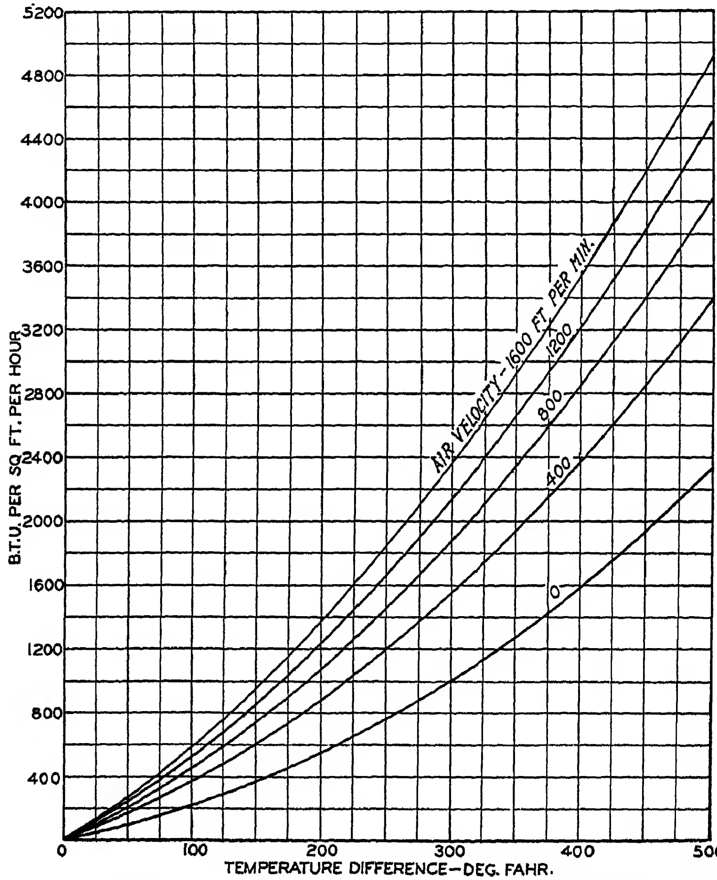


Fig. 15 Effect of Air Velocity on Rates of Heat Transfer from Surfaces of Various Temperatures

(Air temperatures 80 deg. fahr.)

he has shown. See Fig. 16. Many tests by other investigators have shown increases more than double those given by Langmuir's equations. The results given by T. S. Taylor are of particular interest, since he shows that the angle of incidence of the air stream on the surface has a marked effect on the rate of heat transfer. It is not difficult to ascribe highly probable reasons

for many of the most glaring disagreements in the various results from various sources. The differences between effects of stream line and turbulent flow and the failure to define, with reference to the surface, the point at which velocity was measured are outstanding possibilities in this connection.

.62 *Effect of Air Velocity on Heat Transfer Through Insulation.* In the case of well-insulated surfaces the increases in heat losses due to air velocity are very small as compared to the increases just shown for bare surfaces. This is due to the fact that air flowing over the surface of the insulation can increase only the rate of heat transfer from surface to air and cannot change the internal resistance to heat flow inherent in the insulation itself. The effect

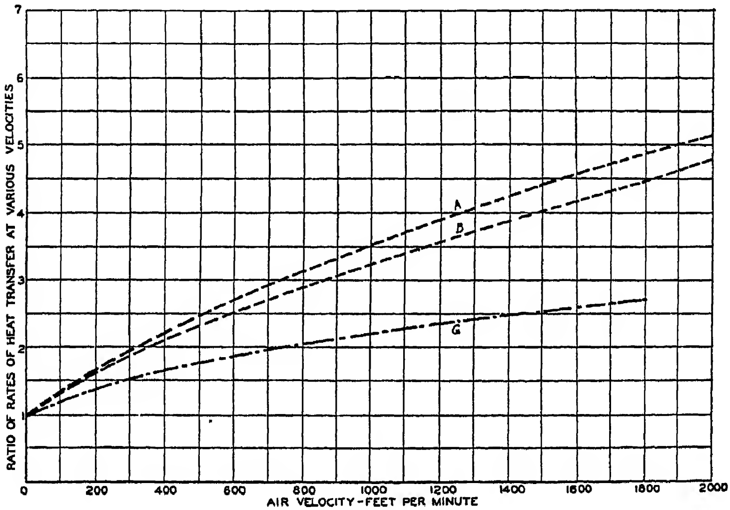


FIG. 16 RELATIVE RATIO OF HEAT TRANSFER AT VARIOUS AIR VELOCITIES AS SHOWN BY DIFFERENT INVESTIGATORS

(Rate of heat transfer under still-air conditions considered as unity.)[1]

of the air circulation, therefore, is to cool the surface of the insulation to a temperature lower than it would have under still air conditions, thereby increasing the temperature drop through the insulation. This is illustrated in Fig. 17 by the decrease in temperature from t_2 to t'_2. Since the heat transfer in a given case is proportional to the temperature gradient, it is obvious that the heat

[1] A T. S. Taylor, Trans. A.S.M.E., vol. 42, p. 243 (surface temperature, 113 deg. fahr.).
B T. S. Taylor, Trans. A.S.M.E., vol. 42, p. 243 (surface temperature, 131 deg. fahr.).
G Langmuir (Equations [13], [14] and [15]). (Surface temperature, 200 deg. fahr.; air temperature, 80 deg. fahr.)

flow will be greater when the temperature difference is $t_1 - t'_2$ than when it is $t_1 - t_2$.

63 In the case of surfaces located out of doors, the combined effect of wind and rain may bring the surface temperature of the insulation practically down to the air temperature, yet even in

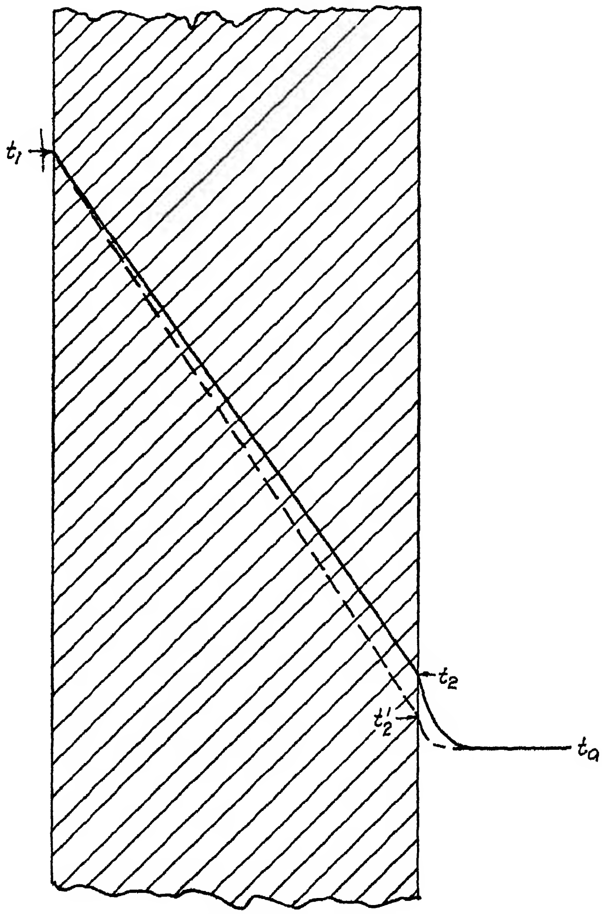


FIG. 17 TEMPERATURE GRADIENTS THROUGH INSULATION

(The dotted curve represents temperature gradient when surface of insulation is subjected to air circulation.)

this extreme case the increase in heat loss through the insulation is not as great as might be expected. This is illustrated by the following example, based on a flat surface insulated with 2 in. thick material, having a conductivity of 0.5 B.t.u. per sq. ft., per deg. temperature difference per 1-in. thickness, per hr., and a rate of heat transfer from its surface to air under still-air condi-

tions of 1.8 B.t.u. per sq. ft. per deg. temperature difference per hr.[1]

$$\text{Internal resistance of insulation} = \frac{2}{0.5} = 4.0$$

$$\text{Surface resistance} = \frac{1}{1.8} = \qquad 0.556$$

$$\text{Total resistance} = \qquad 4.556$$

$$\text{Rate of heat transfer} = \frac{1}{4.556} = 0.22 \text{ B.t.u. per sq. ft.}$$

per deg. temperature difference per hr.

64 If the surface resistance were completely eliminated, due to the cooling action of wind and rain, the internal resistance of 4.0 would still remain, and the rate of heat transfer would be $1/4.0 = 0.25$ B.t.u. per sq. ft. per deg. temperature difference per hr. Therefore, the maximum increase in loss due to wind and rain would be

$$\frac{0.25-0.22}{0.22} = 13.6 \text{ per cent.}$$

65 In like manner, it may be shown that the maximum increase for 1-in. thickness of the same material under the same conditions is 27.9 per cent and, in the case of 3-in. thickness, 9.2 per cent. It is therefore apparent that the thicker or the more efficient an insulation is, the less its rate of heat transfer will be affected by air circulation.

66 Fig. 18 shows graphically the relative increases in rates of heat transfer, due to air circulation, in the case of a bare surface maintained at 400 deg. fahr., and the same surface insulated with 1-in. and 2-in. thickness of an insulation with a conductivity of 0.48 B.t.u. per sq. ft., per deg. temperature difference per 1-in. thickness, per hour. In these curves, the effect of air velocity on rate of heat transfer from surface to air is based on Langmuir's equations.

67 All of the above discussion as to effect of air circulation on losses through insulation is based on flow of air over the surface of the insulation, and applies to cases where the insulation is tightly sealed. If the condition of the insulation is such that the air may circulate through cracks and crevices in the insulation, the increases may be far greater than those given above. Therefore, it is essential that all insulation be sealed as tightly as possible; and this is most particularly true of insulation located out of doors.

68 *Surface Temperature No Satisfactory Measure of Heat Transfer*. The lack of a simple means to measure approximately

[1] It should be noted that the factor 1.8 B.t.u. is not a constant for all cases, but is one of the stated conditions for this particular example.

the amount of heat loss from surfaces has naturally led to a widespread feeling that the degree of effectiveness of insulation may be estimated by the surface temperature. Probably no other conception in connection with heat transfer is so generally misused as this one. Surface temperature considered alone, without reference to temperature of surroundings, is *absolutely no measure of the rate of heat transfer*. A surface at 150 deg. fahr. in a confined space exposed to air at 150 deg. fahr. may be losing no heat at all, whereas a surface at 100 deg. fahr. exposed to air at a temperture of 50 deg. fahr. is losing heat in very considerable quantities.

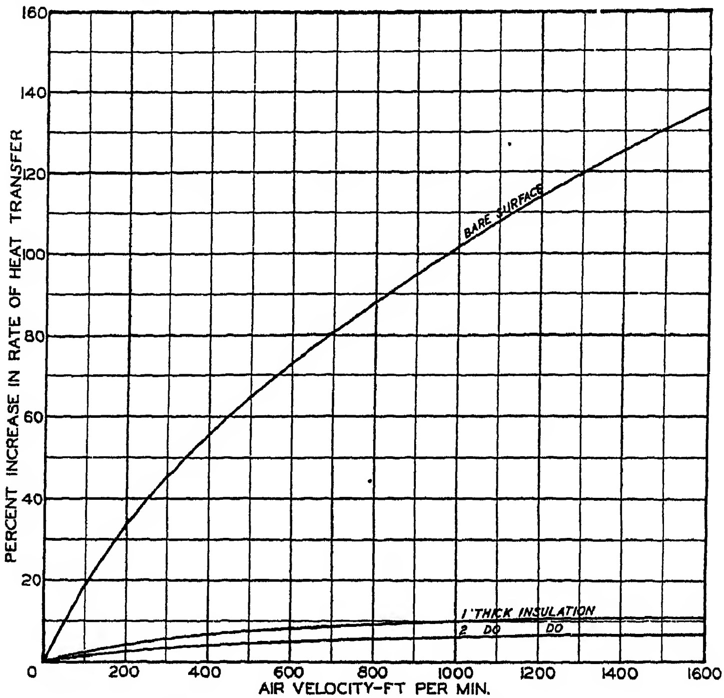


FIG. 18 INCREASE IN HEAT LOSSES DUE TO AIR CIRCULATION

69 Measuring the difference between the surface temperature and the air temperature is a little better, but not much, unless all other conditions are identical — which they rarely are. It is easily possible for the losses to vary over a range of several hundred per cent at the same temperature difference, depending on the air velocity to which the surface may be exposed. Nor is it necessary for that velocity to be very high in order to render either surface temperature or temperature difference, surface to air, entirely valueless as an indication of heat loss.

70 Air circulation over the surface of insulation increases the rate of heat transfer from the surface and, as illustrated in Fig. 17,

this tends to cool its surface and increase the total rate of heat transfer. Therefore, *with lower surface temperature, more heat is being transmitted.* This fact is illustrated more forcibly in Fig. 14, where it may be seen that the rate of heat transfer from a surface 100 deg. fahr. above air temperature and exposed to air circulating at a velocity of 400 ft. per min. (approximately 4.5 miles per hour)

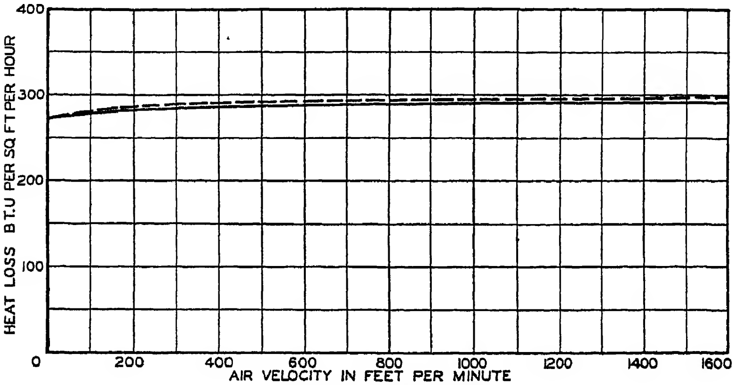


FIG 19 A HEAT LOSSES

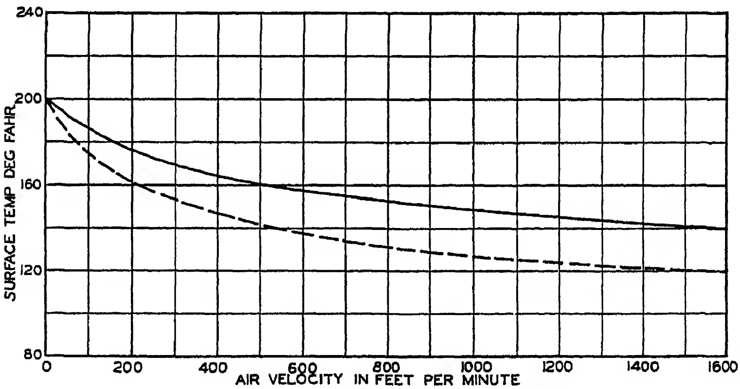


FIG. 19 B SURFACE TEMPERATURES

Fig. 19 Effect of Air Velocity on Heat Losses and on Surface Temperatures of Insulation

is greater than for a surface 150 deg. fahr. above air temperature but exposed only to still air.

71 Reference to Fig. 19 illustrates still more clearly why surface temperatures do not give a reliable measure of heat transfer through insulation. The solid-line curves are based on the factors given in Fig. 14 while the dotted-line curves illustrate conditions which would exist if the increases in surface transmission rate due to air circulation were of the order of the higher values shown in

Fig. 16. It is apparent that the higher rates of increase of surface losses would have little effect on the total rate of heat transfer, while surface temperatures would be still further reduced.

72 Air circulation is not the only cause of unreliability of surface temperature as a measure of heat loss. Exposure of the surface to other nearby hot or cold surfaces, and the nature of the surface itself, may have almost if not quite as noticeable effects on surface temperatures. A bright, polished surface may be losing heat at a rate not much over half as great as that from a dull surface and yet have a higher surface temperature.

73 While these variables will have relatively little effect on the total heat transmitted by the insulation, they may have entirely disproportionate effects on surface temperatures.

Economic Data

74 Perhaps the most valuable use for accurate data on heat transfer in connection with insulation is in the calculation of the thickness of insulation required under various conditions for most economical results. There have been many contributions to the literature on this subject, but most of the methods presented have been graphical. A notable exception is P. Nicholls' contribution[1] on the Economic Thickness of Insulation in the Refrigerating Field. However, long before that paper was presented, the author had been using in his own work a rational analytical method for insulation on flat surfaces, and has just recently extended this to apply to pipe surfaces.

75 Referring to Fig. 20, as the thickness of insulation is increased the cost of heat lost per year (m) is decreased, but the cost per year of insulation (n, first cost multiplied by per cent fixed charges) is increased. Therefore, the thickness at which the sum of these two costs is a minimum is obviously the most economical.

76 For flat surfaces this may be determined from the equation

$$x=\sqrt{\frac{ak}{b}}-Rk \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [16]$$

in which x is the most economical thickness, k is the conductivity, b is the cost of insulation *per inch thickness per year*, R is the sum of the resistances of all other elements in the construction, including surface resistance, and

$$a=\frac{Y(t_0-t_a)M}{1{,}000{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [17]$$

in which Y is hours operation per year, t_0 is inside temperature, t_a is temperature of surrounding air, and M is the value of heat

[1] *Refrigerating Engineering*, Nov. 1922, p. 152.

in dollars per 1,000,000 B.t.u. In Table 1 are given values of a for various temperature differences, various values of heat and 8760 hours per year. See Appendix 4 for derivation of Equation [**16**].

TABLE 1 VALUES OF a

Temp. diff., deg. fahr.	Value of heat in dollars per million available B.t.u.									
	0.10	0 20	0.30	0 40	0.50	0.60	0.70	0.80	0.90	1 00
100	0.088	0.175	0.263	0.350	0.438	0.526	0.613	0.701	0.788	0 876
200	0.175	0.350	0.526	0.701	0.876	1.051	1.226	1.402	1.577	1 752
300	0.263	0.526	0.788	1.051	1.314	1.577	1.840	2.102	2.365	2.628
400	0.350	0 701	1.051	1.402	1.752	2 102	2.453	2.803	3.154	3 504
500	0.438	0.876	1.314	1.752	2.190	2.628	3 066	3.504	3.942	4.380
600	0.526	1.051	1.577	2.102	2.628	3 154	3.679	4.205	4.730	5.256
700	0.613	1 226	1.840	2.453	3 066	3.679	4.292	4.906	5.519	6.132
800	0.701	1.402	2.102	2 803	3 504	4.205	4.906	5.606	6.307	7.008
900	0.788	1.577	2.365	3.154	3.942	4.730	5.519	6.307	7.096	7.884
1000	0.876	1.752	2.628	3.504	4.380	5 256	6.132	7.008	7.884	8.760

77 Equation [**16**] applies to combinations of any number of

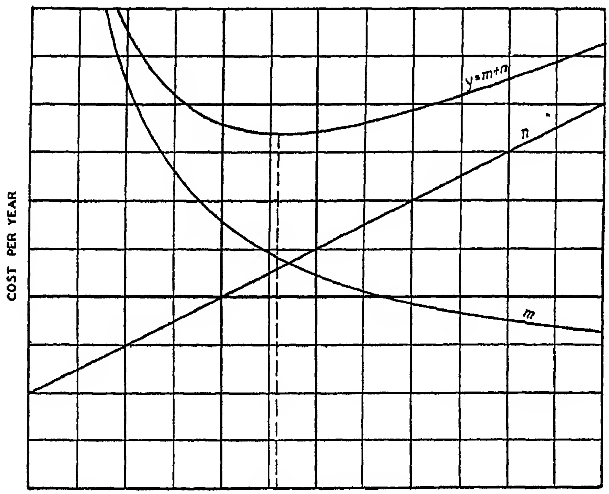


FIG. 20 ECONOMICAL THICKNESS OF INSULATION

materials, the resistance of all but one being provided for in the term R. The physical explanation of the situation is that the value of the term $\sqrt{\frac{ak}{b}}$ gives the thickness of the one material which would be required if there were no other insulating value in the construction than that offered by the material itself. If only one material is involved, the only deduction required is that for surface resistance. The term $R_s k$ is then the thickness of material with conductivity k which has the same resistance as the surface resistance. In like manner, deductions are made for the insulating

values of other elements already present in the construction. If it is desired to increase the thickness of the first layer for some reason, as for example to reduce the temperature to which the second layer is subjected, the solution is very simple. The value of the term $\sqrt{\frac{ak}{b}}$ does not change, but the value of R is increased and the thickness of the outer layer correspondingly decreased.

78 In the case of pipe surfaces, the equation for economical thickness is not quite so simple, yet it is by no means as formidable as it appears at first sight. For one material the cost of which may be expressed by the equation,[1]

$$\text{Cost per linear foot} = \frac{2\pi r_2 b}{12}(r_2 - r_1) + C \quad . \quad . \quad . \quad . \quad [18]$$

$$\left(r_2 \log_e \frac{r_2}{r_1} + R_s k\right)\sqrt{\frac{2r_2 - r_1}{r_2 - R_s k}} = \sqrt{\frac{ak}{b}} \quad . \quad . \quad . \quad [19]$$

All terms in these equations have previously been defined. The derivation of Equation [19] is given in Appendix 5.

79 Recognizing that the first term within the parenthesis is equivalent thickness the close similarity to Equation [16] is at once apparent, since Equation [16] may be written for one material

$$x + R_s k = \sqrt{\frac{ak}{b}}$$

80 Before proceeding with the solution of Equation [19], the equation for economical thickness of the outer layer of insulation over one or more layers of materials having different conductivities will be written, since the same charts may be used for the solution of equations involving all such combinations. This equation, given below, is written for a combination of two materials, but for more than two, the equation is of exactly the same form. The only difference will be the addition of other terms like the second term inside the brackets and the substitution of appropriate values of r where r_s appears in the equation.

$$\left[r_s \log_e \frac{r_s}{r_2} + \frac{r_s}{r_1}\left(\frac{r_1 \log_e \frac{r_2}{r_1}}{k_1}\right)k + R_s k\right]\sqrt{\frac{2r_s - r_2}{r_s - R_s k}} = \sqrt{\frac{ak}{b}} \quad [20]$$

In this equation all terms have been previously defined. Its derivation is given in Appendix 6.

[1] The so-called "Standard List Prices" of sectional pipe insulation are in reasonably close agreement with Equation [18] for thicknesses greater than 1 in. Prices of cork pipe covering follow no regular law. Therefore these Equations, [18], [19] and [20], do not apply to cork.

81 The solution of Equation [20] for 16-in. pipe is illustrated in Fig. 21. a, k and b are established by the conditions of the problem. Knowing the value of $\sqrt{\frac{ak}{b}}$ the economical thickness may be read directly from the chart. For example if $\sqrt{\frac{ak}{b}} = 4.0$, and if no surface resistance is to be considered, the economical thickness is 3.03 in. If $R_s k = 0.3$ in. the economical thickness is

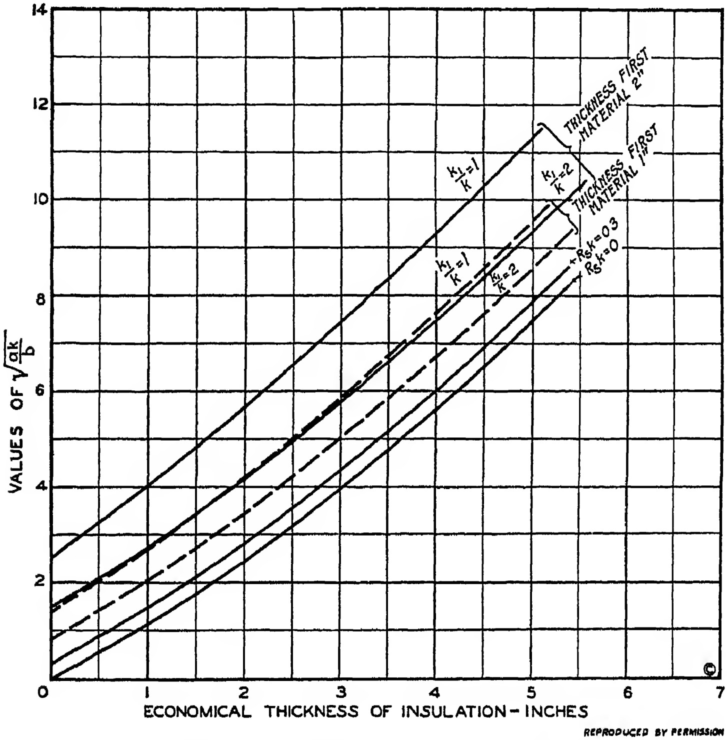


FIG. 21 ECONOMICAL THICKNESS OF INSULATION ON CYLINDRICAL SURFACE 16 IN. DIAMETER

2.80 in. If there is a first layer of material 1 in. thick, and if the conductivity of that layer is 2.0 times that of the second layer (and $R_s k = 0.3$), the economical thickness of the second layer is 2.38 in. For intermediate values interpolations may be made. Naturally the thickness chosen would be the commercially available thickness nearest the thickness found on the chart.

82 The solution is graphical, it is true, and it may be asked why not then make a graphical solution in the first place by plotting the sum of the losses per year and costs per year for a number of

thicknesses and taking the low point as the economical thickness. The answer is that each solution by that or other equivalent methods requires a number of calculations, the plotting of a curve and the location of the minimum point or the point where

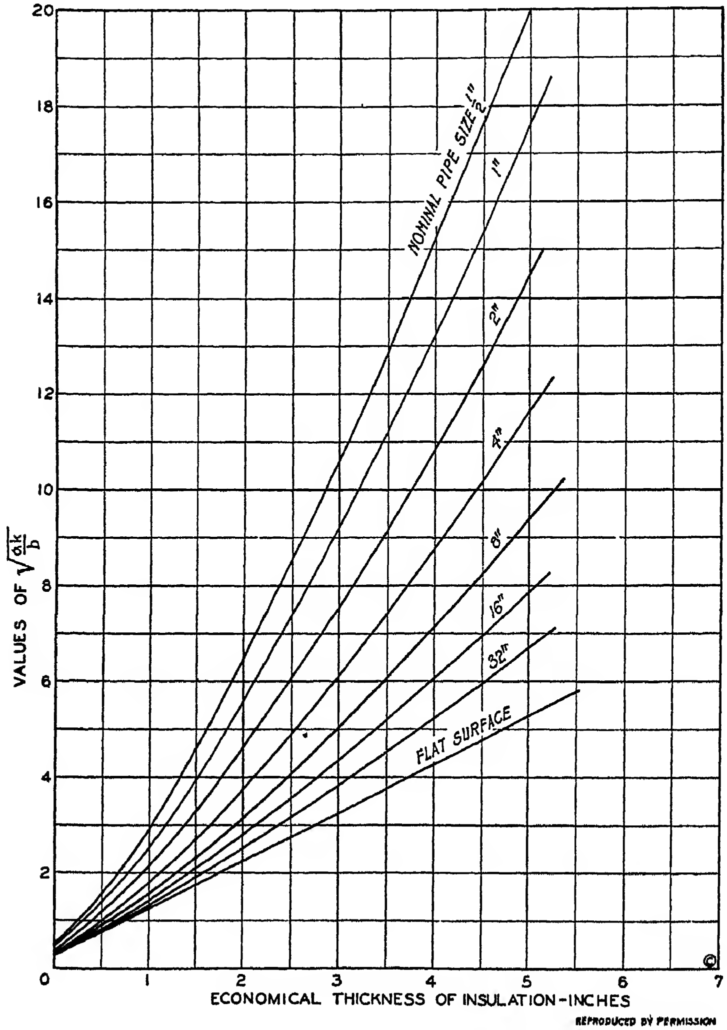


Fig. 22 Economical Thickness of Insulation on Flat and Curved Surfaces. $R_s k = 0.3$ In.

tangents are parallel, all of which is tedious, and the last step of which is likely to be highly inaccurate, while the solution described above requires but a few moments once the charts have been prepared.

83 The chart illustrated in Fig. 22 is applicable to the instant solution of most problems where a single material is involved. It is based on $R_s k = 0.3$ in., which is fairly representative of good insulating materials under average still-air conditions. Even if $R_s k$ for the given case differs considerably from the value of 0.3 in. the change in economical thickness will usually be so small as to be practically negligible However, if somewhat greater accuracy is required, correction may be made by adding to the thickness given by the chart the amount in inches by which $R_s k$ is less than 0.3 in. or subtracting from the thickness the amount by which $R_s k$ exceeds 0.3 in. Where absolute accuracy is required and where combinations of materials are involved, it is necessary to use a chart such as Fig. 21 for each pipe size.

Conclusions

84 It has been shown that the phase of heat transfer in which further research is most urgently needed is in the whole broad field of surface effects. In this connection it is not amiss to suggest that where the pure scientist misses the mark is that he probably feels an entirely justifiable satisfaction in being able to set up an equation which provides adequately for the effect of every variable, and the more complex the equation the more keen the satisfaction, but he fails to extend his analysis a step further to the point which makes the application of the equation to actual cases conveniently useful.

85 This illustrates the necessity of a meeting of the minds of scientists and engineers. Usually the engineer has neither the time nor the inclination to delve into the intricacies of the underlying phenomena to an extent which enables him to establish general laws. He is too much inclined to be satisfied with the apparent relationships of the more prominent variables and to express them in an empirical fashion. Yet he has a very clear idea of the form in which the results should be expressed in order that they be most practically useful.

86 Bringing these extreme points of view into accord would be most profitable. For example, relatively few engineers would be much interested in having to use, for the solution of every problem involving heat transfer from surfaces, equations of the type of Equations [13], [14] and [15], to say nothing of the general equation for convection of which [14] is only a special case. On the other hand, he will use and appreciate the same data presented in the form illustrated by Fig. 15. It is true that the use of the data in such form is limited to the conditions on which the charts are based, but if the limitations are clearly stated, they will generally be heeded.

87 It is highly probable that with the mass of scientific data available on various phases of the subject, the combined efforts

of a group of scientists and engineers could be depended upon to iron out existing contradictions and discrepancies and put the state of knowledge of heat transfer from surfaces to air in very satisfactory shape, with a minimum of actual laboratory work. There is an obvious need for such further investigations dealing with fundamentals. In addition to the effects of air velocity, the effects of shape, extent, position and nature of the surface on the rate of heat transfer should be investigated and the results put in a form for convenient use.

88 It has been pointed out that the true significance of conductivity is not understood by many who have contributed and are contributing to the literature. So long as authors who are classed as authorities use the term *conductivity* loosely to apply to two or more distinctly different units, what hope is there that the layman, or even the experienced engineer who has only occasional contact with heat-transfer problems, will not be confused? It is usually possible for the specialist in heat transfer to sense immediately whether the term conductivity is used in its true significance or not. If he finds that the term is used as an overall unit, including the effects of one or more conductivities and various and sundry surface, shape and thickness effects, his respect for the article in question is likely to be in inverse proportion to the number of effects other than true conductivity which are included in the so-called "conductivity" items. It is not difficult to find abundant examples in the literature illustrating the fact that if the elementary principles of heat transfer were more thoroughly understood the state of the literature would be greatly benefited thereby.

89 If this paper makes any considerable headway in clarifying the meaning and use of the term conductivity, and in emphasizing the importance of a thorough understanding of surface resistance it will have been worth the effort and more. To those who have made a lifetime study of heat transfer, the treatment of the subject in this paper will appear for the most part elementary in the extreme. The presentation is elementary, and that it is such is intentional. The literature of heat transfer is expanding rapidly and it is essential that, if the current literature is to add anything of material value to the fund of knowledge on the subject, it must be based on the sound foundation of fundamental facts. It is useless to attack the more difficult problems without a clear understanding of these basic principles.

APPENDIX NO. 1

DEFINITION OF SYMBOLS

90 The symbols used throughout the paper are defined as follows:

x = thickness of insulation
x_1 = thickness of first material
x_2 = thickness of second material
x_3 = thickness of third material
k = conductivity = B.t.u. per sq. ft., per deg. temp. diff. between surfaces per 1 in. thick, per hr.
k_1 = conductivity of first material
k_2 = conductivity of second material
k_3 = conductivity of third material
S = shape factor
t_0 = temperature of air on warmer side of insulation
t_1 = temperature of warmer surface of insulation
t_2 = temperature of cooler surface of first material
t_3 = temperature of cooler surface of second material
t_a = temperature of air on cooler side of insulation
c = rate of heat transfer from air to surface or from surface to air
c_1 = rate of heat transfer from air to surface
c_2 = rate of heat transfer from surface to air
R = resistance
R_S = surface resistance
r_1 = inner radius of insulation
r_2 = outer radius of first material
r_3 = outer radius of second material
r_x = outer radius of any layer of material
r_S = radius of outer surface of insulation
U = overall rate of heat transfer per hour
U_1 = overall rate of heat transfer per sq. ft. of pipe surface per hr.
U_2 = overall rate of heat transfer per sq. ft. of surface at radius of insulation per hr.
U_S = overall rate of heat transfer per sq. ft. of outside surface per hr.
W_R = rate of heat transfer from surface by radiation, B.t.u. per sq. ft. per hr.
W_C = rate of heat transfer from surface by convection, B.t.u. per sq. ft. per hr.
W_{CV} = rate of heat transfer from surface by convection, B.t.u. per sq. ft. per hr. at any velocity
T_1 = absolute temperature of hot surface
T_2 = absolute temperature of surrounding objects
T_a = absolute temperature of air
E = emissivity
V = air velocity, ft. per min.
Y = hours of operation per year
$a = \dfrac{Y(t_0 - t_a)M}{1{,}000{,}000}$
b = cost of insulation per sq. ft. per 1 in. thick per year
M = value of heat in dollars per million available B.t.u.

NOTE: a and b are used simply as constants in Appendix 3, and C is used in the same manner in Appendices 4, 5 and 6. Such constants a and b have no relation to values of a and b used in connection

with economic data. Neither does the constant C have any relation to the term c representing rate of heat transfer between surface and air, or the term C in connection with convection.

APPENDIX NO. 2

DERIVATION OF EQUATIONS [9] AND [10]

91 *Derivation of Equation* [9]. Expressing Equation [6] in terms of heat transfer per degree difference

$$U_2 = \frac{1}{\dfrac{r_2 \log_e \dfrac{r_2}{r_1}}{k} + \dfrac{1}{c}} \qquad U_1 = \frac{r_2}{r_1} \times U_2 = \frac{1}{\dfrac{r_1 \log_e \dfrac{r_2}{r_1}}{k} + \dfrac{r_1}{r_2} \times \dfrac{1}{c}}$$

Total resistance measured at surface of cylinder

$$R = \frac{r_1 \log_e \dfrac{r_2}{r_1}}{k} + \frac{r_1}{r_2} \times \frac{1}{c}$$

The rate of heat transfer will be a maximum when resistance is a minimum; therefore, differentiating and equating the first derivative to zero gives

$$\frac{dR}{dr_2} = \frac{\dfrac{kr_1}{r_2}}{k^2} - \frac{r_1 c}{r_2^2 c^2} = 0$$

$$\frac{dR}{dr_2} = \frac{r_1}{r_2 k} - \frac{r_1}{r_2^2 c} = 0 \qquad \text{or} \qquad \frac{r_1}{r_2 k} - \frac{r_1}{r_2^2 c}$$

$$\therefore\ r_1 r_2^2 c = r_1 r_2 k \quad \text{and} \quad r_2 = \frac{k}{c}$$

$\therefore$ R is a minimum and U_1 is a maximum when $r_2 = k/c$, or since $\dfrac{1}{c} = R_s$, when $r_2 = R_s k$.

92 *Derivation of Equation* [10]. The rate of heat transfer through the covering on a very small cylinder will be the same as that from bare surface when the total resistance is equal to the surface resistance, or when

$$\frac{r_1 \log_e \dfrac{r_2}{r_1}}{k} + \frac{r_1}{r_2} R_s = R_s \qquad \text{or} \qquad r_1 \log_e \frac{r_2}{r_1} + \frac{r_1}{r_2} R_s k = R_s k$$

$$r_1 \log_e \frac{r_2}{r_1} = R_s k \left(1 - \frac{r_1}{r_2}\right) = R_s k \left(\frac{r_2 - r_1}{r_2}\right)$$

$$r_2 r_1 \log_e \frac{r_2}{r_1} = R_s k (r_2 - r_1)$$

$$\frac{r_2}{R_s k} = \frac{r_2 - r_1}{r_1 \log_e \dfrac{r_2}{r_1}}$$

This equation is readily solved graphically by means of the chart, Fig. 5.

APPENDIX NO. 3

DERIVATION OF EQUATION [11]

93 Let k represent the true conductivity which varies with temperature according to the equation

$$k = aT^n + b$$

in which a and b are constants. (As used here the symbols a and b have no relation to these same symbols used elsewhere in this paper.)

$$H = \int_{T_1}^{T_2} (-kSdT) = \int_{T_1}^{T_2} (-aST^n dT) + \int_{T_1}^{T_2} (-bSdT)$$

In which S = shape factor, a constant dependent upon the size and shape of the specimen.

94 Let K represent the average value of conductivity

$$H = KS(T_1 - T_2)$$

Equating the values of H,

$$KS(T_1 - T_2) = aS\int_{T_1}^{T_2} -T^n dT + bS\int_{T_1}^{T_2} -dT$$

Dividing through by S

$$K(T_1 - T_2) = a\int_{T_1}^{T_2} -T^n dT + b\int_{T_1}^{T_2} -dT$$

Integrating

$$K(T_1 - T_2) = a\,\frac{T_1^{n+1} - T_2^{n+1}}{n+1} + b(T_1 - T_2)$$

$$K = a\,\frac{T_1^{n+1} - T_2^{n+1}}{(n+1)(T_1 - T_2)} + b$$

$$K = k \quad \text{when} \quad T^n = \frac{T_1^{n+1} - T_2^{n+1}}{(n+1)(T_1 - T_2)}$$

or the average conductivity is equal to the true conductivity at the temperature

$$T = \sqrt{\frac{T_1^{n+1} - T_2^{n+1}}{(n+1)(T_1 - T_2)}}$$

95 This relation is general, regardless of the shape of the curve of conductivity with respect to temperature. If this is a straight line, then $n = 1$ and $K = k$ when

$$T = \frac{T_1 + T_2}{2}$$

or the average conductivity is equal to the true conductivity at the arithmetical mean of the surface temperatures.

96 But, the most interesting feature is that $\frac{T_1 + T_2}{2}$ very closely approximates the value of

$$\sqrt[n]{\frac{T_1^{n+1} - T_2^{n+1}}{(n+1)(T_1 - T_2)}}$$

for a considerable range of values of n as illustrated in Table 2, where values are compared for the curve where $n = 2$.

It is evident that, even with a curve in which conductivity varies as the square of the temperature instead of the first power, the error, due

TABLE 2

T_1, absolute deg fahr.	T_2, absolute deg. fahr.	$\frac{T_1+T_2}{2}$ deg fahr	$\sqrt{\frac{T_1^{n+1}-T_2^{n+1}}{(n+1)(T_1-T_2)}}$ deg. fahr	Variation from mean, deg. fahr	Variation in conductivity, B.t.u.
500	600	550	550.8	0.8	0.0004
500	700	600	602.8	2.8	0.0014
500	800	650	655.7	5.7	0.0029
500	900	700	709.5	9.5	0.0047
500	1000	750	763.8	13.8	0.0069
1000	1100	1050	1050.4	0.4	0.0002
1000	1200	1100	1101.5	1.5	0.0008
1000	1300	1150	1153.3	3.3	0.0016
1000	1400	1200	1205.5	5.5	0.0028
1000	1500	1250	1258.3	8.3	0.0041

[1] Based on a rate of change of conductivity of 0.05 per 100 deg. fahr., which is high.

to taking the conductivity at the arithmetic mean of the surface temperatures as the average conductivity, is generally of the order of less than one per cent, which is well within the limits of experimental accuracy.

APPENDIX NO. 4

DERIVATION OF EQUATION FOR DETERMINING THE MOST ECONOMICAL THICKNESS OF INSULATION FOR FLAT SURFACES

97 Let the cost of heat loss per year $= m = \frac{a}{(x/k)+R}$. Let the cost of insulation per sq. ft. per 1 in. thickness per year $= n = bx+C$. Let $y =$ total cost per year $= m+n = \frac{a}{(x/k)+R} + C = \frac{ak}{x+Rk} + bx + C$. Differentiating and equating the first derivative to zero:

$$\frac{dy}{dx} = \frac{-ak}{(x+Rk)^2} + b = 0 = -ak+b(x+Rk)^2 = 0, \quad \text{or} \quad ak = b(x+Rk)^2$$

Dividing through by b and extracting the square root of both terms,

$$\sqrt{\frac{ak}{b}} = x+Rk$$

Therefore, for most economical results the thickness,

$$x = \sqrt{\frac{ak}{b}} - Rk$$

APPENDIX NO. 5

DERIVATION OF EQUATION [19] FOR DETERMINING THE MOST ECONOMICAL THICKNESS OF INSULATION FOR CYLINDRICAL SURFACES—ONE MATERIAL

98 Let the cost of heat loss per sq. ft. of insulation surface per year

$$= \frac{a}{\dfrac{r_2 \log_e \dfrac{r_2}{r_1}}{k} + R_S}$$

Then the cost of heat loss per linear foot,

$$m = \left(\frac{2\pi}{12}\right)\left(\frac{r_2 a}{\dfrac{r_2 \log_e \dfrac{r_2}{r_1}}{k} + R_S}\right) = \frac{\pi}{6}\left(\frac{r_2 a k}{r_2 \log_e \dfrac{r_2}{r_1} + R_S k}\right)$$

The cost of insulation per linear foot per year,

$$n = \frac{2\pi}{12}\, r_2(r_2 - r_1)\, b + C = \frac{\pi}{6}\, r_2(r_2 - r_1)\, b + C = \frac{\pi}{6}\,(r_2^2 - r_1 r_2)\, b + C$$

Then the total cost per year,

$$y = m + n = \frac{\pi}{6} \times \frac{r_2 a k}{r_2 \log_e \dfrac{r_2}{r_1} + R_S k} + \frac{\pi}{6}\,(r_2^2 - r_1 r_2)\, b + C$$

99 The most economical thickness is a function of r_2, and may be found by differentiating the expression for y with respect to r_2 and equating the first derivative to zero.

$$\frac{dy}{dr^2} = \frac{\pi}{6}\; \frac{\left(r_2 \log_e \dfrac{r_2}{r_1} + R_S k\right) ak - r_2 ak \left(1 + \log_e \dfrac{r_2}{r_1}\right)}{\left(r_2 \log_e \dfrac{r_2}{r_1} + R_S k\right)^2} \qquad \frac{\pi}{6}\,(2r_2 - r_1)\, b = 0$$

Clearing of fractions and multiplying through by $\frac{6}{\pi b}$

$$\left(r_2 \log_e \frac{r_2}{r_1} + R_S k\right)\frac{ak}{b} - \frac{r_2 ak}{b}\left(1 + \log_e \frac{r_2}{r_1}\right)$$
$$+ \left(r_2 \log_e \frac{r_2}{r_1} + R_S k\right)^2 (2r_2 - r_1) = 0$$

$$r_2 \frac{ak}{b} \log_e \frac{r_2}{r_1} + \frac{ak}{b} R_S k - r_2 \frac{ak}{b} - r_2 \frac{ak}{b} \log_e \frac{r_2}{r_1}$$
$$+ (2r_2 - r_1)\left(r_2 \log_e \frac{r_2}{r_1} + R_S k\right)^2 = 0$$

Simplifying

$$r_2 \frac{ak}{b} - \frac{ak}{b} R_S k = (2r_2 - r_1)\left(r_2 \log_e \frac{r_2}{r_1} + R_S k\right)^2$$

$$\frac{ak}{b}\,(r_2 - R_S k) = (2r_2 - r_1)\left(r_2 \log_e \frac{r_2}{r_1} + R_S k\right)^2$$

Dividing through by $(r_2 - R_S k)$, and extracting the square root of both sides

$$\sqrt{\frac{ak}{b}} = \sqrt{\frac{2r_2 - r_1}{r_2 - R_S k}}\left(r_2 \log_e \frac{r_2}{r_1} + R_S k\right)$$

APPENDIX NO. 6

DERIVATION OF EQUATION [20] FOR DETERMINING THE MOST ECONOMICAL THICKNESS OF INSULATION FOR CYLINDRICAL SURFACES—TWO OR MORE MATERIALS

100 Let the cost of heat loss per square foot of pipe surface per year

$$= \frac{a}{\dfrac{r_1 \log_e \dfrac{r_s}{r_2}}{k} + \dfrac{r_1 \log_e \dfrac{r_2}{r_1}}{k_1} + \dfrac{r_1}{r_s} + R_s}$$

Then the cost of heat loss per linear foot is

$$m = \left(\frac{2\pi}{12} r_1\right)\left(\frac{a}{\dfrac{r_1 \log_e \dfrac{r_s}{r_2}}{k} + \dfrac{r_1 \log_e \dfrac{r_2}{r_1}}{k_1} + \dfrac{r_1}{r_s} R_s}\right)$$

$$= \frac{\pi}{6}\left(\frac{r_1 a k}{r_1 \log_e \dfrac{r_s}{r_2} + \left(r_1 \log_e \dfrac{r_2}{r_1}\right)\dfrac{k}{k_1} + \dfrac{r_1}{r_s} R_s k}\right)$$

The cost of insulation per linear foot per year is

$$n = \frac{2\pi}{12} r_s (r_s - r_2) b = \frac{\pi}{6} (r_s^2 - r_2 r_s) b + C$$

101 Where it is desired to find the economical thickness of additional insulation to be applied over a given thickness of the first material, the resistance of the first material is provided for in the expression for m by the term

$$\frac{r_1 \log_e \dfrac{r_2}{r_1}}{k_1}$$

102 If more than two materials are involved, all except one are provided for in the expression for m by the addition of terms similar to that for the first material, and in the expression for n by substituting r_3, r_4, or whatever value of r may be required, for r_2.

103 Total cost per year

$$y = m + n = \frac{\pi}{6} \frac{r_1 a k}{r_1 \log_e \dfrac{r_s}{r_2} + \dfrac{k r_1 \log_e \dfrac{r_2}{r_1}}{k_1} + \dfrac{r_1}{r_s} R_s k} + \frac{\pi}{6} b (r_s^2 - r_2 r_s) + C$$

This cost y is the cost of additional insulation which is to be put on over a given thickness of the first material. The most economical thickness is found by differentiating y with respect to r_s and equating the first derivative to zero.

$$\frac{dy}{dr_s} = \frac{\pi}{6} \frac{-r_1 a k \left(\dfrac{r_1}{r_s} - \dfrac{r_1}{r_s^2} R_s k\right)}{\left(r_1 \log_e \dfrac{r_s}{r_2} + \dfrac{k r_1 \log_e \dfrac{r_2}{r_1}}{k_1} + \dfrac{r_1}{r_s} R_s k\right)^2} + \frac{\pi}{6} b (2 r_s - r_2) = 0$$

Clearing of fractions and multiplying through by $\frac{6}{\pi b}$

$$-r_1 \frac{ak}{b}\left(\frac{r_1}{r_s} - \frac{r_1}{r_s^2} R_s k\right) + (2r_s r_2) \times$$
$$\left(r_1 \log_e \frac{r_s}{r_2} + \frac{k r_1 \log_e \frac{r_2}{r_1}}{k_1} + \frac{r_1}{r_s} R_s k\right)^2 = 0$$

$$r_1 \frac{ak}{b}\left(\frac{r_1}{r_s} - \frac{r_1}{r_s^2} R_s k\right) = (2r_s - r_2) \times$$
$$\left(r_1 \log_e \frac{r_s}{r_2} + \frac{k r_1 \log_e \frac{r_2}{r_1}}{k_1} + \frac{r_1}{r_s} R_s k\right)^2$$

Extracting the common factor $\left(\frac{r_1}{r_s}\right)^2$ from both sides of the equation

$$\left(\frac{r_1}{r_s}\right)^2 \frac{ak}{b} (r_s - R_s k) = (2r_s - r_2)\left(\frac{r_1}{r_s}\right)^2$$
$$\left(r_s \log_e \frac{r_s}{r_2} + \frac{r_s}{r_1} \frac{k r_1 \log_e \frac{r_2}{r_1}}{k_1} + R_s k\right)^2$$

Dividing through by $\left(\frac{r_1}{r_s}\right)^2 (r_s - R_s k)$, and extracting the square root of both sides of the equation,

$$\sqrt{\frac{ak}{b}} = \sqrt{\frac{2r_s - r_2}{r_s - R_s k}} \left(r_s \log_e \frac{r_s}{r_2} + \frac{r_s}{r_1} \frac{k r_1 \log_e \frac{r_2}{r_1}}{k_1} + R_s k\right)$$

APPENDIX NO. 7

APPARATUS FOR CONDUCTIVITY DETERMINATIONS

104 The apparatus used for determining the conductivities given in Fig. 7 is shown diagrammatically in Fig. 23. The method is based in principle on that used by the author in his tests at the University of Wisconsin, and described in his paper entitled The Heat Insulating Properties of Steam Pipe Coverings.[1]

105 Briefly, the method consists of applying the material to be tested on a standard steel pipe, provided with an internal electrical heater, and measuring the rate at which electrical energy must be supplied in order to maintain a uniform temperature gradient. Under these conditions the power input is an accurate measure of the total rate of heat transfer through the insulation.

106 However, this earlier apparatus has been simplified and considerably improved. Whereas oil, circulated by means of an impeller, was used at that time for securing uniform temperature throughout the apparatus, this result is now obtained by means of spacing the windings of the heater in such a manner as to insure uniform distribution of heat to all points on the surface of the apparatus. This permits of testing at higher temperatures than was possible when oil was used. The winding is non-inductive, which permits use of alter-

[1] Trans. A.S.M.E., vol. 37, pp. 921 to 969.

nating-current power without the introduction of error due to induced currents. In addition to this double helical winding on a concentric tube inside of the test pipe, auxiliary windings are used to provide for the heat loss from the ends, and these insure maintenance of uniform temperatures from end to end of the apparatus.

107 *The entire power input* to the apparatus *is measured.* The losses from the ends are determined at various temperatures by means of the end-correction apparatus (*E*) shown in Fig. 23. This apparatus and the insulation on it are exact duplicates of the ends of the larger apparatus beyond the portion marked "Test Section."

108 For accurate results a uniform rate of energy input and a uniform room temperature are essential. The first of these requirements is fulfilled by the use of an automatic voltage-regulator and the second by thermostatic control of room temperature.

109 Temperatures at both boundaries of the insulation are determined by the use of copper-constantan thermocouples spaced at the

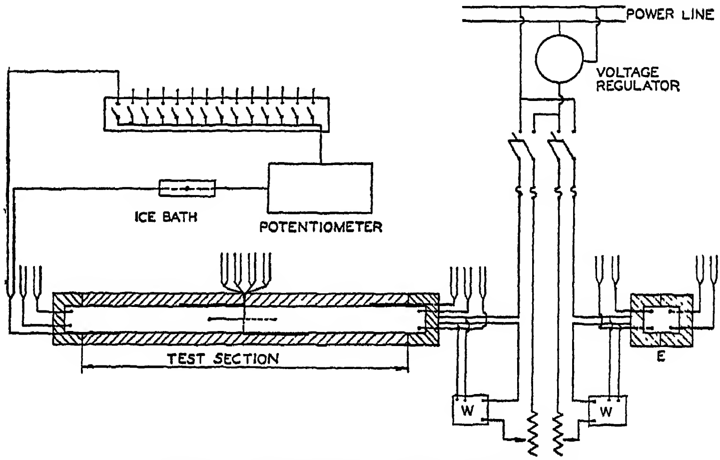


FIG. 23 CONDUCTIVITY TEST APPARATUS

center of each foot of length and distributed around the entire circumference. This is very important, as temperatures on the lower surfaces are usually slightly lower than those on the top. Separate thermocouples are placed under the end caps. This permits of applying the end correction at the exact temperatures of the ends of the larger apparatus and provides against the possibility of error, even if the temperatures of the ends are slightly different from the average temperatures of the whole apparatus.

110 Thermocouple potentials are measured with a Leeds and Northrup precision thermocouple potentiometer which is sensitive to a potential of 0.001 millivolt. This corresponds to an accuracy of temperature measurement of better than 0.03 deg. fahr. Such extreme precision, however, is rarely required, since at a temperature difference of 100 deg. fahr., 0.01 millivolt is less than 0.3 per cent, and at 500 deg. fahr. temperature difference, is less than 0.1 per cent of the total. Therefore, readings are generally taken to the nearest one-hundredth of a millivolt.

111 It is claimed by some that compensating for end losses by means of separate heaters is preferable to the end-correction method.

Theoretically, this is true, but the advantage is purely theoretical. In the author's opinion, it is far more preferable to measure the entire input and make a correction in an item of the order of 10 per cent of the total, which correction if made even to an accuracy of 10 per cent (and much better accuracy is possible) would reduce the total probability of error to the order of 1 per cent, than to feed into the same enclosure *unmeasured* power to the extent of several times 10 per cent, trusting that the temperatures are so nicely balanced that none of this flows into the test section.

112 The ideal apparatus for conductivity determination would be one without ends, edges or corners: one in which the shape of the path for heat flow is the same at every point. The sphere fulfills these requirements but its shape does not lend itself to the testing of materials in other than plastic or pulverized form. The next most suitable form of apparatus is the cylinder, provided with a means of correcting for end loss. If the length is such that the end correction is a proportionately small part of the total loss, this type of apparatus is capable of highly-accurate results.

DISCUSSION

Charles H. Herter.[1] The 1924 Report of the Insulation Committee of the American Society of Refrigerating Engineers shows what research has been done on Heat Insulation in the low temperature field. Sub-committee B, on Heat Transfer of Building and Insulating Materials, of the National Research Council has long realized the need for just such data as the author is now offering in the moderate- and high-temperature field.

In the present paper the subject is treated logically and convincingly. Of particular value is the author's proof that the mere temperature of a surface exposed to air is no reliable guide as to the amount of heat lost, also his Equations [16] to [20] for determining the most economical thickness of insulation.

Appendix 1 containing a model list of symbols seems very useful, and so is Appendix 7, Fig. 23, discussing the author's improved method of testing covering. In this the actual surface temperature apparently is dispensed with, in view of the argument on the use of mean temperature of layers given in Par. 45; but it would doubtless be advisable still to take and record the surface temperature with four thermocouples spaced equally around the circumference. Numerical examples of typical tests with single and two-layer coverings would make clear the method of testing.

The trend of the times is toward higher steam pressures and temperatures, such as 1400 lb. per sq. in. and 800 deg. fahr., and thus the conservation of heat by the use of insulation is now a much more important factor than in the past. A careful study of the paper will show engineers that ample insulation still is the most economical method for reducing loss of heat in engineering applications.

[1] Plant Engineer, Commonwealth Ice Co., New York, N. Y.

For establishing the temperature drop from the exterior surface of a covering to the surrounding air it would be very desirable to have a diagram similar to Fig. 12, with curves showing the rate of heat emission at all temperature differences from 1 deg. up to, say, 100 deg., with various diameters, horizontal and flat black surfaces, and with various rates of air movement. The author observes that this phase requires further experimenting, not enough observations being available today. But it might have been well to expressly point out that the factor 1.8 B.t.u. used in an example in Par. 63, is by no means constant per degree of difference, otherwise all curves in Fig. 16 would be straight lines and parallel.

M. S. Van Dusen.[1] This paper is an excellent elementary exposition of the fundamental facts of heat transfer from the point of view of insulation. It contains nothing essentially new, but it treats certain phases of the subject in such a manner that it is readily grasped by the reader. The appendices might well be omitted, especially Appendix 7, the subject matter of which was published in detail a number of years ago. In any case, Appendix 3 should be revised, since it is liable to be misleading.

The reader is left with the impression that in any practical case the average conductivity over a temperature interval is within one per cent of the true conductivity at the arithmetical mean temperature. This is open to question. The calculations in Appendix 3 are over too small temperature intervals to show much effect, and the temperature coefficient of conductivity is assumed as a constant equal to 0.05, when as a matter of fact this quantity varies with temperature in the hypothetical case assumed.

The Author. The principal objective in the preparation of this paper was to show the effects of the more important variables affecting heat transmission through insulation in the true order of their importance; or in other words, to show in perspective the factors involved. The general comments in the discussions by Mr. Herter and Dr. Van Dusen are appreciated because they seem to show that this objective has been in some measure attained.

Referring to Mr. Herter's comments on Appendix 7, in the method of testing described, temperatures are taken at the cooler surface of the insulation as well as at the warmer surface. This is what was meant by the statement in Par. 108 regarding the location of thermocouples for the measurement of temperatures "at both boundaries of the insulation." Mr. Herter's comment is appropriate, therefore, in calling attention to the need for clarifying the meaning of that statement. The method of testing described in this paper is the same in principle as that described in the author's paper in Trans. A.S.M.E., vol. 37, pages 921 to 969, in which numerical examples were freely given. Therefore, while

[1] U. S. Bureau of Standards, Washington, D. C.

there have been notable improvements such, for example, as the automatic control of power input and room temperature and the adapting of the apparatus to testing at much higher temperatures, it was not considered that using the space required for further numerical examples would be justified.

The author agrees with Mr. Herter that further experimenting is desirable to establish more thoroughly the values of surface resistances, but the usefulness of such data would be principally in providing for more accurate estimates of rates of heat transfer from bare surfaces. As pointed out in the paper, even the maximum values of surface resistances in connection with insulated surfaces are generally small as compared with the resistance to heat flow offered by the insulation itself. The factor 1 8 B.t.u. in Par. 63 was not assumed to be a constant, but was one of the stated conditions applying to the particular numerical example under discussion. Mr. Herter's reference to Fig. 16 in this connection should undoubtedly be to Fig. 12 instead, since Fig. 16 gives ratios and not actual rates of heat transfer.

Referring to Dr. Van Dusen's discussion, the method of determining mean temperatures and the economic data are new and original. Furthermore, while the general equations of heat flow are not new, the author believes that a useful purpose has been served by setting forth the relative magnitudes of the various components of these equations which he believes has not been so completely done before.

The appendices are necessary because the derivations of equations are given there and not in the text of the paper. The need for Appendix 7 has been demonstrated by the difficulties which have been encountered by others who have attempted to use at higher temperatures the apparatus exactly as described in the author's former paper. The principles involved are the same, but as pointed out above, important improvements have been made, knowledge of which will be useful to others.

Referring to Dr. Van Dusen's comments on Appendix 3, there can be no question as to the correctness of Equation [**11**]. It is in connection with the proposition that generally in practical cases, the true conductivity at the arithmetical mean of the surface temperatures may be taken as a representative average conductivity, that the reasoning may not have been made clear. The rate of change of conductivity with respect to temperature was not assumed to be a constant. It is a variable, but the probable maximum rate of change for materials suited to use over considerable ranges of temperature is of the order of 0.05 per 100 deg. fahr. If in any given case the rate of change at the point on the curve in question is less than 0.05 per 100 deg. fahr., and usually it will be less, then the variation in conductivity *will be less* than the values shown in the table which, as was pointed out, were for that example, of the order of less than one per cent.

No. 2035

INDUSTRIAL-BOILER EFFICIENCIES

By Samuel D. Fitzsimmons,[1] Providence, R. I.

Member of the Society

This paper presents results obtained with two boilers installed at the plant of the Brown & Sharpe Manufacturing Company, Providence, R. I., and its purport is to indicate the desirability of low furnace-draft velocities and a more effective use of radiant heat. The premise is assumed that more width in the tube sections of water-tube boilers is desirable. Much that is offered is in the form of conjecture.

GREATER progress has perhaps been made in the last five years toward obtaining boiler efficiencies in the neighborhood of the coveted 80 per cent than was made in the immediately preceding decade. The advent of pulverized fuel stimulated effort, influenced design, and refined long-established theories regarding combustion and heat absorption. Putting these refinements into practice has entirely upset some of the ideas which influenced furnace and boiler design as represented by conventional practice a very few years ago. It is highly probable that higher efficiencies, considered now as being in the realm of the impossible, will be achieved in the next decade. Toward this end much may be accomplished if the problems peculiar to steam generation and utilization for industrial purposes are not confused with, and influenced by, the more extensive requirements of central-station practice.

2 It is assumed here that the industrial engineer's part in the field of experimentation and research is that of intelligently coördinating the contributions revealed through the efforts of the power specialist and the equipment-manufacturer's laboratory. A manufacturer can afford to experiment only for the purpose of improving his commercial product, and the industrial engineer's remoteness from the field of intensive research in matters pertaining to the subject necessarily directs all his effort toward an intelligent application of equipment, which in most cases is that which is commercially offered. The problem, then, is one of a wise selection of equipment of established performance, and an intelligent assembly of that equipment. Practicability is the factor that determines how far we may go in pursuit of the elusive points of boiler efficiency.

[1] Plant Engineer, Brown & Sharpe Mfg. Co.

Presented at the Providence, R. I., Meeting, May 3 to 6, 1926, of The American Society of Mechanical Engineers.

3 Fig. 1 shows the assembly of boiler and combustion equipment installed at the plant of the Brown & Sharpe Mfg. Co. It is the joint effort of Jenks & Ballou, consulting engineers, of Providence, R. I., and the engineering staff of the Brown & Sharpe Mfg. Co. and was placed in regular operation in September, 1924. Table 1 gives the results of a test made in October, 1925, which

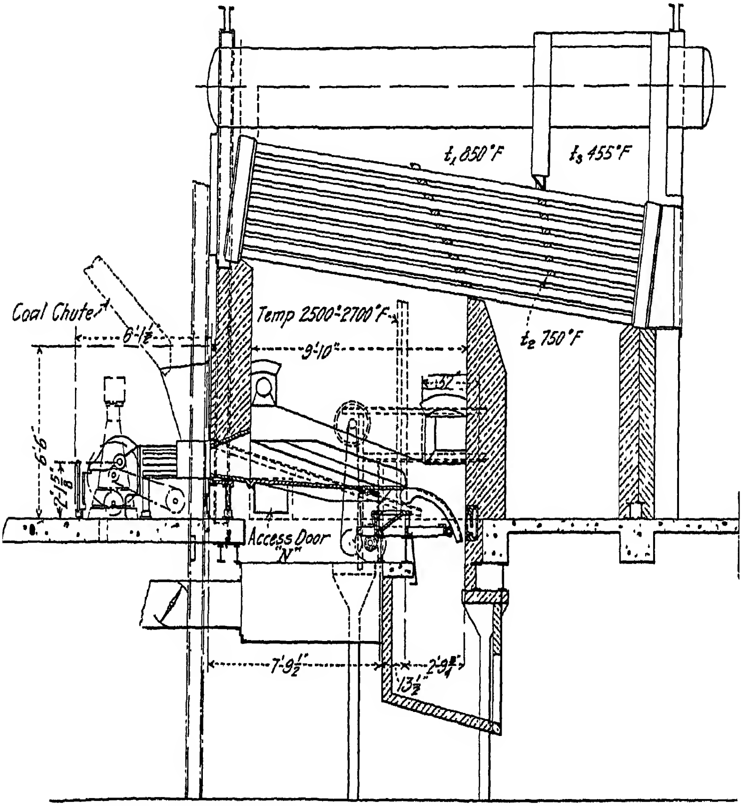


Fig. 1 Assembly of Boiler and Combustion Equipment Installed at Plant of Brown & Sharpe Mfg. Co.

are representative of regular operating performance. Two tests were made by the Brown & Sharpe engineering staff and results are substantially in accord with those of Table 1. The reduction in the amount of coal consumed during the year 1925 indicates an efficiency advantage of 11 to 15 per cent over the replaced equipment, which ranged in efficiency between 68 and 72 per cent, depending on load and condition. The replaced equipment, in use for about twenty years, was highly efficient in the light of contemporary practice.

4 The increased efficiency of the new plant is due largely to the extension of basic principles incorporated in the design of the old plant. The tests were made when the plant demand was confined to the boilers under test and other equipment could be "blanked" off. The test data have thus far withstood sincere effort to discover possible error in them. Daily records indicate efficiencies two to three per cent above and below those given in Table 1. Close checking of the fuel and water consumption for the last eighteen months indicates that the test data represent average performance. The efficiency of the assembly may be attributed to four major factors, and the measure of their respective contribution to the whole is debatable. These factors are —

a Effectiveness of absorption surface
b Effectiveness of grate area
c Effectiveness of air introduction
d Effectiveness of radiant heat.

Effectiveness of Absorption Surface

5 The boilers were designed and built by the Union Iron Works. They are of the inclined water-tube box-header type with 18-ft. 4-in. tubes sloped at an angle of 8 deg. to the steam drum, arranged 9 high and 17 wide and having 3060 sq. ft. of heating surface. The baffling is so arranged that 1300 sq. ft. of tube surface is exposed in the first pass, 645 in the second pass, and 632 in the third pass. The tubes are arranged horizontally, 5⅝ in. on tube centers, giving a tube-to-tube clearance of 1⅝ in. Vertically, the tubes centers are 5⅝ in. apart and the tubes are staggered in the usual manner. This arrangement makes for an unusually wide and low cross-section, and enhances the "scrubbing" effect of the gases. The transverse dimension of the tube section is 94 in. and the vertical dimension 49 in. The cross-sectional area occupied by the tubes is 1920 sq. in. and that by the gas passages is 2686 sq. in. This arrangement exposes more tube surface for radiant-heat absorption than does the regulation design of narrower width and greater height.

6 Owing to the rapidity of circulation it may reasonably be assumed that the temperature is practically uniform throughout the water content of this type of boiler when the feed is introduced at a temperature not over 100 deg. less than the steam-drum temperature. This being so, it would seem that as much of the tube surface as practicable should be placed in the first pass where the temperature head is greatest. The absorption efficiency of the boiler in question declines gradually above 150 per cent of rating, and at 200 per cent of rating the loss is about three per cent. The temperature of the escaping gases given in Table 1 is somewhat higher than those obtained in daily operation. The increase was occasioned by a leaking front baffle in boiler No. 8, discovered

TABLE 1 RESULTS OF BOILER TRIAL

Date of test	Oct. 5-6, 1925
Location	Providence, R. I.
Owner	Browne & Sharpe Mfg. Co.
Maker and type of boilers (2)	Union Iron Works, Water-tube
Rated horsepower of each boiler	300
Fuel used	West Virginia coal
Fuel-burning equipment	2 Riley Stoker Corp. super-stokers, 4 retorts each
Superheater	None
Economizer	None
Test conducted by Riley Stoker Corp.; observed by Brown & Sharpe Mfg. Co.	
Object of test	Determination of efficiency of boiler and stoker

Fuel and Gas Analysis and Data

Fuel Proximate Analyses (per cent)	As fired	Dry
Fixed carbon	72.73	74.52
Volatile matter	19.71	20.20
Ash	5.15	5.28
Moisture	2.41	....
Sulphur (separate determination)	0.89	....
Combustible in dry refuse	9.82	...
B.t.u. per lb.	14,513	14,871

Fuel Ultimate Analysis (per cent)

Carbon	84.86	Nitrogen	1.59
Hydrogen	4.55	Sulphur	0.91
Oxygen	2.91	Ash	5.28

Flue-Gas Analysis (per cent)

Carbon dioxide (CO_2)	13.1	Carbon monoxide (CO)	0.2
Oxygen (O_2)	5.6	Nitrogen (N)	81.1

Pressures, Drafts, and Temperatures

Steam pressure by gage, lb. per sq. in.	127
Pressure in air chamber, inches of water	0.8
Draft in furnace, inches of water	0.01
Draft between damper and boiler, inches of water	0.21
Moisture in steam, per cent	2.02
Temperature of feedwater, deg. fahr.	195.0
Temperature of escaping gases, deg. fahr.	468.0
Temperature of air at blower outlet, deg. fahr.	77.0
Temperature of boiler room, deg. fahr.	73.0

Total and Hourly Quantities

Duration of test, hr.	24
Total weight of fuel fired per hour, lb.	47,968
Fuel as fired per hour, lb.	1,998
Fuel as fired per hour per sq. ft. of projected grate area, lb.	13.9
Total weight of dry refuse, lb.	3.148
Dry refuse, per cent of fuel	6.56
Total weight of water fed to boiler, lb	572,236
Factor of evaporation	1,043
Equivalent evaporation from and at 212 deg. fahr., lb	596,842
Water evaporated from and at 212 deg. fahr. per hr., lb.	24.868
Water evaporated from and at 212 deg. fahr. per hr., per sq. ft. of heating surface, lb.	4.14
Boiler horsepower developed (avg.), 720 hp. for 2 boilers	360
Per cent of rated capacity developed	120

Evaporation and Efficiency

Actual evaporation per lb. fuel as fired, lb.	11.92
Evaporation from and at 212 deg. fahr. per lb. fuel as fired, lb.	12.44
Evaporation from and at 212 deg. fahr. per lb. dry fuel, lb.	12.75
Efficiency of boiler, furnace, and grate, per cent	83.17

Heat Balance

Heat absorbed by boiler	12,373	83.17
Heat loss due to moisture in coal	31	0.21
Heat loss due to water from combustion of hydrogen	520	3.50
Heat loss due to dry chimney gases	1,501	10.10
Heat loss due to incomplete combustion of carbon	128	0.86
Heat loss due to unconsumed combustible in refuse	75	0.51
Heat loss due to unconsumed hydrogen and hydrocarbons, radiation, and unaccounted for	243	1.65
	14,871	100.00

after test was under way. The temperatures in the tube passes at 150 per cent of rating are shown in Fig. 1 and are worthy of comment. The furnace temperature, also given in Fig. 1, was obtained with an optical pyrometer.

7 The author believes that very little heat transmitted by convection is absorbed in the lower three tubes in the front pass, but that the absorption of heat so transmitted may be made more efficient by placing more tube area in the first pass. Boiler width is probably an important factor, as the temperature head between the water and the gases is uniformily greater throughout the wide, shallow, tube section than it is in the narrow and higher cross-section. That is to say, width of tube section or tube length or a combination of both, rather than tube-section height, is responsible for the better efficiencies obtained. Full advantage of the possibilities of heat transmitted by convection cannot be obtained with an 18-ft tube owing to an insufficiency of the time element required for adequate lineal gas travel. Twenty- or twenty-two-foot tubes arranged in a cross-section three times as wide as high would seemingly approximate a more efficient tube surface arrangement.

8 The close tube spacing in the boilers under consideration introduces operating difficulties which partly offset the contributions to efficiency such an arrangement makes. High furnace temperature is a prerequisite to efficiency, and with it the fused-ash problem is always present. The lower tubes in the installation function as do water screens in pulverized-fuel furnaces, and the bonding of the chilled-out ash to the boiler tubes is increased by the sooty surface of the tubes. Closeness of tube spacing facilitates the bridging-over process, and impairment of convection and radiation absorption ensues. The frequent shutdowns necessary for removal of this friable ash impose an appreciable operating burden. While close tube spacing is desirable in the upper tubes for its contribution to efficiency due to an effective scrubbing of the gases, the lower tubes should perhaps be more widely spaced to prevent chilled-out ash from bridging over between tubes.

9 Absorption efficiency is a matter of water circulation and sufficient heating-surface area, properly distributed, and may be obtained in the product of any reliable manufacturer if the prerequisites are recorded in the specifications. With the advantages of lower steam pressures and consequently lower uptake temperatures and lower and more uniform ratings, any properly designed industrial-boiler plant should develop absorption efficiency in excess of that developed in the best of the central-station plants.

Effectiveness of Grate Area

10 The boilers of the plant are equipped with super-type underfeed stokers designed and manufactured by the Riley Stoker

Corporation, and were the first super-type stokers of this particular manufacture to be installed under industrial boilers. The bridge wall is 9 ft. behind the front wall and is equipped with a Riley air back. The projected area of effective grate is 72 sq. ft. Side-wall tuyeres are installed. It will be readily perceived that at the nominal rating of 303 hp., 83 per cent overall efficiency, and a B.t.u. fuel value of 14,500, the combustion rate is approximately 12 lb. per sq. ft. per hour, giving the remarkably low rate of 18 lb. per sq. ft. per hour at 150 per cent of rating. The advance feed stroking of the stokers is $\frac{1}{8}$ in. per min. at 150 per cent of rating. The low combustion rate effects a very complete dissociation of the volatile matter, and the low rate of advance feed stroking effects a nearly complete burn-out of the combustible in the ash. The minimum thickness of fuel is of course that which limits uniformity of air permeability and prevents melting of the grate by radiant heat. Uniformity of air permeability likewise determines the maximum thickness, but between these limits is a range where the thickness of the fuel bed is optional, as combustion acceleration is controlled by air volume and not directly by fuel volume. Combustion acceleration and combustion capacity should not be confused in any consideration of the matter of grate area. The admission of fuel to the grate area is hand-controlled.

11 The importance of and necessity for furnace depth vary directly with the boiler rating. In stoker practice, depth of grate is required for the same reason that furnace volume is required for pulverized fuel. The time element is of paramount importance. In so far as lineal dimension may be identified with function, width of grate determines combustion capacity, and depth or length of grate determines efficiency or amount of combustible in the ash, always assuming the advance feed stroking is properly timed to boiler output. The above fully adheres to the fundamental that coal must be burned rapidly to obtain efficient combustion. The super-type underfeed stoker positively controls the rate of advance toward the bridgewall with less violent eruption of the fuel bed than was possible with the old method. The newer method feeds coal as required and controls its position from the time it enters the furnace until it is deposited as ash at the bridgewall. This all tends to make for a more rapid combustion, in that the combustion is more uniform over the full area of grate.

12 Present stoker design does not permit control of combustible in the ash over wide ranges of load demand. Stoker installations properly designed are efficient in this respect up to rates of driving approximating 200 per cent of rating. Beyond this point the combustible in the ash increases rapidly. The limitation should not affect the use of stokers in industrial plants, as rates of driving in excess of 200 per cent of rating are generally unnecessary and uneconomical. It would seem, however, that for

rates of driving up to 200 per cent the stoker-fired furnace should give as good results as can be obtained with pulverized-fuel furnaces.

Effectiveness of Air Introduction

13 Air is supplied by a turbo-vane fan of Sturtevant design and manufacture. It is steam- and electric-driven for heat-balancing purposes. The fan discharges into a large plenum chamber and thence through individual ducts to the boilers. Each admission duct is equipped with a Ruggles-Klingemann damper actuated by the steam pressure. The uptake dampers are actuated by Engineer Company equipment from over-the-fire pressure.

14 The large grate area and low combustion rate permit a thin fuel bed. The depth of the fuel bed determines air permeability, and therefore air pressure below the grate. The air velocity through the fuel bed is very low, and the assumption is that it is dissipated at the surface of the fuel bed. The combination of large grate and low-velocity air feed effects a very thorough admixture of gas and air at the surface of the fuel bed and not a few feet above it as is the case when the wind velocity is higher, and the velocity of the introduced air must first be absorbed by expansion before thorough admixture occurs. The velocity of air emergence from the surface of the fuel bed determines the horizontal zone of greatest gas and air admixture. *Velocity makes for stratification, and when air-emergence velocity is eliminated, so do we eliminate the need for high settings in stoker-fired boilers.* Air-emergence velocity should not here be confused with velocity propagated by combustion.

15 The claim is advanced that fuel in pulverized form permits of more efficient combustion than does fuel in "solid" form. The author concedes superiority to the pulverized-fuel furnace for rates of driving in excess of 200 per cent of rating because of the inherent limitation heretofore mentioned — combustible in the ash — but for rates of driving below 200 per cent of rating he maintains that stoker-furnace efficiencies commensurate with the best in pulverized-fuel furnaces are obtainable. Theoretically, the propagation of combustion is the same for both methods, and the efficiency of combustion is determined directly by the degree of gas and air admixture. It is not apparent that a more intimate and rapid admixture can be effected in the pulverized-fuel furnace than can be effected in the solid-fuel stoker furnace. Fuel must first be gasified before flame propagation ensues. To the method that accelerates gas and air admixture accrues the theoretical advantage. There is real opportunity for valuable discussion on this point.

16 The idea has been advanced that a stoker furnace must have a volume commensurate with that required for pulverized fuel to obtain a comparable efficiency. However, results of tests

seemingly disallow this claim. The stoker furnace requires a large cross-sectional area in the horizontal dimension to produce complete volatilization of the fuel, and sufficient vertical dimension to gain flame propagation. If the air velocity through the fuel bed is low, thorough admixture of gas and air is effected at the surface of the fuel bed. Flame propagation ensues and the completion of combustion of the volatilized content is instantaneous. The major portion of total heat release is from that incandescent part of the fuel which remains on the grate, and which in the process of combustion projects heat rays in all directions from its positions on the grate area and not while in suspension in the furnace volume. In a stoker furnace the vertical dimension is required for combustion of the volatiles only, and the lower the volatiles are in the coal, the smaller the vertical dimensions that will be required.

17 In published test data covering four pulverized-fuel plants the pressure feeder air recorded is 14.2 in. for 157 per cent of boiler rating, 5.6 in. for 125 per cent of boiler rating, 4.5 in. for 150 per cent of boiler rating, and 9.2 in. for 149 per cent of boiler rating. The average of these values is 8.375 in. and the percentage of boiler-rating average is 145.25. The velocity of the pressure feeder air is obviously the primary reason for large furnace volume, as the velocity must first be absorbed by expansion and stratification eliminated before thorough mixing of air and gas is reached. The velocity incidental to pressure feeder air is directed downward parallel to the vertical dimension of the furnace. It would seem that the excessive vertical dimension of pulverized-fuel furnaces is required to provide the necessary time for combustion or to absorb the momentum of high initial velocity. After this velocity of entry is absorbed, the mixture of incandescent coal and air describes an easy loop and floats upward. The radius at the bottom of the loop surely influences, if it does not determine, the horizontal depth of furnace.

Effectiveness of Radiated Heat

18 The boilers under consideration have 9-ft. settings and the tube slope approximately parallels the grate surface. The incandescent fuel mass on the grates therefore projects most of its heat rays directly on the lower tubes. The large grate area and the close tube spacing and extreme width of the tube section contribute largely to very efficient projection and absorption of radiant heat. The grates, as heretofore stated, are equipped with side-wall tuyeres, and heat radiated to the relatively small area of refractory side wall exposed below the tubes is not destructive as it is reflected and absorbed by the tubes. The fact that the incandescent fuel bed is located below the refractory of the front and side walls means that all radiated heat strikes the exposure of refractories at an angle and reflects to the tube surface. This

is probably the explanation for the very good condition of the refractories after more than eighteen months of service. That unabsorbed radiated heat is highly destructive to refractories is evidenced by the condition of the bridgewall and front baffles. Referring to Fig. 1, it may be shown that direct impingement occurs on the lower part of the bridgewall, and that the angle of the front baffles are repaired at frequent intervals.

19 Effectiveness of radiant heat is perhaps the one real advantage stoker practice has over pulverized-fuel practice. In the stoker furnace, radiant heat is for the most part projected directly to the tube surface and fully absorbed thereby. In the pulverized-fuel furnace, radiant heat is projected in all directions from the incandescent particles of coal suspended in the furnace volume. The heat rays that strike the furnace walls and which cannot be reflected to the tubes are absorbed by the walls and furnace gases, and the idea is advanced that that portion of radiant heat not absorbed by the boiler either directly or indirectly through the gases, effects destruction of the furnace refractory. The pulverized-fuel furnace is efficient because of its high temperature. The high temperature of the gases is seemingly caused by furnace absorption of radiant heat as well as by the correct air supply. If air in sufficient quantity to absorb all released radiant heat were admitted, a large percentage of so-called excess air would be present and the furnace temperature would drop to a point that would make for low efficiency.

20 This assumption therefore credits radiant heat with the desirable features of pulverized-fuel practice, and in so doing necessarily charges refractory destruction to the same agency. Suspended ash made molten by radiant heat and attacking refractory is seemingly another destructive agency. The introduction of water-cooled walls in pulverized-fuel furnaces will no doubt effectively absorb the released radiant heat that destroys refractory in the non-cooled (air or water) walls. The effective radiant-heat absorption by cooled furnace walls will result in a lowered temperature of the furnace gases, as the radiant heat will manifestly not be reflected into the gas volume if it is absorbed by the cooled walls. The suggestion is here advanced that installing water-cooled walls in a pulverized-fuel furnace or placing more tubes in the horizontal dimension of an inclined water-tube boiler set over a mechanical stoker, are pretty much one and the same thing, the object being a more effective absorption of radiant heat, and that a sufficient spread of grate area (and incidental spread of incandescent surface) in the case of stoker installations will result in the same efficient projection of radiant heat that characterizes pulverized-fuel furnaces.

21 Approximately one-half of the heat released from solid fuels is contributed in the form of radiant heat, and it would seem that

the advances made in the art of combustion are traceable to a better understanding of radiant-heat characteristics.

22 The author's purpose in conjecturing as to the relative merits of pulverized-fuel furnaces and mechanical stokers has been to emphasize the inherent features advantageous to the stoker, and which seem to have been lost sight of in the current enthusiasm for pulverized fuel.

DISCUSSION

George W. Bach.[1] While the performance of the Brown & Sharpe boilers is remarkable, there are other Union water-tube boilers operating equally as well in other plants. One of these plants is the new Riverside Pumping Station, Milwaukee, Wis. A comparison of the boilers at Milwaukee with the Brown & Sharpe boilers is consistent, as both installations use the same make of stokers of the same size, and the test results at Milwaukee averaged identically the same CO_2 conditions. The boilers at Milwaukee are somewhat larger, being of 412 rated horsepower. These boilers showed an exit-gas temperature only 27 deg. higher than the saturated-steam temperature, which was 390.2 deg. fahr. at 206 lb. gage operating pressure, at 110 per cent of rating.

Observing the heat balance of the Brown & Sharpe test, it may be noted that there were but two losses attributed in any way to the boiler. These were: (1) loss due to dry chimney gases; (2) loss due to unconsumed hydrogen and hydrocarbons, radiation and unaccounted for, forming, respectively, 10.1 and 1.65 per cent, or a total of 11.75 per cent of the heat given the stoker. Of this amount the item of unconsumed hydrogen and hydrocarbons was not chargeable to the boiler. The remainder of the losses, or those which might be charged against the stoker, were 5.08 per cent. Then the efficiency of the stoker would be 94.92 per cent and the efficiency of the boiler and setting alone, 87.62 per cent.

Analyzing the Milwaukee Water Works test in the same way, the losses due to dry chimney gas were found to be 9.69 per cent, and the unaccounted losses, 4.51 per cent, or a total of 14.20 per cent, leaving the percentage chargeable to stoker 8.29 per cent. The efficiency of the boiler was 84.34 per cent.

In comparing these boiler efficiencies of 87.6 per cent and 84.3 per cent for the Brown & Sharpe and Milwaukee Water Works tests, it should be borne in mind that the settings of the first had the furnace side walls insulated only up to the bottom row of tubes, while the Milwaukee boiler had a steel casing with 2 in. of insulating material between it and the brick, all over. The Brown & Sharpe boilers were operated at 127 lb. pressure and the Milwaukee at 206 lb. gage. The Brown & Sharpe boiler had 10 ft. 6 in. length

[1] Vice-President and General Manager, Union Iron Wks., Erie, Pa. Assoc. A.S.M.E.

of tubes, 17 in number, exposed to radiant heat; the Milwaukee boiler, 9 ft. 8 in. length, 17 in number. The builders' ratings were, respectively, 308 and 412. The heights under the front header were 9 ft. 6 in., and 12 ft. 0 in.; widths of furnace, 7 ft. 7½ in., and 8 ft. 0 in. The Milwaukee boiler had a superheater, while the Brown & Sharpe boilers did not. In general, the writer believes that the difference in efficiency of 3.3 per cent between the Milwaukee boilers and the Brown & Sharpe boilers was due in a large measure to the difference in temperature heads, caused by the different operating pressures in the two plants.

ALFRED VAKSDAL.[1] Three years ago the Corning Glass Works were evaporating about eight pounds of water per pound of coal. The following year they increased this to 9 lb. and the next year to 10 lb. Last year they started with an evaporation of 11 lb. per pound of coal, and last August they increased it to 11½ lb. They called in engineers from the different concerns from whom they had purchased equipment and found, after all meters, gages, and scales had been checked, that this performance was correct.

The writer has been asked how this result had been obtained. He is unable to attribute this performance to anything except that for the last few years the company has not changed the coal; the same men are firing the boilers today as were firing three years ago, and with their complete installation of recording instruments they know very closely what the boilers are doing at all times. The records are posted in the boiler house, and are discussed with the men, who have been complimented for good work. The firemen have ever since been showing a decided interest in doing better.

The plant comprises four Edge Moor water-tube boilers equipped with Westinghouse underfeed stokers, a coal-weighing larry, Bailey flow meters, and a Cochrane V-notch open metering feedwater heater.

The performance for the year 1925 was as follows:

Total boiler horsepower	1,468
Average rate of driving, per cent of builder's rating	141.3
Peak load for four hours, per cent	216
Coal burned per sq. ft. per hr., average, lb	20.7
Coal burned at high rate of driving per sq. ft. per hr., average, lb	31.6
Coal consumed, lb	35,261,619
Water fed to boilers, lb	402,995,000
Water blown down, lb	10,000,000
Net evaporation, lb.	392,995,000
Steam consumed by auxiliaries, lb	50,995,000
Net steam generated for production, lb	342,000,000
Actual evaporation, steam per lb. of coal, lb	11.14
Factor of evaporation	1.047
Equivalent evaporation from and at 212 deg., lb	11.66
Steam per lb. of coal, net generation, lb	9.7
Average steam pressure (gage), lb	178
Average barometer pressure, in. of mercury	29
Average temperature of feedwater, deg. fahr	216
Average temperature of flue gases, deg. fahr	500

[1] Plant Engineer, Corning Glass Works, Corning, N. Y. Mem. A.S.M.E.

HEAT BALANCE

	B t u.	Per cent
Heat absorbed by boilers	11,310	78
Loss due to moisture in coal	22	1 15
Loss due to hydrogen in coal	420	2 90
Loss due to heat carried away in flue gases	1,900	13
Loss due to carbon monoxide	30	0.20
Loss due to combustible in ashes	150	1.07
Loss due to moisture in air	30	0.20
Loss due to radiation and unaccounted for	638	3.48
Heating value of coal	14,500	100.00

GEO H. BARRUS.[1] The author has furnished a telling example of the excellent evaporative results that could be obtained by giving suitable attention to the correct principles of boiler engineering and operation. First: efficient automatic stoking undoubtedly had secured a high degree of uniformity in the feeding, distribution, and combustion of the coal, with a small amount of waste carbon at the dump grate; a low air blast beneath the fuel beds which had further contributed to the efficiency; and a minimum force of draft at the regulating dampers and within the furnace. The arrangement of baffling gave a gradually lessened area of gas passage extending from furnace to flue, whereby short circuits had been obviated and the entire heating surface made effective. In addition, the boilers had not been forced beyond an economical rate of production. These features all combine to secure a high furnace temperature, a low flue temperature, and a good showing in the gas analysis, which are three essential elements required for high evaporative efficiency.

H. M. BURKE.[2] The author presents a splendid example of the worth of a good mechanical stoker, for in considering the furnace volume, heating surface, circulation, and flue-gas temperature, and comparing it with other makes of boilers of equal standing, very little if anything has been found that has a tendency to bring the boiler efficiency much above ordinary good practice. On the other hand, if the stoker and boiler are considered together, it is plainly seen that furnace losses have been reduced. It would be interesting to have similar test results on the same boilers, hand-stoker fired. Hand stokers today are very efficient in permitting a proper mixture of fuel and air. At this rate of driving and with the size of plant cited by the author — a two boiler plant — the labor cost would not change, as about one ton of fuel per hour was used. The furnace volume in this case limited the economical high rate of driving and the saving in first investment and cost of repair parts are reasons in favor of hand stokers. In summing up these advantages and disadvantages, it would seem that in the case of moderate-sized installations, great weight should be given the

[1] Consulting Steam Engineer, Boston, Mass. Mem. A.S.M.E.

[2] Mechanical Engineer, Mt. Hope Finishing Co., North Dighton, Mass Mem. A.S.M.E.

difference in investment between hand stokers and mechanical stokers. Also, the increased efficiency, if any, of the mechanical stoker over the hand stoker, properly operated, if the boilers are to be operated at such a modest overload, warrants serious consideration when small boilers are used. This would especially apply to plants operating for a ten- or twelve-hour day.

Wm. G. Starkweather.[1] The furnaces under these boilers have a volume of $2\frac{1}{2}$ cu. ft. per rated horsepower, which at first was considered rather unfavorable for attaining highest efficiency. Nevertheless, this installation has set a high mark in heat utilization — 83 per cent without superheater, economizer, water walls, preheating, etc.

Under initial conditions, some slag formed in the lower tube rows, but reduced air pressure greatly decreased the slag-carrying capacity of the gases rising from the fuel bed into the tube body. However, the two new units now nearing completion have the lower row of tubes spaced more widely apart, to form a "slag screen," permitting a more gradual reduction of gas temperature. This arrangement will open up more tubes in the second and third rows, for better heat absorption, on a wider angle of the radiant heat rays, than heretofore. There will probably be larger active heating surface in this installation than in usual designs.

Another leading feature of this boiler is the adequate supply of circulating water to all the tubes, particularly to the lower rows in the first pass, where major evaporation occurs. The rear connecting necks have a cross-section of 480 sq. in. each, and the entrances are rounded, thus avoiding contraction. The result is a solid stream of water pouring down each into the rear header. This header is 10 in. wide, and distributes water amply as demanded by the tubes, which of course require different quantities, depending on locations. At the front tube ends, the emulsion of steam and water discharges into another wide header which delivers upward through a large, unrestricted connecting neck with rounded edges, into the front ends of the steam drums, each having a surface of 123 sq. ft., for free release of the steam.

With $6 coal this installation is regularly producing 1000 lb. of steam for less than 25 cents fuel cost, which is a remarkable result for an industrial plant.

James J. Tyrrell.[2] All too often boilers are purchased on the basis of total heating surface in the unit. Little, if any, consideration is given the distribution of heating surface with relation to

[1] President, Starkweather & Broadhurst, Inc., Boston, Mass. Mem. A.S.M.E.

[2] Engineer, Tyrrell & Willett, Pawtucket, R. I. Mem. A.S.M.E.

that which was exposed to the fire, and which absorbed heat by radiation, and that which must depend solely on convection for its absorption power. The author noted particularly the comparatively low settings of these boilers. It should be borne in mind, however, that furnace height was a function of furnace volume. It so happened that in this patricular boiler, ample furnace volume for the fuel burned was provided by this height of setting.

In Par. 16 the author remarks that some one had advanced the idea that equal furnace volumes were required both with stokers and with pulverized fuel. It seems that this obviously is not so, for the reason that pulverized coal is burned in suspension and while the coal is in motion, and larger furnace volumes must be provided for this kind of fuel in order to obtain complete combustion in the furnace, because of the time element involved in the velocity of the fuel in the furnace. Improvements in burning arrangements have been made which permit the use of greatly reduced furnace volumes. The writer believes that while furnace volumes required for pulverized coal would be somewhat in excess of those for stokers, they would be somewhat less than those used in the past and would in general conform to the requirements of fuel oil.

The author has referred to the destructive effect of radiated heat on uncooled furnace walls. It would seem that in many cases furnace designers have not considered refractory linings of furnaces in any other way than as structures which would withstand some degree of heat, whereas properly placed firebrick linings would reflect heat to the boiler after becoming incandescent. By sloping and otherwise arranging walls with this in mind, longer life could be secured from the refractories, and the performance of the boiler could be bettered without sacrificing useful combustion volume to any great degree.

The really important thing brought out by the author is indicated by his statement that much set forth is conjecture. There is little information of value which would indicate the maximum absorption power of a square foot of clean heating surface when exposed to radiant heat. There seems to be no definite knowledge relating to the permissible draft loss through boilers, nor any consistency in the figures available pertaining to the furnace-volume requirements of different kinds of fuels or different firing methods. This lack of information available to the industrial-plant engineer is a serious obstacle to securing the best economies in the boiler plant. Knowledge of combustion, suitable apparatus for insuring proper furnace conditions, and intelligent checking would aid in operating a plant up to the point of its individual high efficiency, but it would seem that it is at least of equal importance to have data available which would make possible an intelligent choice from the equipment offered.

James W. Armour.[1] Pulverized-coal development, as applied to steam boilers, has been very rapid during the past few years, and still further rapid development may be looked for. Stoker development has now extended over a period of about forty years so that, naturally, developments and refinements of the past few years have not been as marked, and therefore have not aroused as great interest. It is interesting to note what the Brown & Sharpe Manufacturing Company has done in the way of obtaining high boiler efficiencies without going to additional expense for those refinements which are generally considered necessary for such high efficiencies. Furthermore, the results obtained at this plant are interesting in view of the tendency, during the past few years, toward higher boiler settings and larger combustion spaces. By keeping the boiler setting comparatively low, they have obtained low maintenance of brickwork and increased effectiveness of the boiler as a radiant-heat absorber, and have not had to spend money for raising the boilers or enlarging the building.

In Par. 12 the author points out that the present design of his setting is very efficient up to rates of driving of approximately 200 per cent of rating, but beyond this point the efficiency dropped off rapidly, due to increased combustible in the ash. Efficiency probably could be kept up to good central-station practice by increased investment. A longer stoker would give a longer fuel travel and, therefore, a longer time in which to burn out carbon before the material reached the dumping point. By the use of clinker grinders, the combustible in the ash would be so reduced that the loss from this source should not be proportionately any greater than at lower rates of driving. It is a question, however, whether the increased return in efficiency would pay as big dividends as an equal investment in other equipment about the plant.

As high efficiencies can be obtained with stokers as with pulverized fuel, provided conditions are equal. However, high efficiencies are more easily obtainable and furnace conditions are more easily controlled with pulverized fuel than with stokers, and pulverized fuel will permit the use of more varied types of fuel without sacrificing efficiency. Comparatively large combustion chambers are necessary at the present time with pulverized fuel, and this, of course, increases the investment. It is quite possible that development in pulverized fuel will evolve a type of burner and improvements in boilers so that smaller combustion chambers may be used effectively, and that the cost of such an installation can be brought down equal to or lower than the cost of similar stoker installation.

[1] Engineering Manager, Riley Stoker Corpn., Worcester, Mass. Assoc. Mem. A.S.M.E.

W. T. HATCH.[1] The emphasis the author has placed on the advantageous use of radiant heat is very timely. The remarkable efficiency secured is no doubt due in a large measure to the judicious arrangement of the heating surface directly over and virtually parallel to the fuel bed. Another important principle illustrated by this installation is that highly efficient combustion can be secured with very low blast pressure. With popular interest running so strongly in favor of the use of pulverized fuel, it is fortunate for the engineering profession to have definite proof that equally high efficiencies can be secured with solid fuel on underfeed stokers, and probably at substantially lower cost both for installation and maintenance. Furthermore, the radiant heat can be more directly utilized with the stoker than with pulverized fuel, and it is possible that a reduction of the distance between the stoker and the boiler tubes would make a still better showing for the stoker-fired boiler.

E. N. TRUMP.[2] A comparison of the author's tests with those on a B. & W. boiler reported by B. N. Bump[3] shows the difference between the two to be largely in the percentage of tube surface exposed to radiation. The author probably is right in his conjecture that improvement in boilers may be found in an increase in the amount of surface exposed to direct radiation from the fire.

However, it would seem that there is no reason why the stoker furnace should give any more effective radiant heat than the pulverized-fuel furnace, if combustion be perfect in the latter. The boiler of the future evidently will consist of heating surface surrounding the furnace on all sides, with an envelope of air which will receive the heat of outside radiation on its way to the furnace. The increase of boiler pressures which has raised the temperature of the water where the gas makes its exit has increased the loss from the furnace gases, and made the air heater or economizers increasingly necessary. If two pressures are used it might, in many cases, be possible to introduce the heating surface for the high pressure so that it would be exposed to direct radiation from the fire and the highest temperature, and then to pass the gas through the low-pressure boilers. Thus by pumping the water in the opposite direction, first into the low-pressure boiler and then into the high-pressure boiler, the heating might be progressive and the gas would leave the boiler at a lower temperature from the low-pressure water than if the last pass were over the heating surface of the high-pressure boiler.

[1] Providence, R. I.

[2] Consulting Engineer, Solvay Process Co.; President, Stump Unaflow Eng. Co., Syracuse, N. Y. Mem. A.S.M.E.

[3] Trans. A.S.M.E., vol. 34 (1912), p. 687.

F. W. Dean.[1] There is no statement in the paper of the method of determining the water used. Without doubt, it was metered. Somehow or other metered water usually gives high results.

The writer strongly disapproves of the author's remarks in Par. 19 concerning radiant heat. Such statements are made frequently now and cause an overvaluation of the importance of radiant heat. The fact of the matter is that radiant heat has no more value than any other heat. It is merely an accompanying feature of high temperature. If this temperature could exist without incandescence the result would be the same. A certain number of units of heat are given to the gas by the combustion of the fuel, and when some of them are absorbed by the surrounding heating surface, some are left to be absorbed further on. A large quantity is absorbed above the grate simply because the temperature is very high and the head of heat is great. The same absorption of heat would be made by the tubes if there were not surface to receive it before the tubes were reached. Where does the mysterious heat in radiant gases come from, and how can there be any but B t.u. from combustion? If there is heat that is anything but B.t.u. from combustion, it comes very near violating the law of conservation of energy. It makes no difference whether a boiler has much or little surface exposed to radiant heat, its economy will be the same. Some years ago, in Germany, two locomotives were fitted with boilers without metal fireboxes but with brick fireboxes in the same location. The results in service were not noticeably different.

The author has said that high furnace temperature is necessary for economy. This is doubtful. The writer believes that economy comes from burning the carbon to CO_2 with minimum air, perfect circulation of the resulting gases around or through the heating surfaces, and the utilization of all the heating surface.

The author has also said that the circulation in this boiler is very rapid and that this is a requisite to economy. The design of this boiler, however, is such that there is considerable resistance to circulation, because the tops of the headers are straight and horizontal, which causes some of the water which would like to go upward, as it comes from the tubes, to turn and move horizontally. Likewise, as it descends from the back end of the drum it is required to move horizontally.

The furnace seems not to be one to give a good mixture of gas, low in air, with gas having an excess of air, and certainly there is no further opportunity for a good mixture before combustion ceases. Any air drawn into the furnace through the hopper cannot be utilized because it immediately turns upward as it enters the furnace, and merely cools the heating surfaces.

[1] Consulting Engineer, Boston, Mass. Mem. A.S.M.E.

The author speculated as to the relative advantages of stoker and pulverized-coal firing, but he did not mention the great amount of unburned carbon in the ash of the former and the small amount in the latter, nor did he mention the great amount of coal put into the stoker furnace at the start, and the great amount left at the end of a run.

ALPHONSE A. ADLER.[1] It is not necessary to have a water-tube boiler in order to get a good efficiency. A horizontal return tubular boiler, 72 in. in diameter by 18 ft. long, was set 7 ft. in the clear above the floor line and had a very low bridge wall — a maximum of 18 in. above the top of the grates. With this setting it has been found possible to burn bituminous coal practically smokelessly except for a very brief interval during firing, the coking method being used.

The results obtained with anthracite also have been encouraging. The fireman was asked first to fire in his own way, and the CO_2 analysis in this case showed an average of 7½ per cent. He was then asked to build up the fire thickness until the CO_2 was a maximum, when it was found that an incandescent fuel bed 1¼ in. thick gave a practically continued average CO_2 of over 14 per cent, reaching as high as 16 per cent at one instant. On two other occasions an incandescent fuel bed of as nearly this thickness as could be estimated without accurate means of measurement, gave the highest CO_2 with practically no CO.

[1] Consulting Engineer, New York, N. Y. Mem. A.S M.E.

No. 2036

MECHANICAL ENGINEERING IN THE CRACKING, HEATING, AND COOLING OF OIL

By B. N. Broido,[1] New York, N. Y.
Member of the Society

The most important development in the last few years in the petroleum industry is unquestionably the method of refining or cracking heavy oils into lighter hydrocarbons. With most processes the cracking of oil occurs at pressures of 600 lb. and above, and temperatures as high as 900 deg. fahr. or higher, and the industry is becoming more than ever dependent upon mechanical engineers to design the apparatus for these pressures and temperatures.

The paper deals with the combustion and furnace design of pipe stills and suggests arrangements that will increase their efficiency. The importance of a knowledge of individual heat-transfer resistances between gases and tubes as well as oil and tubes is pointed out. Data for the heat transmission in each individual case are given also for heat transmission between oil and the tubes in heat exchangers.

The frictional losses in oil flowing through tubes are briefly discussed, and a table is given which shows the frictional loss or pressure drop for different sizes of tubes and different oil velocities and viscosities.

Illustrations and descriptions of newly developed joints between tubes and return headers of pipe stills are given, and desirable arrangements of the tubes are discussed; also various arrangements of heat exchangers.

THE most important development in the last few years in the petroleum industry is unquestionably the commercial application of methods of cracking heavy oils into lighter hydrocarbons. In most processes the cracking of oil occurs at high pressures and high temperatures — of the order of 600 lb. and more for the former, and 900 deg. fahr. and more for the latter.

2 The design of apparatus for these pressures and temperatures is essentially a problem for mechanical engineers. Thus far the

[1] Consulting Engineer, The Superheater Company. Deceased, February 10, 1927.

Presented at the Spring Meeting, San Francisco, Cal., June 28 to July 1, 1926, of The American Society of Mechanical Engineers.

industry seems to have applied comparatively little mechanical engineering to its solution. It is only recently that the refinery has awakened to a recognition of the fact that mechanical engineering can be of considerable assistance. Within the last few years the pipe still has been commercially developed, and only quite recently have bubble towers been applied on an extensive scale with great success, although such towers had been used for many years in other industries.

3 The present paper represents an attempt to cover some of the essential fields of application of mechanical engineering in the cracking, heating, and cooling of oil, particularly the mechanical design and performance of pipe stills and heat exchangers.

Combustion and Furnace Design of Pipe Stills

4 The operating conditions of a furnace for a pipe still are distinctly different from those of a furnace where heat is generated with the purpose of heating or evaporating any other liquid than oil. For instance, the furnace of a steam boiler should be operated at the highest temperature permissible for the refractories, or at temperatures considerably above 2000 deg. fahr. In a pipe still the hot gases reaching the oil tubes should not be higher than 1500 to 1800 deg. fahr., depending upon the processes used and the kind of oil passing through the tubes. Steam boilers, in practically all cases, are provided with baffles which divide the gas path into many passes. In a pipe still, any concentration of heat at any one part of a tube should be avoided. Therefore, the gases should be as uniformly distributed as possible over the entire length of the tubes. These two requirements have considerable influence upon the design of furnaces for pipe stills. Other means than high gas temperatures must be sought to obtain high efficiencies. The combustion space or furnace volume and the fuel-burning devices must be so arranged as to meet the above requirements.

5 In most cases pipe stills are fired with oil. Often the gas formed while oil is being cracked, the so-called "fixed gas," is utilized in the furnace in addition to fuel oil. The two important features of design for oil-fired furnaces are volume and method of introducing the fuel, and air or furnace arrangement and selection of burner. In the construction of the furnace it is very important to utilize every available cubic foot of space as combustion space.

6 Two types of burners can be used in pipe stills successfully: the steam or air atomizing burner and the mechanical or oil-pressure burner. Steam atomizing burners are often used, and reasonably good efficiencies are obtained. The original installation is less expensive and no high-pressure pumps are required. It seems,

however, that the following advantages of the mechanical burner are well established:

a Saving in steam as compared with steam atomizing burners
b Better regulation
c Less trouble from tip stoppage
d Less maintenance cost for burners
e The flame from mechanical atomizers is less intense, fills the furnace more completely with a soft, low-velocity flame, and does less damage to brickwork. This is the most important consideration for pipe stills.

7 More satisfactory results are obtained with oil-burning equipment when the burner used is designed to give a graduated flame and equal efficiency for different quantities of oil burned. It seems that this qualification applies more to mechanical burners than to steam or compressed-air atomizing burners.

8 After the oil supply is regulated to carry the load, the air supply should be adjusted to give a yellow billowing flame in the furnace and a slight haze of smoke from the chimney, this being a rough means of judging efficient combustion. Too much air results in a short dazzling-white flame similar to a tungsten electric lamp, and gives a perfectly clear chimney.

9 The author has seen a number of pipe stills in which the flame was too near to the tubes. It is always advantageous to provide a long travel to insure complete mixing of the products of combustion before they reach the oil tubes. In order to distribute the flame better over the entire furnace, it is more advisable in connection with pipe stills to have more smaller burners than fewer large ones.

10 The furnace volume required for the complete combustion of any fuel is a function of many variables, among the more important of which may be mentioned the chemical composition of the fuel, the type of fuel-burning equipment used, the shape of the combustion chamber, and the means provided for mixing the fuel and air in the furnace. In general, on account of its chemical composition and physical state, fuel oil when burned in modern mechanical atomizing burners requires less volume than any other fuel. In pipe stills, however, due to the fact that any concentration of the flame is to be avoided, liberal furnace volume should be provided. It is good practice to allow between 3 and 5 cu. ft. per gallon of oil to be burned per hour, or between 15 and 20 cu. ft. for every 100 sq. ft. of the outside surface of bare oil-heating tubes, and from 8 to 12 cu. ft. for every 100 sq. ft. of outside surface of the so-called "extended surface."

Furnace Walls

11 In spite of the fact that in pipe-still furnaces, high temperatures are not desirable, due attention should be given to the wall

construction. A perfect furnace wall would be one which under all conditions of service remains impervious to the flow of gas or air, does not crack, spall, or soften, and does not become easily injured by occasional impingement of the flame. Usually the weakest part of a wall is the joining material for the firebrick. Fireclay of approximately the same composition as the brick has been found to be the most satisfactory material with which to bond the wall. Firebrick should be obtained with as close uniformity in dimensions as possible, so that the amount of joining material may be reduced to a minimum. Fireclay brick begins to soften at 600 to 800 deg. fahr. below its melting point. With low settings this is not important, but for larger pipe stills with adequate furnace volume for proper combustion, this characteristic of firebrick must be given consideration. An effective remedy is the air-cooled wall. Many refinery engineers are of the opinion that because high temperatures are not desirable in pipe-still furnaces, preheated air would be of no value. This is entirely contrary to facts. The lower the efficiency of a furnace and the lower the furnace temperatures, the greater the benefit to be derived by preheating the air.

12 Fig. 1 is a graphic representation of the efficiency of a pipe still. It also shows the separate losses for different furnace temperatures or different percentages of CO_2. It is based on 60 deg. fahr. for air entering the furnace and on 600 deg. fahr. exit-gas temperature. Part of the heat produced is lost in the flue gases passing through the stack. In order to account for this loss we must know the temperature of the exhaust gas, the heating value, and volume, as well as the amount of the excess air or surplus air. This loss is represented as area q_1. The heat lost due to the moisture from burning of hydrogen, which moisture is evaporated and passes with the flue gas through the stack, is represented as area q_2. The latent heat of the vapor is a direct loss.

13 Part of the heat is also lost through radiation from the hot furnace walls and roof. This loss is very difficult, if at all possible, to determine directly. It can be found only by very careful tests and computations of heat balances. This loss is designated by the area q_3.

14 The area η represents the heat utilized or imparted to the oil. Depending upon the temperatures of the flue gases, it varies from 35 to 60 per cent of the total heat of the fuel burned. The efficiency, or the percentage of the total heat of the fuel utilized, depends also upon the cleanliness of the tubes inside and outside, upon the relation between the amount of gases and the amount of heating surface passed by them, and upon the arrangement of the heating surface. The more heating surface passed over by the gases and the more intimate the contact between the gases and the heating surface, the more will be the heat absorbed from the

gases, the lower the exit-gas temperatures, and the more the heat of the fuel utilized for heating the oil.

15 At a comparatively low temperature in the furnace, or between 1400 and 1800 deg. fahr., a maximum efficiency of 50 per cent will be obtained. Fortunately there are two ways of increasing the efficiency of a pipe still: First, the stack losses can be reduced, by returning a part of the flue gases to the furnace; second, gases of higher temperature can be generated in the furnace, but before coming in contact with the oil tubes their temperature can be reduced by utilizing their heat for other purposes. The following example will show the saving in fuel that can be accomplished by turning some of the exit gases back to the furnace.

16 Assume a pipe still for heating or cracking 2000 bbl. per day with a fuel consumption of 100 bbl. a day, or about 1350 lb. of oil per hour. Under normal operation the temperature of the gases reaching the steam tubes will not be over 1500 deg. fahr. In order to burn oil of about 18,000 B.t.u. per lb. and have the resulting gases at a temperature of 1500 deg. fahr. with 85 per cent furnace efficiency, it is necessary to have 45 lb. of gases per pound of oil, or 200 per cent of excess air, with a resulting efficiency of the still of 40 to 50 per cent. The lower efficiency is due to the fact that the 200 per cent of excess air which is used has to be heated from atmospheric temperature to 1500 deg. fahr. However, only a part of the heat can be utilized in the still as the flue gases leave the tubes with a temperature of about 700 deg. fahr., containing nearly half of the heat.

17 When the 1350 lb. of oil is burned with 200 per cent of excess air, approximately 6100 lb. of gases will result, at a temperature of 1500 deg. fahr. Assuming that 35 per cent of the flue gases can be returned to the furnace at a temperature of 700 deg. fahr., the other 65 per cent of the flue gases will have to be at a temperature of 1950 deg. fahr. in order to have the same amount of gases at 1500 deg. fahr. To bring this 65 per cent of gases to the temperature mentioned, however, only 1080 lb. of oil will be required. With this proposed arrangement there will be the same amount (6100 lb.) of gases at a temperature of 1500 deg. fahr. at the entrance of the still, but only 1080 lb. of oil will have to be burned, which means a saving of 270 lb. per hour, or 19½ per cent.

18 Another advantage is claimed for this arrangement. The tubes which burn out first are those located nearest the furnace walls, as they receive heat from the furnace as well as radiant heat from the brickwork. At least a part of the returning gases could be introduced into the furnace near these tubes, and would to a certain extent cool them. While this could also be done in a regular furnace, it would, however, reduce the efficiency.

19 In every refinery great quantities of steam are required, and this is usually generated in one or more boiler houses and thence

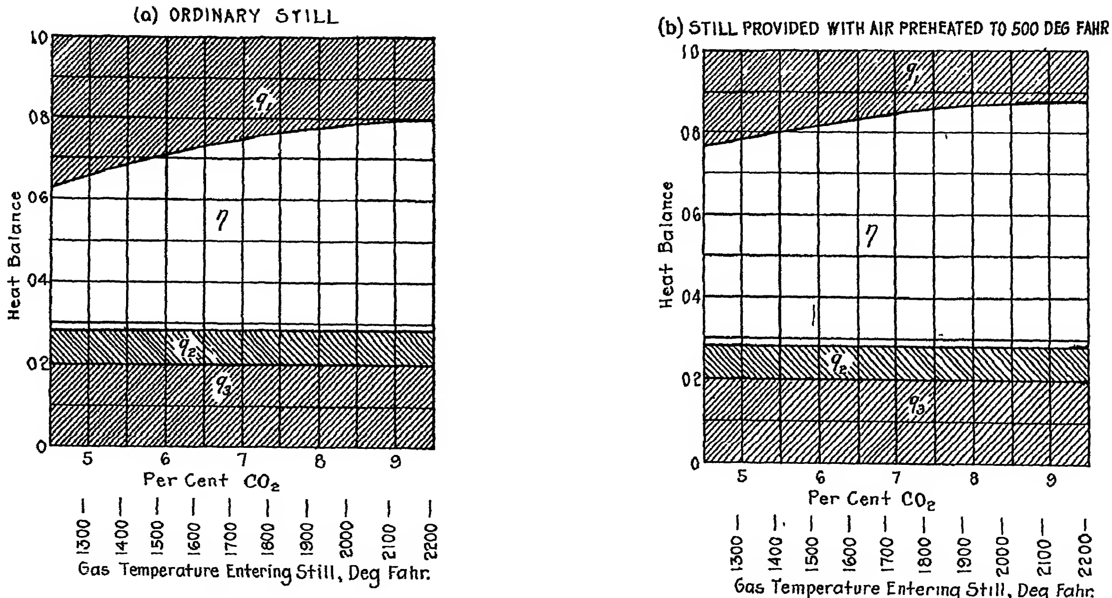


FIG. 1 GRAPHIC REPRESENTATION OF THE EFFICIENCY OF A PIPE STILL

(η = efficiency of still; q_1 = stack loss; q_2 = loss due to moisture from burning hydrogen; q_3 = radiation loss; q_4 = proportion of heat returned to furnace from extra gases; q_5 = proportion of heat absorbed by water tubes.)

(Continued on following page)

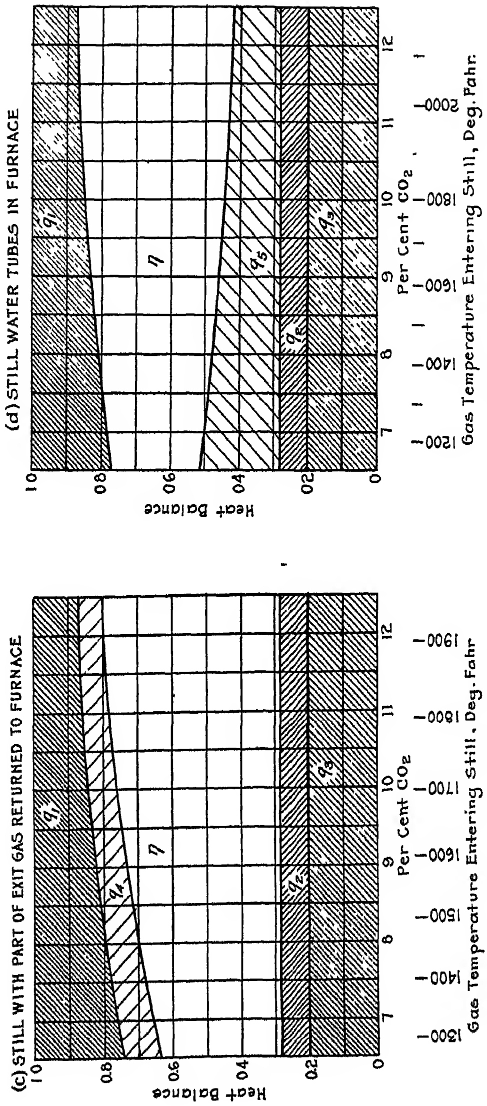


FIG. 1 Continued from preceding page

distributed to the places of use. The efficiency of a pipe still could be increased considerably if the fuel could be burned in the furnace with a low percentage of excess air and a resulting high temperature of the flue gases. In order to reduce the temperature before entering the oil tubes, one or more rows of water tubes could be installed and the steam generated in these tubes.

20 Assume the same example as before: If 6100 lb. of gases per hour are generated in the furnace at a temperature of 2000 deg. fahr., 1500 lb. of oil will have to be burned in the furnace. If the water tubes cool the gases 500 deg., the latter will enter the pipe still at a temperature of 1500 deg. fahr., leaving the tubes at 700 deg. fahr., with the gases cooled from 2000 down to 700 deg. fahr.; and with a furnace efficiency of 85 per cent the total efficiency of the oil and water tubes will be 72 per cent instead of 50 per cent where the gases are generated directly in the furnace with a temperature of 1500 deg. fahr. Taking into consideration, however, the fuel which would be required to generate the same amount of steam in the boiler house, it will be noted that a much better utilization of the heat available in the fuel is accomplished.

21 Fig. 1-*b* is a graphic representation of the efficiency of a pipe still similar to Fig. 1-*a*, but with the air for combustion preheated to 500 deg., either in the brick wall or by a separate air heater utilizing the heat of the escaping flue gases. Fig. 1-*c* shows the efficiency of a pipe still with 35 per cent of the exit gases returned to the furnace, while Fig. 1-*d* represents the efficiency where water tubes are placed in the furnace ahead of the oil tubes. Area q_4 indicates the increase of efficiency due to the returning of the flue gases, and area q_5 that due to the water tubes.

22 Another and better method of utilizing the heat of the fuel with a comparatively low furnace temperature would be to absorb as much heat as possible by radiation. While advantage is taken of this method in steam boilers, experience has shown that it is not advisable to subject oil tubes directly to the flame of the furnace. In recent years a furnace, known as the "carborundum" furnace, has been proposed which makes it possible to take advantage of the radiant heat without exposing the gases directly to the flame of the furnace. Fig. 2 shows a pipe still provided with this type of furnace. The furnace is an internally fired combustion chamber made from a material consisting mainly of carborundum. Carborundum will stand up under any temperature and under all ordinary conditions, and if allowed to radiate its heat will be good for continued use even over a period of years. It has a high heat conductivity, so that the heat generated within the furnace is quickly transferred through the walls from which it is being transmitted by radiation. It is claimed that fully 50 per cent of the heat generated can be made effective as radiant heat, so that the heat of the gases leaving the furnace is materially

reduced. It is also claimed that the material used for this furnace is very dense and strong. The impinging action of the burner flame, therefore, has no effect upon it. The carborundum furnace can be made in almost any reasonable size. The standard sizes are capable of burning from 2 to 100 gal. of oil per hour. They

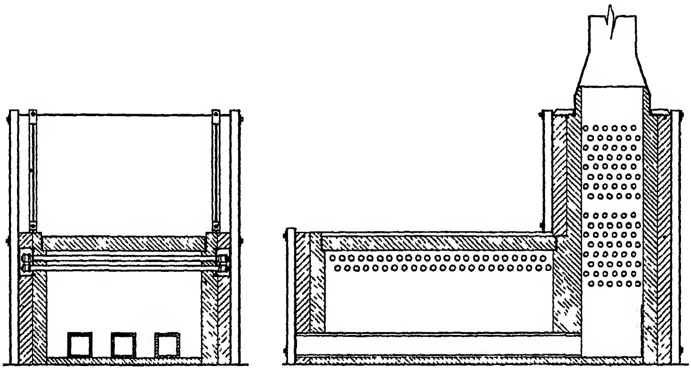


FIG. 2 PIPE STILL WITH CARBORUNDUM FURNACE

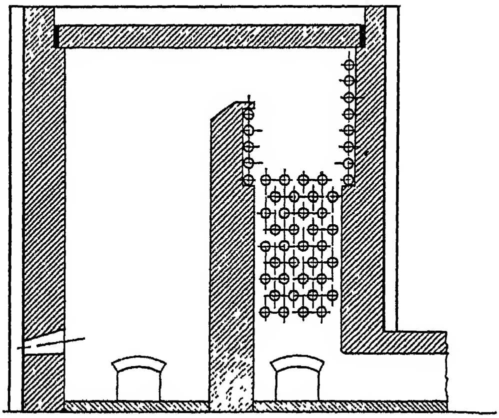


FIG. 3 PIPE STILL WITH A NUMBER OF TUBES ABSORBING HEAT BY RADIATION ONLY

vary in cross-section from approximately 6 sq. in. to 3 sq. ft. They can be set up in lengths of from 3 to 30 ft.

23 The furnace is usually fired in the bottom section, combustion starting close to the burner tip and progressing through the entire length toward the back. If it is necessary to burn more fuel than the capacity of a single carborundum furnace will provide, two or more furnaces should be installed. With this furnace less excess

air seems to be necessary than in general practice. A higher CO_2 content can be obtained with the same temperature of the gases reaching the oil tubes. Due to the fact that a portion of the heat generated by the combustion of the fuel is transmitted through the walls of the furnace and absorbed by oil tubes as radiant heat, the actual temperature of the gases leaving the carborundum furnace is materially reduced, so that the pipe still can be so arranged that gases of a desired comparatively low temperature will come in direct contact with the oil tubes. At the same time, however, a considerably better utilization of the fuel in the furnace can be accomplished. One of the main requirements of a

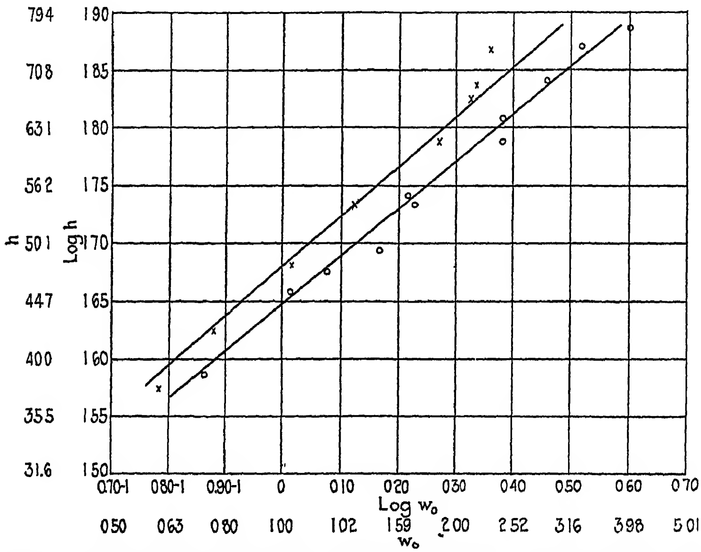


FIG. 4 LOGARITHMS OF HEAT-TRANSFER COEFFICIENT AND OIL VELOCITIES OF PIPE STILLS WITH 4-IN. AND 2⅜-IN. TUBES

pipe still — that the gases be evenly distributed over the entire length of the tubes — is not met by this furnace, as all the gases from the carborundum furnace are ejected from a comparatively small area. Fig. 3 shows another arrangement where radiant heat can be utilized and in which this disadvantage is eliminated.

HEAT TRANSMISSION IN PIPE STILLS AND HEAT EXCHANGERS

24 The whole subject of heat transmission in pipe stills and heat exchangers is as yet comparatively uncertain. Data from heat transfer in boilers cannot be applied to the case of oil stills, because in boilers part of the heat is absorbed by the process of evaporation, while in pipe stills most of it is used to increase the temperature of the oil, and the tube temperature naturally has to

follow this increase. Furthermore a number of tests have been made and formulas established for heat transmission from gases to tubes. Practically all of them, however, were limited to gases flowing inside a tube. Based on theoretical analyses and experimental tests, H. Reiher comparatively recently made a study of heat transmission in Munich, Germany,[2] from the gas flowing outside of tubes with different tube arrangements — in straight and staggered rows, and with different numbers of tubes. For pipe stills, only staggered arrangements of many rows of tubes need be considered. Reiher found that the relation between the heat transmission and the factors influencing it can be expressed in a formula of the following nature:[3]

$$h = C \frac{\lambda}{d} \left(\frac{wd\rho}{\mu}\right)^n$$

where the constant C and the exponent n depend upon the arrangement of the tubes. For staggered tubes in two rows:

$$C = 0.100 \qquad n = 0.69$$

for three rows:

$$C = 0.113 \qquad n = 0.69$$

for four rows:

$$C = 0.123 \qquad n = 0.69$$

for five rows:

$$C = 0.131 \qquad n = 0.69$$

25 H. Reitschel[4] also made a study of heat transmission between gases and tubes, the gases being outside of the tubes. He found the same relation to obtain as did Reiher, but his constants and exponents were somewhat different (See Appendix). Reitscher conducted his tests with rather low gas velocities, while Reiher used considerably higher velocities. It seems that a compromise between the two, particularly for the lower velocities, would give the correct results (see Appendix).

26 The above formula gives the heat transmission between the gases and the tubes per hour per unit of surface area per degree temperature difference between the gas and the metal of the tube. It is independent of how the heat is absorbed from the tube inside, so that it can also be applied to pipe stills. The fluid inside of the tube will influence the temperature of the metal, and in this way also the total heat transmission from the gas.

27 From results of tests with two pipe stills having respectively 4-in. O.D. tubes ⅜-in. thick and 2⅝-in. O.D. tubes ¼-in. thick, the author determined, applying the above formula, the temperature

[2] Mitteilungen über Forschungsarbeiten (Berlin), V.D.I. Heft 269.

[3] For nomenclature see Appendix.

[4] Mit. der Prüfungs Anstalt für Heizungs und Lüftungseinrichtungen der Technischen Hochschule Berlin, Heft 3.

of the tube wall at different parts of the stills, and from this the heat-transmission coefficient between the tubes and the oil. This coefficient varies with the velocity of the oil and its viscosity, and also with the diameter of the tube. No attempt was made to determine the influence of the oil viscosity upon the heat transmission, but a number of readings at different oil velocities with approximately the same viscosity or the same temperatures led the author to certain conclusions which may be of general interest.

28 The oil used in the 4-in. still was heavy Mexican crude oil of 12 deg. Baumé. In the 2⅜-in. pipe still, gas oil of 36-40 deg. Baumé was used. In the former the oil was heated from 400 to 850 deg. fahr.; in the latter, from 600 to 900 deg. fahr. At this high temperature the viscosity of the oil is comparatively low and is not reduced appreciably with the increasing temperature. The coefficient was determined for each of the above two tube sizes separately. An effort was made to develop relations between the heat transfer and the velocity of the oil in the tube. The general relation can be expressed in the following equation:

$$h = x^m$$

The exponent m was determined in the following way. If

$$h = \text{Const.}\, w_0^{\,m}$$

then

$$\log h = m \log w_0 + \log \text{Const.}$$

where h = coefficient of heat transfer in B.t.u. per hr. per sq. ft. per deg. fahr., and

w_0 = velocity of oil inside tube, ft. per sec.

By plotting the logarithm of h against the logarithm of the oil velocity the curve gives the expression for the exponent m. If it is a straight line, the exponent has a constant value. The section on the ordinate gives the logarithm of the constant.

29 In Fig. 4 the logarithms of h have been plotted as ordinates and those of w_0 as abscissas for both pipe stills. While all the points, particularly those for the smaller still, do not follow directly in a straight line, this, however, is probably due to the slight changing of the viscosity with the slightly changing temperature. It is reasonable to assume that if all the readings could be taken at exactly the same temperature or the same viscosity, these points would be closer to the straight line.

30 From this we may conclude that the exponent m for the 4-in. still is 0.84, while for the 2⅜-in. still it is 0.87. The constants are 44.5 and 48.1, respectively, for the two stills. The difference is to be expected, as in general, all other conditions being the same, the heat transmission increases with decreased tube diameter. The above constants or coefficients hold for the sizes of the pipes men-

tioned, for oil temperatures between 400 and 900 deg. fahr., and for velocities of from 2 to 8 ft. per sec. While, as mentioned above, the heat transmission in pipe stills depends upon the viscosity of the oil, for cracking, where the oil temperatures are very high, the viscosity will have a comparatively slight influence. When the pipe still is used for heating oil, particularly heavy oil, lower values for the heat-transmission coefficient should be taken.

31 The heat-transfer coefficient between the gases and the tube, and the oil and the tube, enables us to determine the temperature of the tube wall under certain conditions. The knowledge of the wall temperature is very important in connection with the operation of a pipe still. For instance, in the production of lubricating stocks or in the running of any crude, it is extremely desirable to eliminate the pyrogenic decomposition. The temperature of the oil immediately near the tube wall is higher than the average temperature of the oil. It is therefore important, when the pipe still is used for heating crude oil, to have the temperature of the tube metal not too high in order not to overheat the oil near it.

32 In cracking, it is very essential that the temperature of the oil shall not exceed a certain level. The object of cracking is to alter the heavy molecules so that low-boiling hydrocarbons will be produced that would not appear on ordinary distillation. An excessive temperature will mean a large production of products of decomposition. As soon as the action begins to be a total decomposition, permanent gases will be formed, and both carbon and heavy asphaltic bodies will remain as residue. It is therefore very desirable that the temperature of the metal of the tubes in which oil is being cracked be kept below a predetermined point. As a matter of fact, the author is of the opinion that too little consideration is at present given to the metal temperature of tubes in pipe stills. If this were done, less "fixed" or uncondensable gas and less carbon would be formed, so that the cracking of oil would be more profitable.

33 The foregoing heat-transfer constants and exponents apply to bare steel pipes, clean inside and outside, and when the heat is absorbed by convection only, and in no degree by radiation. If the tubes are covered inside with carbon or outside with soot, the heat-transmission coefficient is accordingly lower. The subject of heat absorption by radiation in furnaces as well as from non-luminous hot gases is discussed in a paper presented by the author at the annual meeting of the Society in December, 1925.[5]

34 A number of pipe stills have been installed with the so-called extended surface, or tubes covered with cast-iron sleeves or gills which increase the surface in contact with the hot gases. As the principal resistance to the heat flow from gases to oil is on the gas

[5] Radiation in Boiler Furnaces, Trans. A.S.M.E., vol. 47 (1925), p. 1123.

side, an increase of the surface exposed to the gases per unit of length seems to be beneficial. The total length of the coil required to absorb a certain amount of heat can be reduced appreciably, and, correspondingly, the pressure through the coil would also be reduced. When the still is used for heating oil to a comparatively low temperature, or when light oil is heated even to higher temperatures, extended surface seems to be advantageous. It should not be forgotten, however, that with the extended surface a new resistance to the heat flow is introduced, namely, that between the cast-iron gills and the steel tube, which resistance increases the longer the still is in operation. As with the extended surface a certain amount of heat can be absorbed with a shorter tube length or with less inside heating surface, the temperature difference between the oil and the tube wall must necessarily be correspondingly greater. As mentioned above, high tube-wall temperatures are undesirable with a pipe still used for cracking. From this one would conclude that for cracking stills, whenever possible, bare tubes should be used.

35 Also with heat exchangers between oil and oil, or between oil and water, it is advisable to consider separately each individual film resistance, or the heat transfer between each of the fluids and the wall.

36 Very interesting and accurate tests with heat transmission in oil-coolers were made by E. Heinrich and R. Stueckle[6] at Stuttgart. Five different arrangements of coolers were tested. The oil was cooled by water. One cooler was a double tube with the inside brass tube about 1¼ in. O.D. and 0.04 in. thick, and the outside steel tube 3¼ in. I.D. The second consisted of a bundle of brass tubes of the same size enclosed in a shell, with water inside and oil outside the tubes, both flowing parallel to each other but in opposite directions. The third, fourth, and fifth coolers were standard commercial heat exchangers with baffles inside, forcing the oil to change its direction and flow more or less perpendicular or normal to the tubes. The oil used for the tests had a specific gravity of 0.86 and a viscosity of 148 Saybolt seconds at 100 deg. fahr. The tests apparently were carried out carefully. The coolers themselves, as well as the weighing and measuring apparatus, were all in good condition. The results, therefore, should be very reliable.

37 From a comparison of exponents for all five coolers in these tests it would appear that where the oil flows normal or practically normal to the tubes carrying the water, the heat-transmission coefficient varies with the 0.3 power of the velocity of the oil. This seems to be independent of other conditions and of the design of the cooler. The exponent, however, increases the nearer the flow

[6] Forschungsarbeiten auf dem Gebiete des Ingenieurwesens. V.D.I., Berlin, Heft 271.

is streamline or parallel to the tubes carrying the water. Where the flow is parallel all the way, the heat transmission increases in the same proportion as the oil velocity. To a certain extent the above is true of all heat exchangers, as will be shown later.

38 By the above-mentioned heat-transmission coefficient in connection with the oil heat exchanger is meant the total heat-transmission coefficient between one fluid and the other. As the heat transmission between water and a tube under different conditions is fairly well established,[7] the author has endeavored to determine,

TABLE 1 HEAT-TRANSFER COEFFICIENT *h* BETWEEN OIL AND TUBES IN OIL COOLERS

	h, B.t.u. per hr. per sq. ft. per deg. fahr.	Water velocity, ft. per sec.	Oil velocity, ft. per sec	Oil temp. deg. fahr.
COOLER I—Oil flow parallel to tube...	23	0.89	1.66	179
	32	0.88	2.38	180
	43	0.89	2.93	181
	51	0.88	3.53	181
COOLER II—Oil flow parallel to tubes almost over the entire length, and normal to tubes at inlet and outlet.	45	0.42	2.21	168
	39	2.30	2.21	162
	35	0.77	1.84	165
	40	0.77	2.16	166
	47	0.77	2.62	168
	52	0.76	3.28	169
COOLER IV—Normal to tubes over the entire length.	53	0.33	0.46	130
	77	0 32	0.86	141
	96	0.33	1.23	141
	92	0.33	1.23	144
	116	0.33	1.53	143
	140	0.33	1.82	144
	153	0.32	1.84	147

first, the water-film coefficient for the most important tests; second, the heat transmission between the metallic walls and the oil; and third, the constants and exponents of the heat transmission depending upon the oil velocities. The results are given in Table 1, which gives the coefficient *h* for oil of three different coolers at different oil velocities. With all coolers *h* increases with the oil velocity.

TABLE 2 CONSTANTS AND VELOCITY EXPONENTS FOR HEAT TRANSMISSION IN OIL COOLERS

(C = constant and m = exponent of heat-transfer coefficient depending on velocity.)

	C	m
COOLER I—Oil flow parallel to tube	14	1.00
COOLER II—Oil flow parallel to tubes almost over the entire length, and normal to tube at inlet and outlet	23	0 72
COOLER IV—Oil flow normal to tubes over the entire length	84	0.63

It also increases with the oil approaching normal or perpendicular flow to the tubes.

39 Table 2 gives the constants and velocity exponents for the same exchangers. It is interesting to note that with cooler I the exponent is 1, which shows that the heat transfer between oil and a tube increases in direct proportion to the oil velocity when

[7] McAdams, *et al.*, in Publications of the Massachusetts Institute of Technology, Serial Nos. 17, 40, 77, 92, and 121.

the oil flows parallel to the tubes. It increases, however, with the 0.63 power of the oil velocity when the oil flows normal or perpendicular to the tube. It is reasonable to assume, and has also been verified in many instances in practice, that this applies to practically all heat exchangers independently of what the other heating or cooling fluid is, so long as the oil velocity is high enough or the flow is turbulent. On the other hand, the constant is smallest with the oil flow parallel to the tubes, and is greatest with the flow normal to the tubes.

40 Heat transmission in heat exchangers also depends upon the viscosity of the oil. Interesting experiments were conducted with oil heated by steam in the Research Laboratory of the Rensselaer Polytechnic Institute.[8] All other conditions being the same, the heat transmission between oil and the tube varied from 49 B.t u. per hr. per sq. ft. per deg. fahr. temperature difference for kerosene with a viscosity of between 36 and 40 Saybolt seconds at 100 deg. fahr., to 40 B.t.u. for crude oil of 53 Saybolt seconds at 100 deg. fahr. and 30 B.t.u. for cylinder oil with a viscosity of 95 Saybolt seconds at 212 deg. fahr.

Friction Losses in Oil Flowing through Tubes

41 Most of the modern pipe stills are arranged so that the oil flows through all tubes in series. In a cracking still, where the temperature of the oil is to be raised 300 to 600 deg., the total length of the tubes through which the oil flows may be 2000 ft. or more, and as it is desirable to have high oil velocity in the pipe still, the friction loss or pressure drop in the still becomes very high. At the same time it is of importance that the oil be heated gradually and uniformly in a pipe still. In cracking oil it is very important that the oil be of the same temperature throughout the cross-section of the pipe. With low oil velocity, or where the flow is parallel to the tubes, the layers of the oil nearer to the tube wall are considerably hotter than those in the center. With high velocity, or in the turbulent region, however, the heated outer layers of the oil are constantly carried into the cooler core of the pipe and a more constant temperature throughout the cross-section of the pipe is obtained. The study of the flow of oil through pipes and the frictional loss is therefore an important factor in connection with pipe stills.

42 It has been found from experiments that the frictional factor f must be some function

$$f = \phi\left(\frac{dw\rho}{\mu}\right)$$

of the pipe diameter d in ft., the average fluid velocity w in ft. per

[8] *Mechanical Engineering*, vol. 43 (1921), no. 9, p. 616.

sec, the density ρ in lb. per cu. ft., and the viscosity μ in poundal-seconds. When plotting f against $\phi\left(\frac{dw\rho}{\mu}\right)$ a single curve, Fig. 6, is obtained which is sufficiently accurate for ordinary purposes.

43 This curve shows that at constant velocity in a given pipe all liquids of the same kinematic viscosity have equal loss of frictional head. Liquids of the same kinematic viscosity have equal Saybolt seconds. From this it follows that for flow at the same velocity in the same pipe, liquids of the same Saybolt viscosity have equal loss of frictional head. This fact is of great importance

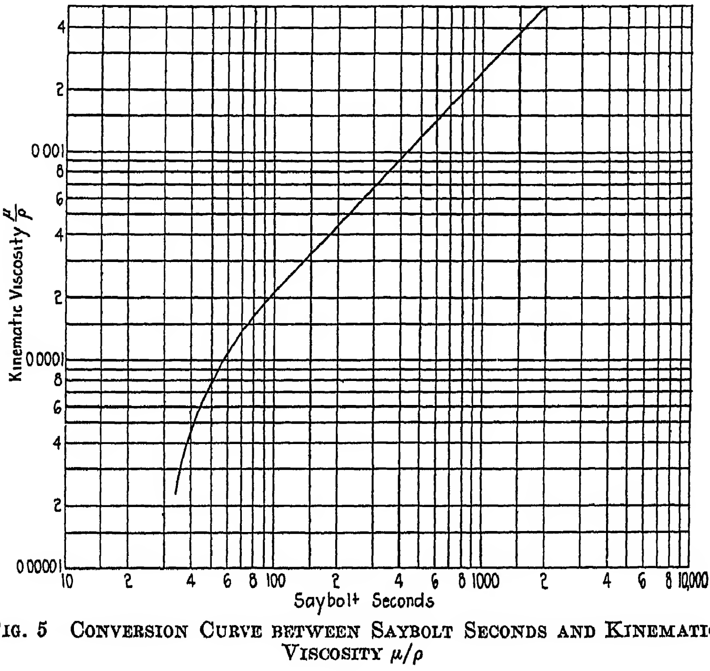


FIG. 5 CONVERSION CURVE BETWEEN SAYBOLT SECONDS AND KINEMATIC VISCOSITY μ/ρ

as it allows a determination of the kinematic viscosity from the viscosimeter readings and Fig. 5, by means of which, for any given flow velocity and pipe diameter, the frictional factor f may be computed.

44 As might have been expected, the curve of Fig. 6 consists of two branches, one for straight-line flow, and the other for turbulent flow. The branch for the straight-line flow is a straight line with a slope of 45 deg., and its intercept on the axis of ordinates is 64. The equation for this line is $f = 64 \div (dw/\mu)$. The other branch is curved and no practical mathematical expression is available. The value of the critical region is clearly seen, and it will be found

that straight-line flow changes to uncertain flow at $dw\rho/\mu = 2000$, and then changes again to regular turbulent flow at $dw\rho/\mu = 3000$.

45 If μ = viscosity of main body of fluid, poundal-sec.
ρ = density of main body of fluid, lb. per cu. ft.
d = actual inside diam. of pipe, ft.
L = length of straight pipe, ft.
w = average velocity of fluid, ft. per sec.
h = pressure drop due to friction, ft., and
g = acceleration due to gravity = 32.2 ft. per sec. per sec.

then

$$h = \phi\left(\frac{dw\rho}{\mu}\right) \times \frac{L}{d} \times \frac{w^2}{2g} \quad \text{[1]}$$

corresponding to the Fanning equation:

$$h = f \times \frac{L}{d} \times \frac{w^2}{2g} \quad \text{[2]}$$

which gives

$$f = \phi\left(\frac{dw\rho}{\mu}\right)$$

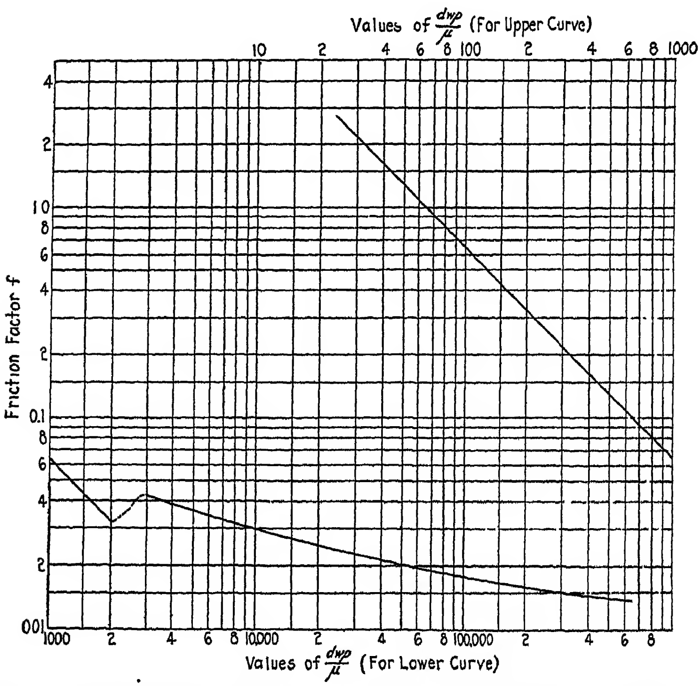


FIG. 6 CURVE SHOWING RELATION BETWEEN THE FRICTIONAL FACTOR f AND $dw\rho/\mu$

The critical region is:

$$2000 < \frac{dw\rho}{\mu} < 3000 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [3]$$

If the frictional loss is to be expressed in pounds per square inch, multiply the above value of h by 0.433 and by the specific gravity of the fluid.

46 Allowance must be made for loss at bends and valves. Most engineering handbooks give the loss in terms of an equal length of straight piping. The equivalent lengths should be added to the straight length of the piping before finally deciding on the total loss, to make sure that this is not greater than permissible.

47 The method of computing the pressure drop or loss of head due to friction will then be:

a Obtain the viscosity and density of the liquid at the required temperature.

b If the obtained value is in Saybolt seconds, convert these into kinematic viscosity μ/ρ by means of Fig. 5.

c Insert μ/ρ together with the assumed values of d and u in [3]. If the value is within the critical region, d or u must be changed.

d Enter the value of $dw\rho/\mu$ in Fig. 6 to obtain the frictional factor.

e Insert the value of f in [2] which will give the desired frictional loss.

48 For rough or corroded pipe or where the fluid may decompose or deposit solid matter on the wall, an amount by which f must be increased should be determined. This can only be a matter of judgment in each particular case as no definite method exists. The formulas do not apply to heavy tar-like oils of very high viscosity, as these oils in some ways behave more like solid matter. It has been shown that the critical velocity is proportional to the kinematic viscosity. If it were attempted to exceed the critical velocity for such heavy oils, this would result in very high pressures and correspondingly excessive frictional losses.

49 The formulas are based upon experiments with drawn steel or brass pipe. When the method is applied to wrought-iron pipe 1 to 6 in. in diameter, add to the frictional losses 12 to 3 per cent.

50 Table 3 gives the pressure drop in pounds per square inch for different pipe diameters, different oil velocities, and different oil viscosities.

Mechanical Design of Pipe Stills and Heat Exchangers

51 The main reason for the rapid development of the pipe still is due to the fact that oil flows through the tubes with high velocity, protecting them from overheating. Particularly in cracking stills, the high velocity of the oil keeps the liberated carbon in suspension and carries it along so that it settles in con-

tainers which are not in contact with the hot gases, and from which it can be removed easily. In earlier pipe stills, various devices were used to protect the bottom of the shell directly exposed to the furnace. None of these devices, however, were as effective as the high velocity of the oil. Tubular heaters on pipe stills are therefore now being used successfully whenever it is necessary to heat oil, particularly for higher pressures. They are used for reheating oil before fractionating, for rerunning light distillates, but more particularly, however, for cracking oil by various processes.

TABLE 3 PRESSURE DROP OF OIL FLOWING IN PIPE—FRICTION HEAD IN FT. PER 100 FT. OF PIPE

Pipe diameter, in.	Oil velocity, ft per sec.	Viscosity in Saybolt Seconds 50	100	150	200	300	400	500	600	800
1....	2	2.60	6.38	10.00	13 60	20.70	27.70	34.60	41.50	55.50
	4	12.40	12.76	20.20	27 20	41.40	55.40	69.20	83.00	111 00
	6	25 00	26.00	30.30	40.80	62.10	83.10	103.80	124.50	166 50
	8	41.35	53.90	40.40	54.40	82.80	110.80	138.40	166.00	222 00
	10	61.40	79.10	75.16	68.00	103.50	138.50	173.00	207.50	277 50
2....	2	1.53	1.60	2.50	3.38	5.14	6 88	8.60	10.34	13 78
	4	5.12	6.65	5 00	6.76	10.28	13 76	17.20	20.68	27 56
	6	10.40	13.42	15.12	11.81	15.42	20.64	25.80	31.02	41.34
	8	17.14	22.12	24.90	26.95	20.56	27.52	34.40	41.36	55.12
	10	25.20	32.85	37.10	40.00	36.73	34.40	43.00	51.70	68.90
3....	2	0.74	0.64	1.00	1.32	2.04	2.72	3.40	4.08	5.44
	4	2 48	3 21	3.31	2.64	4 08	5.44	6.80	8.16	10.88
	6	5.48	7.09	7.95	8.60	6.35	8.50	10.62	12.75	17.00
	8	9.78	12.70	14.35	15.51	17.00	12.80	14.87	17.85	23.80
	10	15.15	19 85	22.10	24.05	26 90	27 30	21.50	22.95	30.60
4....	2	0.64	0.83	0.64	0.84	1.28	1.72	2.16	2.60	3.44
	4	1.91	2.47	2.78	2.88	2.40	3.23	4.03	4.85	6.44
	6	3.84	5.00	5.67	6.10	6.46	4.85	6.04	7.28	9 65
	8	6.32	8.30	9.26	10.10	11.23	11.72	9.03	9.71	12.86
	10	9.35	12.15	13.70	14.78	16.57	17.85	17.70	14.70	16.07
6....	2	0.37	0.48	0.53	0.41	0.56	0.75	0.94	1.12	1.48
	4	1.23	1.60	1.80	1.94	2.14	1.55	1.89	2.24	2.97
	6	2 72	3.54	3.98	4.30	4.80	5.16	5 05	4.28	4.69
	8	3.98	4.97	5.56	6.36	6 96	7.55	7.95	8.54	6.36
	10	5.90	7.14	8.07	9.00	9 93	10 85	11.79	12.40	12.40
8....	2	0.24	0.31	0.36	0.39	0.30	0.39	0.56	0.60	0 86
	4	0.82	1.05	1.20	1.31	1.46	1.57	1.24	1.20	1.61
	6	1.68	2.10	2 35	2.60	2.94	3.11	3.36	3.52	2.78
	8	2 83	3.58	3.88	4.33	4.77	5.21	5.66	5.81	4.62
	10	3.97	5.36	5.83	6.30	7.00	7.46	8.16	8.39	9.32

52 There is liable to be one difficulty with pipe stills. In the old shells any carbon accumulating at the bottom could be removed comparatively easily by having the shell cooled off so that a man could get inside and remove the carbon. The only way to clean pipe stills is from the tube ends. The pipe still must therefore be so constructed that each end of the tubes is easily accessible.

53 One of the first pipe stills built was in connection with the Burton process. It was designed similar to a water-tube boiler where the tubes are rolled in headers having handholes opposite the tube ends. These stills have only natural circulation and the velocity of the oil is therefore comparatively low. However, the

advantage of high velocity in pipe stills is now fully recognized, and those in connection with other processes are arranged so that the oil flows through the tubes which are in series and have return bends on the ends. Stills must be so designed as to allow access to the tubes.

54 The recently developed pipe stills are made of straight lengths of pipes, connected by return headers to form a continuous coil or series of coils. There is a handhole opposite the end of each tube, closed by a removable tapered, screwed, or ball plug. In some designs the entire return head is made removable. All of the return headers and plugs are exposed by opening a door in the setting wall, and the tubes are then opened for their entire length by removing the return head or the plug opposite each end. A cleaner or scraper can be put through every tube without cooling down the setting and with a minimum of effort.

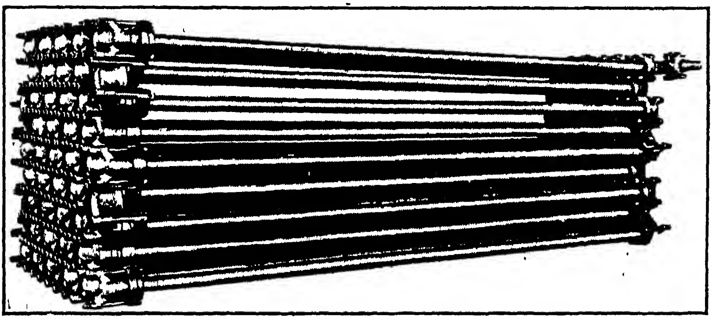


Fig. 7 Assembly of Tubes and Return Bends for a Pipe Still

55 The application of the pipe still has considerable influence upon the design of the return bend and joint. For instance, if heavy oil is heated to high temperatures and pressures, a part of it becomes decomposed or cracked to a lighter oil, in which case carbon necessarily deposits in the tube and has to be removed frequently. For this reason the accessibility of the ends is of utmost importance. When naphtha is heated to a moderate temperature at low pressure, less expensive joints can be applied.

56 Fig. 7 shows the tube assembly of a pipe still applicable for cracking of oil at high temperatures and pressures. Of particular interest is the point, Fig. 8, between the return bend and the pipe. This ball clamp joint consists of a pipe *A*, the ends of which are offset to a ball end in a forging machine and then machined and ground to correct contour. The return bend *H* is of cast steel and the seat to receive the pipe ball ends is machined and ground to 45 deg. The clamp *E* for securing the tubes to the return bend is of drop-forged steel in which is inserted a split washer *F*, the removal of which permits the application and removal of clamp *E*

over pipe ends. The connecting bolts and nuts are made from high-grade alloy steel. This arrangement has proved very satisfactory for the severe service of a pipe still for oil at high pressures and temperatures. Many refineries now consider it the most reliable and safest joint for this service. This arrangement has also the advantage that a tube can be transferred from one location to another in the pipe still and can also be turned in its place.

57 In cases where a comparatively inexpensive construction is desired and where replacement of tubes cannot be accomplished in a short time, the design shown in Fig. 9 offers some advantage. Since commercial pipes can be used, it is not necessary to have them go through a shop. These designs consist of a return bend into which the tubes are expanded and which are provided with ball plugs opposite the two ends. These designs are recommended

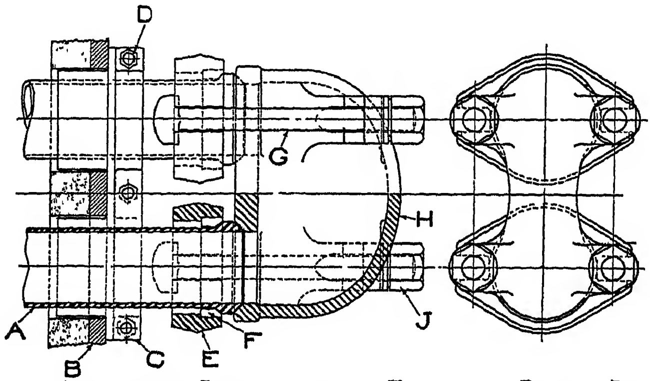


Fig. 8 Detachable Joint between a Tube and a Return Bend

where either heavy or light oil is heated to moderate temperatures. For large pipe stills, sometimes a combination of any two of the above designs can be applied, in which case joints like that of Fig. 9 are placed in the cooler part of the still.

58 Fig. 10 shows return bends made from either drop forgings or cast steel. In this design the closure cup, which is inserted in a tapered seat, is held securely by means of a holding member and setscrew, which are in turn held by overhanging lugs or a yoke. The application of a graphite paste is required on the cup in order to insure tight joints for high pressures and temperatures.

59 The arrangement of the tubes with respect to the gas flow is worthy of serious consideration. Whenever possible it is advisable to have the gas flow downward over the tubes. Any solid carbon precipitated is heavier than the oil, and settles at the bottom of the tubes. With the gases flowing downward, the upper part of the tubes is in better contact with the gases, while the lower part hardly comes in contact with them. The carbon, therefore, does not bake on the tubes and is considerably easier to remove.

60 As in many other heat-transmission apparatus, it is advisable to have the gases flow opposite to the direction of the oil, or have the oil enter where the gases leave the still and the oil outlet

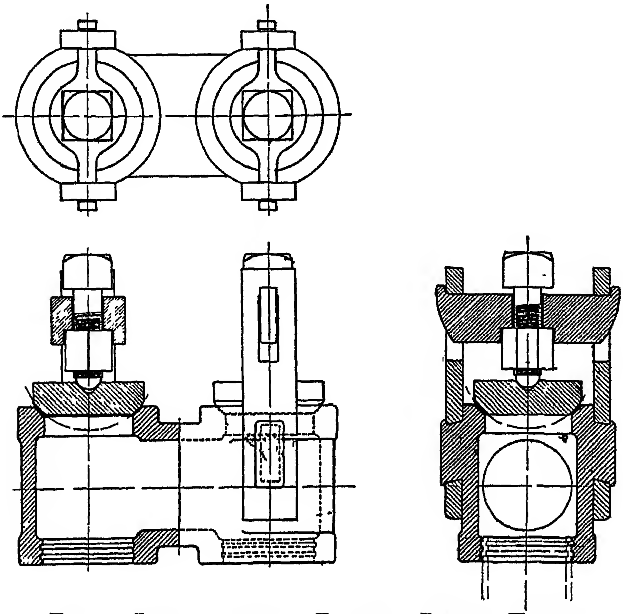


FIG. 9 JOINT BETWEEN A TUBE AND RETURN HEADER

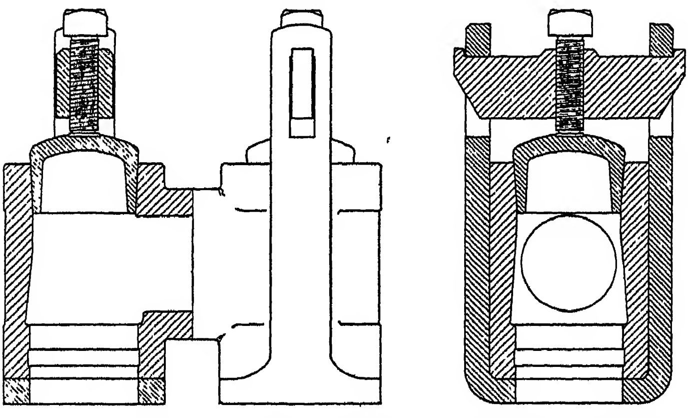


FIG. 10 JOINT BETWEEN A TUBE AND A RETURN HEADER WITH A TAPERED CUP PLUG

where the hottest gases enter the still. With this arrangement, heat transmission is very efficient. The gases can also be cooled off to a minimum, increasing the efficiency of the still. In some

cases, however, in connection with pipe stills, deviations may be advantageous.

61 With highly viscous oil, which would cause a high pressure drop through the still, it may be advisable to arrange the oil to enter the still where the gases first come in contact with the tubes, in order to decrease the viscosity quickly. After the viscosity is reduced a reasonable amount, the oil is carried to the lower part of the still and flows in the direction opposite to the gases.

62 In some cases where heavy oil is used, particularly with the gas flowing upward, a mixed-flow arrangement is found. The oil enters where the gases leave the still. It flows in the direction opposite to the gases for about half-way. It is then carried to the hottest part of the still with a comparatively low temperature and flows in the same direction as the gases, so that the oil leaves the still at a comparatively cool gas zone.

63 Heat exchangers are usually understood to be devices for transmitting heat from a hot fluid which is to be cooled to another liquid which is to be heated. Particular attention has been given to the design of heat exchangers in the past few years due to their wide use in the petroleum industry, especially since the development of the oil-cracking processes. These heat exchangers are of two general designs, the shell and double-tube types, the former being shown in Figs. 11, 12, and 13 and having a bundle of tubes located in a casing or shell. A number of small smooth or corrugated tubes, straight and parallel, are either rolled, threaded, or brazed into two tube sheets, forming a tube bundle. In some cases one of the tube sheets is made integral with the round casing or shell, but in most instances it is independent of the casing. In order to take care of the difference in expansion between the tubes and the shell, the floating-head design is often applied, which can move to a slight degree independently of the shell. In some designs the floating head travels in a packing gland, which permits all joints on the heater to be external so that the smallest leakage can be detected instantly. In other heaters the floating head is merely guided in the shell, which has a separate head at that end.

64 In order to increase heat transmission, the heat exchangers are very often designed with one or more passes as regards the flow through the tubes. The shell is also frequently provided with different arrangements of baffles to bring the liquid outside the tubes in better contact with the surface. In general, heat exchangers are made at least two-pass or multiples thereof in order to have the inlet and outlet to the tubes at the same end, thus making the piping and connections much simpler. Fig. 11 shows one baffle arrangement of an exchanger by means of which the liquid is given a zig-zag flow normal to the tubes, which gives a considerably better heat transmission. Fig. 12 shows a spiral

baffle arrangement. This has practically the same effect as the zig-zag, but it also causes, to a certain extent at least, turbulent flow, which is supposed to increase still further the heat-transfer coefficient.

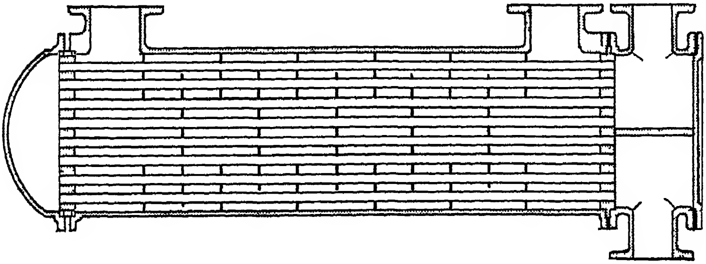


FIG. 11 HEAT EXCHANGER WITH A ZIG-ZAG BAFFLE

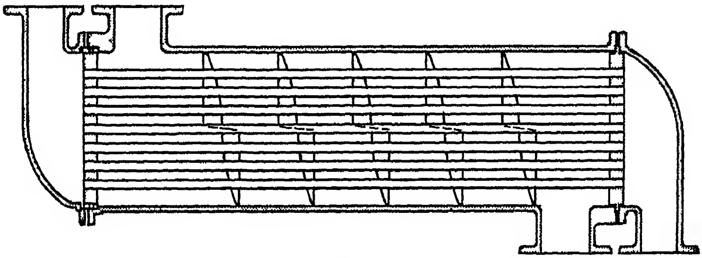


FIG. 12 HEAT EXCHANGER WITH A SPIRAL BAFFLE

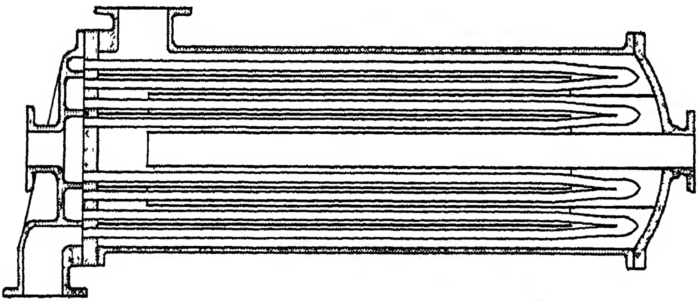


FIG. 13 HEAT EXCHANGER WITH U-TUBES CONNECTED BY FORGED RETURN BENDS

65 A rather novel arrangement of a heat exchanger is that shown in Fig. 13. The heating surface consists of U-tubes connected by return bends made by a special process of forging the pipe itself. With this arrangement each U-tube can expand individually, or independently of the others. Each leg of the U-tube can be cleaned to the very end of the return bend. The number

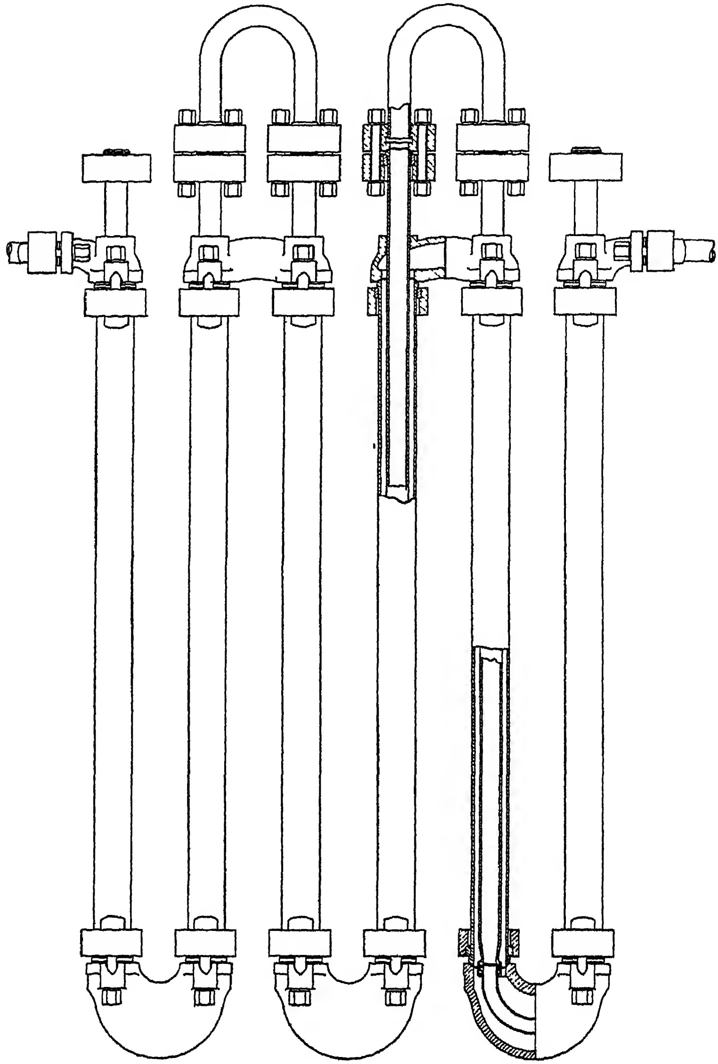


FIG. 14 DOUBLE-TUBE HEAT EXCHANGER

of joints between the tubes and tube sheets for the same length of tubes is reduced one-half. Both liquids flow parallel to each other, but the arrangement of the concentric baffles causes high velocities of both liquids, so that a high degree of heat transfer is secured. With this arrangement each liquid also flows counter to the other through the entire length of their travel.

66 The tube material to be used in oil exchangers depends upon the characteristics of the oil handled. With some oils steel tubes and sheets can be used, while with others various copper alloys must be employed to avoid chemical action of the oil on the tubes. In some cases, even the baffles must be made of non-ferrous metal. When exchangers are used as coolers and the cooling medium is salty water, it is often necessary to line the header with zinc in order to retard the electrolytic action at the tube joints.

67 Where the temperature difference between the liquids is high and a considerable amount of heat is to be transmitted between comparatively small amounts of oil, the double-tube type of heat exchanger is used. One arrangement of an exchanger of this type is shown in Fig. 14. It consists of sections, each having a smaller-diameter pipe concentric inside a larger one. Both pipes have their ends connected by return bends in such a manner as to form two independent flows, one inside of the other. The design of this exchanger is of interest because no gaskets are used. All connections are metal-to-metal ball joints. This design is very flexible. It is especially adaptable for high temperatures and pressures, as a large amount of expansion and contraction can easily be taken care of without objectionable strain on the joints or tubes. Only one end of the inner tube is fixed in the return bend, common to both pipes The other end of the inner tube is joined to a return bend which floats in the larger one of the other pipe, so that each pipe can expand or contract individually. This heat exchanger is simple to clean, as it is only necessary to remove the outside return bends and the inner tube can be withdrawn. A perfect counterflow is possible, so the heat-transmission effect is very high.

Conclusion

68 The foregoing shows why the design of oil-cracking stills lies so largely within the province of the mechanical engineer. For the most part, the problems calling for solution are dependent upon matters with which he is already familiar, such as the efficient generation of heat, heat transmission, the flow of fluids, and mechanical construction for high pressures and temperatures. In furnace design it is imperative to generate heat efficiently and distribute it evenly over the heating surface, and at the same time avoid overheating of the tube bank generally, or in any one part. Heat transmission in pipe stills and heat exchangers brings in the

transfer of heat from gases to liquids of widely varying viscosities, also from one liquid to another of a widely differing temperature.

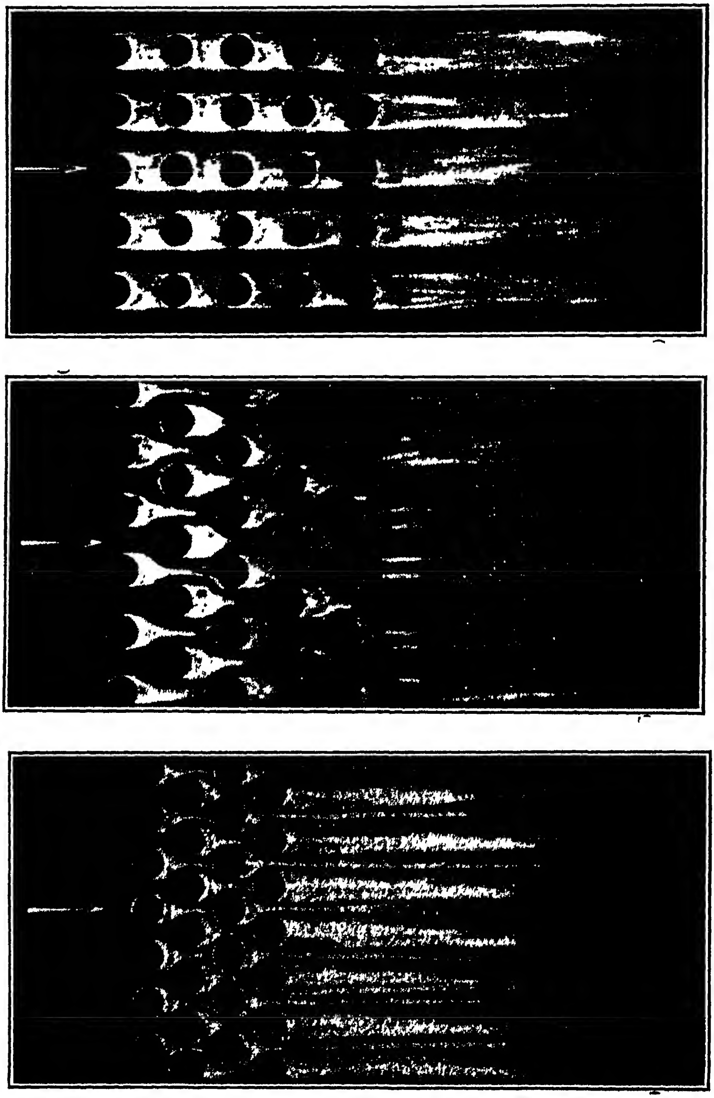


FIG. 15 GAS DISTRIBUTION OVER STRAIGHT AND STAGGERED ROWS OF TUBES

The mechanical details call for a design that will not only withstand the action of high pressures and temperatures, but one that can be readily dismantled for cleaning and renewal of the heating units.

69 It is hoped by the author that he has succeeded in presenting a clearer picture to the refinery man, as well as to the mechanical engineer, of the possibilities held by the problems in this field, and one that will induce them to exert their efforts to further advancement and improvement.

APPENDIX

70 In order to determine the heat transmission from hot gases to a steel tube and to banks of tubes located normal to the flow of the gases and containing a flowing liquid, H. Reiher made extensive tests in the Research Laboratory at Munich, Germany. His apparatus consisted of tubes from 0.46 to 2.8 cm. O.D., and tests were made with air velocities

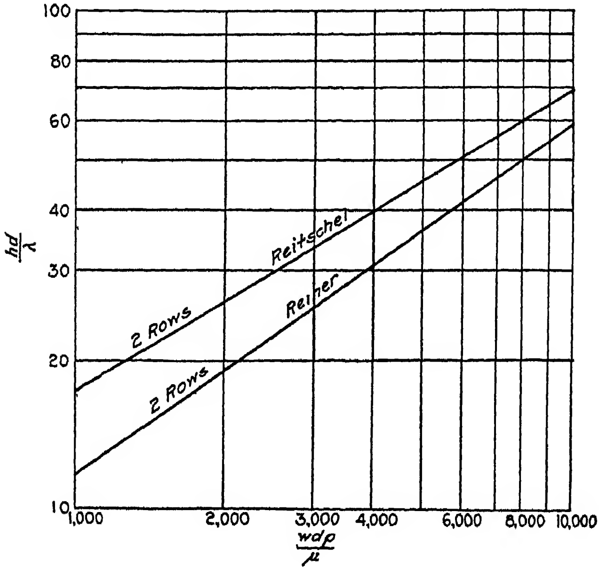


Fig. 16 Curve Showing the Relation between $\frac{hd}{\lambda}$ and $\frac{wd\rho}{\mu}$

of from 5 to 15 meters per second, and gas temperatures up to 200 deg. cent. By means of the law of similarity he extended his investigations to cover also heat transmission between flue gases of considerably higher temperatures and tubes of larger diameters. Coefficients were also established for banks of tubes in straight as well as in staggered rows. Of considerable interest is Fig. 15, showing the distribution of a gas flow over banks of tubes with different tube arrangements.

71 H. Reitschel also made similar tests at the Research Laboratory in the Technical College of Berlin. Both experimenters found that the heat-transmission coefficient h from gas to a pipe is

$$h = C\frac{\lambda}{d}\left(\frac{wd\rho}{\mu}\right)^n$$

where

h = heat-transmission coefficient in kg-cal. per sq. meter per hr. per deg. cent.

λ = conductivity coefficient in kg-cal. per meter per hr. per deg. cent.

ρ = mass density in kg-sec.^{2} per meter^4

μ = viscosity in kg-sec. per sq. meter

d = pipe diameter in meters

w = maximum gas velocity in meters per second.

The exponent n is constant, while C is a function of the number of pipe rows. For the same conditions, however, the values of n and C varied

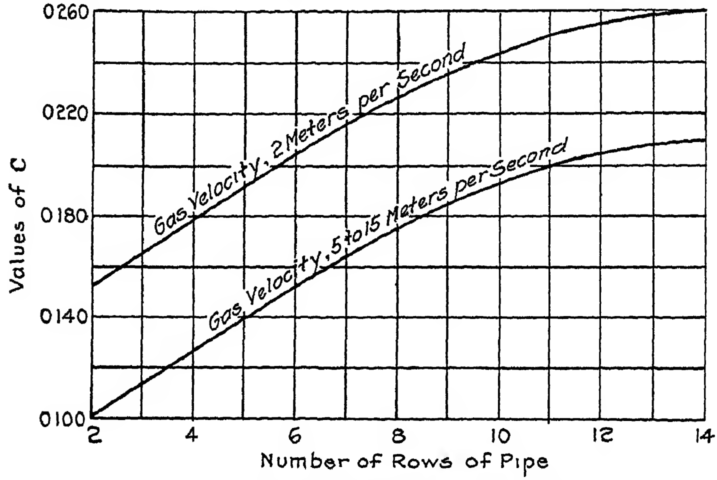


FIG. 17 CURVE SHOWING THE RELATION BETWEEN C AND NUMBER OF ROWS OF PIPE

slightly in the tests of the above experimenters. This seems to be due to the fact that Reitschel's formula is applied to low velocities, while Reiher's experiments do not go below 5 meters per second.

72 In accordance with the above

$$\log \frac{hd}{\lambda} = \log C + n \log \left(\frac{wd\rho}{\mu}\right)$$

This equation for both Reiher and Reitschel's experiments is plotted in Fig. 16. The slope of the lines gives for Reitschel,

$$n = 0.59$$

and for Reiher,

$$n = 0.69$$

The corresponding values for C are given in Table 4.

TABLE 4 VALUES OF C IN REIHER AND REITSCHEL'S FORMULAS

	Number of Tube Rows			
	2	3	4	5
Reiher	0.100	0.113	0.123	0.131
Reitschel	0.322	0.345	0.370	....

These experiments show that the heat-transmission coefficient is larger for pipes in staggered rows than in straight rows. The difference is

smaller for low velocities, but increases with the velocity and with an increasing number of rows.

73 The experiments were not made for more than five rows. On account of the influence of the first effective tubes, the value of h and C increases with the number of rows. It is assumed that h increases up to a certain value where the influence of the first row will be negligible. In other words, C will be constant for a greater number of rows.

74 The lower curve in Fig. 17 shows the relation between C and the number of rows for gas velocities between 5 and 15 meters per second. As Reiher's experiments were not carried below 5 meters per second, Reitschel's formula is more reliable for low velocities. In order to

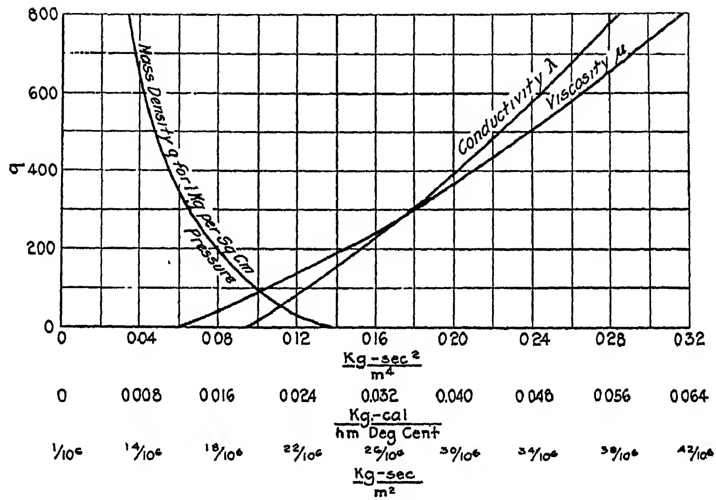


FIG. 18 CONVERSION CURVES BETWEEN DEGREES CENTIGRADE AND ρ, λ, AND μ

apply Reiher's formula to low velocities, the values of the lowest point of the Reitschel curve, Fig. 16, are inserted in Reiher's formula, which gives the value of C corresponding to low velocities. For instance, the lowest point of the Reitschel curve in Fig. 16 is $hd/\lambda = 18$, and $wd\rho/\mu = 1000$ which, inserted in Reiher's formula, gives

$$C = \frac{18}{(1000)^{0.69}} = 0.153$$

75 If we therefore draw a curve in Fig. 17 through the point (1000, 0.153) parallel to the lower curve, this should give the values of C for low velocities. Reiher's formula can be used then for high as well as low velocities, C to be taken from Fig. 17.

76 This method does not apply, however, for velocities lower than one meter per second, as at this velocity there is a considerable effect from natural convection. Fig. 18 gives the relation between the gas temperature in degrees centigrade and the mass density ρ, the conductivity λ, and the viscosity μ, in order to facilitate the use of the formula in figuring the heat transmission between gases and tubes.

77 It should be noted that the temperature in question is that of the gas film nearest the tube, and is assumed to be the mean value of the gas and pipe temperature.

No. 2037

NEW BOILER EQUIPMENT AT THE INTERBOROUGH RAPID TRANSIT CO.'S FIFTY-NINTH STREET POWER STATION

By H. B. Reynolds,[1] J. M. Taggart,[2]
and
R. S. Lane,[3] New York, N. Y.

The original boiler equipment of the Interborough Rapid Transit Company's Fifty-Ninth Street power station comprised 600-hp. boilers with hand-fired grates. With the type of engine then used each boiler could produce about 1000 kw. The paper traces the development of boiler practice, through the introduction of stokers, the development of the "double-end" furnace, and the utilization of the underfeed stoker, showing the successive increases in the kilowatt capacity of a boiler, up to the present installation, in which each boiler has a capacity of 8200 kw. at 350 per cent of rating. The paper outlines the design of the plant and presents a table of test results.

The boiler room of the Fifty-Ninth Street Power Station of the Interborough Rapid Transit Company in New York City, since its initial completion in 1905, has reflected the development which has taken place in coal-burning and boiler equipment in a graphic manner. The original equipment consisted of 600-hp. boilers equipped with hand-fired grates. These boilers could not be operated much in excess of 100 per cent of rating, and in connection with the Manhattan-type engines, which were installed in the engine room, each boiler could produce about 1000 kw.

2 Soon after the station was completed the original hand-fired grates were replaced by Roney automatic stokers, which reduced the labor costs, increased the efficiency, and enabled the boilers to be operated at a slightly higher capacity. In 1910 it was necessary to obtain additional boiler capacity, so the idea was conceived of

[1] Mechanical Research Engineer, Interborough Rapid Transit Co. Mem. A.S.M.E.

[2] Consulting Engineer. Formerly Engineer, Mech. Design, Interborough Rapid Transit Co. Assoc-Mem. A.S.M.E.

[3] Engineer, Mechanical Tests, Interborough Rapid Transit Co. Assoc-Mem. A.S.M.E.

Presented before the Metropolitan Section of The American Society of Mechanical Engineers, New York, February 24, 1926.

installing an additional Roney stoker under the rear of the boilers to supplement the stokers which had been previously installed under the front of the boilers. It is interesting to note that when this proposed addition was discussed with various engineers, and they were told that the boilers were expected to operate at 200 per cent of rating after the alteration, they expressed grave doubts as to whether it would be possible to operate the boilers at such a high overload. However, eighteen of these " double " Roneys were installed, and they gave the required increase in capacity as expected.

3 At 200 per cent of boiler rating and with the increased economy obtained in the engine room by adding low-pressure turbine units with surface condensers to the original engines, the capacity per boiler became approximately 2200 kw.

4 The next increase in boiler capacity was obtained by substituting underfeed stokers in place of the remaining " single " Roney stokers and six of the " double " stokers, thus leaving twelve double Roney stokers in use in 1919. Up to this time no additional boilers had been installed, although the capacity of the station had been more than doubled over the original installation.

5 The installation of the underfeed stokers was made in groups which varied in design of setting and stoker, and in size of stoker. With induced draft, 250 per cent of rating is a fair average of the capacity of the stokers so equipped. These stokers were installed mainly in connection with three 35,000-kw. turbines, and for this reason the boilers feeding these turbines were also provided with superheaters. As a result of these changes and additions, the capacity of each boiler was raised to 3000 kw.

6 Due to the recent purchase of additional subway cars, it became necessary to again increase the boiler-room capacity at the Fifty-Ninth Street Power Station. This could have been obtained by substituting underfeed stokers for the remaining twelve double Roney stokers. However, in view of recent developments in fuel-burning equipment and the fact that the boilers over the Roney stokers were set rather low, this was not desirable. Another objection to increasing the capacity of the old boilers was that these boilers are in the saturated-steam section of the station, and therefore superheaters could not be installed and the resulting economy realized.

7 The Fifty-Ninth Street boiler room was originally designed to house six sections of boilers, each section consisting of twelve 600-hp. boilers discharging into a common stack. However, only five sections and five stacks were installed. All these boilers are on one elevation at 17 ft. 9 in., while economizers were installed on a separate floor at an elevation of 54 ft. 9 in.

8 It was finally decided to obtain the additional capacity by installing four new boilers and stokers in a portion of the vacant section at the west end of the boiler room. This section, like the

others, was originally designed to house twelve 600-hp. boilers set in batteries of two each. As a result of this the existing building columns prevented the installation of new boilers and stokers any wider than the old 600-hp. boilers. Considerable headroom was obtained by removing the economizer floor at the elevation 54 ft. 9 in. The new installation is designed on the basis of ultimately installing four more boilers in the remaining space, so that the final number of boilers in this stack section will be eight. The other four of the twelve available boiler bays are to be used for blowers and the possible installation of economizers or air heaters.

9 The new boilers operating at normal high load of 300 per cent of rating and supplying steam to the 35,000-kw. turbines now in place will develop a capacity of approximately 7000 kw. At 350 per cent of rating and with the same water rate they would have a capacity of 8200 kw. each. Since only eight of the new boilers are proposed for the space occupied by twelve of the old type, the actual ratio of capacity of new and old equipment per stack section would be 65,600 kw. to 36,000 kw. With the addition of turbines with stage heating the capacity of the new boiler equipment per stack section would be approximately double that of the old underfeed stoker and turbine equipment. Thus, for the same boiler-room space the power-plant capacity would be increased about six times over that obtained from the hand-fired furnaces and the reciprocating-engine units.

10 In view of the fact that the old economizer floor at elevation 54 ft. 9 in. was removed in order to provide headroom for these boilers, new methods of bracing the building columns situated between each two adjacent boilers had to be provided. This was accomplished by adding diagonal eccentric bracing at some points, while horizontal bracing girders were provided at other places. The existing columns required no alteration except between the 54-ft. 9-in. and the 69-ft. 3-in. elevations, where it was necessary to strengthen them by adding additional flange plates.

11 The boiler which was selected is of the horizontal longitudinal drum, inclined-tube, vertical-header type, and is 21 sections wide and 24 tubes high, with three longitudinal drums, the tubes being 20 ft. in length and 4 in. in diameter, giving a total heating surface of 11,400 sq. ft. The old 600-hp. boilers were 21 sections wide and 14 tubes high, while the tubes were 18 ft. long. These new boilers are the highest which have ever been installed over a stoker. However, in view of the increasing length of stokers this is a logical development in order to retain the proper ratio between stoker area and boiler-heating-surface area. It is interesting to note that the ratio between the boiler surface and the grate surface in the case of this new installation is almost exactly the same as it is in the case of the old 14-tube-high boilers and the 17-tuyere stokers.

12 The superheater is of the interdeck type and is located between the sixth and seventh rows of boiler tubes from the

bottom. As shown in Fig. 1, the entire length of the lower six rows of tubes is exposed to the fire, and the baffling begins at a point above the superheater.

13 In order to obtain a uniform feedwater supply to all three of the drums and thus prevent any great difference in the water level between them, the individual drum checks are omitted and the feed pipes to each drum are carried to a relatively large-diameter short header across the front of the drums. The water is fed to this header at two points through non-return stop valves and hand-regulating valves. The arrangement was made suitable for automatic feedwater regulators, though these have not yet been installed.

14 The stokers are of the Taylor HC7 type, seven retorts wide and 37 tuyeres long, equipped with double-roll clinker grinders. These stokers have one set of rams which feed the coal into the stoker. The fuel is then worked down the grate by means of a series of six pushers, which may be seen in Fig. 1. Thus the fuel is continuously agitated and forced into the grinder pit. Each stoker is driven by means of a direct-current variable-speed motor through a lineshaft placed under the front of the stoker. The grinder rolls are also driven from this lineshaft. The drive of each stoker is independent. However, each stoker motor is of sufficient capacity to drive two stokers in an emergency, which can be done by throwing in a clutch in the lineshaft. In addition to the variable-speed motor, further flexibility can be obtained by means of the two-speed device with which each one of the two power boxes of each stoker is equipped.

15 Due to the space limitations between the boiler, stoker, and the building columns, adequate protection of the building steel presented a difficult problem in connection with the design of the furnace. However, the ultimate design provided for an exposure of one or two sides of each building column and an air space around the remaining sides.

16 The size of the stoker selected necessitated vertical walls on all four sides of the furnace. A slight arch or outward bulge was given to the walls above the stoker and up to the boiler tubes. The walls were also tied into the sheathing framing at intervals as shown in Fig. 1. Perforated Bernitz blocks were installed in all four walls up to the fire line. The girders at the elevation of the old economizer floor running between the columns at the front of the boiler to the columns at the rear were retained in order to provide the necessary bracing. These girders were put to a further use by allowing them to carry all of the furnace walls above this point. All walls were made up of $\frac{1}{8}$-in. sheathing, a $2\frac{1}{2}$-in. layer of Sil-o-cel insulation, which was carried on the sheathing framing at 3-ft. intervals; a layer of asbestos millboard, and the remainder of firebrick which varied from $13\frac{1}{2}$ in. to $18\frac{1}{2}$ in. in the case of the

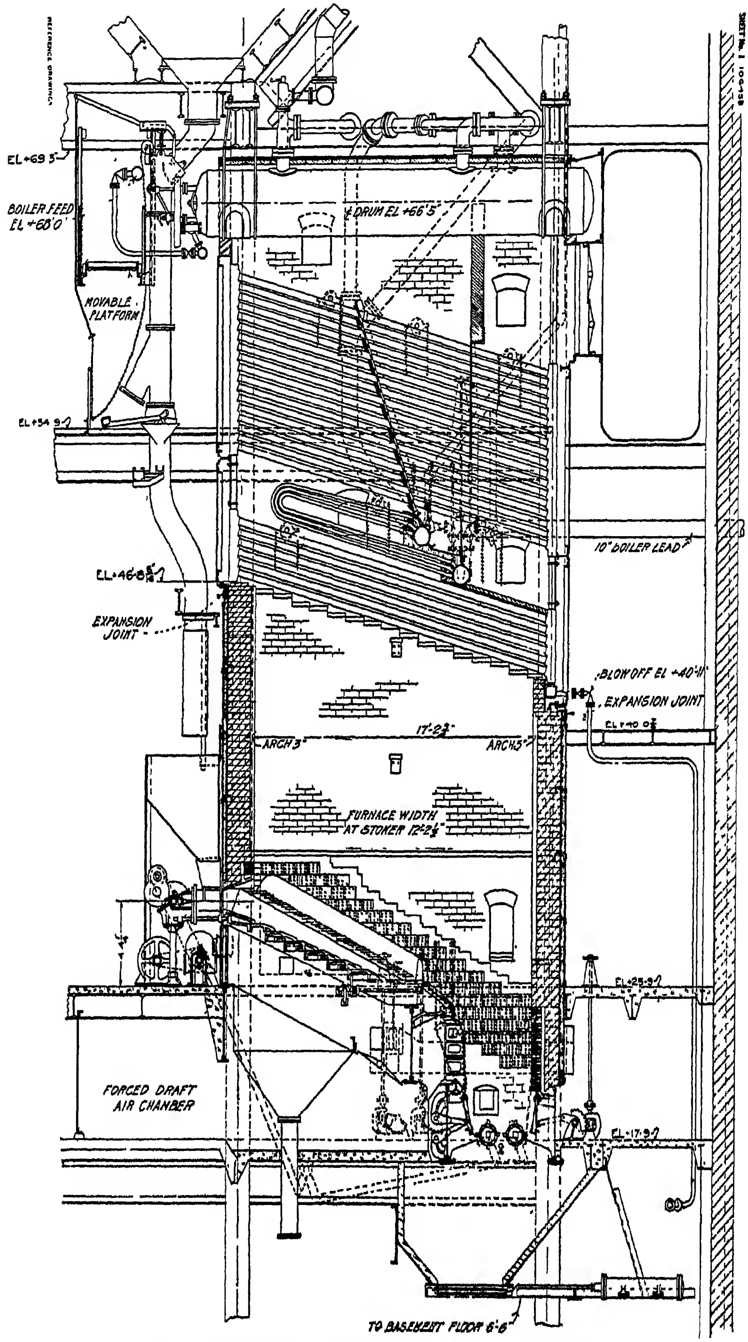


FIG. 1 CROSS-SECTION OF BOILER, STOKER, AND FURNACE

side walls and from 19 in. to 24 in. in the case of the front and rear walls. The purpose of the asbestos millboard was to facilitate the erection of the Sil-o-cel and prevent a bond between the Sil-o-cel and the firebrick.

17 The entire setting including the roof over the boiler is enclosed in sheathing. In order to avoid interference with the column bracing a special type of sheathing framing and joint is used. The angle-iron framing is placed inside of the sheathing. This framing is punched with square holes so that with the use of square-shanked bolts placed with the nuts on the outside, the plates can be removed and replaced after the settings have been completed. The plates were dimensioned so that a space of sufficient width was left between adjacent plates for the bolts to pass through, and thus it was not necessary to punch the plates. In order to close this space and also to form a bearing for the bolt nuts, a strap is placed on the outside overlapping the sheathing plates. A high degree of air tightness is obtained by placing an asbestos-tape gasket between the strap and the sheathing plates.

18 In view of the fact that the upper portion of the sheathing is supported on a girder it is necessary to provide an expansion joint. This expansion joint is formed from two heavy copper strips clamped against the upper and lower sheathing plates so that the sheathing is free to slide back and forth between the copper strips.

19 Due to the existing building columns, it is necessary to place the boilers in groups of two with a passageway on only one side. However, as far as the settings are concerned, a space of about 3 in. is left between each pair of boilers, so that they are entirely independent of each other.

20 In future installations it may be preferable to install one boiler instead of each battery, omitting the center or adjacent walls, if a satisfactory method of protecting the center building columns can be devised. For the present installation this did not work out advantageously from an operating standpoint.

21 Forced draft for each battery of boilers is furnished by one centrifugal single-inlet blower of the backward-curved vane type. This blower has a capacity of 120,000 cu. ft. of air per min. at 7-in. static discharge pressure. It is directly driven by a 250-hp. slip-ring variable-speed motor. As shown in Fig. 2, the blower is arranged for a down-horizontal discharge. The discharge is through a swinging damper, balanced to open or close with the pressure, and into an air duct formed between the original and new boiler-room floors. A partition is placed in this air duct so that the blower discharge can be confined to one battery. Normally it is expected to operate with some panels of this partition removed so that one blower or two blowers can be used for the four boilers, depending on the load. An extension of the installation would simply extend this duct for eight or more boilers. The blowers are

located conveniently for connection to an air heater and the outboard bearing is arranged for water cooling. Their capacity is sufficient for air heating up to about 275 deg. fahr. The control for the forced-draft motor is placed at a control board which serves two boilers The forced-draft dampers of each stoker are operated by motors arranged for push-button control from the control-board.

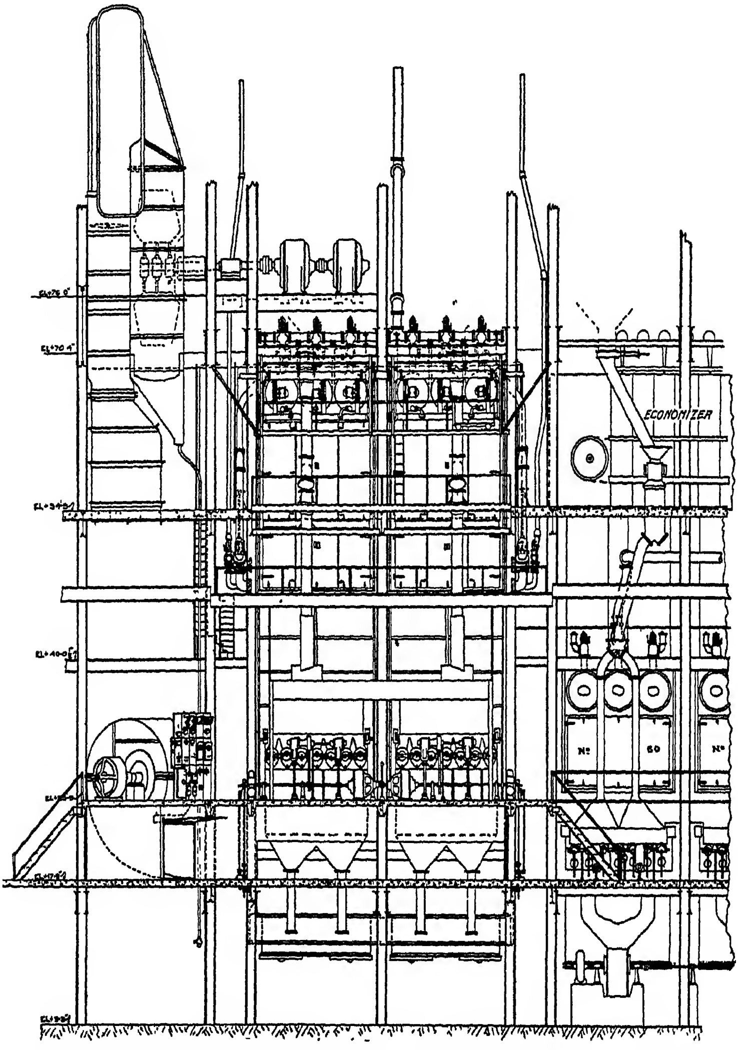
22 The flue gases from each pair of boilers discharge into a common flue which passes to the induced-draft fan. By means of a bypass damper, the gases may pass directly to the stack. The induced draft fans are of the cindervane type. The capacity of each fan is 250,000 cu. ft. per min. against a total head of 3.0 in. of

TABLE 1 PRINCIPAL DATA COVERING BOILERS NOS 61-64, INCLUSIVE, FIFTY-NINTH STREET POWER STATION

Boilers and Superheaters:	
Maker of boilers	Babcock & Wilcox Co.
Horsepower	1110
Heating surface, sq ft	11,400
No. of tubes wide	21
No. of tubes high	24
Length of tubes	20 ft
Diam. of tubes, outside	4 in.
No. of drums	3
Height of bottom of front header above floor	20 ft. 11$\frac{5}{16}$ in
Height of center line of drums above floor	40 ft 8 in.
Heating surface per sq ft. of grate	53 8 sq. ft.
Heating surface per cu. ft. of furnace	3.07 sq. ft.
Area of superheaters, sq. ft	1485
No. of rows of boiler tubes below superheater	6
Stokers and Furnaces:	
Maker of stokers	American Engineering Co.
No. of retorts	7
No. of tuyeres long	37
Width of furnace	12 ft. 2$\frac{1}{4}$ in.
Depth of furnace	17 ft. 2$\frac{3}{4}$ in.
Projected grate area, sq. ft	212
Area of grate per sq. ft. of heating surface	0.0185 sq. ft.
Area of grate per b hp	0.185 sq. ft.
Area of grate per cu ft. furnace volume	0.057
Average height of tubes above grate	17 ft. 6 in.
Total volume of furnace above grate	3710 cu. ft.
Cu. ft. of furnace per sq. ft. grate	17.5
Cu. ft. of furnace per sq. ft. of heating surface	0.326
Cu. ft of furnace per rated b.hp	3.26

water when handling gases at a temperature of 700 deg. fahr. Two squirrel-cage motors are installed on the same shaft in connection with each fan. One motor is of 175 hp. capacity and runs at a speed of 180 r.p.m., while the other motor is of 350 hp. capacity and operates at a speed of 240 r.p.m. Only one motor is used at a time, while the other runs idle. The purpose of the two motors is to provide spare equipment and at the same time furnish simple and economical speed control instead of using an uneconomical or expensive variable-speed motor. The induced-draft fan motors are started and stopped from the control board on the boiler-room floor. The boiler-outlet dampers and the induced-draft fan bypass damper are motor-operated and are controlled by push buttons on the control board. In addition to the remote hand control for

the boiler-outlet dampers an automatic control is provided for these dampers which maintains a constant suction pressure in the furnace.



'IG. 2 VIEW OF TWO OF THE NEW BOILERS, SHOWING ONE OF THE OLD BOILERS AT RIGHT

23 A new stack was build for these boilers, and was placed on a structural-steel platform over the center of the firing aisle. This platform was a part of the original building structure. The five old stacks are of the same height but the internal diameter is only 15 ft. at the top and 18 ft. at the base. The use of a stack

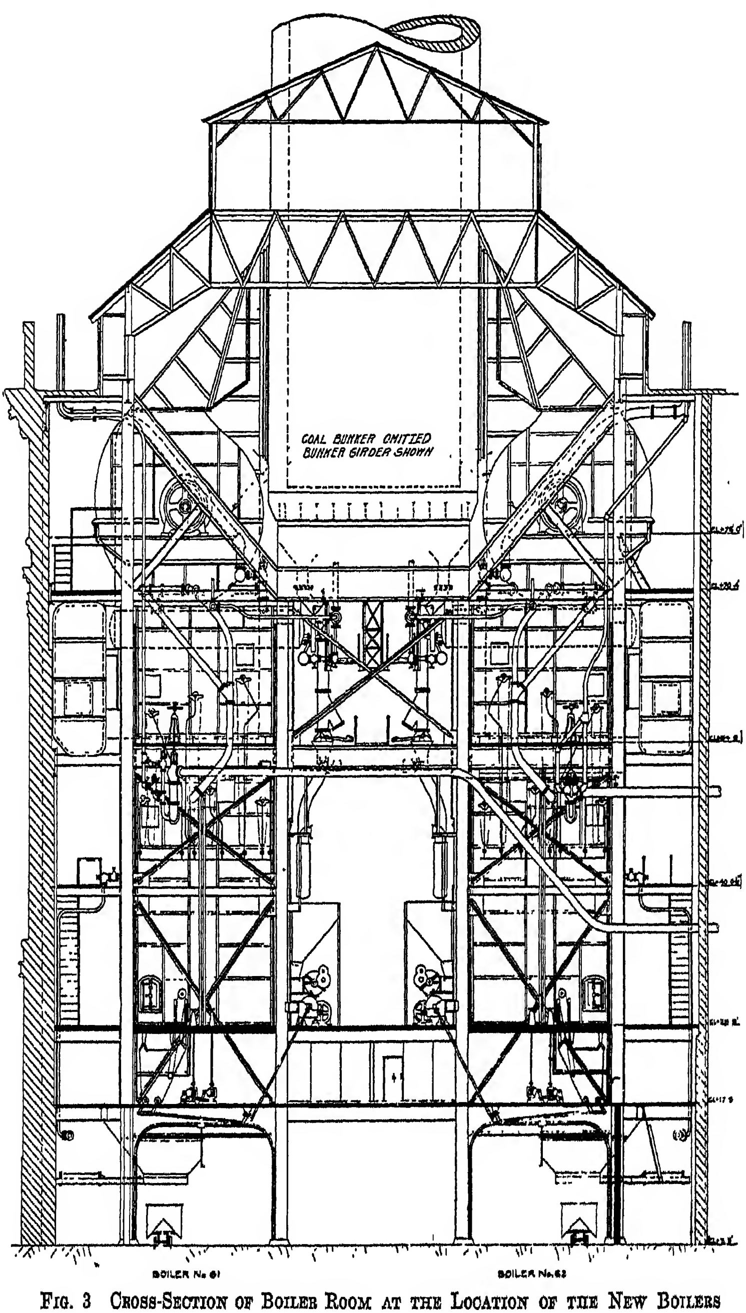


FIG. 3 CROSS-SECTION OF BOILER ROOM AT THE LOCATION OF THE NEW BOILERS

with less taper and a thinner wall made it possible to install one of larger diameter on the existing platform. The stack was designed to receive the gases from eight boilers similar to the four installed and is 162 ft. 11 in. high above its base and 217 ft. $3\frac{1}{8}$ in. high above the boiler-room floor. The internal diameter is 20 ft. $10\frac{3}{8}$ in. at the base and 20 ft., 0 in. at the top. The thickness of the wall is $18\frac{1}{8}$ in. at the base and $10\frac{5}{8}$ in. at the top.

24 The boilers can be operated up to 180 per cent of rating on natural draft and up to 250 per cent can be obtained by using the induced-draft fans with the smaller motors. For operation above 250 per cent of rating it is necessary to use the larger of the induced-draft fan motors.

25 In addition to the various hand and automatic controls which are centralized at the control boards, a Bailey boiler meter is provided for each boiler, together with draft gages indicating the draft at various points. In order to obtain a record as to the condition of the boiler and superheater surface, recording thermometers give the temperatures of the flue gases and the steam leaving the superheater.

26 Economizers or air heaters were not installed in connection with these boilers. However, space was left where such equipment can be installed in the future. As before noted, the forced-draft blowers are of sufficient capacity and are suitably arranged for air heaters. To carry normal high load with air heaters or economizers added, with the induced-draft fans now installed, the present boiler baffling will need to be changed. With this in view the baffle between the second and third passes of the boilers was made of iron plates that can be easily removed in sections.

27 After the installation was completed one of the units was selected on which extensive boiler tests were run. All of these tests were conducted in accordance with the A.S.M.E. test code. Coal and water quantities were determined by means of calibrated scales. The results are given in Table 2, while the principal data are plotted in Fig. 4. The method of taking these data requires no explanation, with the possible exception of the item giving the percentage of combustible in the ash. Due to the large storage capacity of the grinder pit, it was impossible to obtain ash samples which would be representative of each test. In view of this difficulty, samples were taken during each test, but instead of using these samples for their respective tests, the results of all the samples, which varied but little, were averaged and applied to all the tests.

28 The equipment with its accessories is distributed vertically over five floor levels, and some readings had to be taken on each floor. The work of taking observations was reduced considerably by building a shack on the third-floor level and bringing all pressure, draft, temperature, and CO_2 readings above the operating-room

floor to this shack. This was accomplished by means of high-pressure copper tubing for the steam pressures. There was also a supply of high-pressure air provided for blowing out gas lines and for assisting in taking of gas samples. Draft gages were connected to the boiler from the shack with small copper tubing, while the numerous temperatures taken were observed in the shack by using thermocouples and long leads. The temperatures were recorded by use of an automatic Leeds & Northrup 8-point recorder and two manual L. & N. indicators connected to 10-point switches. Thus by means of interchecks between pyrometers, steam-gage readings, and references to steam tables a close watch was continually maintained on all instrument accuracy.

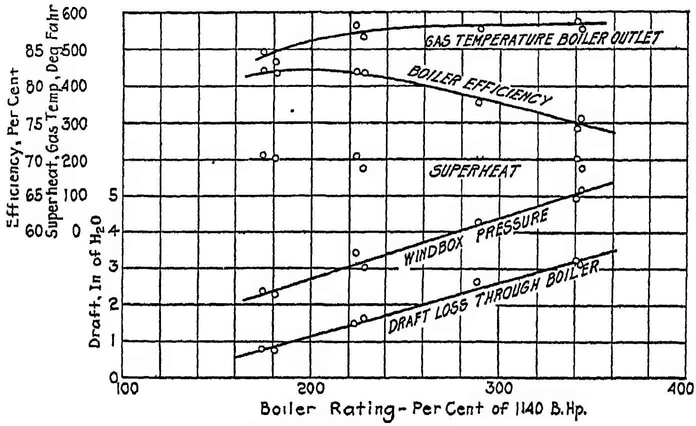


Fig. 4 Test Results

29 The good lighting of the shack and better environment in general contributed much toward the gathering of accurate data with a minimum force of intelligent assistants.

30 In addition to the efficiency tests, a capacity test was run at 400 per cent of rating for one hour. Apparently this rate of operation could have been increased and carried for a longer period as far as the stoker and boiler were concerned. However, it is not intended to operate these units much in excess of 300 per cent of rating during normal operation.

31 The tests show a boiler efficiency fully up to expectations. Some of the test results, however, need further explanation. Thus the low temperature of the gases at the boiler outlet for the high loadings is apparently subject to question. This is probably due to some air leakage in the last pass, where the suction pressure is quite high during the high-rate tests. The large unaccounted-for losses shown at the higher rates of operation would substantiate this view. If the outlet-gas temperatures were assumed to vary directly

TABLE 2 SUMMARY OF RESULTS — TESTS ON 1140-HP. BOILER — I. R. T. CO., FIFTY-NINTH STREET POWER STATION

No.	Item	Test No. (5)	(7)	(8)	(9)	(12)	(14)	(15)
	Rating, Boiler — Per Cent of 1140 Hp	180 8	174.2	223.2	227.8	343 5	288.7	340.2
	Fuel — Proximate Analysis — Dry							
1	Volatile matter, per cent	16 12	16.10	16.41	17 19	16 31	15.78	15.91
2	Fixed carbon, per cent	77.04	77.53	76.92	76.04	76.69	77 24	77 83
3	Ash, per cent	6 84	6 37	6.67	6.77	7.00	6.98	6.26
4	Moisture (as fired), per cent	2.93	3.86	3.43	3.00	3.20	2.74	2.87
5	Heat value (as fired), B.t u. per lb	14,158	14,150	14,212	14,215	14,149	14,236	14,331
6	Heat value (dry), B.t.u. per lb	14 584	14,718	14.716	14,655	14,617	14,638	14,755
	Fuel — Ultimate Analysis — Dry							
7	Carbon, per cent	84.46	84 69	84.84	84.67	84.69	84.05	83.46
8	Hydrogen, per cent	5.12	4 58	5.22	5.53	4.74	5 32	5.76
9	Oxygen, per cent	0.82	1.60	0.59	0.25	0.75	1.11	1.89
10	Nitrogen, per cent	1 22	1 27	1.29	1.28	1.37	1 27	1.21
11	Sulphur, per cent	1 54	1.49	1 39	1.50	1 45	1.27	1.42
12	Ash, per cent	6.84	6.37	6.67	6.77	7 00	6 98	6.26
	Gas Analysis — Boiler Outlet							
13	Carbon dioxide, per cent	14 62	14.15	14 65	14 03	14.28	13 38	13.59
14	Carbon monoxide, per cent	0.24	0.11	0.27	0.16	0 16	0.16	0.48
15	Oxygen, per cent	3 98	4.88	4.36	5 05	4 64	5 56	4.81
16	Nitrogen, per cent	81.16	80.86	80.72	80.76	80.92	80 90	81.12
17	Excess air, coefficient	1.221	1 290	1 210	1 260	1.272	1 327	1.265
	Pressures and Drafts							
18	Steam pressure, gage, superheater outlet, lb. per sq in	230 4	234 5	235.8	227 8	234.9	232.0	234.8
19	Air pressure, ashpit zone, in. H_2O	2.26	2.38	3 44	3 08	5 16	4 30	4 93
20	Draft in furnace, in. H_2O	0 033	0.033	0 055	0 065	0.147	0 086	0.14
21	Draft in boiler outlet, in. H_2O	0 775	0 79	1.60	1.704	3 34	2.76	3 38
22	Draft drop, in. H_2O	0 742	0 757	1 545	1.639	3.193	2.674	3.24

TABLE 2 — CONTINUED

	Temperatures							
23	Steam temp., superheater outlet, deg. fahr.	602.5	613.7	609.8	575.6	579.6	610 6	604.2
24	Superheat, deg. fahr.	203.2	213.0	208.6	177.2	178.7	210.6	203 4
25	Temp. of air for combustion, deg. fahr.	82 7	80.6	81 0	73.3	73.0	87.0	82 2
26	Temp. of gases, boiler outlet, deg. fahr.	475 5	495.1	566.3	539.4	551.6	552.3	573.7
27	Temp. of feedwater, deg. fahr	184.5	187.4	187.1	190.4	188 9	189 8	189 4
	Hourly Quantities							
28	Duration of test, hr.	12	12	12	10	8	7	4
29	Dry fuel per hour, lb.	5784.1	5498.5	7034.2	7230.2	11,848.9	9667.4	11,834.5
30	Dry fuel per sq ft. of grate per hour, lb.	27.28	25.94	33 18	34 10	55.89	45.60	55.82
31	Equivalent evap. per hr., lb.	71,135	68,504	87,781	89,596	135,113	113,572	133,807
	Refuse							
32	Combustible in the refuse,[1] per cent	6.00	6.00	6 00	6.00	6.00	6.00	6 00
	Evaporation							
33	Actual evap. per lb. fuel (dry), lb	10.267	10.380	10.412	10.518	9.653	9.819	9.479
34	Equiv. evap. per lb. fuel (dry), lb.	12 298	12 459	12.479	12.392	11.403	11.748	11.307
35	Boiler rating — per cent of 1140 hp.	180 8	174 2	223 2	227.8	343.5	288 7	340.2
36	Efficiency of boiler, superheater, furnace and grate, per cent	81.82	82 14	82 28	82 05	75 70	77 88	74 36
	Heat Balance							
37	Heat absorbed by water, per cent	81.82	82.14	82.28	82 05	75.70	77 88	74.36
38	Heat loss due to moisture in fuel, per cent	0.25	0.34	0.31	0.27	0.29	0 24	0.25
39	Heat loss due to moisture from burning of H_2O, per cent	3.67	3.46	4.05	4 29	3.70	4.14	4.46
40	Heat loss due to moisture in the air, per cent	0.26	0.18	0.22	0 10	0.15	0 19	0.36
41	Heat loss due to dry chimney gases, per cent	9.37	10.24	11.49	11.61	11.76	12.27	12.05
42	Heat loss due to incomplete burning of C., per cent	0.94	0.45	1.05	0.66	0 63	0 69	1 95
43	Heat loss due to combustible in refuse, per cent	0.43	0.40	0.41	0 43	0 44	0 45	0.39
44	Radiation and losses unaccounted for, per cent	3.06	2.79	0.19	0 59	7 33	4.14	6.18

[1] Owing to the difficulty of obtaining representative ash samples during each test due to the large amount of ash in the grinder pit, 6 per cent was taken throughout all the tests, for the amount of combustible in the refuse, which was based on a great many samples taken at random.

with the load the temperature for test No. 12 would be approximately 690 deg. fahr. This would still leave a fairly high percentage of unaccounted-for losses, taking into consideration the probable heat radiated.

32 Referring to the test curves, Fig. 4, it will be noted that the draft loss through the boiler varied directly with the load. It was anticipated that at the higher rates of operation this curve would show an upward curvature. It is probable that the even velocity flow of the gases provided for by the baffle design eliminated this increasing drop for the loads tested, and that the increasing stack effect in the high first pass tends to offset the draft loss at the high rates of operation.

33 In addition to the data given in the table, it may be of interest to know that during the test at 343.5 per cent of rating, the heat in the fuel fired amounted to about 46,500 B.t.u. per hr. per cu. ft. of furnace volume.

34 The high efficiency of this equipment is of special note. At the time of installation the stokers selected were considerably longer than normal practice. Also the clinker-grinder pit was very much deeper than any that had been installed. It was believed that this would result in a very low combustible in the refuse, and it was hoped that the discharged ash would be at a relatively low temperature. A glance at the test results shows the low percentage of combustible in the refuse. The results of operation prove that quenching the refuse is seldom necessary. It has proved feasible to run the clinker grinder continuously, with very little adjustment. The speed of these grinders varies with the stoker speed since they are driven from the same shaft.

No. 2038

ANNUAL REPORT OF THE COUNCIL AND COMMITTEES FOR 1926

THE A.S.M.E. IN 1926—SUMMARY BY THE COUNCIL

IN THIS brief report for the year 1926 it is possible to set forth only the major activities of the Society. The Council particularly wishes to record its appreciation for and to call attention to the individual reports of the administrative and technical committees which follow. Appreciation may also be expressed here for the especially comprehensive reports of researches represented in the technical papers submitted to the Society during the year.

Industry and the engineering profession owe much to authors and committeemen, and in turn the Society to the leaders in those respective groups who coöperate so fully with the men of their organizations who are giving unsparingly of their time, talents, and the benefit of their experience, for the general good and for the advancement of the engineering profession. It is not superfluous to state that for such unselfish service there is no material compensation of any kind.

The increase in revenues made available October 1, 1925, permitted the restoration of certain services to which members have been accustomed, and also the accumulation of a small working capital which might be used to continue these services should the Society for any reason experience a temporary decrease of its income. We have, therefore, been ready for the varied important projects that have come to us within the year and set the pace for the work ahead.

Standardization, whether called elimination of waste, simplified practice, or by any of the preferred names, has been rightly expressed as the "greatest contribution American economic science has made to modern development."

Within the last year some of the leaders of our largest industries have suggested the possibility of so organizing the standardization procedure and machinery of our country, that it would be authoritative, and have the confidence individually and collectively of all the industries of the Nation, the Government departments and bureaus, the engineering societies, and the other technical bodies. The A.S.M.E., as "the Society of the Industries," has felt obligated to place itself unreservedly at the command of this movement.

Research. The Council considers the stimulation of research an activity of great importance and this year doubled the appro-

priation for speeding up present research activities and developing new projects.

Engineering Education. Coming into prominence as the result of great industrial growth and industrial needs, is the problem of the education of the engineer of the future. The A.S.M.E. is taking its part by representation on the Board of Investigation and Coördination of the Society for the Promotion of Engineering Education, which, under a grant of $118,000 from the Carnegie Foundation, is conducting a survey that is receiving not only the enthusiastic coöperation of the industries and the engineering societies, but also of the educational institutions throughout the United States and Canada. It is the concensus of opinion, after a three years' study, that " The importance of technological education to national security and national prosperity was never more fully appreciated, and it is safe to predict that ample resources can be found for the further development of such education if an adequate and sound program can be evolved."

Meetings — 1892–1926. Thirty-four years ago the Society held a meeting in San Francisco. It was regarded by the leaders at that time as an opportunity to extend the national scope of the Society, which then had a membership of 1424. In 1926, when we again held our Spring Meeting in San Francisco, the members of the Society numbered 17,775. The same spirit actuated the " pioneers " in the trip to Europe in 1889, the nucleus of the ever-widening circle of our international relations.

Further interesting developments during these years may be noted by comparing the reports of the meetings as found in Volume 13 of Transactions and in the present volume. At the former meeting, for instance, nothing was said of hydroelectric plants nor of oil engines, steam engines holding the center of attention.

Public Relations. More and more is the Society being brought into the field of civic and national problems developing in the work of our local sections, professional divisions, research committees, the American Engineering Council, and in the great responsibilities of American industry in safety and accident prevention, through our affiliation with the American Society of Safety Engineers and the National Safety Council.

Never before have the prospects of the Society looked so bright and its possibilities seemed so far-reaching. The splendid foresight of earlier finance committees and Councils have given the Society a sound financial policy, which makes it possible and right that we meet the challenge of the future and play our part well. No longer have we the right to " live unto ourselves."

Meetings of the Council were held as follows: New York, Dec. 4, 1925; Washington, D. C., Jan. 13; New Orleans, La., Mar. 8; Providence, R. I., May 3-4; San Francisco, Cal., June 28; Richmond, Va., Sept. 27; and New York, Dec. 6, 1926.

GENERAL INTERSOCIETY AND PUBLIC RELATIONS

The following brief outline covers intersociety relations and the activities of the Society touching upon the public welfare, national and international, in which more and more the engineering profession, through the engineering societies, is being called upon to take an active part. The A.S.M.E. representatives upon the various boards and committees are given in the Society Affairs section of this volume..

Engineering Foundation represents three gifts from our Honorary Member and Past-President, Ambrose Swasey, in 1915, 1916, and 1920, amounting to $508,850, earning an annual interest of approximately $25,000.

The Engineering Foundation, pursuing the policy adopted in December, 1925, has concentrated its attention upon the following projects endorsed by the societies — arch dam investigations; steel columns (for bridges and buildings); concrete and reinforced-concrete arches; blast-furnace slags; strength of gear teeth; lubrication; steam tables; dielectric absorption; engineering education (coöperating with the S.P.E.E.); fatigue of metals (in coöperation with the National Research Council and University of Illinois); and wood finishing (especially painting).

In these investigations The Engineering Foundation also is in cooperation with Government bureaus and laboratories, universities, associations, industries, etc.

The Henry R. Towne Engineering Fund (established in 1924), the income of which is used for general purposes of the Engineering Foundation, approximates $51,500, giving an annual interest of about $2500.

The John Fritz Medal for 1926 was presented to Edward Dean Adams "For great achievements as an engineer, scientist, financier, whose vision, courage and industry made possible the birth at Niagara Falls of hydroelectric power." The 1927 medal has been awarded to Elmer Ambrose Sperry, Life Member of the A.S.M.E., "For the development of the gyro-compass and the application of the gyroscope to the stabilization of ships and aeroplanes."

During this year the Board adopted the policy of presenting the John Fritz Medal at one of the general meetings of the Founder Societies. The award to Mr. Sperry was made during the A.S.M.E. Annual Meeting. The annual meeting of the Board has been changed from January to October of each year and our representations on the Board change at that time.

Engineering Education Survey. With the other Founder Societies the A.S.M.E. has assisted in the engineering educational survey conducted by the Society for the Promotion of Engineering Education which is now winding up its three-year introductory period of fact-finding, and organizing its two-year period of constructive action. An active campaign is going forward to raise the needed funds for the two years of further effort, with the coöperation of the national engineering societies. If funds are obtained, a program of activities will be launched under five general heads: (1) Revision of curricula and personnel practices in the colleges; (2) Development of activities by the S.P.E.E. to promote the training and development of younger teachers; (3) Joint activities with national engineering societies, aiming to give the societies more active part in the direction of engineering education; (4) Joint activities with industrial organizations to improve conditions surrounding the placement and introductory training of graduates; and (5) Joint activities with representatives of secondary education and other branches of higher education.

The *Division of Engineering and Industrial Research,* one of the several divisions of the *National Research Council,* aims to carry out

the general purpose of the National Research Council in the field of engineering by stimulating research and coördinating the work of the existing agencies. Its major activities may be divided into two groups: promotional, aimed at bringing about a greater appreciation of the value and benefits of research generally; and administration of projects of national scope and importance.

Projects selected for study by the Division are of broad, fundamental character, or of importance to a group of industries, or to some branch of the engineering profession. Typical projects now being investigated are: highways, heat transmission, welding, industrial lighting, and fatigue phenomena of metals.

Other activities of the Division include a Research Information Service in the field of engineering and industrial research.

Through the National Research Council the A.S.M.E. is a contributor to International Critical Tables and Annual Tables of Constants.

Museum of the Peaceful Arts. The A.S.M.E. President and Council granted the request of the Board of Direction of the Museums of the Peaceful Arts in the City of New York, that the secretary of the Society, Calvin W. Rice, who is also secretary of the Museum, make an inspection trip and report on the technical and industrial museums abroad.

As the result of the trip many helpful data were secured that were essential to the preparation of the plans for the projected museum.

American Engineering Standards Committee. From the A.E.S.C. Year Book for 1926, we reproduce the following summary of statistics which indicates the present status of the work going forward under its auspices.

Member-Bodies (organizations or groups of organizations whose representatives form the A.E.S.C.)	24
National organizations included in the Member-Bodies	35
Representatives forming the Main Committee	58
Organizations acting as sponsors for projects......about	60
Trade, technical, or governmental bodies coöperating through representatives on special or sectional committees......about	380
Individuals on sectional committees......over	1600
Sustaining Members	327
Projects having official status (already approved, or on which work is under way)	253
Standards approved to Nov. 22, 1926	94

The 253 projects having official status may be grouped as follows:

Group	Total	Approved
Civil Engineering and Building Trades	36	17
Mechanical	65	12
Electrical	37	9
Automotive	4	2
Transportation	10	9
Shipbuilding	1	—
Ferrous Metallurgy	17	7
Non-Ferrous Metallurgy	14	10
Chemical	13	10
Textile	3	1
Mining	20	3
Wood	5	3
Pulp and Paper	2	1
Miscellaneous	26	10

Engineering Societies Employment Service. As indicative of the activities of the three offices (New York, Chicago, and San Francisco), during the month of September, 1926, a total of 242 men were registered and 125 members were placed; 6377 letters and 982 applications were mailed, and 1252 records of various members sent to prospective employers.

The Service has issued regularly each week two Bulletins, one listing the positions open and the other men available. The latter is sent to employers not represented in the membership of the four Societies and therefore not likely to see similar listings published in the Society journals.

During 1926, up until October 31, 995 members of the four Societies were registered, divided as follows:

A.S.C.E.	254	A.I.M.E.	67
A.S.M.E.	398	A.I.E.E.	276

and there have been placed a total of 777, divided as follows:

A.S.C E.	202	A.I.M.E.	57
A.S.M.E.	352	A.I.E.E.	166

American Engineering Council

Dean Dexter S. Kimball was elected President of American Engineering Council in January, 1926, to serve for two years.

The major activities of American Engineering Council are covered by the following projects:

Inventory of Water Resources. The bill providing for an increasing appropriation for the Water Resources Branch of the United States Geological Survey has been introduced in Congress and received unusually favorable support.

Patent Office. The Committee on Patent Office Procedure during the year completed its survey of the Patent Office conditions. The A.S.M.E. was represented on this committee by Wallace Clark and W. H. Leffingwell. One of the important phases of the report was pointing out the need for the erection of a new building for the Patent Office. Concentrated effort has been made through the Public Buildings Commission and the National Commission of Fine Arts to provide a new building for the Patent Office out of the present appropriation of $50,000,000 for the erection of Government buildings in the District of Columbia.

In coöperation with a number of other organizations, American Engineering Council has aggressively endeavored to secure the passage of a bill to provide increased salaries for federal judges, because of its close bearing upon patent litigation.

Safety on Street and Highway. There are now representatives on seven committees of the Hoover Conference on Street and Highway Safety; through the work of these committees, the principles that have been adopted by the Conference are being put into operation, and a National Conference was held in Washington, October 15, 1926. The A.E.C. was requested to make a study of traffic signs, signals and markings and has been organizing for the execution of this commission.

Safety and Production. A study is being conducted to determine if there is any direct correlation between accidents and production, and for this purpose some 45 local engineering committees were formed and 15 field engineers employed. Through this organization data have been obtained from 14,000 plants. The information is being compiled and an illuminating report will be issued early in 1927.

Reforestation and Fire Prevention. Representatives appeared before a Congressional committee to urge that the $2,000,000 provided for

under the Weeks Law for the purchase of forest preserves be restored to the appropriation bill. These representatives also urged the appropriation provided for under the Clarke-McNary Act, whereby forest-fire prevention might be more adequately taken care of.

Topographic Surveys and Maps. The Temple Act passed two years ago provided for completion in twenty years of the topographic survey of the United States. This Act carries authorization for the work but no appropriation, hence it is necessary to secure an appropriation through the Bureau of the Budget and the Appropriations Committees of the House and Senate. Recently representatives of Council appeared before the Director of the Budget for the purpose of securing recognition of the Temple Act by the Bureau of the Budget.

Government Reorganization. A.E.C. is continuing its activity in the direction of securing the establishment of a Department of Public Works and Domain. A special Committee composed of management engineers has made a thorough study of the present engineering, construction, and architectural organization of the Federal Government and upon this study has predicated a plan of organization for the proposed Department of Public Works and Domain. Wallace Clark, John Price Jackson, and Sanford E. Thompson represent the A.S.M E. on the Committee. An active campaign will be waged for the passage of the Jones-Wyant Bill which provides for the establishment of such a department.

Special Appointments and Exchange of Courtesies

General Assignments. By special invitation the Society has representatives on the following organizations:

National Committee on Metals Utilization, Department of Commerce: Stanley G. Flagg, Jr. (Charles M. Manly, alternate)

National Conference on Street and Highway Safety, organized by the Department of Commerce: E. J. Posselt

National Research Council, Highway Research Committee: H. de B. Parsons

National Screw Thread Commission: Luther D. Burlingame

International High Commission, Advisory Committee to the U. S. Section, concerned with a preliminary study and compilation of a report on the use of Uniform Weights and Measures: R. E. Flanders

Fuel Conservation Board, U. S. Shipping Board: H. L. Seward

National Safety Council, American Society of Safety Engineers, Engineering Section, Study of Low-Voltage Electrical Hazard: John Price Jackson

American Association for the Advancement of Science: Chas. Russ Richards and John T. Faig

International Electrotechnical Commission: W. F. Durand, and F. R. Low (C. Harold Berry, alternate)

Special Committee of the American Welding Society and the American Bureau of Welding — Fusion Welding for Pressure Vessels: W. F. Durand, R. L. Daugherty, E. R. Fish, Sherwood F. Jeter, and D. S. Jacobus; Gas Welding: James Partington and C. W. Obert; Reorganized American Bureau of Welding Advisory Board to the National Research Council: James Partington; Structural Steel Welding, Advisory Committee to the American Bureau of Welding: George A. Orrok

Washington Award of the Western Society of Engineers: Charles Russ Richards and Colonel E. S. Ellicott

American Engineering Standards Committee: representatives on many of the sponsorships of other societies.

In addition to the regular appointment each year, and intersociety relations, the President of the Society during his administrative year is called upon to make appointments of Honorary Vice-Presidents, in response to special invitations from universities, associations, special congresses and similar interests.

For the term of office of President Abbott we have had forty such appointments.

COMMITTEE REPORTS

Finance

Your Society is in good financial condition. The present system and methods of budgeting have proved their effectiveness and reliability. By the splendid coöperation of the Council, the Committees, and the Staff at headquarters, we have been able to place our Society in a sound financial position.

A comparison of the Balance Sheet as of September 30, 1925 and 1926, shows the following changes:

ASSETS

	Sept. 30, 1925	Sept. 30, 1926
Cash	$31,421.35	$40,510.66
Accts. Receivable	133,935.20	127,822.41
Inventory	41,760.43	46,700.44
Deferred Charges	15,736.16	11,971.70
Investments for Trust Funds	75,849.20	87,556.68
Investments for Capital Acct.	134,000.00	226,521.88
Fixed Assets	521,149.09	502,823.96
Total Assets	$953,851.43	$1,043,407.73

LIABILITIES

	Sept. 30, 1925	Sept. 30, 1926
Unfilled Obligations (Current Liabilities)	$30,047.71	$31,464.82
Contributions for Research	4,629.97	30,067.49
Dues Paid in Advance	1,880.08	2,592.04
Trust Funds	75,849.20	87,556.68
Capital	841,444.47	891,726.70
Total Liabilities	$953,851.43	$1,043,407.73

The only item of note in the comparison is the decline in Fixed Assets in 1926 as compared with those of 1925. This is due almost entirely to the decision to adopt Messrs. Ernst & Ernst's recommendation to write the Library Books and the Engineering Index down to $1.00 each.

RECONCILIATION OF FIXED ASSETS

	Sept. 30, 1925	Sept. 30, 1926
Building and Equipment	$498,149.09	$502,821.96
Library Books (Book Value)	13,000.00	1.00
Engineering Index (Book Value)	10,000.00	1.00
Total	$521,149.09	$502,823.96

The comparative summary of the budgets in Table 1 will show the trends and tendencies and reflect the effect of policies adopted by the Council.

It should be noted that while we appropriated $619,000.00, our actual expenditures for the past fiscal year were $613,359.05, or $5640.95 less than the budget, and our income was $657,415.44, or $38,415.44 above the estimated budgeted income. This increase in income was brought about largely by three developments:

1 An increased income of $31,199.51 from the income producing activities due to an increase in appropriation of $9090.64.

2 An almost negligible amount of bad debts — budgeted $10,000, $1458.88 actual.

3 Better handling of our reserves and working capital, so that we earned $3868.24 more in interest than was anticipated.

The difference, then, between actual Net Income Available for Appropriation of $657,415.44 and the actual Expenditures for the fiscal

TABLE 1 COMPARATIVE FINANCIAL STATEMENT

Income

	1925-26 Estimated	1925-26 Actual	1926-27 Budget
Initiation Fees	$25,000.00	$26,846 32	$25,000.00
Membership Dues	285,000.00	279,806 57	295,000.00
Mech. Engrg., Condensed Catalogues, Publications, etc.	334,000.00	365,199.51	342,300.00
Bad Debt Reserve	10,000.00	1,458 88	3,300 00
Interest	10,000.00	13,868 24	11,000 00
Working Capital Reserve	.. .	.... ..	16,269 00
Net Income Available for Appropriation	619,000 00	657,415 44	628,731.00

Expenditures

	1925-26 Estimated	1925-26 Actual	1926-27 Budget
Administration and General	$101,700.00	$95,560.23	$94,781.00
Initiation Fees Account	21,800.00	20,385.61	20,500.00
Service to Members Account	242,940.00	237,945.12	249,400.00
Income Producing and Profession Account	226,050.00	235,140.64	241,500.00
Service to Public Account	26,510.00	24,327.45	22,550.00
Total	$619,000.00	$613,359.05	$628,731.00

year of $613,359.05, is $44,481.39. This sum plus the $26,846.32 from initiation fees, or a total of $71,327.71, is carried over to the Investments for Capital Account.

The largest increase in any appropriation was for Research, which was increased from $6000.00 in the 1926 budget to $14,000 in the 1927 budget. The greatest decrease in any budget item was the appropriation for Council, which was reduced from $27,000 in 1926 to $18,000 in

TABLE 2 MEMBERSHIP CHANGES — OCTOBER 1, 1925, TO SEPTEMBER 30, 1926

	Membership		Losses				Additions		Totals		
	Oct. 1, 1925	Oct. 1, 1926	Trans. from	Resigned	Dropped	Died	Trans. to	Elected	Loss	Gain	Change
Honorary Members	21	21									
Life Members	78	81				3		6	3	6	3 +
Members	8065	8255		133	168	83	345	229	384	574	190 +
Associates	792	729	5	55	68	13	8	70	141	78	63 —
Associate-Members	4182	4185	195	99	269	11	446	131	574	577	3 +
Juniors (20)	1016	752	264	33	60	5	98		362	98	264 —
Juniors (10)	3574	3604	433	95	185			743	713	743	30 +
Totals	17728	17627	897	415	750	115	897	1179	2177	2076	101 —

1927. The Professional Divisions' appropriation was increased from $10,500 in 1926 to $15,000 for 1927, or approximately 50 per cent.

The effect of the policy pertaining to members' dues and prompt collection of these Accounts Receivable is reflected in the decline in the actual receipts of dues from $285,000.00, budgeted, to $279,806.67, the actual amount received. The income from initiation fees, however, was approximately $1850.00 higher than the estimate, indicating that the induction of new members is subject to but slight fluctuations. The financial statements seem to indicate that in so far as income from members is concerned, this is in a state of equilibrium at the present time.

The Finance Committee approved the form of a new voucher, to simplify our accounting procedure, and also approved of the rearrangement and regrouping of certain budget items, as presented by the Secretary, to secure greater simplicity and greater control of accounts.

The Balance Sheet as of September 30, 1926, appears on page 1408.

MEMBERSHIP

The Committee on Membership held eighteen meetings during the year 1925-1926.

The number of applications considered in the transaction of its work, and a summary showing the action taken, are as follows:

Applications pending October 1, 1925	696	
Applications received during fiscal year 1925-1926	1965	
Total applications handled during year 1925-1926		2661
Recommended for membership	1932	
Transfers denied	7	
Deferred	14	
Withdrawn	43	
Applications pending October 1, 1926	665	
Total applications handled during year 1925-1926		2661

Those recommended for membership were divided into the following grades:

Members	242	
Transfers to Member	284	
Associates	29	
Transfers to Associate	8	
Associate-Members	260	
Transfers to Associate-Members	366	
Juniors	307	
Juniors (R 5 Rule 1)	436	
Total		1932

Elections declared void	48
Resignations accepted	377
Dropped from membership	715
Deceased	115

MEMBERSHIP GAIN

Applications approved	1932	
Transfers	658	
Total new Members		1274
Resignations, dropped, deceased and election void	1255	
Net gain in Members[1]		19

Table 2 shows the changes in membership during the fiscal year.

[1] This is not the actual gain of the Society's membership but represents those recommended for admission by the Membership Committee.

Meetings and Program

During the year four meetings of the Society were held which brought into contact with the Society many of the outstanding men in industry, science, and engineering.

The Providence Meeting in May, 1926, presented an elaborate program of technical sessions, and the entertainments and excursions were conducted most successfully. The special program arranged for the ladies was an unusually attractive one.

The San Francisco Meeting in June, 1926, was the first held there in thirty-four years. The San Francisco Section had developed an excellent organization which planned and conducted a fine meeting with attractive entertainment and good technical sessions. Dr. Robert Andrews Millikan received the A.S.M.E. Medal and presented a remarkable address at a large dinner gathering. William Sproule, President of the Southern Pacific Railway, also spoke. The registration was 547.

The San Francisco Meeting furnished the opportunity for 163 members of the Society and their friends to journey by special train from New York and Chicago to the meeting. The trip of 8800 miles by train and 1200 by automobile was made exceedingly attractive by the hospitality of the Local Sections in the cities through which the train passed.

The "Old Dominion" meeting, which opened at Richmond on September 27, combined technical sessions and entertainments in Richmond with excursions to historical shrines and a two-day trip by boat to Washington. The registration of 125 for the meeting showed attendance from eighteen states and Canada.

The Annual Meeting, held in New York December 6 to 10, was the largest in the history of the Society, with a registration of 2213. Joint sessions were held with the American Society of Refrigerating Engineers and the Taylor Society. The second Robert Henry Thurston and Henry Robinson Towne lectures were presented by Dr. Cecil Howard Lander and Dr. Davis Rich Dewey, respectively. As part of the ceremonies of "Presidents' Night" the John Fritz Medal was presented to Dr. Elmer Ambrose Sperry.

More complete reports of this meeting and of the Spring Meeting will be found in the Society Affairs section of this volume.

Publications

The year 1925-1926 continued the broadened policy of publication started in 1920 but curtailed for four years and resumed in 1925.

Transactions, Volume 47, issued in July, contained 1402 pages; 1453 pages of text appeared in *Mechanical Engineering*. This was an increase of 12 per cent over the material appearing last year. The publication of the *A.S.M.E. News*, 1926 Year Book, 1925 Engineering Index Annual, and Condensed Catalogues was conducted in the usual successful manner. The income from the publications showed a substantial increase during the past fiscal year over any previous year.

Professional Divisions

During the current year the constitution of the Standing Committee on Professional Divisions has been changed from that of a Committee elected from the Chairmen of the various Divisions to a Committee appointed by the President, the terms of the members being for five years and one member retiring each year.

This newly constituted Committee has, with the exception of the two summer months, held regular monthly meetings throughout the

year, and in addition has held two general conferences with Chairmen or representatives of the Professional Divisions, together with the Chairmen or representatives of Standing Committees on Local Sections, on Meetings and Program, and on Research. The object of these conferences was to promote a better understanding of the relation of the several Divisions to each other and to the various Standing Committees of the Society and also to develop the cooperation of these several bodies.

As a result of these meetings a statement of the make-up, purpose, duties and organization of the Professional Divisions was drawn up for the guidance of the Executive Committee and transmitted to each Division.

Another important step has been the institution of a section in *Mechanical Engineering* entitled "The Conference Table." In this department are being printed problems of an engineering character which arise in the daily work of the members of the Society and whose solution is not readily available in the text books or literature at hand. These problems are submitted to the several Divisions for solution. Each Division has appointed an editor to assist in preparing the material for publication in *Mechanical Engineering*.

Probably the most important development in the work of the Professional Divisions has been the institution of divisional meetings, national in character and quality, each meeting to be approved by the Standing Committees on Professional Divisions, Local Sections, and Meetings and Program. The papers to be presented are to be approved by the Committee on Meetings and Program.

It is proposed that these meetings should have all the character and dignity of an Annual, Spring, or Regional Meeting of the Society. The Committee on Local Sections has given its hearty approval to this program and is coöperating in its development.

During the conferences with the Division representatives much stress was laid on the fact that research was one of the fundamental objects of the Society and that the Professional Division could render no greater service than the formulation and institution of research projects. Each Division was urged to appoint a Committee on Research and to draw up research programs for consideration by the Standing Committee on Research. Several Divisions have already complied with this request.

Another change which profoundly affects the work of the Divisions and which makes for continuity in their policy is the change in the method of instituting the Executive Committee of each Division. Heretofore the Executive Committee has been elected, but upon request by the Standing Committee on Professional Divisions the Council of the Society has made the Executive Committee of a Professional Division an appointive one, the members being appointed by the President for a term of five years, one member retiring each year.

Local Sections

A Local Section was organized during the year with headquarters at Rockford, Ill., to be known as the Rock River Valley Section. This brings the total number of Sections up to 67. During the year 331 meetings were held by these Local Sections.

More active coöperation was rendered by the various Local Sections to the Professional Divisions and Student Branches than ever before, and one new development in this connection was the offering of prizes for Inter-Student-Branch debates held under the auspices of the Section in the territory where these Student Branches are located. Debates were conducted by the following Sections: Indianapolis, Oregon, Colorado, Washington, D. C., Pittsburgh, and Mid-Continent.

The Local Sections had a unique opportunity to coöperate in the development of interest in the Spring Meeting through the support which they gave to the special Trans-Continental Tour to the San Francisco Meeting, arranged under the general auspices of the Committee on Local Sections and Committee on Meetings and Program. The following Sections cooperated in making the arrangements, and in entertaining the party: Chicago, Kansas City, Colorado, Los Angeles, Oregon, Western Washington, St. Paul, and Minneapolis.

The New Haven Section sponsored for the sixth consecutive year a successful Machine-Tool Exhibition held in conjunction with the Engineering Faculty of Yale University and the New Haven Chamber of Commerce. Coincident with the Exhibition there was held a series of technical sessions developed with the coöperation of the Professional Division on Machine-Shop Practice.

At the Annual and Spring Meetings and also at the Providence Regional Meeting, conferences of Local Sections' Delegates were held; the only one of these which was formal and entitled the delegates to traveling expenses was that held at the Annual Meeting, when 58 out of 66 Sections were represented.

The Committee on Local Sections, which also serves the Society as the Committee on Society Development, was responsible during the year for the issuance of an attractive brochure descriptive of the Society's activities and available for distribution for members or persons interested in having such a statement.

One of the activities which the Committee emphasized during the past years, and which aims to carry on during the ensuing year, was a definite effort to develop all existing Local Sections to a state of efficiency rather than to attempt further extension of the number of Local Stations. Localities, however, which take the initiative to institute new sections will always be given encouragement and assistance.

Education and Training for the Industries

The work of the Committee on Education and Training for the Industries has consisted principally in arranging sessions at the Spring, Regional, and Annual Meetings of the Society, and at the New Haven Machine-Tool Exhibition.

Many of the papers presented at these sessions were published in *Mechanical Engineering* and later will be issued with earlier papers on the subject in book form.

The Committee hopes in this way to bring more and more to the members and the profession generally, the importance of this movement for Education and Training for the Industries, to care for the men below the college grade.

Constitution and By-Laws

This committee acts only as a reviewing committee and advisory to the Council in preparing drafts of by-laws and rules, to carry out policies that have been approved by the Council. This year the Constitution has been amended in the provisions for Junior membership and method of selection of the Treasurer. In order to preserve the same number of persons on the Council, the office of another Vice-President has been created.

The following revision of the Constitution went into effect with the announcement of the report of the Tellers at the Business Session of the Annual Meeting of 1926.

OLD WORDING

ARTICLE C4

Qualifications for Admission

Sec. 6. A Junior must have had such engineering experience as will enable him to fill a subordinate position in engineering work, or he must be a graduate of an engineering school of accepted standing. He must be at least twenty-one (21) years of age, and his connection with the Society shall cease when he becomes thirty (30) years of age unless he be previously transferred to another grade.

REVISED WORDING

ARTICLE C4

Qualifications for Admission

Sec. 6. A Junior must have had such engineering experience as will enable him to fill a subordinate position in engineering work, or he must be a graduate of an engineering school of accepted standing. He must be at least twenty-one (21) years of age, and his connection with the Society shall cease when he becomes thirty-five (35) years of age unless he has been previously transferred to another grade.[1]

OLD WORDING

ARTICLE C7

Directors and Officers (Council)

Sec. 2. The Directors of the Society shall consist of a President. six (6) Vice-Presidents, nine (9) Managers, the last five (5) surviving Past-Presidents, and a Treasurer.

Sec. 4. The President shall be elected for one (1) year, the Vice-President for two (2) years, the Managers for three (3) years, and the Treasurer for one (1) year.

Sec. 6. The Directors may at any time, whenever sufficient cause shall appear to them, delegate to any member of the Society the performance of any duties required by the Constitution to be performed by any Director or by the Secretary.

REVISED WORDING

ARTICLE C7

Directors and Officers (Council)

Sec. 2. The Directors of the Society shall consist of a President, seven (7) Vice-Presidents, nine (9) Managers, the last five (5) surviving Past-Presidents.

Sec. 4. The President shall be elected for one (1) year, the Vice-President for two (2) years, and the Managers for three (3) years.

Sec. 6. At its first meeting after the annual meeting of the Society the Council shall appoint a member of the Society to serve as Treasurer for one (1) year.

The Treasurer shall perform the duties usually pertaining to this office, in accordance with the By-Laws and Rules, and such further duties as may be required by the Council.

Any vacancy in the office of Treasurer shall be filled by appointment by the Council.

Sec. 7. (Same as old Sec. 6.)

---

[1] Inclusive of Junior Member elections from December, 1922.

The By-Laws will be correspondingly amended to carry out the provisions of the amendments to the Constitution.

Minor amendments have been made in the By-Laws regarding Life Membership provisions, and as previously reported in the provisions for the elections of the Professional Divisions Main and Executive Committees.

Awards

The Committee on Awards has been active in the preparation of definite policies for the several Society Awards.

In December, 1925, Major Max Toltz gave $15,000 to endow a Student Loan Fund. The following has been established as the procedure for this gift. Satisfactory recommendation from a responsible officer of an applicant's college and complete study and report of the case by the Committee on Awards are prerequisites.

1 That the gift of $15,000 (fifteen thousand dollars) by Major Max Toltz, a member of this Society, be known as the Major Max Toltz Fund for Assistance to Students.

2 That the full amount be invested by the Treasurer and the Finance Committee, to be kept intact, and that the income be used to restore the fund to $15,000 in case any losses occur.

3 That under ordinary circumstances only the income be used.

4 That in cases of exceptional need where money is not available from income of the Fund small amounts may be advanced by the Society, under guarantee of this special Fund and its income.

5 That for the present the assistance to students be in the form of loans, at five per cent interest, commencing from the date the money is advanced.

6 That loans be made through the Committee on Awards, only to students approved by a responsible officer of the institution to which they belong.

7 That further necessary regulations for the use of the income to assist students be made by the Committee on Awards as that income becomes available and that all rules and cases be reported to the Council.

The Council reported the following awards for 1926:

A.S.M.E. Medal to Dr. R. A. Millikan for his contributions to Science and Engineering.

Student Awards to R. E. Peterson of the University of Illinois for his paper on An Investigation of Stress Concentration by Means of Plaster of Paris Specimens; and to Cecil G. Heard of the University of Toronto for his paper on Pressure Distribution Over U. S. A.-27 Aerofoil with Square Wing Tips. (Both of these papers were published in *Mechanical Engineering* for December, 1926.)

Charles T. Main Award (Subject of paper also selected by Committee and approved by the Council) to W. C. Saylor of the Johns Hopkins University for his paper on Effect of the Cotton Gin upon the History of the United States during Its First Seventy Years.

In addition to the Society's Awards, the Woman's Auxiliary has been helping students and has loaned money for this purpose on two occasions.

The Council has requested the Committee on Meetings to make the necessary arrangements to carry out dignified and appropriate ceremonies in connection with the bestowing of the Society Awards, with medals for distinguished service, special addresses and ceremonies to be provided at a session of one of the general meetings of the Society.

The Committee is now at work on a procedure for the awarding of the Melville Medal. This Medal was provided for in the will of the

late Rear Admiral George W. Melville, Past-President and Honorary Member of the Society.

The Committee records with deep regret the death of one of its members, R. Sanford Riley; James H. Herron of Cleveland was appointed to fill the vacancy to December, 1926.

Relations with Colleges

The Student Branches held 643 meetings during the year, several of these jointly with adjacent Local Sections; debates have also been held at meetings of the following Local Sections: Washington, D. C., Pittsburgh, Colorado, Oregon, Indianapolis, and Mid-Continent.

These have proved successful from every point of view, and is the type of activity the Committee encourages for Student Branches.

The Committee on Relations with Colleges held four meetings during the year. On its recommendation the Council authorized the establishment of the following Student Branches:

University of New Hampshire, Durham, N. H.
University of Florida, Gainesville, Fla.
Princeton University, Princeton, N. J.
Rose Polytechnic Institute, Terre Haute, Ind.
Mississippi A. & M. College, A. & M. College, Miss.

Visits were made to numerous Branches by Professor W. H. Kavanaugh, Dean A. A. Potter, and Dr. H. G. Tyler, all of the Committee on Relations with Colleges. The reports that were received after these visits showed that their talks were highly inspiring and that they made favorable impressions upon all with whom they came in contact.

At the Annual and Spring Meetings of the Society, conferences were held by the Student Branches, at which many interesting matters were discussed.

A new brochure has just been completed and has been distributed to Student Branches. This booklet explains the organization and activities of Student Branches, and also sets forth the relationship between these Branches and the parent Society.

The Committee is always thankful for the coöperation of members who are willing to visit the Branches and tell them of the work of the Society and of the profession.

The Committee records with regret the death of Dr. Tyler on October 27, 1926.

Library

The Council records the special gift of the late George F. Fowler, member of the Society, who by his will left to the Engineering Societies Library several hundred books.

An endowment committee has been organized by the Library Board to seek funds for the Library. The Library Endowment Fund now is approximately $100,000, the amount of the cash gifts by Dr. James Douglas.

Professional Conduct Committee

This committee acts in an advisory capacity to the Council reporting on cases referred to the committee for investigation, involving infractions of professional ethics. There has been only one case where drastic action was recommended to the Council since the adoption of the Code and the establishment of our committee. The reports and records of this committee are confidential and are not published.

STANDARDIZATION

Standardization in the mechanical industries has progressed in a normal way during the past year. The list of Sectional Committees sponsored by the A.S.M.E. has been increased from 14 to 20 and the number of sub-committees from 33 to 52. Throughout the year these committees have worked steadily and many of them have practically completed the projects assigned to them. Summaries of the progress reports of these committees are given herewith.

American Engineering Standards Committee In the report on general intersociety and public relations (see page 1385) will be found a summary of the activities of this committee.

Ball Bearings. William R. Strickland, Chairman. The members of the Sectional Committee on the Standardization of Ball Bearings arranged and carried through an international conference on this project on April 21, 1926, during the Third International Conference of Chairmen and Secretaries of National Standardizing Bodies. As a direct result the international standardization of radial bearings is practically completed.

Shafting. Cloyd M. Chapman, Chairman. This committee has already completed standards for Diameters and Lengths of Shafting and Square and Flat Stock Keys. Sub-Committee No. 3 on a Code for Design of Transmission Shafting this year completed its report which will soon be presented to the Council for approval and transmission to the A.E.S.C. Sub-Committee No. 4 has about completed the standards for Square and Flat Plain Taper Stock Keys and Gib-Head Stock Keys Sub-Committee No. 5 on Woodruff Keys and Keyways has completed the first table of its report covering the dimensions of the keys. The dimensions of the corresponding keyseats and keyways are now under way.

Small Tools and Machine-Tool Elements. Harry E. Harris, Chairman of Central Committee.

Sub-Committee No. 1, T-Slots, Their Bolts, Nuts, and Cutters, Erick Oberg, Chairman, completed its work during the present year. The report has been made ready for submission to the A.S.M.E. Council and the A.E.S.C. for approval.

Sub-Committee No. 2, Tool Holders and Tool-Post Openings, P. M. Mueller, Chairman, was organized in March, 1926. It has completed a draft of the proposed standard which has been distributed for criticism and comment.

Sub-Committee No. 3, on Machine Tapers. A conference on the standardization of machine tapers was held during the Providence Meeting. The conference unanimously approved carrying forward the standardization of this machine-tool element.

Sub-Committee No. 4, Milling Cutters. A general conference held in New Haven in September, 1926, recommended the standardization of milling cutters, and the immediate organization of a sub-committee to develop these standards.

Code for Identification of Piping Systems. Amos S. Hebble, Chairman. It was found desirable to redraft one section of this Code before publication so it is now in the hands of the Editing Sub-Committee for that purpose.

Plain Limit Gages for General Engineering Work. Eugene C. Peck, Chairman. This Sectional Committee's first report entitled Tolerances, Allowances, and Gages for Metal Fits was published in 1925. Its second report, to be known as Methods of Gaging and Specifications for Plain Limit Gages, is now in the hands of a special sub-committee for editing.

National (American) Standard Fire-Hose Couplings Screw Threads. The American Standard for Fire-Hose Couplings Screw Threads was

finally approved in April, 1925. Already 1750 copies of the pamphlet have been sold and we are continually receiving reports of the adoption of this standard hose coupling by the cities and towns throughout the country.

Standardization of Gears. Benjamin F. Waterman, Chairman. The Proposed Standard for Spur-Gear Tooth Form has been made ready for submission to the A.S.M.E. Council and the A.E.S.C. The Sub-Committee on Nomenclature has submitted two sets of symbols and abbreviations to the Sectional Committee for criticism and comment.

Standardization and Unification of Screw Threads. Luther D. Burlingame, Chairman. During the New York meeting of the Chairmen and Secretaries of the National Standardizing Bodies of the World on April 14, 1926, an Anglo-American conference of Screw Threads was held. At this conference the possibility of developing Anglo-American and International agreement was seriously considered and plans were laid for a further study of this important subject.

Pipe Flanges and Fittings. Collins P. Bliss, Chairman. All of the Sub-Committees of this Sectional Committee have been very active during the past year and have made good progress. Sub-Committee No. 3 on Steel Flanges and Flanged Fittings, C. P. Bliss, Chairman, will soon complete dimensional standards for steel flanges and flanged fittings to withstand steam pressures of 250, 400, 600, 900 and 1350 lb. per sq. in. at a temperature of 750 deg. fahr. After approval by the governing boards of the three sponsor societies these standards will be transmitted to the A.E.S.C. for approval and designation as Tentative American Standards.

Sub-Committee No. 1 on Cast-Iron Flanges and Flanged Fittings, Arthur M. Houser, Chairman, has finished its revision of the "1914 Standards" for 125 and 250 lb. steam pressures. These standards are now in the hands of the members of the Sectional Committee for study prior to a general discussion and letter ballot. The Flanged Standard for 25 lb. pressure was resubmitted to the Sub-Group for amplification.

Sub-Committee No. 2 on Screwed Fittings, Stanley G. Flagg, Chairman, presented its reports on the 125- and 250-lb. Cast-Iron Screwed Fittings and the 150-lb. Malleable-Iron Screwed Fittings to the Sectional Committee for study and vote. A proposed standard for Cast-Iron Long-Turn Sprinkler Fittings has been drafted and will shortly be distributed for general criticism.

Bolt, Nut, and Rivet Proportions. Arthur E. Norton, Chairman. This Sectional Committee has been voting on the standards for Wrench-Head Bolt and Nuts, and Wrench Openings, Plow Bolts, Small Rivets, Tinners', Coopers' and Belt Rivets, and Round Unslotted-Head Bolts so they will be presented soon to the sponsors for approval and transmission to the A.E.S.C. The Standard for Track Bolts has been resubmitted to Sub-Committee No. 4.

Scientific and Engineering Symbols and Abbreviations. J. Franklin Meyer, Chairman. This Committee has formed seven new sub-committees during the past year. These sub-committees will cover the following subjects: (1) Mechanics, Structural Engineering and Testing Materials, (2) Symbols for Hydraulics, (3) Heat, Thermodynamics and Thermometry, (4) Photometry and Illumination, (5) Symbols and Abbreviations for Aeronautics, (6) Mathematics and Mathematical Signs, (7) Electrical Symbols and Abbreviations, including Radio.

Pipe Threads. This standard for Pipe Threads was completed in 1919. Revision of this standard at an early date is strongly recommended by the Sectional Committee on the Standardization of Pipe Flanges and Fittings.

Machine Pins. This Sectional Committee was organized on March 19, 1926, to undertake the standardization of straight, taper, split, and dowel pins. Temporary officers were elected and sub-committees were appointed and assigned to the several parts of the work.

Plain and Lock Washers. In March also a Sectional Committee was organized to develop standards for plain and locked washers. Victor E. Bertrandias was chosen as temporary chairman and authorized to appoint three Sub-Committees, one on punched washers, a second for lock washers, and a third for malleable-iron and cast-iron washers.

Transmission Chains and Sprockets. The personnel of the Sectional Committee on the Standardization of Transmission Chains and Sprockets is now nearly completed, so the work on the review of the present standards will begin soon. It will be recalled that this Sectional Committee constitutes a reorganization of the Joint Committee with the same name. Both Committees have been sponsored by the S.A.E., the A.G.M.A., and the A.S.M.E.

Drawings and Drafting-Room Practice. Franklin DeF. Furman was appointed temporary chairman at the organization meeting of the Sectional Committee on Drawings and Drafting-Room Practice which was held on September 23, 1926, and Carl Keuffel temporary secretary. These officers were authorized to appoint three sub-committees on (1) Specifications for Paper and Cloth, (2) Dimensions and Tolerance of Figures, and (3) Lettering and Dimensions.

Pressure Piping. The organization of the Sectional Committee which is to develop a Code for Pressure Piping has progressed satisfactorily during the year and the first meeting of the Committee was held on November 12.

Graphic Presentation. The preliminary organization work of this Sectional Committee has been completed and the Committee organized.

Wire and Sheet-Metal Gages. A conference on Standardization of Wire and Sheet-Metal Gages was held on March 18, 1926. The A.E.S.C. has requested the Society of Automotive Engineers and the A.S.M.E to accept joint sponsorship for this project. Acceptance of this joint sponsorship by the A.S.M.E. is now before the Council.

Representation on Other Committees. The Society has official representatives serving on the following 16 Standards Committees:

Special A.E.S.C. Committee on Hose Specifications
Special A.E.S.C. Committee on Methods of Testing Petroleum Products and Lubricants
Special A.E.S.C. Committee on Size of Publication for Standards
Special A.E.S.C. Committee on Steel Railway Bridges and Specifications for Movable Railway Bridges
Sectional Committee on Methods for Testing Wood
Sectional Committee on Specifications for Special Materials for Use in the Manufacture of Trackwork
Sectional Committee on Outside Coal-Handling Equipment
Sectional Committee on Numbering of Steels
Sectional Committee on Wire Rope for Mines
Sectional Committee on Specification for Cast-Iron Pipe and Special Castings
Sectional Committee on Mine Ventilation, Coal and Metal
Sectional Committee on Fire Tests of Materials and Construction
Sectional Committee on Drainage of Coal Mines
Sectional Committee on Manhole Frames and Covers
Sectional Committee on Units and Bases for Ratings of Rivers for Producing Water
General Committee on Ship Construction and Operation (A.M.S.C.).

Research

No previous year in the history of the Society's research activities has been productive of as great results or has been so full of encouragement as the one just past. It will be recalled that for the year ending October 1, 1926, the Council authorized an expenditure of $6000 on its research program. With this as a working fund your Standing Committee on Research has been able to begin work on six new projects and to stimulate the work of the remaining eleven committees. During this fiscal year four of these Special Research Committees appealed for funds to the industries interested in their work and were successful in securing cash contributions totaling $46,000.

The six new committees will carry on research work on elevators, worm gears, spark arresters, saws and knives for woodworking, and substitute species for domestic woods. On the invitation of the American Welding Society a Joint Research Committee on Fusion Welding for Pressure Vessels is being formed and five members of The American Society of Mechanical Engineers have been appointed to represent it on this Committee. Plans are also being made to organize at an early date a Special Research Committee on Cutting Edges (Thin Metal Plates).

The yearly appropriations of the Council for this activity give substantial support to the presentation and publication of papers, reports, and other publicity matter that stimulates interest in the proposed researches. Last year more than eighty pages of this kind of material were printed in *Mechanical Engineering.*

It is not the policy of the Standing Committee to finance any investigation beyond its beginning. The results of last year's work furnish excellent proof that industrial concerns can be interested in giving financial support to projects that particularly concern them.

Under the present working plan of the Society's research program, practically all new projects originate within the Professional Divisions of the A.S.M.E., or through the efforts of the Standing Committees on Research and Standardization. This committee decides what investigations should be undertaken and, on approval by the Council, organizes Special A.S.M.E. Research Committees, the personnel being chosen from among individuals interested in the particular problems.

Investigations are carried on in commercial and college laboratories and at the Government bureaus. Here the Special Research Committees of the Society maintain and supervise skilled research workers. Progress is reviewed from time to time in published reports to stimulate discussion that is helpful to the most comprehensive conduct of the work.

A review of the work of the various Special Committees is given in the following outline:

Lubrication. The progress report of the Research work on lubrication for the year December 1924-1925 was submitted to Engineering Foundation by Mayo D. Hersey. It is arranged in three parts: (1) Mine-Car Friction Losses, (2) Viscosity of Lubricating Oils under Extremely High Pressures, and (3) Viscosity of Lubricating Oils under both High Pressures and Temperatures.

The Main Research Committee has recommended an enlargement of the personnel, a broadening of the scope of its activities, and has aided slightly its financing.

Bearing Metals. Christopher H. Bierbaum, Chairman. The chairman states that the research work of the Committee during the past year has consisted in the efforts of its individual members directing laboratory research efforts, with two problems successfully solved: (1) the suspension of lead in bronze alloys by new and improved methods; and (2) the improvement of phosphor bronze by increasing the ultimate

tensile strength and elastic limit without increasing the hardness of the softest crystal, or decreasing the hardness of the hardest crystal.

Fluid Meters. R. J. S. Pigott, Chairman. This special Committee was reorganized in July, 1926, with eleven members. They have undertaken a revision of Part I of the Report on Fluid Meters and the compilation of Parts II and III.

Properties of Steam and Extension of the Steam Tables Progress reports presented by the committee at the 1925 Annual Meeting were printed in the February, 1926, issue of *Mechanical Engineering.*

The Executive Committee on the Steam Table Fund reports that approximately $55,800 has been expended to date on this research and that during the year $9750 had been contributed.

Strength of Gear Teeth Wilfred Lewis, Chairman This Committee has been very active during the year. Three Progress Reports have been presented covering the experimental work conducted by John E Nicholas with the gear-testing machine at the Massachusetts Institute of Technology. The splendid response of several of the members of the American Gear Manufacturers Association to requests for test gears and gear materials greatly facilitated this work.

In May, 1926, the Committee issued Progress Report No. 2 entitled Data and Results of Series of Experiments Covering Influence of Error and Elasticity on Strength of Gear Teeth, which in slightly revised and abridged form was printed in the September, 1926, issue of *Mechanical Engineering.* It was reprinted later for distribution as Progress Report No. 3.

Cutting and Forming of Metals. Prof. James A. Hall, Chairman. The sub-committees will develop researches on Cutting Tools, Forming Metals, Turning Processes and Cutting Fluids, Standards of Performance, and Coöperation with Industries and Publicity.

In a joint session of this Special Research Committee and the A.S.M.E. Machine Shop Practice Division at the 1926 Annual Meeting, several papers were presented which appear elsewhere in this volume.

Mechanical Springs. At the Providence meeting Chairman Wood presented a paper on The Specifications and Control of Mechanical Springs which will be found elsewhere in this volume.

A canvass of the industries particularly interested for funds to aid in carrying on this research on mechanical springs is being made. To assist in this a leaflet was prepared outlining the plan and purpose of the work and was turned over to the chairman for distribution.

Effect of Temperature on the Properties of Metals. George W. Saathoff, Chairman. The Committee held meetings at Providence, R. I., on March 17, 1926, and at Atlantic City, N. J., on June 21, 1926.

The response to requests addressed to about 60 laboratories scattered over the country assured the Committee of ample facilities to carry on its research program. These laboratories are known as members of a Coöperating Group and they will develop certain specific tests on materials at high temperatures.

The Secretary of the Committee has for some time been collecting data for the preparation of a bibliography which will be printed in pamphlet form in the near future.

Condenser Tubes. Prof. Albert E. White, Chairman. There have been prepared and distributed to the members of the Committee copies of a bibliography entitled The Causes and Prevention of Corrosion and Other Sources of Deterioration in Condenser Tubes and Condensing Equipment. It was decided that first a study of condenser troubles should be undertaken and a sub-committee on finance was accordingly appointed to solicit funds.

Boiler Feedwater Studies. Sheppard T. Powell, Chairman. The Joint Research Committee on Boiler Feedwater Studies is sponsored by five

organizations, namely, the A.W.W.A., the A.R.E.A., the N.E.L.A., the A.S.T.M., and the A.S.M.E.

The nine Sub-Committees have been very active during the year and progress reports of these Sub-Committees were presented during the Annual Meeting.

Boiler Furnace Refractories. Clarence F. Hirshfeld, Chairman. Immediately following its organization meeting on December 1, 1925, the Committee began an active campaign for funds among those industries especially interested in its research work. Over $13,100 was collected during the year of which nearly $7000 has already been expended principally in support of two correlated investigations on boiler furnace refractories, one in the field and one in the laboratory. S. J. McDowell and R. K. McBerty are carrying on the laboratory work at the Bureau of Standards Ceramic Experiment Station, Columbus, Ohio, and at various points through the country Ralph A. Sherman and William E. Rice of the Bureau of Mines are making the field studies. The members of the Committee are kept closely in touch with this work through monthly reports prepared by the two groups.

From time to time there have appeared in several technical publications papers dealing with the work of this Committee. They are of particular interest and may be enumerated as follows:

Refractories Service Conditions in Boiler Furnaces appearing in the January 19, 1926, issue of *Power*.

Refractories — Investigation of Boiler Furnace Conditions as Related to Refractories Service published in the June, 1926, issue of the N.E.L.A. Bulletin entitled Stokers and Furnaces. Printed in *Mechanical Engineering* for December, 1926.

Refractories Service Conditions in Furnaces Burning Pittsburgh Coal on Chain Grates appearing in the November, 1926, issue of *Mechanical Engineering*.

Elevators. Martin H. Christopherson, Chairman. This Special Research Committee has been organized as a direct result of the work of the Sectional Committee on a Safety Code for Elevators.

It became apparent to the Sectional Committee on the completion of the Safety Code that the design and functioning of many of the modern safety devices now employed on elevators should be given careful study. This Special Research Committee was therefore formed to determine through experimental investigation certain parts of this desired information. John A. Dickinson, formerly with the Safety Section of the Bureau of Standards, has been engaged to carry on this work at the Bureau.

The elevator associations throughout the United States and several industrial concerns are generously supporting the work by contributions. During the year a total of $22,616 has been collected.

Coöperation with Other Societies. During the year the A.S.M.E. has been represented on eight research committees of other societies and associations by a total of thirteen members.

Safety

The Safety Committee of the Society is increasing the rapidity and size of its strides. Greater interest and activity is being aroused in the Society membership. Safety is being introduced into the national and local meetings. Coöperation is being arranged with other organizations. The publications of the Society are giving space to Safety articles, and the Safety Code work is being advanced to wider fields.

Safety Codes. The Society has accepted so far joint sponsorship for six Sectional Committees developing safety codes under the procedure of the American Engineering Standards Committee, and has also repre-

sentation on 20 other similar committees. Two of the six codes were completed previous to this year.

Sectional Committee on Safety Codes for Elevators
Sectional Committee on Safety Codes for Mechanical Power Transmission Apparatus
Sectional Committee on Safety Codes for Machinery for Compressing Air
Sectional Committee on Safety Codes for Conveyors and Conveying Machinery
Sectional Committee on Safety Codes for Cranes, Derricks and Hoists
Sectional Committee on Safety Codes for Mechanical Power Control.

The present safety program of the A.E.S.C. includes 44 codes.

Personnel. On October 1, 1926, these Sectional Committees had organized 9 sub-committees which with the main committee make a total of 16 committees of the Society at work on the Safety program. These committees include 63 A.S.M.E. members and 110 non-members.

The Safety Code for Elevators published in July, 1925, has had a total sale of 2802 copies to October, 1926. The Special A.S.M.E. Research Committee on Elevators has continued its study of the action of safeties, buffers, etc. It will be recalled that this Committee was organized as a direct result of the development of the Safety Code for Elevators and its adoption by various states. A Sub-Committee on Approval and Interpretations of the Sectional Committee on a Safety Code for Elevators having the same personnel is now holding monthly meetings.

The Sectional Committee on Safety Code for Mechanical Power Transmission Apparatus has held several meetings during the past year taking up the revision of this code.

The personnel of the Sectional Committee on the Safety Code for Machinery for Compressing Air has been approved by the A.E.S.C. This Sectional Committee consists of 9 manufacturers, 6 employers, 3 employees, 1 government representative, 4 insurance representatives and 3 qualified specialists.

The personnel of the Sectional Committee on Safety Code for Conveyors and Conveying Machinery has been approved by the American Engineering Standards Committee as submitted in June, 1926.

The preliminary work of organizing the Sectional Committee on Safety Code for Cranes, Derricks and Hoists has been completed and the organization meeting held. Seventeen national organizations have so far appointed representatives on this Committee.

The A.S.M.E. has representation also on the following safety code committees under the procedure of the American Engineering Standards Committee.

Safety Code Correlating Committee
Sectional Committee on Safety Code for Abrasive Wheels
Sectional Committee on Safety Code for Aeronautics
Sectional Committee on Safety Code for Electric Power Control
Sectional Committee on Safety Code for Floor Openings, Railings, and Toe Boards
Sectional Committee on Safety Code for Forging
Sectional Committee on Safety Code for Industrial Sanitation
Sectional Committee on Safety Code for Ladders
Sectional Committee on Safety Code for Laundries
Sectional Committee on Safety Code for Lighting Factories, Mills, and Other Work Places

Sectional Committee on Safety Code for Logging and Sawmill Machinery
Sectional Committee on Safety Code for Machine Tools
Sectional Committee on Safety Code for Mechanical Refrigeration
Sectional Committee on Safety Code for Paper and Pulp Mills
Sectional Committee on Safety Code for Power Presses
Sectional Committee on Safety Code for Rubber Mill Machinery
Sectional Committee on Textile Safety Code
Sectional Committee on Ventilation Safety Code
Sectional Committee on Safety Code for Walkway Surfaces
Sectional Committee on Safety Code for Amusement Parks.

Promotion of Safety. During the year the Committee has endeavored in many ways to advance the cause of "Safety."

In addition to contributions to the technical press the Committee has assisted in drawing the attention of local sections to safety. For instance, the joint meetings in November, 1925, in New York, N. Y., and that in Newark, N. J., in May, 1926, were planned with its help. This activity is in line with the Committee's policy of promoting cooperation in this important field between the A.S.M.E. Local Sections and other bodies such as the local sections of the National Safety Council.

The Societies' Position. Your Committee is unanimous in the opinion that the enormous and growing toll of life and limb caused by accidents in this country makes it imperative that the Society give the work of accident prevention its strongest influence and backing. As the Society contains within its ranks a great proportion of the leaders in all kinds of industry in this country, it is in a strategical position to give service of the utmost value in this important field. The Safety Committee believes that this service should be rendered not only through the activities of the Sectional Committee at work on Codes and through the national and local meetings of the Society, but also in coöperation with the National Safety Council, The American Society of Safety Engineers, the American Engineering Standards Committee, and other bodies which are working toward the reduction of accidents.

The keen interest of many of the A.S.M.E. members in the safety work of the Society, and their willingness to give time and energy in its promotion, augurs well for the future of the work and the possibility of giving even more valuable aid along these indispensable lines of engineering service to the industry of the country.

Power Test Codes

During the year two more of the twenty codes on the program of the Committee on Power Test Codes were issued in final pamphlet form. These were: Code on Definitions and Values, and Test Code for Refrigerating Systems.

The Test Code for Steam Locomotives has passed completely through the procedure laid down by the Main Committee and has had the approval of the Council of the Society, so will soon be available in pamphlet form.

The Test Code for Steam Turbines was presented for discussion at a Public Hearing in December, 1925, and is now receiving its final revision by Individual Committee No. 6. The Test Codes for Solid Fuels, Speed-Responsive Governors, and Gas Producers which were presented for discussion at the Annual Meeting in December, 1924, were completed by their respective committees during the past year and will soon be presented to the Council for approval.

The first printings of several of the twelve test codes which have been published in pamphlet form were exhausted during the present

year and were therefore revised by their respective committees for reprinting.

Further progress made in the formulation of the Code on Instruments and Apparatus relates to Section 6, Glass Thermometers, which was published in the April and May, 1926, issues of *Mechanical Engineering.*

Through its representatives on the U. S. National Committee of the I.E.C., the Society and the Main Committee on Power Test Codes, participated in the New York meeting of the I.E.C. held here in April, 1926. These representatives are Dr. William F. Durand, Dr. Fred R. Low and C. Harold Berry. Their activity centered around the six sessions of the I.E.C. Advisory Committee on Prime Movers and Dr. Durand presided at all of these sessions. The reports of I.E.C. Prime Movers Sub-Committee No. 1 on Water Turbines and I.E.C. Prime Mover Sub-Committee No. 2 on Steam Turbines indicate that considerable progress was made toward the establishment of international agreements on questions affecting these two types of prime movers.

Boiler Code Committee

The special committee on Boiler Code was organized in 1911. The main committee consists of 23 members, with a Conference Advisory Committee composed of state and municipal boiler inspectors, representing states where the A.S.M.E. Boiler Code has been adopted. There are 35 members with nine sub-committees, carrying a membership of 54.

The work of the Boiler Code Committee during the fiscal year 1926 has been continuously devoted to the service of the engineering profession in general, and the steam-boiler industry in particular. The results would seem to indicate that all needs have been met and that the projection of boiler and pressure-vessel rules and requirements into new and untried fields has been accomplished without undue limitation of progress and development. Particularly in the field of construction for extremely high pressure, has the Boiler Code Committee been able to render constructive and helpful assistance.

The outstanding accomplishment of the Boiler Code Committee during the past fiscal year has been the completion and issuance of the Suggested Rules for the Care of Power Boilers which constitutes Section VII of the A.S.M.E. Boiler Construction Code. Following hearings and conferences on certain proposed requirements in this section of the Code that extended over a period of two years, the final report from these Suggested Rules was drawn which reconciled all recommendations with the exception of a single clause in the Appendix pertaining to the question of the so-called "caustic embrittlement" of boiler plate steel. The wording of these references was finally adjusted in such a manner that there was no further objection to the issuance of the Code and it was therefore published and distributed in June, 1926, as Section VII of the A.S.M.E. Boiler Construction Code.

During the year a reprinting of Section III of the Code for Boilers of Locomotives has been completed. In this reprinting there has been incorporated such changes in the matter contained therein which is intended to be identical with the Power Boiler Section, as was found necessary to bring the Locomotive Boiler Section into full conformity with the 1924 revised edition of the Power Boiler Section. Also, certain revisions in the Locomotive Boiler Section that have been recommended by the Committee and approved by the Council in 1922 were incorporated.

The improved method of the Boiler Code Committee of handling recommendations for revisions and addenda to its codes that was announced in the annual report of 1925, has been carried through the

past fiscal year with entire satisfaction This is considered by the Committee a development of great importance as, if adequate provisions were not made readily available for the modification of any particular requirement that may be urgently needed by some current development in the art of boiler and pressure-vessel construction, great harm to the industry might result. Even with the continuous development in the industry looking toward the utilization of new materials, higher pressures, etc., it has been found possible by this new procedure to meet every need of the industry.

During the past fiscal year the Boiler Code Committee has held nine regular meetings of the main Committee, covering its regular interpretation service upon the various sections of the Code, and in addition recommending revisions and addenda. The meetings appear to be of as great importance to the boiler and pressure-vessel industry as ever, because questions of even more vital nature than ever before continue to be submitted and due to recent developments in the industry seem to be more difficult of solution than ever before.

The committee reports with great regret the death of one of its older members, William B. Reed, who represented the cast-iron heating-boiler division of the industry. Mr. Reed passed away on August 4, having served as a member of the Committee for three years since he succeeded to membership following the death of his brother, Richard D. Reed.

The Boiler Code Committee faces an increasing number of difficult technical problems, for many of which extensive research is needed. The problem of developing the work to adequately meet these needs of the industry appears to be one of the Committee's greatest problems. At no time in the past has the Committee failed to meet all reasonable demands made upon it and it is endeavoring to reach out into the new fields toward which the developments of the industry point For much of the developmental work, however, it is necessary for the Committee to go outside of its membership and confer with other organizations or individuals. It may here therefore be desirable to emphasize the need, as expressed a year ago, for coöperation not only with the Research Committee of the Society, but also with the Committee on Meetings in bringing its great problems to the attention of the Society membership as broadly as possible with a view to their solution within the organization.

APPENDIX

REPORT OF ACCOUNTANTS

Wm. J. Struss & Co., certified public accountants, give the results of their examination of the books of the Society for the fiscal year ended September 30, 1926, in the statement of assets and liabilities on page 1408.

BALANCE SHEET AS OF SEPTEMBER 30, 1926

Assets

Cash		$40,510 66
Accounts Receivable:		
Members	$15,312.34	
Non-Members	112,010.07	
		127,322 41
Inventories:		
Supplies	21,989.18	
Publications in process	5,607.57	
Publications for sale	19,103.69	
		46,700.44
Deferred Charges:		11,971.70
Total		$226,505.21
Investments for Trust Funds (see contra)		
Lawyers Mortgage 5½%—1927	42,000.00	
St. Louis, Peoria & N. W. 5%—1948	10,613.89	
N. Y. Central and Hudson River 4%—1942	23,062.50	
S. W. Straus & Co. mortgage bonds—1937	9,000.00	
Cash in banks	2,880.29	
		87,556 68
Investments for Capital Account (see contra)		
Lawyers Mortgage 5½%—1928-29-30	132,000.00	
Lake Shore & Mich. So. 4%—1928	44,521.88	
S. W. Straus & Co. mortgage bonds—1937	50,000.00	
		226,521.88
Fixed Assets (see contra)		
Building and Equipment (Book Value)	502,821 96	
Library Books	1.00	
Engineering Index	1.00	
		502,823 96
TOTAL ASSETS		$1,043,407.73

Liabilities

Unfilled Obligations		$31,464.82
Contributions:		
Lubrication Research	$700.00	
Elevator Safeties Research	15,612.48	
Gear Research	259.58	
Standardization Bolts, Nuts, Rivets	415.44	
Fluid Meter Research	1,875.00	
Steam Table Research	3,686.79	
Metal Spring Research	305.00	
Oil Power National Conference	580.58	
Boiler Refractories Research	6,232.62	
Substitutes for Domestic Woods Research	400.00	
		30,067.49
Dues Paid in Advance		2,592.04
TOTAL		$64,124.35
Trust Funds (see contra)		
Life Membership	50,921.11	
Library Development	5,514.88	
Weeks Legacy	2,201.36	
Melville	1,496.63	
Chas. T. Main	2,796.77	
Hunt Memorial	286.01	
Hess Prize—Juniors and Students	2,000.00	
Easby	445.12	
Westinghouse Bust	163.20	
Holley Medal	6,281.60	
Max Toltz	15,450.00	
		87,556.68
Capital Investment:		
In Fixed Assets (see contra)		502,823.96
Capital Investments (see contra)		226,521.88
Working Capital		162,380.86
TOTAL LIABILITIES		$1,043,407.73

No. 2039

REPORT ON TECHNICAL COMMITTEE PUBLICATIONS

In this section is given a summary of the codes and standards which have been completed during 1926 and are available either in book or pamphlet form upon application, and of preliminary or tentative drafts which have been printed in *Mechanical Engineering* during the year. Other work of the various committees in process is summarized in the reports of committees immediately preceding these pages.

BOILER CODE

Interpretations of the Boiler Code were published in *Mechanical Engineering*, January, p. 66; February, p. 175; March, p. 276; May, p. 529; June p. 629; July, p. 767; September, p. 969; and December, p. 1478. These interpretations are available in data sheet form. Revisions and addenda to several sections of the Boiler Construction Code were published in the issues of June, p. 626; November, p. 1166; and December, p. 1474. These are being prepared for publication in data sheet form.

In addition, Section VII of the Boiler Construction Code, covering Suggested Rules for the Care of Power Boilers, was published in pamphlet form for general distribution, and Section III of the Boiler Construction Code on Boilers of Locomotives was reprinted in pamphlet form.

POWER TEST CODES

Two codes were issued in pamphlet form during 1926, as follows:

Code on Definitions and Values
Test Code for Refrigerating Systems.

Three test codes were revised and reissued in pamphlet form during the year, the first printing having been exhausted. These were

Test Code for Stationary Steam Boilers
Test Code for Reciprocating Steam Engines
Test Code for Internal-Combustion Engines.

The Power Test Code material published in *Mechanical Engineering* during the year was:

Instruments and Apparatus, preliminary draft of Chapter 3, Temperature Measurement, Section 2—Glass Thermometers, April, p. 382, and May, p. 517.

RESEARCH

Steam Tables. The February issue of *Mechanical Engineering* contained reports of the Special Research Committee on the Properties of Steam and the Extension of the Steam Tables covering the progress of this investigation at the Bureau of Standards.

Gear Teeth. The Special Research Committee on the Strength of Gear Teeth issued Progress Report No. 2, entitled Data and Results of Series of Experiments Covering Influence of Error and Elasticity on Strength of Gear Teeth, which in revised and abridged form was printed in the September, 1926, issue of *Mechanical Engineering*, p. 893, as Progress Report No. 3.

Mechanical Springs. At the Providence Meeting, Chairman J. K. Wood of the Special Research Committee on Mechanical Springs, presented a paper on The Specification and Control of Mechanical Springs which later appeared in the August, 1926, issue of *Mechanical Engineering*, p. 808, and will be found elsewhere in this volume.

Boiler-Feedwater. In the Mid-November issue of *Mechanical Engineering* Progress Reports of the Joint Research Committee on Boiler-Feedwater Studies were published as follows:

Sub-Committee No. 2 on Water Softening by Chemicals (External Treatment) — Pretreatment of Boiler-Feedwater, p. 1361

Sub-Committee No. 3 on Zeolite Softness, Internal Treatment, Priming and Foaming, and Electrolytic Scale Prevention — Present Knowledge of Foaming and Priming of Boiler Water, with Suggestions for Research, p. 1364

Sub-Committee No. 6 on Embrittlement of Metals — Embrittlement of Steel, p. 1368

Sub-Committee No. 7 in Relation to Boiler Use — Municipal Water Supplies and the Effect of Trade Wastes in Relation to the Use of Water in Power Plant Practice, p. 1372

Sub-Committee No. 5 on Corrosion of Boilers and Effect of Treated Water in Accelerating or Relieving These Troubles, p. 1373

Sub-Committee No. 9 on Bibliography, p. 1373.

Boiler-Furnace Refractories. The Progress Report of the Special Research Committee on Boiler-Furnace Refractories was published in the November issue of *Mechanical Engineering*, p. 1115, under the title of Refractories Service Conditions in Furnaces Burning Pittsburgh Coal on Chain Grates. In the December issue, p. 1389, there was published the report of an investigation of boiler-furnace conditions as related to refractories service conducted by the Bureau of Mines, under the title of Refractories.

Worm Gears A preliminary report of the Special Research Committee on Worm Gears prepared by Prof. Earle Buckingham was printed in pamphlet form for use at the Annual Meeting under the title of Worm-Wheel Contact, and appears elsewhere in this volume.

SAFETY

During the year the Safety Committee has endeavored to advance the cause of "Safety" and has sent short editorials to *Mechanical Engineering* from time to time.

STANDARDIZATION

Gears. The Tentative American Standard for Spur-Gear Tooth Form, designated as B 6b, was published under the title of Proposed Standard for Spur-Gear Tooth Form in *Mechanical Engineering* for November, 1926, p. 1157.

Small Tools and Machine Tool Elements. The Tentative American Standard for T-Slots, designated as B 5a, was published under the title of T-Slots, Their Bolts, Nuts and Cutters, in *Mechanical Engineering* for November, 1926, p. 1158.

Bolt, Nut, and Rivet Proportions. The December issue of *Mechanical Engineering*, p. 1480, contained the first reports on tinners', coopers', and belt rivets, small rivets, and plow bolts, of the sectional committee on the Standardization of Bolt, Nut and Rivet Proportions. These reports were published under the title of Tentative American Standards for Tinners', Coopers', and Belt Rivets, Small Rivets, (b 18a); and Plow Bolts, (B 18f). The Tentative American Standard for Wrench-Head Bolts and Nuts and Wrench Openings, (B 18b) was published in *Mechanical Engineering* in May, p. 537.

Pipe Flanges and Fittings. The August, 1926, issue of *Mechanical Engineering*, p. 858, contained the Tentative American Standard for Steel Pipe Flanges and Flanged Fittings for Maximum Working Steam Pressures of 250, 400, 600, and 900 Lb. per Sq. In. at a Temperature of 750 Deg. Fahr. (B 16e).

No. 2040

NECROLOGY

ALLEN J. AITKEN

Allen J. Aitken, consulting mechanical engineer on pavement and highway construction machinery, died in Buffalo on May 15, 1926.

Mr. Aitken was born in Buffalo on April 25, 1879. He was educated in the public schools there, and was later tutored for four years in mechanical engineering subjects by a skillful and highly educated Norwegian. He served his apprenticeship under his father who at that time was conducting a machine shop in Buffalo. Later he served as a mechanic of the Buffalo-Pitts Road Roller Co. of Buffalo, and the Kelly-Springfield Road Roller Co. at Springfield, Ohio.

In 1908 Mr. Aitken returned to Buffalo as salesman for the Barber Asphalt Paving Co. In 1911 he became superintendent of the Iroquois Works of this company. During the war he served as supply manager for the Buffalo District of the U. S. Shipping Board.

Mr. Aitken joined the A.S.M.E. in 1926 as an Associate.

HENRY F. ALBRIGHT

Henry F. Albright, vice-president of the Western Electric Co, and general superintendent of their Hawthorne Works, died on May 11, 1926, at the Memorial Hospital in New York City, after a lingering illness.

Henry Fleetwood Albright was born at Lancaster, Pa., on October 5, 1868, and received his education in the public schools of Philadelphia. In 1885, at the age of seventeen, he began work with the Second Geological Survey of Pennsylvania. He then became connected with the Fuel Gas & Electric Engineering Co., with whom he remained from 1886 to 1888. This was followed by two years with the Westinghouse Electric Co. It was with these two concerns that he obtained his shop and drafting-room experience.

In 1890 Mr. Albright joined the Thomson-Houston Co. While with them he secured a large contract covering an electric-lighting plant for the city of Wheeling, W. Va. The fair and able way in which he handled this matter so impressed the three men which the Western Electric Co. had seeking this same contract, that they strongly recommended him to their superiors. As a result Mr. Albright joined the Western Electric Co. as an engineer on March 1, 1892.

In 1894 he was transferred to the construction department of the company in New York City, where his first work was the reconstruction of the power plant. His success in this led to his appointment as factory engineer. A year later he was made assistant superintendent and in 1899 he became superintendent of the New York factory, which became a model manufacturing plant under his supervision.

About this time Mr. Albright fully realized the great future ahead of the concern and began to develop the organization along functional lines and to lay plans for new plants to take care of rapid growth.

The first buildings were erected at Hawthorne in 1905 under plans laid out by Mr. Albright which covered the entire layout of this vast plant as it now stands. This great institution, covering 200 acres, with 102 buildings and 26,000 employees, is very fittingly referred to by high officials of the company as the finest tribute and most impressive memorial to Mr. Albright's memory.

In 1908 Mr. Albright located in Chicago and became general superintendent of the Hawthorne Works. He was elected vice-president of the company in charge of manufacturing in 1917. At the time of his death Mr. Albright was engaged in planning another mammoth plant at Kearny, N. J., which within a few years is expected to rival Hawthorne in size and activity.

Among the societies to which Mr. Albright belonged were the Institute of Electrical Engineers (London), the American Society of Civil Engineers, the American Institute of Electrical Engineers (Fellow), and the Union League Club of Chicago. He had been a member of the A.S.M.E. since 1903.

GEORGE I. ALDEN

Prof. George I. Alden, chairman of the Board of the Norton Company, student and inventor, died on September 13, 1926, at his summer home in Princeton, Mass. He was 83 years old. He was for many years head of the steam and mechanical engineering departments at the Worcester Polytechnic Institute, and recently gave as a gift to the Institute the Alden hydraulic laboratory, which is considered one of the finest of its kind in the world.

Professor Alden became a member of the A.S.M.E. in 1880, and from 1891 to 1893 he was Vice-President. He was also prominent in the activities of the Worcester Branch of the National Metal Trades Association.

LOUIS A. ALEXANDER

Louis A. Alexander, one of the veteran employees of the American Brass Co. of Waterbury, Conn., died in that city on April 4, 1926.

Mr. Alexander was born at Richmond, Me., in 1861, and received his technical education at Worcester Polytechnic Institute. In 1880 he began an apprenticeship as pattern and model maker with Galen Coffin of Boston. This was followed by shop and drafting experience at the Blake Steam and Pump Works, the Hunt Speller Mfg. Co., and the South Boston Iron Works. He was chief draftsman of the last-mentioned works for five years and during that time made the drawings for the turret mechanism, gun mounts, and steering gear of the U. S. Monitor *Terror*.

In 1894 Mr. Alexander went to the Farrel Foundry & Machine Co. as designer of heavy machinery. He had charge of all the drawings for a new brass mill for the Ansonia Brass & Copper Co., and this led to his joining that concern as master mechanic in 1896. The Ansonia Company later became a part of the American Brass Co., with whom Mr. Alexander remained for the rest of his life.

He joined the A.S.M.E. in 1901 and was one of the oldest members of the Waterbury Section.

CLAUDE R. ALLING

Claude R. Alling, Vice-President of the Underwriters' Laboratories of Chicago, died at Evanston, Ill., on October 25, 1926, at the age of 43 years. Following graduation from high school he received his college

training at Denver University, Northwestern University, and Armour Institute, from which he was graduated in 1907. That year he became associated with the Underwriters' Laboratories as assistant to one of the officers. The thorough manner in which he familiarized himself and handled the problems in the different departments in which he served made him the logical choice of the Board when a vice-president was to be chosen last February.

Mr. Alling was active in technical and insurance societies. He became a member of the A.S.M.E. in 1919.

JAMES E. ALLISON

James E. Allison, widely known public-utility valuation engineer of St. Louis, Mo., died September 19, 1926, at the age of 61 years. After being graduated from Harvard University with the degree of A.B. in 1887 he entered the service of the Xenia Gas, Light and Coke Co.

In 1904 Mr. Allison went to St. Louis and was engaged as an engineer of the Louisiana Purchase Exposition. Later he was made chairman of the St. Louis Public Service Commission having regulatory supervision of St. Louis public utilities. He was head of James E. Allison & Co. He became a member of the A.S.M.E. in 1914.

HARRY W. ANDERSON

Harry W. Anderson, branch manager of the Stutz Motor Co. of America, Inc., died in October, 1926. He was born in Baltimore, Md., in December, 1865. Among the first positions he held was that of southern manager of the Harrisburg Foundry and Machine Works of Atlanta, Ga. He was employed by the Speegle Lumber Co. and later made manager of engineering sales and purchasing agent of the N. P. Pratt Laboratory and Fulton Foundry and Machine Works.

From 1909 until the time of his death Mr. Anderson held positions with the following concerns: Anderson-Kent Co. of Atlanta, Ga., district manager of the Regal Motor Co. of the same city, the American Motors Co. as district sales manager, the Stutz Motor Car Co., of Indianapolis, Ind., the Madison Motors Corpn., the Universal Motor Products Co., Templar Motor Corpn., and Duesenberg Auto and Motor Co., Inc. He joined the A.S.M.E. in 1907 as an associate.

FRANK C. ARMSTEAD

Frank C. Armstead of Pittsburgh, Pa., was born in May, 1862, in Cleveland, Ohio. His first position as a skilled workman was with the N. K. Fairbanks Co. of Chicago. He was later employed with the Armour Packing Co. and then with the Chicago City Railway, subsequently becoming their operating engineer. In March, 1888, he was employed by the Westinghouse, Church, Kerr & Co., engaged first in erecting stokers, engines, etc., and later becoming superintendent of construction. He was in the service of the Westinghouse Electric and Manufacturing Co. in 1917 when it absorbed the Westinghouse Machine Co., and was with it in an advisory capacity in stoker work until his death on September 7, 1926.

Mr. Armstead was very well known throughout the country as an expert on combustion matters and boiler-plant operation. He had contributed many inventions in furnaces and under his direction the company introduced many improvements. He became a member of the Society in 1906.

ORTH K. BAKER

Orth K. Baker, engineer upon Portland cement plants, died at Syracuse, N. Y., on April 14, 1926. He was born at Tuscola, Ill., on May 5, 1876. He was educated at the University of Kansas and at the same time, during vacations, worked in machine shops upon gas engines and electrical machinery.

In 1901 he went to work for the Iola Portland Cement Co. at Iola, Kan., as draftsman. He was assistant superintendent of the plant of this company at Dallas, Tex., from 1902 to 1905. Following this he was mechanical engineer of the U. S. Coal & Coke Co. at Gary, W. Va., for a year, and later engineer of construction for the Freeborn Engine & Construction Co. upon the remodeling of the Kosmos Portland Cement Co. plant at Kosmosdale, Ky., and in the drafting office at Kansas City, Mo. Mr. Baker was chief draftsman of the concern from 1909 to 1912.

Subsequent to 1912 Mr. Baker superintended the design and construction of cement plants in Europe and South America. Since 1920 he had served as mechanical engineer in charge of design for the Atmospheric Nitrogen Co. at Syracuse, N. Y.

Mr. Baker joined the A.S.M.E. in 1913.

GEORGE A. BAUER

George A. Bauer, vice-president of the W. E. Shipley Machinery Co., Philadelphia, Pa., died in that city on June 22, 1926.

Mr. Bauer was born at Hamburg, Erie County, N. Y., in March, 1871. After a grammar school education he went to work in 1886 as an apprentice in a small machine shop. After a year in the shop he spent two years with a mill and machinery supply house, and two years as mechanic in a flour mill.

From 1891 to 1899 Mr. Bauer was with the mill and supply firm of E. A. Kinsey Co. He left them to become a salesman for the Harris Automatic Press Co. of Cleveland, manufacturers of special paper machinery and automatic printing presses. He joined the W. E. Shipley Machinery Co. as a salesman about twenty-five years ago, becoming vice-president in 1908.

Mr. Bauer was a member of the Masonic Order, and of the Engineers' Club of Philadelphia. He joined the A.S.M.E. as an Associate in 1917.

LOUIS L. BENTLEY

Louis L. Bentley, who was factory manager of the Armstrong Cork Co., died on May 30, 1926, at Beaver Falls, Pa. At the time of his death he had just patented a method of making artificial cork. Mr. Bentley received his professional education at Cornell University and was graduated with the class of 1890. He was subsequently employed by the Baltimore and Ohio Railroad in the department of tests and inspection. Four years later he was in the U. S. Navy working in the same capacity. From 1902 to 1906 he was employed by the Lehigh Valley Railroad and held various positions, including those of engineer of tests, chief draftsman, and mechanical engineer. Later he served the Oswego Boiler and Engine Co. for a year as general manager. In 1907 he became connected with the Armstrong Cork Co. and during his twenty years of service there held the positions of superintendent and general manager.

Mr. Bentley was born at New Brighton, Pa., in September, 1868. He had been a member of the A.S.M.E. since 1916.

WILLIAM W. BIRD

William W. Bird, professor of mechanical engineering at Worcester Polytechnic Institute, died suddenly at his home in that city on January 24, 1926.

Professor Bird was born at Somerville, Mass., April 11, 1866. He first attended the Cambridge Public Schools, then the Chauncey Hall School in Boston. He entered Worcester Polytechnic Institute, where he specialized in mechanical engineering, and was graduated in 1887 with the degree of B.S.

After graduation he remained at the Institute as assistant and instructor in steam engineering and mechanical drawing. In 1890 his father died, throwing upon his shoulders the management of the Broadway Iron Foundry at Cambridge, Mass., of which his father had been the proprietor for many years.

Professor Bird gave his entire attention to this industry until 1893, when he was able to return to W. P. I. as assistant professor of mechanical engineering. This position he held until 1896, when it again became necessary to turn his entire attention to the affairs of the foundry business in Cambridge for a period of seven years.

In 1903 Professor Bird was again able to return to the Institute, this time as head of the Department of Mechanical Engineering and director of the Washburn Machine Shops. In 1923 his health began to fail and he resigned these positions. He still remained a member of the faculty, however, with the title of Professor of Mechanical Engineering, confining his work to the senior course in Shop Management. He was also president of the Broadway Iron Foundry up to the time of his death.

Professor Bird became a member of the A.S.M.E. in 1889, and so was one of the Society's "Thirty-Fivers."

FRANK C. BLAKE

Frank C. Blake, assistant chemical director of Lazote, Inc., in Wilmington, Del., died on November 13, 1926, at the age of 35 years. He was born in Providence and was graduated from Brown University with the degree of B.S. in M.E. in 1913. He remained at Brown the following year as assistant in physics and was instructor in mechanical engineering from 1914 to 1917. He became a research engineer with the Nitrogen Products Co. of Providence in 1917, and a year later joined the du Pont de Nemours & Co. organization in a similar capacity. In 1924 he became assistant chemical director of Lazote, Inc., a subsidiary of the du Pont Co., organized to manufacture synthetic ammonia, and continued in this position until his death.

Mr. Blake was a member of Sigma Xi and of the American Chemical Society, as well as being a Junior member of the A.S.M.E. since 1915.

MILES CARLISLE BLAND

Miles Carlisle Bland, assistant engineer of the Bridges Bureau of the Engineering Department of Public Works of Philadelphia, died on August 14, 1926, at the age of fifty-one years. After being graduated in 1895 from Colorado College, Mr. Bland began his career with the Pennsylvania Railroad as assistant on the engineer corps. He was employed subsequently by the American Bridge Co., the Dyer Co. of Cleveland, the Pittsburgh Steel Construction Co. of Pittsburgh, Pa., the Massillon Bridge and Structural Co. of Toledo, Ohio, the Canton Bridge Co., Canton, Ohio, and the Bethlehem Steel Bridge Corpn. He

was in general practice for some time and was also research engineer in the County Engineer's Office, Cuyahoga County, Ohio

Mr. Bland held membership in the A.S.C.E. as well as the A.S.M.E., which he joined in 1921.

LOUIS BLOCK

Louis Block, an authority upon problems of refrigeration, died in New York on March 7, 1926.

Mr. Block was born in April, 1850, at Hildesheim, Germany. He received his education at the Gymnasium Andreanum in that city and at the Polytechnic Institute in Hanover. After his graduation he worked for a time as a machinist in a shop at Hildesheim and then as a draftsman at the Gruson Werke in Magdeburg.

He then came to the United States, where he first found work as a machinist at the West Point Foundry, Cold Spring, N Y. This was followed by periods as a draftsman in the Marine Department of the Reading Railroad, with the Carrallo Steam Heating Co., and with the American Zylonite Co.

Mr. Block joined the staff of the De La Vergne Machine Co. of New York in 1882 and remained with them as draftsman, chief draftsman, chief engineer, director, and president until 1905. During this time he designed and patented many machines having to do with refrigeration, and he became widely known as an authority upon this subject.

In 1905 Mr. Block resigned to go into business as a consulting engineer and he had since enjoyed a wide clientele among the largest users of ice and refrigerating machinery.

Mr. Block took an active part in the affairs of the technical societies. He belonged to the Engineers Club of New York, the American Society of Refrigerating Engineers, The American Society of Mechanical Engineers, and the American Association of Ice and Refrigeration. In 1909 Mr. Block was elected president of the American Society of Refrigerating Engineers, being the fifth head of that organization.

After becoming a member of the A.S.M.E. in 1913 he threw himself heartily into the professional work of the Society and did valuable work on committees.

WALLACE E. BOARDMAN

Wallace E. Boardman was born on November 9, 1882. After completing his studies at the Massachusetts Institute of Technology in 1909 he spent a short time with the Greenfield Tap and Die Co., and later associated himself with the late Professor S. Homer Woodbridge, of M. I. T., in connection with work on the Washington Cathedral and other similar undertakings. He was associated with Stone and Webster for ten years, first in the capacity of a draftsman and then as mechanical engineer, which position he held until his death on October 25, 1926.

Mr. Boardman became an associate-member of the A.S.M.E. in 1919, and was also a member of the American Society of Heating and Ventilating Engineers.

CHARLES S. BROWN

Charles S. Brown, Professor of Mechanical Engineering at Vanderbilt University, Nashville, Tenn., died at Woodmont, Conn., on August 30, 1926.

Professor Brown was born August 23, 1860, and in 1883 was graduated from Sheffield Scientific School of Yale University. He received his apprenticeship and shop practice with H. B. Brown, manufacturers

of the Merriman Bolt Cutters, and operated the Hale Gold Mine, in South Carolina. Previous to his professorship at Vanderbilt University he taught at Rose Polytechnic Institute at Terre Haute, Ind. The Johns-Manville steam trap, their volume meter, and radiator modulating valve were invented by Professor Brown.

As Fuel Administrator for Tennessee during the war, member of the advisory committees for the Nashville Water Works and Smoke Abatement, and Secretary, Treasurer and finally President of the Vanderbilt Athletic Association, Professor Brown gained thousands of friends and admirers. He was elected member of the A.S.M.E. in 1890.

HAROLD C. BROWN

Harold C. Brown, at the time of his death on December 13, 1926, was manufacturing analyst of the Western Electric Co., Inc., located at Jersey City. He was born at Chelsea, Mass., on January 28, 1889, and received the degree of S.B. from Massachusetts Institute of Technology in 1911. Upon leaving school in that year, he was employed by the Boston and Albany Railroad as draftsman. He remained with this company for two years and the following two were spent as superintendent of construction for the King Construction Co. of Boston and as draftsman with the Stone & Webster Engineering Corpn. Until July, 1917, he was associated with the Edison Electric Illuminating Co. of Boston as sales engineer and was also in business for himself.

After leaving the Army in March, 1919, he became engineer in the employ of the Holt Manufacturing Co. of Peoria, Ill., in charge of assembling and routing caterpillar tractors. The U. S. Rubber Co. of Providence, R. I., employed him in their development department for a year and from 1921 to 1924 Mr. Brown was superintendent of the Columbia Salvage Corpn. He was elected a member of the A.S.M.E. in December, 1924.

ANDREW C. CAMPBELL

Andrew C. Campbell, widely known as an inventor of automatic metal forming machinery, died at his home in Waterbury, Conn., on February 21, 1926.

Mr. Campbell was born in Brooklyn, N. Y., on June 17, 1856. His education was obtained in the common schools of Brooklyn. In 1874 he began work as draftsman for his father, A. Campbell, who was a brilliant mechanic and inventor of the Campbell Printing Press. The young man's first work was upon the design of an automatic machine to make paper flour sacks. This cut the paper from a roll, printed the label in three colors and folded and glued the sack. He then invented a small printing press which attracted attention at the Centennial Exposition in 1876.

After five years with the Campbell Printing Press Co., he spent nine years as designer and inventor with the Wheeler & Wilson Company, sewing machine manufacturers, at Bridgeport, Conn. From 1888 to 1894, Mr. Campbell was chief draftsman of the Farrel Foundry & Machine Co. of Waterbury, Conn., and from 1894 to 1911 he was secretary and superintendent of the E. G. Manville Machine Co. of the same city. Mr. Campbell left the Manville company to found at Waterbury Andrew C. Campbell, Inc., engineers, which is a subsidiary of the American Chain Co. of Bridgeport, Conn. Since then he had served as president of the Campbell concern and at the same time as consulting engineer for the American Chain Co.

Mr. Campbell was the inventor of an extraordinary number of ingenious machines for automatically converting wire into such articles

as hooks and eyes, safety pins, hair pins, paper fasteners, and his own "Hammer-lock" cotter pins. He also designed many machines for converting rod into rivets, bolts, screws, etc., by the cold heading process. Of late years his unusual talents were concentrated upon the design of the machinery now used in the quantity production of various kinds of chain from wire and sheet metal at the plant of the American Chain Co.

Outside of the mechanical field Mr. Campbell had many interests. He was a great reader, had traveled abroad many times, and was an authority upon Japanese and Chinese art.

Mr. Campbell had been a member of the A.S.M.E. since 1885. He was a graphic writer of descriptions of mechanisms and in 1886 he presented a paper before the Society upon his unique invention, the "Conicograph," which traces all sorts of conic sections. This came to the attention of Lord Kelvin and as a result the instrument aroused wide interest in England.

WILLARD G. CARLTON

Willard G. Carlton, superintendent of power of the electrified divisions of the New York Central Railroad, died at Yonkers, N. Y., on April 15, 1926.

Mr. Carlton was born at Warren, Ill., on February 20, 1869. In 1892 he was graduated from Cornell with the degree of M.E. and went to work immediately for the General Electric Co., where he gained a year's shop experience. In 1893 he joined the Chicago Edison Co. as a draftsman and was soon put in charge of the drafting room. He was engineer in charge of their underground cable and conduit work in 1896 and 1897, and from 1897 to 1901 was also assistant to the chief operating engineer. From 1901 to 1905 he was engineer in charge of designing, constructing, and operating. During his twelve years with the Chicago Edison Co. Mr. Carlton was closely associated with W. L. Abbott, President of the A.S.M.E. during 1926.

Mr. Carlton had served the New York Central as superintendent of power for twenty-one years. He joined the A.S.M.E. in 1906.

WILLIAM L. CATHCART

William L. Cathcart, naval construction engineer, died at Germantown, Pa., on March 31, 1926.

Mr. Cathcart was born at Mystic, Conn., on August 12, 1855, and was graduated from the U. S. Naval Academy, Annapolis, Md. After serving in the Engineer Corps of the U. S. Navy in the North and South Atlantic, South Pacific and Asiatic Squadrons, he resigned in 1891 to enter private business.

He was at first treasurer of a manufacturing concern, then from 1897 to 1899 professor of marine engineering at the Webb Institute of Naval Architecture, New York City. During the Spanish War he served as assistant to the Engineer-in-Chief of the Navy Department. From 1899 to 1903 Mr. Cathcart was assistant professor of mechanical engineering at Columbia. Following this he became a consulting engineer with offices in New York City and later at Germantown, Pa.

In 1918 he again became a professor at Webb Institute and maintained this connection, along with his consulting work, up to the time of his death. During the World War Mr. Cathcart was appointed Lieutenant Commander, U. S. N. R. F. on special duty at the office of the Engineer-in-Chief. He was later promoted to the rank of Commander.

Mr. Cathcart was the author of several textbooks on machine design, of various papers before technical societies and of numerous editorials and special articles. He was recognized as an authoritative writer and lecturer upon war and naval subjects. Mr. Cathcart joined the A.S M.E. in 1899, and was also a member of the American Society of Naval Architects and the Society of Naval Architects and Marine Engineers

GEORGE E. CHAMBERLAIN

George E. Chamberlain, general superintendent of the A. E. Staley Manufacturing Co. died suddenly on October 24 at his home at Decatur, Ill. He had attained nation-wide eminence in his profession in the development of processes diversifying the commercial products from corn and was also recognized as an authority on plant construction and operation.

He was born in Wyalusing, Pa., November 6, 1870, and at the age of nineteen entered Lehigh University, being graduated with the class of 1892. Soon after graduation he secured a position with the American Glucose Co. of Buffalo, N. Y. Resigning this position he entered the employ of the Firmenrich Manufacturing Co. of Marshalltown, Iowa.

After holding various positions for short periods, in 1912 he became connected with the A. E. Staley Manufacturing Co. of Decatur, Ill., and attained the position of chief engineer and general superintendent which he held at the time of his death.

Mr. Chamberlain was a member of the Society of Chemical Industry, the Franklin Institute, the American Chemical Society, Society for the Advancement of Science, and the London Chemical Society, as well as the A.S.M.E., which he joined in 1907.

JOSEPH H. CHAMP

Joseph H. Champ, at the time of his death on May 8, 1926, was president of the Iceless Machine Co., treasurer of The Lincoln Oil and Paint Co., vice-president of the Cleveland Tractor Co., and director of The Standard Tool Co. and of the Heisel Candy Co.

Forty-nine years ago he became associated with the Bishop and Babcock Co. and held the positions of superintendent, general manager, vice-president, president, and treasurer. This company specialized in hydraulic pumps and air compressors.

Mr. Champ was a member of the Cleveland Chamber of Commerce, and became a member of the A.S.M.E. in 1917.

HERBERT W. CHENEY

Herbert W. Cheney, engineer in the electrical department of the Allis-Chalmers Mfg. Co., died at his home in Milwaukee on March 22, 1926, after two months' illness.

Mr. Cheney was born on a farm near Coldwater Lake, Mich., on February 7, 1872. After attending a district school near the farm he attended Valparaiso University. Upon graduation there he taught school for a time near Coldwater, and with the money so earned he was able to take a post-graduate course in engineering at the University of Michigan.

Mr. Cheney began his engineering career with Charles Dunlap, consulting engineer in Detroit. Next he spent some months with the Charles A. Strelinger Co., Detroit, as a machine-tool designer. With this experience he entered the employ of Henry M. Leland, building machine tools and special tools. Mr. Cheney eventually became chief

draftsman and mechanical engineer of the Leland & Faulconer Mfg. Co., which a few years later was merged to form the Cadillac Motor Car Co.

After seven years with this concern Mr. Cheney, in 1899, left to join the staff of the Westinghouse Electric & Mfg. Co. He began with them as draftsman and designer, being placed in charge of electrical controlling apparatus. Later he became mechanical and electrical engineer upon their railway work.

After five years at Pittsburgh, Mr. Cheney went to the Allis-Chalmers Mfg. Co. as designer of electrical details at the Bullock plant at Cincinnati. In 1908, when the engineering work was consolidated at the West Allis works, Mr. Cheney moved to Milwaukee. He specialized in the design of such apparatus as air-brake equipment, small motor-driven air compressors, motor starters, etc.

Mr. Cheney was sent to England in 1925 to investigate the design and manufacture of a new style switch mechanism and was engaged in the adaptation of this to American needs at the time of his death.

Besides being a member of the A.S.M.E., which he joined in 1897, Mr. Cheney was a Fellow of the American Institute of Electrical Engineers and a member of its program and membership committees. and had also served during the past year as chairman of the Committee on Education and Graduate Training of the Milwaukee Engineers' Society.

HARRY BARR COCHRAN

Harry Barr Cochran, a member of the A.S.M.E. since 1922, died on September 2, 1926, at Reedville, Pa., at the age of 53 years. He was educated at Drexel Institute and shortly after graduation took a position with the Westinghouse Electric and Mfg. Co. He held positions with the Union Switch and Signal Co., the Pennsylvania Steel Co., and the Standard Steel Works, where he was promoted from the position of draftsman to that of superintendent and then to mechanical engineer. He had been employed by this company for twenty-seven years. Mr. Cochran was an expert on the design and construction of rolled-steel wheels.

DONALD COLE

Donald Cole was born in Baltimore, Md., November 29, 1885, and died in Detroit, Mich., as the result of a taxicab accident, on December 16, 1926.

Mr. Cole's early engineering education was obtained in the Baltimore City College. His early experience was gained in the drafting room of the Arctic Ice Machine Co., Canton, Ohio, following which he was associated for a time with the Frick Company, Waynesboro, Pa. Returning to the Arctic Ice Machine Co., he held various positions in the engineering department, until in 1909 he became chief engineer, a position which he held for two years, when he became associated with the Timken-Detroit Axle Co., Detroit, Mich., as assistant engineer. In 1914 he became chief of plants and construction for the General Ice Delivery Co. of Detroit. About 1917 Mr. Cole entered the United States Army, where he was commissioned Captain. Following his discharge from the Army, he became a partner in the firm of Wells, Beckwith, and Cole, architects and engineers. At the time of his death he was associated with the General Necessities Corporation in Detroit.

Mr. Cole, in addition to his membership in the A.S.M.E., which he joined in 1921, was also a member of the American Society of Refrigerating Engineers and the Detroit Engineering Society.

WILLIAM CORRY

William Corry, formerly of New York, died at his home in Cincinnati on December 15, 1926, in his eighty-third year. After leaving the Ohio Mechanics Institute, he served an apprenticeship as a machinist and then took a position with the Hall Safe and Lock Co. of Cincinnati, being promoted to the position of superintendent and later to that of manager. In 1902 he entered the employ of the Chrome Steel Works of New York and as vice-president remained there until he resigned from active business in October, 1925.

Mr. Corry had been a member of the A.S.M.E. since 1890 and was also a member of the Engineers' Club of New York City for more than twenty years.

HOWARD S. DAVIS

Howard S. Davis, at the time of his death on October 11, 1926, was assistant engineer at the Public Service Production Company of Newark, N. J., where he compiled schedules for various jobs in the construction department, and estimated costs for field and office work. He was born in Charleston, S. C., in January, 1891, and was educated at Clemson College, S. C., where he received his degree in 1912. Upon graduation he spent a year with the General Electric Co. at Schenectady. In the Government service at the Naval Yard at Charleston, S. C., he planned and estimated cost of building and repairing machinery, under cognizance of the steam-engineering, construction and repair departments. While in the employ of the Atlantic Potash Company, Mr. Davis designed, built, and tested equipment, and while with the Southern Cotton Oil Company he did practically the same work.

Mr. Davis was elected a member of the A.S.M.E. in 1924 and was also a member of the A.I.E.E. and the National Electric Light Association.

GEORGE K. DAVOL

George K. Davol, consulting engineer of San Francisco, Cal., died on January 26, 1926. At the time of his death he was working on a patent for a Diesel engine. Born in Chicago in 1869, he was graduated from the Chicago Manual Training School in 1887. He has held positions with several prominent firms, such as the J. G. Brill Car Co., the Pullman Palace Car Company, Fraser and Chalmers, the Pacific Coast Borax Co., Alameda, Cal., and the Vulcan Iron Works, Denver, Colo. He was also chief engineer of the Davol Engine Development Co. of San Francisco. He became a member of the A.S.M.E. in 1904.

VAHRAM YETTVART DAVOUD

Vahram Yettvart Davoud died suddenly on August 28, 1926. He was born in Constantinople, Turkey, in May, 1881, and educated at Robert College and the Sheffield Scientific School of Yale University, from which he was graduated with honors.

His first practical experience after graduation from Yale was with the Lackawanna Steel Company at Buffalo, N. Y., for one year, as assistant to the construction engineer. Three years were spent in engineering work at Niagara Falls on the design and construction of large hydroelectric generating stations. Mr. Davoud joined the engineering staff of the Telluride Power Co., Provo, Utah, and with this company and its successor, the Utah Power and Light Co., he continued until 1923. During this period, he had wide electrical and mechanical

engineering and operating experience on high voltage electrical transmission lines. Early in 1923 he joined the engineering staff of the Electric Bond and Share Co., New York, N. Y. In this position, his work pertained to investigation and appraisal of public utility properties. He was a member of the A.S.M.E., which he joined in 1906, the A.I.E.E., and the Yale Club.

EDWARD F. DEMING

Edward F. Deming, mechanical engineer with The Pickering Governor Co., Portland Conn., died of pneumonia at the Middlesex Hospital, Middletown, Conn., on March 1, 1926.

Mr. Deming was born in Grove Township, Taylor County, Iowa, on August 18, 1878. He began his education in the public schools of Iowa and completed his school days in Connecticut, to which state he removed in his boyhood. He was unable to attend college, but after going to work he pursued correspondence courses in both steam and electrical engineering with admirable persistence and with most valuable results.

To obtain as much practical experience as possible along the lines of his evening studies, Mr. Deming, prior to 1902, spent four years working in various power plants. It was in 1902 that he went to work for The Pickering Governor Co. He started in their power plant, but his ability was soon recognized and he was transferred to the engineering department where he became a successful designer of steam specialties. Later he took charge of the testing of their apparatus for the control of steam and gas engines, and he also traveled much, studying at first hand the needs of the trade.

Mr. Deming became a member of the A.S.M.E. in 1924.

WILBURN N. DENNISON

Wilburn N. Dennison was born in Toronto, Canada, in 1875. He served an apprenticeship with the Rodgers Typograph Co. of Windsor, Canada, and from 1895 to 1902 he was employed by several companies, including the Lozier Manufacturing Co. of Toledo and the Thomas Edison Co. In 1902 he became connected with the Victor Talking Machine Co. of Camden, N. J.

After nine years with this company, he entered the service of The Gramophone Co., at Hayes, England. In 1917 he became affiliated with the International Fabricating Corp. of Wilkes-Barre and remained there until he retired from active business in 1921. His death occurred on August 6, 1926.

Mr. Dennison joined the A.S.M.E. in 1905.

EDWARD D. DENSMORE

Edward D. Densmore was born on September 1, 1871, and received his early education from the Somerville High School, Somerville, Mass. He attended the Massachusetts Institute of Technology, from which he was graduated in 1893, and studied mathematics and engineering at Harvard University from 1893 to 1894. His shop experience was obtained at the Lynn Works of the General Electric Company. After having two positions with small firms, he entered partnership with Gifford LeClear doing architectural and engineering work. This firm later came to be known as Densmore, LeClear and Robbins. Mr. Densmore was directly connected with a number of important pieces of work among which were the Harvard Medical School and the Carnegie Institute of Technology.

Besides being an associate of the A.S.M.E. since 1898, he was a member of the Harvard Clubs of Boston and New York and the Engineers' Club of Boston. His death occurred on December 25, 1926.

RICHARD CHAMPION DE SAUSSURE, JR.

Richard Champion De Saussure, Jr., of the Grinnel Co. of Atlanta, Ga., was drowned on July 3, 1926. He was a graduate of the Georgia School of Technology of the class of '25, receiving the degrees of B.S. and M.E., and became a junior member of the Society in 1926. He was born in December, 1903.

ARTHUR R. DICKINSON

Arthur R. Dickinson, executive assistant of the Clark Thread Co. of Newark, and a widely known authority in the textile industry, died at his home at Montclair, N. J., on July 12, 1926.

Mr. Dickinson was born at Fall River, Mass., on July 6, 1879. After graduating from the B. M. C. Durfee High School he took the complete manufacturing course at the New Bedford Textile School, being one of the graduates in the first class turned out by this institution.

His practical experience began with a year in the King Philip Mill. He later became a second hand in the carding department of the Utica Spinning Co. at Utica, N. Y. Next he was overseer of carding at the Merchants Mfg. Co. In 1910 he became superintendent of the Nonquitt Spinning Co., New Bedford, Mass., and from 1911 to 1914 was assistant superintendent of the Burgess Mills at Pawtucket, R. I.

Mr. Dickinson joined the organization of Lockwood, Greene & Co. as textile representative at their Atlanta office in 1914. For five years he remained in this Southern territory, most of this time being spent as mill manager for Lockwood, Greene & Co. at their Winnsboro Mills at Winnsboro, S. C. Returning to their Boston office for a short time he next became agent for the Lancaster Mills at Clinton, Mass. Mr. Dickinson became connected with the Clark Thread Co. at Newark on November 1, 1924.

Mr. Dickinson was an ardent golfer and while at Clinton had served as president of the Runaway Brook Golf Club and as director of the Prescott Club. He also had been one of the directors of the Clinton Trust Co. He was prominent in Masonic circles both in Massachusetts and in New Jersey. Mr. Dickinson joined the A.S.M.E. in 1916 as an Associate.

NORMAN PEARCE EARLE

Norman Pearce Earle, of the Smoot-Holman Co. of Inglewood, Cal., died on May 19, 1926. He was born in Des Plaines, Ill., on January 3, 1895. He spent one year at Beloit College and was graduated from the U. S. Naval Academy at Annapolis in June, 1916. His first position, after serving as engineer officer for about two years, was as foreman in charge of development of production for the Benjamin Electric Mfg. Co. of Chicago. From 1921 until his death, Mr. Earle had charge of mechanical phases of manufacturing and enameling sheet-steel and cast-iron articles.

Mr. Earle was a member of the U. S. Naval Institute and the American Society of Marine Engineers. He was elected a junior member of the A.S.M.E. in 1921 and was promoted to the grade of associate member in 1925.

PETER E. EHLERS

Peter E. Ehlers was born in Hamburg, Germany, March 24, 1848, and was educated in that city. His apprenticeship was served in a machine shop at Altona, near his native city of Hamburg. From there he went to Greenock, Scotland, where he was employed at Cairt's Shipbuilding Co. for a year or two. From Scotland he returned to Germany and entered the Karlsruhe Polytechnicum at Baden from which he was graduated. He served in the German Marine Fleet during the Franco-Prussian War of 1870 and received a decoration. At the close of the war he came to America and entered the service of the Havemeyer Sugar Refining Co. of New York as engineer.

A few years later he was employed by the Franklin Sugar Refining Co. as chief engineer, a position which he held until 1883, when he entered into partnership with G. S. Sullivan of Albany, N. Y. In 1893 he returned to Philadelphia as chief engineer of the William I. McCahan Sugar Refining Co. He was there until 1921 when he retired from active business. At the time of his death on October 29, 1926, he had been a member of the A.S.M.E. for over 35 years, having joined in 1890.

EDWARD B. ELLICOTT

Edward B. Ellicott was born in Hartland, New York, on March 28, 1866. During his lifetime he held the following positions: Engineer of the Salina Gas & Electric Co.; superintendent and electrician of the Concordial Electric Light Co.; expert on installation for the Western Electric Co., at Chicago, serving as assistant superintendent and later superintendent of construction; and city electrician of Chicago, having charge of all the city's electrical interests. He entered the Army during the World War and attained the rank of colonel in the chemical warfare division. At the time of his death on October 26, 1926, he had been a member of the A.S.M.E. for 23 years, having joined in 1903.

ALBERT HAMILTON EMERY

Albert Hamilton Emery, nationally noted inventor, died on December 2, 1926, at his home in Glenbrook, near Stamford, Conn., after a brief illness. Mr. Emery was best known for his designing of machinery for testing purposes.

Mr. Emery was born June 21, 1834, at Mexico, N. Y. He attended the Mexico Academy, and after a period of teaching and surveying, entered the Rensselaer Polytechnic Institute, from which he was graduated in 1858 with the degree of civil engineer.

After being engaged in local contracting for several years in Mexico, N. Y., Mr. Emery obtained a position as engineer in the U. S. Army, and spent several years experimenting on guns and projectiles for the War Department. He then undertook the design of a new system of scales and following this was asked by the Ordnance Department to design a large testing machine. This was finished in 1880 and installed at the Watertown Arsenal.

Among other testing machines of his design are two installed at the Bureau of Standards in Washington, one of 230,000 lb. capacity for tension and compression, and the other of 1,150,000 lb. capacity for tension and of 2,300,000 lb. for compression on specimens of all lengths up to 33 ft. He also invented and developed the method of constructing guns by hydraulic radial expansion.

In 1882 Mr. Emery moved to Stamford, Conn., and the Yale & Towne Mfg. Co. took up the manufacture of his scales, gages, and testing machines. During succeeding years he was engaged in designing car dynamometers, apparatus for measuring hydraulic pressure, railway track scales, hydraulically expanded guns, and testing machines. He retired from active business seven years ago, being succeeded by his son.

Mr. Emery joined the A.S.M.E. in 1880. He was also a member of the American Society for Testing Materials and a fellow of the American Association for the Advancement of Science.

LYNN B. EVANS

Lynn B. Evans, a graduate of the University of Kentucky, of the class of 1915, was born on September 9, 1892, at Lebanon, Ky. After finishing his college course, he entered the employ of C. A. Dunham Co. of Marshalltown, Ia., as designing engineer. A year later he joined the sales and engineering force of The Franklin Manufacturing Co. of Franklin, Pa. He was in the U. S. Navy for two years and in 1919 became sales manager and later production manager of the Sturgis Mfg. Co. of Sturgis, Mich. In this same capacity he was affiliated with the Hare Stoker & Furnace Co. of Detroit, Mich. At the time of his death on December 23, 1926, he was manager of the Kansas City Branch of the Studebaker Corporation. Mr. Evans became a junior member of the A.S.M.E. in 1917 and was promoted to the rank of associate member in 1925.

RANDALL E. EVANS

Randall E. Evans, consulting engineer, died suddenly on April 21, 1926, at the Masonic Club, Queens, N. Y., as the result of a heart attack.

Mr. Evans was born at Des Moines, Ia., on August 25, 1898. He received his education in the public schools of St. Joseph, Mo., followed by work in Civil Engineering at the University of Iowa. In 1916 he entered the United States Aviation Service and for the following four years was on active duty in the United States, Mexico, Canada, England, France, and Germany.

During the World War he spent some time as a special student on aviation subjects at the University of Nancy and at the French War School of Aeronautics. His work during the war period was varied, covering drafting and designing, surveying, topographical work and highway construction.

In his engineering work following retirement from the government service, Mr. Evans was connected with the Curtis Aeroplane & Motor Co., upon airplane motor design, with McClellan & Junkersfeld, on structural steel design, and with the firm of Stevens & Wood. In May, 1925, he became a consulting engineer, and from that time till his death he had been actively engaged in the valuation of the Pardee Steel & File Works, Perth Amboy, N. J., and the Blackwood Coal & Coke Co. mines in Virginia.

Mr. Evans was a member of the Association for the Advancement of Science, of the American Legion, and of the Masonic Order. He became a junior member of the A.S.M.E. in 1921.

JOHN J. FLATHER

John J. Flather, head of the Department of Mechanical Engineering at the University of Minnesota, died suddenly on May 15, 1926, at his home in Minneapolis.

Professor Flather was born in Philadelphia on June 9, 1862. He was educated in private schools in Scotland, at the Bridgeport, Conn., High School and at the Sheffield Scientific School of Yale University from which he was graduated in 1883. He carried out graduate studies at Yale, Cornell, and the University of Edinburgh, receiving his Ph.B. degree from Yale in 1885 and his M.M.E. degree from Cornell in 1890.

His early practical experience covered a full machinist's apprenticeship in various New England shops, including Flather & Co., Nashua, N. H., the Howe Sewing Machine Co., Bridgeport, Conn., and the Armstrong Mfg. Co. He served as a foreman at the Ansonia Electric Supply Co. and as superintendent of the Hotchkiss Mfg. Co.

Professor Flather's teaching career began in 1888 when he became instructor in mechanical engineering at Lehigh University, where he remained for three years. In 1891 he went to Purdue University where he was for seven years professor of mechanical engineering. He went to the University of Minnesota in 1898 and was active there as professor of mechanical engineering up to the day of his death. His teaching covered at one time or another practically every subject in the mechanical engineering curriculum.

In the meantime, Professor Flather carried on the practice of consulting mechanical engineering, did a great deal of research work, and was a prolific writer. His consulting engineering work covered some of the important engineering projects of the Northwest from 1891 down to the present. These included municipal water works, electric light plants, factories, and power plants.

His research work began in 1888 and included combustion and gas engine performance with natural gas, artificial gas, gasoline and kerosene, investigations of power required to drive machinery, locomotive tests and train resistance tests, investigations upon rope, belt, and gear drives, and the study of gas movements in chimneys and chimney design in general. Of late he had carried on extensive experiments upon the cooling effect in refrigerators and refrigerator cars.

Professor Flather took deep interest in the affairs of the A.S.M.E., which he joined in 1889, and was also a member of the American Institute of Electrical Engineers, the Society of Industrial Engineers, the American Association of University Professors, the Minneapolis Engineers' Club, the Newcomen Society of London, the Authors' Club, and the honorary societies of Sigma Xi, Tau Beta Pi, and Pi Tau Sigma. He was secretary and vice-president of Section D of the American Association for the Advancement of Science and had served as treasurer and vice-president of the Society for the Promotion of Engineering Education.

GEORGE L. FOWLER

George L. Fowler, consulting engineer upon railway mechanical subjects, died in New York City on July 2, 1926.

George Little Fowler was born at Cherry Valley, N. Y., on August 9, 1855, and was a graduate of Amherst College, class of 1877. Following his graduation he entered the shops of the Miltmore Car Axle Co. at Arlington, Vt., and within three years became their foreman. He left there in 1880 to become head draftsman of the locomotive department, New York Central & Hudson River Railroad.

From 1881 to 1882 he was chief draftsman of the Industrial Works, Bay City, Mich., builders of excavators and traveling cranes. He joined the Flint and Pere Marquette Railroad in 1882 and by 1886 had been promoted to assistant master mechanic at East Saginaw, Mich. He also spent a year as superintendent of the A. F. Bartlett Co., manufacturers of sawmill machinery and steam engines.

Mr. Fowler came to New York in 1887 and spent a year in editorial work on several power and railway journals. From 1888 to 1889 he was superintendent of the Beals Railway Brake Co. and from 1889 to 1892 superintendent of the Pecham Car Wheel Co.

In 1892 Mr. Fowler began the practice of consulting mechanical engineering in New York, specializing in research work upon locomotive and car design, construction, and operation. He served many of the leading railway companies and carried out many important experimental determinations. He made pioneer measurements of the lateral thrust of wheels against rails on curves up to speeds of sixty miles an hour, the temperatures attained by car wheels when running, and many similar determinations.

Mr. Fowler was a prolific writer on railway subjects and had served as associate editor of the *Railroad Gazette* and of the *Journal of Railway Appliances*, contributing editor of the *Railway Age Gazette* and *Railway Age*, and member of the editorial staff of *Railway and Locomotive Engineering*. He was the author of several books, including the Locomotive Dictionary, The Car Wheel, and Locomotive Breakdown Emergencies and Their Remedies.

Mr. Fowler joined the A.S.M.E. in 1886. He had served upon many committees and had contributed many papers and discussions to the Society. In addition to this he was a member of the Mechanics' Association, the Master Car Builders' Association, the Electric Railway Association, the Boiler Makers' Association, and the American Society for Testing Materials.

JOSEPH L. FRAZIER

Joseph L. Frazier, mechanical engineer with the Hendrie & Bolthoff Mfg. & Supply Co., died on February 23, 1926, at St. Joseph Hospital in Denver, Colo., as a result of complications following a serious operation.

Mr. Frazier was born on August 27, 1866, on a farm near Lebanon, Ill. He was almost entirely self educated as his schooling ended at the age of ten years when he was apprenticed to a stationary engineer in the coal mines of Lebanon.

After operating hoist engines in these mines for some time, Mr. Frazier went to St. Louis in 1879. There he went to work for the Consolidated Coal Co. as a boiler washer. Within seven years he rose to be master mechanic of that company.

In 1886 he went to Colorado where at first he was a stationary engineer with mining enterprises in Clear Creek County. During his first fifteen years in Denver he was engaged in operating power plants in large buildings. During that time he became acquainted with General Hale of the General Electric Company, who took an interest in him and instructed him in electrical engineering. This led to his appointment as an erecting engineer with the General Electric Company and later with the Denver Engineering Company. He joined the Hendrie & Bolthoff organization about fifteen years ago.

Among the power plants designed by Mr. Frazier may be mentioned those of the First National Bank Building and the Daniels & Fisher Stores Co., in Denver, and several for the Diamond Coal Co. at Gallup, N. M.

Aside from his professional work Mr. Frazier's greatest contribution to society was through his charity toward orphan children. This was inspired by his own early hardships. He personally brought up in his own home and generally provided for fourteen orphans. These were taken in two or three at a time, generally from orphanages, and were kept until they had completed their public-school education.

Mr. Frazier became a member of the A.S.M.E. in 1924.

CHARLES E. GILES

Charles E. Giles, consulting engineer and one of the close associates of the late George H. Corliss of steam-engine fame, died at his home in Providence, R. I., on May 9, 1926. In spite of his eighty-three years Mr. Giles had been active in his profession up to within ten days of his death.

Mr. Giles was born at Boothbay, Me., on October 14, 1843, and after attending the public schools he served a carpenter's apprenticeship under his father, who was a building contractor. On July 22, 1862, Mr. Giles enlisted as a private in Company E, Fourth Maine Volunteer Infantry. Almost immediately he found himself in the second battle of Bull Run. From that time until the close of the war he participated in at least fifteen battles, including Gettysburg, the Wilderness, Spottsylvania, Cold Harbor, and the siege of Petersburg. He was twice wounded and was present at the surrender at Appomattox.

After being mustered out of the army on May 31, 1865, Mr. Giles worked for his father for a time, attended Mt. Pleasant Academy at Providence, R. I., spent two years at Bates College, and taught school at Boothbay and Boothbay Harbor. In 1869, he again went to Providence to design and build a new building for the Mt. Pleasant Academy. As superintendent for the firm of Moulton & Ingraham, contractors, he later built several large buildings in Providence, including the Providence County Court House.

In 1879 Mr. Giles became associated with George H. Corliss as foreman of the pattern shop at the Corliss Steam Engine Co., later becoming general superintendent and a director of the company. Following Mr. Corliss' death in 1888, Mr. Giles carried on the works until their sale two years later to the American & British Mfg. Co. He then served many years as agent of this latter concern.

Mr. Giles in 1900 designed and built the largest Corliss-type engine ever constructed, this being still in daily operation in the Albany Street power plant of the Boston Elevated. He also installed the first steam turbines in manufacturing plants both in Rhode Island and Massachusetts.

Mr. Giles was a member of the Masonic Order, and was one of the last of the A.S.M.E. members who was also a member of the Grand Army of the Republic. He joined the A.S.M.E. in 1891.

J. M. GRAVES

J. M. Graves was born at Lexington, Ky., on August 23, 1878, and was graduated from the State University of Kentucky in 1901 with the degree of M.E. After serving a short time with Field & Hinchman of Detroit, he went to Pittsburgh where he became associated with Heyl & Patterson. In 1903 he became assistant to the general engineer of the Allegheny County Light Co. and later was made assistant superintendent of power stations. In 1906 he was advanced to the position of superintendent of power stations. Mr. Graves continued in the service of this company until it became the Duquesne Light Co. with which company he remained until his death on July 23, 1926. During these years of service he held the positions of assistant general manager, general manager, and vice-president.

Mr. Graves was a prominent member of the National Electric Light Association, being a member of its Prime Movers Committee and instrumental in developing the Duquesne Light Company Section of that organization. He was elected a member of the A.S.M.E. in 1915. He was also a member of the Engineering Society of Western Pennsylvania, the American Institute of Electrical Engineers, and the Pittsburgh Chamber of Commerce.

KEPPELE HALL

Keppele Hall, well known as a consulting engineer in management, died in New York City on April 24, 1926, after a brief illness with influenza.

Colonel Hall was born on June 10, 1872, at Newark, N. J., and was a graduate of Princeton University, from which he received his E.E degree in 1894. After his graduation and a period of post-graduate work, he became a member of the firm of Elmer & Hall, engineers and contractors at Trenton, N. J. From 1898 to 1900 he was with the Western Electric Co. upon electrical laboratory work at New York, from 1900 to 1901 was assistant manager for Wm. Browe & Sons Co., electrical contractors at Newark, N. J., and from 1901 to 1902 he was general manager of the Shore Electric Co., light and power producers at Red Bank, N. J.

Colonel Hall joined the National Cash Register Co., Dayton, Ohio, as works engineer in 1902. He left here in 1906 to organize The Keppele Hall Co. (subsequently the Hall Cronan Co.), of which he served as president for five years. This concern had its headquarters at Dayton, Ohio, and was devoted to reinforced-concrete construction work. It was while with this concern that Colonel Hall first became interested in the Taylor System, which he was eminently successful in applying to the work at hand.

He resigned in 1911 to become superintendent of motive power for the Dayton & Troy Electric Railway Co. After two years in this position he spent four years with Sanford E. Thompson, consulting engineer at Boston. When the United States entered the World War he entered the army and was commissioned Lieutenant Colonel in the Ordnance Department and attached to the General Staff War College upon organization and methods work. Upon his release from the service in 1919 he spent two years associated with Morris L. Cooke, H. K Hathaway, and John H. Williams upon consulting management work.

In 1921 Colonel Hall went to Cleveland, Ohio, to become superintendent of planning for The Joseph & Feiss Co. At the time of his death he had just completed his work with that concern and had been in New York but a very short time.

Besides his membership in the A.S.M.E., which he joined in 1911, Colonel Hall held membership in the Taylor Society. He was deeply interested in the work of the Taylor Society and his death removes one of their most valued and active members.

JAMES V. HAMILTON

James V. Hamilton, who became a member of the A.S.M.E. in 1893, died August 18, 1926. He was born in St. Louis, August 29, 1859, and at the age of thirteen was sent to Germany to complete his education. On his return to America he was an apprentice with Ferris and Miles. In later years, while with Krajewski and Pesant, he designed and superintended the erection of sugar machinery and mills and spent a great deal of his time in Cuba. He travelled extensively in Europe, South America, and the United States.

CHARLES HANCOCK

Charles Hancock, for eighteen years professor of mechanical engineering at the University of Virginia, died on May 11, 1926, at his home in University, Va. He was born at "Edgeworth" in the eastern end of Albemarle County, Virginia. He was educated at Miller School and

the University of Virginia, from which he received the degree of Bachelor of Science in Engineering in 1902.

Professor Hancock was an instructor in mechanical engineering at the University of Tennessee, 1888 to 1889; a teacher in shop work and elementary mathematics in the Miller School, for which he also designed and installed a heating and power plant, from 1889 to 1896; an instructor in mechanical engineering, South Carolina Agricultural and Military College, 1898 to 1900; and professor of civil engineering and drawing, Mississippi Agricultural and Military College, 1903 to 1908. In 1908 he returned to the University of Virginia as professor of mechanical engineering. For many years he was engaged in consulting engineering practice in town planning, sanitation, power development, and heating.

Professor Hancock was a member of Phi Beta Kappa, Lambda Pi, the Raven Society, Tau Beta Pi, the American Association of University Professors, and the Society for the Promotion of Engineering Education. He became a member of the A.S.M.E. in 1922.

ALPHONS O. HARANG

Alphons O. Harang was born in Lafourche Parish, La., on May 7, 1874, and died on October 7, 1926. He had a very wide experience in the sale of machinery, having been connected with the Erie City Iron Works for a number of years and subsequently with A. Baldwin & Co. in the capacity of mechanical engineering salesman. In September of 1925 he was elected vice-president of the Carbine-Harang Machinery and Supply Co., Inc., of New Orleans, La., and had charge of the mechanical department. He became a member of the A.S.M.E. in 1922.

FREDERICK G. HARRISSON

Frederick G. Harrisson, assistant sales and advertising manager for the McIntosh & Seymour Corpn., Auburn, N. Y., died on March 6, 1926, at the Robert Packer Hospital, Sayre, Pa.

Mr. Harrisson was born at Chiswick, Middlesex, England, on July 19, 1886. He was educated in both public and private schools and at Technical Acton and Chiswick Polytechnic. In 1903 he began his machinist's apprenticeship in the steam truck shops of John I. Thornycroft & Co., Ltd., at Chiswick. When this automotive plant was discontinued he was transferred by the company to their marine equipment plant and completed his apprenticeship as a draftsman on Diesel engines.

Upon the completion of his apprenticeship in 1911, Mr. Harrisson left Thornycroft and became a draftsman for the Diesel Engine Co., Ltd., and their successors. In 1913 and 1914 he was a draftsman with Babcock & Wilcox, Ltd., of London. He resigned to enlist but failed to pass the physical tests, so he became chief designer in the oil engine department of Brazil, Straker & Co., Ltd., at Bristol.

He remained with them until 1916, when he became engineer for The Commercial & Marine Engine Co. at Hanwell, London. From 1919 to 1921 he was chief designer for Petters, Ltd., and Vickers, Petters, Ltd., at Yeovil in Somerset.

On account of the depression in the engineering trades in England, Mr. Harrisson came to the United States in the fall of 1922. He went to work for the McIntosh & Seymour Corpn. immediately upon his arrival in this country, at first as draftsman and designer of Diesel

engines. It was not long before his familiarity with this subject led to his being concentrated upon advertising and sales work.

Mr. Harrisson was an Odd-Fellow, an associate member of the Institution of Mechanical Engineers (Great Britain), and had belonged to the A.S.M.E. since 1923.

JOHN HENRY HASLAM, JR.

John Henry Haslam, Jr., vice-president of the Georgia Supply Co., died November 3, 1926, at the age of 28 years. Upon graduation from Fordham University in 1920 he was employed as shipping clerk, the highest position in the warehouse of the Georgia Supply Co. In 1922 he was promoted to salesman and in July, 1923, to treasurer. In April, 1925, he was elected to the position of vice-president and general manager which he held until his death. He was elected to junior membership in the A.S.M.E. in 1925.

JOHN HENNEY

John Henney of New Haven, Conn., died on February 26, 1926. He was born at Moosup, Conn., on June 21, 1843. In his study of mechanical engineering, he was much assisted by his father, having entered his employ at a very early age. Later he was employed by the Hartford, Providence and Fishkill Railroad, and then by the New York, New Haven and Hartford Rr. Co., starting as a mechanic and afterwards becoming a locomotive engineer. After many years of faithful service, the company promoted him to the position of superintendent of motive power of the entire road.

Upon his retirement from active business, the American Society of Locomotive Engineers, of which he was an honorary member, presented Mr. Henney with a loving cup in appreciation of his services. He became a member of the A.S.M.E. in 1884.

HENRY G. HERZOG

Henry G. Herzog was born in Philadelphia on October 4, 1898, and received his education at the Collegiate School in New York, N. Y., and Drexel Institute, Philadelphia, Pa. He served a special apprenticeship with the Baldwin Locomotive Works, and was also employed by the Pennsylvania Railroad at Altoona, Pa., and H. K. Porter of Pittsburgh as a locomotive designer. In 1920 he had the opportunity to become chief draftsman and designer for the U. S. Aluminum Co., New Kensington, Pa. and later was offered a position to supervise the development and design of special machinery at the Debay Research & Manufacturing Co. Upon resigning this position in 1922, he became affiliated with G. H. Elmore, a consulting engineer of Philadelphia. At the time of his death on September 1, 1926, he was in the employ of the Cuban-Dominican Sugar Co. He was elected a junior member of the A.S.M.E. in 1922.

ADELBERT C. HIGGINS

Adelbert C. Higgins, chief inspector for the Employers Liability Assurance Corpn. of Philadelphia, died there on April 25, 1926.

Mr. Higgins was born at Lowell, Mass., on March 31, 1881, and received a grammar school education, supplemented by evening studies on engineering subjects. In 1900 he went to work for the Edison Illuminating Co. of Boston, remaining with them for three years.

Following this he was for a year a general repair man for the Interborough Rapid Transit Co. in New York City. From 1904 to 1912 Mr. Higgins was engaged upon steam fitting, plumbing, and refrigeration installation with various building contractors in Chicago.

Following this he was for two years chief repair man with the Illinois Maintenance Co., during the reconstruction of the Edison Building, and two years as chief engineer of the Metz Furniture Mfg. Co. He entered the insurance field in 1914 as boiler and engine inspector with the Hartford Steam Boiler Inspection & Insurance Co. He had been with the Employers Liability since 1917.

Mr. Higgins joined the A.S.M.E. in 1922.

GUSTAV HILLMANN

Gustav Hillmann, retired naval architect, died at Brooklyn, N. Y., on March 15, 1926.

Mr. Hillmann was born at Wismer, Germany, on February 11, 1839. For eight years he studied in the Latin School there and then in 1856 began a three years' apprenticeship in a shipyard at Rostock, Germany. In 1859 he embarked as ship's carpenter on a voyage around Cape Horn to the west coast of South America. Upon his return in 1860 he spent a year in the Construction Chamber of the Danish Navy at Copenhagen, studying naval architecture.

Mr. Hillmann followed this course with a year upon construction of iron ships at a Liverpool shipyard. In 1862 he went to New York. After spending a year as a ship carpenter he entered the office of an inspector of ironclads and during the rest of the Civil War period he was engaged in drafting work concerned with U. S. monitors.

Upon the close of the war Mr. Hillmann became the proprietor of his own shipyard at City Island, N. Y. He carried this on until 1875 when he concentrated upon naval architecture and for the next forty years built up an enviable reputation as a designer of yachts. He retired about twelve years ago on account of ill health.

Mr. Hillmann was interested in educational matters and in the work of the professional societies. He was particularly interested in the Department of Naval Architecture at Cornell and in the Webb Institute of Naval Architecture, where some of his finest drawings are now preserved.

He was a member of the Society of Naval Architects and Marine Engineers, and was a life member of the A.S.M.E., which he joined in 1883.

FRED A. HITT

Fred A. Hitt, foreman of the inspection department of the Hartford Screw Co., was born in Kingston, N. Y., on December 1, 1885. In 1912 he entered the service of the Crane Co. of Bridgeport, Conn., in whose service he remained until 1917 when he accepted a position with Pratt & Cady Co., Inc., of Hartford, Conn., as assistant foreman and later foreman. He died at his home in East Hartford, Conn., on November 24, 1926.

Mr. Hitt joined the A.S.M.E. in 1919 and was an associate-member at the time of his death.

F. AMOS JOHNSON

F. Amos Johnson, a consulting mechanical engineer and patent attorney of New Haven, Conn., died on October 19, 1926, at the age of 65 years. He was a graduate of the class of 1886 of the University of

Minnesota. In 1887, after eighteen years of experience in designing and shop superintending, he was employed by the Johnson-Crane Matrix Co. He remained with this company until 1903, at which time he became connected with the Tachytype Manufacturing Co. Before entering the consulting field he was also in the service of the Johnson Typesetter Co. at New Bedford, Mass.; the Unitype Co. in charge of their experimental department; the Vita-a-Type Co. of Jersey City as secretary and engineer; and the Pope-Johnson Corpn. of the same city as treasurer. He became a member of the A.S.M.E. in 1907.

ROBERT B. JOHNSTON

Robert B. Johnston was born in Syracuse, N. Y., August 21, 1875. He attended the local public schools and then served a four-year apprenticeship as toolmaker with the H. H. Franklin Die Casting Co. of Syracuse. The next three years were spent in acquiring practical experience with various firms in Syracuse. From 1903 until 1911 he was connected with the Smith Premier Typewriter Co. of Syracuse, as toolmaker, model maker, layout man on tool work, and foreman of tool work. He was subsequently connected with the Meriden Firearms Co., Meriden, Conn., Remington Arms & Ammunition Co., Ilion, N. Y., Weston Electrical Instrument Co., Newark, N. J., and Miller Lock Company, Philadelphia, Pa., in supervisory positions. In August, 1918, Mr. Johnston became superintendent of the metal shop of the Naval Aircraft Factory, at the Philadelphia Navy Yard. At the time of his death he was superintendent of the Audubon Wire Cloth Co., at Audubon, N. J.

Mr. Johnston was elected to associate membership in the A.S.M.E. in 1917, and promoted to full membership in 1920.

THOMAS HENRY KANE

Thomas H. Kane accepted the position of president of the Berger Manufacturing Co. at Canton, Ohio, two weeks before his death in October, 1926. Mr. Kane was born in Toledo, Ohio, in 1872, and received his early training in the machinist trade in Detroit, Mich. He later became associated with the Bronson-Walton Co., of Cleveland, Ohio, and then with the Toledo Machine and Tool Co.

In 1908 Mr. Kane accepted the position of general manager of the Truscon Steel Co., at Youngstown, Ohio, and later became vice-president of the company. He became a member of the A.S.M.E. in 1914.

CLARENCE V. KELLOGG

Clarence V. Kellogg, president of the Kellogg-Mackay Company, Chicago, died suddenly at his home in that city, May 9, 1926.

Mr. Kellogg was born in Cicero, N. Y., January 19, 1860. He attended the local grammar school, supplementing it later with an academic education. Up to the age of eighteen, he worked on his father's farm, developing a physical strength which carried him through many years of an unusually active business life. His first outside employment was with a tea store in Syracuse, N. Y., but after a brief experience in the tea business he began the study of law, and was admitted to the bar in Syracuse in 1881. He continued his law practice in New York State, making commercial law his specialty until February 1, 1893.

Tiring of law as a vocation, he came to Chicago in 1893 as the treasurer of the American Boiler Company. In 1898 the company was succeeded by the Kellogg-Mackay-Cameron Co. The Pierce, Butler &

Pierce Mfg. Co. took over the eastern business of the American Boiler Co., buying up the entire capital stock, and selling to Mr. Kellogg the western end of the business. The Kellogg-Mackay-Cameron Co. continued until January 1, 1909, when Mr. Cameron retired and the concern became the Kellogg-Mackay Co.

Aside from his regular business connections, Mr. Kellogg was active in other companies. His interest in trade associations was unusually strong. He was president of the Central Supply Association from 1913 to 1916, an honor never before or since duplicated. At one time he was president of the National Tubular Boiler Co., and also the Illinois and Wisconsin Jobbers' Credit Bureau. For some time, he was vice-president of the Pierce, Butler & Pierce Mfg. Corpn., and a director of the Kewanee Boiler Co. He also had been president of the American Boiler Manufacturers' Association, and was a member of the National Pipe & Supplies Association.

Mr. Kellogg, a 32d degree Mason, was active in club and fraternal circles, and belonged to the following organizations: The Chicago Athletic Association; Park Lodge, No. 843, A. F. & A. M.; Park Chapter No. 213, R. A. M.; Oriental consistory, Medinah temple; Medinah Country Club, and Briergate Golf Club. He became an associate of the A.S.M.E. in 1922.

ERNST KOPKE

Ernst Kopke, who was consulting engineer on sugar-house machinery of the Honolulu Iron Works, Honolulu, died on May 27, 1926. He was born in Hanover, Germany, in January, 1854, and received his early education in this country, attending the Worcester Free Institute of Technology from 1878 to 1879. From 1886 to 1888 he attended a technical school in Germany. He was machinist apprentice, and journeyman with the Honolulu Iron Works from 1872 to 1878, after which he returned for a year to the Worcester Polytechnic Institute.

From 1880 to 1899 Mr. Kopke was employed by the Lihue and Hanamanlu Mills as superintendent engineer; the Kehaha Sugar Co. as contracting engineer; Ewa Plantation Co. as superintendent engineer; the Honolulu Iron Works as superintendent of shops; the Kahuku Plantation and Pioneer Mill Co.; and the Oahu Sugar Co. From 1899 until his death, Mr. Kopke was connected with the Honolulu Iron Works as mechanical engineer. He was elected a member of the A.S.M.E. in June, 1913.

ALMERON M. LANE

Almeron M. Lane, inventor of clock mechanisms, died at his home in Plantsville, Conn., on January 25, 1926. Mr. Lane was born at Forestville, Conn., on August 8, 1847, and received his early education in the common schools of that town. This was supplemented by private studies in mechanical engineering carried on under the direction of Professor Honey of New Haven. In this way Mr. Lane became a skilful draftsman.

He began his engineering career as an apprentice in a clock factory at Forestville, and it was not long before his inventive ability became apparent. Among the concerns for whom he worked may be mentioned the National Clock Co. of San Francisco, 1875-1880; the Gilbert Clock Co. of Winsted, Conn., 1880-1885; and the Parker Clock Co. and Parker-Whipple Co. of Meriden, Conn., 1885 until retirement sixteen years ago. While with the Gilbert Clock Co., as designer, Mr. Lane supervised the manufacture of their first marine clocks. He became a life member of the A.S.M.E. in 1892.

JOHN V. LANNON

John V. Lannon, chief dispatcher for the Broad River Power Co. at Columbia, S. C., met with a fatal accident there on April 29, 1926.

Mr. Lannon was born at Hazelton, Pa., on December 29, 1892, and was graduated from the high school in that city in 1913. In 1921 he was graduated from the Pennsylvania State College as an electrical engineer.

Before entering college he worked for four years in the Maintenance of Way Department on the M. & H. Division of the L. V. R. R. After graduation he took a position with the Pennsylvania Edison Co. at Easton, Pa., remaining there for a year and a half. He was then transferred to Reading, Pa., where he spent a year with the Metropolitan Power Co. He was then sent to South Carolina, first to Spartanburg and then to Columbia.

Mr. Lannon became an associate-member of the A.S.M.E. in 1925.

LOUIS R. LEMOINE

Louis R. Lemoine, chairman of the board of the United States Cast Iron Pipe & Foundry Co., died at his home in Philadelphia on April 23, 1926, as a result of pneumonia.

Mr. Lemoine was born in St. Louis, Mo., on July 3, 1858. After completing his schooling he entered the iron business, which he learned under Joseph Wharton, who was later one of the sponsors of both the New Jersey Zinc Co. and the Bethlehem Steel Co. In 1883 Mr. Lemoine went to Philadelphia, where he became associated with R. D. Wood & Co. in the cast-iron-pipe business.

He literally grew up with the cast-iron-pipe industry and in 1896 became vice-president of the McNeal Pipe & Foundry Co. of Burlington, N. J. This concern was later absorbed by the United States Cast Iron Pipe & Foundry Co., of which Mr. Lemoine served as president from 1911 until he became chairman of the board in 1923. While serving as a director of this concern, Mr. Lemoine was at the same time general manager of the New Jersey Zinc Co., from 1906 to 1910.

He was very active in the improvement of cast-iron-pipe manufacture and was largely instrumental in the adoption of the de Lavaud process by this company.

Mr. Lemoine became an associate in the A.S.M.E. in 1887.

JOHN B. LINDSAY

John B. Lindsay, chief engineer of the Public Service Electric and Gas Company of Burlington, New Jersey, died on February 13, 1926. From 1903 to 1908 he was employed by the General Electric Company of Schenectady, New York, having charge of power-plant equipment for electric, steam, and mechanical installations. In 1908 he became associated with the Westinghouse Electric & Manufacturing Co. of East Pittsburgh, in the capacity of trouble engineer, having charge of electrical and mechanical apparatus in the erection department of the company. From 1912 to 1917 he was superintendent of generation in charge of apparatus and construction of the Citizens Light, Heat and Power Company of Johnstown, Pa. Subsequently, he held the position of chief engineer with the Lowellville Power Station of Youngstown, Ohio, and the Burlington Power Station, Public Service Electric and Gas Company of Burlington, N. J.

Mr. Lindsay became a member of the A.S.M.E. in May, 1925, and was also a member of the New Jersey State Engineers, and president of the Burlington County Industrial Association. He was born in Chester, S. C., on October 17, 1882.

ERNEST J. LORING

Ernest J. Loring, ordnance expert, died at Washington, D. C., on January 19, 1926.

Mr. Loring was born at Somerville, Mass., on February 18, 1874, and attended the schools in that city. He later studied architecture at the Massachusetts Institute of Technology, from which he was graduated in 1895. For three years he worked as an architect with his father, who was a member of the firm of Loring & Phipps in Boston.

He first became well known in the mechanical field as the inventor and manufacturer of the speedometer known as the Loring Speed Gauge. Mechanical engineering held his interest and after spending five years in the Turbine Department of The General Electric Co., at Lynn, Mass., he joined the Lamson Service Co. While with them he installed their pneumatic tubes in the New York Stock Exchange, doing away with the messenger service on the floor of the Exchange.

During the World War Mr. Loring served as a Captain in the Ordnance Department. One of the problems upon which he was engaged was the production of helium. Upon his discharge from the army he worked for a time with the Hartford Fairmount Co., designing glass-working machinery.

In 1921 Captain Loring returned to the Ordnance Department in Washington and since that time was engaged as ordnance engineer in the Washington offices and at Aberdeen Proving Grounds. Captain Loring's special field covered the design and manufacture of bombs, bomb sights, and cameras.

Captain Loring was a member of the Masonic Order, of the Cosmos Club of Washington, the Army Ordnance Association, and the Society of Automotive Engineers. He became a member of the A.S.M.E. in 1912.

ANDREW C. LOUDON

Andrew C. Loudon, long recognized as an authority upon railway mechanical engineering, died quite suddenly of pneumonia at his home in Burlington, Vt., on April 11, 1926.

Mr. Loudon was born on July 7, 1883, at Valleyfield, Que., and attended the public high school. In 1906 he was awarded the degree of B.S. by McGill University. In 1901 he began his engineering work as an apprentice with the Grand Trunk Railroad at Montreal and at Portland, Me. In 1906 he joined the American Locomotive Co., Schenectady, N. Y., where he spent a year in the calculating department.

Mr. Loudon returned to the Grand Trunk in 1907 as engine-house foreman at Island Pond, Vt. This was followed by a year as machinist and draftsman with the Central Vermont and the Delaware & Hudson at Green Island, N. Y., a year with the Achison, Topeka & Santa Fe in the test department, and another year with the Grand Trunk as foreman of construction and locomotive and car repairs.

He left railway work in 1912 to become associated with the Simmons-Boardman Publishing Co. At first he was assistant editor in charge of the *Car Builders' Dictionary,* and subsequently associate editor of the *Railway Mechanical Engineer.*

In 1917 Mr. Loudon became associated with The Superheater Co. of New York. He rapidly worked up through the service and engineering

departments and in 1920 was given charge of The Superheater Co., Ltd., of Canada, of which he was elected vice-president in 1921. This position he held at the time of his death.

Mr. Loudon joined the A.S.M.E. in 1913.

FRANK OTIS LOVEJOY

Frank Otis Lovejoy, factory manager of the Fiberloid Corporation, Indian Orchard, Mass., died February 23, 1926. He was born at Springfield, Mass., in June, 1865. From 1882 until 1897 Mr. Lovejoy was employed by the Hampden Watch Company, the Seth Thomas Clock Company, the Waltham Watch Company, and the Keystone Watch Company. From 1897 until his death, Mr. Lovejoy had been employed by the Fiberloid Company as foreman, superintendent, assistant manager, and factory manager.

Mr. Lovejoy became an associate of the A.S.M.E. in 1917.

MATTHEW LUND

Matthew Lund, an authority on safety in industry, died at the Blodgett Hospital in Grand Rapids, Mich., on June 12, 1926, following an operation.

Mr. Lund was a native of Denmark, where he was born in September, 1865. In 1879 he began a machinist's apprenticeship and at the same time carried on engineering studies in the Danish Government Technical Evening School. Here he was awarded first prize in mechanical drawing in 1884. From 1884 to 1887 he was a student at the Polytechnic Institute in Copenhagen, where he also won an award in drafting.

As a result of his excellent record at the Polytechnic Institute he was awarded the Danish Government's traveling stipend. From 1887 to 1889 he was thus enabled to travel in several European countries where he worked in the principal engineering plants for short periods, and also as a marine engineer, to gain experience.

Mr. Lund came to the United States in 1889 and spent his first four years here as erecting engineer at Philadelphia with Bement, Miles & Co., Wm. Sellers & Co., the Baldwin Locomotive Works, and Wm. Cramp & Sons, and at Alliance, Ohio, with the Morgan Engineering Co.

In 1893 he went to Grand Rapids as engineer and superintendent of the Fox Machine Co., where he remained until 1908. He was awarded a certificate of honorable mention as an expert artisan from the World's Columbian Exposition at Chicago.

From 1908 until 1913 Mr. Lund was in charge of several small manufacturing enterprises and completely rebuilt the plant of the American Clay Working Machinery Co. at Willoughby, Ohio. He became vice-president and manager of the Valley City Machine Works at Grand Rapids in 1913 and also carried on his consulting work under the name of the Lund Engineering Co. He was intensely interested in industrial safety work and for the past three years had been safety engineer for the Furniture Mutual Insurance Co.

Mr. Lund joined the A.S.M.E. in 1918 and was one of the organizers of the Peninsula Section, which he served as a member of the Executive Committee in 1924 and 1925. He had recently been appointed field engineer for the Grand Rapids Territory in the nation-wide safety investigation being conducted by the American Engineering Council. He was also a member of the Grand Rapids Engineering Society, the Grand Rapids Safety Council, and the Grand Rapids Industrial Executives Club, and was a prominent Mason.

CHARLES A. R. MAKEPEACE

Charles A. R. Makepeace, designer of textile plants, died at his home in Providence, R. I., on February 10, 1926, as the indirect result of being struck by an automobile.

Mr. Makepeace was born at Fayetteville, N. C., on May 20, 1860. After attending public schools in that section he entered Trinity College, now known as Duke University. During his senior year he was forced to leave college because of the death of his father, and he was unable to return. Later he was awarded his degree, however, as of the Class of 1880, and also was graduated from the Rhode Island Institute of Technology and School of Design.

Mr. Makepeace worked for a time at his father's mill at Fayetteville and at various bleacheries and dye works between 1877 and 1881, and followed this with a six-months' inspection trip to textile plants in Great Britain, France, and Germany. Upon his return to the United States he joined the staff of Thompson & Nagle (later D. M. Thompson & Co.), architects and engineers, of Providence. He was with them for six years as draftsman and supervisor of plant construction.

In 1889 Mr. Makepeace bought out D. M. Thompson & Co. and organized a new company as C. R. Makepeace & Co. Since that time Mr. Makepeace has been at the head of this well-known organization, which has to its credit the design of a great number of the largest and finest textile plants in New England, the Middle West, and the South.

Mr. Makepeace was prominent in the civic and social life of Providence and was a member of many clubs and associations. He was officer and director of several prosperous financial and manufacturing organizations. He was a life member of the A.S.M.E., which he joined in 1915.

EDWARD H. MAROT

Edward H. Marot, sales engineer with the Bower Roller Bearing Co., died at Detroit, Mich., on May 29, 1926.

Edward Heald Marot was born at Philadelphia, December 17, 1875. While in the last year of grammar school his father died and he was obliged to leave school and go to work. This educational deficiency he later made up by persistent attendance at evening schools and by home study.

He began his engineering career as an apprentice in the shops of Bement, Miles & Co. at Philadelphia, gradually working his way up to the drafting room. In the meantime he was studying enginering at the Spring Garden Institute during the evenings. His next experience was as a draftsman at the Philadelphia Engineering Works and with the American Steel & Wire Co. Later he spent some time as draftsman and designer with the Cambria Steel Co., the Reading Iron Co., the United Coke & Gas Co., and the Saxton Furnace Co.

In 1904, Mr. Marot joined the Philips Pressed Steel Pulley Works at Philadelphia. After a year upon the design of special machinery for pulley manufacture, he became a salesman for the concern. In 1907 he joined the Hyatt Roller Bearing Co. as sales engineer, becoming a district sales manager.

From 1913 to 1915 Mr. Marot was employed as sales engineer with the Standard Roller Bearing Co. and the Royersford Foundry & Machine Co. He returned to Hyatt in 1915 and was located in Cleveland, where he specialized upon machine-tool and motor-truck applications of roller bearings.

Mr. Marot joined the Bower Roller Bearing Co. at Detroit soon after the War. He had travelled widely both in this country and

Europe, had a very wide acquaintance in the engineering field, and was looked upon as an authority in the industrial application of roller bearings. He joined the A.S.M.E. in 1903.

WILLIAM E. MARSHALL

William E. Marshall, mechanical engineer with the Western Electric Co., in New York, died on March 12, 1926.

He was born on April 7, 1891 at Philadelphia, and attended the Heffely Preparatory School in Brooklyn. Following this he entered Stevens Institute of Technology, at the age of seventeen, and was graduated in 1912 with the degree of M.E.

He began his engineering career with the General Acoustic Co., Jamaica, L. I., upon telephone work. This was followed by a year as draftsman with Walter Kidde & Co., New York, and five months as draftsman with the American Sugar Refining Co., of Brooklyn.

In December, 1913, Mr. Marshall returned to Stevens Institute as instructor of drafting and descriptive geometry. He remained there until 1918. In the meantime he and his brother, M. W. Marshall, invented several office specialties, including an automatic check endorser. His patents were assigned to The Marshall Bros. & Griffin Mfg. Co., of which he was president and chief engineer. When this concern leased its patent rights to the Mailometer Company of Detroit, Mr. Marshall became an engineer for this latter concern.

Mr. Marshall joined the Western Electric Co. in 1923, being employed at first as a designing draftsman upon machine switching apparatus. He later entered the building maintenance department, and had charge of heating, ventilating, and refrigerating installations.

Mr. Marshall was active in the Masonic Order. He joined the A.S.M.E. in 1914.

CLYDE H. McCLINTOCK

Clyde H. McClintock, mechanical engineer with Arthur L. Mullergren, consulting engineer, Kansas City, Mo., died April 25, 1926, at Kansas City. He was educated at the University of Colorado, receiving an M.E. degree in 1921. He was formerly connected with the Denver Tramway Company, Federal Light and Traction Company, and the Ideal Portland Cement Company. He joined the Society as a junior in 1916 and was promoted to associate member in 1921. He was born in 1894.

EDMUND B. McCORMICK

Edmund B. McCormick, authority upon highway construction, died at Alameda, Cal., on January 15, 1926.

Mr. McCormick was born at Normal, Ill., on November 24, 1870, and was graduated from the Illinois State Normal University in 1889. In 1897 he was awarded a degree of B.S. in Mechanical Engineering by the Massachusetts Institute of Technology. During his college vacations and for several months following his graduation Mr. McCormick worked as a journeyman machinist and erector. As such he was associated with the C. & A. R. R. at Bloomington, Ill., from 1889 to 1893.

After receiving his degree from M. I. T. Mr. McCormick spent four years as assistant, instructor, and assistant professor of mechanical engineering at the Montana State College, Bozeman, Mont. In 1901 he went to the Kansas State Agricultural College, Manhattan, Kans., as professor of mechanical engineering, becoming Dean of the Engineer-

ing Division in 1907. He also served as professor of power and experimental engineering and director of the Engineering Experiment Station.

Professor McCormick became associated with the U. S. Department of Agriculture, Bureau of Public Roads, as consulting engineer in 1907. In 1913 he joined this Bureau as mechanical engineer. From that time until 1925 he remained in that Department serving as Chief of the Division of Rural Engineering and Chief of the Equipment Division. He was also consulting editor of the Agricultural Engineering series for the McGraw-Hill Book Co.

Mr. McCormick was an active member of the Masonic Order, belonged to the Sigma Tau fraternity, and was a member of the Society for Promotion of Engineering Education. He joined the A.S.M.E. in 1913.

HUGH R. McGREGOR

Hugh R. McGregor, long the mechanical superintendent of the Brown & Sharpe Mfg. Co., died at his home in Providence, R. I., on March 18, 1926. His health had been failing for some time.

Mr. McGregor was born at Brookville, Nova Scotia, on March 29, 1859. He came to Providence in 1871 and began his mechanical work at the Providence Tool Co., who were then making quantities of rifles for the Turkish Government.

After six years with that concern, Mr. McGregor, on June 4, 1877, joined the Brown & Sharpe Mfg. Co. as an apprentice. Before he had completed his apprenticeship period, he was made assistant foreman of milling machine manufacture.

From 1893 to 1898 Mr. McGregor was with the Curtis Electric Co. at Jersey City and the Sprague Electric Co. at Bloomfield, N. J. He returned to Brown & Sharpe in March, 1898, and since that time his influence upon the design and manufacture of the products of that company had been profound.

Mr. McGregor joined the A.S.M.E. in 1903.

HARRY PARR MEREDITH

Harry Parr Meredith, who at the time of his death on October 13, 1926, was general works manager of the Maxwell Motor Co. of Detroit, Mich., was born in Gloucester County, Virginia, on January 12, 1879. For twenty years he was in the employ of the Pennsylvania Railroad Co., having risen from the position of machinist's apprentice at the Altoona, Pa., works to master mechanic of the Baltimore Division, the Williamsport and Stonbury Divisions, and the Maryland and Delaware Divisions. In 1917 he became master mechanic of the Maryland and Delaware Division of the Philadelphia, Baltimore and Washington Railroad Co. Later in that year he was engineer in charge of mechanical maintenance of the E. I. DuPont de Nemours & Co., and was promoted in 1918 to the position of mechanical division engineer. In 1920 he was made works manager of the Saxon Motor Car Corpn. of Detroit, and in 1922 joined the Maxwell company.

Mr. Meredith became a member of the A.S.M.E. in 1917.

ARTHUR A. MERRITT

Arthur A. Merritt, former mechanical engineer for the Willcox & Gibbs Sewing Machine Co., died November 19, 1926. He was born at Milford, Mass., August 12, 1860, attended the public schools of Worcester, Mass., and learned the machinists trade in the L. L. Pollard Co., of Worcester. He remained with this concern from 1877 to 1886 and

then entered the employ of Barrett Bros., general machinists and manufacturers of sewing machines. After leaving that firm he sold sewing machines for the Pentucket Variable Stitch Machine Co. of Haverhill, Mass., and three years later became affiliated with the Jones Special Machine Co., of Boston. Subsequently he was employed by the Union Special Machine Co. of Chicago, Ill.

Mr Merritt became the representative of the Willcox & Gibbs Sewing Machine Co in 1906 and spent much of his time at the works of the Brown & Sharpe Manufacturing Co, building machines for Willcox & Gibbs. During his long career as a specialist on sewing machines, he had more than 50 patents issued to him. Mr. Merritt became a member of the A S M.E. in 1907. He retired from active business on January 1, 1925.

LEONARD METCALF

Leonard Metcalf, well-known water works engineer of Boston, died at his home in Concord, Mass., on January 29, 1926. While Mr. Metcalf was born at Galveston, Texas, August 26, 1870, his home for the greater part of his life was in Concord. There he attended the high school, after which he went to the Massachusetts Institute of Technology, where he was graduated from the Civil Engineering Department in 1892, with the degree of B.S.

For three years following his graduation he was employed by Wheeler & Parks of Boston, civil engineers and operators of water companies Following this he served for two years as professor of mathematics and engineering at the Massachusetts Agricultural College. In 1897 Mr. Metcalf began the practice of consulting civil engineer in Boston under his own name, which he continued until 1907, when he went into partnership with Harrison P. Eddy under the present name of Metcalf & Eddy, hydraulic and sanitary engineers.

Mr. Metcalf's outstanding achievements were in the field of municipal water works. In the thirty years of his active career he handled the design and construction of a great number of successful projects, including reservoirs, wells, pumping stations, filters, and distribution systems. He also served as manager of public-service corporations, both water companies and electric-light plants.

Mr. Metcalf was a prolific writer upon hydraulic and related subjects, his writings frequently appearing in the Journals of the American and the New England Water Works Associations and of the American Society of Civil Engineers. He was also either author or co-author of many books and manuals well known to the engineering profession. He even wrote two valuable papers while confined to his bed by his final illness.

Upon the entrance of the United States into the World War Mr. Metcalf became an active member of the Sub-Committee upon the Emergency Construction of Buildings and Engineering Structures, under the Council of National Defence.

In accordance with his firm belief that engineers should make their work a profession, not merely a business, Mr. Metcalf took an active part in the work of the professional and educational societies. He had served as president of the M. I. T. Alumni Association, and at the time of his death was a member of the Institute Corporation. He served as a director and as vice-president of the A.S.C.E., was president of the Boston Society of Civil Engineers in 1919, of the American Water Works Association in 1916-17, and of the New England Water Works Association in 1915. He was very instrumental, immediately following the War, in the conception and organization of the present successful Affiliated Engineering Societies of Boston. Mr. Metcalf became a member of the A.S.M.E. in 1904.

CHESTER B. MILLS

Chester B. Mills, chief engineer of the Sperry Gyroscope Company, died at his home in Rockville Center, Long Island, September 26, 1926. Mr. Mills was born at Lindsay, Ontario, Canada, December 1, 1876. He received his education in the public schools; Collegiate Institute, Toronto; Toronto School of Science; and Mechanics Institute, Rochester, N. Y. He entered the employ of the Westinghouse Electric and Manufacturing Co., East Pittsburgh, Pa., in 1898. From the time of his entering the Westinghouse Co., Mr. Mills steadily advanced and in 1910 was made engineer in charge of mechanical design of all apparatus manufactured by the industrial branch of the company. He was also consulting engineer on shop processes and methods embracing practically all the company's varied products. In 1918, he resigned from the Westinghouse Co., and entered the employ of the Sperry Gyroscope Co., Brooklyn, N. Y., as assistant chief engineer and was made chief engineer of that company October 28, 1919. Mr. Mills had been a member of the A.S.M E. since 1912.

THOMAS H. MIRKIL, JR.

Thomas H. Mirkil, Jr., who at the time of his death on November 12. 1926, was district sales manager of the Treadwell Engineering Co. of Philadelphia, Pa., was born August 15, 1858, at Chester, Pa. He was educated in Chester and served four years as an apprentice in the shops of the Robert Wetherill & Co. before entering the engineering school of the University of Pennsylvania. He returned to business in his junior year. From 1883 to 1910 he held the positions of draftsman, chief draftsman, general foreman, superintendent, and general manager of the Southwark Foundry and Machine Co. of Philadelphia. From 1910 to 1914 he was vice-president and general manager of the Poole Engineering and Machine Co. of Baltimore.

Mr. Mirkil was a member of the A.S.M.E. for 42 years and was also a member of the Engineers' Club of Philadelphia.

WILLIAM A. NETTLETON

William A. Nettleton, born in Hannibal, Mo., on October 24, 1863, was educated at Phillips Academy, Andover, Mass., and Sheffield Scientific School of Yale University, New Haven, Conn., from which he was graduated with the class of 1885. His early career was spent in surveying for the Kansas City, Fort Scott & Memphis Railroad, and bridge-inspection work. After serving as an apprentice in the Westinghouse Air Brake Co. he inspected the installation of the air brakes on the railroad by which he was formerly employed.

He held prominent positions in the Atchison, Topeka and Santa Fe, the Frisco System, the Chicago, Rock Island and Pacific and the Chicago and Eastern Illinois railroads. He died in London on August 30 at the age of 63, having been inactive in business since 1912.

Mr. Nettleton became a member of the A.S.M.E. in 1905.

MARTIN F. NEWMAN

Martin F. Newman, manager of the water purifying department of Wm. B. Scaife & Sons Co., died at Oakmont, Pa., on May 25, 1926.

Mr. Newman was born at Waynesboro, Pa., in February, 1871. He received his education in the public schools, and in 1890 entered upon

a machinist's apprenticeship. He became experienced upon saw mills, steam engines, and refrigerating machinery.

From 1891 until 1903 Mr. Newman was in the engineering department of the Frick Company at Waynesboro, where he carried through many original designs along the lines mentioned. Following this he spent a year with the Carnegie Steel Co. at Pittsburgh in charge of drafting the first dry blast installation.

He joined Wm. B. Scaife & Sons Co in 1905 and designed and patented the original water-purifying apparatus manufactured by them

Mr. Newman joined the A.S.M.E. in 1911 and had served upon several committees.

WILLIAM H. ODELL

William H. Odell, for many years known as a specialist in steam economy, died at a private home for the aged in Brooklyn on February 15, 1926. He was born on August 12, 1840, at Greenburg, Westchester County, New York. After attending the local schools he went to Brooklyn, where he became a machinist's apprentice. At the same time he spent his evenings studying drafting at Cooper Union.

After working in various marine machine shops around New York City, during which time he became a foreman, Mr. Odell in 1868 became engineer for Baldwin & Flagg, hat manufacturers at Yonkers, N. Y. Here he originated a novel and successful method of heating dye kettles with the exhaust steam, formerly wasted. This improvement cut the coal bills of the company in half.

After leaving this concern in 1870, Mr. Odell was for a few months with the Remington Arms Co., Ilion, N. Y. He then became superintendent of William Wright & Co., of Newburgh, N. Y. He had in the meantime been studying the subject of steam under Charles J. Emery, and recognizing his broad knowledge of this subject Mr. Wright encouraged him to become a consulting specialist in the economic generation and application of steam. Mr. Odell was "ahead of his time," and the business was rather slow in growing. He was, however, recognized by such men as C. H. De Lamater, Capt. John Ericsson, Henry R. Worthington, Sr., George H. Corliss, and Thomas A. Edison, and carried on work for them. He also did work for such corporations as Babcock & Wilcox and the Otis Elevator Co.

In 1880 Mr. Odell was engaged by the firm of Sweet & Young to select the machinery for their work of driving a tunnel under the government area at West Point, N. Y. One of the innovations which he introduced on that job was the use of electric lights in the workings instead of the miners' headlamps which had been previously used.

During his lifetime Mr. Odell invented and patented many devices connected with steam engineering which are in general use today. In 1874 he became acquainted with Professor Phinn, who encouraged him to write an article for the *Technological* upon the value of the steam indicator. This was one of the earliest articles upon this subject and was the first of Mr. Odell's quite voluminous technical writing. His last was Reminiscences of a Mechanical Engineer, an interesting series published from time to time in *Power* during 1923, 1924, and 1925.

Mr. Odell was a life member of the A.S.M.E. and was one of the small group which joined the Society at the time of its inception in 1880.

HARRY A. OESTERLE, JR.

Harry A. Oesterle, Jr., died on May 24, 1926, at Niquero, Cuba, as a result of a premature dynamite explosion. He was born in Newark

N. J., December 23, 1892, and was educated at the Newark Technical School.

For about a year Mr. Oesterle was connected with the Research Corporation of New York, designing electrical precipitators, and with the Guarantee Construction Company dealing with general plant construction and ash-handling systems. From December, 1914, to May, 1915, he was engaged on the design and construction of an electrical precipitator for copper blast-furnace fumes for the Balbach Smelting and Refining Company, Newark, N. J. From May, 1915, to December, 1916, he was a designing draftsman on power-house work with Westinghouse, Church, Kerr and Company. Since 1916 he had been resident engineer in charge of construction and assistant in administration for the New Niquero Sugar Company, Niquero, Cuba.

Mr. Oesterle became an associate member of the A.S.M.E. in 1922 He was also a member of the American Society of Civil Engineers, and the American National Engineering Societies of Cuba.

JOHN F. O'NEIL

John F. O'Neil, former president of the Fulton Iron Works, died at his home in St. Louis, December 3, 1926. He was born at Brockport, N. Y., April 23, 1857. At the age of 22 he entered the employ of the Dubuque Iron works and remained there for several years, going from there to St. Louis to become superintendent of the Fulton Iron Works. He attained the position of president of the company in 1905. Mr. O'Neil was the originator of many improvements in sugar-mill machinery. He retired from active business in 1917, and had been a member of the A.S.M.E. since 1908.

JOHN OVERN

John Overn, manager of the Dominion Engineering Works, Ltd., of Montreal, Canada, died at Ocean City, N. J., on July 2, 1926.

Mr. Overn was born at Philadelphia in October, 1868. He was educated in the public schools of Philadelphia and served his apprenticeship at the Baldwin Locomotive Co. Later, he was in charge of the pattern shop of the Barr Pump Engine Co. From 1898 to 1923 he was employed in the I. P. Morris Department of William Cramp and Sons in charge of the department which produced the hydraulic machinery.

Mr. Overn became a member of the A.S.M.E. in 1914. He was a member of the Masonic Order and of many other organizations, including the Union League Club of Philadelphia.

WILLIAM BRECKENRIDGE REED

William Breckenridge Reed, vice-president of the H. B. Smith Company of Westfield, Mass., died on August 4, 1926, at Edgartown, Mass., where he had been for a few weeks, seemingly recovering from an illness which had kept him at home for several months.

Mr. Reed was born on March 3, 1869, at Westfield, Mass. After being graduated from the Westfield High School he attended the Massachusetts Institute of Technology for two years, leaving school to enter the employ of the H. B. Smith Company, with which company he remained up to the time of his death, advancing step by step with it and becoming its vice-president in 1918. He knew the heating business as do few men.

Mr. Reed was intensely interested in the affairs of his home town and quietly did many things for it. He was general chairman of the many committees that planned the 250th anniversary celebration of Westfield Academy. His latest public service to Westfield was his active interest in the construction of the Whitney Library building. For this project he gave not only ideas and directions, but raised much of the money to make possible a structure monumental for Westfield.

Mr. W. B. Reed became a member of the A.S.M.E. in 1921.

MAXWELL L. REIDINGER

Maxwell L. Reidinger, methods and equipment engineer with the Singer Manufacturing Co., Elizabethport, N. J., and a prolific inventor of needle-making machinery, died in Elizabeth, N. J., on April 12, 1926.

Mr. Reidinger was born at Elizabeth on December 23, 1870, was educated in the public schools there, and spent three years in college in the study of mechanical drawing and machine design.

He joined the Singer Manufacturing Co. about forty years ago, spending two years on production work and then being transferred to the tool room. In 1895 he was transferred to the needle-making department, where his unusual ingenuity and mechanical ability attracted immediate attention.

Some of Mr. Reidinger's outstanding inventions were: a machine for packing twelve needles at a time in a waxed envelope, wrapping this in tin foil and enclosing this in turn in an outer envelope; an automatic needle threading machine which runs polishing cord through the eyes of 224 needles per second; an automatic eye-piercing machine which punches 150 needles per minute; an automatic needle straightening machine, which does its work by the taps of a light hammer controlled through a sensitive indicator by the crook in the needle; and an automatic pointing machine, which points 350 needles per minute.

Mr. Reidinger was a veteran of the Spanish-American War, serving with Company C, Third New Jersey Volunteers. He joined the A.S.M.E. in 1921.

R. SANFORD RILEY

R. Sanford Riley, combustion engineer of international repute, who was the founder of the Riley Stoker Corpn. and president of this and its several subsidiary companies, died at his home in Worcester, Mass., on May 6, 1926.

Mr. Riley was born at Hamilton, Ont., on July 26, 1874. He attended the public schools of Hamilton and, after a preparatory course at the Collegiate Institute at Winnipeg, he entered Worcester Polytechnic Institute. He played center on the football team, was editor of the class book, became president of his class and in 1896 graduated third in his class and with the degree of B.S.

While still in school Mr. Riley began his practical training as an apprentice machinist in the Canadian shops of the Northern Pacific Railroad, during the summers of 1890 to 1893. After graduation he spent a year at Atlanta University, then returned to Worcester Polytechnic Institute for a year as instructor in woodworking at the Washburn Shops.

After a short period as machinist at the Baldwin Locomotive Works, Mr. Riley became a draftsman at Cramp's Shipyard. In 1899 and 1900 he was in Winnipeg as foreman in the shops of the Canadian Pacific Railroad.

In 1900 he began a trip around the world. He set forth from Cramp's shipyard on a ship which he had helped to design, crossed the Pacific

on a ship of the Empress Line, and at Hong Kong joined the United States Navy. He served through the Boxer Rebellion and was a chief engineer with the Asiatic Squadron for a year and a half. He finally returned home by way of the Indian Ocean as chief engineer of the naval auxiliary *Arethusa*.

From his return in 1903 until 1905 Mr. Riley served as assistant to the chief engineer of the New York Shipbuilding Co. Following this he was for a year mechanical engineer for the Union Metallic Cartridge Co. In 1906 he became manager of the American Ship Windlass Co. at Providence, R. I.

He began work upon the design of hoisting and steering mechanisms, in which his naval training stood him in good stead. Soon he turned his attention to the improvement of the Taylor stoker, which had been left with the concern in rather crude form upon the death of its inventor. With his wide fireroom experience Mr. Riley had a thorough knowledge of combustion and was able successfully to redesign the Taylor stoker and to build up a large business by it. Thus he became known as a pioneer in the commercial development of high-capacity underfeed stokers.

In 1911 Mr. Riley sold out the interests which he had acquired in the American Ship Windlass Co. He then organized at Worcester the Sanford Riley Stoker Co. for the manufacture of the Riley self-dumping underfeed stoker which he had invented. The success of this concern was assured by the ability and the dynamic personality of its founder and its growth was rapid.

During the World War Mr. Riley was called upon to organize the testing and performance department for the Emergency Fleet Corpn. He was one of the leading citizens of Worcester and as president of its Chamber of Commerce gave a splendid demonstration of what an engineer can do in civic affairs. He was a trustee and director of two Worcester banks and was a director of the Norton Company.

Mr. Riley loved the sea, he delighted in horseback riding, and was an ardent mountaineer, having climbed some of the highest of the Swiss Alps and Canadian Rockies. He belonged to the Appalachian Club and the Alpine Club of America. He was also a member of the Engineers' Clubs of Boston and New York, and the Society of Naval Architects and Marine Engineers.

Mr. Riley's active interest in the A.S.M.E. dated back to 1906 when he became a member. Since that time he had served with distinction upon many of the Society's committees and had contributed many valuable papers.

C. C. ROBBINS

C. C. Robbins, general manager and secretary of The Cleveland Crane & Engineering Co., of Wickliffe, Ohio, died on March 23, 1926, as a result of an automobile accident the evening before.

Mr. Robbins was born in Cleveland December 16, 1877, and was educated in the elementary schools there. At the age of 18 he began an apprenticeship at the Cleveland Punch & Shear Works as a pattern-maker. He gained machine shop and drafting experience at the same works.

Leaving there in 1896, Mr. Robbins worked for a time with the Cleveland Shipbuilding Co., Reid & Warren, W. C. Thayer, consulting engineer, The Cleveland Machine Screw Co., and the Kilby Mfg. Co. He then was connected with the Julian Kennedy organization in Pittsburgh and with the Kelley-Jones Co. at Greensburg, Pa.

In 1904 Mr. Robbins returned to Cleveland as manager of the Cleveland Machine Co. While there he was invited to assume direction of

The Cleveland Crane & Engineering Co. Under his direction this concern grew rapidly and is now very well known in its field.

During the War Mr. Robbins worked tirelessly upon munitions orders in addition to the many heavy-duty cranes for navy and ordnance purposes. His organization produced a very large number of shell forgings during the war period.

In addition to his managership of The Cleveland Crane & Engineering Co. Mr. Robbins served as treasurer of the Clark Chemical Co. of Wickliffe, and director of the First National Bank of Willoughby, Ohio. He joined the A.S M.E. in 1900.

EDWIN W. ROBINSON

Edwin W. Robinson, president of the Punxsutawney Foundry and Machine Co. at Punxsutawney, Pa., died on March 6, 1926, at the age of 60 years. Directly after graduation from Yale University in 1888 he was employed by the Rochester Mines at DuBois, Pa. as assistant engineer. A year later he became associated with the Mexican Central Railway in the same capacity. As an employee of the Rochester & Pittsburgh Coal and Iron Co. from 1891 to 1901, he was engaged in designing and erecting coke-oven plants. A year was spent in Australia with Bewick, Moreing & Co. examining various coal and iron mines. In 1904 Mr. Robinson became president and general manager of the Punxsutawney Foundry and Machinery Co. He held this position until his death. He had held membership in the A.S.M.E. since 1916.

CHARLES B. ROWLAND

Charles B. Rowland, a member of the A.S.M.E. since 1885, and vice-president of the Continental Iron Works, died at his home in Greenwich, Conn., on November 1, 1926. He was born in the Greenpoint section of Brooklyn, N. Y., on May 22, 1863, and received his early education there. He was a graduate civil engineer of the class of 1884 of the School of Mines of Columbia University. Immediately after his graduation he entered the Continental Iron Works, which had been carried on by his father for many years, and began his career in the machine shop. He followed this by several years' experience in the drafting room and later became vice-president of the company, which position he held at the time of his death. He exercised a general supervision over the firm's activities, being particularly connected with the building of illuminating-gas-producing plants, supervising the construction of refrigerating machinery, and also that of special machinery used in the production of paper pulp.

Mr. Rowland was also a member of the American Society of Civil Engineers, the Society of Naval Architects and Marine Engineers, The Franklin Institute of Philadelphia, and other organization of prominence.

VERNON LE ROY RUPP

Vernon Le Roy Rupp, superintendent of The Williamson Trade School, died suddenly on August 11, 1926. Mr. Rupp was born in October, 1878, and his early education was obtained in Harrisburg, Pa. Following graduation from The Williamson Free Trade School, he was appointed assistant on the instructing staff and was intimately associated with the work there throughout his life. He also served a considerable time as teacher of power-plant engineering at the Y. M. C. A.

Mr. Rupp was a member of the Engineers' Club of Philadelphia, the American Association of Steam Engineers, and the National Association of Stationary Engineers. He became an associate member of the A.S.M.E. in 1922.

GUSTAVE A. SACCHI

Gustave A. Sacchi, manager of the Stoker Sales Dept. of the Westinghouse Electric and Manufacturing Co., died suddenly on November 4, 1926, at his home in Lansdowne, Pa. He was born in New York, N. Y., in October, 1878, and received his early school training there.

Mr Sacchi virtually devoted his entire active life to the Westinghouse organization with which he became associated in 1893, when it was known as Westinghouse, Church, Kerr & Co. He was gradually promoted in the Westinghouse Machine Co. until he was made district engineer. In 1909 he engaged in the sale of stokers and large turbines, and was appointed manager of the stoker sales department in 1918.

In April, 1926, Mr. Sacchi was elected president of the Stoker Manufacturers' Association, which honor he held until the time of his death. He became a member of the A.S.M.E. in 1917. He was also a member of the Pennsylvania Electric Association and the Engineers' Club of Philadelphia.

JOHN R. SAYLOR

John R. Saylor was born at Parkerford, Pa., on February 4, 1862. His trade training as machinist was acquired at the Ellis Keystone Agricultural Works at Pottstown, Pa., after which he began a service of fifteen years with the Baldwin Locomotive Works at Philadelphia. In 1893 he had charge of erecting the Baldwin exhibit at the World's Fair at Chicago. Under the name of the Pottstown Machine Company he started in business for himself. This business flourished from 1895 to 1926 when it was incorporated. He was elected a member of the A.S.M.E. in August, 1917. He died at his home in Pottstown on October 21, 1926.

CHARLES R. SCHMIDT

Charles R. Schmidt, retired mechanical engineer and an early advocate of modern methods of plumbing, died at Baltimore on March 31, 1926.

Mr. Schmidt was born on November 3, 1855, in Tanna, a small town in Germany. After serving some time in Germany as a machinist's apprentice he came to the United States in 1871. While working during the day as an instrument maker with A. Schmidt & Bros. of New York, he continued his studies in the evening. In 1873 he went to the Kensington Iron Works, Philadelphia, as a draftsman, and here he studied evenings at the Franklin Institute. After about one year he joined the Plumbers' Metal Works of the same city, and in ten years rose to be manager.

Mr. Schmidt then went to Baltimore, where for six years he was a partner in the Baltimore Sheet Metal Works. From 1894 to 1895 he was general manager of the Henry McShane Mfg. Co., from 1895 to 1900 proprietor of the Lansdale Iron Works, Lansdale, Pa., and later he was vice-president in charge of plants of the Central Foundry Co. of New York City.

Mr. Schmidt retired in 1910 and since that time had done much travelling. He became a member of the A.S.M.E. in 1895.

PETER S. SCHOTT

Peter S. Schott, assistant engineer with the Roessler and Hasslacher Chemical Co., located in Perth Amboy, N. J., died in that city on October 30, 1926. Mr. Schott was born in Manchester, England, on March 23, 1885, but received his education in Frankfurt, Germany. He also attended the Zurich University in Switzerland, where he received his degree for electrical and mechanical engineering. He held one position as apprentice before he came to America and that was in Saint Quentin, France.

In 1912 Mr. Schott came to the United States and entered the employ of Roessler & Hasslacher Chemical Co. He was in the employ of this firm at the time of his death. He specialized mainly in power-plant installations He was elected a member of the A.S.M.E. in November, 1919.

GEORGE N. SCOTT

George N. Scott, marine engineer and naval architect, died at Saranac Lake, N. Y., on March 7, 1926, after a lingering illness.

Mr. Scott was born at Glasgow, Scotland, on March 26, 1883. He received his technical education at the Glasgow and West of Scotland Technical College His practical work began in 1898 when he began a four-year apprenticeship upon the design and construction of engines with Campbell, Binnie & Co. of Glasgow. This was followed, from 1902 to 1905, with work with Alex Stephen & Sons of the same city upon ship and engine building.

Mr. Scott went to sea in 1905 and attained the rank of chief engineer. He came to the United States in 1912, becoming outside manager of ship repairs for the Washington Engine Works of New York. Here he began to specialize upon oil-burning equipment. From 1915 to 1917 he was chief engineer for the Chester Shipbuilding Co. From 1917 until his health failed Mr. Scott was chief engineer for the Standard Shipbuilding Corpn., Shooter's Island, N. Y.

Mr Scott was a member of the Society of Marine Engineers and Naval Architects. He joined the A.S.M.E. in 1919.

JOHN FREW SCOTT

John Frew Scott died at his home in Pittsburgh, Pa., on November 1, 1926. Mr. Scott had been in the real-estate business with his father for about a year. Previous to this time he was employed by the Duquesne Light Co. of Pittsburgh to do power-plant supervision work.

Mr. Scott was born in Pittsburgh, May 23, 1897, and attended the public schools of that city. He entered Lafayette College with the class of 1919, but left in the spring of 1918 to enter the Navy. He was a junior member of the A.S.M.E. since 1921 and was also a member of the Engineers' Society of Western Pennsylvania.

CLINTON HOMER SCOVELL

Clinton Homer Scovell was born in Manchester, N. H., July 15, 1876. He attended the public schools in Manchester, and later went to Harvard, being graduated with the class of 1903. He was awarded the C.P.A. degree in New York and Massachusetts. From 1904 to 1906 he was employed in the accounting departments of the Great Northern Railroad and the United States Smelting and Refining Co. In 1907 and 1908 he was connected with the accounting firm of Gunn, Richards & Co. and later was a partner in the firm of Harvey S. Chase & Co. of

Boston. In 1910 he organized Clinton H. Scovell & Co. which was later called the Scovell, Wellington Co., of which Mr. Scovell was president. During the World War, Mr. Scovell was associated with General Goethals in the reorganization of the Quartermaster's Department. His chief professional interests were along the lines of cost accounting.

Mr. Scovell was a director of the National Association of Cost Accountants from 1920 to 1925, at which time he became president. He held membership in the Boston School of Physical Education, the American Institute of Accounts, the Society of Industrial Engineers, the University and Engineers' Clubs of Boston, the Harvard Clubs of Boston and New York, and the Chambers of Commerce of Boston and the U. S. A. He became an associate member of the A.S.M.E. in 1915. He died in Newton, Mass., December 31, 1926.

ROBERT L. SIMPSON

Robert L. Simpson died on July 31, 1926. He was born in Columbus, Miss., on August 14, 1901, and received his education at the Central High School and the Alabama Polytechnic Institute, from which he was graduated in May, 1924. Upon leaving the institute he applied for junior membership in the A.S.M.E. and a few months later entered the graduate apprenticeship course of the Allis-Chalmers Co. of Milwaukee, Wis., where he was employed at the time of his death.

WILLIAM W. SIVEWRIGHT

William W. Sivewright, well known as an engineer upon tropical machinery such as that used in sugar, coffee, and cocoa production, died at San Domingo City on February 22, 1926. He left New York a few weeks before on a business trip to Porto Rico and San Domingo and was stricken with typhoid fever.

Mr. Sivewright was born in London, England, on June 18, 1882, and his technical education was received in Scotland at the Glasgow & Western Scotland Technical College. In 1901 he began work as a draftsman on sugar-mill machinery at the works of A. & P. W. McOnie in Glasgow. After three years with them he was successively draftsman and engineer upon similar lines for Blair, Campbell & McLean, Ltd., Duncan, Stewart & Co., and Mirrlees, Watson Co., all of Glasgow.

Mr. Sivewright came to the United States in 1906 and from that time until 1912 he was connected with the Kilby Manufacturing Co. of Cleveland, the Honolulu Iron Works Co., and the Marcus Mason Co. of Boston. With all of these concerns he was mechanical engineer upon sugar machinery and allied lines.

In 1912 Mr. Sivewright organized the Angus Engineering Co., Inc., at New York. Since that time he had served as president, and as consulting and sales engineer of this firm which has designed and equipped many of the largest sugar and coffee plants in the West Indies and Central America. In addition to his consulting work with this firm Mr. Sivewright since 1922 had been manager of the New York office of the Farrel Foundry & Machine Co. of Ansonia, Conn.

Mr. Sivewright became a member of the A.S.M.E. in 1921.

OBERLIN SMITH

Oberlin Smith, President of The American Society of Mechanical Engineers in 1890, passed away early in the morning, July 19, 1926, at his home, Lochwold, Bridgeton, N. J., following an attack of heart failure. He was the last of a notable group of fifteen engineers serving

as presidents of the Society between the time of its founding in 1880 and the year 1897. Active up to three days before his death, the eminent inventor and successful manufacturer had passed his eighty-sixth birthday and had rounded out sixty-three years of devotion to a rather unique business career in the little city of Bridgeton.

He was born in Cincinnati, Ohio, on March 22, 1840. When he was very young the family moved from Cincinnati to Bridgeton, N. J. He attended the public schools, and during vacations worked on farms and also learned carpentry. His education was continued at the West Jersey Academy, and his technical training began at the Philadelphia Polytechnic Institute. This latter was of a broad and practical nature. To use his own words in speaking of the period from 1856 to 1861, he ". . . learned drafting, patternmaking, gas and steam fitting, blacksmithing, architectural ironwork, machine work, die making, etc., with schooling mixed in." The "schooling" here referred to was largely home study undertaken on his own initiative.

In the early days of the Civil War Mr. Smith became particularly interested in the die working of metals and in the design of dies and presses for this purpose. This interest crystallized in 1863 when he established at Bridgeton a concern for the manufacture of improved dies and presses which he had invented. This business, which in 1877 was incorporated as the Ferracute Machine Company, had been a powerful factor in the commercial development of the die working of metals. Its influence has extended from the minting of Chinese money to the mass production of automotive products.

In the sixty-three years of Oberlin Smith's continuous service as mechanical engineer and president of this concern, he became widely known as an authority on presswork. During this time he designed and built more than five hundred types of presses, and obtained more than fifty patents. For the Atlantic Refining Company and other Standard Oil concerns he worked out in the early eighties can-making devices which were responsible in no small way for the development of the package oil trade with the Far East. In his paper on Shop Management read before the Society in 1903, Frederick W. Taylor gave Mr. Smith credit for the mnemonic symbol system which is such a characteristic and important feature of scientific management.

While most of Mr. Smith's inventions related to presswork, there were several in entirely outside fields. Among them were such widely divergent lines as improved looms, dump carts, keyless locks, and egg boilers. In 1883 he achieved considerable publicity through the invention of a magneto-electric phonograph, and this same publicity prevented him from obtaining a patent upon it. Citizens of Bridgeton bear witness to Mr. Smith's having frequently driven through the streets of that town in a motor-propelled vehicle long before the days of automobiles.

Mr. Smith was a prolific writer and lecturer, his works in this direction covering science, fiction, and even theology. Two of his books on widely different subjects were Press Working of Metals, and The Material, Why Not Immortal? His active physical and mental characteristics were reflected in his favorite recreations, which included swimming, rowing, motoring, dancing, and golf, most of which he entered into with enthusiasm to the last of his life.

Some years ago he equipped his phonograph with a mechanism by which records could be changed automatically. In his last years, when his sight was failing, through this means he could sit in his favorite chair and by pressing buttons, with the location of which he had become thoroughly familiar, change the tune being played on an instrument in a nearby room.

His intellectual interests not only covered a very wide range, but he espoused many of them with the utmost vivacity. He was particularly given to astronomical discussions. Mrs. Smith tells of bringing such a debate which Mr. Smith was enjoying with a house guest to an end at one o'clock on a Sunday morning. To her surprise and amusement at six-thirty the next morning she heard the resumption of the heated discussion. Mr. Smith had gotten into bed with his guest and the two were hard at it once more!

It was natural that Oberlin Smith should have been one of the early and active members of The American Society of Mechanical Engineers. He joined the Society in 1881, within a year of its founding. He served as a Manager from 1883 to 1886, and in 1889 succeeded the late Henry R. Towne as the ninth President of the Society. In 1901 his prominence in the engineering field led to his appointment as New Jersey Commissioner to the Pan-American Exposition in Buffalo. He made several European tours for purposes of engineering observation. His acquaintanceship was very wide; among his intimate friends he numbered such well-known national characters as Thomas A. Edison and Henry Ford.

Mr. Smith belonged to numerous clubs and societies. In addition to the A.S.M.E., he was a member of the American Institute of Mining and Metallurgical Engineers, the American Institute of Electrical Engineers, the American Society of Civil Engineers, the American Iron and Steel Institute, the Franklin Institute, the American Association for the Advancement of Science, and the Engineers' Clubs of both New York and Philadelphia. He also belonged to the American Automobile Association, the Philadelphia Art Club, The Atlantic Union, the New York Lotus Club, the Luther Burbank Society, the National Academy of Political and Social Science, the National Geographic Society, and the Advisory Council of the Simplified Spelling Board, and had served as vice-president of the Men's League for Woman Suffrage.

JAMES M. SNODGRASS

James M. Snodgrass, professor of railway engineering and member of the faculty of the University of Illinois, Urbana, Ill., for 24 years, died on December 4, 1926. Professor Snodgrass was appointed as instructor in the department of mechanical engineering in 1902. He held this position until 1906, when the department of railway engineering was formed. He was then transferred as an instructor in the newly-organized department. In 1906 he resigned to accept a position with the American Locomotive Co. He also held positions with the Illinois Central Railroad, and the C. C. C. and St. L. Railroad.

Professor Snodgrass returned in 1908 as an associate in mechanical engineering, and in 1909 was made an assistant professor in the department. In 1910, he accepted an assistant professorship in the department of railway engineering, and retained that position until 1918, when he was made associate professor. The following year he was appointed as a full professor, which position he held until the time of his death.

He was born in Coldspring, Wis., September 1, 1874. He was educated in Urbana, and entered the University of Illinois in 1898, receiving his degree in 1902. He had been a member of the A.S.M.E. since 1904.

JAMES B. STANWOOD

James B. Stanwood was born in Arlington, Mass., in December, 1855, and died at his home in Cincinnati on October 22, 1926. Mr. Stanwood the inventor of numerous improvements on steam engines and boilers, was educated at M. I. T., from which he was graduated with the class

of 1875 His apprenticeship was served at Lane & Bodley Co. of Cincinnati, where he remained until 1886 in the position of mechanical engineer. Later he was employed by the Ice Machine Co. of Cleveland. Until 1924 he was vice-president and consulting engineer of the Houston, Stanwood, and Gamble Co., at which time it merged to become the Stanwood Corporation.

During his 37 years of membership in the A S.M.E. he was very active, being a manager from 1897 to 1900, a member of the Nominating Committee in 1906, and vice-chairman of the Cincinnati Local Section in 1913-1914 Mr. Stanwood was also a member of the American Society of Heating and Ventilating Engineers and the Engineers' Club of Cincinnati.

JULIAN B. STRAUSS

Julian B. Strauss, consulting engineer, died in New York on January 25, 1926.

Mr. Strauss was born at Heilbrown, Germany, in February, 1878. After attending the public schools he entered the shop of Schwartz & Co as a machinist apprentice. This concern built sugar machinery and also did general repair work on heavy mechanical equipment.

He became an expert upon the building and equipping of sugar mills, and as such joined the firm of Hauptman & Loeb Co. Here, between 1902 and 1907, he was successively machinist, foreman and general manager.

After coming to the United States, Mr. Strauss served as consulting engineer upon many large sugar-mill, canning-plant, and drainage projects and he specialized upon conveying machinery. This work took him to New Orleans, Sidney, Australia, Buenos Aires, and Germany at various times.

Mr. Strauss had been a member of the A.S.M.E. since 1910.

ALLAN CLYDE SULLIVAN

Allan Clyde Sullivan, who at the time of his death on May 28, 1926, was northwest manager of the Chain Belt Co. at Portland, Ore.; was born in Genoa, Ohio, on June 4, 1881. After being graduated from the University of Washington in 1906 he was employed by the Allis-Chalmers Manufacturing Co. at Milwaukee for 12 years, first as a student apprentice and later as draftsman, salesman, and engineer. In 1912 he became chief engineer and assistant manager of the Smith and Watson Iron Works and in 1921 mechanical engineer for the Mountain Timber Co. About July, 1923, he became affiliated with the Pacific Power and Light Co. as mechanical engineer. He left their employ in November, 1923, to join the Chain Belt Co. as northwest manager.

He was made an associate member of the A.S.M.E. in March, 1918, and promoted to the grade of member in 1924.

GUY C. SYLVESTER

Guy C. Sylvester, engineer of the Link Belt Pacific Co. of Los Angeles, Cal., died on July 16, 1926. After attending the University of Pennsylvania, he became staff engineer in charge of the mechanical-testing department of the Midvale Steel Co., Philadelphia, Pa. In 1913 he became associated with E. S. Cobb, a consulting engineer of Los Angeles. From 1915 up until the time of his death Mr. Sylvester was connected with the Link Belt Pacific Co., holding the positions of sales engineer, local manager, and engineer. He was elected a member of the A.S.M.E. in March, 1924.

MERL N. TABER

Merl N. Taber, metallurgist with the National Supply Co., Toledo, Ohio, died of pneumonia at Houston, Texas, on April 13, 1926.

Mr. Taber was born at Bowling Green, Ohio, on October 24, 1889 He was graduated at the Toledo High School, after which he entered the University of Michigan, where he studied metallurgical engineering He was graduated there in 1913 with the degree of Bachelor of Chemical Engineering.

He immediately entered the employ of the National Supply Co., large manufacturers of oil-well supplies at Toledo, as assistant metallurgist and chemical engineer. Within about a year he was placed in full charge of these departments.

In 1923 he was put upon research work and this carried him to practically every oil field in the United States. It was upon one of these trips that he contracted his fatal illness.

Mr. Taber was a member of the American Chemical Society, the American Institute of Mining and Metallurgical Engineers, the American Institute of Metals and the American Society for Testing Materials. He joined the A.S.M.E. in 1917.

STEVENSON TAYLOR

Stevenson Taylor, president of the American Bureau of Shipping, died at his home in New York on May 19, 1926. He was seventy-eight years old, having been born in New York City on February 12, 1848. He was educated in the public schools of New York, including a period at the Free Academy, now the College of the City of New York.

At the age of sixteen Mr. Taylor became an apprentice with Fletcher, Harrison and Company, North River Iron Works, New York City, where he learned the trade of machinist. From 1868 to 1883 he was chief draftsman of the Fletcher, Harrison and Company and their successors, the W. and A. Fletcher Company, during which time the firm constructed the machinery for many famous river, sound, and lake steamers. In 1881 he prepared the plans and specifications for the Fall River steamer *Pilgrim*, one of the first steamers to use electric lighting. The installation was made under Mr. Taylor's supervision, and consisted of 869 lamps, by far the largest equipment installed on any vessel or in any place on shore up to that time. In 1883 he became vice-president and responsible technical head of the W. and A. Fletcher Company, a position which he held until 1904.

In 1904, Mr. Taylor disposed of his interest in the W. and A. Fletcher Company, being appointed receiver of the United States Shipbuilding Company to appraise the property and yards of the concern. Upon completion of this service in July, Mr. Taylor joined the Quintard Iron Works as vice-president, a position which he held until 1914.

When William H. Webb founded the Webb Academy and Home for Shipbuilders in New York City in 1889, Mr. Taylor was made a charter member of the board of trustees, and upon the death of Mr. Webb in 1899 he became president of the board. He was president of the American Bureau of Shipping from 1916 to 1926.

Mr. Taylor was an original member of the Society of Naval Architects and Marine Engineers. He served two terms as president, 1909 to 1912, and 1915 to 1918. He became a member of the A.S.M.E. in 1886, and served as vice-president from 1899 to 1901.

On March 30, 1917, he was appointed to the Board of Appraisal for Merchant and Private Vessels for the United States Navy. On April 2, 1917, he was appointed Lieutenant-Commander, U. S. N. R. F., and on April 7 he was appointed by the United States Shipping Board to be

a member of the Board of Surveys and Consulting Engineers to take charge of German steamers here. President Wilson appointed him on July 2, 1917, as a member of the board to determine just compensation for the owners of vessels commandeered by the United States Government.

JAMES W. THOMAS

James W. Thomas, engineer with the United Gas & Electric Engineering Corpn., died at his home in Plandome, Long Island, on April 13, 1926.

Mr. Thomas, who was born at Boston on February 13, 1874, attended Exeter Academy and was graduated from the Massachusetts Institute of Technology in 1895, with the degree of B.S. Upon his graduation he entered upon a machinist's apprenticeship in the Cambridge, Mass., shops of the Boston & Maine Railroad. From there he went to the Wyoming Shovel Works at Wyoming, Pa., where he served as assistant manager and superintendent for seven years.

He resigned to join the Westinghouse, Church, Kerr Co. and was given charge of many important construction projects, including the Westinghouse Lamp Works at Watsessing, N. J., and the Westinghouse Stoker Works at Attica, N. Y. Before joining the United Gas & Electric Engineering Corpn., Mr. Thomas spent a short time as engineer with the New England Engineering Co.

Mr. Thomas, who was a Fellow of the American Institute of Electrical Engineers, a member of the Technology Club of New York, and a member of Phi Beta Epsilon, Boston, joined the A.S.M.E., in 1905.

WILLIAM THORNE

William Thorne, engineer of William Sellers & Co. of Philadelphia, died on February 23, 1926, after an absence from his work of only six days. He was born in Philadelphia on January 28, 1847, and educated at the private school of the Society of Friends, North West Grammar School, and Central High School, from which he was graduated in 1863.

His apprenticeship was served with the Delaware River Ship and Engine Building Co. and between the years 1867 and 1877 he was associated with the following firms: Neafie & Levy, William Cramp and Sons, Bement & Daugherty, Richards, Thorne & Co., and Ferris & Miles, all of Philadelphia. He also founded Thorne & DeHaven. In 1878 he was employed by Wm. Sellers Co., which position he held until his death.

Mr. Thorne was director of the evening school of Mechanical Arts of the Franklin Institute and professor of mechanical drawing. He published several books on mechanical drawing which have been used by many engineering schools including the U. S. Military Academy. In 1874 Thorne, DeHaven & Co. were awarded a medal by the Franklin Institute for Mr. Thorne's portable drills. In 1876 the same machines were awarded a medal by the U. S. Centennial Commission. He was a member of the Committee on Science and the Arts of the Franklin Institute for almost fifty years and a member of the A.S.M.E. since 1884.

FREDERIC THUMAN

Frederic Thuman, a life member of the A.S.M.E., died in May, 1926. He was born near Evansville, Ind., February 7, 1867. After completing a four-years' apprenticeship with a firm of engineers and mill-wrights, he entered Stevens Institute of Technology, being graduated in 1890 with the degree of Mechanical Engineer.

Mr. Thuman was then for a short period employed in the Standard Oil Company's Atlantic Refinery in Philadelphia. From 1890 to 1892, he was employed by the United Gas Improvement Company of Philadelphia in various capacities and particularly in drafting and erecting water-gas apparatus. He was employed by the London firm of Humphreys and Glasgow in 1892, at which time this firm was organized. At first he was engaged in designing and erecting water-gas plants and in tests looking to the increase in economy of these water-gas plants and particularly in meeting European conditions.

In 1894 he permanently located in the London office of Humphreys and Glasgow as manager of the construction department and principal assistant to Arthur G Glasgow. Later he became chief engineer of the London office. He developed a number of patents covering improvements in carburetted water-gas apparatus During the last few years, his health having definitely failed, he had retired from business and had returned to his family in the United States.

WILLIAM L. TOBEY

William L. Tobey was born in Worcester, Mass., November 4, 1860, and died in Needham, Mass., on October 24, 1926. He was graduated from the Worcester Polytechnic Institute in 1879 and immediately entered the employ of the Atlantic Works in East Boston. After some years he changed to affiliate with the Lockwood Manufacturing Co., also of Boston, where he remained until 1912. At this time he entered into business for himself, under the name of the Thermex Silencer Works, which was located in Boston. He remained in this work for about five years, devoting himself to manufacturing and selling his own inventions. In 1917 he joined the Boston Engineering Co. and remained there until 1921, at which time he retired from active business except for consulting work. He became a member of the A.S.M.E. in 1890.

W. J. TOLLERTON

W. J. Tollerton, general superintendent of motive power of the Rock Island Line of the Chicago, Rock Island and Pacific Railway Co., died at his home in Chicago, Ill., on March 3, 1926.

Mr. Tollerton was born in St. Paul, Minn., on January 2, 1870, and after a high-school education, began his railroad career as a machinist apprentice with the Northern Pacific Railway. He later became a fireman on the Chicago, St. Paul, Minneapolis and Omaha Railway. In 1890, at the age of 20 years, he was made foreman in the Union Pacific shops. Six years later he was master mechanic on the Utah Division of the Oregon Short Line and was later transferred to Pocatello, Idaho, in the same capacity. He came to the Rock Island Lines in 1906 as superintendent of motive power in charge of the lines west of the Mississippi River. In 1907 he was transferred to Chicago as assistant general superintendent of motive power, where he was promoted to mechanical superintendent in 1912. He was made general superintendent of motive power of the Rock Island Lines in 1913, which position he held until his death.

Mr. Tollerton was a prominent member of the American Railway Master Mechanics Association and was elected president in 1919. In 1922 he was a delegate to the International Railway Congress held in Rome, Italy. He became a member of the A.S.M.E. in 1915.

HAZEN G. TYLER

Hazen Greeley Tyler, professor of experimental engineering in the College of Engineering, and director of the Evening Engineering Division, New York University, died on October 27, 1926, following an operation. His death brought to a close a life filled with enthusiastic activity in the teaching profession, to which he brought a point of view whetted by close contact with engineering practice and an unusual amount of administrative ability. His untiring efforts in behalf of its College of Engineering place him deep in the gratitude and respect of New York University. Coming to it as a vigorous young man and at a time expansion of the College of Engineering was beginning, Dr. Tyler joined with Professor Bliss in the work of providing the Sage Laboratory with its equipment and in laying out the plans for the building. He lived to see the first two of a group of fine engineering buildings completed, and to mold the character of the work done in them by undergraduate engineering classes and research workers. He was largely responsible for the work done by the Popular Science Institute of Standards. When the Evening Engineering Division of New York University was established in 1922, Dr. Tyler was chosen to organize and administer it.

Dr. Tyler was born on March 21, 1890, in Brooklyn, New York. There he attended the public schools, and the Polytechnic Institute, from which he was graduated, in 1911, with the degree of E.E. Remaining at the Institute as an assistant in mechanical engineering from 1911 to 1916, he received the degrees of M.S. in 1912 and M.E. in 1913. During this period he also studied at New York University, from which he received the degree of Sc.D. in 1916. In 1916 he went for a year to the Rensselaer Polytechnic Institute, Troy, New York, as an instructor in mechanical engineering.

In October, 1917, Dr. Tyler went to New York University as assistant professor of mechanical engineering. In 1921 he was made associate professor of mechanical engineering. In 1924 he was made associate professor of experimental engineering, and in 1926 professor of experimental engineering. He was a member of the A.I.E.E., the S.P.E.E., and the American Physical Society.

Dr. Tyler's professional career has been closely linked with the A.S.M.E. From an enthusiastic member of the Student Branch at Brooklyn Polytechnic Institute, he became a junior member of the Society in 1913; was promoted to associate-member in 1918, and to member in 1925.

FRANK F. VATER

Frank F. Vater, power plant engineer in Chicago, was instantly killed in an automobile accident near Milwaukee on January 9, 1926.

Mr. Vater was born in Indianapolis, Ind., on December 24, 1859. His education was obtained in the public schools of Indianapolis, augmented by courses in night schools, while employed during four years as a bank clerk.

In 1878 Mr. Vater began his engineering work as a draftsman at the Atlas Engine Works. After a year on the board he was appointed a salesman by that company. He left in 1886 to open a machinery agency of his own at Minneapolis, where he also carried on consulting engineering work. He remained there about ten years. During that period he designed and equipped many water and electric light plants, including steam machinery and pole lines.

In 1898 Mr. Vater worked with the Westinghouse, Church, Kerr Co. upon the Municipal Lighting plant at Owensboro, Ky., which contract was secured by his efforts.

Following this he was a salesman at the Chicago office of the Harrison Safety Boiler Works.

Mr. Vater in 1904 organized the Power Plant Specialties Co., at Chicago, of which concern he was president at the time of his death He designed oil separators and water separators for steam plants, feedwater heaters, water softeners, filters, etc. In fact, he specialized in water-treatment apparatus, and designed and built several large installations for industrial plants and cities.

Mr. Vater became a member of the A.S.M.E. in 1919.

JAMES HOLLIS WELLS

Colonel James Hollis Wells, senior member of the architectural firm of Clinton & Russell of New York and former commander of the 71st Regiment, New York National Guard, died at his home in Jersey City, on Friday, September 24, 1926, at the age of sixty-three.

Colonel Wells received the degree of C.E. from Lehigh University in 1885. In 1886 he was inspector of pavements in New York and then became engineer for the contracting firm of J. D. & T. E. Crimmins. His later activities were in the capacity of clerk of the works for Cornelius Vanderbilt, then he became associated with the firm of Clinton & Russell. He was a member of the Engineers' Club of New York; member of the Board of Governors and chairman of the House Committee of the Railroad Club of New York; vice-president of the New York State Rifle Association; a director of the National Rifle Association of the State of New York and a member of other clubs and organizations of importance. He became a member of the A.S.M.E in 1904.

GEORGE BURR WHAREN

George Burr Wharen, for six years head of the Mechanical Engineering Department, University of North Dakota, died May 1, 1926. Professor Wharen was born on October 30, 1883, at Wilkes-Barre, Pennsylvania. He received the degree of Bachelor of Science from Pennsylvania State College in 1910, and the degree of Master of Science from the same institution in 1913.

From 1902 to 1905, Professor Wharen worked as an apprentice machinist for the Lehigh Valley Railroad, and during his summer vacations while at college he was a journeyman machinist and an airbrake inspector for the same road, and also a machinist for the Matheson Motor Car Co. From July 1910, to February 1911, he was an engineering apprentice instructor with the American Locomotive Company, Schenectady, N. Y., and for the next two years he conducted classes at the Pennsylvania Railroad Company, Altoona, Pa., in elementary locomotive design and operation, in coöperation with the engineering school of the Pennsylvania State College. In 1914 he was an instructor in mechanical drawing and elementary machine design at Wentworth Institute, Boston, Mass., going in September of that year to the University of Pennsylvania, Philadelphia, Pa., as an instructor in mechanical engineering in the Experimental Division, where he remained until September, 1920.

In 1920 Professor Wharen went to Grand Forks, North Dakota, as head of the department of mechanical engineering, University of North Dakota, a position which he held at the time of his death. He was one of the ablest combustion engineers in the Northwest, and had become an authority on the use of North Dakota lignite in pulverized form for power-plant use. He was a member of the Society for the Promotion of Engineering Education, Sigma Tau, and Sigma Xi. He became an associate-member of the A.S.M.E. in 1914, and a member in 1924.

ERNEST F. WIEDERHOLDT

Ernest F. Wiederholt, president of the Wiederholt Construction Co., died on April 13, 1926, at his home in St. Louis, Mo. He had become prominent as a builder of unique chimneys. The huge hollow tile stacks on the Cahokia Power Plant of the Union Electric Co. are an example

He was born in St. Louis on June 21, 1878, and received his education at Washington University, from which he was graduated as mechanical engineer in 1901. From the time he was graduated until he became connected with the Wiederholdt Construction Co. he was employed by various companies, among them being the American Car & Foundry Co., the Commonwealth Steel Co., the Gilsonite Construction Co., and the Atlas Construction Co. He had been a member of the A.S M.E. since 1924.

WILLIAM EDWARD WILLSON

William Edward Willson, a draftsman in the machinery division at the Navy Yard, Brooklyn, N. Y., died in August, 1926. Previous to holding this position, Mr. Willson was in charge of the drawing room and chief draftsman of the Navy Yard located at Portsmouth, N. H., from 1902 to 1915. Here he had complete charge of machinery and of converting the Spanish War vessels into receiving ships. From 1880 to 1894 he was assistant draftsman at the Quintard Iron Works. The next year was spent with the Morgan Iron Works designing engines and boilers. The years 1896 to 1902 were spent in experimenting and developing oil engines.

He was born in Brooklyn, N. Y., August 12, 1862, and elected a member of the A.S.M.E. in 1923.

No. 2041

INDEX TO VOLUME 48

NOTE

1 Names of authors and discussers, also those of deceased members preceding an obituary notice, are in caps and small caps A discusser is distinguished from the author of a paper by (*D*), placed after the subject of the discussion Illustrations are denoted by (*I*), curves by (*C*), and tables by (*T*).
2. Titles of papers, where placed after the name of the author, and appearing in their exact form, are in italics Papers are indexed not under their titles but under their subject-matter
3 On page 1483 et seq. will be found an alphabetical list of (*a*) 1926 Spring, Annual, and Regional Meeting papers not included in this volume but published in *Mechanical Engineering*, (*b*) of papers presented before sections of the Society and published in *Mechanical Engineering*; and (*c*) of other leading articles and technical reports published in *Mechanical Engineering* during 1926.

REFERENCES TO 1926 PAPERS, ARTICLES, AND REPORTS NOT PUBLISHED IN THIS VOLUME

NOTE

Below will be found an index to (*a*) 1926 Spring, Annual, and Regional Meeting papers which have been published in *Mechanical Engineering* but are not included in this volume; (*b*) papers presented before Sections of the Society and published in *Mechanical Engineering*; and (*c*) other leading articles and technical reports published in *Mechanical Engineering* during 1926.
Mechanical Engineering has been abbreviated to M.E.

Aerofoil, pressure distribution over U. S. A.-27. M.E., vol. 48, Dec. 1926, p. 1401
Aeronautics, progress in. M.E., vol. 48, Dec. 1926, p. 1432
Air, flow of in pipes. M.E., vol. 48, Oct 1926, p. 1025
Air mail, operation of. M.E., vol. 48, May 1926, p. 465
Aircraft construction, fusion-joining of metallic materials in. M.E., vol. 48, Mid-Nov. 1926, p. 1240
Aircraft engines, research on oil-injection. M.E., vol. 48, Nov. 1926, p. 1123
Airship, Zeppelin, safety of. M.E., vol. 48, Feb. 1926, p 115
Apprentice training. M.E., vol. 48, Nov. 1926, p 1110
Apprentices, education and training of on Pacific Coast. M.E., vol. 48, June 1926, p. 686
Apprenticeship, has the need for passed? M.E., vol. 48, May 1926, p. 409
Apprenticeship system, district. M.E., vol. 48, Sept. 1926, p. 885
Aviation, civil, in U. S. M.E., vol. 48, Nov. 1926, p. 1092

Babcock, C. L. Refinements in Woodworking-Machinery Design. M.E., vol. 48, May 1926, p. 415
Bailey, C. F. Apprentice Training. M.E., vol. 48, Nov. 1926, p. 1110
Bailey, E. G. Limiting Factors in Reducing Excess Air in Boiler Furnaces. M.E., vol. 48, July 1926, p. 703
Ballistics, interior, experiments in. M.E., vol. 48, Aug. 1926, p. 819
Bancel, Paul. Operating Performance of Some Modern Surface Condensers. M.E., vol. 49, Mar. 1927, p. 219
Barges, effect of end shape on towing resistance. M.E., vol. 48, Aug. 1926, p. 833
Barre, H. A. Water Power and Steam Power in California Utilities. M.E., vol. 48, June 1926, p. 662
Bato, A. A. Flue-Gas Computations. M.E., vol. 48, Apr. 1926, p. 330
Baud, R. V. The Strength of Gear Teeth. M.E., vol. 48, Nov. 1926, p. 1103
Baynton, R. S. The Design of High-Pressure Industrial Power Plants. M.E., vol. 48, Oct. 1926, p. 1030
Bell, W. C. Interconnections in Virginia and North Carolina. M.E., vol. 48, Oct. 1926, p. 1036
Berger, L. B. Refractories. M.E., vol. 48, Dec. 1926, p. 1389
Black, Archibald. Civil Aviation in the United States. M.E., vol. 48, Nov. 1926, p. 1092
Blauvelt, W. H. The Distillation of Coal. M.E., vol. 48, Mid-Nov. 1926, p. 1210